Frugal Materialism and Risk Preferences

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Abstract: Frugal materialism is a tendency of consumer demand to become more elastic in product durability in response to a tightening budget constraint. This paper proposes a model of frugal materialism, and establishes a theoretical link between frugal materialism and the slope of risk aversion the function would exhibit if it were a von Neumann Morgenstern function in expected utility theory: for both their absolute and relative versions, frugal materialism and increasing risk aversion are nearly equivalent to each other. This relationship is surprising because the proposed model of buying durable goods does not involve any risk. An extension of the proposed model to allow for income risk in the future provides implications for the relationship between frugal materialism and the precautionary savings motive: spending on the durable good can substitute for precautionary savings whenever precautionary motives are present, but the precautionary motive is largely incompatible with frugal materialism.

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1. Introduction

Understanding the drivers of consumer demand for durable goods is an important goal of both managers and policy makers, especially during recessions (e.g. Auernheimer et al. 1977, Bernanke 1981, Levinthal and Purohit 1989, Leamer 2007, Koenigsberg et al 2010, Berger and Vavra 2015). Several well-known forces reduce demand for durables relative to perishables during economic downturns (e.g., durables tend to be bigger-ticket items and their purchases are more easily postponed). This paper explores a recently discovered force in the opposite direction – the *frugal materialism* property of consumer preferences. Frugal materialism is my terminology for the heretofore unnamed effect discovered by Tully, Hershfield, and Meyvis (2015, hereafter "THM") who found that making people feel more financially constrained results in higher stated demand for more durable material goods relative to perishable versions of the same goods. THM's consumers exhibit frugal materialism in that a reduction in their disposable incomes (forcing them to become more "frugal") increases their demand for *more* durable material goods ("materialism" in the sense of demand's elasticity in durability[1]). Other results in the behavioral literature conceptually replicate this finding (e.g., Bardhi and Eckhardt 2017, Saatcioglu and Ozanne 2013, or Lee et al 2018).

Frugal materialism seems like an intuitive property of consumer preferences: the poorer you are, the more attractive is an increase in the durability of the goods you buy, especially if the more durable goods are not more expensive: the durability simply helps stretch your limited funds further. A natural question thus arises whether frugal materialism

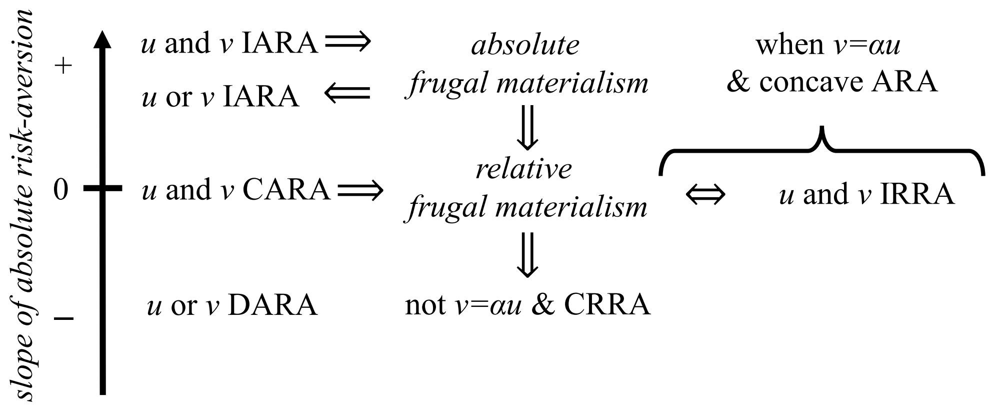
[1] Note that throughout this paper, "materialism" means an individual-level increase in revealed preference for acquiring more durable material possessions, not a personal value or a belief system as in Richins (2011). In other words, materialism is just a label for the marginal effect of increased durability on demand.

is a generic behavior predicted under standard economic modeling assumptions, or whether frugal materialism places testable restrictions on preference models. And if frugal materialism occurs only under restrictive assumptions about preferences, what other behaviors should it coincide with and which other behaviors does it rule out?

This paper proposes a parsimonious model of frugal materialism, and shows that the behavior implies a restriction on the consumer utility function nearly equivalent to increasing risk-aversion the function *would exhibit if* it were a von Neumann-Morgenstern function in expected utility theory. In other words, the scale-free measure of curvature driving purchases of risky assets under the expected utility model as described by Pratt (1964) is also useful in predicting purchases of durable goods under financial constraints in a model without any risk. To be more specific about the aforementioned "near equivalence" of frugal materialism and increasing risk aversion (i.e., accelerating diminution of log marginal utility), I need to introduce my main modeling assumptions.

Consider a consumer who faces a budget-constrained choice between two divisible goods. One of the goods is a material good in that it can have various degrees of durability, and the other good is perishable (called "experience" throughout the paper for consistency with the behavioral literature that discovered frugal materialism). I analyze the most parsimonious canonical model of such a consumer's demand for different amounts of the two goods—an additively separable utility with one utility function u for the material good and a possibly different second utility function v for the experience. Let *absolute (relative) frugal materialism* be an increase in the absolute (relative) budget allocated to the material good in response to the joint event of (1) increasing the durability of the material good and (2) shrinking the overall budget. Figure 1 summarizes my findings.

Figure 1: Summary of results



The first major finding of this paper is that absolute frugal materialism implies at least one of the two good-specific utility functions u and v defined in the previous paragraph (if interpreted as a von Neumann-Morgenstern function) exhibits increasing absolute risk aversion (IARA), and both utilities being IARA in turn implies absolute frugal materialism.[2] Therefore, when the two utility functions are affine transformations of each other (denoted as $u = \alpha v$ for some constant α>0 in Figure 1), absolute frugal materialism is equivalent to IARA.

The second major finding of this paper is a somewhat restricted analogue of the above relationship that applies to the relative versions of the two constructs: I show that when the two utility functions are affine transformations of each other and the absolute risk aversion is concave, relative frugal materialism is equivalent to increasing relative risk aversion (IRRA). When the two utility functions are distinct, CARA (which are IRRA) preferences imply relative frugal materialism, but CRRA preferences do not.

[2] Note that researchers studying risk preferences often consider utility functions over different amounts of money, whereas the focus here is on the amount of a good as the argument of each utility function. The math is identical.

The main findings have immediate implications for modeling consumer preferences for durable goods using popular assumptions. For example, if consumers exhibit any type of frugal materialism, their preferences cannot be captured by the Stone-Geary, Cobb-Douglas or any isoelastic model (including the popular multinomial logit model). If they exhibit relative but not absolute frugal materialism, then their preferences can be captured by the exponential utility function, but not by Dixit's quadratic one which implies both types of frugal materialism. Within the popular HARA family of utilities, the presence of different types of frugal materialism implies specific restrictions on the parameters.

Additional implications of my results present themselves if we assume that a particular consumer's acceleration of diminishing marginal utility matches their acceleration of risk aversion – a plausible and testable assumption I call *unified acceleration assumption*. Such a correspondence between the shape of von Neumann-Morgenstern function over money and the shape of the consumer utility function over different quantities of the durable good implies that the demand for risky assets of a frugally materialistic consumer should exhibit predictable patterns associated with increasing risk-aversion. The implied deep relationship between two seemingly unrelated categories of goods (risky assets and durables) is important in the context of recessions that motivate this paper's study of durable goods because recessions involve increases in economic risk in addition to lowering real incomes of consumers.

