Hysteresis in Market Response:

When is marketing spending an investment ?

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Abstract

Conventional wisdom says that it takes ever increasing marketing budgets and/or lower prices to create sustained growth in sales, which may or may not be profitable in the long run. However, some documented cases and conceptual evidence suggest that marketing spending or prices can exhibit hysteresis in market response, a condition where temporary changes in the marketing mix are associated with permanent movements in sales performance. As such, hysteresis creates a long-run investment benefit from marketing actions, which can be highly beneficial to the firm, or detrimental to competition. Yet the assessment and formal implications of market response hysteresis for marketing resource allocation are unexplored.

This paper addresses the modeling of marketing hysteresis in three ways: one, it shows how the presence of full vs. partial hysteresis in response may be assessed from longitudinal data on sales and marketing-mix allocations. Two, it develops the link between sales hysteresis and profit hysteresis, and derives the long-run optimal spending rules for a firm whose marketing efforts exhibit some form of hysteresis. Three, it illustrates both the measurement of hysteretic marketing effects and their optimal spending implications for a leading supplier of computer printers. The paper concludes with an agenda for future research in this important, yet under-researched area of marketing strategy.

Keywords: marketing mix, long-term marketing effects, marketing resource allocation, econometric models, time-series analysis

Introduction

Motivated by the financial expectations of their private or public stakeholders, businesses continuously search for new ways to achieve profitable growth. The marketing component of this search is focused on reaching higher plateaus of sales and market share that are preferably *sustainable*. However, these higher levels are not necessarily more profitable, as the sales response function is typically characterized by decreasing returns to scale. In such cases, higher sales or market share inevitably require sustained higher marketing spending, which may or may not be profitable in the long run.

A far more attractive scenario to the marketer is when only *temporary* marketing efforts are needed to achieve sustained sales growth, a condition sometimes called *hysteresis*. Hysteresis is the tendency of a stimulus-induced response to stay at a higher level even after the stimulus is removed. The concept was first used in the nineteenth century in the physical sciences and has since been used to explain economic phenomena such as unemployment and trade deficits. In marketing, Little (1979) first suggested that sales response to advertising could exhibit hysteresis. An overview of the use of the hysteresis concept in various scientific disciplines in shown in Table 1.

Marketing-induced sales hysteresis creates two ways in which marketing spending can foster sales growth over time: one is the *triggering* function, i.e. the temporary - if costly - investment that brings about a new, higher sales plateau and, two, the *maintenance* function, or the sustained spending that preserves the higher sales. All else equal, the stronger the marketing hysteresis, the more budget can be allocated to short-term triggering (investing) and the less to sustained maintenance, which creates more profitable long-run economics. Under full hysteresis, marketing is pure investment, which triggers an increase in market performance that is sustainable without subsequent marketing spending. Without hysteresis, sustained sales growth needs to be nurtured by permanently higher marketing spending, so that marketing loses its one-time investment character. Figure 1 illustrates the difference between full hysteresis and partial hysteresis in marketing.

Table 1 – History of Hysteresis

Figure 1 – Full and Partial Marketing Hysteresis

While hysteresis is not a common part of the managerial or even academic marketing vocabulary, its existence is *implied* by several well-known marketing phenomena. For example, when a financial services provider engages in an effective customer acquisition campaign, the incremental revenue stream that is perpetually generated by loyal new customers points to a hysteretic effect of that campaign. Similarly, when new-product trial is followed by repeat purchasing or when new-product adoption by an innovator leads to positive word-of-mouth and adoption by imitators, any role that marketing plays in temporarily stimulating the trial or adoption may have a hysteretic sales effect over time. Hysteresis is also at the core of the current US government allegation that, in the mid-nineties, American Airlines competed unfairly in certain regional markets in which new low-cost entrants were trying to establish a foothold. For example, according to the *New York Times* (May 14, 1999), prior to competitive entry in the Dallas – Colorado Springs route, American flew an average of 3,723 monthly passengers between these cities, at an average fare of $158. When low-cost competition entered, American reacted with a combination of more flights and lower prices. This resulted in 19,909 monthly passengers on the route, at an average fare of $88, which drove the new entrants out of the market. Finally, American allegedly reestablished its higher price point, an average of $133 in this case, but was now able to draw 9, 237 monthly passengers on average. Taking these numbers at face value, we would conclude that American's *temporary* marketing activity resulted in a *permanent* gain of over 5,500 passengers per month, with incremental monthly revenue exceeding $600,000.

Causes and Implications of Marketing Hysteresis

Why would marketing-induced consumer response stay at a higher level even after the stimulus is removed ? When a marketing stimulus such as an ad campaign or a sales visit is effective, it changes consumer attitudes, for example by creating an awareness or preference for the advertised product. We expect such changes to be self-sustaining because, once a consumer is made familiar with an offering, he or she cannot easily be made unfamiliar with it. Attitudes and/or attitude components may be stored in long-term memory to be retrieved for later use, rather than reconstructed on every occasion (Fishbein & Ajzen 1975). Consequently, at the level of attitudinal response, we expect effective marketing efforts to be hysteretic, i.e. temporary stimuli can have sustained response.

However, attitudinal change in and of itself may be insufficient to justify a marketing investment. Only under certain conditions (e.g. an attitude is sufficiently strong) will changes in attitude result in changes in behavior (e.g. purchase behavior) that are economically beneficial to the marketer. Once engaged, such behavioral changes *can* become self-sustaining, even independent of attitudes, if it is advantageous to the consumer to do so. For example, marketing-induced product trial may lead to user satisfaction, which in turn creates habit-forming repeat purchase behavior, because the satisfied consumer saves time and effort in exploring other alternatives. However, new market conditions may emerge (for example competitive activity) that interfere with the self-sustained behavior and diminish or even remove the hysteretic effect. The more intense that interference, the more the marketer has to engage in *maintenance* marketing spending in order to protect the hysteretic effect, which is of course costly. In the limit, there is no hysteresis at all, and any desired sustained sales growth will require permanently higher spending. Thus a priori we expect the hysteresis in marketing-induced behavior change to be *partial* at best, and we will need a metric that represents a range from zero to partial to full hysteresis. Marketing conditions that *favor* hysteresis include newer product categories with growing demand, product and marketing differentiation, price flexibility and high consumer switching costs. Conditions that *disfavor* hysteresis are price wars and other forms of intensive competitive reaction, undifferentiated products and marketing, stable demand patterns and low consumer switching costs.

For the marketer, the first critical task in engaging hysteresis is to obtain initial consumer response (attitudinal or behavioral), i.e. there should be short-run marketing effectiveness. Now, it is well known that most sales-to-marketing elasticities are below unity, and some are quite small on average (e.g. advertising elasticities average .1 to .2). Other short-run marketing elasticities are higher, but it takes a substantial and sustained effort to execute such marketing initiatives, as in the case of sales force and distribution. Both of these observations suggest that hysteresis is probably not the *rule* in market response. In fact, Simon (1997) argued that *single* marketing actions may not be potent enough to create a hysteretic effect on sales. However, the *combination* of timely marketing effort and advantageous circumstances – both of which are temporary – can trigger a stronger immediate behavioral change (a higher response elasticity) which is more likely to be hysteretic. From a customer perspective, such temporary circumstances include marketing incentives such as a price cut, an invitation to join a loyalty program, or a chance to acquire a limited-supply product. They also include environmental conditions that make the marketing activity unusually salient. For example, Simon (1997) illustrates that the consumption of the Gorbatschow-brand vodka in Germany rose fivefold after Mikhael Gorbachev rose to power in the (then) Soviet Union. When the brand's environmental conditions later returned to normal (i.e. the Gorbachev regime ended in 1991), the behavioral change in their vodka sales was self-sustaining, presumably because the target market had become habitual Gorbatschow consumers and saw no reason to switch brands. In conclusion, the combination of a favorable external situation, along with innovative company action across several marketing-mix instruments, is more likely to result in hysteresis.

The *antithesis* of hysteresis is mean-reverting sales response to marketing, which can be zero-order or distributed over time (carryover effects). Zero-order (current) effects have been reported in several cases involving price response, whereas distributed-lag effects are more typically associated with advertising response (see Hanssens, Parsons &

Schultz 2001). In either case, even though marketing is effective in that there is a non-zero response elasticity, once the marketing stimulus is removed, the behavioral change dissipates, either immediately or gradually, and sales adopt a stationary (e.g. mean-reverting) pattern over time. In such cases, achieving *sustained* growth in market performance will require either permanently lower prices and/or permanently higher marketing spending .

The study of marketing hysteresis therefore raises new modeling opportunities and challenges, both empirical and analytical. From an empirical perspective, we must establish evolution (permanent change) in market performance, and we must relate that evolution to temporary changes in marketing support, possibly mitigated by external circumstances. We must also develop a metric that quantifies the hysteretic effects on a continuous scale. From an analytical perspective, we must set up the marketing resource allocation problem as a long-run optimization in which trigger actions may well have permanent effects on performance, with or without the need for follow-up (maintentance) actions. These are the objectives of our research.

We begin with a brief discussion of existing market-response models and how they are ill equipped to capture hysteresis. That motivates the design of an implementable modeling approach for hysteresis: we formulate empirical conditions on longitudinal marketing data, and we develop a metric that gauges the strength of hysteresis on a 0-to-1 scale, i.e. ranging from zero to partial to full hysteresis. Next, we study dynamic resource allocation under hysteresis, in particular the question of how much to spend on triggering vs. maintenance, where the former is upfront, temporary marketing investment and the latter is sustained spending. We then combine measurement and optimization in an actual case study of a leading supplier of a high-technology product. We econometrically estimate and validate the relative strength of hysteresis in marketing campaigns for this durable product in its evolving market. We also derive marketing investment prescriptions that exploit hysteresis to create a substantial gain in long-term profitability. The paper ends with a number of recommendations for future research in this new area.

