Managing Product-reusability under Supply Disruptions

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We model and analyze product-reusability (executed through refurbishing), in the presence of supply disruptions. In this setting, consumers could trade-in their used units and receive new ones when made available while a firm can attract consumers to do so through a trade-in fee. We develop a discrete-time Markov chain model to determine the degree of product-reusability, the price-premium on refurbished units during supply disruption, and the trade-in fee that the firm should offer, when it can refurbish trade-ins through an external refurbishment facility.

Our analysis of this model shows that increasing product's reusability through better product-design is beneficial as the likelihood of supply disruption increases to a certain extent. However, increasing reusability further when the disruption risk is high could hurt the firm's profit due to increased design-costs and decreased expected revenues. Besides, we show that as the supply risk increases and when sufficient number of customers trade-in, the firm exclusively chooses between the "*risk-absorbing strategy*" (i.e., absorb supply-risk by proactively increasing product-reusability) and the "*risk-transferring strategy*" (i.e., transfer supply-risk to customers reactively by increasing prices). The specific choice between these strategies is driven by the magnitude of supply-risk. Through our analysis, we obtain the firm's optimal decisions and examine how these decisions are influenced by various problem parameters. Finally, we obtain the optimal trade-in fee that the firm should offer to customers and show that it benefits the firm to encourage more trade-ins through a higher fee as the supply disruption risk increases.

Key words: Product-reusability, product-design, price-premium, supply-risk, refurbishing, product trade-ins

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1. Introduction

One of the Sustainable Development Goals (SDG) set by the United Nations in 2015 is "Responsible consumption and production." It aims to reduce and manage waste by properly reusing products and resources that have been returned, discarded, or disposed of at various stages of a product's life cycle. According to regulatory policies set by governments worldwide, the adoption of such reuse practices is on the rise in many industries. Recently, there has been an increased discussion about how such reuse practices can help businesses withstand supply disruptions like the Suez Canal blockade and the COVID-19 pandemic (UNEP-report 2020). An emerging perception is that product and resource reuse practices can provide a back-up supply for manufacturers in the event of a supply disruption. Thus, reuse practices are increasingly being seen as local value-retention strategies that can aid firms in coping with and recovering from disruptive events (EIONET-report 2021). In this paper, we analyze how a firm should manage its product-reusability decisions in the presence of supply disruptions. Specifically, we address the question of how much reusability should a firm design in its product (along the dimensions of modularity or disassemblability) that helps the firm in its reuse practices in the presence of supply-risk. Incorporating reusability into product design benefits the firm by lowering its unit refurbishment cost, but it incurs significant product-design costs to increase the ease of product's reusability.

Supply chains are not unfamiliar with disruptions. Natural disasters, sanctions (such as on imports and exports), employee unrest, political conflicts, sudden economic fluctuations, and other similar events are examples of common disruptions. During such disruptions, ensuring a secure supply for continued operations is the most common concern. A recent example of a supply disruption caused by the Coronavirus outbreak is the shortage of computer chips (commonly referred to as "chipageddon") that impacted a variety of products, including computers, automobiles, and mobile phones (Kelion 2021). The industries that make and use these products are still trying to recover from these shortages.

Existing risk mitigation strategies that address supply or sourcing disruptions, such as order splitting and multi-sourcing, are advantageous for securing supply (Anupindi and Akella 1993, Tomlin 2006). However, from a long-term perspective of resource availability and responsible consumption, reuse practices can aid in sustainably coping with disruptions of varying sizes. Reuse practices such as refurbishing and remanufacturing are advocated by expert practitioners and organizations as a way for businesses to better manage disruptions (Alicke et al. 2020, Fitzsimons 2020). During the pandemic, for instance, HP launched its remanufactured laptops alongside its mainstream products with the slogan "*We believe in reincarnation – at least when it applies to HP Notebooks*" (REMATEC 2020). In another instance, post-pandemic, Nike launched its refurbishment program in a number of stores (Salpini 2021).

Our own interviews with a few senior refurbishing and remanufacturing professionals from firms in different geographical regions (both developed and emerging economies) revealed the importance of reuse practices in general, and particularly during supply disruptions.[1] One of the professionals commented, *"refurbishing and remanufacturing surely is playing a critical role in the recent times as we have seen imports have been banned and there was a huge supply chain disruption going on [during the COVID pandemic]. Having said that companies still have not cracked on refurbishing or remanufacturing in big way yet. It's just a start and has a long way to go till refurbishing and*

[1] We approached multiple professionals around the globe. Three professionals (from the UK, Singapore, and India) responded to our interview invitations. They held positions of Director-remanufacturing, Group manager-advanced remanufacturing, and Senior research fellow at their respective organisations.

remanufacturing become an everyday norm. Recent supply chain disruptions has given it a big boost though." Another professional pointed, *"This [i.e., using reuse practices as a tool to address supply disruption] is true but only possible for a short to medium term fix."* These diverse opinions from practitioners strongly motivated us to investigate how a firm can manage product-reusability under supply disruptions, and potentially mitigate the impact of supply shortage risks that can result from such disruptive events.

Based on industrial practice and academic literature, we classify reuse practices broadly into two categories, namely, (1) *refurbishment*, and (2) *remanufacturing* & *recycling*. The primary distinction between these two categories is the product quality after processing of the used units returned (or traded-in) by consumers. The quality of a refurbished unit is inferior to the original's and consumers' willingness to pay for refurbished units decreases as a result. On the contrary, in remanufacturing and recycling, a used unit's quality is restored to that of a newly manufactured unit of the product (Thierry et al. 1995, Chen and Chen 2019). Nevertheless, refurbishing is one of the most widely used methods to reuse products and resources in an efficient manner. Additionally, post the COVID-19 pandemic, the market for refurbished products has grown considerably (Rawuf 2022, Allied-Market-Research 2023). Thus, the focus of this paper will be on refurbishment practices.

A carefully designed product helps a firm to extract greater value from used units during reuse operations (Gershenson et al. 2003, Souza 2017). This is typically achieved by enhancing key product characteristics such as "disassemblability", "modularity", and "reusability". For example, Dell's product-design methodology articulates the role of modular product-design as a means of providing "easy access and disassembly" in implementing their initiatives for circular economy (Shrivastava and Schafer 2018). Interactions with industry professionals revealed a significant demand for "design for reuse." One of our respondents commented, *"surely if a firm is going for refurbishing or remanufacturing process they need to think on the designs right away."* However, it is important to note that although product-design techniques such as modularization aid a firm's reuse operations, they are usually accompanied by substantial costs of the design processes. Thus, such costs should be carefully considered in any analysis of product-reusability under supply disruptions.

In this paper, we address the following questions when a firm faces possible supply disruptions:

1. *Is it always beneficial to increase product-reusability to facilitate refurbishing as supply disruptions get more likely?*

2. *Does it benefit the firm to charge a price-premium on refurbished units during supply disruptions?*

3. *What is the best strategy for the firm to jointly manage product-reusability and price-premium on refurbished units in the event of supply-risk?*

4. *What is the optimal trade-in fee that the firm should offer to motivate customers to trade-in used units when there is supply-risk?*

To the best of our knowledge, this is the first paper that studies product-reusability executed via refurbishment, in the presence of supply disruptions. We develop a stylized discrete-time Markov chain model to analyze this problem to answer the above mentioned questions. Our analysis provides managerial insights, which can assist firms in designing efficient products for reuse when there are supply disruptions.

In order to address the above questions, we consider a model setting where a firm is faced with potential supply disruption. In such setting, the firm can sell both new and refurbished units during the periods of normal supply. However, during the periods when the supply of key raw material is disrupted, the firm can only sell refurbished units that are obtained by refurbishing the trade-ins received from consumers, through an external refurbishing facility. In order to manage supply disruptions, the firm strategically decides on (i) the optimal product design (reusability level that facilitates refurbishing), (ii) the price-premium to charge on refurbished units during disruption, and (iii) the trade-in fee offered to customers. Following are some key findings from our analysis:

1. Increasing product-reusability as supply disruption risk increases could be beneficial up to a certain extent. However, it would hurt the firm's net profits when the disruption probability is high due to increased design costs and reduced revenues. This is true even when the firm is allowed to set a price-premium on its refurbished units during supply disruption.

2. As the likelihood of supply disruption increases and if sufficient number of customers trade-in their used product units, the firm exclusively chooses between a "*risk-absorbing strategy*" and a "*risk-transferring strategy.*" Under the proactive risk-absorbing strategy, the firm finds it beneficial to increase product's reusability through its design and reduce the price-premium charged on refurbished units during the supply disruption, thereby absorbing the increased risk. Whereas under the reactive risk-transferring strategy, the firm reduces its investment in product's reusability and increases the price-premium charged on refurbished units, thereby transferring the risk to consumers. The choice of the firm between these two strategies is driven by the magnitude of supply disruption risk apart from product design cost.

3. In order to maximize its profit, it is beneficial for the firm to offer a higher trade-in fee to its customers as the likelihood of the supply disruption increases. An increased trade-in fee motivates more customers to trade-in their units thereby enabling the firm to increase its sales of refurbished-units when supply is disrupted. Thus, when the supply-risk is high, the firm may find it optimal to offer a higher trade-in fee while choosing a low product-reusability to generate more sales of refurbished units, albeit at leaner margins. However, setting too high a fee will degrade the firm's profit due to a high trade-in fee expenditure.

This paper is organized as follows. In Section 2, we discuss the literature related to our work. In Section 3, we present the model preliminaries capturing the firm-setting and consumer-behavior. In Section 4, we develop and formulate the model that describes firm's execution of product's reusability via refurbishment. In Section 5, we analyze the model and draw insights on product-reusability, price-premium, and trade-in fee decisions of the firm. In Section 6, we present additional insights on the impact of key problem parameters on firm's decisions through a set of numerical studies. In the concluding Section 7, we summarize our work and provide future research directions. For the readers' convenience, we summarize all the assumptions in Appendix A. We provide model extensions in Appendix B and auxiliary results in Appendix C. All proofs are provided in Appendix D.

2. Literature Review

This paper falls in the intersection of "closed loop supply chains" (CLSC) and "supply disruption" domains.

CLSC can be defined as *the design, control, and operation of a system to maximize value creation over the entire life cycle of a product with dynamic recovery of value from different types and volumes of returns over time* (Guide Jr and Van Wassenhove 2009). We use the five-phase framework proposed by Guide Jr and Van Wassenhove (2009) to focus the literature review of this paper. Our study falls under phases four and five. Phase four considers CLSC issues across the entire product life-cycle (i.e., both forward and reverse logistics), such as product-design decisions, while phase five addresses market dynamics (such as cannibalization issues between new and refurbished units) and product valuation (i.e., remanufactured/refurbished product's prices). Since our paper pertains to product-design (for its reusability) and product pricing decisions of a firm, we discuss existing studies that have dealt with these issues in the CLSC literature. We introduce Table 1 that summarizes a few significant studies (in phases 4 and 5) in the CLSC domain and highlights the positioning of our work. Table 1 highlights that none of the studies in CLSC literature have focused on product-design and pricing decisions in the presence of supply disruption along the three primary dimensions: (i) product design, (ii) refurbished-product pricing during supply disruption, and (iii) trade-in fee, which we consider in our study.

Next, we discuss these key studies in the CLSC domain around the three primary dimensions considered in our study. The main idea behind product-design aspects in the CLSC literature is that a well-designed product can help firms to extract maximum value out of the returned product units. Mukhopadhyay and Setoputro (2005) study the value of product modularity for a firm implementing a return policy for its build-to-order product. They consider the trade-off between the cost of modularity and the salvage value of the returned and recycled product. Similarly, Wu

Paper	CLSC setting	SD*	PM*	RPP*	TF*
Guide Jr et al. (2003)	Remanufacturing	N	N	N	N
Mukhopadhyay and Setoputro (2005)	Recycling	N	Y	N	N
Ray et al. (2005)	Remanufacturing	N	N	N	Y
Ferrer and Swaminathan (2006)	Remanufacturing	N	N	N	N
Vorasayan and Ryan (2006)	Refurbishing	N	N	Y	N
Geyer et al. (2007)	Remanufacturing	N	N	N	N
Zikopoulos and Tagaras (2007)	Refurbishing	N	N	N	N
Wu (2012)	Remanufacturing	N	Y	N	N
Subramanian et al. (2013)	Refurbishing	N	Y	N	N
Wang et al. (2017)	Remanufacturing	N	N	N	N
Raz and Souza (2018)	Recycling	N	N	N	N
Alev et al. (2020)	Recycling	N	N	N	N
Borenich et al. (2020)	Refurbishing	N	N	Y	Y
Gui (2020)	Recycling	N	Y	N	N
Agrawal et al. (2021)	Leasing	N	Y	N	N
Rahmani et al. (2021)	Recycling	N	Y	N	N
Liu et al. (2022)	Remanufacturing	N	N	N	N
Hu et al. (2023)	Refurbishing	N	N	N	Y
Our Study	***Refurbishing***	***Y***	***Y***	***Y***	***Y***

Table 1 Positioning our research in the extant CLSC literature (Analytical studies). (*SD: Supply Disruption, *PM :Product Modularity, *RPP: Refurbished Product Price, TF*: Trade-in Fee)

(2012) considers the disassemblability of a product that helps a firm in reducing remanufacturing costs but requires upfront fixed cost. Subramanian et al. (2013) study a multi-product setting and evaluate the value of component commonality (CC) in a refurbishing setting. They explore the trade-off between higher production cost for the low-end product and lower refurbishment cost for the high-end product. More recent studies (Gui 2020, Rahmani et al. 2021) explore the role of product-design in the recycling context.

In the context of refurbishment operations, the pricing decision of refurbished product in relation to a new product becomes a key factor for a firm. A few studies in the CLSC literature based on refurbishment operations investigate the optimal price for refurbished products (Vorasayan and Ryan 2006, Borenich et al. 2020).

Next, product-return in the CLSC literature is a fundamental part of the problem settings considered in various studies (Guide Jr et al. 2003, Ferrer and Swaminathan 2006, Vorasayan and Ryan 2006, Geyer et al. 2007). However, managing product-design together with trade-ins during supply disruptions is not the focus of most of the papers in the CLSC literature. Some of the studies (Zikopoulos and Tagaras 2007, Wang et al. 2017) have focused on the impact of quality of returned units on the reuse practices of a firm. Likewise, a few studies have considered the role of trade-in fee in the reuse operations of a firm. Borenich et al. (2020) study the impact of unit refund offered by a manufacturer to the retailer in lieu of providing the used units on the manufacturer's refurbishing operations. However, the authors do not consider the unit refund as a decision variable. Alternatively, Ray et al. (2005) and Hu et al. (2023) consider a setting where the firm optimally decides the trade-in fee while managing its reuse operations.

In summary, it is important to note that none of the above listed papers in the CLSC literature related to product design, refurbished product pricing, and trade-in fees consider the risk and impact of supply disruptions on the above decisions.

Next, without elaborating much on the details of supply uncertainty literature, we define the context of supply disruption that we adopt in our paper. Broadly, the literature classifies uncertainty into two broad categories: (1) yield uncertainty and (2) supply disruption. We refer the reader to Fang and Shou (2015) and Kumar et al. (2018) for a comprehensive review of literature on yield uncertainty and supply disruption, respectively. Under yield uncertainty, the supply is not entirely stalled; however, the quantity supplied remains uncertain (Dada et al. 2007, Chintapalli 2021). On the other hand, under supply disruption, a supplier either supplies the entire order when there is no disruption or supplies nothing in case of a disruption (Tomlin 2006, Kumar et al. 2018, He et al. 2020). Out of the above two types of supply uncertainties, in this paper, we focus on the second type that usually have a low occurrence probability but are highly detrimental when they occur (Sheffi and Jr. 2005, Kleindorfer and Saad 2005).

3. Model Preliminaries and Consumer Behavior

In this section, we formulate the model to analyze a firm's product-reusability decision when it is executed through its refurbishment. To incorporate supply disruptions, we let the parameter $\alpha \in [0,1]$ denote the risk of supply disruption during a period. When supply is disrupted, the designated supplier fails to supply key raw-materials to the firm due to which the firm cannot produce new units; however, the firm can still refurbish used units that are traded in by its past customers.

Let N and D denote the state of normal operation when there is no supply disruption and the state when supply is disrupted, respectively. The discrete-time Markov chain with state-space $\{N,D\}$ that models the above context is given in Figure 1.

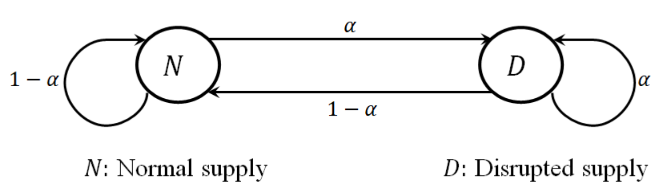


Figure 1 State transition diagram with state-space $\{N,D\}$ (N: Normal supply; D: Disrupted supply).

