Managing Product Reusability under Supply Disruptions

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We model and analyze product reusability (executed through either refurbishing or remanufacturing), in the presence of supply disruptions. By assuming that consumers could exchange their used units for new ones and firms can attract consumers to do so through an exchange fee, we develop a discrete-time stochastic model to determine the degree of product-reusability, price, and exchange fee in steady state.

Our analysis of this model shows that increasing product's reusability through better product design is beneficial to a certain extent when the probability of supply disruption increases. However, increasing reusability further, when the disruption probability is high, could hurt a firm's profit due to increasing design costs. We further show that, when consumers are more likely to exchange used units, increasing product-reusability is beneficial only when the price is high. Next, we discuss how a manufacturer should set the exchange fee to entice consumers to exchange their old units and set the discounted price for refurbished units that it sells in the market. We also show that increasing product-reusability need not be always beneficial even when manufacturer can set its product's price. Hence, a firms needs to carefully choose the right level of product-reusability to maximize manufacturer's profits.

Finally, we compare product refurbishing and remanufacturing and observe that refurbishing could be more viable than remanufacturing when supply disruption is less likely, because of the additional expenditure that remanufacturing incurs when compared to refurbishing.

Key words: Product reusability, refurbishing, remanufacturing, supply disruption, product design, product exchanges

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1. Introduction

One of the Sustainable Development Goals (SDG) set by the United Nations in 2015 is "Responsible consumption and production." It aims to reduce and manage waste by properly reusing products and resources that have been returned, discarded, or disposed of at various stages of a product's life cycle. Refurbishing, remanufacturing, and recycling are the three most widely used methods to reuse products and resources. According to regulatory policies set by governments worldwide, the adoption of such practices is on the rise in many industries. Additionally, there has been a recent increase in the discussion about how such reuse practices can help businesses withstand supply disruptions like the Suez Canal blockade and the COVID-19 pandemic (UNEP-report 2020). An emerging perception is that product and resource reuse practices can provide a back-up supply for companies in the event of a disruption. Thus, reuse practices are increasingly being seen as local value-retention strategies that can aid firms in coping with and recovering from a disruptive event (EIONET-report 2021). In this paper, we analyze product reusability decisions in the presence of supply disruptions.

Supply chains are not unfamiliar with disruptions. Natural disasters, sanctions (such as on imports and exports), employee unrest, political conflict, sudden economic fluctuations, and other similar events are examples of common disruptions. During such disruptions, ensuring a secure supply for continued operations is the most common concern. A recent example of a supply disruption caused by the Coronavirus outbreak is the shortage of computer chips (commonly referred to as "chipageddon"), that impacted a variety of products, including computers, automobiles, and mobile phones (Kelion 2021). The industries that make these products are still trying to recover from these shortages.

Existing risk mitigation strategies that address supply or sourcing disruptions, such as order splitting and multi sourcing, are advantageous for securing supply (Anupindi and Akella 1993, Tomlin 2006). However, from a long-term perspective of resource availability and responsible consumption, reuse practices can aid a company in coping with disruptions of varying sizes. Reuse practices, such as remanufacturing, are advocated by expert practitioners and organisations as a way for businesses to better manage disruptions (Alicke et al. 2020, Fitzsimons 2020). During the pandemic, for instance, HP launched its remanufactured laptops alongside its mainstream products with the slogan "*We believe in reincarnation – at least when it applies to HP Notebooks*" (REMATEC 2020).

Our own interviews with a few senior remanufacturing professionals from firms in different geographical regions (both developed and emerging economies) revealed the importance of reuse practices in general, and particularly during supply disruptions.[1] One of the professionals commented, *"recycling and remanufacturing surely is playing a critical role in the recent times as we have seen imports have been banned and there was a huge supply chain disruption going on. Having said that companies still have not cracked on recycling or remanufacturing in big way yet. It's just a start and has a long way to go till recycling and remanufacturing become everyday norm. Recent supply chain disruptions has given it a big boost though."* Another professional pointed, *"This [i.e., using reuse practices as a tool to address supply disruption] is true but only possible for a short to medium term fix."* These opinions from practitioners further motivated us to investigate how to manage product-reusability under supply disruptions, and potentially mitigate the supply shortage risks that can result from such disruptive events.

[1] We approached multiple remanufacturing professionals around the globe. Three professionals (from the UK, Singapore, and India) responded to our interview invitations. They held positions of Director-remanufacturing, Group manager-advanced remanufacturing, and Senior research fellow at their respective organisations.

Based on industrial practice and academic literature, we classify reuse practices into two categories, namely, (1) *refurbishment*, and (2) *remanufacturing* and *recycling*. The primary distinction between these two categories is the product quality after processing of the returned unit. The quality of the refurbished product is inferior to that of the original, and consumers' willingness to pay decreases as a result. On the contrary, in remanufacturing and recycling, the returned unit's quality is restored to that of a newly manufactured unit of the product (Thierry et al. 1995, Chen and Chen 2019).

A carefully designed product helps a firm to extract greater value from used products during reuse operations (Gershenson et al. 2003, Souza 2017). This is typically achieved by enhancing key product characteristics such as "disassemblability" and "reusability". For example, Dell's product-design methodology articulates the role of modular product-design as a means of providing "easy access and disassembly" in implementing their initiatives for the circular economy (Shrivastava and Schafer 2018). Interactions with remanufacturing professionals revealed a significant demand for "design for remanufacturing." One of our respondents commented, *"surely if a firm is going for recycling or remanufacturing process they need to think on the designs right away."* However, it is important to note that although product-design techniques such as modularization aid a firm's reuse operations, they are usually accompanied by significantly higher costs of the modularization process. Thus, such costs should be carefully considered in any analysis of product reusability under supply disruptions.

In this paper, we address the following specific questions when a firm faces possible supply disruptions:

1. *Is it always beneficial to increase product-reusability for easy refurbishability or remanufacturability as supply disruptions get more likely?*

2. *Should a firm increase its product-reusability when its customer's are more likely to exchange older units?*

3. *Should a firm opt for refurbishing or remanufacturing when there is possibility of supply disruptions?*

To the best of our knowledge, this is the first paper that studies product-reusability with reuse practices such as refurbishment and remanufacturing in the presence of supply disruptions. In practice, it is often very difficult to obtain confidential and proprietary product-level data on product-designs and reuse mechanisms from manufacturing companies. Therefore, we elected to develop a stylized discrete-time stochastic model to analyze this problem in a more general manner. Such analysis can then be employed to derive useful business insights, which can assist firms in designing efficient products for reuse when there are supply disruptions.

The following are some key findings based on our analysis of product reusability under supply disruptions:

1. We show that increasing product-reusability as supply disruption risk increases could be beneficial to a certain extent. However, it would hurt the net profits when the disruption probability is high due to design costs. This is true even when manufacturer can set its product's price.

2. We find that if consumers' propensity to exchange used units increases, it is beneficial to increase reusability only when the price is sufficiently high.

3. We show that refurbishing can be beneficial than remanufacturing whenever the supply disruption probability is low.

This paper is organised as follows. In Section 2, we discuss the literature related to our work. In Section 3, we develop the model for the case when reusability is executed via refurbishment. In Section 4, we analyse this model and draw insights on product-reusability and pricing decisions of the firm. In Section 5, we discuss the model along with the analysis for the case when reusability is executed through remanufacturing (and recycling). In Section 6, we numerically demonstrate how to choose price of new units when we have product reusability and supply disruption. In the concluding section, we summarize our work and provide future research directions. All proofs are provided in the appendix.

2. Literature Review

This paper falls in the intersection between the literature on closed loop supply chains (CLSC) and that on supply disruption. Table 1 summarizes a few significant studies in the CLSC domain and highlights the positioning of our work.

Paper	**PM***	**Supply Disruption**	**CLSC setting**
Guide Jr et al. (2003)	N	N	Remanufacturing
Mukhopadhyay and Setoputro (2005)	Y	N	Recycling
Ray et al. (2005)	N	N	Remanufacturing
Ferrer and Swaminathan (2006)	N	N	Remanufacturing
Vorasayan and Ryan (2006)	N	N	Refurbishing
Geyer et al. (2007)	N	N	Remanufacturing
Zikopoulos and Tagaras (2007)	N	N	Refurbishing
Wu (2012)	Y	N	Remanufacturing
Subramanian et al. (2013)	Y	N	Refurbishing
Wang et al. (2017)	N	N	Remanufacturing
Raz and Souza (2018)	N	N	Recycling
Alev et al. (2020)	N	N	Recycling
Borenich et al. (2020)	N	N	Refurbishing
Gui (2020)	Y	N	Recycling
Agrawal et al. (2021)	Y	N	Leasing
Rahmani et al. (2021)	Y	N	Recycling
Our Study	***Y***	***Y***	***Refurbishing vs. Remanufacturing & Recycling***

Table 1 Positioning our research in the extant CLSC literature (Analytical studies). (*PM stands for Product Modularity.)