Market Response Models and Hysteresis

The majority of market-response models are of the 'level-level' form, where a certain level of spending is associated with an equilibrium level of performance, possibly after some temporal adjustment (see, e.g. Hanssens, Parsons and Schultz 2001). Therefore, optimization rules lead to spending prescriptions that are increasing in sales, i.e. the higher the desired result, the higher the required spending. Among those, the best known rule, due to Dorfman and Steiner (1954), states that the short-run profit maximizing monopolist optimally spends on advertising (for example) as follows :

$$\frac{\text{ADVERTISING}}{\text{REVENUE}} = \frac{\varepsilon_{ADV}}{|\varepsilon_{PRICE}|}$$

where ε refers to elasticity. The rule implies that, with certain price and advertising elasticities (for example, -2.5 and 0.1, respectively) there exists an optimal advertising-to-revenue ratio, in casu 4 %. Numerous extensions of the Dorfman-Steiner conditions exist, for example those accommodating long-term profit maximization (e.g. Schmalensee 1972), competitive reactions (e.g. Lambin, Naert and Bultez 1975) and goodwill accumulation and depletion (e.g. Vidale and Wolfe 1957, Nerlove and Arrow 1962). These extensions, however, do not change the basic premise that marketing resources should be allocated in proportion to their response elasticities, and that higher sales performance always requires higher spending levels.

Some market response models incorporate the possibility that optimal spending allocations are uneven over time. Specifically, the literature on *pulsing* and *chattering* – mostly in the context of advertising – has demonstrated conditions under which it is optimal to pulse:

- with an S-shaped response curve, pulsing may allow the manager to allocate a restricted budget in the concave portion of the curve, whereas even spending may result in perennial advertising below the threshold response level (in the convex portion of the curve) (e.g. Sasieni 1971, 1989),

- even with a concave response curve, the presence of advertising wearout justifies a pulsing strategy, as it helps avoid the loss of advertising effectiveness due to excessive repetition (e.g. Naik, Mantrala and Sawyer 1998).

The resulting dynamic allocation rules are more complex, allowing, for example, for substantial frontloading of spending, followed by longer periods of lower-level efforts. Similarly, in a diffusion-of-innovation framework, conditions exist that lead to aggressive optimal marketing spending early in the product life cycle (e.g. Horsky and Simon 1983). This aggressive early spending shifts the sales pattern forward, though total lifetime sales remain the same. Despite their temporal complexity, these models still have the property that marketing effects on sales are temporary and therefore, higher equilibrium sales require higher marketing spending.

Hysteresis changes this basic premise, because *there is no longer a one-to-one mapping of marketing effort into sales performance.* Instead, temporary spending deviations *from any level* could result in permanent increases in sales. Therefore, in order to capture hysteresis in response, (1) some specific temporal sales and marketing behaviors should be observed, and (2) the response model itself should include the effect on sales of a *change* in marketing from any existing *level* of spending. Formally, the hysteresis conditions on the data and the response model are:

- sales (or the relevant market response measure) should change over time, and that change should be of a permanent nature. Indeed, in the opposite case of stationary sales behavior, all sales fluctuations are temporary and the sales level eventually reverts back to the mean (or a deterministic trend) ;
- the hysteretic marketing action should be temporary. By contrast, if the marketing action is permanent, then it represents sustained spending which, however effective, always implies higher spending for higher results ;
- sales evolution should be linked to the temporary marketing action. Without that link, there is of course no market-response effect, even though the temporal conditions on the data are met.

Assessing these conditions can be done by using modern time-series techniques and incorporating the results in the specification of a market-response model. For example, a unit-root test on sales should reveal evolution or I(1) behavior, and the same test on the marketing variable should reveal stationarity or I(0) behavior (see e.g. Hanssens, Parsons and Schultz 2001 for details on such tests). An empirical generalizations study has revealed that about two thirds of sales time series, one third of market- share time series, and half of marketing-mix variables are evolving (Dekimpe and Hanssens 1995b). Therefore, a priori the univariate conditions for marketing hysteresis exist on a substantial subset of marketing data. These conditions are summarized in the 'hysteresis' cell of Dekimpe and Hanssens' four strategic marketing scenarios described in Figure 2 (Dekimpe and Hanssens 1999).

Figure 2 – Evolution and Stationarity in Marketing Effort and Market Performance

Given evolving sales and stationary marketing, a hysteresis response model would establish a significant relationship between changes in sales and temporary marketing activity. Such a result was recently discovered on marketing scanner data: temporary price promotions for a private-label brand of soup were found to have a long-run market-expansive effect on the category (Dekimpe, Hanssens and Silva-Risso 1999).

In many cases, though, the observed sales evolution is caused by factors other than marketing hysteresis. Among them, there could be evolution in the external environment, e.g. rising disposable personal incomes, or evolution in the marketing mix such as steadily higher distribution (e.g. Bronnenberg et al. 2000). Such co-evolution can also be assessed by time-series methods, in casu the presence of *cointegration* between sales and other evolving variables (external or marketing) (see, e.g. Powers, Hanssens, Hser and Anglin 1991; Franses 1994). We therefore need to construct a metric for hysteresis that can distinguish the hysteretic effects of marketing from the long-term effects of evolution in the environment or in the marketing mix. Furthermore, if the hysteresis is *partial*, a mixture of temporary and sustained marketing efforts should enter into the response function. Temporary efforts (changes) trigger higher sales, but subsequent maintenance efforts (levels) are needed to preserve the sales gains. Partial hysteresis is not an explicit scenario in Figure 2. Instead, it combines the two sales-evolving scenarios in the figure, and motivates our development of a new hysteresis metric.

A Hysteresis Metric

A hystereris metric, ρ, should accommodate a mixture of evolving[i] and stationary time series, along with the possibility that actual hysteresis is absent, partial or full. For that purpose we postulate a general class of market-response functions that are log-linear to accommodate nonlinear returns[ii]. We shall first describe the hysteresis response model and later address the temporal properties of the data. Let

S = sales,

A = log marketing spending of any kind,

Z = other sales drivers, which may or may not have hysteretic effects,

then marketing hysteresis ρ is derived from the following response function

$$S(t) = c + \varphi S(t-1) + \alpha [A(t) - \rho A(t-1)] + \theta Z(t) + u(t), \qquad (1)$$

where u(t) is an error term with the standard assumptions, and effective marketing implies $\alpha > 0$ (see e.g. Gordon (1989) for a similar specification in macro-economics). This model is made empirically tractable by rearranging terms as:

$$S(t) = c + \varphi S(t-1) + \alpha [(1 - \rho) A(t) + \rho \Delta A(t)] + \theta Z(t) + u(t) \quad (2)$$

where Δ is the difference operator, i.e. $\Delta A(t) = A(t) - A(t-1)$. Therefore, in the estimation equation

$$S(t) = c + \varphi S(t-1) + b_1 A(t) + b_2 \Delta A(t) + \theta Z(t) + u(t) \qquad (3)$$

and the degree or strength of hysteresis ρ is estimated from

$$\rho = b_2 / (b_1 + b_2),$$

so hysteresis is conveniently measured as the magnitude of the change effect relative to the total (level + change) effect. Absent hysteresis ($\rho \leq 0$), only the levels of marketing affect sales. Under ρ=0, equation (1) reduces to the conventional current-effects response model in levels

$$S(t) = c + \varphi S(t-1) + \alpha A(t) + \theta Z(t) + u(t) \qquad (4)$$

and under ρ<0, it becomes a distributed-lag effects model in levels:[iii]

$$S(t) = c + \varphi S(t-1) + \alpha A(t) + \beta A(t-1) + \theta Z(t) + u(t) \qquad (5)$$

where $\beta = - \alpha \rho$. Conversely, full hysteresis (ρ=1) implies that sales are driven *only* by changes in marketing spending, regardless of levels:

$$S(t) = c + \varphi S(t-1) + \alpha [A(t) - A(t-1)] + \theta Z(t) + u(t). \qquad (6)$$

Finally, partial hysteresis is obtained when both levels and changes of marketing impact sales, i.e. $0<\rho<1$. This metric is not only empirically tractable, it also provides an intuitive definition of marketing hysteresis: marketing is decomposed as level plus change, and *hysteresis is the relative importance of marketing change in driving sales evolution.*

Can hysteresis be detected in combination with temporary response effects ? More complex distributed-lag versions of (1) and (3) can be specified if there was reason to believe that “change in marketing spending” involved more than one period (as in the case of advertising wear-in), and/or if market response consists of hysteresis with carryover effects. A more general version of our hysteresis estimation equation (3) accommodates this as follows:

$$S(t) = c + \varphi S(t-1) + b_1 (L) A(t) + b_2 (L) \Delta A(t) + \theta Z(t) + u(t) \quad (7)$$

where L is the lag operator and the b_i (L) (i=1,2) are lag polynomials representing temporary, delayed response effects of marketing on sales, i.e. $b_i (L) = b_{0i} + b_{1i} L + b_{2i} L^2 + \dots$ In that case the hysteresis metric would be estimated as

$$\rho = b_2 (1) \ / \ [b_1 (1) + b_2 (1)]$$

The generalized form (7) does not change the substance of hysteresis measurement and its resource implications. We will only use it in empirical application and determine the shapes of the distributed-lag functions b_i (L) (i=1,2) by direct-lag search.

In conclusion, a hysteresis metric can be developed starting from conventional market-response models, by imposing specific conditions on the dynamic response parameters. This approach offers the advantage that current and decaying marketing - effects models are *nested* in the hysteresis model. For the empirical detection of hysteresis we must first turn to the temporal properties of the variables.

