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Small Business Equity Returns: Empirical Evidence from the Business Credit Card Securitization Market

MATTHIAS FLECKENSTEIN and FRANCIS A. LONGSTAFF*

ABSTRACT

We present a new approach for estimating small business equity returns. This approach applies the Merton (1974) credit model to the returns on entrepreneurial business credit card debt securitizations and solves for the implied equity returns for the small businesses owned by the cardholders. The estimated small business equity premium is 10.74%. The standard deviation of small business equity returns is 56.37%. We validate the methodology by applying it to investment-grade corporate bonds and recovering a public equity premium of 6.17%.

WHAT IS THE EXPECTED RETURN from investing in a small business? This question is of fundamental importance since the total value of noncorporate entrepreneurial equity in the United States is more than $12 trillion and represents a substantial fraction of aggregate household wealth.[1] Furthermore, the Small Business Administration reports that there were 31.7 million small businesses in the United States in 2020, collectively accounting for 47.1% of total private sector employment.[2]

Despite the importance of this question, however, relatively little is known about small business equity returns. This may simply be due to the fact that

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Correspondence: Francis A. Longstaff, UCLA Anderson School of Management, 110 Westwood Plaza, Los Angeles, CA 90095-1481; e-mail: francis.longstaff@anderson.ucla.edu

[1] See Federal Reserve Statistical Release Z.1 Financial Accounts of the United States, First Quarter 2020, B.101.

[2] See https://cdn.advocacy.sba.gov/wp-content/uploads/2020/11/05122043/Small-Business-FAQ-2020.pdf. The Census Bureau's Statistics of U.S. Businesses (SUSB) data set indicates that firms with fewer than 10 employees account for about 10% of nongovernment employment, while firms with fewer than 100 employees account for roughly 33%. Schlingemann and Stulz (2021) show that the ratio of employment by publicly traded firms to total employment has decreased significantly over time.

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small business equity does not trade in the capital markets: Without actual market prices, it is difficult to estimate small business equity returns directly. Previous researchers have thus turned to alternative methodologies. For example, Moskowitz and Vissing-Jørgensen (2002) and Kartashova (2014) impute small business returns using the self-reported values of private firms included in the Survey of Consumer Finances (SCF).[3] While it is tempting to use estimates from the extensive literature on private equity and venture capital returns as proxies for small business equity returns, the typical small business is several orders of magnitude smaller than the firms receiving venture capital or private equity financing.[4] Thus, results from the private equity and venture capital markets may not be representative of the small business sector.

This paper introduces a novel approach for estimating the investment returns on small entrepreneurial firms. The key to our approach is the use of secondary market prices for business credit card securitizations, where the credit cards specifically target small businesses such as those with fewer than 10 employees and less than three million dollars in annual revenues. From these secondary market prices, we can directly identify the returns for debt claims on the assets of these small entrepreneurial firms. We then invert the Merton (1974) structural credit model to solve for implied risk and return measures for equity investments in these firms.

Credit card borrowing represents an important source of debt capital for many small entrepreneurial firms. The pools of assets underlying the various business credit card securitizations in our sample include receivables from several million cardholders. These credit cards can only be used for business purposes. The entrepreneur that owns the firm is jointly and severally liable with the business for all transactions on the account. Thus, the pricing of credit inherent in these receivables is a direct reflection of the risk of entrepreneurship. The personal liability of the entrepreneur, however, introduces a unique dimension to the capital structure of a small business. To capture this, we extend the standard Merton (1974) framework to allow for the possibility that the entrepreneur/small business may have other debt, some of which might in effect be senior to business credit card debt.

To estimate small business equity returns, we collect secondary market pricing data for the credit card debt securitizations of several major small business card issuers for the 2000 to 2018 period. We then compute monthly returns for these debt issues and use them in calibrating the extended Merton (1974) framework to the asset dynamics and capital structure of a representative small business. Finally, we use the model to solve for the moments of the corresponding implied small business equity returns.

[3] Other methodologies that potentially could be applied to small businesses include the cash-flow replication approach of Gupta and van Nieuwerburgh (2021) and the generalized public market equivalent approach of Korteweg and Nagel (2016).

[4] Axelson et al. (2013) report that the average enterprise value of private companies receiving buyout financing was more than $600 million. Chemmanur, Krishnan, and Nandy (2011) report that private firms that received venture capital funding typically have more than 250 employees.

Several important findings emerge from our analysis. First, the results suggest that the small business equity premium may be substantially higher than the public equity premium. In particular, the baseline calibration implies that the small business equity premium is 10.74% during the 2000 to 2018 sample period. In contrast, the realized excess return on public equity as measured by the Center of Research in Security Prices (CRSP) value-weighted index over the same period is 5.24%. Our estimate of the small business equity premium is slightly lower than the 12.21% premium implied by the results in Moskowitz and Vissing-Jørgensen (2002), as well as the 12.83% premium suggested by Kartashova (2014). We show that the results are robust to leverage assumptions and alternative calibrations of model parameters.

Second, the results indicate that small business entrepreneurial investment is very risky. In particular, the estimated standard deviation of small business equity returns is 56.37%. This suggests that small business investors face substantial idiosyncratic risk given that Moskowitz and Vissing-Jørgensen (2002) find investment in private equity to be significantly underdiversified. In particular, they show that households with private entrepreneurial equity invest an average of more than 70% of their private holdings in a single firm in which they have an active management interest. The volatility of small business equity is higher than the 45.70% average volatility of publicly traded stocks during the sample period. This makes sense since small businesses are orders of magnitude smaller than publicly traded firms and volatility is negatively correlated with firm size.

To further validate the methodology, we apply it to a broad index of corporate bond returns over a similar sample period and examine the implications for the stock returns of the firms issuing these bonds. We find that the properties of the implied stock returns given by our approach closely match those of the actual stock returns over the sample period. In particular, the model implies an equity premium of 6.17%, an average standard deviation of individual stock returns of 38.48%, and an average market beta of 0.862.

Finally, the results demonstrate that using prices from securitized debt markets can provide new windows into measuring risk and returns for asset classes that have traditionally been difficult to study. In particular, our approach could be applied to a broad set of securitized entrepreneurial and consumer/household debt claims such as Small Business Administration loans, business loan syndications, student loans, auto loans, home equity lines of credit, and personal lines of credit.

This paper is related to the literature on estimating the returns on small business entrepreneurial investment. A key paper in this area includes Moskowitz and Vissing-Jørgensen (2002), who use self-reported data from the SCF to estimate the returns on private entrepreneurial investment. This data set provides unique insights into small business returns since the vast majority of firms included in the survey are operated as sole proprietorships and/or family businesses, which are more representative of small businesses than the much larger firms studied in the private equity and venture capital literature. Moskowitz and Vissing-Jørgensen (2002) find that returns on aggregate

private equity are similar to those on public equity during their 1989 to 1998 sample period, a result that they characterize as the private equity premium puzzle. Kartashova (2014) extends the original data set and finds that private equity returns are higher than public equity returns during the subsequent decade. A number of recent papers, however, raise questions about the reliability of self-reported entrepreneurial data (see, e.g., Tedds (2010), Astebro and Chen (2014), and Hurst, Li, and Pugsley (2014)). This paper advances this literature by introducing a new empirical approach based on the secondary market prices of securitized small business debt.[5]

This paper is also related to the literature that models the relation between the debt and equity of a firm using a structural credit framework, such as Merton (1974). Important examples include Campbell and Taksler (2003), Vassalou and Xing (2004), Campbell, Hilscher, and Szilagyi (2008), Bharath and Shumway (2008), Campello, Chen, and Zhang (2008), Chen, Collin-Dufresne, and Goldstein (2009), Schaefer and Strebulaev (2008), Coval, Jurek, and Stafford (2009), Chava and Purnanandam (2010), Garlappi and Yan (2011), He and Xiong (2012), Diamond and He (2014), Friewald, Wagner, and Zechner (2014), Culp, Nozawa, and Veronesi (2014), and Nagel and Purnanandam (2020). This paper contributes to this literature by showing that the Merton (1974) framework can be applied to a much broader set of asset classes for which securitized debt is traded.

The paper is organized as follows. Section I provides background on business credit card securitizations. Section II describes our data. Section III describes the approach we use to identify small business equity returns. Section IV discusses the model calibration. Section V presents the results for the estimated small business equity returns. Section VI describes how we apply the model to corporate bonds. Finally, Section VII concludes.

I. Business Credit Card Securitizations

Business credit cards represent an important source of financing for many entrepreneurs.[6] For example, total transaction volume for small business credit cards was $245 billion in 2017.[7] Robb and Robinson (2014) report that credit card debt represents 46.3% of total outsider debt for the nearly 5,000 start-up firms in the Kauffman Firm Survey data set. The Small Business Credit Survey published by the Federal Reserve reports that applying for bank

[5] Several recent papers use secondary market prices for listed private equity funds and funds-of-funds or limited partnership transactions. See, for example, Jegadeesh, Kräussl, and Pollet (2015), Boyer et al. (2018), and Nadauld et al. (2019).

[6] Section II.B of the Internet Appendix discusses the role of business credit cards in financing small entrepreneurial firms and Section II.C provides a detailed discussion of business credit card securitizations. See also Fleckenstein and Longstaff (2022). The Internet Appendix may be found in the online version of this article.

[7] See Statista (www.statista.com) at https://www.statista.com/statistics/936159/leading-small-business-credit-cards-usa-by-purchase-volume/.

lines of credit/loans and credit cards are the two most common ways in which small entrepreneurial firms seek financing.[8]

The underlying collateral for a business credit card securitization consists of short-term unsecured loans made to cardholders as they use their credit cards to purchase goods and services. The business credit card issuer pools these loans into a master trust that then issues series of notes to investors that are typically referred to as asset-backed securities (ABS).

To illustrate, imagine that an issuer has a portfolio of loans made to its business credit card customers with a total notional amount of $100. There are at least two ways in which these loans can be securitized. Under the first approach, the issuer issues a single class of notes with a total notional value of $100 backed by the payments made by its business credit card customers. An example of this type of securitization is a mortgage-backed pass-through security in which an investor receives the principal and interest payments from an underlying pool of mortgages.

