PSYCHOMETRIKA—VOL. 48, NO. 4.
DECEMBER, 1983

TWO LOGIT MODELS FOR EXTERNAL ANALYSIS OF PREFERENCES

LEE G. COOPER

UNIVERSITY OF CALIFORNIA, LOS ANGELES

MASAO NAKANISHI

KWANSEI GAKUIN UNIVERSITY, JAPAN

A logit vector model and a logit ideal point model are presented for external analysis of paired comparison preference judgments aggregated over a homogeneous group. The logit vector model is hierarchically nested within the logit ideal point model so that statistical tests are available to distinguish between these two models. Generalized least squares estimation procedures are developed to account for heteroscedastic sampling error variances and specification error variances. Two numerical illustrations deal with judgments concerning employee compensation plans and preferences for salt and sugar in the brine of canned green beans.

Key words: multidimensional scaling, external analysis of preferences, generalized least squares estimation.

INTRODUCTION

"External analysis of preferences" is the phrase Carroll [1972, 1980] used for the process of relating preference judgments to a pre-existing configuration of stimuli or choice alternatives. While internal analysis of preferences attempts to display the multidimensional structure underlying preferential judgments themselves, external analysis of preferences relates preference judgments to stimulus configurations developed by other means. In marketing research, product maps have come from factor analysis, discriminant analysis, cluster analysis, conjoint measurement, and multidimensional scaling of dissimilarity judgments, as well as physical measurement of product attributes. External analysis of preferences allows the researcher to choose the information base for the mapping and to integrate preference judgments into the chosen information base.

Coombs' [1964] unidimensional unfolding model, and the multidimensional generalizations of it by Bennett and Hays [1960], held that the individuals agreed on the similarity structure underlying the objects, and individuals disagreed only on the preference orderings of the objects. Implicit in external analysis of preferences, however, is the flexibility to represent systematic differences in perception of the objects as well as differences in preferences. Carroll used a weighted euclidean "distance" formulation to allow for individual variability in the importance of each dimension in accounting for the preferences of each individual. With all positive weights, there could be differential shrinking or stretching of the dimensions of a common perceptual space. Isopreference ellipsoids displayed how preference dropped off with increasing departure from an ideal point. With all negative weights, these same ellipsoids displayed how preferences increased with increasing departure from an "anti-ideal" point. With a mixture of positive and negative

The authors would like to thank Yoshio Takane, Robert J. Meyer and anonymous reviewers for their helpful comments on earlier drafts of this manuscript.

Requests for reprints should be sent to Lee G. Cooper, Graduate School of Management, University of California, Los Angeles, California, 90024.

0033-3123/83/1200-5001$00.75/0

weights, saddle points described how preferences decreased with departures in certain directions from the ideal and increased in other directions. While the case of positive and negative weights can provide an appealing representation of preferences, it does make it somewhat awkward to speak of external analysis of preferences as a distance model. The "distances" in the mixed weight case could possibly be negative (i.e., violate the minimality assumption of distances), and the triangle inequality could be violated in certain cases.

Overcoming this awkwardness is part of the rationale for Srinivasan and Shocker's [1973] linear programming model for external analysis of preferences. Their LINMAP procedure allows the researcher the choice of whether or not to restrict weights to being strictly positive. Davison's [1976a] quadratic programming approach to Carroll's weighted unfolding model also allows the researcher the option of constraining dimensional weights to being nonnegative.

We believe that in consumer research and possibly many other contexts the flexibility for representing preferences in terms of saddle points is an asset. Consider d_{ij}^*

$$d_{ij}^* = \sum_{h=1}^{H} \beta_{hi}(X_{hi} - X_{hj})^2, \tag{1}$$

where X_{hj} are the known coordinates of the choice alternatives $j = 1, 2, \ldots, m_i$ on given dimensions $h = 1, 2, \ldots, H$; X_{hi} are coordinates to be estimated of an ideal point for individual or homogeneous group $i = 1, 2, \ldots, I$; and β_{hi} are dimensional weights to be estimated. This is what Carroll calls a squared distance. If the lack of an absolute origin dissuades us from calling this a distance, there is nonetheless a great deal of interval scale information in d_{ij}^*. To understand what this measure tells us about preferences one must first specify a relation of d_{ij}^* to preferences. In his linear-quadratic hierarchy of models Carroll [1972, 1980] specified a method for obtaining dimensional preference weights (i.e., a vector preference model) and a procedure for obtaining estimates of ideal points for the set of alternatives. The paired comparison judgments are aggregated over repeated administrations to a single individual or homogeneous group into a matrix of choice frequencies of each alternative over each other alternative. Twice the column sums of this aggregated dominance matrix are used as preference scale values p_{ij}. For the ideal point model, the preference scale values are considered to be a linear function of d_{ij}^*.