We let p denote the unit-price of new units. Likewise, let δp ($\delta \in [0,1]$) be the price of refurbished units during the normal supply state N. Hence, the unit-discount offered for refurbished units in state N is $(1-\delta)p$. Further, let $x \cdot \delta p$ denote the price of refurbished units during the disrupted supply state D. Thus, x denotes the premium-factor that the firm charges on refurbished units during supply disruption vis-à-vis when there is none.

We assume that the prices p and δp of new and refurbished units during the normal state are set based on various extraneous market factors that include firm's market strategy, product segmentation, competition, among others, and are thus exogenous to our model. Consequently, we primarily analyze the firm's decisions on:

1. *Price-premium* x on refurbished units during supply disruption D, which captures the differential price of refurbished units during state D vis-à-vis during normal state N.
2. *Product-reusability* θ, which reduces the cost of refurbishing through better product-design.
3. *Trade-in fee factor* v, which is the fraction of price p that firm refunds as "*trade-in fee*" (or discount) to the past customers who trade-in their units.

We make six assumptions in developing our model. These are discussed in the following two sections and are also summarized in Appendix A. We start with the following assumption about the refurbishment process:

ASSUMPTION 1 **(Refurbishing operations)**. *The firm accepts trade-ins of units that are newly purchased and used for 1 period. It delegates trade-ins to an external refurbishing facility that provides refurbished units in the next period. For parsimony, we normalize the unit transfer-price between the firm and the facility to 0. Furthermore, we assume that the external facility puts the leftover inventory for its own use without strategically storing it for the firm's future benefit.*

The above assumption first means that a customer who purchases a new unit at the beginning of a period can trade it in for another new one only at the end of the period, and no later. This simplifying assumption aligns with the practice that firms do not entertain very old units to be traded-in for brand-new ones due to the deteriorated residual value of the former. In Appendix B.2, we explain how the above assumption can be relaxed to extend our model to accommodate trade-ins of older units with age k for a return fee of $v_{\{k\}} \cdot p$, with additional notation. Second, the assumption helps in modelling the common business set-up where refurbishing is outsourced to external facilities that specialize in refurbishment due to the substantial reverse-logistics and capital costs involved in refurbishing operations[2] (Liu et al. 2018, Zhou et al. 2020). Such a refurbishing

[2] Examples include:
1. https://www.repairsvc.com/mobile-repair-services
2. https://www.cordongroup.com/en/our-services/refurbished-electronics/

facility uses its leftover inventory to earn revenues (through recycling) by avoiding holding and obsolescence costs. Thus, it does not strategically store inventory for the benefit of the firm. This also seems consistent with practice in which firms do not withhold inventories of products with high industry clock-speeds. We defer the case when trade-in inventory can be strategically stored to future research.

Let ξ_e denote the number of trade-ins and ξ_n denote the demand for new-units in a period of state N during which the firm can undertake new production. Likewise, let ξ_o^N and ξ_o^D denote the demands for refurbished units in states N and D, respectively. Using this information, Figure 2 provides the sequence of events during the transition from any period t to $t+1$.

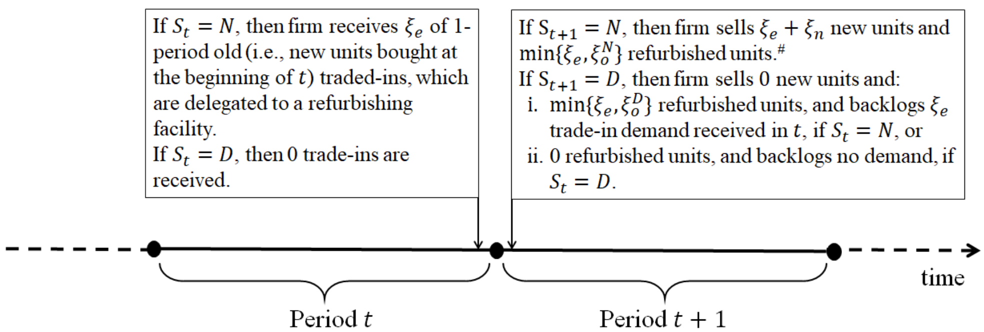


Figure 2 **Timeline of state transitions; S_t denotes the state in period t.**

Note that the ξ_e new-unit demand in period $t+1$ is due to the trade-ins in period t if $S_t = N$, whereas it is due to the ξ_e backlogged trade-ins from the latest $N \to D$ transition if $S_t = D$.

3.1. State Transitions

Now, using Figure 2, we will explain the 4 types of transitions that can occur from any period t to period $t+1$ in the Markov chain described in Figure 1. All the expressions for ξ_e, ξ_n, ξ_o^N, and ξ_o^D will be obtained later through a utilitarian model of the consumers.

1. $N \to N$ **transition.** During the transition $N \to N$ the firm will receive ξ_e trade-ins at the end of period t. Hence, in period $t+1$, the firm will make $\min\{\xi_o^N, \xi_e\}$ sales of refurbished units at unit price δp and a total of $\xi_n + \xi_e$ sales of new units at unit price p, as it meets the demand due to ξ_e trade-ins (in period t) and the demand ξ_n for new units in period $t+1$. We denote the stage-profit earned during this transition by g_{NN}.

2. $N \to D$ **transition.** Likewise, during a $N \to D$ transition, since the firm cannot undertake production in period $t+1$, it backlogs the demand ξ_e due trade-ins at the end of period t, and forgoes the demand ξ_n during period $t+1$. However, the firm makes $\min\{\xi_o^D, \xi_e\}$ sales of refurbished units in period $t+1$ at a unit price of $x \cdot \delta p$. We let g_{ND} be the stage-profit during this transition.

3. $D \to N$ **transition.** During the transition $D \to N$, the firm satisfies the demand ξ_e that was backlogged during its latest $N \to D$ transition along with ξ_n demand for new units at a unit-price p in period $t+1$ (because the firm is in N and can undertake production in $t+1$). However, since the firm cannot obtain any trade-ins in period t (because it was in state D in period t), no refurbished units are sold in period $t+1$. Let g_{DN} be the stage-profit during this transition.

4. $D \to D$ **transition.** This transition is straightforward and the firm cannot make any sales during this transition due to the lack of new production and trade-ins. It is important to note that the trade-ins received in previous periods cannot be used during $t+1$ because no leftover trade-in inventory is carried over across periods (Assumption 1). Therefore, the profit earned during this transition is $g_{DD} = 0$.

We determine all the stage-profits g_{NN} through g_{DD} corresponding to each of the above 4 transitions later in (13) to (16) in subsection 4.2, after discussing the consumers' behavior in subsections 3.2 and 3.3. Table 2 summarizes the demands, backlogs, and firm's stage-profits during each of the 4 transitions described above:

Table 2 State transition descriptions for Figure 1.

State in t	State in $t+1$	# trade-ins in t	New-unit sales in $t+1$	Refurb sales in $t+1$	New-unit backlog in $t+1$	New unit-price in $t+1$	Refurb unit-price in $t+1$	Stage profit
N	N	ξ_e	$\xi_n+\xi_e$	$\min\{\xi_o^N,\xi_e\}$	0	p	δp	g_{NN}
N	D	ξ_e	0	$\min\{\xi_o^D,\xi_e\}$	ξ_e	p	$x\cdot\delta p$	g_{ND}
D	N	0	$\xi_n+\xi_e$	0	0	p	δp	g_{DN}
D	D	0	0	0	0	p	$x\cdot\delta p$	g_{DD}

By using the prices and sales provided in Table 2, and by considering the two states N and D, we obtain the expected discounted profit in steady-state in Section 4.2.

Finally, we use the following notation for exposition: $f^{(t)} = \partial f/\partial t$ and $t^+ = \max\{0,t\}$, and suppress the arguments of functions to improve readability, unless required for clarity. For readers' convenience, Table 3 summarizes the notation used in this paper. Next, in the following subsections, we discuss the consumer's behavior during each state: N and D.

3.2. Consumer's Behavior in State N

In the absence of supply disruption, due to the simultaneous availability of new and refurbished units, *first-purchasers* (i.e., consumers who do not own the product before) choose to purchase one or neither of them. On the other hand, the *pre-owners* of the product who are eligible for a trade-in choose to do so depending on their valuation of the product and its future availability due to supply disruption. We now describe these decisions in the following subsections, in detail.

Variable	Definition
N	State of normal supply (see Figure 1).
D	State of disrupted supply (see Figure 1).
V, rV	Consumer's value for a newly manufactured and a refurbished or old unit, respectively.
$p, \delta p$	Price of a newly manufactured and a refurbished unit, respectively.
x	Premium-factor for refurbished units during supply disruption.
c	Unit-cost of manufacturing a new unit.
k	Product-design cost factor (i.e., cost of designing reusability in product).
θ	Product-reusability level.
ϕ	Efficacy of reusability; $c\phi(1-\theta)$ is the net refurbishment cost.
α	Probability of supply disruption.
v, vp	Trade-in fee factor and trade-in fee, respectively.
λ	Consumer's intrinsic trade-in propensity.
β	Discounting factor.
ξ_n	Demand for new units (only in state N).
ξ_o^N, ξ_o^D	Demand for refurbished units in states N and D, respectively.
ξ_e	Number of pre-owned units traded-in by consumers.
$\gamma(t)$	$= (1-t)/(1-\beta t)$.
ρ	$= \lambda/(1-\lambda)$.
$f^{(t)}$	$= \partial f/\partial t$
t^+	$= \max\{0, t\}$

Table 3 Summary of Notation

3.2.1. Purchase decision of first-purchasers. First-purchasers choose between purchasing a new and a refurbished unit. We assume that first-purchasers are boundedly rational when purchasing the product and account for the value derived from their purchase in the current period. (We relax this assumption in the Appendix B.1 and consider consumers who are more forward-looking.)

Let $V \sim U[0,1]$ denote a consumer's valuation for a new unit (Tirole and Jean 1988, Huang et al. 2013) and let their valuation for an old (or refurbished) unit be $r \cdot V$, where $r \in (0,1)$. Therefore, a consumer with valuation V will purchase a new unit when $V - p > \max\{0, rV - \delta p\}$ and purchases a refurbished unit when $rV - \delta p > \max\{0, V - p\}$. On the other hand, if $V < p$ and $rV < \delta p$, the consumer will not purchase the product. To avoid trivializing the problem, we assume the following:

Assumption 2 **(Non-zero demand for refurbished units)**. *The percentage reduction in the price of a refurbished unit over a new unit is higher than the relative reduction in a consumer's valuation; i.e.,* $1-\delta > 1-r \Leftrightarrow r > \delta$.

It is easy to observe that in the absence of Assumption 2 the demand for refurbished units never exists. Thus, by using the above assumption, we can obtain the demands for new and refurbished units in state N, which we denote by $\xi_n(p,\delta)$ and $\xi_o^N(p,\delta)$, respectively, as:

$$\xi_n(p,\delta) = 1 - \left(\frac{1-\delta}{1-r}\right) p, \text{ and} \tag{1}$$

$$\xi_o^N(p,\delta) = \left(\frac{1-\delta}{1-r} - \frac{\delta}{r}\right) p = \frac{p(r-\delta)}{r(1-r)}. \tag{2}$$

The above demand model is illustrated in Figure 3.

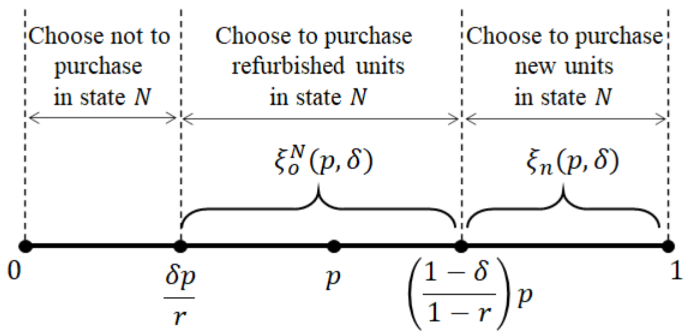


Figure 3 **Product demands in each period.**

3.2.2. Trade-in decision of pre-owners. The consumers who owned a new unit in the previous period are entitled to trade-in their unit for a brand-new one at a trade-in fee vp paid by the firm (see Assumption 1). Furthermore, since supply disruption is critical and affects product's availability, we assume that consumers account for product's future availability when deciding to trade-in.

Next, in practice, firms can backorder trade-in demand when new units are unavailable. Examples include mobile operators like T-mobile (T-Mobile 2022) and Verizon (Verizon 2021), while others like Apple (Oberoi 2021), Dell (Dell 2022), and BestBuy (BestBuy 2022) offer credit toward a new purchase (when available), which is equivalent to backordering the trade-in demand. Hence, we assume that the demand from consumers who opt for trade-in is backlogged in the case of supply disruption and is fulfilled whenever the supply resumes. Moreover, such consumers who trade-in will pay for the new unit only when it is delivered by the firm. Hence, consumers are cautious about product's availability when trading in their units, as explained below.

A consumer prefers to trade-in their owned unit whenever the net utility from exchanging it for a new one is higher than from using the owned, old unit; that is,

$$vp + (V-p)\left((1-\alpha) + (1-\alpha)\alpha\beta + (1-\alpha)\alpha^2\beta^2 + \cdots\right) > rV, \tag{3}$$

where $\beta \in (0,1)$ is the discount factor. The right-hand side of (3) is the consumer's utility by using the old unit that they own. The left hand side denotes the net utility that the consumer gains when they trade-in their owned unit for a new one, after accounting for uncertainty in product's availability due to supply-risk α. While the first term vp is the instantaneous trade-in fee that the firm pays to the consumer, the second term denotes the present value of net utility that is obtained from the new unit for which the old one has been traded-in.

Next, we let λ denote the exogenous probability that consumers are intrinsically inclined to trade-in their pre-owned units. While some trade-ins could be contractual off-lease, some other waste-averse consumers (see Desai et al. (2016)) choose to trade-in so long as they plan to use the product and are driven by sense of exclusivity and vogue for new units, more so, in a *waste-averse* (i.e., environmentally and socially responsible) manner (Stanley 2020, Kate 2020, Grauer 2021, Sapra et al. 2010). Nonetheless, some consumers may choose not to trade-in due to behavioral factors (Thaler 1985, Desai et al. 2016) or their plans to discontinue to use the product. Therefore, we let the parameter λ to capture this diversity in consumers' idiosyncratic trade-in behavior. It should be noted that a consumer may choose to use a new unit over multiple periods by trading-in their owned unit with probability λ in every period, as long as such trade-in is affordable according to (3). For convenience, we term λ as a consumer's *intrinsic trade-in propensity.*

Hence, by using (3) and the fact that product trade-ins are backlogged whenever there is a supply disruption (i.e., whenever transition occurs to state D in Figure 1), we can obtain the number of consumers requesting product trade-in in $(n+1)$th period of normal supply (i.e., after n transitions to state N in Figure 1)[3] as:

$$\sum_{i=1}^{n}\left\{\begin{matrix}\text{\# of consumers who purchased a new unit in the} \\ i\text{th transition to } N \text{ and continued to trade-in since then}\end{matrix}\right\}$$
$$=\sum_{i=1}^{n}\lambda^i\cdot\mathbb{P}\left[V>\left(\frac{p(1-\delta)}{1-r}\right), V(\gamma(\alpha)-r)>(\gamma(\alpha)-v)p\right],\text{ where we let }\gamma(t)=\frac{1-t}{1-\beta t}\in[0,1],\forall t\in[0,1], \tag{4}$$

for ease of exposition. We introduce the following result that explains consumers' trade-in behavior for any supply-risk α:

LEMMA 1. *Let $\gamma(\cdot)$ be as defined in (4) and let:*

$$v_1(\alpha)=\frac{[r-(1-p)\gamma(\alpha)]^+}{p}\ \textit{and} \tag{5}$$
$$v_2(\alpha)=\frac{r(1-\gamma(\alpha))}{1-r}+\frac{\delta(\gamma(\alpha)-r)}{1-r}. \tag{6}$$

It is easy to verify that $v_1(\alpha)\geqslant v_2(\alpha)\Leftrightarrow\alpha\geqslant\gamma(r)$.

The following consumers choose to trade-in depending on the magnitude α of the supply-risk:

1. *High-valuers trade-in if supply-risk α is low (i.e., $V>\max\left\{\frac{p(1-\delta)}{1-r},\frac{p(\gamma(\alpha)-v)}{\gamma(\alpha)-r}\right\}$ trade-in if $\alpha<\gamma(r)$). Moreover, all consumers choose to trade-in if $v\geqslant v_2(\alpha)$ whereas no consumer chooses to trade-in if $v<v_1(\alpha)$.*

[3] Note that if there are k disruption periods, i.e., k transitions to state D, during $n+k$ periods, the number of product trade-ins in the $(n+k+1)$th period, if we are in state N, is given by (4).

2. *Mid-valuers trade-in if supply-risk* α *is high (i.e.,* $\frac{p(1-\delta)}{1-r} \leqslant V \leqslant \frac{p(v-\gamma(\alpha))}{r-\gamma(\alpha)}$ *trade-in if* $\alpha \geqslant \gamma(r)$*). Moreover, all consumers choose to trade-in if* $v \geqslant v_1(\alpha)$ *whereas no consumer chooses to trade-in if* $v < v_2(\alpha)$.