CLSC can be defined as *the design, control, and operation of a system to maximize value creation over the entire life cycle of a product with dynamic recovery of value from different types and volumes of returns over time* (Guide Jr and Van Wassenhove 2009). The CLSC literature revolves around various activities involved in such a process, including product returns/acquisition, reverse logistics, parts recovery and reuse (remanufacturing/refurbishing/recycling), and remarketing. We use the five-phase framework proposed by Guide Jr and Van Wassenhove (2009) to narrow down the literature focus for our paper. Our study falls in the realm of phases four and five discussed. Phase four considers CLSC issues over the entire product life-cycle (i.e., both forward and reverse logistics), such as product-design decisions, while phase five addresses the market dynamics (such as cannibalization issues) and product valuation (i.e., remanufactured/refurbished product's prices). Since our paper pertains to product-design (for its reusability) and product pricing decisions of a firm, we discuss the extant studies that have dealt with these issues in the CLSC literature. Table 1 highlights that none of the studies in CLSC literature have focused on product-design decisions in confluence with supply disruption, as we do in this paper. In addition, unlike many of these papers, we also compare the options of "refurbishing" and "remanufacturing & recycling" to examine their viability under different levels of supply disruption risk.

The fundamental premise behind product-design aspects in the CLSC literature is that a well-designed product can help firms to extract maximum value out of the returned product units. Mukhopadhyay and Setoputro (2005) study the value of product modularity for a firm implementing a return policy for its build-to-order product. They consider the trade-off between the cost of modularity and the salvage value of the returned and recycled product. Similarly, Wu (2012) considers the disassemblability of a product that helps a firm in reducing remanufacturing costs but requires upfront fixed cost. Subramanian et al. (2013) study a multi-product setting and evaluate the value of component commonality (CC) in a refurbishing scenario. They explore the trade-off associated with a higher CC, i.e., trade-off between higher production cost for the low-end product and lower refurbishing cost for the high-end product. More recent studies (Gui 2020, Rahmani et al. 2021) explore the role of product-design in the recycling context. However, none of the above papers explore the role of product-reusability in CLSC operations, when a firm faces supply disruptions.

In the context of refurbishment operations, the pricing decision of refurbished product in relation to a new product becomes a key lever for a firm. A few studies in the CLSC literature based on refurbishment operations investigate the optimal price for refurbished products (Vorasayan and Ryan 2006, Borenich et al. 2020). However, here again, these studies do not consider the product-reusability aspects under supply disruptions.

Next, without elaborating the details of supply uncertainty literature, we define the context of supply disruption that we adopt in our paper. Here, the literature classifies uncertainty into two broad categories: (1) yield uncertainty and (2) supply disruption. Under yield uncertainty, the supply is not entirely stalled; however, the quantity supplied remains uncertain. On the other hand, under supply disruption, the suppliers either supply the entire order when there is no disruption or supply nothing in case of a disruption (Tomlin 2006, Kumar et al. 2018). We refer the reader to Fang and Shou (2015) and Kumar et al. (2018) for a comprehensive review of literature on yield uncertainty and supply disruption, respectively. Out of the above two uncertainties, in this paper, we focus on the second type of supply disruptions that usually have a low occurrence probability but are highly detrimental when they occur (Sheffi and Jr. 2005, Kleindorfer and Saad 2005).

3. Model for Refurbishment

In this section, we present a model to analyze reusability when it is executed through refursbishment. We start by introducing the consumers' valuation model through which we derive the demand for *new* (i.e., freshly manufactured) and *old* (i.e., refurbished) units of a product. We adopt the linear-city model (Tirole and Jean 1988, Huang et al. 2013) to describe a consumer's valuation V for the new product, and we assume that V is uniformly distributed in the interval $[0,1]$. We also assume that a consumer, who values a new unit at V, will value an old (i.e., refurbished) unit at $r \cdot V$, where $0 < r < 1$. Let p and δp, where $0 < \delta \leqslant 1$, denote the unit prices a firm charges for new and refurbished units, respectively. Therefore, a consumer with valuation V will choose to purchase a new unit of the product when, $V - p > rV - \delta p$ and $V > p$ and chooses to purchase a refurbished unit when, $V - p < rV - \delta p$ and $rV > \delta p$. On the other hand, if $V < p$ and $rV < \delta p$, the the customer will not purchase the product. To avoid trivializing the problem, we make the following assumption:

Assumption 1. *The percentage reduction in price of a refurbished unit over a new unit is higher than the relative reduction in consumer's valuation. That is, $r > \delta$.*

It is easy to observe that, in the absence of the above assumption, only the demand for new units exists whereas the demand for refurbished units does not exist. By using the above assumption, we can observe that the demands for *new* and *old* (or refurbished) units, which we denote by $\xi_n(p,\delta)$ and $\xi_o(p,\delta)$, respectively, are given by:

$$\xi_n(p,\delta) = 1 - \left(\frac{1-\delta}{1-r}\right) p, \text{ and} \tag{1}$$

$$\xi_o(p,\delta) = \left(\frac{1-\delta}{1-r} - \frac{\delta}{r}\right) p = \frac{p(r-\delta)}{r(1-r)}, \tag{2}$$

The above demand model is illustrated in Figure 1.

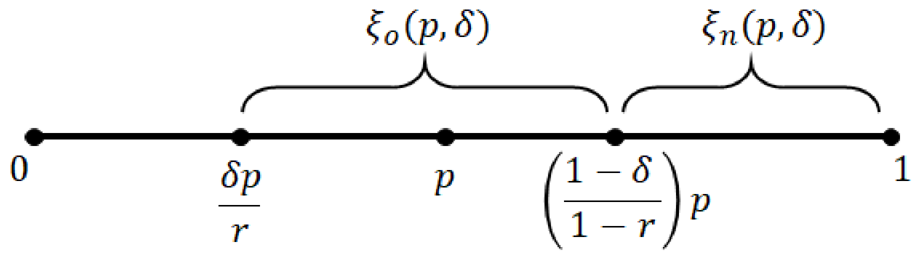


Figure 1 **Product demands in each period**

Next, we let the parameter $\alpha \in [0,1]$ denote the supply risk. We assume that the designated supplier fails to supply the crucial raw-materials to the firm with probability α during a period due to which the firm cannot produce new units whenever there is supply disruption. Nevertheless, despite the shortage of raw-materials to manufacture new units, the firm can continue to refurbish older units that are exchanged from its past customers for new units, at a discount of v offered by the firm. The discrete state transition diagram of the firm across time period is given in Figure 2.

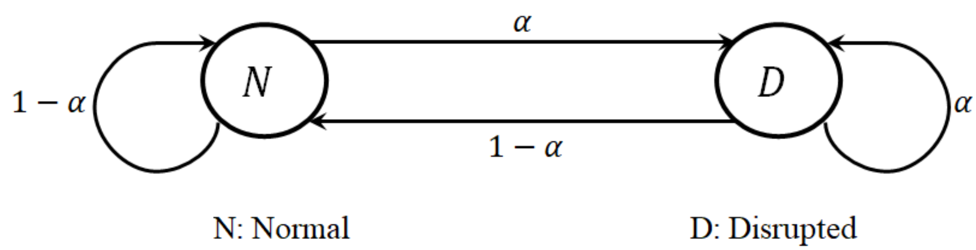


Figure 2 **State transition diagram for infinite-period problem (N: Normal supply; D: Disrupted supply).**

The next assumption explains the sequence of manufacturer's decisions that is commonly followed in practice:

Assumption 2. *The manufacturer chooses product's prices before state transitions occur and should freeze the product design before the product is sold in the market.*

Now, in the case of supply disruption, we assume that the demand for new units from the customers who intended to exchange their old units is backlogged and is fulfilled whenever the raw-material supply is restored. Such consumers pay for the new unit whenever it is delivered by the firm. By using β to denote the discount factor, we can conclude that a customer prefers to exchange a used product that they own for a new unit when the net utility from exchanging the used unit is higher than that derived from using it, or:

$$v + (V-p)\big((1-\alpha)+(1-\alpha)\alpha\beta+(1-\alpha)\alpha^2\beta^2+\cdots\big) > rV. \tag{3}$$

The right-hand side of (3) is the customer's utility by using the old unit that they own. The left hand side denotes the net utility that the customer gains by exchanging the owned old unit for a new one. The first term v is the instantaneous exchange fee that the firm pays the customer for the returned unit. Next, since the demand is backlogged in the event of supply disruption and fulfilled whenever there is supply of raw materials, the second term denotes the present value of net expected utility that is obtained from the new unit for which the old one has been exchanged. In practice, manufacturers can backorder trade-in demand when new units are unavailable. Examples include mobile operators like T-mobile (T-Mobile 2022) and Verizon (Verizon 2021), while others like Apple (Oberoi 2021), Dell (Dell 2022), and BestBuy (BestBuy 2022) offer credit toward a new purchase (when available), which is equivalent to backordering the trade-in demand. Such a situation may also arise in automobiles

We make the following assumption about the type of exchanges the firm will entertain:

Assumption 3. *The firm accepts only the units that are one period old for exchange.*

The above assumption means that a customer who purchases a new product in the current period can exchange it for another new unit only in the following period and no later. This simplifying assumption is practical because manufacturers usually do not entertain very old units to be exchanged for brand-new units keeping in view the deteriorating residual value of the unit being exchanged.[2]

Next, we let λ denote the exogenous probability that an owner of a used unit is intrinsically inclined to trade-in the old unit for a new one in a period. While some exchanges could be contractual off-lease, some consumers would choose to exchange due to their preference for new units driven by sense of exclusivity and vogue, more so in an environmentally and socially responsible manner (Stanley 2020, Kate 2020, Grauer 2021, Sapra et al. 2010). However, some consumers may choose not to exchange due to behavioral factors (Thaler 1985, Desai et al. 2016).