Under the second and more common way of securitizing business credit card loans, the issuer issues a series of tranched notes. For example, the loans could be securitized by issuing a series of A, B, and C tranches with notional amounts of $70, $20, and $10, respectively. The C tranche absorbs the first $10 of credit losses, the B tranche absorbs the next $20 of credit losses, while the A tranche absorbs the remaining credit losses. Alternatively, the C, B, and A tranches can be described as attaching and detaching at 0% and 10%, 10% and 30%, and 30% and 100%, respectively. The attachment point represents the percentage of the loan pool balance that can default before the tranche experiences its first losses. The detachment point represents the level of credit card defaults that leads to a total loss of the tranche. Note that if an investor were to purchase all of the A, B, and C tranches issued in the securitization, the resulting portfolio would be equivalent to owning the single class of notes issued in the first type of securitization described above.

The process by which cash flows are allocated to investors has two distinct periods: a revolving period and a controlled amortization (in some cases, controlled accumulation) period. If there are no losses, this structure mimics a traditional bond in the sense that interest is distributed every month and principal is paid in a single "bullet" cash flow on the maturity date.

Specifically, upon issuance, a business credit card ABS begins the "revolving period," during which investor coupon cash flows are paid from finance charge collections on the credit card accounts, and principal collections are used to purchase new loans. This latter feature has important implications for the securitization. In particular, since the principal payments received as cardholders pay off their balances are reinvested in new receivables, the principal balance of the underlying receivables pool does not decrease over the revolving period. This means that because the revolving collateral pool is continually replenished, there is no uncertainty about the length of the

[8] See https://www.newyorkfed.org/medialibrary/media/smallbusiness/2016/SBCS-Report-EmployerFirms-2016.pdf.

revolving period or the maturity date of the securitization. In other words, the maturity date of the securitization does not depend on how rapidly cardholders pay off their balances. This is in stark contrast to nonrevolving securitizations such as mortgage-backed securities in which the collateral balance declines over time and prepayment-related timing risk plays a major role. Any residual cash flows after paying investor interest and other expenses is referred to as "excess spread" and, depending on the master trust, may be released to the issuer.[9]

The revolving period continues for a predetermined length of time, and then the controlled amortization (accumulation) period begins when principal collections are distributed to investors. For instance, a business credit card ABS with a 60-month maturity would revolve for 48 months and then enter amortization over the final 12 months. Again, because the collateral pool is continually replenished, there is no uncertainty about the length of the revolving period and the actual maturity date of the securitization (the exception being the case of sustained defaults triggering early amortization—see the discussion below). In the case of controlled amortization, principal cash flows are distributed in equal installments, for instance, 1/12 of the invested amount every month for 12 months. In the case of controlled accumulation, principal cash flows are deposited into a collection account (principal funding account) every month and then paid out as a single cash flow at the end of the accumulation period.

Sustained defaults or charge-offs on the pool of credit card accounts trigger early amortization, independent of whether the business credit card ABS is in the revolving period or in controlled amortization (accumulation). Typical early amortization trigger events include collateral performance deterioration (e.g., the three-month average excess spread falls below zero, or the collateral balance falls below the investor invested amount), seller/servicer problems (e.g., seller interest falls below the required minimum level, or the seller fails to transfer new receivables into the trust when necessary), and legal issues (e.g., breach of representation or warranties by the issuer, or default, bankruptcy, and insolvency of the seller or servicer). Early amortization is a reflection of financial stress within the collateral pool, and the business credit card ABS immediately starts to amortize with principal balances being paid to investors according to their seniority. In this case, the effective maturity of the securitization might differ slightly from its stated or expected maturity.

Industry sources provide some insight about the types of investors who may hold credit card ABS. For example, the Federal Deposit Insurance Corporation (FDIC) Credit Card Securitization Manual states that "The primary investors in credit card ABS are pension funds, insurance companies, foreign banks, large domestic banks, and other investment managers who require predictable cash flows … Very few credit card ABS are marketed to retail customers,

[9] Note, however, that the issuer does not bear any credit risk in an early amortization event. Thus, the issuer's claim to the excess spread does not play the role of an "equity" tranche. Section II.C in the Internet Appendix provides additional details.

primarily due to the complex nature of the transactions and the need to continually monitor various performance indices on the underlying receivables." Riley (2019) provides a more detailed breakdown of the types of investors who own general ABS (not necessarily specific to credit card ABS). The latter source reports that 43% of ABS is purchased by investment companies (such as broker/dealers, banks, hedge funds, closed-end funds, etc.), 22% by insurance companies, 16% by asset managers, 5% by government entities, 4% by corporations, 3% by mutual funds, 2% by pension funds, and 5% by others.

II. The Data

Our approach is to use secondary market prices of debt claims on the underlying assets of private entrepreneurial firms to estimate the equity returns of these small businesses. Although data for this asset class can be difficult to obtain, we are able to use a combination of industry sources to construct an extensive data set of the securitizations of two major business credit card issuers, namely, Advanta and American Express. In this section, we provide background information on small businesses, we discuss the Advanta and American Express small business credit card programs, and we describe the securitization data.[10]

A. Small Businesses

As we discuss above, private investment in small businesses represents one of the most important asset classes for U.S. households. Based on data from the 2019 SCF, Bhutta et al. (2020) report that 13.4% of all surveyed households owned a privately held business. Of these, 78% had fewer than five employees. The SCF also reports that the average business equity of privately held firms with fewer than five employees was $447,400 and that the average business equity of firms with five or more employees was $4,081,500. Taken together, these statistics suggest that firms with fewer than five employees could account for as much as 28% of all private equity in the United States. Furthermore, small businesses may be very different in nature from larger private firms or publicly traded firms. To provide evidence on this, we estimate employment betas for firms in different size categories. The employment beta is the slope coefficient in a regression of the percent change in employment for firms in different employment categories on the percent change in aggregate U.S. nonfarm employment (see, e.g., Berry and Blackwell (2005)). The employment betas for firms with 1 to 9, 10 to 99, 100 to 499, and 500 or more employees are 0.446, 0.919, 1.054, and 1.022, respectively. These results suggest that the smallest firms may be less cyclical than other firms.

[10] Section I of the Internet Appendix provides detailed additional information about the data and methodology used in this section.

B. The Advanta Business Credit Card

Advanta Corp. (Advanta) was a monoline credit card bank and one of the largest issuers of credit cards to small business entrepreneurs in the United States, with more than one million accounts at the end of 2006. As summarized in Table I, Advanta's business credit cards targeted small firms with fewer than 10 employees and less than three million dollars in annual revenues. Among Advanta's core customers were small independent brick-and-mortar and online retailers, small business start-ups, and business professionals such as consultants, lawyers, physicians, contractors, television writers and producers, and online content developers. Because of its focus on the small business market, Advanta provided its cardholders an array of tools and services tailored toward small business owners. For instance, Advanta offered payroll management, employee expense tracking, online tools for credit card accounting and bookkeeping, business and health insurance, discounts on business travel, tools to create websites, tutorials on developing business plans, marketing, tax, and legal advice on business and personal finance, and many other services of interest to small businesses.[11]

Advanta accounts were restricted to business owners, and applicants needed to provide proof of business ownership or involvement.[12] Small business owners signed as personal guarantors of the business credit card. Specifically, under the cardholder agreement, the entrepreneur and the business were jointly and severally liable for all transactions on the business credit card account.

Table I provides summary statistics about Advanta cardholders from Advanta's 2006 10-K filing. Account balances typically ranged from zero to $10,000, with an average balance of $4,540. Credit limits for Advanta accounts ranged from about $5,000 to $25,000, with an average value of $14,894. In addition, Advanta cardholders had relatively high credit scores. For example, Table I shows that nearly 45% had FICO scores in excess of 720. Table I also shows that the largest concentrations of Advanta cardholders were in California, Florida, Texas, and New York.

C. The American Express Business Credit Card

American Express is also one of the largest issuers of business credit cards in the United States, with nearly 1.6 million accounts at the end of 2013. As summarized in Table I, American Express business credit cards target small businesses and business professionals. For instance, the OPEN business credit card targets small businesses with fewer than 100 employees and less than $10 million in annual revenue. American Express business credit card customers have access to various online tools and services to manage their small businesses, including account management, expense tracking, vendor payment

[11] See www.advanta.com, accessed via https://archive.org/web/.

[12] Applicants were also asked to provide their Federal tax ID number and business phone and address before finalization of card acceptance. See https://www.financeglobe.com/credit-cards/card-205/.

Table I

Description of the Advanta and American Express Small Business Credit Card Programs

This table describes the small business credit card programs of Advanta and American Express. Employment Range and Annual Revenue Range of Targeted Businesses are the average number of employees and the average annual revenue range of the targeted business credit card users, respectively. Average Account Balance and Average Credit Limit of Business Card Users are the average account balance and the average credit limit on the business credit cards of account holders. Recourse indicates the type of pledge that the business credit card account holder provides to the business credit card issuer. Average Number of Businesses in Securitizations gives the average number of businesses underlying the pool of credit card receivables of the securitizations by Advanta and American Express, respectively. Program Description gives descriptions of the business card programs of Advanta and American Express, respectively. Panel B shows the distribution of small business credit card account balances as a percentage of the total number of customer accounts. Panel C shows the distribution of FICO scores as a percentage of the total credit card receivables in the securitizations. Panel D shows the geographical distribution of small business credit card receivables as a percentage of the total credit card receivables in the securitizations. Data are from Prospectus Supplements of the Advanta Business Card Master Trust (Series 2006-A1) and the American Express Issuance Trust (Series 2013-1).