I now briefly outline the implications of frugal materialism under the unified acceleration assumption. Because most previous research has either found absolute risk aversion to be decreasing or argued a priori that it should be so (e.g. Bernoulli 1738, Pratt 1964, Arrow 1971, Rapoport, Zwick, and Funk 1988, Levy 1994, Gollier and Pratt 1996, and others) a finding of absolute frugal materialism would be surprising on its own. A finding of relative frugal materialism but not absolute frugal materialism in the context of two closely related goods would zero in on non-IARA and IRRA preferences in accordance with Arrow's (1971) famous hypothesis.

It is immediate that future studies of consumer demand for durable goods can be sharpened with a parallel analysis of risk preferences, and policy responses to recessions need to consider both frugal materialism and its associated risk preferences. One situation in which durability and risk interact is the classic context of *precautionary savings* - saving more as a precaution to a turbulent future, which has been thoroughly documented (e.g. Kennickell and Lusardi 2004, Carroll and Kimball 2007). As shown by Leland (1968) and Kimball (1990), the precautionary saving motive can only be active when the von-Neumann Morgenstern utility function is "prudent" – a property implied by non-increasing absolute risk-aversion. Given the prominence of increasing absolute risk aversion in the first main finding of this paper, the precautionary motive is thus largely incompatible with absolute frugal materialism – only a consumer with prudent IARA preferences can exhibit both. In the second part of this paper, I provide an exact characterization of this condition.

The rest of the paper proceeds by first presenting the general results for the absolute and relative versions of frugal materialism / risk aversion, then illustrating the results on several concrete examples, and then explaining the relationship with models of precautionary savings.

2. Summary of empirical evidence for frugal materialism

As outlined in the Introduction, the phenomenon was first described by Tully, Hershfield, and Meyvis (2015) who used a series of experiments to establish their main finding that making people feel more financially constrained increases their "concern about the lasting utility of their purchases" (p. 59) and results in results in higher stated demand for durable material goods relative to perishable versions of the same goods. THM's Study 6 provides the clearest example of their finding: Subjects were to imagine they are walking around the city when it starts to rain, and they can either stop for a coffee in Starbucks or buy a poncho that is either described as "disposable" or "reusable" between subjects. Price variation is not an issue in the scenario, because the coffee costs the same as the poncho in all conditions. In a control group, the subjects expressed approximately the same strength of preference for both types of poncho over coffee. However, asking the subjects to "keep in mind their financial constraints" before making the decision dramatically increased their relative preference for the reusable poncho over coffee, while decreasing their relative preference for the disposable poncho.

Other recent findings in the consumer behavior literature echo frugal materialism: for example Bardhi and Eckhardt (2017) argue that economic precarity strengthens demand for "solid" material long-lasting products. Saatcioglu and Ozanne (2013) describe how the preference for owning a house (the quintessential durable) is much stronger among people who have fallen on hard times compared to similarly situated people who have lived under tight budget constraints for a while. Lee, Hall and Wood (2018) find that compared to richer consumers who derive more happiness from experiences, poorer consumers derive greater happiness from durable material purchases. As a recession pulls everyone down the economic ladder, all of the above results suggest that consumers should shift their demand to durables and become more sensitive to increased durability. The goal of the present paper is to formally model this phenomenon and analyze its theoretical implications for economic modeling of consumer preferences.

It is also not clear whether THM found only relative or also absolute frugal materialism as defined in the Introduction because their dependent variable is only a single choice between an experience and a material good. I analyze both versions of the phenomenon to provide a complete characterization, and I hope future empirical research explores the distinction between the two types of the phenomenon.

3. Model of frugal materialism and its relationship to risk aversion

Let there be two goods, one called an experience and one called a material product.[3] Both goods cost the same per unit, and a consumer has a budget B – the total units of both goods he can afford. The material product can be durable in that an expected number λ of future consumption opportunities exists during which a unit purchased today will still be available, with λ including any potential temporal discounting of future consumption. The utility of consuming E of experience and M of the material product is additively separable, assuming away potential complementarities. In addition, the consumer experiences diminishing marginal utility (concavity of each univariate utility):

Assumption: $Utility(E, M; \lambda) = (1 + \lambda)u(M) + v(E)$ with u and v increasing and concave.

[3] Both the "experience" and the "product" are just generic goods in this paper; consumer framing of goods as either experiences or products is not modeled. I label the good with variable durability a "material product."

To determine his demand, the consumer selects the amount M^* of product and the amount E^* of the experience to purchase to maximize his utility such that the budget constraint $M + E \leq B$ holds:

$$\{M^*, E^*\} = \underset{M\geq 0, E\geq 0}{argmax}\ (1+\lambda)u(M) + v(E) \text{ subject to } M + E \leq B \tag{1}$$

In terms of the above notation, THM find tightening the budget constraint B increases the difference between demand for a durable version (high λ) and demand for the disposable version (low λ) of the material product. Considering a small change in durability, we can employ the tools of calculus to define local absolute (relative) frugal materialism in terms of the cross partial of (percentage) demand for the material product in budget and durability:

Definition: A consumer exhibits *absolute frugal materialism* when $\frac{\partial^2 M*}{\partial\lambda\ \partial B} < 0$, and exhibits *relative frugal materialism* when $\frac{\partial^2}{\partial\lambda\ \partial B}\left(\frac{M^*}{B}\right) < 0$.

The goal of this paper is to explore what the sign of this cross partial teaches us about the shape of *u* and *v*. The first main result of this paper follows (see the Appendix for all proofs):

Proposition 1*: In terms of the absolute risk aversions of utilities u and v evaluated at the optimal consumption bundle* $A_u(M^*) = \frac{-u''(M^*)}{u'(M^*)}$ *and* $A_v(E^*) = \frac{-v''(E^*)}{v'(E^*)}$*, the demand cross partial driving absolute frugal materialism can be expressed as:*

$$\frac{\partial^2 M^*}{\partial\lambda\ \partial B} = -\frac{A_v' A_u + A_u' A_v}{(1+\lambda)(A_u + A_v)^3}$$

.

Note that there is no apriori reason to believe that the key cross-partial should be expressible in terms of absolute risk-aversions alone, so the proposition is immediately surprising. The implications of Proposition 1 are straightforward: Because absolute risk aversions of concave functions are positive by construction, it follows that $\frac{\partial^2 M^*}{\partial\lambda\,\partial B} < 0 \Leftrightarrow A_v^{'} A_u + A_u^{'} A_v > 0$; that is, consumers exhibit absolute frugal materialism iff a weighted average of their absolute risk aversions of the two goods is increasing. Thus, either *u* or *v* of absolute frugal materialists must be IARA; the popular CARA and DARA specifications rule out frugal materialism. See Figure 1 for a summary of these implications.

Note that Proposition 1 makes no claim about the relationship between frugal materialism and risk preferences: the utilities in equation 1 are consumption utilities of different quantities of M and E, they are not von Neumann – Morgenstern functions (vNM) in expected utility theory. Nevertheless, measuring the risk-aversions of *u* and *v* as if they were vNM functions generates the result. One could also call the quantity $\frac{-u''(x)}{u^{'}(x)}$ the acceleration of the diminishing log marginal utility, but I find the term "absolute risk aversion" easier, and I propose its relevance is surprising in this modeling context without any risk. The second main result of this paper is:

Proposition 2: *In terms of the absolute risk aversions A_u and A_v and the relative risk aversions* $R_u \equiv M^* A_u$ and $R_v \equiv E^* A_v$, *the percentage-demand cross partial driving relative frugal materialism can be expressed as:*

$$\frac{\partial^2}{\partial\lambda\,\partial B}\left(\frac{M^*}{B}\right) = -\frac{\left(R_u^{'} + R_v^{'}\right)(A_u + A_v) + \left(A_u^{'} - A_v^{'}\right)(R_v - R_u)}{B^2(1+\lambda)(A_u + A_v)^3}$$

.