Temporal Behavior

Model (1) and its estimation version are sufficiently general to accommodate various temporal behaviors in marketing and sales, each with their own unique set of conditions on the parameters. First, and foremost, sales must be evolving, i.e. its time series has a unit root. Given that S(t) is I(1):

- under full hysteresis, only temporary changes in marketing spending have an impact. There is no long-term relationship between sales and marketing levels, i.e. no cointegration between their time series. The conditions on the parameters are $\varphi=1$, $\rho=1$ and $\alpha>0$. This is the “hysteresis” cell in Figure 2. Its strategic implication is that only trigger or temporary investment spending is needed to permanently increase sales. An example is a customer acquisition campaign that yields new and loyal accounts without the need for subsequent customer retention spending.
- absence of hysteresis, $\rho \leq 0$. In this case, there may be a long-term marketing effect in levels, i.e. $\alpha>0$ and $0<\varphi<1$, the “co-evolution” cell in Figure 2 which necessitates ever higher marketing spending for higher sales goals. An example is increasing spending on brand advertising in order to obtain higher distribution levels which, in

turn, enable higher sales results. It is also possible that sales evolution is altogether unrelated to marketing spending, i.e. α=0 and φ=1.

- partial hysteresis involves right-hand-side terms in (1) that are all contributing to sales growth, i.e. $0<\varphi<1$, $0<\rho<1$ and $\alpha>0$. A "trigger" marketing campaign (temporary spending hike) lifts sales to a higher plateau, and a subsequent adjustment in maintenance spending prevents mean reversion in sales, i.e. a return to lower levels. *Both triggering and maintenance are necessary*: without trigger, there is no sales lift to a higher level, and without maintenance, that higher level is not sustainable. Cast in terms of Figure 2, this scenario combines the lower-left and lower-right quadrants: evolving marketing levels ("maintenance") plus temporary marketing changes ("triggering") drive the evolution of sales performance. This combined trigger/maintenance effect of marketing breaks the one-to-one relationship between marketing spending and sales performance.   An example is successful customer acquisition spending followed by sustained retention spending in order to prevent attrition of the newly acquired customers.

Equation (3), estimated with ordinary least squares, produces consistent estimates of the parameters (φ b_1 b_2 θ), even though sales are I(1). This is because there exist values for the coefficients for which the error term is I(0) (see e.g. Hamilton 1994, Chapter 18). Equation (3) is preferred over a specification in the changes of sales, because the latter would remove potentially useful information contained in the levels of marketing, in particular the marketing maintenance function. As an alternative, we could specify a vector-error correction response model (e.g. Powers et al. 1991), which combines changes and levels, however such a model requires the cointegration condition between marketing and sales to hold, which we do not want to impose on the data a priori.

Hysteresis of Marketing Campaigns

The response model (1) accounts for hysteretic effects of both increases and cuts in marketing spending. In our context, however, it is more appropriate to work with an asymmetric form of (1) that isolates the hysteretic effects of marketing investments or campaigns, which are identified as temporary *increases* in marketing spending. Indeed, managers generally aim at *growth* in financial performance, for which they are willing to engage in certain marketing actions, as in the American Airlines example in the introduction. Hysteresis in marketing campaigns is also consistent with the theoretical considerations we discussed earlier. For example, the attitude-storage theory of Fishbein and Ajzen (1975) may be used to hypothesize a hysteretic effect of positive changes in marketing (in casu, advertising) spending only. Under this hypothesis, the advent of an advertising campaign causes sales to evolve to a higher level. When the campaign ends, sales do not return to their pre-campaign levels. Instead, they fluctuate at a different level, depending on the amount of maintenance spending and the strength of hysteresis.

Model (1) and its estimation version (3), are easily adapted to measure the hysteresis of marketing campaigns by restricting changes in spending to positive values:

$$S(t) = b_0 + \varphi S(t-1) + b_1 A(t) + b_2 \max\{ [A(t) - A(t-1)] , 0 \}$$
$$+ \theta Z(t) + u(t) \qquad (8)$$

Note that a similar specification was used in the *Adpuls* model (Simon 1982), except of course that sales are not mean reverting in our case. The strength of hysteresis is once again measured by the quantity $b_2 / (b_1 + b_2)$.

Can there be hysteresis in marketing spending cuts ? Many marketers are understandably concerned about reducing marketing support, lest it permanently damages the brand's market performance. If the spending cuts are sustained, their sales impact will be measured by the level term and its parameter b_1 in equation (8). Beyond the level-impact, if we want to test the hypothesis that even a temporary reduction in support has a permanently damaging effect on sales, we could augment equation (8) with a *temporary spending cut* variable defined, for example, as $\min\{ [A(t) - A(t-1)] , 0 \}$. We are not aware of any a priori reasons for hypothesizing such an effect, though.

Difference with impulse-response modeling.

Our hysteresis metric, ρ, is not the only way in which hysteresis can be assessed empirically. Dekimpe and Hanssens (1995a, 1999) have used a vector-autoregressive (VAR) time-series model for this purpose, in which the impulse-response function of any pair of variables (X,Y) is used to detect hysteresis, defined as the permanent effect on Y of a one-unit (or one standard-deviation) shock in X. That approach produces a numerical and a visual path of impulse-response weights that can be used in a simulation of long-term sales and profit consequences of certain marketing actions (see Dekimpe and Hanssens 1999). However, as reduced-form parameters the impulse-response weights cannot be used directly in a dynamic optimization framework, nor can they accommodate partial hysteresis.

By contrast, the present approach combines level (maintenance) and difference (trigger) effects of marketing on sales in a single-equation model that lends itself to optimal resource allocation over time in evolving markets, with either zero, full or partial hysteresis. Marketing spending is therefore a control or exogenous variable, whereas in the VAR modeling approach, there are no a priori exogenous variables. The two metrics use the same initial stage of testing for evolution vs. stationarity in the data, and they complement each other in addressing different hysteresis modeling objectives. The VAR model offers a complete description of a dynamic multi-equation marketing system, (i.e. without exogeneity), whereas the hysteresis response model addresses the long-run optimal allocation of a marketing resource under control of the firm (i.e. with exogeneity).

Optimal Triggering and Maintenance Spending

Optimal marketing resource allocation is affected by hysteresis because economic returns may be captured quickly and permanently. Ceteris paribus, we expect higher hysteresis to imply more frontloading of marketing investments and less maintenance spending. The economic dynamics of this intuitive result are complex, as they involve both response and economic parameters in an *evolving* environment. We will use the well-established principles of unit-root detection and modeling in time series to characterize such evolving environments (e.g. Dekimpe and Hanssens 1995).

The key to hysteresis estimation is the term [A(t) - ρ A(t-1)] in Equation (1). The value of ρ indicates how a "*change in A*" , i.e. a temporary marketing action, generates permanent (non-decaying) effects on sales. An optimization model with hysteresis should, therefore, capture the spirit of the expression [A(t) - ρ A(t-1)].

Dynamic problem formulation

Let

S(t): sales or demand for a product at time *t*, a positive function of goodwill;

g: goodwill, a positive function of current and previous marketing, similar to the specification in Nerlove and Arrow (1962);

A (t): marketing spending in general;

A^T(t): marketing investment for triggering the hysteresis effect;

A^M(t): marketing spending for maintaining the triggered hysteresis effect. Absent hysteresis, this spending is the same as general marketing spending;

P: price of the product;

C: unit variable cost of the product;

r: capital cost (discount rate);

M(A(t)): marketing cost function;

B(t): budget constraint.

Marketing resource allocation is motivated by long-run profitability. Therefore, we use *A*(t) as a control variable and *g* as a state variable to maximize the firm's profit stream Π from periods *t*=0 to *t*=*T*:

$$\max \Pi = \max_{\{A(t)\}} \Sigma \sum_{t=0}^{T} \left(\frac{1}{1+r} \right)^{t} \left[(P-C) S(t) - M(A(t)) \right], \qquad (9)$$

$$with \qquad S(t) = f\left(g(A(t), A(t-1))\right).$$

The first line of Equation (9) is the standard formulation for choosing a marketing spending policy *A(t)* to maximize a discounted profit stream Π over a planning horizon (0, …, T) (e.g. Welam 1982). The second line of Equation (9) characterizes hysteresis through a goodwill specification in the tradition of Nerlove-Arrow (1962). They treat marketing (in casu, advertising) as an investment in the firm's goodwill which, in turn, affects sales (Liu and Forker 1990). The general Nerlove-Arrow model is expressed as *S(t) = f(g(A(t)),* but our goodwill specification is *g(A(t), A(t-1)),* which captures hysteresis as explained in the previous section[iv]. It is therefore a more abstract expression of the term [A(t) - ρ A(t-1)] in Equation (1). Furthermore, marketing budgets are consumed between current trigger and future maintenance spending, so:

$$A^{T}(t) + A^{M}(t+1) \leq B(t) \qquad (10)$$

Equations (9) and (10) form our resource allocation problem. The dynamic optimization is designed around a budget cycle with two sub-periods, one for triggering and one for maintenance. This setting distinguishes our hysteresis modeling from conventional marketing optimization studies. Indeed, the common nature of optimal policies in the literature - including even spending, pulsing, chattering and their combinations - is that they all recommend *repetitive* marketing actions (as summarized by Naik et al. 1998). In contrast, we expect the optimal policy derived from this modeling to be *non-repetitive*, so long as our strict hysteresis conditions are met.

The system in (9) and (10) can be simplified and made into a dynamic programming problem over a discrete time horizon. Let:

parameter π = P-C stand for contribution margin,

time preference $(1+r)^{-1} = \beta$ ($\beta > 0$),

goodwill be a Cobb-Douglas function[v]: $g = A^{\xi}(t)\, A^{-\mu}(t-1)$,

sales be a log function of goodwill: $S(t) = \ln g = \xi \ln A(t) - \mu \ln A(t-1)$, which allows for the assessment of zero, partial or full hysteresis as in the previous section. It is important to note that our hysteresis metric ρ developed from (1) equals μ / ξ in this sales response function,[vi]

the marketing cost function $M(\mathrm{A(t)}) = \ln A^{\delta}(t) = \delta \ln A(t)$. The cost function alows for nonlinearities, for example due to quantity discounts in media buying.