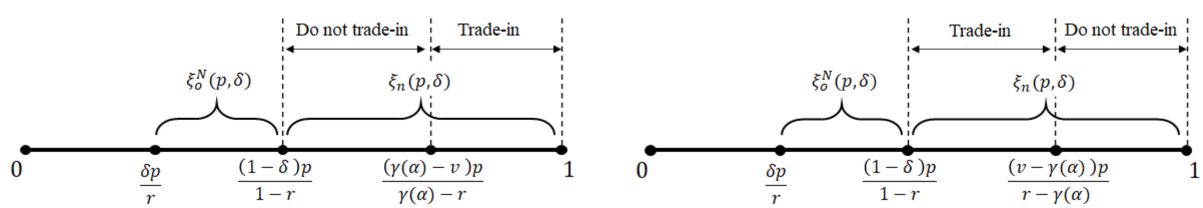
3. *Low-valuers (i.e.,* $V < \frac{p(1-\delta)}{1-r}$*) cannot trade-in as they do not own units eligible for trade-in.*

Furthermore, for any trade-in fee factor v*, the maximum supply-risk for which the consumers choose to trade-in is:*

$$\alpha_{max}(v) = \begin{cases} 0 & \text{if } v < \frac{r+p-1}{p}, \\ \min\{1, \gamma\left(\frac{r-pv}{1-p}\right)\} & \text{if } \frac{r+p-1}{p} \leqslant v \leqslant r, \text{ and} \\ \min\{1, \gamma\left(\frac{r(1-\delta)-v(1-r)}{r-\delta}\right)\} & \text{otherwise,} \end{cases} \tag{7}$$

where $\gamma(\cdot)$ *is as defined in* (4).[4]

When the supply-risk is low (i.e., $\alpha < \gamma(r)$), the likelihood of future unavailability of new units is also low. Therefore, consumers who value the product highly (i.e., $V > \max\left\{\frac{p(1-\delta)}{1-r}, \frac{p(\gamma(\alpha)-v)}{\gamma(\alpha)-r}\right\}$) more readily trade-in for new units, from which they derive higher utility. However, when the risk is high (i.e., $\alpha \geqslant \gamma(r)$), the high-valuers (i.e., $V > \frac{p(v-\gamma(\alpha))}{r-\gamma(\alpha)}$) refrain from trading in due to the increased product's unavailability in the future. On the other hand, the mid-valuers (i.e., $\frac{p(1-\delta)}{1-r} \leqslant V \leqslant \frac{p(v-\gamma(\alpha))}{r-\gamma(\alpha)}$) who can tolerate the potential risk of not possessing the product due to higher supply-risk choose to trade-in their units for new ones. Finally, the lemma shows that no consumer chooses to trade-in their unit when the supply-risk is sufficiently high (i.e., $\alpha > \alpha_{max}(v)$) due to the low likelihood of the product's future availability. Figures 4a and 4b graphically summarize the trade-in behavior of consumers when supply-risk is low (i.e., $\alpha \leqslant \gamma(r)$) and high (i.e., $\alpha \geqslant \gamma(r)$), respectively.



(a) High valuers trade-in when α is low (i.e., $\alpha \leqslant \gamma(r)$). (b) Mid valuers trade-in when α is high (i.e., $\alpha > \gamma(r)$).

Figure 4 **Impact of supply disruption on trading-in behavior.**

Thus, by using Figures 4a and 4b, we can obtain the trade-in volume $\xi_e(p, v, \delta)$ in steady-state as:

[4] Note that the inverse-functions of $v_1(\cdot)$ and $v_2(\cdot)$ are $v_1^{-1}(v) = \gamma\left(\frac{r-pv}{1-p}\right)$ and $v_2^{-1}(v) = \gamma\left(\frac{r(1-\delta)-v(1-r)}{r-\delta}\right)$, which define $\alpha_{max}(v)$; therefore, $\alpha_{max}(v)$ can be rewritten as $\alpha_{max}(v) = \min\{1, v_2^{-1}(v), [v_1^{-1}(v)]^+\}$

1. **If supply-risk is low** (i.e., $\alpha \leqslant \gamma(r)$, or equivalently $r \leqslant \gamma(\alpha)$):

$$\xi_e(p,v,\delta) = \rho \cdot \mathbb{P}\left[V > \max\left\{\frac{p(1-\delta)}{1-r}, \frac{(\gamma(\alpha)-v)p}{\gamma(\alpha)-r}\right\}\right] = \begin{cases} 0, & \text{if } v \leqslant v_1(\alpha), \\ \rho\left(1 - \frac{(\gamma(\alpha)-v)p}{\gamma(\alpha)-r}\right), & \text{if } v_1(\alpha) < v \leqslant v_2(\alpha) (\leqslant r), \text{ and} \\ \rho\,\xi_n(p,\delta), & \text{otherwise, and} \end{cases} \tag{8}$$

2. **If supply-risk is high** (i.e., $\alpha \geqslant \gamma(r)$, or equivalently $r \geqslant \gamma(\alpha)$):

$$\xi_e(p,v,\delta) = \rho \cdot \mathbb{P}\left[\frac{p(1-\delta)}{1-r} \leqslant V \leqslant \frac{(v-\gamma(\alpha))p}{r-\gamma(\alpha)}\right] = \begin{cases} 0, & \text{if } v \leqslant v_2(\alpha), \\ \rho\, p\left(\frac{v-\gamma(\alpha)}{r-\gamma(\alpha)} - \frac{1-\delta}{1-r}\right), & \text{if } (r \leqslant) v_2(\alpha) < v \leqslant v_1(\alpha), \text{ and} \\ \rho\,\xi_n(p,\delta), & \text{otherwise,} \end{cases} \tag{9}$$

where $\rho = \frac{\lambda}{1-\lambda}$, and $v_1(\alpha)$ and $v_2(\alpha)$ are given in (5) and (6), respectively.

Next, since we want to evaluate the role of trade-ins and the impact of supply-risk on them, we make the following assumptions in our analysis so that there are non-zero trade-ins:[5]

Assumption 3 **(Viability of trade-in)**. *The supply-risk is not too high so that at least some consumers choose to trade-in their old units; i.e., $\alpha \leqslant \alpha_{max}(v)$, which is defined in Lemma 1.*

Figure 5 pictorially summarizes the above discussion on consumers' trade-in decisions for different values of supply-risk α and return-fee v. It shows that all consumers choose to trade-in when the trade-in fee is high (i.e., $v \geqslant \max\{v_1(\alpha), v_2(\alpha)\}$), a few trade-in if v is moderate (i.e., $\min\{v_1(\alpha), v_2(\alpha)\} \leqslant v \leqslant \max\{v_1(\alpha), v_2(\alpha)\}$), and none trades-in if v is low (i.e., $v < \min\{v_1(\alpha), v_2(\alpha)\}$, which is equivalent to $\alpha \geqslant \alpha_{max}(v)$).

3.3. Consumer's Behavior in State D

In state D, the firm cannot produce new units due to the disrupted supply of raw materials. However, it can still refurbish the trade-ins, so long as they are available. More importantly, the firm can price the refurbished units in state D differently from those in state N keeping in view the product's scarcity caused due to supply disruption. We let x denote the factor by which the firm changes the price of refurbished units so that the selling price of these units is $x \cdot \delta p$ in state D as opposed to δp in state N. As only refurbished units are available in state D, a consumer will purchase it as long as $rV - x\delta p \geqslant 0$. Thus, the total demand for refurbished units in state D is:

$$\xi_o^D(p,x,\delta) = \left(1 - \frac{x\delta p}{r}\right). \tag{10}$$

[5] In practice, the firm will set v sufficiently high in order to have non-zero trade-ins so that Assumption 3 holds true. This is also evident from the fact that $\alpha'_{max}(v) \geqslant 0$.

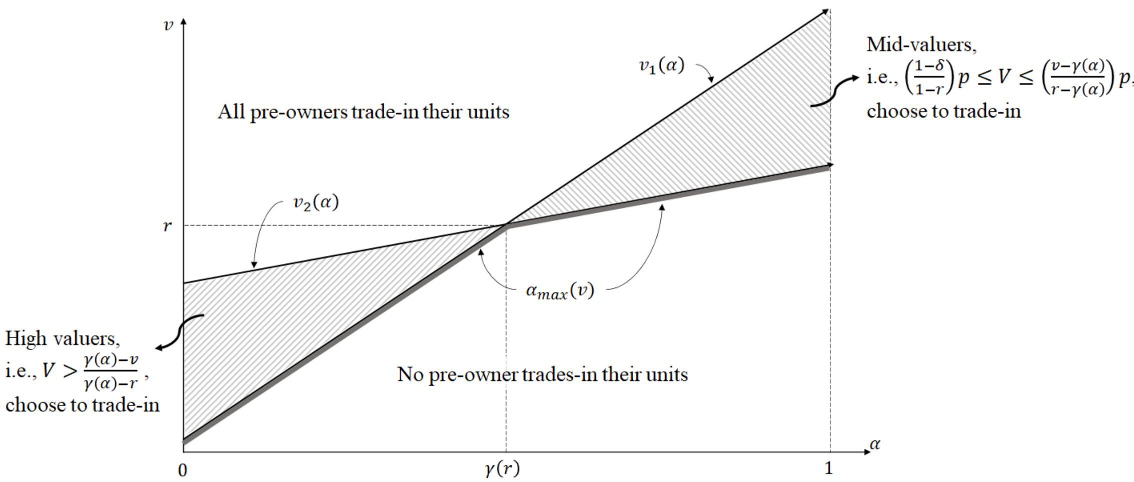


Figure 5 Consumers' trade-in behavior for different values of supply-risk α and trade-in fee v.

4. Model for Product Refurbishment

In this section, we present the model for product refurbishment. Here, we assume that the unit production cost is lower for a refurbished product than for a product that is entirely manufactured from raw materials. The lower cost of a refurbished unit can be attributed to various factors. The most important of these is reusing modular components reduces the machine and labor time in processing a refurbished unit.

Refurbishing a product entails "*involves anything from running a few simple tests to undertaking a thorough clean-up and rebuild of the product*" (Canon 2021); thus, the unit-cost of refurbishing is less than that of manufacturing. Therefore, we let c and ϕc, where $\phi \in (0,1)$, denote the unit-costs of manufacturing and refurbishing, respectively. Additionally, we let $\theta \in [0,1]$ denote the "*product-reusability*" (i.e., the ability of product's design to facilitate easy refurbishability) so that the net refurbishment cost is $\phi c(1-\theta)$. Clearly, the higher the reusability θ of the product, the lower is its net refurbishment cost $\phi c(1-\theta)$. Here, we term the parameter ϕ as the *efficacy of reusability* because it "moderates" the extent to which the reusability level θ can result in cost savings. Next, we note that incorporating better reusability θ in product-design is not free but entails some cost. We assume that such product-design cost is convex increasing in θ, indicating that it is increasingly costlier to enhance product-reusability through its modular design, an assumption that is widely used in the literature (Rahmani et al. 2020). For functional convenience we assume that this cost is given by $k\theta^2$. Finally, we make the following non-restrictive assumption:

ASSUMPTION 4 **(Feasibility of positive margin)**. *The highest valuer (i.e., consumer with $V = 1$) always values a refurbished unit more than its refurbishment cost; i.e., $r \geqslant c\phi$.*

It is easy to note that no consumer values a refurbished unit higher than its refurbishment cost if Assumption 4 does not hold true. Thus, the firm will never be able to sell a refurbished unit at a positive margin, which can dissuade the firm from selling refurbished units and which is unlikely to hold true in practice.[6]

4.1. Firm's Decisions

Keeping in view the consumer's behavior discussed in Section 3, the firm should decide its product-reusability θ and the premium-factor x for refurbished units during supply disruption (i.e., in state D) in order to maximize its profit, for any p, v, and δ. As explained in Section 3, the new-unit price p and discount factor δ during normal periods (i.e., in state N) are largely determined by exogenous market factors and firm's strategic pricing policies. Hence, in order to focus on (i) the firm's strategic decision of product-reusability θ and (ii) its differential pricing x of refurbished units during supply disruption, we let p, v, and δ be exogenous to our analysis.[7]

4.2. Firm's State-dependent Profits

Let $\pi_N(\theta, x; p, v, \delta)$ and $\pi_D(\theta, x; p, v, \delta)$ denote the firm's steady state profits when in states N and D, respectively. By using the state transitions depicted in Figure 1, we can obtain the expressions of π_N and π_D as follows:

$$\begin{aligned}\pi_N(\theta,x;p,v,\delta) &= (1-\alpha)\cdot[g_{NN}(\theta,p,v,\delta)+\beta\cdot\pi_N(\theta,x;p,v,\delta)] \\ &\qquad + \alpha\cdot[g_{ND}(\theta,x,p,v,\delta)+\beta\cdot\pi_D(\theta,x;p,v,\delta)] \text{ and,} \qquad (11)\\ \pi_D(\theta,x;p,v,\delta) &= (1-\alpha)\cdot[g_{DN}(p,v,\delta)+\beta\cdot\pi_N(\theta,x;p,v,\delta)]+\alpha\cdot[g_{DD}+\beta\pi_D(\theta,x;p,v,\delta)], \qquad (12)\end{aligned}$$

where

$$\begin{aligned}g_{NN}(\theta,p,v,\delta) &= (\delta p-\phi c(1-\theta))\cdot\min\{\xi_o^N(p,\delta),\xi_e(p,v,\delta)\}-vp\cdot\xi_e(p,v,\delta) \qquad (13)\\ &\qquad + (p-c)\cdot(\xi_e(p,v,\delta)+\xi_n(p,\delta)),\\ g_{ND}(\theta,x,p,v,\delta) &= (x\delta p-\phi c(1-\theta))\cdot\min\{\xi_o^D(p,x,\delta),\xi_e(p,v,\delta)\}-vp\cdot\xi_e(p,v,\delta), \qquad (14)\\ g_{DN}(p,v,\delta) &= (p-c)\cdot(\xi_e(p,v,\delta)+\xi_n(p,\delta)), \text{ and} \qquad (15)\\ g_{DD} &= 0. \qquad (16)\end{aligned}$$

If the current state is N (i.e., there is no supply disruption), then the future state can be either N or D with probabilities $(1-\alpha)$ and α, respectively. In a state transition from N to N, the firm can sell both refurbished and newly manufactured units, which includes trade-ins. Therefore, the profit earned by the firm, which we denote by g_{NN}, is the sum of $(\delta p - \phi c(1-\theta))\cdot\min\{\xi_o^N, \xi_e\}$, which

[6] Note that this assumption is non-restrictive because if a product-reusability level of at least $\theta_{min} > 0$ is required to ensure $r > c\phi(1-\theta_{min})$, then the range of feasible reusability levels can be restricted to $\theta \in [\theta_{min}, 1]$ instead of $\theta \in [0, 1]$.

[7] We endogenize the trade-in fee factor v later in the paper.

is the profit earned through refurbished products, and $(p-c)\cdot(\xi_e+\xi_n)$, which is the profit from new units, less $vp\cdot\xi_e$, which is the fee paid for trade-ins.[8] This explains (13). Likewise, a transition from N to D, will allow the firm to sell only refurbished units at a differential price without any new production, so that the firm's profit is $(x\delta p-\phi c(1-\theta))\cdot\min\{\xi_o^D,\xi_e\}$. The demand for new units due to trade-ins is backlogged and will be satisfied when the next transition occurs from D to N, which explains (14). By using the probabilities to weight the above profit expressions, we obtain π_N as given in (11).

Similarly, then the system is in state D, transitioning again to D that happens with probability α will fetch no profit (i.e., $g_{DD}=0$). However, transitioning from D to N with probability $(1-\alpha)$ will enable firm to earn profit only through newly produced units, which includes satisfying the backlogged demand during the latest transition from N to D, so that the profit earned is $(p-c)\cdot(\xi_e+\xi_n)$, as shown in (15). Therefore, the expected profit at state D is given by $\pi_D(p,\delta)$, which is defined in (12). Next, by solving (11) and (12) to obtain the conditional profits π_N and π_D and by using the steady-state probabilities of states N and D, we can obtain the firm's expected profit as:

$$\begin{aligned}\pi(\theta,x;p,v,\delta) &= (1-\alpha)\cdot\pi_N(p,\delta)+\alpha\cdot\pi_D(p,\delta)-k\theta^2\\ &= \begin{pmatrix}(p-c)\cdot(\xi_n+\xi_e)-vp\cdot\xi_e+\alpha(x\delta p-\phi c(1-\theta))\cdot\min\{\xi_e,\xi_o^D\}\\ +(1-\alpha)(\delta p-\phi c(1-\theta))\cdot\min\{\xi_e,\xi_o^N\}\end{pmatrix}\cdot\left(\frac{1-\alpha}{1-\beta}\right)-k\theta^2.\end{aligned} \tag{17}$$

Then, the firm should decide the optimal level of refurbishability θ, which should be designed in the product, and the premium-factor x for refurbished units in state D that maximizes its expected profit that is defined in (17). Thus, the firm's optimization problem for any p, δ, and v is given by:

$$\max_{x,\theta} \quad \pi(\theta,x;p,v,\delta) \tag{18}$$

5. Product-reusability, Premium-factor, and Trade-in fee Decisions

In this section, we first discuss the firm's decisions on pricing its refurbished units during supply disruption (i.e., the premium-factor x for refurbished units) and product-reusability θ, by solving (18) for any p, v, and δ values. Later, we also determine the optimal trade-in fee the firm should set. Additionally, we discuss the impact of various problem-parameters on these decisions to draw some managerial insights.