The implications of Proposition 2 are less stark than those of Proposition 1 because the expression in the numerator is more complicated. Nevertheless, it is immediate that CARA utilities imply $\frac{\partial^2}{\partial\lambda\,\partial B}\left(\frac{M^*}{B}\right) < 0$ because the second term in the numerator is zero and the first term is positive due to CARA implying IRRA.

When $v = \alpha u$, the sign of the slope of relative risk aversion is tightly connected to relative frugal materialism. It is immediate from Proposition 2 that CRRA implies $\frac{\partial^2}{\partial\lambda\,\partial B}\left(\frac{M^*}{B}\right) = 0$ because both terms are zero ($v = \alpha u$ implies $R_v = R_u$). It turns out CRRA is precisely the boundary case under the $R_v = R_u$ assumption as long as ARA is concave:

Corollary to Proposition 2: *When $v = \alpha u$ and ARA is concave, consumers exhibit relative frugal materialism iff the relative risk aversion of u is increasing.*

The requirement that ARA be concave is not necessary for frugal materialism to coincide with IRRA as evidenced by the quadratic utility function with a convex ARA, IRRA, and relative frugal materialism. When u and v are CRRA with $R_v \neq R_u$, the sign of the key cross partial for relative frugal materialism varies with λ: for example, when $u(x) = \sqrt{x}$ and $v(x) = log(x)$, then $\frac{\partial^2}{\partial\lambda\,\partial B}\left(\frac{M^*}{B}\right)\Big|_{B=1} < 0 \Leftrightarrow \lambda > \sqrt{2\left(1+\sqrt{2}\right)} - 1$.

4. Concrete Examples of Popular Utility Functions

This section illustrates the two main results on several concrete and popular examples of consumer-preference models. Imagine a meeting of famous economists at a conference, and overhearing the following reactions to the THM Study 6 described above: Avinash says "This result is obvious, poorer people should not waste their scarce money on coffee when they can get a durable poncho instead". Charles says "This result is surprising, richer people should just buy more of everything, they should not shift their demand towards one particular good", and his friend Paul agrees. Finally, Roy says "Coffee is a necessity, so making someone poorer shifts their demand to coffee, and the shift is faster with durable ponchos because they obviously represent a bigger chunk of the discretionary budget". This section reveals the last names of all these discussants by showing all of their statements hold true under the utility functions named after them. Table 1 at the end of the section collects all the formulae for easy reference, and includes additional functional forms not discussed in detail in the text.

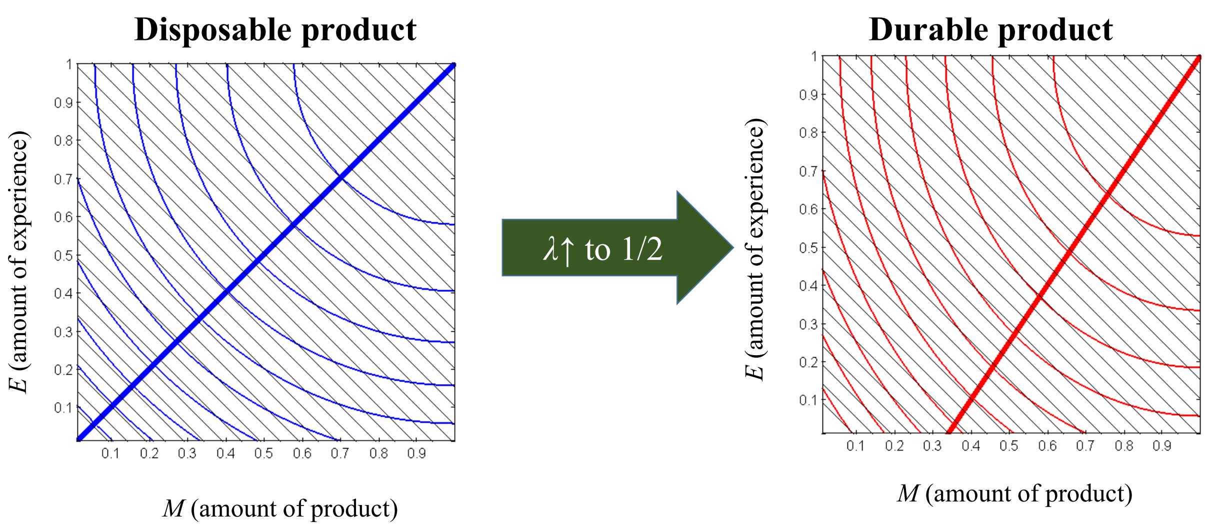
4.1 Quadratic utility (example of IARA implying frugal materialism)

Consider the quadratic utility $U(M,E;\lambda) = (1+\lambda)\left(M - \frac{M^2}{2}\right) + \alpha\left(E - \frac{E^2}{2}\right)$ of Dixit (1979), where α>0 is a constant that weighs the relative importance of the experience. Note these preferences involve $v = \alpha u$ in equation 1, and both u and v are IRRA and IARA. The consumer solves

$$\max_{M\geq 0, E\geq 0} (1+\lambda)\left(M - \frac{M^2}{2}\right) + \alpha\left(E - \frac{E^2}{2}\right) \text{ subject to } M + E = B \qquad (2)$$

The solution to the FOC is $M* = \frac{1+\lambda+\alpha(B-1)}{1+\alpha+\lambda}$, which is less than B whenever $B > \frac{1+\lambda-\alpha}{1+\lambda}$. For smaller budgets, the consumer spends the whole budget on the material good. The consumer exhibits both absolute frugal materialism as predicted by IARA and Proposition 1: $\frac{\partial^2 M*}{\partial\lambda\,\partial B} = \frac{-\alpha}{(1+\alpha+\lambda)^2} < 0$ and relative frugal materialism: $\frac{\partial^2}{\partial\lambda\,\partial B}\left(\frac{M*}{B}\right) = -\frac{2\alpha}{B^2(1+\alpha+\lambda)^2} < 0$. Since the absolute risk-aversion of a quadratic is the convex function $A(x) = \frac{1}{1-x}$, this example shows that the "ARA is concave" sufficient condition in the Corollary to Proposition 2 is not necessary.

Figure 2: Quadratic preferences with α=1



Note to figure: The curves are indifference curves. The thin downward-sloping straight lines are budget constraints. The thick upward-sloping line is the locus of solutions to equation 2.

To gain insight into quadratic preferences, consider the slope of relative demand in the budget: $\frac{\partial}{\partial B}\left(\frac{M*}{B}\right) = \frac{\alpha-(1+\lambda)}{B^2(1+\alpha+\lambda)} < 0 \Leftrightarrow 1+\lambda > \alpha$. In words, given sufficient durability to make buying a unit of the material good preferable to buying a unit of the experience, an increase in the budget *decreases* the proportion of the budget spent on the material good.

Figure 2 assumes the consumer values the non-durable versions of the two goods equally (i,e., $\alpha = 0$), and shows what happens when the budget increases and the product is durable: For small budgets, the consumer buys only the material good. As his budget increases, he adds some experience into the mix. In this sense, quadratic preferences capture the idea of perishable "experience" as a luxury, and the possible intuition that the THM result is obvious because poorer people should not waste their scarce money on coffee when they can get a durable poncho instead.