	Advanta	American Express
	Panel A: Credit Card Program Characteristics	
Targeted Businesses	Small Business and Business Professionals	Small Business and Business Professionals
Employment Range of Targeted Businesses	<10 employees	<100 employees
Annual Revenue Range of Targeted Businesses	<$3 million	<$10 million
Average Account Balance of Business Card Users	$4,540	$2,683
Average Credit Limit of Business Card Users	$14,894	–
Recourse	Personal Guarantee	Personal Guarantee
Average Number of Businesses in Securitizations	1,054,097	1,574,294
Programm Description	"The sponsor offers business purpose credit cards to the small business market…" "The sponsor's sources for prospective cardholders include credit reporting agencies, lists from data compilers and customer lists from establishments that have small business customer bases."	"OPEN From American Express… referred to… as small business…" Most small businesses in the OPEN portfolio are companies with fewer than 100 employees and less than $10 million in annual revenue. FSB, as account owner for small business charge accounts, offers various small business charge card products, including but not limited to the Business Green Rewards Card, the Business Costco Charge Card, the Business Gold Rewards Card, the Executive Business Card, the Business Platinum Card, and the Business Centurion Card." "Small business cards are accepted at service establishments worldwide and are intended to be used solely for the purchase of merchandise and services related to the operation of small businesses."

(*Continued*)

Table 1—*Continued*

	Advanta	American Express
Panel B: Distribution of Credit Card Account Balances		
$0.00	40.47	43.65
$0.01–$5,000	33.90	51.23
$5,001–$10,000	12.67	3.07
$10,001–$20,000	10.13	1.33
$20,001–$25,000	1.63	0.31
>$25,000	1.20	0.41
Total	100.00	100.00
Panel C: Distribution of FICO Scores		
No FICO Score	0.00	0.51
<600	5.73	3.75
600–720	49.62	39.66
>720	44.65	56.08
Total	100.00	100.00
Panel D: Geographical Distribution of Credit Card Receivables.		
California	13.82	13.48
Florida	7.51	9.78
New York	6.69	16.79
Texas	7.23	8.65
New Jersey	3.58	7.22
Other	61.17	44.08
Total	100.00	100.00

management, bookkeeping, employee business card management, business purchase protections, and dispute resolution tools. In addition, cardholders earn rewards points on business purchases and business travel.

American Express business cards are restricted to business owners, and applicants need to prove business ownership or involvement.[13] In addition, small business owners sign as personal guarantors of the business credit card. Specifically, under the cardholder agreement, the entrepreneur and the business are jointly and severally liable for all transactions on the business credit card account.[14]

[13] Credit card applicants are required to provide information including the business name and address, the legal structure of the business, the number of employees, annual business revenues, the Federal tax ID number of the business, or the business owner's Social Security number if the business is a sole proprietorship. See https://thepointsguy.com/guide/amex-business-card-application/.

[14] "Liability for charges made on accounts is joint and several between the primary cardmember on the account and the company, which means that the applicable account owner may pursue payment for all charges from both the primary cardmember on the account and the company." See American Express Issuance Trust, Series 2008-1, Prospectus Supplement dated April 1, 2008 to Prospectus dated March 27, 2008.

Table I also provides summary statistics about American Express business cardholders from the prospectus supplement. Account balances typically range from zero to $10,000, with 51.23% of accounts having balances between zero and $5,000. Moreover, American Express cardholders have relatively high credit scores, with 56.08% of receivables originating from card owners with FICO scores of 720 or greater. The table further shows that the largest concentrations of American Express cardholders are in New York, California, Florida, Texas, and New Jersey.

D. The Securitization Data

We collect secondary market price data for all A, B, and C tranches from the securitizations of the Advanta Business Card Master Trust from August 2000 to December 2010 and the American Express Issuance Trust (AEIT) I/II from August 2005 to September 2018. The securitizations of both issuers were highly rated. In particular, using the Ferri, Liu, and Majnoni (2000) numerical mapping, the weighted-average rating across the tranches is AAA/AA+ for both issuers. The data are obtained primarily from the Bloomberg system ("Bloomberg"). For the AEIT II, however, we supplement the Bloomberg data with data obtained from Markit for the period from May 2015 to September 2018. We note that there is a small gap in the time series for American Express. The reason for this is that American Express replaced the AEIT with the AEITT II in March 2013. The final maturity date for the longest securitization issued by AEIT was September 2012, while the first securitization issued by AEITT II was in March 2013. Since we filter out tranches with maturities of less than six months, this results in missing observations from the 13-month period from March 2012 to April 2013.

To provide some assurance that the market is sufficiently liquid for the data to be reliable, we employ a number of metrics. First, we use data provided by the Securities Industry and Financial Markets Association (SIFMA) and the New York Federal Reserve Bank to compute the average annual trading turnover of credit card ABS. These sources imply that the total trading volume of credit card ABS during 2018 represented 44.11% of the outstanding notional amount of all credit card ABS. To put this into perspective, we note that the comparable turnover ratio for corporate bonds during 2018 was 95.58%. Thus, the intensity of trading activity in the credit card ABS market, while not quite as high, appears to be roughly on the same order of magnitude as that in the corporate bond market. Similar results hold throughout the sample period. Second, to rule out the possibility of stale pricing, we examine the frequency of changes in tranche prices in the data set. We find that more than 96% of the monthly price observations were based on updated and revised data. Furthermore, the average number of price revisions during a month for the various tranches ranged from about 6 to 20.[15] Third, we are able to obtain bid-ask

[15] Specifically, 96.29% of the month-end price observations for Advanta are based on daily data that changed during the month. For American Express, 97.30% of the month-end prices are based on updated intramonth data from 2005 to 2012.

spreads for the subsample of American Express tranches sourced from Markit. The average bid-ask spreads for the A, B, and C tranches are 0.098, 0.147, and 0.243 per $100 notional amount, respectively. These relatively small bid-ask spreads compare well with those in other active fixed income markets such as off-the-run Treasuries, agency bonds, and corporate bonds.[16] Finally, Markit provides a proprietary liquidity score for its pricing data based on the quality of the source, for example, actual trading prices, covers, two-way quotes, prices scraped from broker screens/phones, etc. Pricing data based exclusively on actual trades receives a score of one, while pricing data based exclusively on scraped broker data receives a score of five.[17] The weighted average Markit liquidity score for the subsample of American Express tranches is 2.39. Again, this compares well with the average Markit liquidity score of 2.30 for corporate bonds (see, e.g., Colvin and Mehta (2015)).

We also hand-collect data on attachment/detachment points and the floating coupons paid by each tranche from 424(b)(5) filings with the Securities and Exchange Commission (SEC) and servicer reports from 10-D filings with the SEC.[18] Each of the Advanta and American Express tranches in the sample pays a floating coupon that equals the one-month London Interbank Offered Rate (LIBOR) plus a fixed spread. For consistency throughout the analysis, we swap these floating coupon rates into their fixed coupon rate equivalents.

As discussed earlier, a portfolio consisting of all the A, B, and C tranches issued in a securitization is economically equivalent to a single-class pass-through securitization. Intuitively, this follows since by holding the portfolio of all tranches, an investor essentially "undoes" the original tranching of credit risk and receives the same total cash flows as if there were only a single tranche. Since we are interested in the entrepreneurial credit risk associated with the business credit cards, we make use of this equivalence by focusing specifically on the risk and return characteristics of the composite portfolio of all tranches (rather than on the individual tranches). For expositional convenience, we will refer to this composite portfolio simply as a "bond."

Figure 1 plots the time series of the bond prices. As can be seen, these prices were generally close to par except for the crisis period from 2008 to 2009, during which the bond prices were often less than $90. The minimum price of $79.068 occurred in March 2009. The maximum price of $100.854 occurred in August 2001.

Figure 2 plots the monthly time series of average credit spreads for the bonds. For perspective, the figure also plots the credit spread for AAA corporate bonds with comparable maturities. As can be seen, the credit spreads for the business credit card and AAA corporate bonds are similar during the precrisis and postcrisis periods, but are very different during the crisis period. Specifically, the average credit spread for business credit card bonds increased

[16] For example, Edwards, Harris, and Piwowar (2007) estimate the average bid-ask spread for a corporate bond for a trade size of $100,000 to be about 30 basis points ("bps"). Campbell, Li, and Im (2014) find that the average bid-ask spread for 10-year Treasury bonds and agency mortgage-backed securities to about 2 and 5 bps, respectively. Assuming a five-year duration, these values

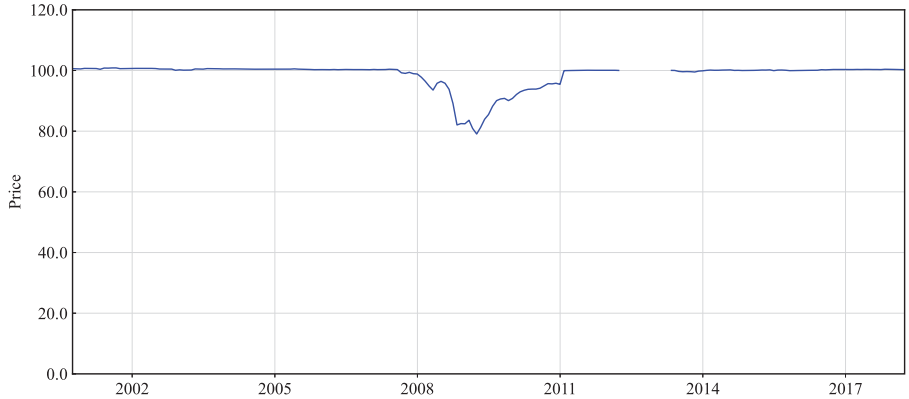

Figure 1. Small business credit card securitization prices. This figure plots the average prices of the Advanta and American Express credit card bonds. Prices are based on a $100 notional amount.
(Color figure can be viewed at wileyonlinelibrary.com)

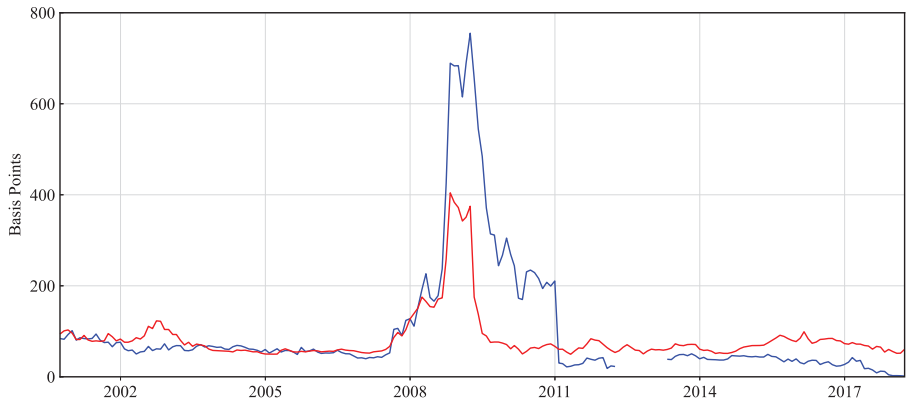

Figure 2. Credit spreads for the small business credit card securitizations. This figure plots the average credit spread for the Advanta and American Express credit card bonds (in blue) as well as the option-adjusted credit spread for the Bloomberg index of AAA-rated corporate bonds (in red).
(Color figure can be viewed at wileyonlinelibrary.com)

to roughly 750 bps during the peak of the financial crisis. In contrast, the credit spread for AAA bonds increased to only about 400 bps during the same period.

map into dollar bid-ask spreads for corporate bonds, Treasuries, and agency mortgage-backed securities of roughly 1.500, 0.100, and 0.250 per $100 notional amount, respectively.