It should be noted that a consumer may choose to use a new product over multiple periods by exchanging their used unit in every period with probability λ, as long as such exchange is affordable according to (3). Thus, λ denotes consumer's affinity to use new product in a responsible way, and we term it as *exchange propensity* of a consumer. Therefore, by using (3) and the fact that product exchanges are backlogged whenever there is supply disruption (i.e., whenever transition occurs to

[2] Our model can be extended to accommodate exchange of older units with age i at a return fee of v_i, where $v_1 > v_2 > \cdots > 0$, with additional notation.

state D in Figure 2), we can obtain the number of consumers requesting product exchange after $n+1$ periods of normal supply (i.e., after $n+1$ transitions to state N in Figure 2)[3] as:

$$\sum_{i=1}^{n}\left\{\begin{matrix}\text{\# of consumers who purchased a new unit in the}\\ i\text{th transition to } N \text{ and continued to exchange since then}\end{matrix}\right\}$$
$$=\sum_{i=1}^{n}\lambda^i\cdot\mathbb{P}\left[V>\left(\frac{1-\delta}{1-r}\right)p, V(\gamma-r)>\gamma p-v\right], \tag{4}$$

where we define $\gamma=\frac{1-\alpha}{1-\alpha\beta}(<1)$ for ease of exposition. Next, we make the following assumption to focus our analysis on more pragmatic case of supply disruption.

Assumption 4. *The probability of supply disruption α is low so that $\gamma>r$, where $\gamma=\frac{1-\alpha}{1-\alpha\beta}(<1)$.*

The above assumption is realistic in the context of supply disruptions because, as stated in our literature review, supply disruptions are rare but have substantial negative impacts on supply chains when they occur.[4] Therefore, in the steady state, the demand for new units due to product exchanges is given by:

$$\xi_e(p,v,\delta)=\left(\frac{\lambda}{1-\lambda}\right)\cdot\mathbb{P}\left[V>\max\left\{\left(\frac{1-\delta}{1-r}\right)p, \frac{\gamma p-v}{\gamma-r}\right\}\right].$$

To ensure in every period that the number of customers opting to exchange old units is less than the number of customers who owned new units in the previous period, we require v to be set such that $\left(\frac{1-\delta}{1-r}\right)p\leqslant\frac{\gamma p-v}{\gamma-r}$, so that:

$$\xi_e(p,v)\equiv\xi_e(p,v,\delta)=\left(\frac{\lambda}{1-\lambda}\right)\cdot\left(1-\frac{\gamma p-v}{\gamma-r}\right). \tag{5}$$

All the notations used in this paper are listed in Table 2.

3.1. Market clearing exchange fee

We now determine the exchange fee that the firm should set in order to match the supply of exchanged old units with the demand for refurbished units that are sold at the reduced price of δp per unit, when operating in the normal state N.[5] We characterize the market-clearing exchange fee in the following result:

Lemma 1. *The exchange fee offered by the firm is given by:*

$$v^*(p,\delta)=\min\left\{\gamma p-\left(\frac{1-\delta}{1-r}\right)p(\gamma-r), \gamma p-(\gamma-r)\left(1-\frac{(1-\lambda)\xi_o(p,\delta)}{\lambda}\right)\right\}. \tag{6}$$

[3] Note that if there are k disruption periods, i.e., k transitions to state D, during $n+k$ periods, the following will be the number of product exchanges in the $(n+k+1)$th period.

[4] Our analysis largely holds true when $\gamma\leqslant r$, but we skip it to focus on more practical situation of supply disruptions.

[5] We do not consider the case when a manufacturer chooses to strategically hold the exchanged units for multiple future periods. Manufacturers tend to reuse the exchanged units within a period to avoid holding costs.

Variable	Definition
N	State when the supply is normal (see Figure 2).
D	State when the supply is disrupted (see Figure 2).
V, rV	Consumer valuation for a new and a refurbished unit, respectively.
$p, \delta p$	Price of a new and a refurbished unit, respectively.
$c, \phi c$	Unit cost of manufacturing and refurbishing, respectively.
k	Product design cost factor.
θ	Product-reusability level.
α	Probability of supply disruption.
v	Exchange fee offered by the firm.
λ	Intrinsic inclination of a consumer to exchange old unit for a new one.
β	Discounting factor.
ξ_n	Demand for new units.
ξ_o	Demand for refurbished units.
ξ_e	Number of used units exchanged by consumers.

Table 2 Summary of Notation

From Lemma 1 it is clear that to satisfy a higher demand for refurbished units (i.e., higher $\xi_o(p,\delta)$), the firm should offer a higher return fee $v^*(p,\delta)$ to obtain adequate supply of old units through exchanges. The following result further explains the characteristics of the market-clearing exchange fee $v^*(p,\delta)$:

LEMMA 2. *The return-fee $v^*(p,\delta)$ defined in (6) satisfies the following:*

1. *A lower discount on refurbished units requires a lower exchange fee (i.e., $\frac{\partial v^*}{\partial \delta} < 0$).*
2. *A higher price p requires a higher exchange fee (i.e., $\frac{\partial v^*}{\partial p} > 0$).*

Since a lower discount on refurbished units will the decrease the demand for these units, firms set a lower exchange fee to reduce the number of consumers who choose to exchange their used units. Likewise, since a higher price dissuades consumers to exchange their old units (as observed from (3)), the firm offers a higher exchange fee to entice its consumers to exchange their units, to ensure the supply of old units for refurbishment.

3.2. Product Refurbishment

We assume that the unit production cost is lower for a refurbished product than for a product that is freshly manufactured from raw materials. The lower cost of a refurbished unit can be attributed to various factors of which the primary being that reusing modular components reduces the machine and labor time in processing a refurbished unit.

Refurbishing a product entails "*involves anything from running a few simple tests to undertaking a thorough clean-up and rebuild of the product*" (Canon 2021); so, unit cost of refurbishing is less than that of remanufacturing. Therefore, we let c and ϕc, where $\phi \in (0,1)$, to denote the unit costs of manufacturing and refurbishing, respectively. Next, we let $\theta \in [0,1]$ denote the "*product-reusability*" (i.e., the ability of product's design to facilitate easy refurbishability) so that the total production cost of a refurbished unit is $\phi c(1-\theta)$. Clearly, the higher the reusability of the product, the lower is its refurbishing cost $\phi c(1-\theta)$. However, we note that incorporating better reusability θ in the product-design entails some cost. We assume that such product-design cost is convex increasing in θ, indicating that it is increasingly costlier to enhance product reusability through its modular design, an assumption that is widely used in the literature (Rahmani et al. 2020). For functional convenience we assume that the cost is given by $k\theta^2$.

3.3. Firm's State-dependent Profits

Let $\pi_N(p,\delta,\theta)$ and $\pi_D(p,\delta,\theta)$ denote the firm's steady state profits at states N and D, respectively, when the price of a newly manufactured unit is p, the discount percent for refurbished unit is $1-\delta$, and the refurbishability built into the product-design is θ. By using the state transitions depicted in Figure 2, we can obtain the expressions of $\pi_N(p,\delta)$ and $\pi_D(p,\delta)$ as follows:

$$\begin{aligned}\pi_N(p,\delta) &= (1-\alpha)\left[(\delta p-\phi c(1-\theta)-v^*(p,\delta))\cdot\xi_o(p,\delta)+(p-c)\cdot(\xi_o(p,\delta)+\xi_n(p,\delta))+\beta\pi_N(p,\delta)\right]\\ &\quad+\alpha\left[(\delta p-\phi c(1-\theta)-v^*(p,\delta))\cdot\xi_o(p,\delta)+\beta\pi_D(p,\delta)\right] \text{ and,} \qquad (7)\\ \pi_D(p,\delta) &= (1-\alpha)\left[(p-c)\cdot(\xi_o(p,\delta)+\xi_n(p,\delta))+\beta\pi_N(p,\delta)\right]+\alpha\left[\beta\pi_D(p,\delta)\right]. \qquad (8)\end{aligned}$$

If the current state is N (i.e., normal operation without disruption), then the future state can be either N or D with probabilities $(1-\alpha)$ and α, respectively. In a state transition from N to N, the firm can sell both refurbished and new products, which includes exchanges as well. Therefore, the profit earned by the firm is the sum of $(\delta p-\phi c(1-\theta)-v^*(p,\delta))\cdot\xi_o(p,\delta)$, which is the revenue earned through refurbished products, and $(p-c)\cdot(\xi_o(p,\delta)+\xi_n(p,\delta))$, which is the revenue earned from new units.[6] Likewise, a transition from N to D, will allow the firm to sell only refurbished units without any new production, so that the firm's profit is $(\delta p-\phi c(1-\theta)-v^*(p,\delta))\cdot\xi_o(p,\delta)$.[7] The demand for new units due to exchanges of old units is backlogged and will be satisfied when the next transition occurs from D to N. By using the probabilities to weight the above profit expressions, we obtain $\pi_N(p,\delta)$ as given in (7).

Likewise, then the system is at state D, transitioning again to D, which happens with probability α, will earn no profit. Next, transitioning from N to D will enable firm to earn profit through newly produced units only (along with the demand due to exchanges backlogged during the latest transition from N to D) so that the profit earned is $(p-c)\cdot(\xi_o(p,\delta)+\xi_n(p,\delta))$, with probability $(1-\alpha)$. Therefore, the expected profit at state D is given by $\pi_D(p,\delta)$, which is given in (8).

[6] It should be recalled that the number of exchanges of old units is equal to the refurbished demand due to $v^*(p,\delta)$.