4.2 Cobb-Douglas utility (example of CRRA, and so DARA, ruling out frugal materialism)

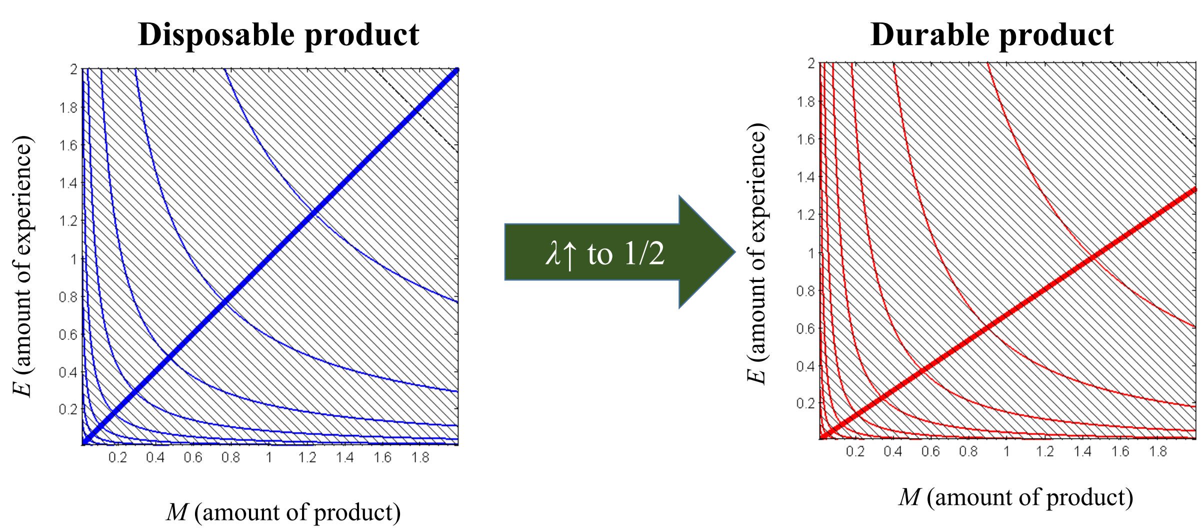
Another textbook example of preferences is the Cobb-Douglas utility function $U(M, E; \lambda) = (1 + \lambda)\, log(M) + \alpha\, log(E)$, where α>0 again represents the relative weight of the experience. Note that Cobb-Douglas preferences involve $v = \alpha u$ in equation 1, and both u and v are CRRA and DARA. The consumer solves

$$\max_{M,E} (1 + \lambda)\, log(M) + \alpha\, log(E) \text{ subject to } M + E = B \tag{3}$$

The solution to this problem is $M *= \frac{1+\lambda}{1+\alpha+\lambda} B$, so the consumer splits his budget according to the effective weight of each good in overall utility. Durability simply increases the effective weight of the material good in the joint utility. Clearly, these preferences support the potential intuition that relative frugal materialism is surprising because richer people should just buy more of everything proportionally instead of shifting their relative demand towards one particular good: Because the percentage demand does not depend on the budget, it is immediate that $\frac{\partial^2}{\partial\lambda\, \partial B}\left(\frac{M*}{B}\right) = \frac{\partial}{\partial B}\left(\frac{M*}{B}\right) = 0$, so Cobb-Douglas preferences rule out relative frugal materialism as predicted by Proposition 2. Because $\frac{\partial^2 M*}{\partial\lambda\ \partial B} = \frac{\alpha}{(1+\alpha+\lambda)^2} > 0$, Cobb-Douglas preferences also rule out absolute frugal materialism as predicted by DARA and Proposition 1.

Preferences that imply $\frac{M*}{B}$ does not depend on B are called "homothetic.", and the previous paragraph shows that homothetic utility functions are inconsistent with relative frugal materialism. Graphically, homothetic preferences have indifference curves whose slopes are constant along rays beginning at the origin (see Figure 3). Formally, a utility function is homothetic when a monotonic transformation of it (i.e., an alternative representation of the same underlying preferences) exists that is homogeneous of degree 1: $U(cM, cE) = cU(M, E)$. A well-known example of homothetic utility functions is the isoelastic function shown in Table 1.

Figure 3: Cobb-Douglas preferences with α=1



Note to figure: See note to Figure 2, but replace equation 2 with equation 3.

4.3 Stone-Geary utility (DRRA, and so DARA, opposite of frugal materialism):

So far, we have seen two examples with the crucial cross partials that are either negative or zero. Another example is needed to show the relative-demand cross partial can also be positive, and so its sign is thus not a priori even weakly constrained by standard consumer theory. Consider the following generalization of the Cobb-Douglas preferences, due to Geary (1950) and used in empirical work by Iyengar et al. (2011): $U(M, E) = log(M - m) + \alpha\, log(E - e)$, where $m \geq 0$ and $e \geq 0$ represent minimum amounts of M and E that the consumer needs to purchase (Cobb-Douglas is the special case of $e = m = 0$), with the utility only valid for $M > m$ and $E > e$. Note that Stone-Geary preferences involve u and v that are both DRRA and DARA. The solution to the consumer problem is $M* = m + \frac{1+\lambda}{1+\alpha+\lambda}(B - e - m)$, and the key cross partial for relative frugal materialism is $\frac{\partial^2}{\partial\lambda\, \partial B}\left(\frac{M*}{B}\right) = \frac{\alpha(m+e)}{B^2(1+\alpha+\lambda)^2} > 0$. Therefore, Stone-Geary preferences cannot exhibit relative frugal materialism. An analogous calculation shows absolute frugal materialism is also ruled out (see Table 1).

When we set $m = 0 < e$, we obtain a model of a consumer for whom increased durability makes him spend a greater part of his *discretionary* budget (B-e) on ponchos while very financially constrained consumers (i.e., $B \approx e$) spend all their money on coffee. Such a consumer's intuition may be that coffee is a necessity, so a budget reduction shifts their demand to coffee, and the shift is faster with durable ponchos because they represent a bigger chunk of the discretionary budget whenever they are purchased.

4.4 Hyperbolic absolute risk-aversion utility (a general family consistent with frugal materialism under some parameter settings)

A popular utility function in the study of risk preferences is the HARA function $U(x) = \frac{1-\gamma}{\gamma}\left(\frac{ax}{1-\gamma}+b\right)^{\gamma}$, known to allow all three possible combinations of increasing and decreasing absolute and relative risk aversions. For tractability, I consider the following three-parameter *u*=*v* example (a, b, and γ are parameters):

$$\max_{M,E}(1+\lambda)\left(\frac{aM}{1-\gamma}+b\right)^{\gamma}+\left(\frac{aE}{1-\gamma}+b\right)^{\gamma} \text{ subject to } M+E=B \qquad (4)$$

It is well known that a HARA function is DARA if γ<1, IARA if γ>1, and CARA as $\gamma\to\infty$. As Proposition 1 predicts, the sign of the absolute-demand cross partial hinges only on γ because

$\frac{\partial^2 M*}{\partial\lambda\ \partial B}=\frac{1}{1-\gamma}F(\gamma,L)$, where $F(\gamma,L)=\frac{(1+L)^{\frac{2-\gamma}{\gamma-1}}}{\left(1+(1+L)^{\frac{1}{\gamma-1}}\right)^2}>0$ for all γ, and so $\frac{\partial^2 M*}{\partial\lambda\ \partial B}<0 \Leftrightarrow \gamma>1$

It is also well known that a HARA function is IRRA iff b>0. Indeed, the sign of the relative-demand cross partial hinges only on b: $\frac{\partial^2}{\partial\lambda\ \partial B}\left(\frac{M*}{B}\right)=-\frac{2b}{aB^2}F(\gamma,L)<0 \Leftrightarrow b>0$.