[17] Section I of the Internet Appendix provides a more detailed discussion of Markit's liquidity score.

[18] On average, the Advanta A, B, and C tranches represent 83.0%, 9.5%, and 7.5% of the capital structure of the securitization, respectively. Similarly, the American Express A, B, and C tranches represent 92.6%, 2.9%, and 4.5% of the capital structure of the securitization, respectively.

Table II

Summary Statistics for the Credit Spreads of the Securitizations

This table reports summary statistics for the credit spreads of the business credit card bonds along with their average maturities. Credit spreads are measured relative to the yield on a Treasury bond with the identical maturity and coupon rate and are expressed in basis points. First Month and Last Month are the month and year of the first and last observation, respectively. Mean maturity is expressed in months. Statistics for the combined data are based on the average values across issuers for months in which data for more than one issuer are available. *N* denotes the number of monthly observations.

			Credit Spread						
Issuer	First Month	Last Month	Mean	Std Dev	Min	Med	Max	Mean Maturity	*N*
Advanta	Sep 2000	Dec 2010	180.15	219.70	39.11	68.10	966.36	38.18	124
American Express	Oct 2005	Mar 2018	83.92	130.77	1.69	43.59	750.50	33.74	138
Combined	Sep 2000	Mar 2018	101.62	138.52	1.69	54.68	755.07	34.14	199

This suggests that there may be fundamental differences in the nature of small business and large corporate credit risk, particularly during periods when financial markets experience major macroeconomic shocks. Table II provides summary statistics for the credit spreads.[19]

III. Identifying Small Business Equity Returns

In this section, we describe the approach we use to identify small business equity returns. We begin by applying the Merton (1974) structural credit model to debt claims on the assets of these entrepreneurial firms. We then invert the model to solve for the implied moments of equity positions in these firms.[20]

A. *The Merton Framework*

Using the Merton (1974) structural credit framework to model the relation between a firm's debt and equity has become standard in the literature. Key examples include Campbell and Taksler (2003), Vassalou and Xing (2004), Campello, Chen, and Zhang (2008), Campbell, Hilscher, and Szilagyi (2008), Schaefer and Strebulaev (2008), Coval, Jurek, and Stafford (2009), He and Xiong (2012), Diamond and He (2014), Culp, Nozawa, and Veronesi (2014), and Nagel and Purnanandam (2020).

The basic Merton (1974) framework assumes that the firm's debt consists of a single bond issue. In applying this framework to small entrepreneurial firms,

[19] Figures 1 and 2 show that there are discontinuities in the average price and credit spreads at the end of 2010 when the Advanta data end. The discontinuity, however, is a simple composition effect and has no impact on the results below since they are based on the return series for the individual issuers, which do not have discontinuities.

[20] Section III of the Internet Appendix provides a detailed discussion of the methodology used in identifying small business equity returns along with the full derivation of all key results.

however, it is important to account for some key features of small business capital structures. In particular, there may be few if any boundaries between the assets and liabilities of a small business and those of the entrepreneur. For example, small business owners typically sign as personal guarantors of business credit cards, and the entrepreneur and the business may be jointly and severally liable for all transactions on the account. As a result, the other types of debt such as bank loans and mortgages that entrepreneurs use to finance their businesses need to be considered. However, since some of this debt may be collateralized by specific assets, it can represent a senior claim on the business and personal assets of the entrepreneur. To model this feature, we extend the Merton (1974) framework to allow for the possibility that a firm may have both senior and junior debt claims.[21]

The basic Merton (1974) framework also assumes that the riskless interest rate is constant and that the term structure is flat. As it turns out, riskless interest rates declined by several hundred basis points during the 2000 to 2018 sample period covered by our data set. Thus, a significant part of the realized business credit card bond returns may be due to interest rate effects rather than the pricing of small business credit risk. To account for this possibility, we extend the Merton (1974) framework to a stochastic interest rate environment in which the Vasicek (1977) term structure model holds. This extension is a simple case of the stochastic interest rate option pricing model presented in Merton (1973) and Rabinovitch (1989).

Recall that the Merton (1974) framework uses an option-based approach to model the debt and equity claims on a firm's asset. Let V denote the value of the underlying assets of the firm, and let r denote the riskless rate. We assume that V and r follow the dynamics

$$dV = (r + \mu)\, V\, dt + \sigma\, V\, dZ_V, \tag{1}$$

$$dr = (\xi - \gamma\, r)\, dt + \eta\, dZ_r \tag{2}$$

under the objective measure, and

$$dV = r\, V\, dt + \sigma\, V\, dZ_V, \tag{3}$$

$$dr = (\alpha - \beta r)\, dt + \eta\, dZ_r \tag{4}$$

under the risk-neutral measure, where Z_V and Z_r are uncorrelated Brownian motions.

As discussed above, the basic Merton (1974) framework assumes that the firm's debt consists of a single bond issue. We extend the Merton (1974) framework by assuming that the capital structure of the firm consists of equity with

[21] We are grateful to the referee for this suggestion.

a market value of S and two debt issues. The first debt issue is a zero-coupon bond with face amount F, maturity date T, and market value A. This bond is a senior debt claim on the assets of the firm. Intuitively, we can think of this bond as representing bank loans or mortgages that are secured by, or have recourse to, the personal and business assets of the entrepreneur. The second debt issue is a junior zero-coupon bond with face amount $H - F$, maturity date T, and market value B. This bond can be viewed as representing unsecured debt claims such as business credit card debt and other forms of debt ranking pari passu with business credit card debt.

Following Merton (1974), the value at time T of the senior bond A_T, the junior bond B_T, and equity S_T can be expressed in terms of the cash flows received at time T,

$$A_T = V_T - \max\left(0, V_T - F\right), \tag{5}$$

$$B_T = \max\left(0, V_T - F\right) - \max\left(0, V_T - H\right), \tag{6}$$

$$S_T = \max\left(0, V_T - H\right). \tag{7}$$

The results in Merton (1974) can now be used to show that the current or time-zero values of these securities can be expressed as

$$A = V - C(F), \tag{8}$$

$$B = C(F) - C(H), \tag{9}$$

$$S = C(H), \tag{10}$$

where $C(F)$ and $C(H)$ are the values of call options on the assets of the firm with strike prices F and H, respectively. Let $F^* = F\ D(T)/V$ and $H^* = H\ D(T)/V$. The values of these call options in this Black-Scholes-Vasicek framework are given by

$$C(F) = V\,N(f) - F\,D(T)\,N(f - \phi), \tag{11}$$

$$C(H) = V\,N(h) - H\,D(T)\,N(h - \phi), \tag{12}$$

where $N(\cdot)$ is the cumulative standard normal distribution function,

$$f = \frac{-\ln\left(F^*\right) + \phi^2/2}{\phi}, \tag{13}$$

$$h = \frac{-\ln(H^*) + \phi^2/2}{\phi}, \tag{14}$$

and where

$$\phi^2 = \sigma^2 T + \frac{\eta^2}{\beta^2}\left(T - \frac{2}{\beta}\left(1 - e^{-\beta T}\right) + \frac{1}{2\beta}\left(1 - e^{-2\beta T}\right)\right), \tag{15}$$

$$= \sigma^2 T + \int_0^T \mathrm{Var}\left[\frac{dD(t)}{D(t)}\right] dt. \tag{16}$$

Finally, since A and B denote the market values of the senior and junior debt of the small business, A/V represents the fraction of the firm's capital structure consisting of secured/senior debt, and B/V represents the fraction consisting of unsecured/junior debt. From equations (11) and (12), these two leverage ratios can be expressed as

$$A/V = N(f) - F^* N(f - \phi), \tag{17}$$

$$B/V = N(h) - H^* N(h - \phi). \tag{18}$$

B. Small Business Debt and Equity Returns

Given these closed-form expressions, an application of Itô's Lemma allows us to solve for the moments of returns for the debt and equity components of the firm's capital structure under the objective measure. The expected excess return and the variance of returns for the junior bond can be expressed as

$$\mathrm{E}\left[\frac{dB}{B}\right] - r = \frac{\mu\left(N(f) - N(h)\right)}{B/V} + \frac{H^* N(h-\phi) - F^* N(f-\phi)}{B/V}\left[\mathrm{E}\left[\frac{dD(T)}{D(T)}\right] - r\right], \tag{19}$$

$$\mathrm{Var}\left[\frac{dB}{B}\right] = \left(\frac{\sigma\left(N(f) - N(h)\right)}{B/V}\right)^2 + \left(\frac{H^* N(h-\phi) - F^* N(f-\phi)}{B/V}\right)^2 \mathrm{Var}\left[\frac{dD(T)}{D(T)}\right]. \tag{20}$$

Similarly, the expected excess return for equity and the variance of equity returns can be expressed as

$$\mathrm{E}\left[\frac{dS}{S}\right] - r = \frac{\mu N(h)}{S/V} - \frac{H^{*}N(h-\phi)}{S/V}\left[E\left[\frac{dD(T)}{D(T)}\right] - r\right], \tag{21}$$

$$\mathrm{Var}\left[\frac{dS}{S}\right] = \left(\frac{\sigma N(h)}{S/V}\right)^{2} + \left(\frac{H^{*}N(h-\phi)}{S/V}\right)^{2}\mathrm{Var}\left[\frac{dD(T)}{D(T)}\right], \tag{22}$$

where $S/V = 1 - A/V - B/V$. Finally, the market beta β_S of the equity returns can be expressed as

$$\beta_S = \left(\frac{B/V}{S/V}\right)\left(\frac{N(h)}{N(f)-N(h)}\right)\beta_B + \left(\frac{1}{S/V}\right)\left(\frac{F^{*}N(h)\,N(f-\phi) - H^{*}N(f)\,N(h-\phi)}{N(f)-N(h)}\right)\beta_D, \tag{23}$$

where β_D and β_B are the market betas for the risk-free bond and the junior bond, respectively. Since the market beta of equity returns depends on the market betas of both the risk-free and the junior bond, it is not simply just a leverage-scaled version of the market beta of the junior bond.