[7] It is important to note that although the demand for refurbished units is higher that $\xi_o(p,\delta)$ in state D because a few consumers who prefer new to refurbished units may choose to purchase refurbished ones due to the unavailability of new ones, the manufacturer can only sell $\xi_e(p,v^*(p,\delta),\delta)=\xi_o(p,\delta)$ units.

Next, by solving (7) and (8) for the conditional profits $\pi_N(p,\delta)$ and $\pi_D(p,\delta)$, we can then obtain the expected profit of the firm as:

$$\begin{aligned}\pi(p,\delta,\theta) &= (1-\alpha)\cdot\pi_N(p,\delta)+\alpha\cdot\pi_D(p,\delta)-k\theta^2\\ &= \{(\delta p-\phi c(1-\theta)-v^*(p,\delta))\cdot\xi_o(p,\delta)+(p-c)\cdot(\xi_o(p,\delta)+\xi_n(p,\delta))\}\cdot\left(\frac{1-\alpha}{1-\beta}\right)-k\theta^2, \quad (9)\end{aligned}$$

because the steady state probabilities of the Markov chain in Figure 2 residing in states N and D are $(1-\alpha)$ and α, respectively.[8] For any price p, the firm has to decide the optimal levels of refurbishability θ that should be incorporated in the product, along with the discount factor δ that should be offered for refurbished units. Hence, the optimization problem of the firm is given by:

$$\max_{\delta,\theta}\pi(p,\delta,\theta) \quad (10)$$

4. Discount and Product-reusability Decisions for Refurbishment

In this section, we discuss the firm's decisions on discount δ for refurbished units (i.e., pricing the refurbished units) and reusability θ, by solving (10) for a given price p. We also examine the impact of various parameters on these decisions and draw some practical insights.

4.1. Optimal Discount for Refurbished Units

Now, we obtain the optimal discount factor $\delta^*(\theta)$ the firm offers for refurbished units, for any product-reusability level θ. It is easy to observe that $\pi(p,\delta,\theta)$ is concave in δ so that $\delta^*(\theta)$ is given by the first order condition. The following result characterizes $\delta^*(\theta)$:

LEMMA 3. *Let* $\delta_0=\frac{r(p-(1-r)\lambda)}{p(1-(1-r)\lambda)}(<r)$. *The optimal discount factor* $\delta^*(\theta)$ *that the firm offers for purchasing refurbished units is given by:*

$$\delta^*(\theta)=\min\left\{r,\max\{\delta_0,\tilde{\delta}(\theta)\}\right\}, \quad (11)$$

where $\tilde{\delta}(\theta)$ *is the unique solution of the first order condition (i.e.,* $\frac{\partial\pi}{\partial\delta}=0$*), and whose closed form is given as:*

$$\tilde{\delta}(\theta)=\frac{r}{2p}\cdot\frac{p(\lambda(r(5-\gamma-2r)-(\gamma+1))+2(\gamma-r))-\lambda(1-r)(\gamma-r-c(1-r+\phi(1-\theta)))}{\gamma(1-\lambda)-r(1-\lambda(2-r))}. \quad (12)$$

For any product refurbishability θ, choosing the discount factor $\delta^*(\theta)$ above maximizes the firms profit while ensuring the clearance of refurbished units. The following lemma further characterizes the optimal discount factor:

LEMMA 4. *The discount-factor* $\delta^*(\theta)$ *defined in* (11) *satisfies the following:*

[8] This can be obtained through obtaining the steady state probabilities, using the transition probability matrix for the Markov chain given in Figure 2.

1. *The firm charges a lower price for refurbished units when the product-reusability is high (i.e.,* $\frac{\partial \delta^*}{\partial \theta} \leqslant 0$*).*

2. *The firm charges a higher price for refurbished units whenever the unit production cost is high (i.e.,* $\frac{\partial \delta^*}{\partial c} \geqslant 0$*).*

3. *If the reusability* θ *is high, (so that* $\delta^*(\theta) = \delta_0$*), then lower discount rate is offered for refurbished units whenever the price* p *is higher (i.e.,* $\frac{\partial \delta^*}{\partial p} > 0$*). On the other hand, if* θ *is low (so that* $\delta^*(\theta) = \tilde{\delta}(\theta)$*), then a higher discount rate can be offered at higher price* p *if, and only if, the production cost is low (i.e.,* $\frac{\partial \delta^*}{\partial p} < 0 \Leftrightarrow c > \frac{\gamma - r}{(1-\theta)\phi + 1 - r}$*).*

The first two statements of Lemma (4) are straightforward. Since the firm's production cost of a refurbished unit decreases as the product-reusability increases, the firm can afford to offer higher discount (or equivalently charge lower price) for refurbished units, as explained in the first statement. Likewise, if the unit cost is high, then a higher price is charged for a refurbished unit due to its increased production cost as indicated in the second statement. However, the impact of the price p on the discount percent, which is given in the third statement, is more intricate and needs some discussion.

Note that refurbished units are highly discounted when the reusability θ is high (because from the first statement $\delta^*(\theta)$ decreases to its least value δ_0, when θ is sufficiently high). Next, when the price p increases, the demand for new units decreases due to which the number of consumers available to exchange old units also decreases. Therefore, in order to match supply of old units with demand for refurbished units, the firm decreases the discount on refurbished units as the price p of new units increases (i.e., $\frac{\partial \delta_0}{\partial p} > 0$).

On the other hand if the reusability θ is low (so that $\delta^*(\theta) = \tilde{\delta}(\theta) > \delta_0$), then the impact of price p on optimal discount rate $\delta^*(\theta)$ is moderated by the unit production cost c. First, we note that the firm's overall sales decrease when p increases. Next, when both θ and c are low, the total cost saving $\theta\phi c$ in refurbishment is too insignificant to compensate for the revenue lost due to the decreased sales when p increases. Hence, the firm chooses to lower the discount offered on refurbished units (i.e., $\frac{\partial \tilde{\delta}}{\partial p} > 0$ if $c < \frac{\gamma - r}{(1-\theta)\phi + 1 - r}$). However, if c is high, the cost saving $\theta\phi c$ is significant and this enables the firm to offer a higher discount rate as p increases (i.e., $\frac{\partial \tilde{\delta}}{\partial p} \leqslant 0$ if $c \geqslant \frac{\gamma - r}{(1-\theta)\phi + 1 - r}$).

4.2. Optimal Product-reusability

In this section, we observe the optimal reusability $\theta^*(\delta)$, for a predetermined discount factor δ. By noting from (9) that the profit is concave in θ (for any δ), we provide the optimal product-reusability and its sensitivity in the result below:

Lemma 5. *For any discount factor* δ*, the optimal product-reusability is given by:*

$$\theta^*(\delta) = \min\left\{1, \frac{cp\phi(1-\alpha)(r-\delta)}{2kr(1-\beta)(1-r)}\right\}. \tag{13}$$

Moreover, the above optimal reusability level is:

1. *increasing in* p *and* c *(i.e.,* $\frac{\partial\theta^*}{\partial p} > 0$ *and* $\frac{\partial\theta^*}{\partial c} > 0$*), and*
2. *decreasing in* δ*,* k*, and* α *(i.e.,* $\frac{\partial\theta^*}{\partial\delta} < 0$*,* $\frac{\partial\theta^*}{\partial k} < 0$ *and* $\frac{\partial\theta^*}{\partial\alpha} < 0$*).*

Note that an increase in price p decreases the product's demand, which results in fewer sales. Hence, the firm commensurately decreases its refurbishment cost through higher reusability (i.e., $\frac{\partial\theta^*}{\partial p} > 0$) in order to compensate for the revenue loss due to fewer sales. All the other sensitivities, except with respect to the disruption probability α, are straightforward, and we skip their discussion.

It is surprising to note that the optimal product-reusability $\theta^*(\delta)$ is decreasing when the probability of disruption α is higher (i.e., $\frac{\partial\theta^*}{\partial\alpha} < 0$), because one would expect the converse to hold true for easy product refurbishment when the disruption probability is high (as opined in the interviews mentioned in the introduction). However, the decrease in $\theta^*(\delta)$ is due to the fact that as α increases, the firm's profit will decrease because the firm stays in disruption state (i.e., D in Figure 2)for longer. This observation poses the question if it benefits to increase product-reusability in the presence of supply disruption, and, if yes, then to what extent. We explore this question in the next section (i.e., in 4.3.1).