Substitute the above in Equation (9), then scale the parameters to unity by setting $\pi \xi - \delta = 1$, and let $(\pi \mu) / (\pi \xi - \delta) = \pi \mu = \gamma$.

After rearranging items, our objective function becomes:

$$\max_{\{A(t)\}} \Sigma \sum_{t=0}^{T} \beta^{t} \left[\ln A(t) + \gamma \ln A(t-1) \right] \qquad (11)$$

This objective function corresponds to the hysteresis response function (1)[vii], and the analytical hysteresis parameter γ is related to the empirical hysteresis coefficient ρ in equation (1), as we demonstrate below. Note also that we make abstraction of separable exogenous demand drivers other than marketing spending, such as product quality and prices, so our results will be conditional on their levels.

Profit hysteresis (γ)

The new parameter γ can be interpreted as “profit weighted” hysteresis, as it combines response hysteresis and profitability. Indeed, substituting $\rho = \mu / \xi$ in the definition of γ reveals that $\gamma = (\pi / (\pi - \delta/\xi)) \rho$. Now, δ/ξ is a measure of marginal marketing cost, i.e. marginal cost of effort divided by responsiveness. The higher the marketing cost, either because the effort is expensive (δ is high) or its response is low (ξ is small), the more important the impact of response hysteresis on profit, i.e. ρ is amplified. Likewise, when contribution margin is higher, profit hysteresis is increased. On the other hand, when marketing is free (e.g. unpaid media publicity), $\delta=0$ and response and profit hysteresis are the same. As one would expect, zero response hysteresis ρ or zero contribution margin π always imply zero profit hysteresis γ.

The marketing budget, *B*, is a given amount of money for marketing spending at time *t*. The marketing manager could invest the entire budget immediately, or could allocate only a portion for current spending and reserve another portion for future use. Absent hysteresis, there would be no trigger spending, and the budget would be allocated as in the extant literature, represented as[viii]: $B(t) = A^M(t)$. The constraint now becomes:

$$A^T(t) + A^M(t+1) \le B(t) = A^M(t), \qquad (12)$$

This budget says that a lump-sum available resource, A^M(t), will be allocated into two parts: trigger (investment) spending A^T(t) for the current period and maintenance spending A^M(t+1) for the future. Equations (11) and (12) form a dynamic system which can be solved by Bellman's equation.

Analytical Solution

Chow (1997) shows that the rationale of dynamic programming is that the value of the objective function at the end of the planning horizon is maximized first, and then this maximized value is used as a constraint to determine the value of the present choice variable. In our model, we first ensure that the (higher) sales plateau will last to the end of *T*, no matter how long it is, and then we determine the current triggering action.

With given initial conditions: $A^M(0)>0$ and $A^T(-1)$ known (for example, let $A^T(-1) = 0$), a recursive method can be used to find solutions for this system. The *state* for this problem is the pair $\{A^M(t),\ A^T(t-1)\}$, with the transition equation for the first element given by the budget constraint, whereas the transition equation for the second element is a simple function of the control $A^T(t)$. Then the Bellman equation for this system is[ix]:

$$v\left(A^M, A^T(-1)\right) = \max_{\{A^T, A^M\}} \left\{\ln A^T + \gamma \ln A^T(-1) + \beta v((A^M)', A^T)\right\} \qquad (13)$$

$$s.t. \qquad A^T + (A^M)' \le A^M$$

where $A^T = A^T(t)$, and $(A^M)' = A^M(t+1)$. From the structure of the Bellman equation, we conjecture that the value function should be a form of[x]:

$$v(A^M, A^T(-1)) = \Phi + \Omega \ln A^M + \Psi \ln A^T(-1)$$

To verify this conjecture we must find a triplet (Φ,Ω,Ψ) such that the corresponding value function satisfies Bellman's equation, that is:

$$\begin{aligned} &\Phi + \Omega \ln A^M + \Psi \ln A^T(-1) = \\ &\max_{\{A^T,(A^M)'\}} \left\{ \ln A^T + \gamma \ln A^T(-1) + \beta\Phi + \beta\Omega \ln (A^M)' + \beta\Psi \ln A^T \right\} \\ &\quad s.t. \qquad A^T + (A^M)' \le A^M \end{aligned} \tag{14}$$

After the constraint has been imposed at equality, the first-order condition of the maximization problem on the right-hand side yields [xi]:

$$\frac{1}{A^T} + \frac{\gamma}{A^T} + \frac{\beta\Psi}{A^T} = \frac{\beta\Omega}{A^M - A^T} \tag{15}$$

This gives us:

$$(A^T)^* = \frac{1 + \beta\Psi + \gamma}{1 + \beta\Psi + \beta\Omega + \gamma} B \tag{16}$$

and:

$$(A^M)'^* = \frac{\beta\Omega}{1 + \beta\Psi + \beta\Omega + \gamma} B \tag{17}$$

where B is the budget. The triplet (Φ,Ω,Ψ) can be identified by substituting Equation (11) back to Equation (10), and they can be expressed in terms of parameters (β, γ) [xii]. The optimal solutions for partial profit hysteresis are given as:

$$(A^T)^* = \left(1 + \frac{\beta\gamma}{1+\gamma}\right)(1-\beta)B; \qquad \gamma \neq -1 \qquad (18)$$

$$(A^M)^* = B - (A^T)^*. \qquad (19)$$

Under full profit hysteresis (γ = 1) a corner solution exists where all marketing effort is triggering[xiii], i.e. $A^{T*} = B$.

This set of solutions for the firm's long-run profit maximization enables us to allocate a given budget *B* into triggering and maintaining. The optimal policy is *non-repetitive,* i.e. there is no need to trigger again once a firm arrives in the maintenance stage, and this scenario is unique to hysteresis. Compared to conventional spending rules, the optimal policy with hysteresis requires a smaller marketing budget for a given sales target, *or* it could generate and maintain higher sales and profits for a given marketing budget. The two-period model adequately characterizes the non-repetitive spending nature, but does does *not* imply that the planning horizon must consist of two equal-length periods. Instead, these two periods characterize two regimes, which may have different lengths. In practice, the maintenance period will likely be significantly longer than the triggering period.

Specifically, the results imply the following:

(i) Equations (18) and (19) show how to split a given budget into current triggering and subsequent maintenance, which depends on the strength of hysteresis, γ. For a given budget *B*, the greater the triggering, the smaller the maintenance spending, and vice versa; under full profit hysteresis, all spending is triggering, and no maintenance is required;

(ii) $\partial A^T/\partial\gamma > 0$, or the stronger the profit hysteresis, the greater portion of *B* should be allocated to triggering. Note that higher profit hysteresis can be obtained either by higher response hysteresis, or by higher profitability;

(iii) $\partial A^T/\partial r > 0$, or the greater the discount rate, the more budget dollars should be allocated to current trigger spending ;

Triggering Without Constraint

We can also use the solution to address the managerially important question of resource allocation with given sales targets. Given the hysteresis response function (11),

$$A_t^{T*} = \exp\left\{ S_t^* \;\; + (-\gamma)^t \ln A_0 + \Sigma \sum_{i=1}^{t-1} (-\gamma)^i S_{t-i} \right\} \qquad (20)$$

for a given target level of sales S^*, the necessary triggering amount of marketing is [xiv]:

In interpreting the important economic ramifications of Equation (20), recall that triggering is upfront, temporary marketing spending, and that maintenance is permanent, recurring spending. Hysteresis allows for a portion of the required maintenance spending to be *substituted* by triggering, so that substantial marketing cost savings emerge in the long run. The "penalty" for these savings is that the required trigger spending will be higher, as all the hysteresis terms in Equation (20) are positive. Specifically, the determinants of trigger spending are:

(i) the target itself (S_t^*). Ceteris paribus, the higher the target, the higher the spending. Furthermore, absent profit hysteresis ($\gamma = 0$), this target level is the *only* determinant of marketing spending. In that case, however, the spending has to be sustained in every subsequent period, i.e. it becomes permanent maintenance spending.

(ii) the profit-weighted strength of hysteresis (γ). We have already established that stronger response hysteresis (ρ) justifies higher trigger spending. Furthermore, the higher the profitability ($\pi / (\delta/\xi - \pi)$) generated from response hysteresis, the more can be allocated to temporary trigger spending as well.

(iii) the historical sales path, up to the most recent sales level (S_{t-1}). If, for example, historical sales and hysteresis are high, more can be spent on triggering, resulting in fewer required maintenance dollars in the future. Note also that, the longer the time horizon (e.g. the older the product), the smaller the upfront penalty for trigger spending. Indeed, in the limit ($t \rightarrow \infty$), optimal triggering approaches the asymptote of permanent spending. This is the lowest marketing-cost solution in that an infinite spending stream has been fully replaced by a one-time investment of the same amount. To the best of our knowledge, this is the first analytical evidence that *the evolution of historical sales itself* can drive optimal long-run marketing spending policies.

(iv) the inaugural marketing spending level (A_0). Ceteris paribus, if initial sales for a product were generated with little or no marketing support, that reduces the required trigger spending penalty toward future sales targets, and vice versa. The influence of this term, too, dies out as time progresses. This connection between initial marketing conditions and future spending is also a new insight in the theory of marketing resource allocation.

The general behavior of the optimal triggering expression (20) is illustrated in Figure 3, where optimal trigger spending toward a given sales target varies with the strength of profit hysteresis and inaugural marketing spending. Exogenous drivers of demand naturally affect optimal triggering in (20) as well, via the sales terms in the exponent.