[8] Note that if the unit-revenue from the refurbishing facility is R, then the net trade-in fee paid by the firm is $(vp-R)\cdot\xi_e$ and the net refurbishment cost is $[R+c\phi(1-\theta)]\cdot\min\{\xi_e,\xi_o^i\}$, for $i\in\{N,D\}$; however, we normalize the transfer-price R to 0 as explained in Assumption 1.

5.1. Optimal Premium-factor

We start by obtaining the optimal premium-factor $x^*(\theta)$ the firm offers for refurbished units, for any product-reusability level θ. One should note that the product-reusability level θ is a strategic decision that is decided at the time of designing the product. On the other hand, the premium-factor x is a more tactical decision that can be made at the beginning of a period when supply is disrupted. Hence, examining the optimal premium-factor decision $x^*(\theta)$ for any given reusability level θ can provide valuable insights on managing refurbished-unit prices of already existing products with a predetermined reusability level θ.

It is easy to observe from (17) that $\pi(\theta, x; p, v, \delta)$ is concave in x (for any θ). Hence, the first order condition defines the optimal premium-factor $x^*(\theta)$, which is explained in the following result:

Lemma 2. *Let $x_0 = \frac{r(1-\xi_e)}{p\delta}$, so that $\xi_e \geqslant \xi_o^D$ if, and only if, $x \geqslant x_0$. For any product-reusability θ, the optimal premium-factor $x^*(\theta)$ for the refurbished units during supply disruption is:*

$$x^*(\theta) = \max\left\{x_0, \frac{c(1-\theta)\phi + r}{2\delta p}\right\}. \tag{19}$$

From the above result, it is evident that the firm can charge a premium for refurbished units during supply disruption so long as consumers do not considerably undervalue refurbished units (i.e., $x^*(\theta) > 1$ if, and only if, r is sufficiently high). Otherwise, the firm should offer a deeper discount on refurbished units (i.e., set $x^*(\theta) < 1$) to encourage their sales. Next, the following result further characterizes the optimal premium-factor $x^*(\theta)$:

Lemma 3. *The premium-factor $x^*(\theta)$ for refurbished units defined in (19) satisfies the following:*
1. *The firm charges a higher premium when unit production cost c is high (i.e., $x^{*(c)} \geqslant 0$).*
2. *The firm charges a lower premium when product-reusability θ is high (i.e., $x^{*\prime}(\theta) \leqslant 0$).*
3. *The firm charges a higher premium when supply-risk α is high (i.e., $x^{*(\alpha)} \geqslant 0$).*

First, a higher unit production cost c that decreases the firm's margins prompts the firm to increase the prices of refurbished units through a higher premium-factor, as evident from (19).

Second, a higher product-reusability θ leads to a lower refurbishment cost that enables the firm to decrease its price of refurbished units. Through a lower premium-factor the firm can also achieve more refurbished-unit sales that will positively affect its profit.

Next, and more importantly, a higher supply-risk α results in an increased likelihood of the firm selling refurbished units to high-valuers whenever there is supply disruption. This cannibalization of new-units' demand by refurbished-units dilutes the firm's revenue. Additionally, a higher risk α will also increase the steady state probability of being in D, which further degrades the firm's expected profit. Therefore, in order to compensate for these dilutions in its revenues, the firm chooses a higher premium-factor whenever there is supply disruption. This fact is summarized in the third statement of Lemma 3.

5.2. Optimal Product-reusability

In this section, we observe the optimal reusability $\theta^*(x)$ for any predetermined premium-factor x. Sometimes, it could be the case that firms should efficiently design their products when their prices, even during supply disruption, are largely determined by exogenous market factors and regulator's and company's policies. Therefore, in this section, we analyze the firm's decision on product-reusability θ, for any predetermined premium-factory x. Furthermore, we find that this will also help us to conveniently analyze the scenario when both product-reusability and premium-factor are jointly decided, in Section 5.3. It is easy to verify that the profit in (17) is concave in θ (for any x). We provide the optimal product-reusability $\theta^*(x)$ and its sensitivity to various problem-parameters in the result below:

LEMMA 4. *For any premium-factor x, the optimal product-reusability $\theta^*(x)$ is given by:*

$$\theta^*(x) = \min\left\{1, \frac{c\phi(1-\alpha)\left[\alpha\min\{\xi_e,\xi_o^D\} + (1-\alpha)\min\{\xi_e,\xi_o^N\}\right]}{2k(1-\beta)}\right\}. \tag{20}$$

Furthermore, the firm lowers the above reusability level whenever:

1. *production cost c decreases or design cost k increases (i.e., $\theta^{*(c)} \geqslant 0$ and $\theta^{*(k)} \leqslant 0$),*
2. *it charges a higher premium-factor (i.e., $\theta^{*\prime}(x) \leqslant 0$),*
3. *the supply-risk α increases, when the risk is sufficiently high (i.e., $\theta^{*(\alpha)} < 0$ if α is high), or*
4. *the sales of refurbished units (i.e., $\min\{\xi_e,\xi_o^N\}$ or $\min\{\xi_e,\xi_o^D\}$) decreases.*

The firm lowers the product-reusability to save on its design cost whenever k is high (i.e., $\theta^{*(k)} \leqslant 0$), but increases the reusability level as its unit-cost c increases (i.e., $\theta^{*(c)} \geqslant 0$) in order to lower its refurbishment cost. Thus, the firm trades off between the unit-savings in refurbishment cost against the upfront product design cost to maximize its profit, as explained in the first statement of Lemma 4. Next, the firm can afford a higher refurbishment cost when the premium-factor x is high. Hence, the firm decreases its product-reusability in order to save on its product design cost $k\theta^2$, while incurring a higher refurbishment cost whenever x is high. This is explained in Statement 2 of the lemma. The third statement of Lemma 4 is more intriguing and states that it benefits the firm to decrease product-reusability $\theta^*(x)$ as supply-risk α increases, when the risk α is high. This contradicts the common beliefs of many business experts (as discussed in the introduction) that a higher product-reusability should be adopted as supply-risk increases. The rationale for this is as below.

A high supply-risk α drastically dilutes the firm's expected profit by increasing the steady-state probability of state D, in which the firm cannot sell the product (both new and refurbished units) whenever there are successive periods of supply disruptions (i.e., $D \to D$ transitions in Figure 1). Hence, the firm chooses to save upfront on its product design cost $k\theta^2$ by reducing the reusability level θ. Thus, we can conclude that, contrary to popular belief, *it may not be always beneficial to increase product-reusability as supply-risk increases, and it hurts to do so especially when the supply-risk is high and the firm cannot decide the refurbished-units' price during supply disruption.*

However, it still remains to verify if the above finding is true when the firm can charge a premium on the refurbished units during supply disruption. That is, is it beneficial to increase product-reusability if supply-risk increases when the firm can decide the premium-factor x along with the reusability level θ, especially when α is high. We discuss this in the next section.

According to the last statement, the firm increases the reusability level $\theta^*(x)$ as the number of refurbished units sold increases (i.e., $\min\{\xi_e, \xi_o^N\}$ or $\min\{\xi_e, \xi_o^D\}$ increases) in order to decrease the total refurbishment cost. These savings in refurbishment cost offset the increase in design cost incurred by the firm.

5.3. Jointly Optimal Premium-factor and Product-reusability

In this section, we obtain (i) the optimal premium-factor for refurbished units during supply disruption and (ii) the optimal product-reusability level, when the firm can decide both of them simultaneously. It can be shown from (17) that the firm's profit $\pi(\theta, x; p, v, \delta)$ is concave in θ and x when the design-cost factor k is significant, which is largely true in practice due to the high research & developmental, capital, and other such costs involved in designing a product. Therefore, we make the following assumption:[9]

ASSUMPTION 5 **(Product-design is costly)**. *Reusability design is sufficiently costly; i.e.,* $k > k_0 \equiv k_0(\alpha) = \frac{c^2\phi^2\alpha(1-\alpha)}{4r(1-\beta)}$. *This ensures concavity of the firm's profit* $\pi(\theta, x; p, v, \delta)$ *given in* (17).

Furthermore, in order to draw practical insights, in what follows in the paper, we analyze a more practical scenario wherein the trade-in fee v is not so high that all consumers choose to trade-in their units; this is hardly true in practice. Therefore, by letting $v < \max\{v_1(\alpha), v_2(\alpha)\}$, as discussed in Figure 5, we obtain the following result that characterizes the firm's optimal decisions:

LEMMA 5. *Let* x_0 *and* $\theta^*(\cdot)$ *be as defined in Lemmas 2 and 4, respectively, and let:*

$$\tilde{x} = \frac{r}{\delta p} \cdot \frac{\left\{2k(1-\beta)(r+c\phi) - c^2\phi^2(1-\alpha)[\alpha + (1-\alpha)\cdot\min\{\xi_e, \xi_o^N\}]\right\}^+}{4kr(1-\beta) - c^2\phi^2\alpha(1-\alpha)} \text{ and} \tag{21}$$

$$\tilde{\theta} = c\phi(1-\alpha)\left[\frac{\alpha(r - c\phi) + 2r(1-\alpha)\cdot\min\{\xi_e, \xi_o^N\}}{4kr(1-\beta) - c^2\phi^2\alpha(1-\alpha)}\right]. \tag{22}$$

The joint optimal premium-factor x_{opt} *and product-reusability* θ_{opt} *are given by:*

$$x_{opt} = \max\left\{\frac{r}{2\delta p}, x_0, \tilde{x}\right\} \text{ and} \tag{23}$$

$$\theta_{opt} = \min\{1, \theta^*(x_0), \tilde{\theta}\}. \tag{24}$$

[9] For completeness, we discuss the case when $k \leqslant k_0$ in Lemma EC.2 in Appendix C.

It can be easily verified from (23) and (24) that firm always chooses a lower reusability level and higher premium-factor whenever the design cost k is high (i.e., $\theta_{opt}^{(k)} < 0$ and $x_{opt}^{(k)} > 0$). (We formalize this result in Lemma EC.1 in Appendix C.) While it is expected that firm reduces product-reusability θ_{opt} as it gets costlier to design it (i.e., when k is high), the increase in refurbishment cost $c\phi(1-\theta_{opt})$ due to low θ_{opt} prompts the firm to increase the premium-factor x_{opt} in order to increase its margin on refurbished units.

In the next subsection, we address our research question by analyzing the impact of supply-risk α on the firm's decisions x_{opt} and θ_{opt}, which are given in Lemma 5. We verify if it benefits to increase reusability θ_{opt} as the risk α increases, when the firm can decide the premium x_{opt} on the refurbished units during supply disruption.

5.3.1. Is reusability beneficial during supply disruption? In order to evaluate the benefit of reusability during supply disruption, we first analyze how the firm complements: (i) the more strategic product-reusability decision θ_{opt} and (ii) the more tactical premium-factor decision x_{opt}, with each other as the supply-risk α increases. We introduce the following result to explain the firm's strategy depending on the number of trade-ins ξ_e the firm receives:

LEMMA 6. *Let $\overline{\xi}_e \equiv \overline{\xi}_e(\alpha) = \max\{\frac{2k(1-\beta)(r-c\phi)+c^2\phi^2(1-\alpha)^2\xi_o^N}{4kr(1-\beta)-c^2\phi^2\alpha(1-\alpha)}, \frac{2k(1-\beta)(r-c\phi)}{4kr(1-\beta)-c^2\phi^2(1-\alpha)}\}$. The firm's optimal strategy is as follows:*[10]

1. *If there are many trade-ins (i.e., $\xi_e > \overline{\xi}_e$), then the firm exclusively chooses one of the following two strategies when supply-risk α increases:*

   (a) ***Risk-absorbing strategy.*** *Absorb the increasing risk through increased product-reusability level and benefit consumers through decreased premium-factor, or*

   (b) ***Risk-transferring strategy.*** *Transfer the increasing risk to consumers through increased premium-factor and save on design cost through decreased reusability level.*

2. *Alternatively, if there are few trade-ins (i.e., $\xi_e \leqslant \overline{\xi}_e$), the firm always increases the premium factor (i.e., $x_{opt}^{(\alpha)} = x_0^{(\alpha)} \geqslant 0$). Moreover, there exists a threshold $\hat{\alpha}$ ($\leqslant \frac{1}{2}$) such that the firm increases product-reusability if, and only if, $\alpha \leqslant \hat{\alpha}$ (i.e., $\frac{d}{d\alpha}\{\theta^*(x_0(\alpha),\alpha)\} \geqslant 0 \Leftrightarrow \alpha \leqslant \hat{\alpha}$).*

Lemma 6 first explains the impact of supply-risk on firm's decisions and states the two exclusive strategies that firm adopts when the risk increases and when there are enough number of trade-ins. In the first strategy – the risk-absorbing strategy – that is stated in Statement 1 of the lemma, the firm chooses to invest upfront more in product design (to obtain higher reusability θ_{opt}) to decrease its refurbishment cost $c\phi(1-\theta_{opt})$. These savings in refurbishment cost enable the firm to set a lower premium-factor x_{opt}, which increases the demand $\xi_o^D(p, x_{opt}, \delta)$ for refurbished units

[10] It is easy to verify that $\tilde{x} \leqslant x_0 \Leftrightarrow \xi_e \leqslant \overline{\xi}_e(\alpha)$. Hence, $x_{opt} = x_0 \Leftrightarrow \xi_e \leqslant \overline{\xi}_e(\alpha)$. See the proof of Lemma 6 for details.

during supply disruption. We describe the risk-absorption strategy as "proactive" because the firm upfront invests in the product-design in order to incur lower refurbishment cost in the future.

Whereas, in the second strategy – the risk-transferring strategy – that is opposite to the first one, the firm forgoes the refurbished-unit sales by setting a higher premium-factor x_{opt}, but compensates for the lost revenue (due to the above forgone sales) through decreasing it design cost $k\theta_{opt}^2$ by choosing lower reusability level θ_{opt}. We term the risk-transferring strategy as "reactive" because the firm chooses to reduce its upfront investment in product-reusability but reacts to the increased supply risk by setting higher prices after the supply is disrupted.

The firm strategically chooses between these two contrasting strategies whenever supply-risk α increases. Moreover, the firm's choice between these two strategies is primarily governed by the magnitude of α, as explained later in Propositions 1 and 2.

On the other hand, the firm always chooses to increase the premium-factor x_{opt} when it is constrained on trade-in inventory (i.e., when $\xi_e \leqslant \overline{\xi}_e$). Moreover, the firm finds it beneficial to increase product-reusability θ_{opt} if, and only if, the supply-risk is not high (i.e., $\alpha \leqslant \hat{\alpha}$). The low revenues earned when supply-risk is high (i.e., when $\alpha \geqslant \hat{\alpha}$) will discourage the firm from investing more in increasing its product's reusability θ_{opt}.

Next, before proceeding further with our analysis to examine the impact of supply-risk α on the firm's decisions x_{opt} and θ_{opt}, we make the following assumption to focus on more practical scenarios for managerial insights:

ASSUMPTION 6 **(Trade-ins can meet refurbished-unit demands)**. *The consumers' trade-in propensity λ is sufficiently high so that the trade-ins are sufficient to meet the refurnished-unit demands; i.e., $\xi_e(\alpha) \geqslant \xi_o^N$ and $\xi_e(\alpha) \geqslant \overline{\xi_e}(\alpha)$.*

The above is often true in many electronic commodities like cell-phones where consumers choose to trade-in their used units frequently. Besides, it is important to note that the firm may opt to receive more trade-ins than it chooses to refurbish (by offering a sufficiently high trade-in fee $v \cdot p$) if it can improve its total profit by selling new-units to the customers who trade-in.[11]

Now, having discussed the impact of supply-risk α on product's reusability when the firm is constrained on trade-in volume (i.e., $\xi_e \leqslant \overline{\xi}_e$) in Lemma 6 (statement 2), we next analyze how supply-risk affects the firm's decisions when there is sufficient number of trade-ins (i.e., when Assumption 6 holds true), in the following result:[12]

[11] In Lemma EC.4, in Appendix C, we formally show that firm chooses δ such that it meets the entire demand for refurbished units during normal periods (i.e., $\xi_e \geqslant \xi_o^N$), whenever its unit-margins are positive. (We provide the corresponding sufficient conditions in Lemma EC.4.)

[12] For completeness, we analyze the case when $\overline{\xi_e}(\alpha) < \xi_e < \xi_o^N$ and discuss the impact of supply-risk on the firm's decisions in Lemma EC.5 in Appendix C. We note that the firm's strategy broadly remains similar to that explained in Propositions 1 and 2, although the product-design cost factor k also plays a role.