C. Applying the Framework to Securitizations

The basic Merton (1974) framework applies to an individual firm. In reality, however, business credit card securitizations are typically based on a master trust portfolio that may include debt claims on more than one million individual small businesses. We therefore need to adapt the Merton (1974) framework to a securitization setting.

To do this, our approach is to allow the returns on entrepreneurial assets to be correlated across firms, and to then solve for the moments of portfolios of debt claims on these assets. Specifically, we assume that for any two firms, the correlation of changes in the Brownian motions driving the dynamics of V is simply ρ. This implies in turn that the correlation between the returns on the bonds for these firms is likewise ρ.

Assuming now that the master trust portfolio consists of an equally weighted portfolio of the junior bonds of a large number N of homogeneous small businesses, the expected excess return on the portfolio is still given by the expression in equation (19). The variance of returns on the portfolio, however, is given by

$$\frac{1}{N^2}\sum_{i=1}^{N}\mathrm{Var}\left[\frac{dB_i}{B_i}\right] + \frac{1}{N^2}\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N}\mathrm{Cov}\left[\frac{dB_i}{B_i}, \frac{dB_j}{B_j}\right]$$

$$= \frac{1}{N^2}\sum_{i=1}^{N}\mathrm{Var}\left[\frac{dB}{B}\right] + \frac{1}{N^2}\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N}\rho\,\mathrm{Var}\left[\frac{dB}{B}\right], \tag{24}$$

$$= \left(\frac{1}{N^2} + \rho\,\frac{N^2-N}{N^2}\right)\mathrm{Var}\left[\frac{dB}{B}\right], \tag{25}$$

which converges to

$$\rho\,\mathrm{Var}\left[\frac{dB}{B}\right] \tag{26}$$

as $N \to \infty$. Thus, given the correlation parameter ρ, we can directly infer the variance for a debt claim on an individual firm from the variance for the master trust portfolio underlying a securitization.[22]

IV. Model Calibration

In this section, we describe how we calibrate the Merton (1974) framework used to estimate small business equity returns. Although the framework has numerous parameters, a number of these can be substituted out using the observed moments of riskless bond returns. We therefore only require estimates of the leverage ratios A/V and B/V and of the mean and variance of monthly returns on the business credit card bonds to identify the moments of small business equity returns.[23]

A. *Estimating the Leverage Ratios*

Recall that A and B denote the market values of the senior and junior debt of a small business firm. Thus, A/V represents the fraction of the firm's capital structure consisting of senior/secured debt, and B/V represents the fraction consisting of junior/unsecured debt. To obtain estimates of the leverage ratios for small business firms, we use nonfinancial noncorporate business assets and liabilities data from the Federal Reserve's Financial Accounts of the United States Z.1 Release for the 2000 to 2018 sample period. The Financial Accounts data show that the amount of secured mortgage debt is consistently very close to the amount of nonmortgage debt (such as bank loans, trade payables, business credit card borrowing, etc.) throughout the sample period. Accordingly, we assume that $A = B$, which means that the total leverage of small business firms consists of a 50-50 mix of senior/secured and junior/unsecured debt.

[22] We are grateful to the referee for suggesting this approach. We acknowledge that the cross-issuer correlation we estimate may represent an upper bound on the cross-firm correlation because of the diversification within issuer portfolios.

[23] Internet Appendix Sections I and IV provide full details about the data sources and methodology used in the calibration process, respectively.

The Financial Accounts data further indicate that the ratio of total liabilities to total assets for nonfinancial noncorporate businesses varies from roughly 35% to 50% during the sample period, with an average value of 41.09%. This range of leverage ratios is consistent with those discussed in Heaton and Lucas (2000), Robb and Robinson (2014), and Kartashova (2014). The average total leverage ratio of 41.09% implies average values for A/V and B/V of approximately 20% during the sample period.[24]

To provide a broader perspective on the leverage ratios of small business firms, we also collect annual data from the Internal Revenue Service's Statistics of Income reports. This data set implies that the total leverage ratios for firms with assets in the range of \$1 to \$500,000, \$500,000 to \$1,000,000, and \$1,000,000 to \$5,000,000 are 47.0%, 46.8%, and 47.6%, respectively, for the 2000 to 2013 period. We also collect data from the SCF on the leverage ratios for self-employed individuals for the 1989 to 2019 period. The average ratio of total liabilities to total assets from this source is 45%. Finally, to ensure that the results are robust with respect to the assumptions about leverage, we also calibrate the model using a wide range of total leverage ratios. Estimates of the moments of small business equity returns are very consistent over this range.

B. Maximum Likelihood Estimation

As the second step in the calibration process, we estimate the mean and variance of securitization returns along with the correlation between the returns of different securitizations. Since the Merton (1974) framework is based on zero-coupon bonds, we compute monthly returns for the bonds as follows. For each issuer, we calculate the average yield across all bonds for each month.[25] Using these averages, we calculate the price of a zero-coupon bond as of the beginning and end of a month, where the bond has an initial maturity of 36 months. From these prices, we then compute the continuously compounded return for the bond for that month.

For much of the sample period, the data consist of monthly returns for the securitizations of only one issuer (either Advanta or American Express). For the 2005 to 2010 period, however, we have returns for the securitizations of both Advanta and American Express. An important advantage of having simultaneous returns for two securitizations is that we can identify the correlation between the returns of independent master trust portfolios of debt claims. This correlation can also be interpreted as the correlation between the returns on the debt claims of individual small businesses. Figure 3 plots the time series of the monthly returns for the two issuers. Note the similarity in the returns

[24] We acknowledge that we are implicitly assuming that the leverage ratios are relatively stationary over time. To show that this is a reasonable assumption, we plot the time-series patterns of leverage ratios in Figure IA.1 of the Internet Appendix.

[25] Using the average yield minimizes the effect of potential measurement error in the yields of individual bonds. Since monthly returns are determined primarily by the change in the yield over the month, the use of the average should not adversely affect the results as long as the change in the average yield mirrors the change in individual bond yields.

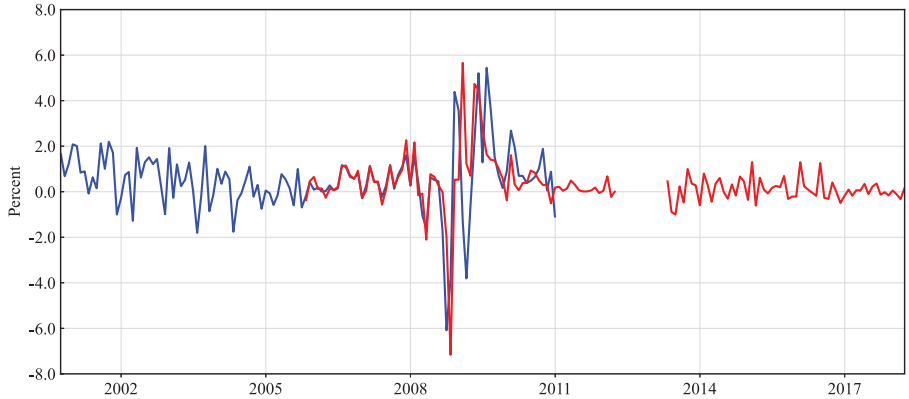

Figure 3. Small business credit card bond returns. This figure plots the monthly returns of the Advanta (in blue) and American Express (in red) credit card bonds.
(Color figure can be viewed at wileyonlinelibrary.com)

for the two issuers during the overlapping period from October 2005 to December 2010. This makes a strong case for viewing the two return series as drawn from a common distribution.

Given the unbalanced nature of the time-series data, however, it is important to estimate return moments using an approach that weights individual months in an appropriate way and that accounts for the correlation between the returns of the issuers. Accordingly, we use a maximum likelihood approach to estimate these return moments. Specifically, we assume that monthly securitization returns are normally distributed with mean m and variance v^2. Furthermore, we assume that the returns from different securitizations are jointly bivariate normally distributed with correlation coefficient ρ.

Let N_1 denote the number of months in the sample period for which the return for only one securitization is available. For notational convenience, we index these observations by $t = 1, 2, \ldots, N_1$. Let N_2 denote the number of months in the sample period for which returns for two securitizations are available. These observations are indexed by $t = N_1 + 1, N_1 + 2, \ldots, N_1 + N_2$. Let R_t denote the securitization return for months in which there is only one return observed, and let R_{1t} and R_{2t} denote the securitization returns in months for which two returns are observed. Given this notation, the log likelihood of the securitization return data is

$$\sum_{t=1}^{N_1} -\frac{1}{2}\ln\left(2\pi v^2\right) - \frac{1}{2}\left(\frac{R_t - m}{v}\right)^2 + \sum_{t=N_1+1}^{N_1+N_2} -\ln\left(2\pi v^2\sqrt{1-\rho^2}\right) - \frac{1}{2(1-\rho^2)}$$
$$\times\left[\left(\frac{R_{1t} - m}{v}\right)^2 - 2\rho\left(\frac{R_{1t} - m}{v}\right)\left(\frac{R_{2t} - m}{v}\right) + \left(\frac{R_{2t} - m}{v}\right)^2\right]. \tag{27}$$

Table III

Results from the Maximum Likelihood Estimation

This table reports results from the maximum likelihood estimation of the bond return moments. The monthly returns of the Advanta and American Express bonds are modeled as bivariate normally distributed with common mean m and standard deviation v. The correlation of the returns is ρ. The sample is monthly from September 2000 to March 2018.