Figure 3 – Drivers of Trigger Spending

Optimal Marketing Budgeting in Practice

The above analysis is based on a theoretical logical order, i.e., starting with an objective function and a constraint, we derive a solution to this system. The practice of marketing strategy, however, often proceeds in a reverse order. First, a marketing campaign is designed in order to achieve a given target sales level, and the marketing support budget is negotiated with senior management. However, the optimal marketing budgets for a given target are rarely assessed in practice. If they were, then our solution could be used in the following order[xv]:

Step 1. *Optimal triggering*: for a given target sales, S^*, Equation (20) can be used to determine the optimal triggering, $A^{T*} = A^T(S^*)$;

Step 2. *Optimal budgeting*: use Equation (18) to determine the optimal budget, $B^* = B(A^{T*})$;

Step 3. *Optimal maintaining*: use Equation (19) to split the optimal budget: $A^{M*} = B^* - A^{T*}$.

As a result, A^{T*}, A^{M*}, and B^* can be expressed in terms of S^* and parameters.

The remaining task in the paper is to demonstrate the use of our hysteresis metric in an actual case setting, leading to a diagnosis of the company's existing spending patterns relative to long-term optimal.

The Hysteresis of Value Adjusted Advertising: A Case Study

The empirical detection of hysteresis

Hysteresis can be measured empirically either by direct or indirect sales-response methods. If marketing is of a direct-response nature, longitudinal records of marketing campaigns, customer acquisition and customer retention could be used to measure the *lifetime value of a customer* and derive the spending policy that maximizes this quantity (see, e.g. Blattberg & Deighton 1996). In this case, the more hysteretic sales response, the more would be allocated to customer acquisition (triggering) at the expense of customer retention (maintenance) efforts, and vice versa. This approach is relatively simple to execute so long as the acquisition and retention-response parameters are stable over time. On the other hand, its application is restricted to direct-response marketing and it may overestimate marketing productivity, as each customer acquisition is fully attributed to marketing action, which could have been redundant. For these reasons we will not explore direct-response models in this context.

Indirect sales-response methods do not require a separate long-term metric of market performance, as we isolate the permanent (long-term) component in sales performance itself. As argued earlier, this method is anchored in the rich econometric market-response literature and requires equal-interval data on sales and the marketing mix for several years. The data interval should be response- and decision relevant, most likely weekly to monthly in the context of marketing campaigns and budget allocations. There is no restriction on the types of marketing activities that could generate hysteresis, though our example will focus on the *quantitative* (spending) dimension of marketing, consistent with the dynamic optimization model. Thus we require substantial detail on the evolution of prices, period-by-period spending levels and results. Future research should investigate whether or not *qualitative* changes in the marketing mix have hysteretic effects on sales, as well as possible segment differences in hysteretic market response.

Data description

Our empirical application is set in the evolving market for computer printers in the early to mid-nineties. This market development was spurred by the growing installed base of personal computers, along with the advent of newer printing technologies such as laser and inkjet. We sample monthly sales of a leading supplier of printers from 1990 to 1994, along with its marketing mix comprised of retail price, print and electronic advertising spending, and a one-time expansion in distribution from specialty-only to specialty plus general outlets (Figure 4).

A large-scale conjoint measurement was executed near the end of the time period, so we have estimates of end users' utilities for product features. This offers a unique opportunity to quantify the relative attractiveness of the product relative to competition in this fast-moving technology market. Indeed, by retroactively changing the product and its competitors' attributes to coincide with actual historical market changes (such as an improvement in print resolution from 300 to 600 dpi, or a reduction in product weight or footprint), we can generate a time series of consumer utilities for the product. Figure 5 shows the history of these conjoint-inferred consumer preferences for the product. From a customer value perspective, the product goes through phases of higher and lower values, relative to its price and competitive offerings. We expect these relative-quality variations to have a positive effect on consumer sales, while controlling for the other elements in the marketing mix. Following the attitude and behavioral change framework discussed earlier, we also expect communications efforts that coincide with periods of higher product value to have a stronger effect on product sales.

Figure 4 – Sales and Marketing Spending

Figure 5 – Price and Consumer Inferred Product Value

Marketing hysteresis model

The combined availability of advertising spending and perceived-value data allows us to measure the hysteretic effects of ‘value adjusted advertising’. We build on Simon’s (1997) argument that single marketing-mix actions are unlikely to create hysteresis in sales in and of themselves. However, temporary conditions may exist that create unusual but short-lived opportunities for hysteretic marketing effects. In this case, the hypothesis is that, *the higher the product’s perceived value, the stronger the hysteresis in sales induced by advertising*. The reason is that purchases made under high-value conditions create higher customer satisfaction which stimulate subsequent sales due to purchasing of additional units (e.g. by businesses) and positive word-of-mouth (e.g. by individual consumers).

Since the competitive arena and the retail prices in this high-technology industry evolve rapidly, the strategic implication of our hypothesis would be to trigger sales to higher levels precisely at times when the product has a value advantage, and vice versa. In order words, the firm has an opportunity to accelerate its sales growth with advertising during certain windows of opportunity. As argued earlier, we also hypothesize that advertising’s hysteretic effects are asymmetric, i.e. they only exist in marketing campaigns (spending increases), not in cuts.

These considerations lead to the following hysteresis market response specification, which is an application and extension of model (8) as previously discussed. The exogenous variables in logarithms are indicated with a prime (‘) :

$$S(t) = b_0 + \{ b_1(L) * M'(t) + b_2(L) * \max\{ [M'(t) - M'(t-1)], 0\} \} * Q'(t)$$

$$+ \varphi S(t-1) + b_3 P'(t) + b_4 Q'(t) + b_5 D(t) + u(t) \qquad (21)$$

where S is sales, P is price, Q is conjoint derived product utility, M is marketing communications spending and D is distribution. As per our previous discussion, both the level and the trigger efffect of marketing communications are hypothesized to interact with perceived product value in generating sales response: the marketing effects and hysteresis are the strongest in periods when the product offers good competitive value relative to price, and vice versa.

Predictive Testing

Table 2 summarizes the temporal behavior of the variables, in the form of various unit-root tests (see, e.g., Dekimpe and Hanssens 1995a for a more detailed discussion on methods). The necessary conditions for long-term and possibly hysteretic marketing effects are met, i.e. sales are evolving, the marketing mix (price, product value and media spending) is evolving and marketing campaigns are stationary. Given the evolution in sales, we now examine to what extent it is driven by evolution in the marketing mix and by marketing hysteresis.

Model (21) provides a predictive test for the sources of sales evolution, as follows:

- if sales evolution is independent of the marketing mix, then $\varphi = 1$ and the other response parameters are zero.
- if sales levels evolve with marketing levels only, then $\varphi < 1$, and all $|b_i| > 0$, except $b_2 = 0$. Furthermore, a cointegration test between sales and marketing should reveal that the series are cointegrated, i.e. there exists a long-run equilibrium among them. This would be the ‘co-evolution” scenario in Dekimpe and Hanssens (1999), implying that higher sales necessitate lower prices and/or sustained higher marketing spending.

- if sales evolution is linked uniquely to temporary changes in the marketing mix, in this case advertising triggers, a condition of full hysteresis emerges, with $\varphi = 1$, $b_2 > 0$ and the other response parameters are zero. In this case, the prescription would be to expose the market to periodic bursts of short-lived advertising when conditions are favorable, and to withhold marketing maintenance spending in between.
- finally, a mixture model where $\varphi < 1$ and all response parameters significant, implies partial hysteresis. Temporary advertising triggering under favorable market conditions drives sales upward, but some increase in sustained marketing spending is necessary to preserve the sales growth. The prescription in this case calls for a balanced marketing trigger/maintenance spending strategy, where the terms of the balance are dictated by the strength and profitability of hysteresis.

Empirical Results and Validation

The estimation results for model (21) are summarized in Table 3. The best lag specification (obtained by direct search) was simple: $b_1(L) = b_1$ and $b_2(L)= b_2 L$, i.e. marketing campaigns take about one month to achieve maximum short-run impact. All marketing effects are statistically significant with the expected signs, except for spending in electronic media which are insignificant and are therefore omitted in the final model and discussion.

Table 2 – Hysteresis Model Tests

Table 3 – Hysteresis Model Parameters

Overall, the estimated parameters strongly support the mixture-model scenario of sales evolution. Using the parameters of the levels and positive changes in value-adjusted advertising, we estimate the degree of hysteresis (ρ) as 349 / (356 + 349) = 49.5 %. Thus we find evidence of *partial hysteresis* of value-adjusted marketing spending: when competitive conditions are favorable, then triggering spending on print advertising helps propel sales to higher levels in the evolutionary path of the product. At the same time, some maintenance spending will be needed to sustain the path of growth, so the success associated with higher revenues comes with a burden on future marketing spending[xvi].

How robust are these partial-hysteresis results against other possible explanations ? First, in unreported experiments, we find no statistical evidence of hysteresis in other elements of the marketing mix on which data were available. Second, a symmetry test reveals that advertising hysteresis exists only in marketing campaigns, not in spending cuts. Furthermore, the hysteretic advertising effects lose their statistical significance when the interacting variable (Q) is omitted[xvii]. This finding is in line with Simon's (1997) conjecture that marketing spending in and of itself does not generate hysteresis. For the company under study, it implies that, the more it engages in product- and technology enhancement that creates even temporary competitive advantage, the more it can generate long-term benefits from its marketing spending.

We further test for the presence of reverse causation that could bias our hysteresis estimates. A Hausman specification test reveals no endogeneity in advertising spending. Furthermore, print advertising spending does not Granger cause value shocks, and value shocks do not Granger cause print advertising spending.

Lastly, we conduct a validation test of the partial hysteresis result by examining the stability of the parameters b_1 and b_2 (and, therefore, ρ) in (21), estimated over various time subsamples. Indeed, if we wish to derive optimal spending rules from the presence of hysteresis, it is important to establish that the hysteresis effect can be replicated over time. The test computes recursive estimates of hysteresis along with the other response parameters, starting from a minimum sample size of 30 observations, and expanding the sample one observation at a time. We find that the time path of the 13 successive estimates of ρ is centered around 58.7 %, with a standard deviation of 4.9 %. Furthermore, all 26 parameter estimates have the correct sign.