Table 1: Summary of concrete examples under the *v=αu* assumption

Name of utility function	$\boldsymbol{u}(\boldsymbol{x})$	$\frac{\partial^2}{\partial\lambda\,\partial B}\left(\frac{M*}{B}\right)$	$\frac{\partial^2 M*}{\partial\lambda\,\partial B}$
Stone-Geary (DARA, DRRA)	$log(x - x_{min}())$	$\frac{\alpha\left(M_{\min}+E_{\min}\right)}{B^2\left(1+\alpha+\lambda\right)^2}>0$	$\frac{\alpha}{(1+\alpha+\lambda)^2} > 0$
Cobb-Douglas (DARA, CRRA 1)	$log(x)$	0	$\frac{\alpha}{(1+\alpha+\lambda)^2} > 0$
Isoelastic (DARA, CRRA *r*)	$\frac{x^{1-r}-1}{1-r}$	0	$\frac{\sqrt[r]{\alpha(1+\lambda)}}{r(1+\lambda)\left(\sqrt[r]{\alpha}+\sqrt[r]{1+\lambda}\right)^2} > 0$
Exponential (CARA *a,* IRRA)	$\frac{1-e^{-ax}}{a}$	$\frac{-1}{2a(1+\lambda)B^2} < 0$	0
Quadratic (IARA, IRRA)	$x - \frac{x^2}{2}$	$\frac{-2\alpha}{B^2(1+\alpha+\lambda)^2} < 0$	$\frac{-\alpha}{(1+\alpha+\lambda)^2} < 0$
Hyperbolic risk aversion (IARA iff γ>1, IRRA iff *b*>0)	$\left(\frac{ax}{1-\gamma}+b\right)^{\gamma}$	$-\frac{2b}{aB^2}\underbrace{F(\gamma,L)}_{>0} < 0$ $\Leftrightarrow b > 0$	$\frac{1}{1-\gamma}\underbrace{F(\gamma,L)}_{>0} < 0$ $\Leftrightarrow \gamma > 1$

5. Relationship between frugal materialism and precautionary savings

Leland (1968) shows that risk-aversion is not sufficient for precautionary savings, and demonstrates that a positive third derivative of utility (i.e. convex marginal utility) is necessary. Kimball (1990) sharpens the necessary and sufficient conditions for precautionary savings by defining *absolute prudence of utility u* as $\eta_u(x) = -\frac{u'''(x)}{u''(x)}$, and showing that prudence drives precautionary saving behavior isomorphically to the way risk aversion drives investments in risky assets. Specifically, he shows that precautionary saving happen iff $\eta(x) > 0$, and the dollar amount of precautionary saving increases with $\eta(x)$, ceteris paribus.

5.1 Relationship between frugal materialism and precautionary savings in the basic model

Since absolute frugal materialism is related to the slope of absolute risk aversion, and the slope in turn involves a third derivative of utility, frugal materialism should also be related to precautionary saving motives. Specifically, since absolute frugal materialism implies that either *u* or *v* is IARA and Kimball (1990) shows[4] that

$$A'_u(x) > 0 \Leftrightarrow \eta_u(x) < A_u(x) \quad (4)$$

we immediately obtain the following corollary of Proposition 1:

Corollary to Proposition 1: *Absolute frugal materialism implies that for u or v (or both), absolute risk aversion exceeds absolute prudence.*

The next immediate corollary connects the two behaviors directly to each other. Since CARA and DARA preferences imply positive prudence and rule our absolute risk-aversion, we obtain the following characterization of how rare a potential co-occurrence of absolute frugal materialism and precautionary savings is:

Corollary 2: *When preferences exhibit both precautionary savings and absolute frugal materialism, then for u or v (or both),* $0 < \eta_u(x) < A_u(x)$.

An example of preferences that satisfies Corollary 2 are HARA preferences $u(x) = \left(\frac{ax}{1-\gamma} + b\right)^{\gamma}$ with $\gamma > 1$ to guarantee IARA and $\gamma < 2$ to guarantee positive prudence because the prudence of this HARA specification is $\eta_u(x) = \frac{a(2-\gamma)}{ax-b(\gamma-1)}$. While the co-

[4] By showing that $A'_u(x) = \frac{[u''(x)]^2 - u'''(x)u(x)}{[u'(x)]^2} > 0 \Leftrightarrow \frac{u''(x)}{u(x)} - \frac{u'''(x)}{u''(x)} < 0 \Leftrightarrow \eta_u(x) < A_u(x)$.

occurrence of both precautionary savings and absolute frugal materialism is thus possible, it is walking a narrow path in the broad landscape of plausible preference specifications.

It is also important to note that actual precautionary savings may not happen in a model that allows for durability: because one gets to enjoy a durable product right after purchasing it, increasing the spending on a durable product in response to a future income shock is even more attractive to a prudent consumer than putting more money aside. We now illustrate this point formally in a two-period model.

5.2 Two-period model with precautionary motives and product durability

Generalize the main model of this paper to time periods t=1,2 which I will sometime denote "today" and "tomorrow" for ease of exposition. In every period, the consumer receives a regular income B (for "budget"), and can spend it on either the material good or the experience. The second period ("tomorrow") involves a mean-preserving income shock I_2. The income shock takes the form of receiving or losing some amount σ with equal probability (and so the variance of the income shock is σ^2). As discussed in the precautionary savings literature, the consumer may want to set aside some of today's budget until tomorrow as precautionary saving *s.* Durability of the material product is defined as the probability that the material product survives unscathed until the second period.

Tomorrow, the consumer will observe both the outcome of the durability and the income shock, and allocate any remaining budget $B + s + I_2$ optimally between purchasing additional material product and purchasing tomorrow's experience. The consumer's problem can be described in terms of the amounts M_t of the material product purchased in each period t as follows:

$$\underset{M_1,s}{max}u(M_1) + v(B - M_1 - s) + E_{I_2}\begin{Bmatrix} \lambda \underset{M_2}{max}\left[u\left(M_1 + M_{2,1,I_2}\right) + v\left(B + s + I_2 - M_{2,1,I_2}\right)\right] + \\ +(1-\lambda)\underset{M_2}{max}\left[u\left(M_{2,0,I_2}\right) + v\left(B + s + I_2 - M_{2,0,I_2}\right)\right] \end{Bmatrix}$$

where the curly brackets capture the second period, the second-period amount is contingent on the realization of product durability and the income shock, and the expectation E_{I_2} averages over the possibility of a beneficial shock $I_2 = +\sigma$ and the possibility of a negative shock $I_2 = -\sigma$.