4.3. Jointly Optimal Discount Factor and Product-reusability

In this section, we obtain the optimal discounted price for refurbished product and optimal product-reusability when the firm can make both these decisions simultaneously. By substituting $\theta^*(\delta)$ from (13) in $\pi(p,\delta,\theta)$ given in (9), we can show that $\pi(p,\delta,\theta^*(\delta))$ is concave in δ when the reusability cost k is high, which is largely true in practice due to the high research & developmental, capital, and other such costs involved in designing a product. Therefore, we make the following assumption:

ASSUMPTION 5. *The reusability cost* k *is substantial so that* $k > k_0$*, where* $k_0 = \frac{c^2\phi^2\lambda(1-\alpha)}{4(1-\beta)((1-\lambda)(\gamma-r)+\lambda(1-r)r)}$. [9]

The following result provides the optimal decisions of the firm:

LEMMA 6. *Let* δ_0*,* k_0*, and* $\theta^*(\delta)$ *be as defined in Lemma 4, Assumption and* (13)*, respectively. The optimal discount factor* δ_{opt} *and product-reusability* θ_{opt} *are given by:*

$$\delta_{opt} = \min\left\{r, \max\left[\delta_0, r\left(1 - \frac{k\lambda(1-r)((1-\gamma)p + (\gamma-r) - c(1-r+\phi))}{2p(k-k_0)((1-\lambda)(\gamma-r)+\lambda(1-r)r)}\right)\right]\right\} \text{ and} \tag{14}$$

$$\theta_{opt} = \min\left\{\theta^*(\delta_0), \frac{c\lambda(1-\alpha)((1-\gamma)p + (\gamma-r) - c(1-r+\phi))}{4(1-\beta)(k-k_0)((1-\lambda)(\gamma-r)+\lambda(1-r)r)}\right\}. \tag{15}$$

[9] It should be noted that the condition $k > k_0$ is equivalent to the Hessian of $\pi(p,\delta,\theta)$, which given in (9), being negative-semidefinite in (θ,δ), for any given p; i.e., $k > k_0 \Leftrightarrow \frac{\partial^2\pi}{\partial\theta^2}\cdot\frac{\partial^2\pi}{\partial\delta^2} - \left(\frac{\partial^2\pi}{\partial\theta\partial\delta}\right)^2 < 0\,\forall p$, which indicates that $\pi(p,\delta,\theta)$ is concave in (θ,δ) whenevever $k > k_0$.

In order to draw useful practical insights, in what follows in the paper, we analyze the interior equilibrium and ignore the unrealistic extreme case when all owners of old units can exchange their product in a period.

The following result provides important insights by summarizes the impacts of reusable design cost k, which captures the cost efficiency of the firm in designing a product, and consumer's exchange propensity λ, which captures consumers' behavior and market characteristics, on the (interior) optimal discount factor and reusability level that are given in Lemma 6:

PROPOSITION 1. *Let* $c_0 = \frac{\gamma - r + p(1-\gamma)}{1-r+\phi}$. *The decisions* δ_{opt} *and* θ_{opt} *given in* (14) *and* (15), *respectively, satisfy the following:*

1. *A higher reusability cost (i.e., k) persuades the firm to increase refurbished unit's price (i.e., δ_{opt}) and decrease product-reusability (i.e. θ_{opt}) if, and only if, the production cost c is low (i.e., $\frac{\partial \delta_{opt}}{\partial k} \geqslant 0$ and $\frac{\partial \theta_{opt}}{\partial k} \leqslant 0 \Leftrightarrow c \leqslant c_0$).*
2. *A higher consumer's exchange propensity (i.e., λ) persuades the firm to decrease refurbished unit's price (i.e., δ_{opt}) and increase reusability (i.e. θ_{opt}) if, and only if, the production cost c is low (i.e., $\frac{\partial \delta_{opt}}{\partial \lambda} \leqslant 0$ and $\frac{\partial \theta_{opt}}{\partial \lambda} \geqslant 0 \Leftrightarrow c \leqslant c_0$).*

According to the proposition, reusability cost k and consumers' exchange propensity have countering effects on the product's discount factor δ_{opt} and product-reusability θ_{opt}.

Usually, one would anticipate that an increase in the reusability cost k will prompt the firm to decrease the optimal product-reusability level θ_{opt}. However, according to the first statement of the above proposition, this is true only when the unit production cost c is low (compared to the price p). If c is high (i.e., $c > c_0$), then firm will choose higher reusability as the design cost k increases (i.e., $\frac{\partial \theta_{opt}}{\partial k} \geqslant 0$), because the cost saving obtained from refurbishing the efficiently designed units outweighs the increased reusability cost. Moreover, the firm, diverts a fraction of its cost saving accrued from refurbishing to the consumers of refurbished units through higher discount ($\frac{\partial \delta_{opt}}{\partial k} \leqslant 0$) in order to encourage the demand for refurbished units, which, in turn, increases the firm's profit from the sales of refurbished units.

Next, in the second statement, a high consumer's exchange propensity λ results in more used units being exchanged for new ones and enables the firm to offer a lower return-fee. Therefore, when c is low (i.e., $c \leqslant c_0$), the increased profit from new units, due to low value of c, will encourage the firm to invest more in product-reusability (i.e., $\frac{\partial \theta_{opt}}{\partial \lambda} \geqslant 0$) so that the refurbishing cost is reduced. These increased savings on refurbished units and higher profit from new units will also enable the firm to offer higher discount on refurbished units (i.e., $\frac{\partial \delta_{opt}}{\partial \lambda} \leqslant 0$). On the other hand, a high unit production cost c (i.e., $c > c_0$) decreases the profit earned from new units, which dissuades the firm from choosing a high product-reusability (i.e., $\frac{\partial \theta_{opt}}{\partial \lambda} \leqslant 0$). The lower reusability chosen by the firm increases its cost of refurbishing that causes the firm to increase the price of refurbished units (i.e., $\frac{\partial \delta_{opt}}{\partial \lambda} \geqslant 0$).

4.3.1. Value of Reusability during Supply Disruption In this section, we answer the following question: *Does it benefit a firm to increase its product-reusability to counter supply disruption, when it can choose the discount it offers for refurbished units?* We introduce the following result, which helps in answering the above question:

PROPOSITION 2. *The impact of disruption probability α on product-reusability θ_{opt} choice is as follows. As the disruption probability α increases:*

1. *It is beneficial to increase product-reusability if, and only if, the price p is high (i.e., $\frac{\partial \theta_{opt}}{\partial \alpha} > 0$ if, and only if, p is sufficiently high).*

2. *It is not beneficial to increase product-reusability , when α is high (i.e., $\frac{\partial \theta_{opt}}{\partial \alpha} \leqslant 0$ when α is high).*

3. *It is not beneficial to increase product-reusability if the design cost k or unit production cost c is high (i.e., $\frac{\partial \theta_{opt}}{\partial \alpha} \leqslant 0$ when k or c is high).*

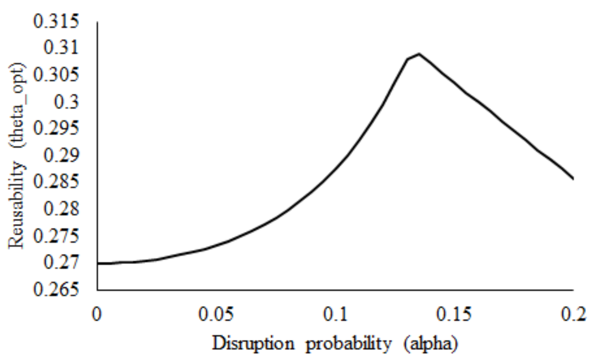
The above proposition highlights the important finding that it is not always beneficial to increase product-reusability as the disruption probability increases. The first statement proves that a sufficiently high price is required to increase product-reusability as disruption probability increases. Such a high price enables the firm to obtain sufficient profit from new units so that it can sustain its investment in increasing product-reusability in order to further save on its refurbishing cost. However, it is important to observe that the lower profits that firm earns by spending longer time in a disrupted state (i.e., state D in Figure 2) when the disruption probability α is high causes the firm to reduce product-reusability, in order to save on its reusability cost. This is demonstrated by the second statement. The third statement is more intuitive. The decreased profits when disruption probability α increases, along with the increased production cost when c is high or the increased reusability cost when k is high, make efficient product design less affordable.

Finally, we observe the impact of customer's exchange propensity λ on reusability θ_{opt}. Specifically, we address the question: *Should firm increase product-reusability when its customer's exchange propensity increases?* The following result helps in answering this question:

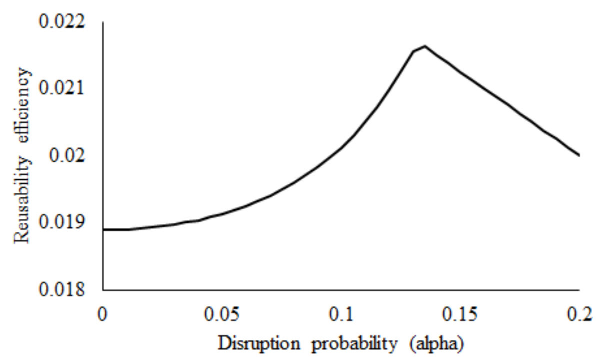
PROPOSITION 3. *If consumers are more likely to exchange used units for new ones (i.e., consumer's exchange propensity λ increases), then it is beneficial to increase product-reusability if, and only if, price is high (i.e., $p > \frac{[c(1-r+\phi)+r](1-\alpha\beta)-(1-\alpha)}{\alpha(1-\beta)}$).*

The higher profit earned when product's price is high will enable the firm to invest more in increasing its product-reusability. The high consumer's exchange propensity λ will result in more supply of old units and hence in more sales of refurbished units, when the exchange fee is set in a manner to match supply of old units with the demand for refurbished units.

4.3.2. Discussion Figures 3a and 3b provide the plots of product-reusability choice and the corresponding cost-benefit ratio, respectively, when p is high (corresponding plots when p is low are given in Figures 4a and 4b). As shown in Proposition 2, it is beneficial to reduce product-reusability when disruption probability α is high (see Figure 3a). Next, Figure 3b illustrates that the net benefit (i.e., savings in the production cost) to cost (i.e., the sum of exchange fee and the price discount) ratio, i.e., $\frac{c\theta_{opt}}{(1-\delta_{opt})p+v^*(p,\delta_{opt})}$, which can be termed as the "*return on reusability*". The plot illustrates that there exists a threshold value of α until which the ratio increases and decreases thereafter indicating that the returns from reusability diminish when α is high.