The conclusion of these validation tests is that our sample estimates indeed capture a hysteresis effect that is robust and replicable over time, and that is therefore amenable to optimization.

Long-term Marketing Spending Implications

The long-term profitability impact of hysteresis and trigger-maintenance marketing spending can be demonstrated by comparing the required multi-period marketing budgets under two scenarios, with and without hysteresis. Suppose that, given the market conditions near the end of the evolutionary path in our example, sales are targeted to grow to 50,000 monthly units for the next six months. Without hysteresis, the monotonic sales-response function would dictate that monthly advertising spending of $148,400 would be required to achieve that goal (obtained by log inversing the response function), holding prices and other market conditions constant. The required marketing budget over the planning horizon is therefore $890,400.

By contrast, our partial hysteresis finding (ρ = 0.495) indicates that such repetitive spending is not necessary. A superior strategy is to take advantage of the (partial) investment quality of marketing spending: trigger sales to the desired target level, and keep it there for six months with a *reduced* marketing maintenance budget. While we do not have the exact data on profit margin π and marketing cost function δ, we know that the product is profitable and we use a conservative estimate of profit hysteresis γ = 0.5. Under this scenario, we can apply equation (20) to the sales target and the unique sales path leading up to the present. We find that the optimal trigger investment is $181,100, and the maintenance spending is $85,200. As a result, the semi-annual profit increase due to hysteresis is $890,400 - $181,100 - 5 * $85,200 = $283,300, or approximately 32 % of the original budget. The longer the planning horizon and/or the more favorable the current market conditions, the more economic benefits will accrue. As proposed in the introduction to the paper, the long-run profitability implications of hysteresis can indeed be meaningful.

Conclusions

When temporary marketing activities affect the long-term evolution of sales, a condition called hysteresis is created. Hysteresis in marketing changes the prevailing resource allocation paradigm. Instead of allocating marketing investments in accordance with a one-to-one mapped marketing-sales function, hysteresis creates two distinct roles of marketing spending: one is triggering sales performance to higher levels, the other is maintenance spending to sustain these higher levels. As such, hysteresis is economically beneficial to managers aspiring to meet their sales growth targets.

The *intuition* behind this economic benefit is as follows. Absent marketing hysteresis, the firm's achievement of sustained sales growth requires permanently higher (repetitive) marketing budgets, notwithstanding possible uneven short-term allocations such as pulsing or chattering. In these cases, the one-to-one functional relationship between sales and marketing spending drives the allocation rules. By contrast, under full hysteresis, all of this repetitive spending is replaced by a single upfront temporary investment and the resulting sales increases are self-sustaining. Finally, under partial hysteresis, a portion of the required permanent marketing spending can be substituted by the upfront trigger investment. The full or partial substitution of permanent marketing spending by temporary investments gives hysteresis its powerful economic benefit to the marketer. Indeed, an argument could be made that hysteresis creates long-term assets out of current marketing expenditures and that these expenditures should therefore be capitalized for accounting and financial reporting purposes.

Our paper has formalized these fundamental results in two ways. First, we have shown how the strength of marketing-induced hysteresis, ρ, can be measured on a simple zero-to-one scale from readily available sales and marketing-mix time-series data. Second, we have derived the long-term optimal marketing resource allocation rules in function of γ, a metric that combines response hysteresis ρ with profitability. These rules split any given marketing budget in a triggering and a sustenance allocation, and they can also help set the unconstrained optimal level of triggering and maintenance. Using dynamic programming, we have obtained exact expressions for the amount of triggering that is needed to replace costly marketing maintenance budgets.

One corollary of hysteresis is that future marketing spending optimally depends not only on the intended sales and profit target, but also on the historical time path of sales. Furthermore, the more a brand's historical sales success has depended on past initial marketing spending, the more will have to be spent to reach higher future targets. These findings provide new quantitative evidence of the role of past sales evolution in the determination of future sales targets and required marketing budgets.

Our paper has focused on the analytical and empirical modeling of marketing hysteresis, without formally investigating the market conditions or behavioral rationales that lead to its existence. Some conditions that favor hysteresis have already been hypothesized in the literature, but until they are thoroughly tested and disseminated, we should not expect management to fully anticipate hysteresis, and allocate their marketing resources accordingly. Instead, management should rely maximally on current information on sales or other performance measures and, when hysteresis is detected, be prepared to act on it quickly. Meanwhile, both behavioral and marketing strategic research is needed in order to create a *knowledge base* for hysteresis and its ramifications. Specific topics include the hysteresis of qualitative marketing initiatives, segment response differences, impact of competition, and the combination of hysteretic and pulsing effects of marketing. Our empirical investigation provides evidence in one important case: when, in the course of a product's evolution, there are windows of opportunity created by the product's high customer value relative to the competition, then temporary advertising triggering in support of the product can have a beneficial hysteretic effect on sales, and this effect allows management to shift permanent marketing budgets into temporary ones. The resulting gains in marketing effectiveness and long-term profitability can be substantial.

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APPENDIX A

PARAMETER IDENTIFICATION

Assume the optimal allocation takes the form of♣:

$$\ln A^M(t+1) = I + H \ln A^M(t)$$

Where I and H are unknown constants. Then:

$$\frac{A^M(t+1)}{(A^M(t))^H} = e^I$$

We impose: $H = 1$, then:

$$A^M(t) = \frac{A^M(t+1)}{e^I} \qquad (A1)$$

Combine (A1) with the (binding) constraint (12), we have:

$$A^T(t) = A^M(t) - A^M(t+1) = (1 - e^I)A^M(t) \qquad (A2)$$

Rearrange items:

$$A^M(t) = \frac{A^T(t)}{1 - e^I} \qquad (A3)$$

From the (binding) constraint (12) we know:

$$A^T(t-1) + A^M(t) = A^M(t-1)$$

Therefore:

$$A^T(t-1) = A^M(t-1) - A^M(t)$$

Combine it with (A1) and (A3), we have:

♣ This conjecture can be easily verified by substituting (A5) back into it.

$$A^T(t-1)=\frac{1-e^I}{e^I}\left(\frac{A^T(t)}{1-e^I}\right)$$

That is:

$$A^T(t-1)=\rho A^T(t) \qquad (A4)$$

Where ρ is a constant.

Substitute (A3) and (A4) into our conjecture (14), (and note $A^T = A^T(t), A^T(-1) = A^T(t-1),$ $A^M = A^M(t), (A^M)' = A^M(t+1)$), the right-hand-side of (14) becomes:

$$RHS=\ln A^T+\gamma\ln\rho A^T+\beta\Phi+\beta\Omega\ln(A^M-A^T)+\beta\Psi A^T$$

Then the first-order-condition yields:

$$\frac{1}{A^T}+\frac{\gamma}{A^T}+\frac{\beta\Psi}{A^T}=\frac{\beta\Omega}{A^M-A^T} \qquad (*)$$

Rearrange items, we get:

$$(A^T)^*=\frac{1+\beta\Psi+\gamma}{1+\beta\Psi+\beta\Omega+\gamma}A^M$$

and

$$(A^M)'^*=A^M-(A^T)^* \qquad (A5)$$

Substitute (A5) back to the *RHS* expression, and rearrange items give us:

$$\Omega=1+\beta\Psi+\beta\Omega$$
$$\Psi=\gamma$$

Therefore we can identify:

$$\Omega=\frac{1+\beta\gamma}{1-\beta} \qquad (A6)$$

Substitute (A6) back to (A5), we get:

$$(A^T)^* = \frac{1+\gamma+\beta\gamma}{1+\gamma}(1-\beta)B = \left(1+\frac{\beta\gamma}{1+\gamma}\right)(1-\beta)B$$
$$(A^M)^* = B-(A^T)^*$$

where B is the budget, and γ is the profit-weighted hysteresis parameter.

Q.E.D.

APPENDIX B

DETERMINING UNCONSTRAINED TRIGGERING

To achieve a targeted level of sales, S_t, how much marketing spending, A_t, is necessary? A general marketing hysteresis response model can be characterized as:

$$S_t = f(g(A_t, A_{t-1}, \gamma))$$

Where g is goodwill, which serves as an intermediate variable, and γ is the hysteresis parameter. The time index t = 1, 2, …. The inverse of this function gives us:

$$A_t^* = f^{-1}(S_t, S_{t-i}, A_0, \gamma), \quad i=1, 2, 3, \ldots \quad (B0)$$

This relationship says that the triggering is determined by a targeted level of sales, S_t, all previous sales, S_{t-i}, as well as the degree of hysteresis, γ, and the initial condition, A_0. With assumptions of a competitive market condition (i.e., the firm is a price taker), fixed unit production costs, and a Cobb-Douglas specification for the goodwill function, the hysteresis sales-marketing response function takes the form of Equation (11):

$$S_t = lnA_t + \gamma lnA_{t-1}, \quad t = 1, 2, 3, \ldots$$

Therefore we have the following:

$$\begin{aligned}
S_1 &= \ln A_1 + \gamma \ln A_0 && (B1) \\
S_2 &= \ln A_2 + \gamma \ln A_1 && (B2) \\
&\ldots \\
S_{t-1} &= \ln A_{t-1} + \gamma \ln A_{t-2} && (Bt-1) \\
S_t &= \ln A_t + \gamma \ln A_{t-1} && (Bt)
\end{aligned}$$

In order to cancel out items of lnA_i, i=1, 2, …, and get an expression like ($B0$), we use a weight factor $(-\gamma)^{t-i}$ to weigh and sum Equations (Bt), (Bt-1), .. ($B2$), ($B1$):

$$(Bt) - \gamma(Bt\text{-}1) + \gamma^2(Bt\text{-}2) - \gamma^3(Bt\text{-}3) + \ldots + (-\gamma)^{t-2}(B2) + (-\gamma)^{t-1}(B1)$$

The summation gives us:

$$LHS:$$

$$S_t - \gamma S_{t-1} + \gamma^2 S_{t-2} - \gamma^3 S_{t-3} + ... + (-\gamma)^{t-2} S_2 + (-\gamma)^{t-1} S_1$$

$$= S_t + \Sigma \sum_{i=1}^{t-1} (-\gamma)^i S_{t-i}$$

And

$$RHS:$$

$$\ln A_t + (-\gamma)^{t-1} \ln A_0$$

Let *RHS* = *LHS*, and rearrange items:

$$\ln A_t = (-\gamma)^t \ln A_0 + S_t + \Sigma \sum_{i=1}^{t-1} (-\gamma)^i S_{t-i}$$

Therefore the triggering is:

$$A_t^* = \exp\left\{ S_t + (-\gamma)^t \ln A_0 + \Sigma \sum_{i=1}^{t-1} (-\gamma)^i S_{t-i} \right\}$$

The triggering amount is jointly determined by the target sales level, S_t, the system's initial condition, A_0, the history of market performance, S_{t-i}, and of course, the degree of profit hysteresis, γ.