A complete analysis of this model is not tractable in full generality. Nevertheless, several special cases provide clear evidence about the interaction of the precautionary savings motive and durability:

Proposition 3: When u''' and v''' have the same sign and the utility functions are well-behaved to guarantee a solution at first-order conditions, then precautionary behavior depends on durability as follows:

a) when the material product is not durable at all ($\lambda = 0$), then $s^* > 0$ and $\frac{ds^*}{d\sigma}$ has the same sign as the prudence of *u* and *v.*

b) when the material product is perfectly durable ($\lambda = 1$), then $s^* = 0$ and $\frac{dM_1^*}{d\sigma}$ has the same sign as the prudence of *u* and *v.*

The a) part replicates the classic result dating back to Leland (1958) within the present model: as long *u* and *v* have the same sign of their third derivatives, the consumer has a precautionary saving motive iff the third derivative is positive. The b) part then extends the model to durable goods, and shows that the strength the precautionary motive for buying durable goods is governed by the same utility feature (prudence) as the precautionary savings motive. The only difference between the two situations is that the demand for precautionary savings manifests itself as an increase in the amount of durable good purchased in the early period when $\lambda = 1$ as opposed to as an increase in the amount of actual savings set aside when $\lambda = 0$.

6. Discussion

In an economic downturn, consumer demand for durables seems to be more elastic in the products' durability – a behavior recently documented in the behavioral literature. Such "frugal materialism" can be rationalized in a canonical microeconomic model with additively separable utility, but it is not a generic property of standard preferences. This paper documents a close relationship between frugal materialism and the deceleration of the utility functions over quantities of the goods in the market: the frugal materialism behavior implies a restriction on the consumer utility function nearly equivalent to increasing risk-aversion the function *would exhibit if* it were a von Neumann-Morgenstern function in expected utility theory. In other words, the scale-free measure of curvature driving purchases of risky assets under the expected utility model is also useful in predicting purchases of durable goods under financial constraints in a model without any risk.

I show that frugal materialism rules out several popular assumptions about the shape of utility over quantity of goods, namely Stone-Geary, Cobb-Douglas, and iso-elastic. More generally, frugal materialism of either kind is incompatible with homothetic preferences commonly used in the empirical literature (e.g., the multinomial logit model of consumer demand). Future modelers need to develop non-homothetic models, especially when attempting to model demand for durable goods under varying financial constraints. Such models are rare in the literature; the seminal example is Allenby and Rossi (1991) extended in Allenby, Garratt, and Rossi (2010). Moreover, the sensitivity of THM's cross partial to the curvature of the utility function suggests specific functional forms of the non-homothetic models matter a lot for matching basic patterns of the data; for example, the popular Stone-Geary model used by Iyengar et al (2011) is inconsistent with relative frugal materialism.

Under the additional plausible and testable unified acceleration assumption that a consumer's acceleration of diminishing marginal utility matches their acceleration of risk aversion, the result in this paper imply that the demand for risky assets of a frugally materialistic consumer should exhibit predictable patterns associated with increasing risk-aversion in the seemingly unrelated domain of decision-making under risk. Broadening the implications of the THM findings, I show that frugal materialism is potentially very surprising if consumers, in fact, exhibit both versions of it: The theoretically appropriate and empirically relevant slope of risk aversion has received much discussion since Pratt's (1964) definition of the concept. Regarding the slope of absolute risk aversion, most research to date has either found it to be negative (i.e., DARA, e.g., Rapoport, Zwick, and Funk 1988, Levy 1994, and others), or argued a priori that it should be so (Bernoulli 1738, Pratt 1964, Arrow 1971, Gollier and Pratt 1996, and others). Arrow (1971) advanced a DARA-IRRA hypothesis as the most plausible pair of slopes of absolute and relative risk aversion, and recent work by Brocas et al. (2018) finds empirical evidence of Arrow's hypothesis. Given the unified acceleration assumption, a finding of absolute frugal materialism would thus be quite surprising because subjects who exhibit it should have IARA preferences over at least one of the goods in question. On the other hand, a finding of relative but not absolute frugal materialism would be consistent with Arrow's hypothesis and the prevailing understanding of risk aversion in the literature. A finding of no frugal materialism would suggest DRRA preferences, found by a relative minority of work to date (for an example of a DRRA finding, see Ogaki and Masao 2001).

This paper thus amply delivers on Kimball's classic observation that "The empirical context of an economic theory lies in its ability to connect two or more different observable phenomena." (Kimball 1993, p. 606): it both connects frugal materialism to restrictions on preference models, and also connects frugal materialism to seemingly unrelated domain of decision making under risk such as choice among risky assets and precautionary savings.

The newly discovered relationship documented herein suggests further directions for empirical work at the intersection of risk and demand for durable goods. The implications for further empirical work are at least threefold: First, we need to find whether and when consumers exhibit both forms of frugal materialism or only the relative version. Second, we need to conduct within-subject measurements of both the intensity of frugal materialism and the slope of risk aversion to empirically test the proposed link via the unified acceleration assumption. Finally, we need to explore potential relationships between frugal materialism and other important behaviors under risk, such as precautionary savings (Kimball 1990).

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Appendix: Proofs of Propositions

Proof of Proposition 1: Because both utilities are increasing in quantity consumed, the budget constraint binds and the consumer's problem in equation 1 is equivalent to $\max_{M}(1+\lambda)u(M)+v(B-M)$, which has the first-order condition

$$(1+\lambda)u^{'}(M*)=v^{'}(B-M*) \qquad . \qquad \text{(FOC)}$$

To derive the cross partial of interest, differentiate the FOC twice, starting with the budget, and

and express in terms of the absolute risk aversions of u and v denoted $A_u=\frac{-u''}{u'}$and $A_v=\frac{-v^{''}}{v^{'}}$respectively:

$$(1+\lambda)u^{''}\frac{\partial M^*}{\partial B}=v^{''}\left(1-\frac{\partial M^*}{\partial B}\right)\Rightarrow\frac{\partial M^*}{\partial B}=\frac{v^{''}}{(1+\lambda)u^{''}+v^{''}}=\frac{A_v}{A_u+A_v}, \qquad (5)$$

where the arguments of u, v, and their derivatives have been suppressed for clarity (from this point in, u, A_u, and their derivatives always haveM^*as the argument, whereas v, A_v, and their derivatives always have $E^*\equiv B-M^*$). The formula is intuitive: When the budget increases, the consumer buys more of the material good when the utility of the experience is diminishing faster (larger v'') relative to the effective (durability-weighted) utility of the material good. The (1+λ) weight drops out when $\frac{\partial M^*}{\partial B}$is expressed in terms of the absolute risk aversions, because the implicit M^* and E^*arguments satisfy the FOC.

To finish the derivation of $\frac{\partial^2 M^*}{\partial B\partial\lambda}$, differentiate equation 5 with respect to λ, remembering the argument of A_v is $B-M^*$, and hence $\frac{\partial A_v}{\partial\lambda}=-\frac{\partial M^*}{\partial\lambda}A_v^{'}$:

$$\frac{\partial^2 M^*}{\partial B \partial \lambda} = \frac{-\frac{\partial M^*}{\partial \lambda} A_v'(A_u+A_v) - A_v\left(A_u' - A_v'\right)\frac{\partial M^*}{\partial \lambda}}{(A_u+A_v)^2} = -\frac{\partial M^*}{\partial \lambda}\frac{A_v' A_u + A_u' A_v}{(A_u+A_v)^2}. \qquad (6)$$

Finally, differentiate the *FOC* with respect to λ, and again express the result in terms of the risk aversions:

$$u' + (1+\lambda)\frac{\partial M^*}{\partial \lambda} u'' = -\frac{\partial M^*}{\partial \lambda} v'' \Rightarrow \frac{\partial M^*}{\partial \lambda} = \frac{-u'}{(1+\lambda)u'' + v''} = \frac{1}{(1+\lambda)(A_u+A_v)}. \qquad (7)$$

Plugging equation 7 into equation 6 completes the proof.