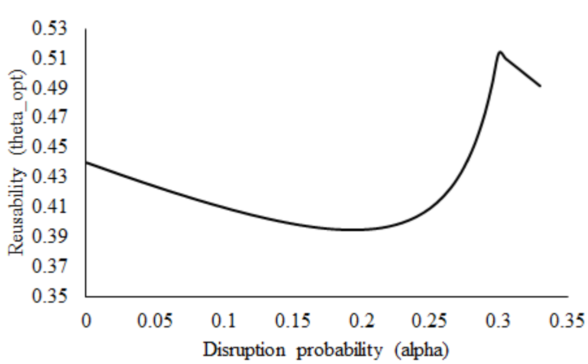


(a) Optimal reusability with probability of disruption

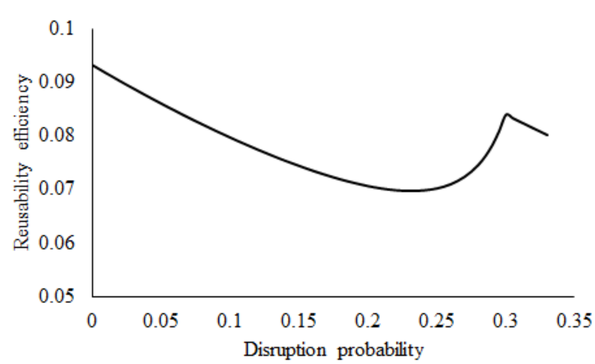


(b) Cost savings and discount cost with probability of disruption

Figure 3 Impact of probability of disruption on reusability choice when p is high.



(a) Optimal reusability with probability of disruption



(b) Cost savings and discount cost with probability of disruption

Figure 4 Impact of probability of disruption on reusability choice when p is low.

5. Model and Analysis for Remanufacturing

In the earlier sections, we modeled and discussed the benefits of product refurbishing and its role in mitigating risk of supply disruption faced by a manufacturing firm. In this section, we model and analyze remanufacturing to compare its performance against that of refurbishing. Recall that under refurbishing the quality of old and used units is improved to a level less than that of newly manufactured units. In contrast, remanufacturing ensures that an old unit is renewed and is brought up to the quality of a new unit (Thierry et al. 1995, Chen and Chen 2019, Canon 2021).

We use the following unit cost structure for remanufacturing:

$$c_r = (1-\psi)\cdot c + \psi \cdot \tilde{\phi} c(1-\theta), \tag{16}$$

where $\psi \in [0,1]$. The fraction $(1-\psi)$ denotes the portion of the cost incurred due to replaced components, whereas ψ denotes the fraction of cost due to reusing from old units, which is usually lower than replacement cost (i.e., $\tilde{\phi} < 1$), but possibly higher than the refurbishing cost (i.e., $\tilde{\phi} > \phi$) due to the extra work required in renewing the product (Canon 2021).[10] Furthermore, the product-reusability θ can further reduce the reusing cost to $\psi\tilde{\phi}c(1-\theta)$, as shown in (16). A similar decomposition of cost structure is proposed by Raz and Souza (2018) and the fact that product-reusability can reduce remanufacturing/recycling cost is adopted by Gui (2020).

Next, because remanufactured units are as good as newly manufactured ones and there are no old units, all units are priced at p. A consumer chooses to exchange the product for a new one when the consumer purchased a new unit in the past (i.e., $V \geqslant p$) and finds it beneficial to exchange it (i.e., $v + \gamma(V-p) \geqslant rV$). Using these two facts, and the customer's exchange propensity λ, we can obtain the number of consumers opting for exchange in steady state as:

$$\tilde{\xi}_e(p,v) = \left(\frac{\lambda}{1-\lambda}\right)\cdot \mathbb{P}\left[V > \max\left\{p, \frac{\gamma p - v}{\gamma - r}\right\}\right]. \tag{17}$$

To ensure inventory balance, so that the number of customers exchanging old units is always less than the number who own it in every period, we make the following assumption:

Assumption 6. *The firm offers return fee v such that $v \leqslant pr$.*

By using the state transitions given in Figure 2, and noting that remanufacturing required some new parts, due to which it cannot be undertaken when a transition occurs from N to D, we can obtain the expected profit of the firm as:

$$\tilde{\pi}(p,v,\theta) = \{(p-c)\cdot\tilde{\xi}_n(p) + (p - c_r - v)\cdot \tilde{\xi}_e(p,v)\}\cdot\left(\frac{1-\alpha}{1-\beta}\right) - k\theta^2, \tag{18}$$

[10] Note that the value of recycling can be captured by using our model for remanufacturing. The cost proportion $(1-\psi)$ will be higher under recycling than that under remanufacturing, because the objective under recycling is to break-down the used product to its bare raw materials (identity and functionality of the intermediate modules are lost) and manufacture new units from this raw material, which entails higher manufacturing cost (Thierry et al. 1995)

where $\tilde{\xi}_n(p) = (1-p)$ is the demand for new units in a period and $\tilde{\xi}_e(p,v)$ is demand from customer's exchanging old units, and is given in (17). The return fee that maximizes the firm's profit is:

$$\tilde{v}^*(p,\theta) = \min\left\{pr, \frac{p+r-(1-p)\gamma - c(1-\psi+\psi\phi(1-\theta))}{2}\right\}. \tag{19}$$

By substituting $\tilde{v}^*(p,\theta)$ into $\tilde{\pi}(p,v,\theta)$, we obtain the optimal product-reusability as:

$$\tilde{\theta}^* = \frac{c\lambda\psi\phi(1-\alpha)(p(1-\gamma)+\gamma-r-c(1-\psi(1-\phi)))}{4k(1-\beta)(1-\lambda)(\gamma-r)-c^2\psi^2\phi^2\lambda(1-\alpha)}, \tag{20}$$

for all high values of k (i.e., $k \geqslant \frac{c^2\psi^2\phi^2\lambda(1-\alpha)}{4(1-\beta)(1-\lambda)(\gamma-r)}$). We make the following observations about $\tilde{\theta}^*$:

1. The firm installs reusability when the price p is sufficiently higher than the cost c (i.e., $\tilde{\theta}^* > 0 \Leftrightarrow p > \frac{c(1-\psi(1-\phi))-(\gamma-r)}{1-\gamma}$),
2. The firm chooses higher reusability as customer's exchange propensity λ increases (i.e., $\frac{\partial\tilde{\theta}^*}{\partial\lambda} \geqslant 0$), and
3. The firm chooses lower reusability as its cost k increases (i.e., $\frac{\partial\tilde{\theta}^*}{\partial k} \leqslant 0$).

The second statement above (which is analogous to Proposition 3) shows that unlike the case of refurbishing, it is always beneficial to increase product-reusability when consumer's exchange propensity increases in remanufacturing. Furthermore, it can be easily shown that Proposition 2 holds true for the product-reusability choice $\tilde{\theta}^*$ under remanufacturing. We avoid restating the proposition and repeating its discussion for brevity.

5.1. Discussion

Figure 5 shows that product-reusability under remanufacturing follows a trend similar to the case of refurbishing, with respect to the disruption probability α. It is beneficial to decrease product-reusability when α is high, although it helps to increase it when α is low. Moreover, we can observe that the reusability chosen under remanufacturing is lower than that chosen under refurbishing because of its decreased cost advantage due to ψ (see equation (16)).

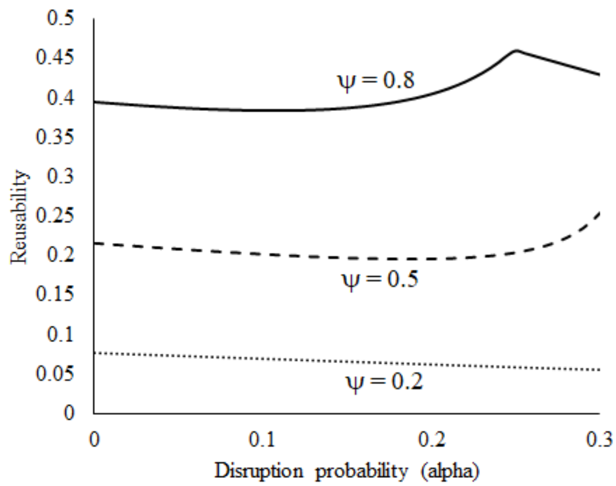


Figure 5 **Optimal reusability levels under remanufacturing for different values of ψ.**

According to industrial practices, remanufacturing costs are typically higher than refurbishment costs (Canon 2021). We find, after accounting for the additional costs associated with remanufacturing , that refurbishment could be preferable to remanufacturing when the disruption probability α is low, and vice-versa when the probability is high. We illustrate this through Figure 6 that provides the difference between the profits under refurbishing and under remanufacturing strategies (i.e., $\pi - \tilde{\pi}$), for different values of disruption probability. When α is low, the higher expenditure incurred for remanufacturing makes it less lucrative than refurbishing, while when α is high, the discounted price for refurbished units makes refurbishing less lucrative than remanufacturing.