		Parameter Estimate	Standard Error
Mean Return	m	0.003902	0.000842
Std Deviation of Returns	v	0.012658	0.000556
Correlation of Returns	ρ	0.390916	0.071177
Log-Likelihood			778.2548
Mean Log-Likelihood			3.9108
Number of Months			199
Number of Returns			262

Table IV

Baseline Calibration

This table reports the baseline calibrated values of the indicated parameters of the model and the annualized moments of the returns on the credit card bonds and the corresponding Treasury zero-coupon bonds.

Parameter		Calibrated Value
Senior Leverage Ratio	A/V	0.20000
Junior Leverage Ratio	B/V	0.20000
Equity Ratio	S/V	0.60000
Expected Bond Return	$E\,[\,dB/B\,]$	0.04683
Std Dev Bond Returns	$SD\,[\,dB/B\,]$	0.04385
Market Beta Bond Returns	β_B	0.03549
Expected Treasury Return	$E\,[\,dD/D\,]$	0.03672
Std Dev Treasury Returns	$SD\,[\,dD/D\,]$	0.02713
Market Beta Treasury Returns	β_D	−0.07737
Integral of Treasury Variance	$\int_0^T \mathrm{Var}\,[\,\mathrm{dD/D}\,]\,\mathrm{dt}$	0.00072
Average Treasury Bill Rate	r	0.01483
Maturity	T	3.00000
Correlation	ρ	0.39092

Maximum likelihood estimates of the moments of the securitization returns are now easily obtained by maximizing this log-likelihood function with respect to m, v^2, and ρ. Table III reports the results from this estimation.

C. The Baseline Calibration

Table IV presents the baseline calibration used in our analysis. In addition to the leverage ratios and maximum likelihood estimates of the moments of the securitization returns, the baseline calibration requires a number of other

inputs. In particular, we require estimates of the mean and variance of returns of a matched-maturity riskless Treasury bond over the sample period. We also use the average short-term one-month Treasury bill rate in calculating excess returns.

We use a calibrated value of T of three years, which corresponds closely to the average maturity of the securitizations during the sample period. We use the maximum likelihood estimate of ρ as the calibrated value of the correlation coefficient. The integral of the variance of returns on riskless bonds that appears in equation (16) is calibrated by calculating the variances for zero-coupon Treasury bonds with maturities ranging from one to T months and then summing the variances.

Finally, we compute the market betas β_D and β_B using the Fama-French market factor. We note that the estimate of β_D is negative during our sample period. This is consistent with the results reported in other studies such as He, Nagel, and Song (2022).

While our estimate of β_B is positive, the numerical value of 0.03549 is relatively small. The standard error for this estimate is 0.01750. One possible concern is whether illiquid or stale prices could be a factor leading to a downward bias in the estimated beta. As discussed previously, however, a number of metrics suggest that this market is reasonably liquid. To provide additional perspective, in Table IA.IV of the Internet Appendix we report the market betas for 10 maturity-sorted Treasury bond portfolios based on the data used in He, Kelly, and Manela (2017). As can be seen, the market betas for all 10 Treasury portfolios are negative and comparable in magnitude to our estimate. Table IA.IV also reports the market betas for 10 portfolios of corporate bonds sorted by credit spreads based on the data used in He, Kelly, and Manela (2017). The betas for the first three portfolios, which represent the highest-credit-quality corporate bonds, are either negative or comparable in magnitude to our calibrated value. Since the credit card securitizations had high credit ratings throughout the sample period, this suggests that the low beta may be due more to credit quality than to the effects of potential illiquidity.[26]

V. Small Business Equity Returns

In this section, we present the results for the estimated small business returns. For robustness, we also report results using a variety of alternative calibrated values for key model parameters.

A. *The Baseline Results*

Given the baseline calibration presented in Table IV, equations (17) through (20) now represent a system of four nonlinear equations that can be solved

[26] As a robustness check, we also estimate the beta by regressing the bond returns on the contemporaneous and first three lagged values of the Fama-French market factor, and then sum the coefficients. The resulting value of 0.0387 is very similar to our estimate. Furthermore, we note that the bond returns do not display the negative serial correlation pattern characteristic of illiquidity. Indeed, the bond returns are slightly positively serially correlated.

Table V

Baseline Small Business Equity Return Results

This table reports the small business equity return results using the baseline calibration of the model. The equity premium and standard deviation of equity returns are expressed as percentages.

	Baseline
Equity Premium	10.74
Standard Deviation	56.37
Sharpe Ratio	0.191
Market Beta	0.878
Fitted Parameters	
μ	0.0753
σ	0.3520
F^*	0.2003
H^*	0.4124

numerically for μ, σ, F^*, and H^*. From these values, we can then solve for the implied moments of small business equity returns. Table V reports the small business equity premium (expected excess return), standard deviation, Sharpe ratio, and market beta for the small business equity returns. For completeness, Table V also reports the implied model parameters μ, σ, F^*, and H^*.

A.1. The Small Business Equity Premium

As shown, the estimated small business equity premium over the 2000 to 2018 sample period is 10.74%. To put this into perspective, it may be useful to compare this estimate with previous estimates of the small business equity premium in the literature. As discussed earlier, the most relevant comparisons are the results presented in Moskowitz and Vissing-Jørgensen (2002) and Kartashova (2014), which are based on the small businesses underlying the SCF data set used in their analysis. In particular, the results reported in Moskowitz and Vissing-Jørgensen (2002, Table 4, Line 1) imply a small business equity premium of 12.21% (average annual return of 17.17% minus average riskless rate of 4.96%) for their 1990 to 1998 sample period. Similarly, the results in Kartashova (2014) imply a small business equity premium of 12.82% (average annual return of 16.50% minus average riskless rate of 3.68%) for their 1990 to 2010 sample period. These estimates of the small business equity premium are slightly higher but similar in magnitude to our estimate.

Following Moskowitz and Vissing-Jørgensen (2002) and Kartashova (2014), we can also compare our estimate of the small business equity premium with the public equity premium. The realized excess return on the CRSP value-weighted index over the 2000 to 2018 period is 5.24%. In contrast, the realized excess value-weighted return for the firms in the smallest size decile of the CRSP index over the same period is 19.95%. Thus, the realized small business equity premium is roughly twice as large as the public equity premium during

the sample period, but only about half as large as the premium on the smallest size decile of publicly traded firms.

Although the businesses reflected in our data set are much smaller than those studied in the private equity and venture capital literature, contrasting our results with those in that literature may provide useful additional perspective. Table IA.V shows the estimated excess return on private equity or venture capital reported in a set of nearly 40 studies. The average estimated excess return taken over all of these studies is 13.22%.

It is also useful to examine the robustness of the estimated small business equity premium to the riskless benchmark we adopt. If Treasury bonds trade at a premium because of their liquidity, this would widen the spread between the average realized returns on credit card bonds and Treasuries, resulting in a higher estimate of the small business equity premium. Thus, using an alternative risk-free benchmark that was less liquid (such as Treasury Inflation Protected Securities [TIPS] combined with inflation swaps as in Fleckenstein, Longstaff, and Lustig (2014)) might result in a more conservative estimate of the equity premium. To explore this possibility, we consider scenarios in which the liquidity of Treasury bonds widens the spread between the yields (and expected returns) of credit card bonds and Treasuries by 10, 20, or 30 bps.[27] After controlling for these liquidity effects (by adding 10, 20, or 30 bps to the calibrated value for the expected Treasury return in Table IV), the resulting estimates of the small business equity premium are 10.00%, 9.25%, and 8.50%, respectively. Thus, the results are fairly robust across a realistic range of assumptions about the size of the liquidity premia in the risk-free benchmark.[28]

A.2. Small Business Return Risk

The model also allows us to estimate the standard deviation of small business equity returns. Using the baseline calibration, the annualized standard deviation of small business equity returns is 56.37%. Since our algorithm accounts for the correlation between the firms in the loan portfolio underlying the small business credit card securitizations, this value can be interpreted as the volatility of returns of an individual small business. Our results confirm that small business entrepreneurial investment is a highly risky venture, particularly given the lack of diversification that entrepreneurs typically face in their personal portfolios (see, e.g., the discussion in Moskowitz and Vissing-Jørgensen (2002) and Vereshchagina and Hopenhayn (2009)).

While our estimate of the volatility of small business equity returns may appear large relative to the volatility of broader stock indexes, it is actually comparable to the average volatilities of many individual stocks. For example, the average volatility of monthly returns during the 2000 to 2018 period for the 3,715 firms listed on the NYSE, Amex, and NASDAQ exchanges is 45.70%.

[27] These values are consistent with those given in Fleckenstein, Longstaff, and Lustig (2014) for the precrisis period.

[28] We are grateful to the referee for raising this issue and suggesting this analysis.