QED Proposition 1

Proof of Proposition 2: To calculate the cross partial of percentage demand, first differentiate with respect to budget:

$$\frac{\partial}{\partial B}\left(\frac{M^*}{B}\right) = \frac{B\frac{\partial M^*}{\partial B} - M^*}{B^2} = \left(\frac{1}{B^2}\right)\left(B\frac{\partial M^*}{\partial B} - M^*\right).$$

Now differentiate by durability, and plug in the result of Proposition 1:

$$B^2 \frac{\partial^2}{\partial \lambda\, \partial B}\left(\frac{M^*}{B}\right) = -\frac{\partial M^*}{\partial \lambda} + B\frac{\partial^2 M^*}{\partial \lambda\, \partial B} = -\frac{(A_u+A_v)^2 + B\left(A_v A_u' + A_u A_v'\right)}{(1+\lambda)(A_u+A_v)^3}.$$

When the relative risk aversions are denoted $R_u \equiv M^* A_u, R_v \equiv E^* A_v$, the denominator can be expressed in terms of $R_u' = M^* A_u' + A_u, R_v' \equiv E^* A_v' + A_v$ as follows to prove the result:

$$A_v\left(MA_u' + A_u + EA_u' - EA_v' + EA_v' + A_v\right) + A_u\left(EA_v' + A_v + MA_v' - MA_u' + MA_u' + A_u\right)$$

$$= A_v\left(R_u' + R_v' + E\left(A_u' - A_v'\right)\right) + A_u\left(R_u' + R_v' + M\left(A_v' - A_u'\right)\right) =$$

$$= \left(R_u' + R_v'\right)(A_u + A_v) + \left(A_u' - A_v'\right)(R_v - R_u) \qquad .$$

QED Proposition 2.

Proof of Corollary to Proposition 2: To show IRRA $\Rightarrow$ relative frugal materialism, it is enough to focus on the DARA case because we already know IARA and CARA are sufficient on their own. IRRA makes the first term in the numerator of $\frac{\partial^2}{\partial\lambda\,\partial B}\left(\frac{M^*}{B}\right)$ positive, so making the second term also positive, that is, $\left(A_u' - A_v'\right)(R_v - R_u) > 0$, is sufficient for relative frugal materialism. There are two cases:

1) When *M>E*, IRRA and $u = \alpha v$ means $R_v < R_u$, so we need $A_u' < A_v'$. From DARA, both slopes of ARA are negative. Because *M>E*, $A_u' < A_v'$ when ARA is steeper at the higher of the two consumption amounts, for which global concavity of ARA is sufficient.
2) When *M<E*, IRRA means $R_v > R_u$, and so we need $A_u' > A_v'$. From DARA, both slopes of ARA are negative. Because *M<E*, $A_u' > A_v'$ is steeper at the higher of the two consumption amounts, for which global concavity of ARA is sufficient.

To show relative frugal materialism $\Rightarrow$ not DRRA, note DRRA makes the first term in the numerator of $\frac{\partial^2}{\partial\lambda\,\partial B}\left(\frac{M^*}{B}\right)$ negative, so making the second term also negative, that is, $\left(A_u' - A_v'\right)(R_v - R_u) < 0$, is sufficient to rule out relative frugal materialism. There are two cases:

1) When *M>E*, DRRA and $u = \alpha v$ means $R_v > R_u$, so we need $A_u' < A_v'$. Because DRRA implies DARA, both slopes are negative, and the same argument as in the above case 1 shows global concavity of ARA is sufficient.
2) When *M<E*, DRRA and $u = \alpha v$ means $R_v < R_u$, so we need $A_u' > A_v'$. Because DRRA implies DARA, both slopes are negative, and the same argument as in the above case 2 shows global concavity of ARA is sufficient. *QED Corollary to Proposition* 2

Proof of Proposition 3: The solution to the two-period problem proceeds by backward induction. It is obvious that tomorrow, the consumer will equalize the marginal benefits of the experience and the material product by setting

$$u'\left(M_1 + M^*_{2,1,sgn(I_2)}\right) = v'\left(B + s + sgn(I_2)\,\sigma - M^*_{2,1,sgn(I_2)}\right)$$

whenever the material product turns out to be durable, and

$$u'\left(M^*_{2,0,sgn(I_2)}\right) = v'\left(B + s + sgn(I_2)\,\sigma - M^*_{2,0,sgn(I_2)}\right)$$

otherwise, where $M^*_{2,durability,sgn(I_2)}$ is the optimal level of additional material product purchased tomorrow. I assume that *u* and *v* are well-behaved and *B* is large-enough relative to σ such that these first-order conditions indeed describe the optimal state-contingent choice of M_2. Inducting backward to today, the first-order conditions in M_1 and *s* are:

$$u'(M_1) + \lambda E_{I_2}\left[u'\left(M_1 + M^*_{2,1,sgn(I_2)}\right)\right] = v'(B - M_1 - s)$$

$$E_{I_2}\left[\lambda u'\left(M_1 + M^*_{2,1,sgn(I_2)}\right) + (1-\lambda)u'\left(M^*_{2,0,sgn(I_2)}\right)\right] = v'(B - M_1 - s)$$

where the envelope theorem implies that we do not need to consider the marginal effects of M_1 and *s* on the four possible $M^*_{2,durability,sgn(I_2)}$, and tomorrow's first-order conditions allow us to express the marginal benefit of *s* in terms of the slope of *u.*

Consider the two extremes of durability, starting with complete perishability: When $\lambda = 0$, the material product only gives a contemporaneous utility flow, and the first-conditions become:

$$u'(M_1) = v'(B - M_1 - s) = E_{I_2}\left[u'\left(M^*_{2,0,sgn(I_2)}\right)\right]$$

$$= \frac{v'\left(B + s + \sigma - M^*_{2,0,+}\right) + v'\left(B + s - \sigma - M^*_{2,0,-}\right)}{2}$$

where the first term is the marginal benefit of today's material product, the second term is today's marginal opportunity cost of not buying today's experience, the third term is the marginal benefit tomorrow from saving money today, and the last term both explicates the expectation over income shocks and uses tomorrow's first-order condition to express the third term in terms of *B* and *s.* It is immediate that when σ=0, the equations are satisfied with *s*=0 and $M_1^* = M_2^*$: without tomorrow's income shock, there is no need to save money today because the per-period incomes and utilities are the same. What happens to the optimal *s* as σ increases? Differentiating all parts of the first-order condition with respect to σ yields the following result:

Lemma: Let $u''_{sgn(I_2)} = u''\left(M^*_{2,0,sgn(I_2)}\right)$ and $v''_{sgn(I_2)} = v''\left(E^*_{2,0,sgn(I_2)}\right)$. Then, the sign of the slope of the optimal precautionary saving amount is $sgn\left(\frac{ds^*}{d\sigma}\right) = sgn\left(u''_+u''_-(v''_+ - v''_-) + v''_+v''_-(u''_+ - u''_-)\right)$. When $u''' > 0$and $v''' > 0$, then $sgn\left(\frac{ds^*}{d\sigma}\right) = +1$. When $u''' < 0$and $v''' < 0$, then $sgn\left(\frac{ds^*}{d\sigma}\right) = -1$.