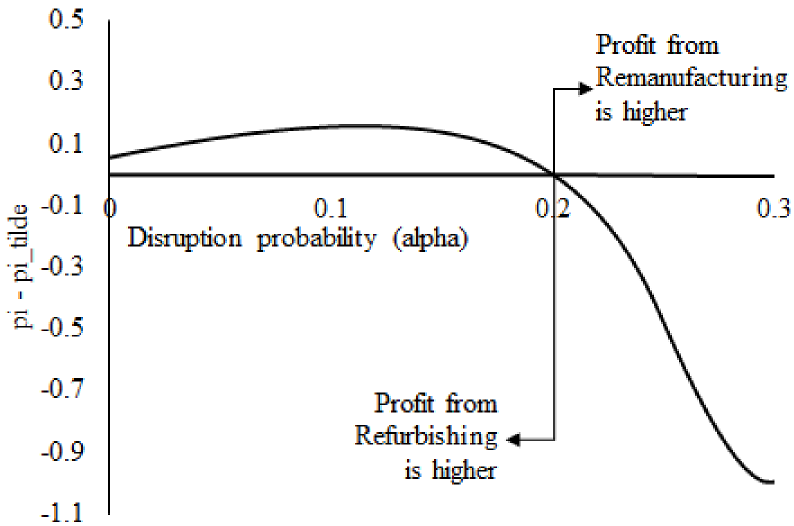
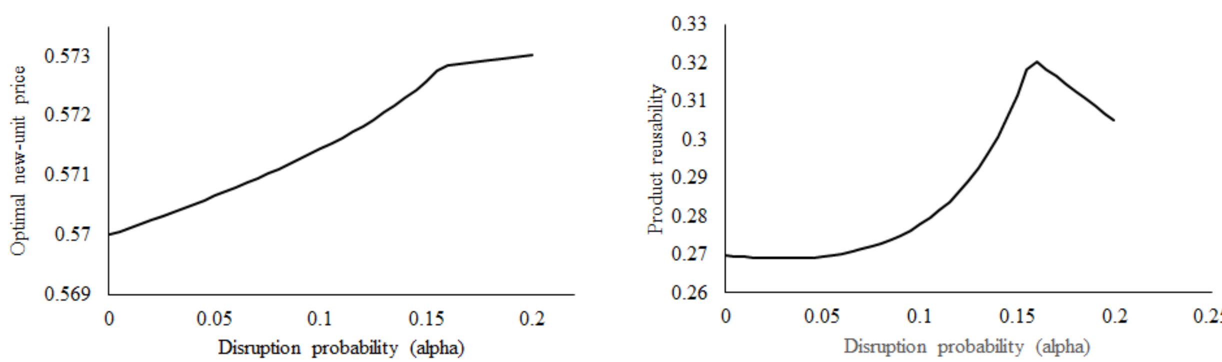


Figure 6 **Difference between the profits under refurbishing and under remanufacturing (i.e., $\pi - \tilde{\pi}$).**

6. Choosing Price of New Units

In order to analyze if product reusability via refurbishing always adds value when the manufacturer can decide the prices of both new and old units, we endogenize the price p of the new units as a decision variable in this section. By substituting δ_{opt} and θ_{opt}, given in (14) and (15), respectively, in the firm's profit that is given in (9), we obtain the firm's profit as $\pi(p, \delta_{opt}, \theta_{opt})$. It can be shown that the profit is concave in the price p when the disruption probability α or the exchange propensity λ is low.[11]

Illustrative plots of optimal product-reusability (θ) and price (p) values for different disruption risk probabilities are given in Figures 7b and 7a, respectively. It is evident from Figure 7a that manufacturer marginally increases its new-unit price as the disruption probability increases in order to protect its revenue. However, Figure 7b clearly indicates that increasing product-reusability is not always beneficial even if the manufacturer can set the prices of new and old units. Here, as in the case of exogenous price (i.e., in Figures 4a and 3a), the lower profit due to longer time spent in disrupted state (i.e., D in Figure 2) when α is high will dissuade the manufacturer to retain higher reusability level, despite the increase in price. It should also be noted that the higher price decreases the total demand for the product. This reduced demand also diminishes the manufacturer's profits thereby prompting them to decrease its investment in reusability.



(a) Optimal product price with probability of disruption (b) Optimal reusability with probability of disruption

Figure 7 Optimal reusability level and new-unit price under refurbishing.

We find similar trends in the case of remanufacturing: it is not always beneficial to increase reusability level even though the manufacturer is able to choose the new and old unit prices. We provide the corresponding plots in Figure 8 in the appendix.

[11] Through differentiation, we can obtain that $\frac{d^2}{dp^2}\{\pi(p, \delta_{opt}(p), \theta_{opt}(p))\} = -\frac{4(k-k_0)\big(4\{(1-\lambda)(\gamma-r)+\lambda(1-r)r\}(k-k_0)-(1-\gamma)^2 k\lambda\big)}{8\{(1-\lambda)(\gamma-r)+\lambda(1-r)r\}(k-k_0)^2}$, where γ and k_0 are given in assumptions 4 and 5, respectively. Thus, $\frac{d^2}{dp^2}\{\pi(p, \delta_{opt}(p), \theta_{opt}(p))\} < 0$ if the disruption probability α or the exchange propensity λ is low; note that $\alpha \to 0 \Rightarrow \gamma \to 1$. Therefore, the optimal price is obtained by the first order condition $\frac{\partial \pi}{\partial p}(p, \delta_{opt}(p), \theta_{opt}(p)) = 0$ when α or λ is low.

7. Conclusions

In this paper, we modeled and analyzed the role of product-reusability (executed via refurbishing and remanufacturing), in the presence of supply disruption. We examined if it is always beneficial to increase product-reusability to enable easier refurbishing and remanufacturing so that the firm can mitigate the risk of supply disruption. We also examined how firms can set the exchange fee for the returned objects in Section 3.1, and the optimal discounted price for the refurbished units in Section 4.1.

Through our analysis in Section 4.3.1, we showed that conventional wisdom of enahancing product-reusability via refurbishing to counter increasing supply disruption risk is true only when the probability of disruption is low. We showed that this strategy could be counter productive when the supply disruption probability is high. This is because the manufacturer will earn lower revenues because of the longer time it spends in disrupted state, as the disruption probability increases. Therefore, it is beneficial for the manufacturer to decrease product-reusability and save on reusability cost, when disruption is more likely. Next, we proved in Section 4.3 that when consumers are more likely to exchange used units, it benefits the firm to increase product-reusability only when the price is sufficiently high. This is because the firm can afford investing in reusability to obtain higher margins on its sales.

In Section 5, we conduct the corresponding analysis when product reusability is executed through remanufacturing. We also numerically compared refurbishing and remanufacturing with respect to their net profits. Here we observed that refurbishing could be better than remanufacturing when the disruption probability is high due to the additional investment that remanufacturing requires when compared to refurbishing.

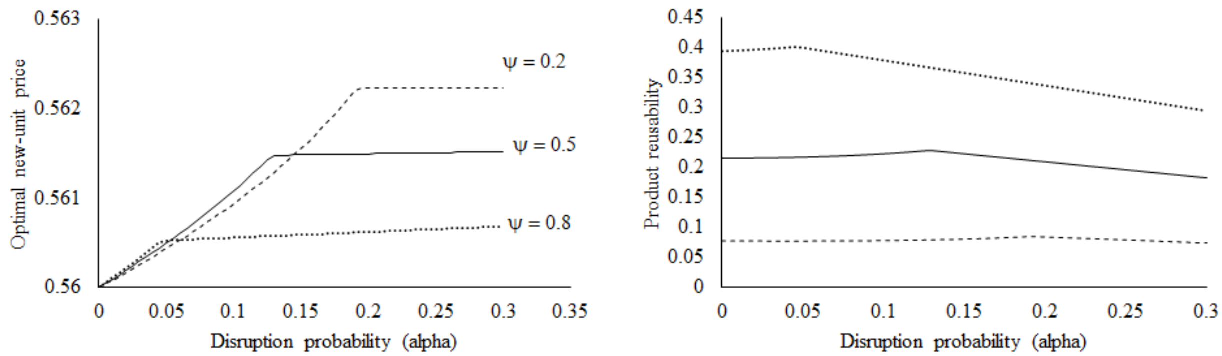
Finally, in Section 6, we demonstrated through numerical examples that it is not always beneficial to increase product-reusability even when the manufacturer can set the prices for both new and old units. Thus, the lever of product-reusability should be chosen carefully. Further, the probability of disruption plays an important role in a manufacturer's choice of the level of product-reusability for refurbishing and remanufacturing.

Our paper is an initial step towards integrating product-reusability design decisions with supply disruption management strategies. There are several interesting avenues for future research. First, a manufacturer's decision making process changes substantially if they can decide the discount for refurbished-units after the state (i.e., normal N or disrupted D) is realized. It is interesting to examine the role of product-reusability when manufacturer can make such an informed pricing decision. Second, in this paper, we only analyzed the decisions of a single manufacturer and we relegated the role of competition to future research. It is important to explore how the various factors like difference in design costs (i.e., k), price (i.e., p), and consumer valuations (i.e., V) affect manufacturers' decisions on product-reusability and price choice in the presence of competition. Another potential topic of future research includes exploring the impact of governmental policies on waste-recycling and *manufacturer take-back* programs on manufacturer's choice of product-reusability in the presence of supply disruptions.

In conclusion, we believe this paper presents a useful framework to model, analyze, and manage product-reusability under supply disruptions.

Appendix A: Technical Appendix

A.1. Plots



(a) Optimal product price with probability of disruption (b) Optimal reusability with probability of disruption

Figure 8 Optimal reusability level and new-unit price under remanufacturing.