The volatility of smaller firms, however, tends to be higher than that of larger firms. Accordingly, we regress the log of volatility (expressed as a percentage) on the log of firm size (market value expressed in millions of dollars). The estimated regression is

$$\begin{aligned} \ln \mathrm{Vol} &= 4.13650 - 0.07734 \ln \mathrm{Size}, \\ &\quad (0.0404) \quad (0.0062) \qquad\qquad R^2 = 0.0401 \end{aligned} \tag{28}$$

(standard errors in parentheses). Substituting into the fitted regression values for small business size of $250,000, $500,000, and $1,000,000 implies volatilities of 69.67%, 66.03%, and 62.58%, respectively. Thus, our estimate of the volatility of small business equity returns, while slightly lower, appears to be fairly consistent with the estimated volatility-size relation for individual NYSE and NASDAQ firms.[29] Our estimate is also consistent with Herskovic et al. (2016), who show that the volatility of publicly traded stocks in the smallest size quintile can range from 60% to 120%. Our estimate is also broadly consistent with deal-level estimates of the volatility of private equity and venture capital returns, which range from roughly 25% to more than 100%.[30]

A.3. The Sharpe Ratio

Table V shows that the estimated Sharpe ratio for a representative individual firm is 0.191. This value is broadly consistent with the results in Moskowitz and Vissing-Jørgensen (2002), who argue that the relatively low expected return on entrepreneurial investment relative to its overall risk poses a significant puzzle. We note that the Sharpe ratio is also comparable to that of an investment in an individual publicly traded firm. In particular, assuming an expected excess return of 5.24% (the realized excess return on the CRSP value-weighted index for the 2000 to 2018 period) and an average volatility of 45% results in a Sharpe ratio of 0.116 for an individual publicly traded firm.

Note that the Sharpe ratio for investing in an individual firm is smaller than investing in a diversified portfolio of firms. In particular, the Sharpe ratio for the CRSP value-weighted index for the 2000 to 2018 period is 0.273. The Sharpe ratio for the firms in the smallest size decile in the CRSP index for the same period is 0.431.

A.4. Small Business Systematic Risk

Finally, we solve for the implied market beta of small business equity returns using the baseline calibration. As shown in Table V, the estimated market beta

[29] We are grateful to the referee suggesting this analysis.

[30] These results are also consistent with Pflueger, Siriwardane, and Sundaram (2020), who find that high-volatility public firms are better proxies for private firms than low-volatility private firms.

is 0.878. We can use this estimate to provide a simple back-of-the-envelope estimate of the implied excess return from investing in a small business. Assuming a market equity premium of 5.24%, the single-factor Capital Asset Pricing Model (CAPM) implies that the small business equity premium is about $0.878 \times 0.0524 = 5.60\%$. Thus, the excess return appears to be on the order of $10.74 - 4.60 = 6.14\%$.

Given the important role that the market beta plays in understanding the economics of small business equity returns, it is useful to examine its sensitivity to potential measurement error in the estimated credit card bond beta. To do so, we solve for the small business equity beta using values for the bond beta of 0.00049 and 0.07049, which represent the endpoints of the 95% confidence interval for β_B. The resulting estimates of β_S are 0.583 and 1.177, respectively.

This confidence interval for the small business equity beta is comparable to the range of market betas for publicly traded NYSE, Amex, or NASDAQ firms (see, e.g., Fama and French (1997)). In contrast, Korteweg and Nagel (2016) report that estimates of venture capital market betas in the literature are consistently above one (see their footnote 4). Furthermore, Korteweg (2019) provides an extensive list of private equity and venture capital market beta estimates reported in that literature. The average market beta taken over all the studies surveyed by Korteweg (2019) is 1.55.

Finding that the market beta for small business equity returns is lower than that for publicly traded equity or for private equity and venture capital leaves us with some puzzles. Could this result be due to the much smaller size of the firms we study? This seems unlikely since it is well known that smaller-capitalization firms tend to have higher market betas than other publicly traded firms (see, e.g., Fama and French (1995), Lewellen and Nagel (2006), and Bali, Engle, and Murray (2016)). Thus, the size difference actually makes the result more puzzling. Alternatively, could the result be due to differences in capital structure? For example, could the result be explained by small businesses having lower leverage ratios than other firms?[31] Again, this seems unlikely since we show later that the market beta for small business equity actually decreases with leverage, and that publicly traded firms tend to have higher leverage than the firms we study. Thus, the impact of leverage goes in the wrong direction to provide an explanation. A third possibility might be systematic differences in the industry composition of the small businesses we study. This explanation is at least plausible given the information in Table I suggesting that business credit cards target entrepreneurs in specific types of industries such as personal services. Furthermore, the Small Business Credit Survey by the Federal Reserve (Kramer Mills, Terry, and Wiersch (2016)) suggests that smaller nonemployer firms tend to concentrate in the real estate, professional services, business support, consumer services, and nonmanufacturing goods production industries. Since data on the industry composition of

[31] For example, Dinlersoz et al. (2019) find that smaller private firms are less levered than larger private firms.

Table VI

Leverage Ratios and Small Business Equity Returns

This table reports small business equity return results for a range of assumptions about senior and junior leverage ratios, where the leverage ratios are expressed as a percentage of the capital structure. The equity premium and the standard deviation of equity returns are expressed as percentages.

	Senior/Junior Leverage Ratios				
	10/10	15/15	Baseline	25/25	30/30
Equity Premium	12.05	11.23	10.74	10.46	10.34
Standard Deviation	64.39	59.29	56.37	54.83	54.46
Sharpe Ratio	0.187	0.189	0.191	0.191	0.190
Market Beta	1.023	0.932	0.878	0.846	0.833
Fitted Parameters					
μ	0.1019	0.0868	0.0753	0.0658	0.0576
σ	0.5224	0.4256	0.3520	0.2910	0.2378
F^*	0.1005	0.1504	0.2003	0.2502	0.3001
H^*	0.2062	0.3093	0.4124	0.5157	0.6195

the Advanta and American Express receivables are not available, however, we leave this as an open question to be resolved in future research.[32]

B. Robustness Results

To ensure that the results are robust with respect to the key calibration inputs, we repeat the analysis using alternative sets of model parameters.

B.1. Leverage Assumptions

We begin by examining robustness of the results with respect to the senior/secured and junior/unsecured leverage ratio assumptions. In particular, Table VI reports results for senior/junior leverage ratios ranging from 10% to 30%. These values imply total small business leverage ratios over a broad spectrum ranging from 20% to 60%. As can be seen, the estimated equity premium is highly robust to the underlying leverage assumptions. Specifically, the equity premium estimate ranges from 12.05% for the 10% leverage scenario to 10.34% for the 30% leverage scenario.

It is interesting to observe that the equity premium actually declines slightly as the leverage ratio increases. We observe similar patterns for the estimated standard deviations and market betas. These patterns may seem counterintuitive at first, since we would generally expect equity risk to increase with firm leverage, leading to a higher equity premium. What is different in this Merton framework, however, is that there is a strong offsetting effect as leverage increases. In the Merton framework, equity can be viewed as a call option on the

[32] We are grateful to the referee for raising this issue and suggesting these potential explanations.

Table VII

Return Correlation and Small Business Equity Returns

This table reports small business equity return results for a range of assumptions about the correlation between the bond returns of different small businesses. The equity premium and the standard deviation of equity returns are expressed as percentages.

	Correlation Coefficient				
	0.20	0.30	Baseline	0.50	0.60
Equity Premium	8.69	9.84	10.74	11.73	12.59
Standard Deviation	61.64	58.37	56.37	54.58	53.29
Sharpe Ratio	0.141	0.169	0.191	0.215	0.236
Market Beta	0.648	0.776	0.878	0.988	1.083
Fitted Parameters					
μ	0.0630	0.0699	0.0753	0.0813	0.0864
σ	0.3902	0.3663	0.3520	0.3393	0.3302
F^*	0.2009	0.2005	0.2003	0.2002	0.2002
H^*	0.4199	0.4150	0.4124	0.4104	0.4090

assets of the firm. As leverage increases, however, the call becomes deeper out of the money, and its sensitivity or delta with respect to changes in the value of the firm's assets declines. In contrast, the debt of the firm—which can be viewed as including a put option on the firm's assets—becomes more sensitive to changes in the value of the firm's assets. Thus, the excess return for equity relative to that of junior/unsecured debt depends not only on the leverage ratio, but also on the relative deltas of equity and debt.[33]

B.2. Correlation Assumptions

As discussed in Section III.C, the correlation parameter ρ is an important factor in mapping from the observed variance of returns on the portfolio of debt claims underlying the securitization to the variance of returns on a debt claim for an individual firm. To explore the robustness of the results to this parameter, Table VII reports the results that obtain when the correlation coefficient varies from 0.20 to 0.60.

As can be seen, the estimated equity premium is fairly robust to the correlation parameter. In particular, the estimated equity premium varies from 8.69% to 12.59% over this wide range of correlation parameters. Similarly, the standard deviation of returns decreases with the correlation coefficient. This latter result is intuitive since the higher the correlation coefficient, the lower the effective diversification in the securitization portfolio and therefore the smaller the implied volatility of the firm's assets needs to be to match the volatility of the observed debt returns. Finally, the estimated market beta is an increasing

[33] Section VI.A of the Internet Appendix shows that the relative sensitivity or delta effect largely offsets the direct leverage effect, resulting in the robustness of the estimated equity premium to the leverage assumption.

Table VIII

Single-Debt-Class Leverage Ratios and Small Business Equity Returns

This table reports small business equity return results for a range of assumptions about the total leverage ratio for the special case when there is only a single class of debt in the capital structure. The total leverage ratio is expressed as a percentage of the capital structure. The equity premium and the standard deviation of equity returns are expressed as percentages.

	Total Leverage Ratios				
	20	30	40	50	60
Equity Premium	13.39	12.64	12.21	11.99	11.95
Standard Deviation	72.99	68.28	65.73	64.59	64.67
Sharpe Ratio	0.184	0.185	0.186	0.186	0.185
Market Beta	1.174	1.089	1.041	1.017	1.012
Fitted Parameters					
μ	0.1136	0.0981	0.0861	0.0760	0.0670
σ	0.5970	0.4977	0.4208	0.3561	0.2985
F^*	0.0000	0.0000	0.0000	0.0000	0.0000
H^*	0.2128	0.3199	0.4275	0.5358	0.6451

function of the correlation parameter. In particular, the market beta is 0.648 for a correlation of 0.20 and 1.083 for a correlation of 0.60.