Proof of Lemma: The following three first-order conditions jointly characterize M_1^*,$s^*$and $M^*_{2,0,sgn(I_2)}$:

$$u'(M_1^*) = v'(B - M_1^* - s^*)$$

$$u'\left(M^*_{2,0,sgn(I_2)}\right) = v'\left(B + s + I_2 - M^*_{2,0,sgn(I_2)}\right)$$

$$2v'(B - M_1^* - s^*) = v'\left(B + s^* + \sigma - M^*_{2,0,+}\right) + v'\left(B + s^* - \sigma - M^*_{2,0,-}\right).$$

Differentiating both sides of the first equation with respect to σ yields:

$$\frac{dM_1^*}{d\sigma} = \left[\frac{-v''(E_1^*)}{u''(M_1^*) + v''(E_1^*)}\right]\frac{ds^*}{d\sigma}$$

The expression in the square brackets is clearly negative, so M_1^*increases in σ whenever s^*decreases in σ, and vice versa. Differentiating both sides of the second equation with respect to σ yields:$\frac{dM_{2,0,+}^*}{d\sigma} = \frac{v_+^{''}}{u_+^{''}+v_+^{''}}\left(1+\frac{ds^*}{d\sigma}\right)$ and $\frac{dM_{2,0,-}^*}{d\sigma} = -\left(1-\frac{ds^*}{d\sigma}\right)\frac{v_-^{''}}{u_-^{''}+v_-^{''}}$

Both $\frac{dM_{2,0,sgn(I_2)}^*}{d\sigma}$ have the same sign as $\frac{ds^*}{d\sigma}$because the whole point of first-period savings is to reserve some money for spending tomorrow. Finally, differentiating both sides of the third equation with respect to σ yields:

$$-2\left(\frac{ds^*}{d\sigma}+\frac{dM_1^*}{d\sigma}\right)v^{''}(E_1^*) = \left(1+\frac{ds^*}{d\sigma}-\frac{dM_{2,0,+}^*}{d\sigma}\right)v_+^{''} - \left(1-\frac{ds^*}{d\sigma}+\frac{dM_{2,0,-}^*}{d\sigma}\right)v_-^{''}$$

Plugging $\frac{dM_1^*}{d\sigma}$ and $\frac{dM_{2,0,sgn(I_2)}^*}{d\sigma}$ into yields the following solution for $\frac{ds^*}{d\sigma}$in terms of only the second derivatives of *u* and *v* at different points:

$$\frac{ds^*}{d\sigma} = \frac{\frac{u_+^{''}v_+^{''}}{u_+^{''}+v_+^{''}}-\frac{u_-^{''}v_-^{''}}{u_-^{''}+v_-^{''}}}{-2\frac{u^{''}(M_1^*)v^{''}(E_1^*)}{u^{''}(M_1^*)+v^{''}(E_1^*)}-\frac{u_+^{''}v_+^{''}}{u_+^{''}+v_+^{''}}-\frac{u_-^{''}v_-^{''}}{u_-^{''}+v_-^{''}}} \qquad \text{(A1)}$$

Since *u* and *v* are concave, the denominator is clearly positive, and so the sign of $\frac{ds^*}{d\sigma}$boils down to the sign of the numerator:

$sgn\left(\frac{ds^*}{d\sigma}\right) = sgn\left(\frac{u_+^{''}v_+^{''}}{u_+^{''}+v_+^{''}}-\frac{u_-^{''}v_-^{''}}{u_-^{''}+v_-^{''}}\right) = sgn\left(u_+^{''}v^{''}(u_-^{''}+v_-^{''}) - u_-^{''}v_-^{''}(u_+^{''}+v^{''})\right) =$

$sgn\left(u_+^{''}u_-^{''}(v_+^{''}-v_-^{''}) + v_+^{''}v_-^{''}(u_+^{''}-u_-^{''})\right)$ where the last equality merely rearranges terms. It is obvious that the signs of $(v_+^{''}-v_-^{''})$ and $(u_+^{''}-u_-^{''})$, when the same, determine the sign of the entire expression. Since a positive income shock leaves more money to be spent on both goods in the second period, it follows that $M_{2,0,+}^* > M_{2,0,-}^*$ and $E_{2,0,+}^* > E_{2,0,-}^*$, so $(v_+^{''}-v_-^{''}) > 0$whenever $v^{''}$is increasing, i.e. whenever $v^{'''} > 0$. Analogously, $(u_+^{''}-u_-^{''}) > 0$ whenever $u^{'''} > 0$. *QED Lemma*

When $\lambda = 1$, the first-order conditions imply that the consumer will not put aside any money in the form of precautionary savings:

$$u^{'}(M_1) + E_{I_2}\left[u^{'}\left(M_1 + M^*_{2,1,sgn(I_2)}\right)\right] = v^{'}(B - M_1 - s) \qquad \left(FOC_{M_1}\right)$$

$$E_{I_2}\left[u^{'}\left(M_1 + M^*_{2,1,sgn(I_2)}\right)\right] = v^{'}(B - M_1 - s) \qquad (FOC_s)$$

The RHS of both FOCs is today's marginal opportunity cost of not buying today's experience, and it is clearly the same for M_1 and *s.* The $E_{I_2}\left[u^{'}\left(M_1 + M^*_{2,1,sgn(I_2)}\right)\right]$ of the LHS is tomorrow's benefit of spending money today on anything other than today's experience, and it is also the same for M_1 and *s.* However, spending money on the durable product today has an additional benefit of getting a utility flow from it today. Therefore, perfect durability cannot involve a positive level of saving, and the optimal investment into the material product is governed by the first FOC with s=0. As above, we can investigate the effect of increasing σ from zero: When σ=0, the combination of today's and tomorrow' FOCs becomes:

$$u^{'}(M_1^*) = v^{'}(B - M_1^*) - v^{'}(B - M_2^*) \text{ where } v^{'}(B - M_2^*) = u^{'}(M_1^* + M_2^*)$$

Since u and v are concave, the requirement that $v'(B - M_1^*) > v'(B - M_2^*)$ immediately implies that $M_1^* > M_2^*$. In words, the durability of the material product makes the consumer buy more of it today than tomorrow because the marginal benefit of buying more material good today is both the marginal contemporaneous utility flow from the good today and the marginal benefit of allocating more of tomorrow's budget to experiences. For the same reason, the consumer buys more of the durable material good than if the good were completely perishable.

The previous paragraph has shown that increased durability increases today's expenditure on the material good because of the associated increase in tomorrow's resources: not having to spend as much on the material good tomorrow leaves more money for buying experiences. But while such a comparative static can be interpreted as a saving motive whereby the consumer gives up some amount of experiences today in order to benefit in the future, it is not a *precautionary* savings motive because the future is not uncertain. To examine such a precautionary motive in today's demand for the durable good, we need to investigate the effect of increasing σ from zero on M_1^*. Restate the expected marginal benefit of future resources in terms of the marginal utility of tomorrow's experience, and the FOC becomes:

$$u^{'}(M_1) + \frac{v^{'}\left(B + \sigma - M_{2,1,+}^*\right) + v^{'}\left(B - \sigma - M_{2,1,-}^*\right)}{2} = v^{'}(B - M_1) \qquad \left(FOC_{M_1}\right)$$

Since the uncertainty only affects the expected marginal utility of the increase in tomorrow's resources, M_1^* increases from the σ=0 level whenever the expected marginal benefit of future resources (the second term on the LHS) increases in σ, i.e. whenever $v''\left(B + \sigma - M_{2,1,+}^*\right) > v''\left(B - \sigma - M_{2,1,-}^*\right)$, which is the same condition for precautionary savings in the completely perishable case: $v^{'''} > 0$and $u''' > 0$ are necessary. *QED Proposition* 3