Table 1: Hysteresis and Its Development

Year	Investigator	Discipline	Object	Methodology	Contribution	Application
1887*	L. Rayleigh	Physics	Ferromagnetic	Analytical model describing current-magnetic intensity	Discovery of ferric-memory capacity	All kinds of electrical and electronic appliances and equipment
1864*	H. Tresca	Mechanics	Plastics	Stress analysis	Theory of elastic and plastic properties	Various material & structural engineering applications
1979	J. Little	Operations Reserach	Advertising Effectiveness	Conceptual description	Argues that advertising may exhibit hysteresis	Introduces the concept to marketing
1983	P. Jones	Operations Research	Sales-advertising relationship	Static game theory	Shows existence of hysteresis in asymmetric games	Introduces the concept to economics and business disciplines
1986	O. Blanchard & L. Summers	Economics	Labour markets	Theoretical analysis	Explains persistent high unemployment in Europe	Supports New Keynesian multi-equilibria theory
1987	R. Baldwin, A. Dixit, & P. Krugman	Economics	International Trade	Theoretical analysis	Explains persistence in the US trade deficit	Provides theoretical rationale for President Reagan's trade policy
1989	R. Gordon	Economics	Labour markets	Empirical analysis	Estimates Phillips curve in various OECD economies	Finds traces of hysteresis in the unemployment/inflation relationship
1995, 1999	M. Dekimpe & D. Hanssens	Marketing	Sales-marketing relationship	Multivariate Time-Series Analysis	Unit- root based diagnostics for persistent marketing effects	Long-run marketing effects on sales, profits
1997	H. Simon	Marketing	Marketing Decision Making	Conceptual description	Call for awareness	Encourages more studies in the field
2001	Current Study	Marketing	Marketing Decision Making	Econometric & theoretical analysis	Quantifies strength of marketing hysteresis & optimal resource allocation	Marketing resource allocation for long-run profitability

* Publication date of a formal model, though earlier studies had already made conceptual descriptions. Due to its importance as a theoretical framework and in practical application, hysteresis studies in these disciplines continue to appear.

Table 2

Hysteresis Model Tests*

Variable	ADF	Verdict
Sales	2.086	Evolving
Price	-1.189	Evolving
Value	-1.257	Evolving
Print Advertising	-2.432	Evolving
TV Advertising	-1.497	Evolving
Print Triggering	-26.661	Stationary
TV Triggering	-5.205	Stationary

* The number of lags in the ADF tests was determined by the Schwartz criterion and ranged between 0 and 5. The critical ADF values at $p<.05$ ranged from –2.93 to –2.94.

Table 3

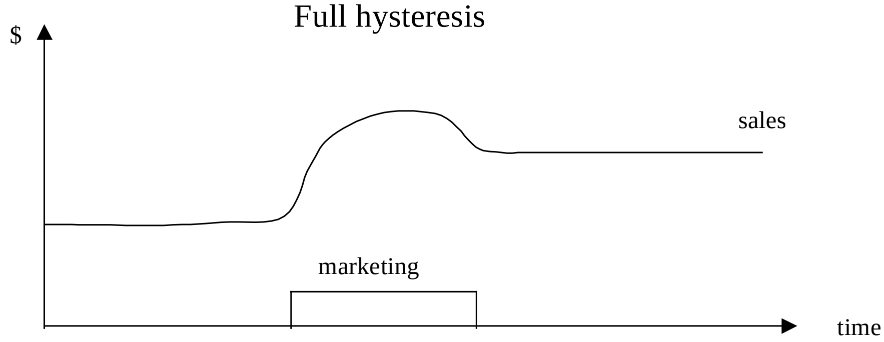
Hysteresis Model Parameters *****

Variable	Response Parameter (st.err.)
Intercept	357,533 (97,899)
price	-62,545 (17,001)
preference	6,330 (4,474)
distribution	12,146 (6,753)
lagged sales	0.377 (0.160)
ad level	356 (188)
ad trigger	349 (132)

* Parameter estimates by least squares with White's correction for heteroskedasticity. All are statistically significant in unidirectional tests at $p<.05$, except TV advertising effects (which were subsequently excluded). The overall R-squared is 0.884 and the Durbin-Watson is 1.740. The model is robust in specification, for example a log-log (constant-elasticity) formulation provides a similar set of results.

Figure 1. Full and Partial Hysteresis

Full hysteresis



Partial hysteresis

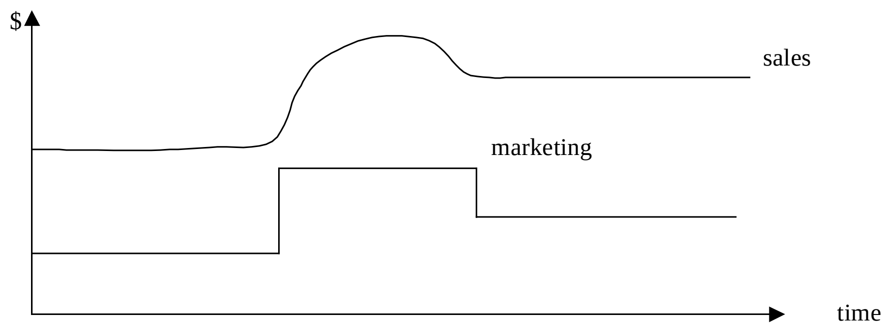
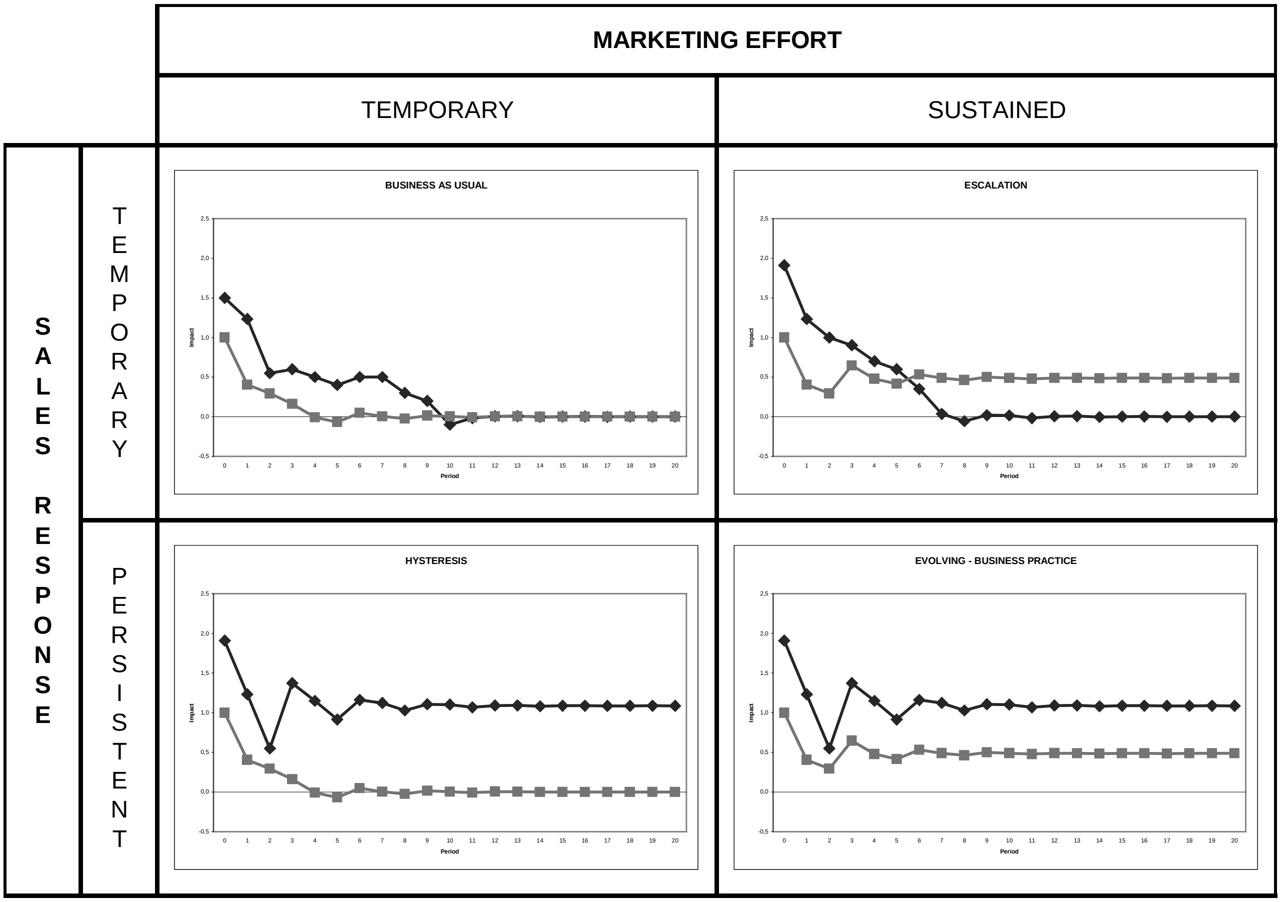