B.3. The Senior/Junior Debt Assumption

In applying the Merton framework, we allow entrepreneurial firms to have both senior/secured debt, such as mortgages, and various types of junior/unsecured debt that ranks pari passu with unsecured credit card borrowing. To explore the implications of this assumption, we consider the alternative scenario in which all debt ranks pari passu with unsecured credit card debt and hence there is effectively only one class of debt. This scenario can be nested within our Merton framework by setting $F = 0$ and interpreting H as the total notional amount of the firm's debt. Table VIII reports the results based on total leverage ratios ranging from 20% to 60%.

Comparing the results in Table VIII to those in Table VI shows that allowing for both senior and junior debt in the capital structure of entrepreneurial firms reduces the estimated small business equity premium by about 1.00% to 1.50%. Intuitively, this follows since by modeling unsecured credit card debt as a junior and therefore riskier claim on the firm's assets, the Merton framework is able to match the observed moments of securitization returns at a lower implied risk level for the underlying assets of the firm. In turn, this leads to lower estimates of the small business equity premium as well as the standard deviation and market betas of small business equity returns.

B.4. Potential Tranche Mispricing

An extensive literature examines whether the credit risk of subprime mortgage-backed securities, collateralized debt obligations, and other types of

structured products may have been underestimated by market participants prior to the financial crisis of 2008 (see, e.g., Coval, Jurek, and Stafford (2009)). In light of this research, it is worth considering whether mispricing could play a role in the context of this study.

Figure 2 shows that the credit spreads on credit card bonds were relatively low prior to the financial crisis, reached high levels during the crisis, and then remained at somewhat elevated levels after the crisis. This pattern is consistent with the commonly expressed view in the industry that structured products were initially mispriced and that the financial crisis provided a "wake-up" call to the market. While providing a full analysis of this issue lies beyond the scope of the present paper, we note that there are other possible explanations for the pattern of credit spreads illustrated in Figure 2.

Specifically, a number of important recent papers such as Adrian, Etula, and Muir (2014) and He, Kelly, and Manela (2017) suggest that the frictions and constraints imposed on financial intermediaries by postcrisis regulation may play a central role in explaining the wider spreads observed during the postcrisis period. Fleckenstein and Longstaff (2022) present evidence that the major repricing of unsecured consumer credit card risk that occurred after the financial crisis is directly related to intermediary constraints. For example, they find that the consumer credit risk premium is significantly related to measures of intermediary constraints such as the intermediary leverage ratio of He, Kelly, and Manela (2017) and the turn-of-the-year measure of balance sheet costs of Fleckenstein and Longstaff (2020). In summary, the observed patterns in spreads may not necessarily be evidence of mispricing, but could be consistent with a rational response to the increased costs now faced by financial intermediaries.

VI. Applying the Model to Corporate Bonds

As an alternative way of validating the methodology, we apply the Merton (1974) framework to the returns of a broad index of investment-grade corporate bonds. This allows us to evaluate the methodology by comparing the resulting implications of the model for stock returns with the properties of historical stock returns.[34]

In applying the Merton (1974) model to corporate bonds, we use a calibration approach paralleling that used for small business credit card securitization returns. First, we search the Bloomberg system to identify a broad set of fixed-rate investment-grade nonfinancial U.S. corporate bonds. Corporate bond pricing data are available from this source for the January 2005 to December 2018 period. Since not all bonds have prices in the Bloomberg system (or have only a small number of observations), we exclude all bonds with fewer than 10 monthly observations. We also require that CRSP stock return data be available for each issuer. In cases with multiple bonds for an issuer, we include the most-actively-traded of these bonds in the sample. We then calculate monthly

[34] We are grateful to the referee for suggesting this approach.

returns for each bond. The final sample consists of monthly returns for the most-actively-traded bonds of 85 distinct issuers. The average maturity of the bonds in the sample is 20.5 years. We compute the average return, standard deviation of returns, and market beta for the equally weighted index of these bond returns. An important advantage of this approach is that since we know the composition of the index, we can compare the equity returns implied using the Merton (1974) model directly with the actual equity returns for the firms used in the calibration process.

We also compute the pairwise correlation of returns across these bonds and take the arithmetic average, which is 38.27%. This value is almost identical to the 39.09% correlation coefficient used in estimating small business equity returns. This value is also comparable to estimates of the average pairwise correlation coefficient among individual stock returns reported in the literature (see, e.g., Pollet and Wilson (2010)).

To calibrate the moments of the matched-maturity Treasury bond, we use the index of monthly returns for 20-year Treasury bonds from the well-known Ibbotson and Harrington (2021) Stocks, Bonds, Bills, and Inflation data set. Using this matched-maturity index facilitates comparison of corporate and Treasury bond return moments.

Next, we make the simplifying assumption that all types of corporate debt rank pari passu with corporate bonds, and apply the Merton framework using a single class of debt. We also use a total leverage ratio of 50%. This value is consistent with the estimates of total corporate leverage for nonfinancial firms reported in Fleckenstein, Longstaff, and Strebulav (2020). This value is also broadly consistent with estimates based on data from the Financial Accounts of the United States as reported in the Federal Reserve's Z.1 report of balance sheet statistics for nonfinancial corporate business (Table B.103). As in the previous section, we find that the results hold across a broad range of leverage assumptions. Finally, the other inputs and parameters used in applying the Merton (1974) framework to corporate bond returns are calibrated in a similar way as described earlier. Table IX reports the results and parameter values obtained from this calibration process.

Given this calibration, we use the Merton (1974) framework to solve for the implied return moments for the equity of a representative individual firm represented in the corporate bond index. Since our index of corporate bonds includes a broad set of issuers, we can interpret the representative firm as similar to the typical firm in an index such as the S&P 500 that includes the larger types of firms that have access to the corporate bond markets. Table X reports the resulting equity premium, standard deviation of returns, Sharpe ratio, and market beta for the representative firm.

As can be seen, the estimated equity premium is 6.17%. This implied value is identical to the realized equity premium of 6.17 during the sample period for the firms represented in the corporate bond index used in the calibration of the model. This estimate also compares well with estimates of the historical equity premium over long horizons, which are typically on the order of 6% to 7%. Furthermore, this estimate is also similar to the realized equity premia of

Table IX
Corporate Bond Calibration

This table reports the calibrated values of the indicated parameters of the model and the annualized moments of corporate and Treasury bond returns. The calibration period is 2005 through 2018.

Parameter		Calibrated Value
Senior Leverage Ratio	A/V	0.00000
Junior Leverage Ratio	B/V	0.50000
Equity Ratio	S/V	0.50000
Expected Bond Return	$E\,[\,dB/B\,]$	0.06404
Std Dev Bond Returns	$SD\,[\,dB/B\,]$	0.07891
Market Beta Bond Returns	β_B	0.09847
Expected Treasury Return	$E\,[\,dD/D\,]$	0.06000
Std Dev Treasury Returns	$SD\,[\,dD/D\,]$	0.11135
Market Beta Treasury Returns	β_D	−0.22165
Integral of Treasury Variance	$\int_0^T \mathrm{var}\,[\,\mathrm{dD/D}\,]\,\mathrm{dt}$	0.23894
Average Treasury Bill Rate	r	0.01200
Maturity	T	20.00000
Correlation	ρ	0.38269

Table X
Corporate Equity Return Results

This table reports the corporate equity return results inferred from the model. The equity premium and standard deviation of equity returns are expressed as percentages.

	Baseline
Equity Premium	6.17
Standard Deviation	38.48
Sharpe Ratio	0.160
Market Beta	0.862
Fitted Parameters	
μ	0.0569
σ	0.2461
F^*	0.0000
H^*	0.8417

7.75% for the S&P 500 and 8.11% for the CRSP value-weighted index over the 2005 to 2018 period, respectively.

Table X also shows that the estimated standard deviation of returns for the representative firm is 38.48%. This value is somewhat higher than the estimated average volatility of 30.01% for the firms represented in the corporate bond index, but is close to the 38% median volatility of NYSE, Amex, and NASDAQ stock returns during the 2005 to 2018 period.

Finally, Table X shows that the implied market beta for the equity of the representative firm in the index is 0.862. By way of comparison, the average

market beta for the corporate bond issuers used in the calibration of the model is 0.844. Note that the firms that typically raise capital through the corporate bond markets tend to be larger and more established than other firms. Thus, it is not surprising that the average market beta for these firms is less than one.

In summary, these results provide strong support for the methodology we use to identify small business equity returns. In particular, applying the extended Merton (1974) framework to a broad index of corporate bonds leads to realistic estimates of the equity premium, the average volatility of individual stock returns, and the market beta of corporate bond issuers. These results are also consistent with those in Schaefer and Strebulaev (2008), who show that the Merton (1974) framework is able to recover the relative volatilities of corporate bonds and stocks (hedge ratios).

VII. Conclusion

We introduce a new approach for estimating small business equity returns. Our approach uses the secondary market prices of small business credit card debt securitizations in conjunction with an extended version of the Merton (1974) structural credit modeling framework to infer small business equity returns.

Using this methodology, we estimate that the average excess return on small business equity is 10.74% during the 2000 to 2018 period, which is nearly twice the average excess return on public equity over the same period. We estimate that the volatility of equity returns for a representative individual small business is 56.37%, which is consistent with the volatility of smaller publicly traded firms. Finally, we estimate that the market beta of small business equity is 0.878. We show that our results are robust to a range of assumptions about small business leverage and other calibration parameters.

As an alternative way of validating the methodology, we apply it to the returns of corporate bonds and solve for the implied returns on the underlying equity of the bond issuers. The results provide strong support for the methodology. In particular, the results give an estimated equity premium of 6.17%, an average equity return volatility of an individual firm of 38.48%, and a market beta of 0.862. These values implied from corporate bond returns are very close to the values obtained using the actual equity returns for these issuers during the sample period.

Finally, an important advantage of our methodology is that it is scalable and can potentially be applied to other types of entrepreneurial, consumer, and household asset classes. In particular, our approach could be applied to other active securitization markets such as those for Small Business Administration loans, business loan syndications, student loans, auto loans, home equity lines of credit, and personal lines of credit.

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Supporting Information

Additional Supporting Information may be found in the online version of this article at the publisher's website:

Appendix S1: Internet Appendix.
Replication Code.