Proposition 1. *Let k_0 be as defined in Assumption 5 and let:*

$$\overline{\delta} \equiv \overline{\delta}(\alpha) = r - \frac{2kr(1-r)(1-2\alpha)(1-\beta)(r-c\phi)}{(1-\alpha)p\left(8kr(1-\beta) - c^2\phi^2(1-\alpha)\right)}. \tag{25}$$

An increase in supply-risk α impacts the product-reusability level $\tilde{\theta}$ that is given in (22) *as follows:*

1. *If the risk $\alpha \geqslant \frac{1}{2}$ then the firm decreases product-reusability (i.e., $\tilde{\theta}^{(\alpha)} \leqslant 0$).*
2. *If the risk $\alpha < \frac{1}{2}$, then the firm increases product-reusability if, and only if, $\delta \geqslant \overline{\delta}$.*

The first statement of Proposition 1 shows that if the supply-risk is high (i.e., $\alpha \geqslant \frac{1}{2}$), then firm always decreases product-reusability $\tilde{\theta}$ and transfers supply-risk to consumers through a higher premium-factor $\tilde{x}$ as the risk α increases. The firm adopts this risk-transferring strategy because a high supply-risk ($\alpha \geqslant \frac{1}{2}$) negatively impacts the firm's expected profit by increasing the steady-state probability of supply disruption (i.e., of state D). However, according second statement, when the risk $\alpha < \frac{1}{2}$, the firm will increase product-reusability if, and only if, the unit-price of refurbished units is sufficiently high (i.e., $\delta \geqslant \overline{\delta}(\alpha)$). Such a high margin will help the firm to offset its product-design cost and encourage it to increase product-reusability.

At this point, we revisit and address the following question: *Does it benefit the firm to increase its product-reusability to counter supply disruption, when it can mark-up refurbished units during supply disruption?* We answer this question through the following proposition:

Proposition 2. *Consider the firm's decisions $\tilde{\theta}$ and $\tilde{x}$ that are given in* (22) *and* (21)*, respectively. The firm absorbs supply-risk by increasing product-reusability and decreasing premium-factor if, and only if, α is low; i.e., $\tilde{\theta}^{(\alpha)} \geqslant 0 \geqslant \tilde{x}^{(\alpha)} \Leftrightarrow \alpha \leqslant \overline{\alpha} \equiv \overline{\alpha}(k) (\leqslant \frac{1}{2})$, for any $k > k_0$.*[13]

Proposition 2 explains that the firm will absorb an increase in supply-risk and benefits its consumers through lower premium-factor on refurbished units (i.e., adopts the risk-absorbing strategy) if, and only if, the supply-risk is low (i.e., $\alpha \leqslant \overline{\alpha}$). On the other hand, if the risk is high (i.e., $\alpha \geqslant \overline{\alpha}$) the firm adopts the opposite risk-transferring strategy through a higher premium-factor and a lower investment in product-reusability design. This shift in firm's strategy from risk-absorbing to risk-transferring as the supply-risk increases can be attributed to: (i) the decrease in firm's profit due to the increased steady-state probability of supply disruption D, and (ii) the increased consumers' reluctance to trade-in due to the higher risk of product's unavailability.

Thus, it benefits the firm to increase reusability if, and only if, the supply-risk is low. Additionally, we can easily show that the threshold $\overline{\alpha}$, which is defined in Proposition 2, decreases as the design cost k increases (i.e., $\overline{\alpha}^{(k)} < 0$). Hence, we can conclude that the firm can absorb more risk when

[13] $\overline{\alpha} \equiv \overline{\alpha}(k)$ is the unique value that satisfies $\overline{\delta}(\overline{\alpha}) = \delta$, where $\overline{\delta}(\alpha)$ is defined in (25); the closed-form expression of $\overline{\alpha}$ is given in (EC.6), in the proof of Proposition 2.

the design cost k is low; i.e., the firm adopts risk-absorption strategy until a higher value of $\overline{\alpha}(k)$, when k is low.

While we discuss the impact of supply-risk on the firm's product-reusability decision when the design cost is sufficiently high (i.e., $k \geqslant k_0$) in Lemma 6 and Proposition 2, we discuss the case when k is low (i.e., $k < k_0$) in Lemma EC.2 in Appendix C. Again, we observe that the firm increases reusability if, and only if, the risk is low.

Therefore, we conclude that *it is beneficial for a firm to improve its product-reusability design as the supply disruption risk increases only when the risk is limited (i.e., below a threshold); otherwise, it is beneficial to reduce the reusability level.*

5.3.2. Impact of unit-prices p and δp. Finally, it is important to understand how the unit-price p and discount δ in normal periods (i.e., in state N) affect the reusability $\tilde{\theta}$ and premium-factor $\tilde{x}$ decisions, which crucially affect the firm's profits during supply disruption (i.e., in state D). We discuss this in the following lemma:

LEMMA 7. *The firm chooses lower reusability when the discount on refurbished units is lower (i.e., $\tilde{\theta}^{(\delta)} \leqslant 0$) but chooses higher reusability when the new-unit price is higher (i.e., $\tilde{\theta}^{(p)} \geqslant 0$).*

A higher price p that increases the firm's unit-margins on both new and refurbished units enables the firm to invest more in product-reusability. However, it is intriguing to observe from Lemma 7 that although a lower discount (i.e., higher δ value) also increases the firm's margin on refurbished units, unlike the new-unit price p, it prompts the firm to decrease product-reusability. The reason for this is as follows. It is evident from Figure 4a that the demand ξ_o^N for refurbished units decreases when a lower discount (i.e., higher δ) is offered, whereas the demand ξ_n for new units, which fetch a higher unit-margin than refurbished units, increases. Hence, the firm finds it beneficial to save on design cost $k\tilde{\theta}^2$ through a lower reusability $\tilde{\theta}$. [14]

5.3.3. Optimal Trade-in Fee. Finally, by substituting $\tilde{\theta}$ and $\tilde{x}$ in (17) we can obtain the firm's optimal profit $\pi(\tilde{\theta}, \tilde{x}; p, v, \delta)$ for any value of p, v and δ, when the number of trade-ins is high (i.e., $\xi_e \geqslant \max\{\xi_o^N, \overline{\xi_e}\}$, due to Assumption 6); we relax this assumption in our discussion in Section 6. On maximizing the resultant profit $\pi(\tilde{\theta}, \tilde{x}; p, v, \delta)$ with respect to the trade-in fee v, we can obtain the optimal trade-in fee as:

$$v_{opt} = \begin{cases} \max\left\{v_1(\alpha), \min\left\{v_2(\alpha), \frac{p-c}{2p} + \frac{v_1(\alpha)}{2}\right\}\right\} & \text{if } \alpha \leqslant \gamma(r), \text{ and} \\ \max\left\{v_2(\alpha), \min\left\{v_1(\alpha), \frac{p-c}{2p} + \frac{v_2(\alpha)}{2}\right\}\right\} & \text{otherwise,} \end{cases} \tag{26}$$

where $v_1(\alpha)$ and $v_2(\alpha)$ are given in (5) and (6), respectively. From (26), it is easy to conclude that the firm offers a higher trade-in fee v_{opt} when:

[14] The impact of p and δ on the premium-factor $\tilde{x}$ is more intricate and is explained in detail in Lemma EC.3 in Appendix C, for brevity.

(i) *the risk α is high.* Consumers are more reluctant to trade-in their units due to the high risk of product's unavailability in the future, when α is high. Therefore, the firm offers a higher trade-in fee to encourage trade-ins in order to sell more refurbished units, especially during periods of supply disruption.

(ii) *the discount on refurbished units is low (i.e., δ is high).* The higher margin that is earned from refurbished units enables the firm to offer a higher trade-in fee. Doing so will encourage more consumers to trade-in, which helps the firm to earn higher profit by selling new units to these consumers.

(iii) *the consumers valuation of refurbished units (i.e., r) is high.* The increased demand for refurbished units (due to high r) will enable the firm to sell more refurbished units at a higher price. Therefore, the firm will offer higher trade-in fee to encourage more trade-ins for refurbishment.

(iv) *the production cost c is low.* Finally, the lean unit-margins when the unit-cost c is high will make high trade-in fee unaffordable for the firm. Hence, the firm offers lower trade-in fee when c is high and offers high trade-in fee when c is low.

6. Numerical Study

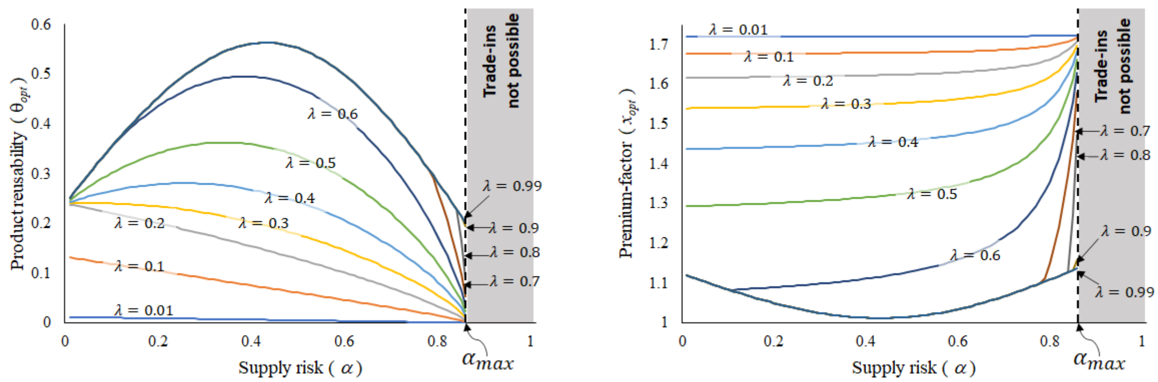
In this section, we provide further analysis on the impact of key model parameters through numerical examples. We focus our discussion on the impact of consumer's trade-in propensity λ (§6.1) and the efficacy of reusability ϕ (§6.2), which has not been analyzed thus far.

6.1. Consumers' trade-in propensity λ

Recall that the parameter λ denotes the intrinsic trade-in propensity of consumers. As λ increases (or equivalently, as ρ increases) it is evident from (8) and (9) that the number of trade-ins ξ_e also increases. The firm's resuability and premium-factor decisions for different values of trade-in propensity λ and supply-risk α are provided in Figures 6a and 6b, respectively. Figure 6a shows that for any value of λ, the product-reusability decreases in supply-risk α whenever α is high (as proved in Proposition 2). Furthermore, as the number of trade-ins ξ_e increases due to higher λ value, the firm increases reusability to save more on its refurbishment costs.

Next, as Figure 6b indicates, the premium-factor is high when λ is low due to the limited number of trade-ins ξ_e, and the firm reduces the premium-factor as the supply of trade-ins increases, as λ increases.

Next, Figure 7 provides the plots of optimal trade-in fees for different values of α and λ. The plots indicate that, for any λ, the firm offers higher fee to encourage more trade-ins as the supply risk α increases. This helps the firm to sell more refurbished units during supply disruption. Additionally, it is interesting to observe that for any given α, firm increases the trade-in up to a certain value of λ and decreases the fee thereafter, as λ increases. A very low value of λ (for example, $\lambda = 0.01$



(a) Product-reusability level. (b) Premium-factor for refurbished units.

Figure 6 Impact of trade-in propensity λ on firm's decisions.

to 0.1 in Figure 7) dissuades the firm from offering a high fee v^* because the consumers' intrinsic reluctance to trade-in results in a minuscule number of trade-ins (i.e., $\xi_e < \xi_o^N$) although v^* is high. Hence, the firm opts to save on its trade-in fee expenditure than to encourage trade-ins. This strategy reverses when λ is high (i.e., $\lambda > 0.1$ in Figure 7 so that (i.e., $\xi_e > \xi_o^N$)) and the firm encourages trade-ins through high v^* as λ increases.

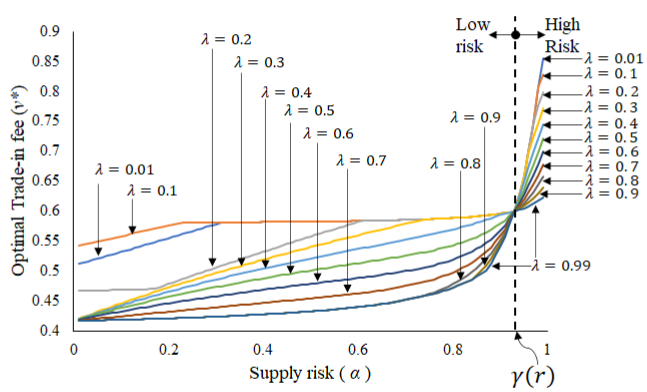
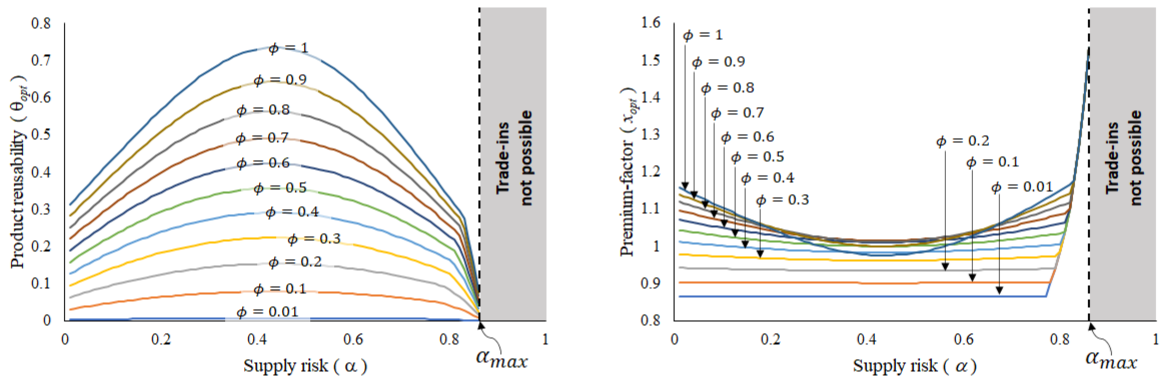


Figure 7 Optimal trade-in fee v^* for different trade-in propensities λ and supply-risks α

6.2. Efficacy of reusability ϕ

In this section, we numerically investigate the impact of the efficiency of reusability, which is given by ϕ. The plots for reusability level and premium-factors for different values of ϕ and α are given in Figures 8a and 8b, respectively. Figure 8a shows that a higher reusability efficacy ϕ encourages the firm to choose higher reusability levels θ_{opt} due to the higher savings in refurbishment cost, which is as expected. Likewise, the plots of premium-factors for different values of ϕ and α are given in Figure 8b. The firm takes into account the interaction between the reusability level θ_{opt} along with its efficacy ϕ when deciding the appropriate premium-factor x_{opt}.



(a) Product-reusability level. (b) Premium-factor for refurbished units.

Figure 8 Impact of reusability efficacy ϕ on firm's decisions.

7. Conclusions

In this paper, we modeled and analyzed the role of product-reusability executed through refurbishment by an external facility in the presence of supply disruptions. We examined if it always benefits to increase product-reusability whenever the supply risk increases. First, we determined how firms can optimally set the price-premium for refurbished units and product-reusability in the presence of supply disruption risk. Next, we determined the optimal trade-in fee that the firm should offer its customers to encourage trade-ins, while maximizing its profit.

Through our analysis in Section 5.2, we showed that the conventional wisdom of enhancing product-reusability for easy refurbishment as a strategy to counter increasing supply disruption risk is beneficial only when the likelihood of disruption is low. However, we showed that this strategy of increasing reusability could be counter-productive when the disruption probability is high. This is because the firm earns a lower expected revenue as it spends a longer time in the state of disrupted supply whenever the disruption probability is high. Therefore, it is beneficial for a firm to decrease product-reusability and save on its product-design cost whenever disruption is highly likely. Next, in Section 5.3, we analyzed the firm's joint-decisions when it can simultaneously decide the product-reusability level and the price-premium on refurbished units during supply disruption. We showed that when there are sufficient number of trade-ins, the firm exclusively chooses between the proactive "*risk-absorbing strategy*" and the reactive "*risk-transferring strategy*", and its choice between these strategies is governed by the magnitude of the supply-risk. In the risk-absorbing strategy, the firm counters the increased supply-risk by increasing the product-reusability upfront and reducing the price-premium on refurbished units during disruption. Whereas, in the risk-transferring strategy, the firm counters the increased disruption risk by reducing product-reusability and increasing price-premium on the refurbished units during disruption, thus transferring the risk to its customers. Later, in Section 5.3.3, we also analyzed the firm's optimal choice of trade-in fee.

We observed that the firm encourages trade-ins through a higher trade-in fee when the supply-risk is higher. Moreover, a higher trade-in fee is also offered when the customers highly value refurbished units.