Figure 2: Strategic scenarios resulting from temporary versus permanent effort/response



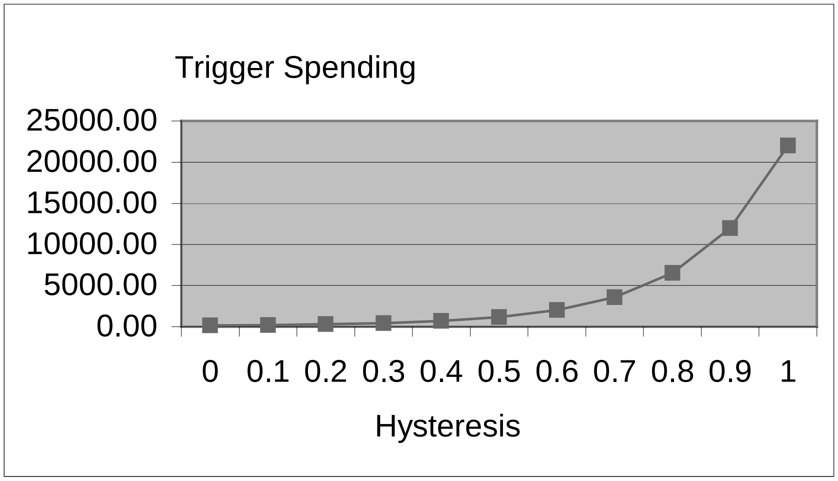
◆ ◆ ◆ **Sales**

■ ■ ■ ■ **Marketing Mix**

Source*: Dekimpe and Hanssens**, ***Journal of Marketing Research, **November 1999**

Figure 3

Required Trigger Spending to Reach a Given Sales Target



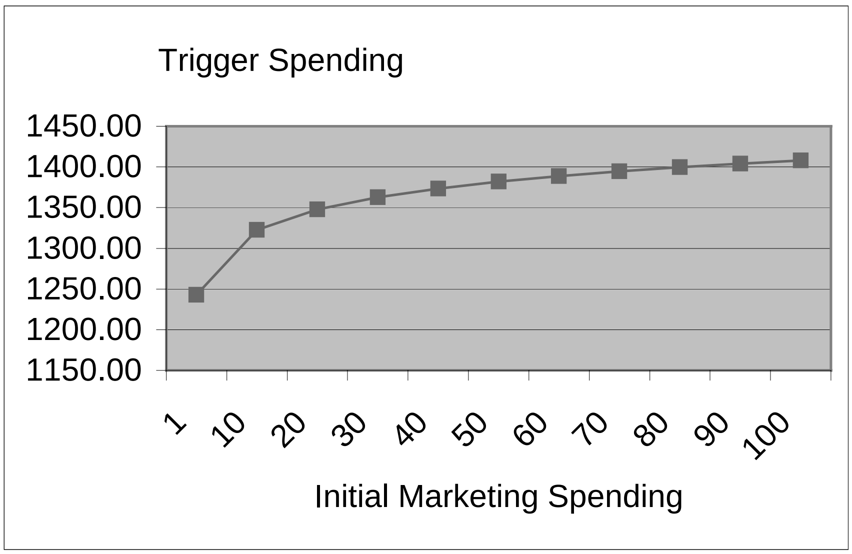


Figure 4. Sales and Marketing Spending

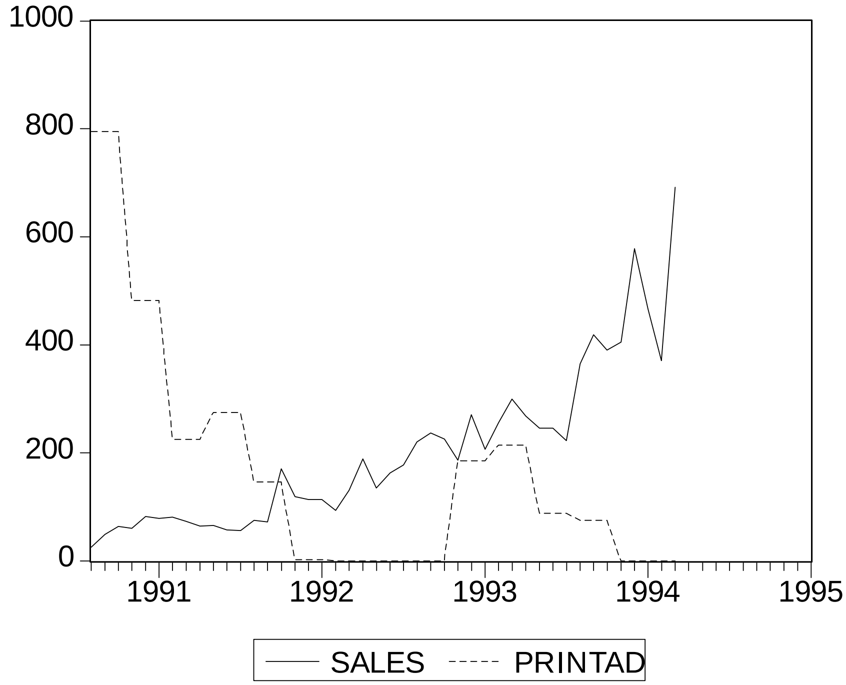
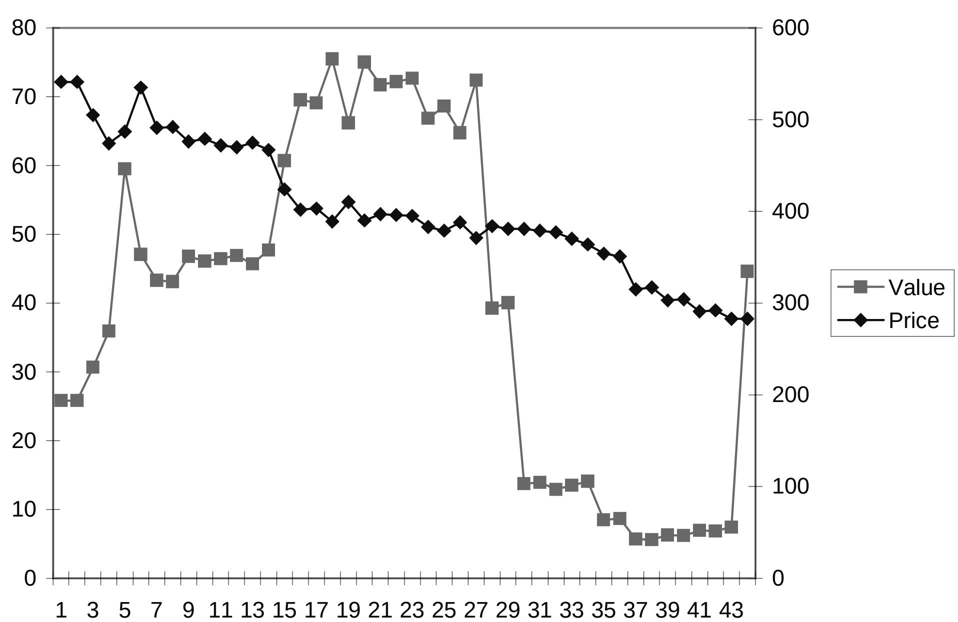


Figure 5. Price and Competitive Value



Footnotes

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[i] We assume throughout that the order of integration of the data is at most 1 (i.e. evolution)

[ii] The literature on hysteresis in unemployment and international trade has used a similar approach, see for example Gordon (1989), Franz (1990), Fortin (1991), Ouyang (1997)

[iii] In our subsequent development, we will only discuss the case of ρ=0 as absence of hysteresis. The distributed-lag case (ρ< 0) is not conceptually different

[iv] A similar treatment may be found in the economics literature, for example, Sargent (1987)

[v] The Cobb-Douglas function transforms marketing spending, both present and previous, into goodwill monotonically, so the parameters ξ and μ can be transformed to γ (to be consistent with ρ, the hysteresis metric). The Cobb-Douglas function requests the summation of the power parameters to be positive, representing their returns to scale. We impose a minus sign in front of μ to serve our purpose, i.e., characterizing our hysteresis metric. In order to make the objective function behave as a standard Cobb-Douglas function (so that dynamic programming can yield meaningful solutions), we subsequently offset the minus sign we imposed, as shown in Equation (11). See Varian (1992) for further discussion.

[vi] The lagged-sales term and the presence of exogenous drivers of demand in equation (1) do not affect this definition.

[vii] The parameter scaling to unity was done for the purpose of simplifying the theoretical analysis. In empirical work, δ and π are known, and the response parameters are estimated from the data, so that the unity restriction can be removed.

[viii] We adopt this notation to show what happens when hysteresis is absent. In the theoretical demonstration, we assume a budget is given as an exogenous variable. Optimal budgeting will be discussed in a later section.

[ix] Chow (1997) demonstrates that the Bellman equation can be used for two unequal-length periods dynamic programming.

[x] For discussions on how and why the conjecture is set, see the literature on dynamic programming, for example Bellman and Dreyfus (1962) and Sargent (1987).

[xi] See Appendix A for details

[xii] See Appendix A for details

[xiii] (γ = -1) is an empty point in (A^T-γ) space. Although our model allows for negative hysteresis (i.e. γ < 0), that would be irrelevant for optimal resource allocation. Therefore we rule out full negative hysteresis (γ = -1) without affecting the generality of the solution.

[xiv] See Appendix B for details

[xv] If a concavity assumption is imposed on the response function, we could derive a general optimal solution without knowing a given budget or a given target level of sales.

[xvi] We also verified empirically that the levels of sales and marketing are cointegrated, i.e. they co-evolve.

[xvii] An additional specification test reveals that value shocks (in Q) by themselves do not explain the evolution in sales (i.e. the series are not cointegrated). The statistical details of this and other tests, all of which were conducted at p<.05, are available upon request