A.2. Proofs of lemmas

Proof of Lemma 1: By equating the supply of old units (given for exchange) with the demand for refurbished units, we obtain:

$$\xi_o(p,\delta) = \xi_e(p,v,\delta) \Rightarrow v = \gamma p - (\gamma - r)\left(1 - \frac{(1-\lambda)\xi_o(p,\delta)}{\lambda}\right). \tag{21}$$

Next, since we require v such that $\left(\frac{1-\delta}{1-r}\right)p \leqslant \frac{\gamma p - v}{\gamma - r}$, we obtain the return fee as given in (6). ■

Proof of Lemma 2: By differentiating $v^*(p,\delta)$ we obtain the following sensitivities:

$$\frac{\partial v^*}{\partial \delta} = -\frac{(1-\lambda)p(\gamma - r)}{\lambda(1-r)r} < 0$$

$$\frac{\partial v^*}{\partial p} = \frac{\lambda\left((1-\gamma)r^2 + \delta(\gamma - r)\right) + (\gamma - r)(r - \delta)}{\lambda(1-r)r} > 0, \text{ and}$$

$$\frac{\partial v^*}{\partial r} = \frac{(1-\lambda)p\left(\gamma\delta + r^2(\gamma + \delta - 1) - 2\gamma\delta r\right) + \lambda(1-r)^2 r^2}{\lambda(1-r)^2 r^2},$$

which sequentially prove the three statements of the lemma. ■

Proof of Lemma 3: First, through differentiation of (9), we can obtain that $\frac{\partial^2 \pi}{\partial \delta^2} = -\frac{2p^2((1-\lambda)(\gamma-r)+\lambda(1-r)r)}{\lambda(1-r)^2 r^2}$. Therefore, the optimal discount factor is obtained by the first order condition $\frac{\partial \pi}{\partial \delta} = 0$, which yields the optimal discount factor as $\tilde{\delta}(\theta)$ that is given in (12). Now, to ensure *in every period* that the number of customers opting to exchange old units is less than the number of customers who owned new units previously, we require δ to be set such that $\left(\frac{1-\delta}{1-r}\right)p \leqslant \frac{\gamma p - v^*(p,\delta)}{\gamma - r}$. Using Lemma 2, we can observe that the RHS of the above inequality is increasing in δ whereas, the LHS is decreasing in δ. Therefore, there exists δ_0 such that the above inequality holds if, and only if, $\delta > \delta_0$. By solving the inequality as equality, we obtain $\delta_0 = \frac{r(p-(1-r)\lambda)}{p(1-(1-r)\lambda)} (< r)$. Next, by using Assumption 1, we obtain (11). ■

Proof of Lemma 4: By differentiating, we can obtain that $\frac{\partial \tilde{\delta}}{\partial \theta} = -\frac{c\lambda(1-r)r\phi}{2p((1-\lambda)(\gamma-r)+\lambda(1-r)r)} < 0$ and $\frac{\partial \tilde{\delta}}{\partial c} = \frac{\lambda(1-r)r((1-\theta)\phi - r + 1)}{2p((1-\lambda)(\Gamma-r)+\lambda(1-r)r)} > 0$. Next, by using the fact that $\frac{\partial \tilde{\delta}}{\partial \theta} < 0$, we can conclude that there exists a threshold θ_0 such that $\delta^*(\theta) = \delta_0$ if, and only if, $\theta \geqslant \theta_0$. Therefore, if $\theta \geqslant \theta_0$, then $\frac{\partial \delta^*}{\partial p} = \frac{\partial \delta_0}{\partial p} = \frac{\lambda(1-r)r}{p^2(1-\lambda(1-r))} > 0$. On the other hand, if $\theta < \theta_0$, then $\frac{\partial \delta^*}{\partial p} = \frac{\partial \tilde{\delta}}{\partial p} = \frac{\lambda(1-r)r(\gamma - r - c((1-\theta)\phi + 1 - r))}{2p^2((1-\lambda)(\gamma-r)+\lambda(1-r)r)} < 0 \Leftrightarrow c > \frac{\gamma - r}{(1-\theta)\phi + 1 - r}$. ■

Proof of Lemma 5: The proof follows directly from first order condition $\frac{\partial \pi}{\partial \theta} = 0$ and noting that $\theta \in [0,1]$. ■

Proof of Lemma 6: The proof follows directly from first order conditions $\frac{\partial \pi}{\partial \theta} = 0$ and $\frac{\partial \pi}{\partial \delta} = 0$ and using the facts that $\delta \in [\delta_0, r]$ and $\theta^*(\delta)$ is decreasing in δ. ■

A.3. Proof of propositions

Proof of Proposition 1: By differentiating the interior optimal solution given in (14) and (15), we obtain:

$$\frac{\partial \delta_{opt}}{\partial k} = \frac{2(1-\alpha)(1-\beta)c^2\lambda^2(1-r)r\phi^2(-c(-r+\phi+1)+\gamma-\gamma p+p-r)}{p\left(4(1-\beta)k((1-\lambda)(\gamma-r)+\lambda(1-r)r)-(1-\alpha)c^2\lambda\phi^2\right)^2},$$
$$\frac{\partial \delta_{opt}}{\partial \lambda} = -\frac{8(1-\beta)^2k^2(1-r)r(\gamma-r)(-c(-r+\phi+1)+\gamma-\gamma p+p-r)}{p\left(4(1-\beta)k((1-\lambda)(\gamma-r)+\lambda(1-r)r)-(1-\alpha)c^2\lambda\phi^2\right)^2},$$
$$\frac{\partial \theta_{opt}}{\partial k} = -\frac{4(1-\alpha)(1-\beta)c\lambda\phi((1-\lambda)(\gamma-r)+\lambda(1-r)r)(-c(-r+\phi+1)+\gamma-\gamma p+p-r)}{\left(4(1-\beta)k((1-\lambda)(\gamma-r)+\lambda(1-r)r)-(1-\alpha)c^2\lambda\phi^2\right)^2}, \text{ and}$$
$$\frac{\partial \theta_{opt}}{\partial \lambda} = \frac{4(1-\alpha)(1-\beta)ck\phi(\gamma-r)(-c(-r+\phi+1)+\gamma-\gamma p+p-r)}{\left(4(1-\beta)k((1-\lambda)(\gamma-r)+\lambda(1-r)r)-(1-\alpha)c^2\lambda\phi^2\right)^2},$$

from which the proof follows directly. ■

Proof of Proposition 2: By substituting that $\gamma = \frac{1-\alpha}{1-\alpha\beta}$, we obtain through differentiation that:

$$\frac{\partial^2 \pi}{\partial\theta\partial\alpha}(p,\delta^*(\theta),\theta) = \frac{-(1-\beta)\cdot g(c)}{2(1-\alpha)^2\left[r(1-\alpha\beta)-(1-\alpha)(1-\lambda)-\lambda r(2-r)(1-\alpha\beta)\right]^2}, \tag{22}$$

where $g(c) = (1-\alpha)^2(1-\lambda)\lambda\phi(1-r+\phi(1-\theta))\cdot c^2 - (1-\alpha)^2\lambda\phi(1-r)(p(1-\lambda(1-r))-\lambda r)\cdot c + 4k\theta\left[r(1-\alpha\beta)-(1-\alpha)(1-\lambda)-\lambda r(2-r)(1-\alpha\beta)\right]^2$, which is a convex quadratic.

First, whenever $p > \frac{(1-\alpha)^2c\lambda\phi(c(1-\lambda)(1-r+\phi(1-\theta))+\lambda(1-r)r)+4\theta k(r(1-\alpha\beta)-(1-\alpha)(1-\lambda)-\lambda r(2-r)(1-\alpha\beta))^2}{(1-\alpha)^2c\lambda(1-r)\phi(1-\lambda(1-r))}$, then $g(c)<0$. Likewise, when k or c is high, then $g(c)>0$ always. Next, when $\alpha\to 1$, we obtain that $g(c)>0$; on the other hand, if the interior optimizer is increasing in α and $\theta_{opt}=\theta^*(\delta_0)$ at sufficiently high α, then $\frac{\partial\theta^*(\delta_0)}{\partial\alpha}<0$, because $\theta^*(\delta_0)$ is linear and decreasing in α. Thus, whenever $g(c)>0 \Rightarrow \frac{\partial^2\pi}{\partial\theta\partial\alpha}(p,\delta^*(\theta),\theta)<0 \Rightarrow \frac{d\theta_{opt}}{d\alpha}<0$. This completes the proof of the proposition for interior optimizer.

Next, if the interior optimizer θ_{opt} is increasing in α, then it is easy to observe from (14) that the interior optimizer δ_{opt} is decreasing in α. Now, if α is sufficiently large such that $\delta_{opt}=\delta_0$, then ■

Proof of Proposition 3: By differentiating θ_{opt} with respect to λ we obtain:

$$\frac{\partial\theta_{opt}}{\partial\lambda} = \frac{4ck\phi(1-\alpha)(1-\beta)(1-\alpha\beta)(\gamma-r)((1-\alpha)-c(1-\alpha\beta)(-r+\phi+1)+\alpha(1-\beta)p-r(1-\alpha\beta))}{\left((1-\alpha)c^2\lambda\phi^2(1-\alpha\beta)-4(1-\beta)k(-\alpha+\lambda(\alpha+(r-2)r(\alpha\beta-1)-1)+\alpha\beta r-r+1)\right)^2}, \tag{23}$$

so that $\frac{\partial\theta_{opt}}{\partial\lambda}\geqslant 0$ if, and only if, $p\geqslant p_0$. ■

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