Finally, in Section 6, through various numerical examples, we illustrated the impact of consumers' trade-in propensity and efficacy of reusability on the firm's reusability decision. We showed that as the number of trade-ins increases due to increased trade-in propensity, the firm increases product-reusability. Doing so enables the firm to save more on refurbishment costs. These savings, in turn, allow the firm to reduce the price-premium on refurbished units. Next, we showed that as the efficacy of reusability increases the firm invests more in product-reusability, which is as expected.

Our paper is an initial step towards integrating product-reusability design decisions with supply disruption management strategies. There are several interesting avenues for future research. In this paper, we analyzed the scenario when refurbishment is outsourced to an external facility and did not consider in-house refurbishment. The latter scenario provides the firm with higher flexibility to strategically manage its trade-in inventory by withholding and carrying it forward, if required. Developing a multi-period inventory policy to efficiently manage trade-ins would be a fruitful topic for future research. Next, we analyzed the decisions of a single firm in this paper and did not study the multi-firm competitive version of this problem. Here, it would be interesting to explore how the various factors like difference in design costs (i.e., k), prices (i.e., p), and consumer valuations (i.e., V) affect a firms' decisions on product-reusability and premium-factor in the presence of competition. Another potential topic of future research includes exploring the impact of governmental policies on waste-recycling and *firm take-back* programs on firm's choice of product-reusability in the presence of supply disruptions.

In conclusion, we believe that this paper presents a useful framework to model, analyze, and manage product-reusability under supply disruptions, and serves as a basis for further research in the area of sustainable production and consumption.

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Online Appendix (E-Companion)

Appendix A: Assumptions

The following are the important assumptions that we make in the paper:

Assumption 1: Refurbishing operations. The firm accepts trade-ins of units that are purchased brand-new and used for one period. It delegates trade-ins to a refurbishing facility that provides refurbished units in the next period. We normalize the unit transfer-price between the firm and the facility to 0, for parsimony. Furthermore, we assume that the facility will use the leftover inventory for its own benefit and will not strategically store inventory for the firm.

Assumption 2: Non-zero demand for refurbished units. The percentage reduction in price of a refurbished unit over a new unit is higher than the relative reduction in consumer's valuation; i.e., $r > \delta$.

Assumption 3: Viability of trade-in. The supply-risk is not too high so that at least some consumers choose to trade-in their old units; i.e., $\alpha \leqslant \alpha_{max}(v)$, which is defined in Lemma 1.

Assumption 4: Feasibility of positive margin. The highest valuer (i.e., consumer with $V = 1$) always values a refurbished unit more than its refurbishment cost; i.e., $r \geqslant c\phi$.

Assumption 5: Product-design is costly. Reusability design is sufficiently costly; i.e., $k > k_0 \equiv k_0(\alpha) = \frac{c^2\phi^2\alpha(1-\alpha)}{4r(1-\beta)}$. This ensures concavity of the firm's profit $\pi(\theta, x; p, v, \delta)$ given in (17).

Assumption 6: Trade-ins can meet refurbished-unit demand. The consumers' trade-in propensity λ is sufficiently high so that the trade-ins are sufficient to meet the demands for refurnished units; i.e., $\xi_e(\alpha) \geqslant \xi_o^N$ and $\xi_e(\alpha) \geqslant \overline{\xi_e}(\alpha)$.

Appendix B: Extensions

B.1. Forward-looking First-time Purchasers

To incorporate complete forward-looking behavior of consumers, one can assume that each consumer gains utility $r^t \cdot X$, where $X \sim U[0,1]$ and $r \in [0,1)$, in period t. Therefore, the consumer's total value from a new unit of the product is given by $V = X/(1 - r\beta)$ so that $V \sim U[0, \frac{1}{1-r\beta}]$. However, one should continue to assume that consumers do not commit to trade-in the product at the end of the period when valuing a newly manufactured unit; in practice, consumers usually tend evaluate the option to trade-in their units in course of time after they purchase the product.

B.2. Allowing Trade-ins through Product's Lifetime

We relax the Assumption 1 so that firm can allow trade-ins throughout the product's lifetime. However, we note that it can be difficult to obtain a closed-form expressions of the results.

Let the product's life be L periods long. Let $v_{\{k\}}p$, $k = 1, 2, \cdots, L-1$, be the trade-in fee that the firm provides for k-period old trade-ins. Let $\tilde{f}_k$ denote the fraction of consumers who purchase the new units (i.e., among ξ_n in a period) and choose to trade them in k periods later. Likewise, let $p_k \equiv \mathbb{P}[k \text{ periods ago was } N] = (1-\alpha)$. By assuming that the value of the product depreciates by the fraction r in each period and the refurbished unit is always valued at rV, we can obtain:

$$\tilde{f}_k = \mathbb{P}\left[\begin{array}{c} V > \left(\frac{p(1-\delta)}{1-r}\right), V(\gamma(\alpha) - r^k) > (\gamma(\alpha) - v_{\{k\}})p, \\ k = \arg\max_j \{[V(\gamma(\alpha) - r^j) - (\gamma(\alpha) - v_{\{j\}})p]\beta^{j-1}\} \end{array}\right] \cdot p_k$$

$$= \mathbb{P}\left[\begin{array}{c} V > \left(\frac{p(1-\delta)}{1-r}\right), V(\gamma(\alpha)-r^k) > (\gamma(\alpha)-v_{\{k\}})p, \\ k = \arg\max_j\{[V(\gamma(\alpha)-r^j)-(\gamma(\alpha)-v_{\{j\}})p]\beta^{j-1}\} \end{array}\right]\cdot(1-\alpha). \tag{EC.1}$$

Note that when $k=1$, $f_1 = \frac{\tilde{f}_1}{1-\alpha}$ is the probability that is given in (5), given the previous period is N. Now, let $\psi_{e,k}$ denote the expected number of trade-ins of k-period old units in any period (irrespective of N or D) in the steady state. Next, the total expected number of trade-ins in any period is $\psi_e = \sum_{k=1}^{L-1}\psi_{e,k}$.[15] The expected total trade-in demand that is satisfied in a period of type N is $\omega = \sum_{k=1}^{L-1}\omega_k$, where ω_k is the expected number of k-period old trade-ins that are satisfied in the period. By taking that the k-period trade-ins can be forwarded through backorders from previous periods due to supply disruptions and the trade-ins can only be served if the current period is N, we obtain ω_k in an N type period (w.p. $(1-\alpha)$) as:

$$\omega_k = \psi_{e,k}\sum_{i=1}^{\infty}\alpha^i\cdot(1-\alpha) = \psi_{e,k}.$$

Now, by using the fact that k-period old trade-ins in a period (irrespective of N or D) can occur only if the period k periods ago is N, we obtain:

$$\psi_{e,k} = \lambda\left[\tilde{f}_k + \omega_k\right] = \lambda\left[\tilde{f}_k + \psi_{e,k}\right],$$

so that $\psi_{e,k} = \left(\frac{\lambda}{1-\lambda}\right)\tilde{f}_k = \rho\tilde{f}_k$, after taking into account the intrinsic propensity λ to trade-in. Now, we can compute the transition profits for the Markov-chain in Figure 1 as:[16]

$$\begin{aligned}
\tilde{g}_{NN} &= (\delta p - \phi c(1-\theta))\sum_{k=1}^{L-1}\min\{\xi^N_{o,k}\cdot p_k, \psi_{e,k}\} + \sum_{k=1}^{L-1}(p-c-v_{\{k\}}p)\psi_{e,k} + (p-c)\xi_n \\
\tilde{g}_{ND} &= (x\delta p - \phi c(1-\theta))\sum_{k=1}^{L-1}\min\{\xi^D_{o,k}\cdot p_k, \psi_{e,k}\} + \sum_{k=1}^{L-1}(p-c-v_{\{k\}}p)\psi_{e,k} \\
\tilde{g}_{DN} &= (\delta p - \phi c(1-\theta))\sum_{k=1}^{L-1}\min\{\xi^N_{o,k}\cdot p_k, \psi_{e,k}\} + \sum_{k=1}^{L-1}(p-c-v_{\{k\}}p)\psi_{e,k} + (p-c)\xi_n = \tilde{g}_{NN} \\
\tilde{g}_{DD} &= (x\delta p - \phi c(1-\theta))\sum_{k=1}^{L-1}\min\{\xi^D_{o,k}\cdot p_k, \psi_{e,k}\} + \sum_{k=1}^{L-1}(p-c-v_{\{k\}}p)\psi_{e,k} = \tilde{g}_{ND},
\end{aligned}$$

where

$$\xi^N_o = \sum_{k=1}^{L-1}p_k\cdot\xi^N_{o,k} \quad \text{and} \quad \xi^D_o = \sum_{k=1}^{L-1}p_k\cdot\xi^D_{o,k}. \tag{EC.2}$$

Thus, the state-specific expected profits are $\tilde{\pi}_N = (1-\alpha)[\tilde{g}_{NN}+\beta\tilde{\pi}_N] + \alpha[\tilde{g}_{ND}+\beta\tilde{\pi}_D] = (1-\alpha)[\tilde{g}_{DN} + \beta\tilde{\pi}_N] + \alpha[\tilde{g}_{DD}+\beta\tilde{\pi}_D] = \tilde{\pi}_D$, so that $\tilde{\pi}_N = \tilde{\pi}_D = \frac{(1-\alpha)\cdot\tilde{g}_{NN}+\alpha\cdot\tilde{g}_{ND}}{1-\beta}$. Therefore, the firm's expected net-profit is given by $\tilde{\pi}(x,\theta) = (1-\alpha)\tilde{\pi}_N + \alpha\tilde{\pi}_D - k\theta^2 = \tilde{\pi}_N - k\theta^2$. It can be verified easily that if $L=1$, then $\tilde{\pi}(x,\theta) = \pi(x,\theta)$ that is given in (17). However, closed-form expressions for $L>1$ can be difficult to obtain due to the complexity involved in the computation of $\tilde{f}_k$, $k=1,\cdots,L-1$, which is given in (EC.1) and due to the demand-supply balance equations (EC.2).

[15] Note that the average number of "useful" trade-ins that can be refurbished can be obtained as $\psi_e = \sum_{k=1}^{L-1}\zeta_k\psi_{e,k}$, if $\zeta_k < 1$ is the probability of a k-period trade-in being successfully refurbished. Here, we let $\zeta_k = 1$.

[16] For exact computation, the firm has to account for the number of periods for which the periods were and decide how to differentially price the refurbished units depending on the number of trade-ins accumulated. Such an analysis warrants modifying the state-space into a vector of length L that includes the states of all the previous L periods. However, when $L=1$, the state-space and transitions are still given as in Figure 1.

B.2.1. Brand-new versus old product If the differentiation is only two-fold, that is brand-new and used (or refurbished) units, with the former valued at $V\ U[0,1]$ and the latter at rV. Then from (EC.1), we obtain $\tilde{f}_k = 0$, for all $k > 1$ and $\tilde{f}_1 = f_1(1-\alpha)$, so that the trade-ins are always 1-period old by the choice of consumers, so that Assumption 1 holds true.

Appendix C: Auxiliary results

LEMMA EC.1. *Let θ_{opt} and x_{opt} be as given in Lemma 5. Then:*

1. $\theta_{opt}^{(k)} \leqslant 0$ *and* $x_{opt}^{(k)} \leqslant 0$.
2. $\theta_{opt}^{(p)} \geqslant 0$.

LEMMA EC.2. *Let x_0 be as given in Lemma 2 and $\theta^*(\cdot)$ be as given in* (20). *The optimal reusability level and premium-factor when $k \leqslant k_0$ are given by:*

$$\theta_{opt} = \theta^*(x_0) \quad \text{and} \quad x_{opt} = \max\left\{x_0, \frac{r}{2\delta p}\right\}. \tag{EC.3}$$

Therefore, for any $0 \leqslant k \leqslant k_0$, there exists a threshold $\hat{\alpha} \equiv \hat{\alpha}(k) (\leqslant \frac{1}{2})$ such that the firm increases the reusability level if $\alpha \leqslant \hat{\alpha}$, and lowers it if otherwise (i.e., $\theta_{opt}^{(\alpha)} \geqslant 0 \Leftrightarrow \alpha \leqslant \hat{\alpha}$). However, the firm always increases the premium-factor as supply-risk α increases (i.e., $x_{opt}^{(\alpha)} \geqslant 0$).

Discussion of Lemma EC.2: As in the case when $k \geqslant k_0$, when the firm's profit is concave in its decisions θ and x, we observe that when $k < k_0$, i.e., when that the profit is not concave in the decisions, the firm's strategy is to decrease its reusability level whenever the supply-risk is high and increase it when the risk is low. However, the firm always increases its premium-factor as supply-risk increases irrespective of the magnitude of the risk, because the supply of units traded in, which forms a bottleneck to the sales of refurbished units during disruption (since $\xi_e = \xi_o^D$), decreases as the risk α increases.

LEMMA EC.3. *The firm increases premium-factor with price p if, and only if, $(k_0 <) k \leqslant \frac{2rk_0}{r+c\phi}$ and with δ if, and only if, $k \leqslant \frac{c^2\phi^2(1-\alpha)((1-\alpha)p+\alpha(1-r))}{2(1-\beta)(1-r)(c\phi+r)} (\geqslant \frac{2rk_0}{r+c\phi})$.*

Discussion of Lemma EC.3: Lemma EC.3 shows that if k is low (i.e., $k_0 < k < \frac{2rk_0}{r+c\phi}$), then higher premium $\tilde{x}$ is chosen when p or δ is higher to improve profit through higher margin but fewer sales, because the total design cost is low. Likewise, if k is high (i.e., $k \geqslant \frac{(1-\alpha)c^2\phi^2((1-\alpha)p+\alpha(1-r))}{2(1-\beta)(1-r)(c\phi+r)}$), then lower premium is chosen when p or δ is higher in order to benefit through more number of sales. Finally, if k is moderate (i.e., $\frac{2rk_0}{r+c\phi} < k < \frac{c^2\phi^2(1-\alpha)((1-\alpha)p+\alpha(1-r))}{2(1-\beta)(1-r)(c\phi+r)}$), firm chooses a lower premium to spur demand when p is high because it earns higher margins but on lower demand, and chooses a higher premium when δ increases to earn higher margin but on fewer sales.

LEMMA EC.4. *The firm sets the discount δ such that it is always possible to meet the demand for refurbished goods during normal supply (i.e., $\xi_e \geqslant \xi_o^N$) if one of the following is true:*

1. *The supply-risk is low; i.e., $\alpha \leqslant \gamma(r)$.*
2. *The firm always obtains a net positive margin from trade-ins and refurbished units; i.e., $p(1-v) \geqslant c$ and $\delta p \geqslant \phi c(1-\tilde{\theta})$.*

Therefore, if the unit price p is high or costs c and ϕc are low, then $\xi_e \geqslant \xi_o^N$.

Lemma EC.5. *The impact of increase in supply-risk α on product-reusability level $\tilde{\theta}$ when $\xi_e < \xi_o^N$ is as follows:*

1. *If $\alpha \geqslant \frac{1}{2}$ then the firm decreases reusability (i.e., $\tilde{\theta}^{(\alpha)} \leqslant 0$ if $\alpha \geqslant \frac{1}{2}$).*
2. *If $\alpha < \frac{1}{2}$, then the firm increases reusability if k is low.*
3. *If $\alpha < \frac{1}{2}$ and k is high, then firm decreases reusability if α is sufficiently high.*

Discussion of Lemma EC.5: The firm chooses to reduce the reusability level if the supply-risk is high (i.e., $\alpha \geqslant 1/2$). However, when the risk is low (i.e., $\alpha < 1/2$), the firm's strategy is governed by the design cost k and the consumers' trade-in propensity λ that, in turn, governs the number of units traded-in by the consumers (i.e., ξ_e). When $\alpha < 1/2$, the firm always chooses to increase the reusability level when the design cost is low and gain cost advantage in refurbishing all the units traded-in and selling them during normal periods (because $\xi_e < \xi_o^N$) when there is no supply disruption. Finally, when the design cost k is high it hurts the firm's to increase reusability when the risk is sufficiently high (i.e., $\alpha \to \frac{1}{2}$ in $[0, \frac{1}{2})$).

Lemma EC.6. *The optimal trade-in fee factor v^* that the firm sets when it chooses the premium-factor and reusability level as $\tilde{x}$ and $\tilde{\theta}$ is given as in* (26).

Appendix D: Technical Appendix

D.1. Proofs of lemmas

Proof of Lemma 1: The proof follows directly from (4) for each case: (i) $\alpha < \gamma(r) \Leftrightarrow r < \gamma(\alpha)$ and (ii) $\alpha \geqslant \gamma(r) \Leftrightarrow r \geqslant \gamma(\alpha)$. When $\alpha < \gamma(r) \Leftrightarrow r < \gamma(\alpha)$, we obtain as shown in (4) that consumers who own new units choose to trade-in their used units if, and only if, $V > \left(\frac{p(1-\delta)}{1-r}\right) \wedge V(\gamma(\alpha) - r) > (\gamma(\alpha) - v)p \Leftrightarrow V > \left(\frac{p(1-\delta)}{1-r}\right) \wedge V > \frac{(\gamma(\alpha)-v)p}{\gamma(\alpha)-r} \Leftrightarrow V > \max\left\{\frac{p(1-\delta)}{1-r}, \frac{(\gamma(\alpha)-v)p}{\gamma(\alpha)-r}\right\}$. The segment of consumers who choose to trade-in when $\alpha < \gamma(r)$ is as illustrated in Figure 4a. Therefore, it is easy to observe that no consumer chooses to trade-in if, and only if, $\frac{(\gamma(\alpha)-v)p}{\gamma(\alpha)-r} > 1 \Leftrightarrow v < v_1(\alpha)$. Likewise, all consumers trade-in their units if, and only if, $\frac{(\gamma(\alpha)-v)p}{\gamma(\alpha)-r} < \frac{(1-\delta)p}{1-r} \Leftrightarrow v > v_2(\alpha)$. Furthermore, for any v, the maximum risk that can ensure trade-ins is given by $v = v_1(\alpha) \Leftrightarrow \alpha = [v_1^{-1}(v)]^+ = \gamma\left(\frac{r-pv}{1-p}\right) \cdot \mathbb{I}_{v > \frac{r+p-1}{p}}$, where $\mathbb{I}_C = 1$, if the condition C is true and 0 otherwise. In a similar manner, the second statement can be proved when $\alpha \geqslant \gamma(r)$ and the demand structure in this case is illustrated as in Figure 4b. The maximum risk that can ensure trade-ins when $\alpha \geqslant \gamma(r)$ can be obtained as $\alpha = v_2^{-1}(v)(> \gamma(r) > 0)$. The third statement follows from the fact that $V < \frac{p(1-\delta)}{1-r}$ either buy refurbished units, or do not purchase the product, as shown in Figure 3.

Next, by noting that (i) $v_1(\alpha) = v_2(\alpha) \Leftrightarrow \alpha = \gamma(r)(> 0)$, (ii) $v_1'(\alpha) > 0$ and $v_2'(\alpha) > 0$, (iii) $v_1(\alpha) \geqslant v_2(\alpha) \Leftrightarrow \alpha > \gamma(r)$, and (iv) $\alpha \in [0, 1]$, we can conclude that $\alpha_{max}(v)$ is as defined in (7) so that no consumer chooses to trade-in if, and only if, $\alpha > \alpha_{max}(v)$, for any trade-in fee v. ■

Proof of Lemma 2: Through differentiating, we can conclude that $\pi^{(x,x)} = \frac{-2(1-\alpha)\alpha\delta^2 p^2}{(1-\beta)r} < 0$, whenever $\xi_e \geqslant \xi_o^D \Leftrightarrow x \geqslant x_0$, and $\pi^{(x)} = \frac{\alpha(1-\alpha)\delta p \xi_e}{1-\beta} > 0$, whenever $\xi_e < \xi_o^D \Leftrightarrow x < x_0$, so that the optimal premium-factor $x^*(\theta)$ is given by (19). ■

Proof of Lemma 3: By differentiating (19), we obtain $x^{*\prime}(\theta) \leqslant 0$, $x^{*(\alpha)} = 0$ if $x^*(\theta) = \frac{c(1-\theta)\phi + r}{2\delta p}$ and $x^{*(\alpha)} = \frac{-r\xi_e^{(\alpha)}}{\delta p} \geqslant 0$ if $x^*(\theta) = x_0$, where x_0 is defined in Lemma 2, because if (i) $v \notin [\min\{v_1(\alpha), v_2(\alpha)\}, \max\{v_1(\alpha), v_2(\alpha)\}]$, then $\xi_e^{(\alpha)} = 0$ and (ii) if $v \in [\min\{v_1(\alpha), v_2(\alpha)\}, \max\{v_1(\alpha), v_2(\alpha)\}]$, then $\xi_e^{(\alpha)} = \frac{\rho p|r-v|\gamma'(\alpha)}{(\gamma(\alpha)-r)^2} < 0$, which proves the result. Recall that, $v_1(\alpha) < v_2(\alpha) < r$ if $\alpha < \gamma(r)$, $v_1(\alpha) = v_2(\alpha) = r$ if $\alpha = \gamma(r)$, and $r < v_2(\alpha) < v_1(\alpha)$ if $\alpha > \gamma(r)$. ■

Proof of Lemma 4: Through differentiation, it is easy to check that $\pi^{(\theta,\theta)} = -2k$ so that the optimal reusability level is obtained through the first-order-condition. By imposing the constraint $\theta \leqslant 1$, we obtain the optimal reusability level as in (20).

Next, the proof of Statement 1 of the lemma follows from direct differentiation of (20). Next, by noting that $\xi_o^{D\,(x)} \leqslant 0$, $\xi_o^{N\,(x)} = 0$, and $\xi_e^{(x)} = 0$, we obtain that $\theta^{*\prime}(x) \leqslant 0$, which is Statement 2 of the lemma.

Finally, by differentiating (20) with respect to α, we obtain:

$$\theta^{*(\alpha)} = \frac{-\theta^*}{1-\alpha} + \frac{c\phi(1-\alpha)}{2k(1-\beta)} \cdot \frac{\partial}{\partial\alpha}\Big\{\alpha\min\{\xi_e,\xi_o^D\} + (1-\alpha)\min\{\xi_e,\xi_o^N\}\Big\} \tag{EC.4}$$

so that $\lim_{\alpha\to 1}\theta^{*(\alpha)} = \lim_{\alpha\to 1}\frac{-\theta^*}{1-\alpha} = \frac{-c\phi\min\{\xi_e,\xi_o^D\}}{2k(1-\beta)} < 0$, by using (20) and the fact that $\frac{\partial}{\partial\alpha}\{\alpha\min\{\xi_e,\xi_o^D\} + (1-\alpha)\min\{\xi_e,\xi_o^N\}\} < \infty$. This proves Statement 3 of the lemma. The last statement is straightforward by noting that the sales of refurbished units are given by $\min\{\xi_e,\xi_o^D\}$ and $\min\{\xi_e,\xi_o^N\}$ during D and N, respectively. ■

Proof of Lemma 5: Through differentiation, it can be shown that the Hessian of π with respect to (θ, x) is negative semidefinite if, and only if, $k \geqslant k_0$, which is defined in Assumption 5. Next, by solving the first order conditions $\pi^{(x)} = 0$ and $\pi^{(\theta)} = 0$, and by noting that (i) $x \geqslant x_0$ for any θ from Lemma 2, and (ii) $x^{*\prime}(\theta) < 0$ with $\theta \in [0,1]$, we can obtain the optimal premium-factor as in (23). Next, by noting that $\theta \in [0,1]$ and $\theta^{*\prime}(x) < 0$ (from Lemma 4), we obtain the optimal reusability level as in (24). ■

Proof of Lemma 6: First, we note that:

$$\begin{aligned}
\xi_e \geqslant \xi_o^D(p,\tilde{x},\delta) \Leftrightarrow \xi_e \geqslant 1 - \frac{\tilde{x}\delta p}{r} &= 1 - \frac{2k(1-\beta)(r+c\phi) - c^2\phi^2(1-\alpha)\left(\alpha + (1-\alpha)\min\{\xi_e,\xi_o^N\}\right)}{4kr(1-\beta) - c^2\phi^2\alpha(1-\alpha)} \\
&= \frac{2k(1-\beta)(r-c\phi) + c^2\phi^2(1-\alpha)^2\min\{\xi_e,\xi_o^N\}}{4kr(1-\beta) - c^2\phi^2\alpha(1-\alpha)}.
\end{aligned}$$

Next, therefore, if $\xi_e \geqslant \xi_o^N$, then $x_{opt} = \tilde{x} \Leftrightarrow \xi_e \geqslant \xi_o^D(p,\tilde{x},\delta) \Leftrightarrow \xi_e \geqslant \frac{2k(1-\beta)(r-c\phi)+c^2\phi^2(1-\alpha)^2\xi_o^N}{4kr(1-\beta)-c^2\phi^2\alpha(1-\alpha)}$.
On the other hand, if $\xi_e < \xi_o^N$, then:

$$\begin{aligned}
\xi_e \geqslant \xi_o^D(p,\tilde{x},\delta) \Leftrightarrow \xi_e \geqslant \frac{2k(1-\beta)(r-c\phi) + c^2\phi^2(1-\alpha)^2\min\{\xi_e,\xi_o^N\}}{4kr(1-\beta) - c^2\phi^2\alpha(1-\alpha)} &= \frac{2k(1-\beta)(r-c\phi) + c^2\phi^2(1-\alpha)^2\xi_e}{4kr(1-\beta) - c^2\phi^2\alpha(1-\alpha)} \\
\Leftrightarrow 4kr(1-\beta) - c^2\phi^2(1-\alpha) > 0 \wedge \xi_e \geqslant \frac{2k(1-\beta)(r-c\phi)}{4kr(1-\beta) - c^2\phi^2(1-\alpha)}&.
\end{aligned}$$

By combining the above two conditions, we can conclude that $\xi_e \geqslant \xi_o^D \Leftrightarrow \xi_e \geqslant \overline{\xi}_e$. Note that if $\xi_e < \xi_o^N$ and $4kr(1-\beta) - c^2\phi^2(1-\alpha) \leqslant 0$, then $\frac{2k(1-\beta)(r-c\phi)+c^2\phi^2(1-\alpha)^2\xi_o^N}{4kr(1-\beta)-c^2\phi^2\alpha(1-\alpha)} \geqslant \xi_o^N$ and therefore, $\xi_e < \xi_o^D(p,\tilde{x},\delta) \Leftrightarrow \tilde{x} \leqslant x_0 \Leftrightarrow x_{opt} = x_0$.

Now, when $\xi_e \geqslant \overline{\xi}_e$, we have the interior optimizer $x_{opt} = \tilde{x}$ and $\theta_{opt} = \tilde{\theta}$, which satisfy the first-order-condition (19). Therefore, through differentiation of the interior optimal solution, we can obtain that $x_{opt}^{(\alpha)}/\theta_{opt}^{(\alpha)} = \tilde{x}^{(\alpha)}/\tilde{\theta}^{(\alpha)} = -\frac{c\phi}{2\delta p} < 0$. This proves the first part of the result.

Next, when $\xi_e \leqslant \bar{\xi}_e$, we have $x_{opt} = x_0$ and $\theta_{opt} = \theta^*(x_0)$. Hence, by differentiating we obtain $x_0^{(\alpha)} = \frac{(1-\beta)\rho r|r-v|}{\delta(\alpha-\alpha\beta r+r-1)^2} \geqslant 0$ whenever $v \leqslant \max\{v_1(\alpha), v_2(\alpha)\}$ from (8) and (9), and $x_0^{(\alpha)} = 0$, otherwise (since $\xi_e^{(\alpha)} = 0$). Next, by substituting x_0 in (20),we obtain:

$$\theta^*(x_0(\alpha),\alpha) = \frac{c\phi(1-\alpha)\left(\alpha\xi_e + (1-\alpha)\min\{\xi_e,\xi_o^N\}\right)}{2k(1-\beta)}$$

$$\Rightarrow \frac{d}{d\alpha}[\theta^*(x_0(\alpha),\alpha)] = \frac{c\phi}{2k(1-\beta)}\left[(1-2\alpha)\xi_e - 2(1-\alpha)\min\{\xi_e,\xi_o^N\} + \alpha(1-\alpha)\xi_e^{(\alpha)} + (1-\alpha)^2\frac{\partial}{\partial\alpha}\min\{\xi_e,\xi_o^N\}\right],$$

Now, we consider two cases:

Case (i): $\xi_e \leqslant \xi_o^N$ ($\Leftrightarrow \alpha \geqslant \tilde{\alpha}_1$, which is the unique value such that $\xi_e(\tilde{\alpha}_1) = \xi_o^N$, as $\xi_e^{(\alpha)} = \frac{p\rho|r-v|\gamma'(\alpha)}{(\gamma(\alpha)-r)^2} \leqslant 0$), and
Case (ii): $\xi_e > \xi_o^N$ ($\Leftrightarrow \alpha < \tilde{\alpha}_1$).

In case (i) since $\xi_e \leqslant \xi_o^N$, we obtain $\frac{d}{d\alpha}[\theta^*(x_0(\alpha),\alpha)] \leqslant 0$, because $\xi_e^{(\alpha)} = \frac{p\rho|r-v|\gamma'(\alpha)}{(\gamma(\alpha)-r)^2} < 0$.

In case (ii), when $\xi_e > \xi_o^N$, we have $\frac{d}{d\alpha}[\theta^*(x_0(\alpha),\alpha)] = \frac{c\phi}{2k(1-\beta)}\left[(1-2\alpha)\xi_e - 2(1-\alpha)\xi_o^N + \alpha(1-\alpha)\xi_e^{(\alpha)}\right] = \frac{c\phi}{2k(1-\beta)}\left[(1-2\alpha)(\xi_e-\xi_o^N) - \xi_o^N + \alpha(1-\alpha)\xi_e^{(\alpha)}\right]$. Clearly, $\frac{d}{d\alpha}[\theta^*(x_0(\alpha),\alpha)] \leqslant 0$ if $\alpha \geqslant \frac{1}{2}$. Moreover, if $\alpha \leqslant \frac{1}{2}$, then $\frac{d^2}{d\alpha^2}[\theta^*(x_0(\alpha),\alpha)] = \frac{c\phi}{2k(1-\beta)}\left[-2(\xi_e-\xi_o^N) + 2(1-2\alpha)\xi_e^{(\alpha)} + \alpha(1-\alpha)\xi_e^{(\alpha,\alpha)}\right] \leqslant 0$ because $\xi_e^{(\alpha,\alpha)} = \frac{p\rho|r-v|(\gamma''(\alpha)(\gamma(\alpha)-r)-2(\gamma'(\alpha))^2)}{(\gamma(\alpha)-r)^2} \leqslant 0$ as $\gamma''(\alpha)(\gamma(\alpha)-r) - 2(\gamma'(\alpha))^2 = \frac{-2(1-\beta)(1-\beta r)}{(1-\alpha\beta)^3} < 0$. Therefore, there exists a threshold $\tilde{\alpha}_2 \leqslant \frac{1}{2}$ such that $\frac{d}{d\alpha}[\theta^*(x_0(\alpha),\alpha)] \leqslant 0$ if and only if $\alpha \geqslant \tilde{\alpha}_2$. The threshold $\tilde{\alpha}_2$ is given by the stationary point of the concave function $\theta^*(x_0(\alpha),\alpha)$; i.e., the unique solution of $\frac{d}{d\alpha}[\theta^*(x_0(\alpha),\alpha)] = 0 \Leftrightarrow (1-2\alpha)\xi_e - 2(1-\alpha)\xi_o^N + \alpha(1-\alpha)\xi_e^{(\alpha)} = 0$. Now, the proof of the lemma follows by taking $\hat{\alpha} = \min\{\tilde{\alpha}_1, \tilde{\alpha}_2\}$ so that $\frac{d}{d\alpha}[\theta^*(x_0(\alpha),\alpha)] \leqslant 0 \Leftrightarrow \alpha \geqslant \hat{\alpha}$. ■

Proof of Lemma 7: By differentiating the expression for $\tilde{\theta}$ we obtain: $\tilde{\theta}^{(\delta)} = \frac{-2c\phi p(1-\alpha)^2}{(1-r)(4kr(1-\beta)-c^2\phi^2\alpha(1-\alpha))} \leqslant 0$ and $\tilde{\theta}^{(p)} = \frac{2c\phi(1-\alpha)^2(r-\delta)}{(1-r)(4kr(1-\beta)-c^2\phi^2\alpha(1-\alpha))} \geqslant 0$. ■

Proof of Lemma EC.1: First, we note that x_{opt} and θ_{opt} are continuous functions. Next, by differentiating the interior solution, we obtain: $\theta_{opt}^{(k)} = -\frac{4(1-\beta)r\theta_{opt}}{4kr(1-\beta)-c^2\phi^2\alpha(1-\alpha)} \leqslant 0$ and $x_{opt}^{(k)} = \frac{2cr\phi(1-\beta)\theta_{opt}}{\delta p(4kr(1-\beta)-c^2\phi^2\alpha(1-\alpha))} \geqslant 0$. Likewise, when $x_{opt} = x_0$, then $x_{opt}^{(k)} = 0$ and $\theta_{opt}^{(k)} = \theta^{*(k)}(x_0) = -\theta^*(x_0)/k \leqslant 0$. ■

Proof of Lemma EC.2: Before proceeding, we note that $\theta_0 = \frac{2\delta p(x^*(0)-x_0)}{c\phi} = \frac{c\phi+r-2x_0\delta p}{c\phi}$ is the unique value that satisfies $x^*(\theta_0) = x_0$ so that $\theta_0 = x^{*-1}(x_0)$. Now, we consider two cases: (i) $x^*(0) < x_0$ (i.e., $\theta_0 < 0$) and (ii) $x^*(0) \geqslant x_0$ (i.e., $\theta_0 \geqslant 0$).

In case (i), when $x^*(0) < x_0$, we obtain that $x_0 > x^*(0)(\geqslant x^*(1) = \frac{r}{2\delta p})$, because $x^{*\prime}(\theta) \leqslant 0$, so that $x_{opt} = x_0(=\max\{x_0, \frac{r}{2\delta p}\})$. Now, since $\pi(\theta, x_0; p, v, \delta)$ is concave in $\theta \in [0,1]$, we obtain $\theta_{opt} = \theta^*(x_0)$.

In (ii), when $x^*(0) \geqslant x_0$, we can obtain that $\lim_{\theta\to 0}\pi^{(\theta)}(\theta, x^*(\theta); p, v, \delta) = c\phi\left(\frac{1-\alpha}{1-\beta}\right) \cdot [\alpha\xi_e + (1-\alpha)\min\{\xi_e,\xi_o^N\}] \geqslant 0$, which indicates that $\theta = \min\{\theta_0, 1\}$ is the optimal solution in the interval $\theta \in [0, \min\{\theta_0, 1\}]$ and $x_{opt} = \max\{x_0, \frac{r}{2\delta p}\}$. Next, in the interval $\theta \in [\theta_0, 1]$, we obtain that $\frac{r}{2\delta p} = x^*(1) \leqslant x^*(\theta) \leqslant x_0$, so that $x_{opt} = x_0(=\max\{x_0, \frac{r}{2\delta p}\})$. Then $\pi(\theta, x_0; p, v, \delta)$ is concave in $\theta \in [\theta_0, 1]$. Therefore, the optimal solution in the interval $[\theta_0, 1]$ is given by the first order condition $\pi^{(\theta)}(\theta, x_0; p, v, \delta) = 0$, that is $\theta = \max\{\theta_0, \theta^*(x_0)\}$. Therefore, the optimal θ value in the interval $[0,1]$ is given by $\theta_{opt} = \max\{\theta_0, \theta^*(x_0)\}$. Now, since $x_{opt} = x_0$ we have $\xi_e = \xi_o^D$; therefore, we can obtain that

$$\begin{aligned}
\theta^*(x_0) - \theta_0 &= \frac{c\phi(1-\alpha)}{2k(1-\beta)}\left[\alpha\xi_e + (1-\alpha)\min\{\xi_e,\xi_o^N\}\right] - \frac{c\phi + r - 2x_0\delta p}{c\phi} \\
&= \frac{c^2\phi^2(1-\alpha)\left[\alpha\xi_e + (1-\alpha)\min\{\xi_e,\xi_o^N\}\right] - 2k(1-\beta)\left[c\phi + r - 2r(1-\xi_e)\right]}{2kc\phi(1-\beta)} \\
&= \frac{\left[c^2\phi^2\alpha(1-\alpha) - 4kr(1-\beta)\right]\xi_e + c^2\phi^2(1-\alpha)^2\min\{\xi_e,\xi_o^N\} + 2k(1-\beta)\left[r - c\phi\right]}{2kc\phi(1-\beta)} \geqslant 0,
\end{aligned}$$

because $k \leqslant k_0 \Leftrightarrow c^2\phi^2\alpha(1-\alpha)-4kr(1-\beta)\geqslant 0$. Therefore, $\theta_{opt}=\theta^*(x_0)$.

Next, as shown in the proof of Lemma 6, we can obtain that $x_0^{(\alpha)}=\frac{(1-\beta)\rho r|r-v|}{\delta(\alpha-\alpha\beta r+r-1)^2}\geqslant 0$ whenever $v\leqslant \max\{v_1(\alpha),v_2(\alpha)\}$ from (8) and (9), and $x_0^{(\alpha)}=0$, otherwise (since $\xi_e^{(\alpha)}=0$), so that $x_0^{(\alpha)}\geqslant 0$, always.

Next, again as shown in the proof of Lemma 6, we can conclude that $\frac{d}{d\alpha}[\theta^*(x_0(\alpha),\alpha)]=\frac{c\phi}{2k(1-\beta)}\left[-\xi_e+(1-\alpha)\xi_e^{(\alpha)}\right]<0$ if $\xi_e\leqslant\xi_o^N$ ($\Leftrightarrow \alpha\geqslant\tilde{\alpha}_1$, as explained in the proof of Lemma 6). Likewise, in the case when $\xi_e>\xi_o^N$ we can obtain that $\frac{d}{d\alpha}[\theta^*(x_0(\alpha),\alpha)]=\frac{c\phi}{2k(1-\beta)}\left[(1-2\alpha)(\xi_e-\xi_o^N)-\xi_o^N+\alpha(1-\alpha)\xi_e^{(\alpha)}\right]$ so that $\frac{d}{d\alpha}[\theta^*(x_0(\alpha),\alpha)]<0$, for all $\alpha\geqslant\frac{1}{2}$. When $\alpha<\frac{1}{2}$, we can argue as in the proof of Lemma 6 that $\theta^*(x_0(\alpha),\alpha)$ is concave in α so that $\frac{d}{d\alpha}[\theta^*(x_0(\alpha),\alpha)]<0$ if, and only if, $\alpha>\tilde{\alpha}_2$, where $\tilde{\alpha}_2$ is the unique stationary point of $\theta^*(x_0(\alpha),\alpha)$, which is defined by the unique solution of $\frac{d}{d\alpha}[\theta^*(x_0(\alpha),\alpha)]=0\Leftrightarrow(1-2\alpha)(\xi_e-\xi_o^N)-\xi_o^N+\alpha(1-\alpha)\xi_e^{(\alpha)}=0$, and is independent of k. Clearly, $\tilde{\alpha}_2<1/2$ because $\lim_{\alpha\to 1/2}\frac{d}{d\alpha}[\theta^*(x_0(\alpha),\alpha)]=-\xi_o^N+\frac{\xi_e^{(\alpha)}}{4}<0$. Next, the proof follows by setting $\hat{\alpha}=\min\{\tilde{\alpha}_1,\tilde{\alpha}_2\}$ ■

Proof of Lemma EC.3: Through differentiation, we obtain $\tilde{x}^{(p)}=\frac{r\left((1-\alpha)\alpha c^2\phi^2-2(1-\beta)k(c\phi+r)\right)}{\delta p^2(4(1-\beta)kr-(1-\alpha)\alpha c^2\phi^2)}\geqslant 0\Leftrightarrow k\leqslant\frac{2rk_0}{r+c\phi}$ and $\tilde{x}^{(\delta)}=\frac{r\left((1-\alpha)c^2\phi^2((1-\alpha)p+\alpha(1-r))-2(1-\beta)k(1-r)(c\phi+r)\right)}{\delta^2 p(1-r)(4kr(1-\beta)-c^2\phi^2\alpha(1-\alpha))}\geqslant 0\Leftrightarrow k\leqslant\frac{c^2\phi^2(1-\alpha)((1-\alpha)p+\alpha(1-r))}{2(1-\beta)(1-r)(c\phi+r)}$. Also, $\frac{2rk_0}{r+c\phi}\geqslant k_0$ because $r\geqslant c\phi$ and $\frac{(1-\alpha)c^2\phi^2((1-\alpha)p+\alpha(1-r))}{2(1-\beta)(1-r)(c\phi+r)}-\frac{2rk_0}{r+c\phi}=\frac{c^2\phi^2p(1-\alpha)^2}{2(1-\beta)(1-r)(c\phi+r)}\geqslant 0$. ■

Proof of Lemma EC.4: By using the FOC (19) and implicit function theorem to differentiate (17) and the optimal solution $\tilde{\theta}$ and $\tilde{x}$, we obtain:

$$\frac{d\pi}{d\delta}(\tilde{\theta}(\delta),\tilde{x}(\delta);p,v,\delta)=(p-c)\xi_n^{(\delta)}+(1-\alpha)p\xi_e+\left[p(1-v)-c+(1-\alpha)(\delta p-c\phi(1-\tilde{\theta}))\right]\xi_e^{(\delta)}. \tag{EC.5}$$

Clearly, from (1), (2), (8), and (9), we can conclude that $\xi_n^{(\delta)}\geqslant 0$ and $\xi_e^{(\delta)}\geqslant 0$. Furthermore, $\xi_e^{(\delta)}=0$, if $\alpha\leqslant\gamma(r)$. Therefore, $\frac{d\pi}{d\delta}>0$ if $\alpha\leqslant\gamma(r)$. Now, since $\xi_e^{(\delta)}\geqslant 0$ and $\xi_o^{N(\delta)}<0$, we can infer that the firm will always choose δ high enough so that $\xi_e\geqslant\xi_o^N$. Next, when $\alpha>\gamma(r)$, we can infer that $\frac{d\pi}{d\delta}>0$ if $p(1-v)\geqslant c$ and $\delta p\geqslant\phi c(1-\tilde{\theta})$. Therefore, using the above argument, we can conclude that $\xi_e\geqslant\xi_o^N$. Note that whenever p is sufficiently high, or costs c and $c\phi$ are sufficiently low, we obtain $p(1-v)\geqslant c$ and $\delta p\geqslant\phi c(1-\tilde{\theta})$, from which the result follows. ■

Proof of Lemma EC.5: By differentiating $\tilde{\theta}$ that is given in (22) with respect to α when $\xi_e<\xi_o^N$, we obtain:

$$\tilde{\theta}^{(\alpha)}=2cr\phi\left[\frac{(1-\alpha)^2\xi_e^{(\alpha)}}{(4(1-\beta)kr-(1-\alpha)\alpha c^2\phi^2)}-\frac{(1-\alpha)\xi_e\left(8(1-\beta)kr-(1-\alpha)c^2\phi^2\right)+2k(2\alpha-1)(1-\beta)(r-c\phi)}{\left(4(1-\beta)kr-(1-\alpha)\alpha c^2\phi^2\right)^2}\right].$$

Clearly, if $\alpha\geqslant 1/2$, then $\tilde{\theta}^{(\alpha)}<0$, since $\xi_e^{(\alpha)}\leqslant 0$ and $8(1-\beta)kr-(1-\alpha)c^2\phi^2>0$, for $k>k_0$. Next, if $\alpha<1/2$, we can express $\tilde{\theta}^{(\alpha)}$ as follows:

$$\tilde{\theta}^{(\alpha)}=\frac{2cr\phi}{\left(4(1-\beta)kr-(1-\alpha)\alpha c^2\phi^2\right)^2}\left[\begin{array}{c}2k(1-\beta)(1-2\alpha)(r-c\phi)+\rho(1-\alpha)\{c^2\phi^2(1-\alpha)\left(\frac{\xi_e}{\rho}-\alpha(1-\alpha)\frac{\xi_e^{(\alpha)}}{\rho}\right)\\-4kr(1-\beta)\left(2\frac{\xi_e}{\rho}-(1-\alpha)\frac{\xi_e^{(\alpha)}}{\rho}\right)\}\end{array}\right]$$

so that if k is sufficiently small; i.e., $k\leqslant\frac{c^2\phi^2(1-\alpha)\left(\frac{\xi_e}{\rho}-\alpha(1-\alpha)\frac{\xi_e^{(\alpha)}}{\rho}\right)}{4r(1-\beta)\left(2\frac{\xi_e}{\rho}-(1-\alpha)\frac{\xi_e^{(\alpha)}}{\rho}\right)}$ (recall that $\xi_e^{(\alpha)}\leqslant 0$), then $\theta^{(\alpha)}\geqslant 0$. On the other hand, if $k>\frac{c^2\phi^2(1-\alpha)\left(\frac{\xi_e}{\rho}-\alpha(1-\alpha)\frac{\xi_e^{(\alpha)}}{\rho}\right)}{4r(1-\beta)\left(2\frac{\xi_e}{\rho}-(1-\alpha)\frac{\xi_e^{(\alpha)}}{\rho}\right)}$, then $\theta^{(\alpha)}<0$ if α is sufficiently high, i.e., $\alpha\to 1/2$ in $[0,\frac{1}{2})$. ■

Proof of Lemma EC.6: By substituting $\tilde{\theta}$ and $\tilde{x}$ in (17), and by differentiating, we obtain $\pi^{(v,v)}(\tilde{\theta},\tilde{x};p,v,\delta)=\frac{-2(1-\alpha)p^2\rho}{(1-\beta)(\gamma(\alpha)-r)}\leqslant 0$, so that the optimal value of v is obtained by the first-order condition $\pi^{(v)}(\tilde{\theta},\tilde{x};p,v,\delta)=0\Rightarrow v=\frac{p-c+r-(1-p)\gamma(\alpha)}{2p}$. Then, by noting that $v\leqslant v_2(\alpha)$ ■

D.2. Proofs of propositions

Proof of Proposition 1: By differentiating $\tilde{\theta}$ that is given in (22) with respect to α, we obtain:

$$\tilde{\theta}^{(\alpha)} = 2c\phi\left[\frac{(1-\alpha)p(\delta-r)\left(8(1-\beta)kr-(1-\alpha)c^2\phi^2\right)}{(1-r)\left(4(1-\beta)kr-(1-\alpha)\alpha c^2\phi^2\right)^2}+\frac{2(1-2\alpha)(1-\beta)kr(r-c\phi)}{\left(4(1-\beta)kr-(1-\alpha)\alpha c^2\phi^2\right)^2}\right]$$

Now, we consider two cases: (i) $\alpha<\frac{1}{2}$, and (ii) $\alpha\geqslant\frac{1}{2}$. In (i) when $\alpha<\frac{1}{2}$, we obtain that, if $k\leqslant\frac{k_0}{2\alpha}=\frac{c^2\phi^2(1-\alpha)}{8r(1-\beta)}\Leftrightarrow 8kr(1-\beta)-c^2\phi^2(1-\alpha)\leqslant 0$, then $\tilde{\theta}^{(\alpha)}\geqslant 0$ since $r>\delta$. On the other hand, if $k>\frac{k_0}{2\alpha}=\frac{c^2\phi^2(1-\alpha)}{8r(1-\beta)}\Leftrightarrow 8kr(1-\beta)-c^2\phi^2(1-\alpha)>0$, then:

$$\tilde{\theta}^{(\alpha)}\geqslant 0\Leftrightarrow\delta\geqslant\overline{\delta}\equiv\overline{\delta}(\alpha)=r-\frac{2kr(1-r)(1-2\alpha)(1-\beta)(r-c\phi)}{(1-\alpha)p\left(8kr(1-\beta)-c^2\phi^2(1-\alpha)\right)}(<r).$$

Note that, when $k\leqslant\frac{k_0}{2\alpha}\Leftrightarrow\overline{\delta}\geqslant r$. Therefore, we can conclude that $\tilde{\theta}^{(\alpha)}\geqslant 0\Leftrightarrow\delta\geqslant\overline{\delta}$. Next, in case (ii) when $\alpha\geqslant\frac{1}{2}$ we always have $k\geqslant k_0/2\alpha$; hence, $\tilde{\theta}^{(\alpha)}\leqslant 0$, because $r>\delta$. ■

Proof of Proposition 2: Clearly if $\alpha\geqslant\frac{1}{2}$ then $\tilde{x}^{(\alpha)}\geqslant 0\geqslant\tilde{\theta}^{(\alpha)}$ from Proposition 1. Next, when $k\geqslant\frac{k_0}{2\alpha}$ and $\alpha\leqslant\frac{1}{2}$, then $\tilde{\theta}^{(\alpha)}\geqslant 0$ if, and only if, $\delta\geqslant\overline{\delta}(\alpha)$. Next, from (25), we can observe that $\overline{\delta}'(\alpha)\geqslant 0$ so that $\overline{\alpha}$ is the unique value that satisfies $\overline{\delta}(\overline{\alpha})=\delta$. Noting that $\overline{\delta}(\alpha)-\delta=0$ can be written as a concave quadratic in α, we can conclude that:

$$\overline{\alpha}\equiv\overline{\alpha}(k)=\frac{\left[\begin{array}{c}c^2\phi^2p(r-\delta)+2kr(1-\beta)((1-r)(r-c\phi)-2p(r-\delta))\\-\sqrt{2kr(1-\beta)\left[c^2\phi^2p(r-\delta)(1-r)(r-c\phi)+2kr(1-\beta)((1-r)(r-c\phi)-2p(r-\delta))^2\right]}\end{array}\right]}{c^2\phi^2p(r-\delta)},\tag{EC.6}$$

which is the smaller root of the quadratic $\overline{\delta}(\alpha)-\delta=0$. Next, we can observe that (i) $k_0(\alpha)$ and $\frac{k_0(\alpha)}{2\alpha}$ intersect at $\alpha=\frac{1}{2}$ and when $k_0(\alpha)=\frac{c^2\phi^2}{16r(1-\beta)}=\frac{k_0(\alpha)}{2\alpha}$, and (ii) $\overline{\alpha}\geqslant\frac{1}{2}\Leftrightarrow k\leqslant\frac{c^2\phi^2}{16r(1-\beta)}$, so that we can conclude that $\tilde{\theta}^{(\alpha)}\geqslant 0\geqslant\tilde{x}^{(\alpha)}\Leftrightarrow\delta\geqslant\overline{\delta}(\alpha)\Leftrightarrow\alpha\leqslant\overline{\alpha}(k)\leqslant\frac{1}{2}$. ■