Rather than compressing $m_i(m_i - 1)/2$ paired comparisons into m_i preference scale values, p_{ij}, prior to obtaining dimensional weights and ideal points, we propose two models for estimating these parameters directly from the pairwise relative choice frequencies, f_{ijk}.

The Models

Let $\bar{f}_{ijk}$ be the expected relative frequency of choice of alternative j over alternative k in the paired comparison preference judgments of homogeneous group i.

$$\bar{f}_{ijk} + \bar{f}_{ikj} = 1, \qquad (j, k) = 1, 2, \ldots, m_i; \qquad i = 1, 2, \ldots, I. \tag{2}$$

The model for $\bar{f}_{ijk}$ is given by:

$$\bar{f}_{ijk} = \frac{\exp(d_{ij}^*)\varepsilon_{ij}}{\exp(d_{ij}^*)\varepsilon_{ij} + \exp(d_{ik}^*)\varepsilon_{ik}}, \tag{3}$$

where d_{ij}^* is defined in (1) and ε_{ij} is a log-normally distributed, specification error term unique to the alternative and the homogeneous group. Although Yellott [1977] did not consider ulitity functions such as d_{ij}^* in (1), we note that the function in (3) is formally derivable from the generalized extreme value distribution for a random utility model [cf. Yellott, 1977]. Figure 1 displays the ideal distance model implied by (3). In this figure and the two subsequent figures, the ideal point is considered to be at (0, 0) for dimensions one and two. Since preference decreases in any direction away from the origin the preference density on the z axis is exp (d_{ij}^*) where the signs of both weights β_{hi} in (1) are negative. Figure 2 displays the anti-ideal distance model in which the signs of both weights β_{hi} in (1) are positive. Figure 3 displays the saddle point which results from one negative weight and one positive weight.

Exponential density functions were introduced into multidimensional scaling by Shepard [1957]. The exponential of minus the interobject distance provides for rapid exponential decay in any direction away from the location of an object and is a very good representation of stimulus generalization gradients. Shepard [1957] used this representation to relate the probability of interobject confusion to interpoint distance. For relating distance to preference we are more attracted to the exponential of minus the squared "distance." Rather than being sharply peaked, as in Shepard's representation, there is a region of almost maximum preference before preference declines more rapidly. Rather than abruptly terminating to zero preference, as in the external ideal point analysis in Carroll's linear-quadratic hierarchy of models [Carroll & Chang, Note 1; Carroll 1972, 1980], this function tails off with increasing distance from the ideal point.

The logit ideal point model in (3) can be compared to a corresponding logit vector model given by

$$\bar{f}_{ijk} = \frac{\exp\left(\sum_{h=1}^{H} \alpha_{hi} X_{hj}\right)\varepsilon_{ij}}{\exp\left(\sum_{h=1}^{H} \alpha_{hi} X_{hj}\right)\varepsilon_{ij} + \exp\left(\sum_{h=1}^{H} \alpha_{hi} X_{hk}\right)\varepsilon_{ik}}, \tag{4}$$

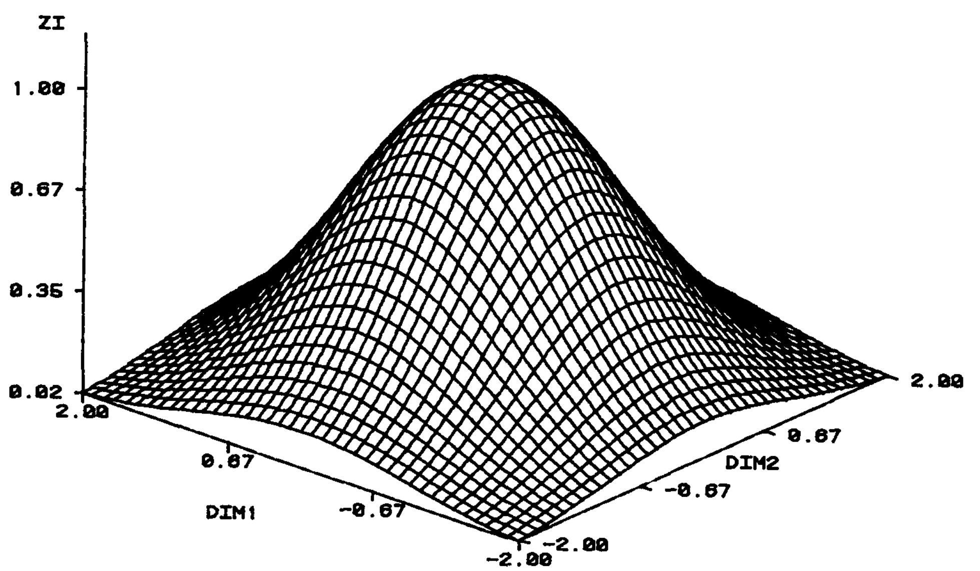

FIGURE 1.
Preference density, ZI, as a function of departure from an ideal point at (0, 0).

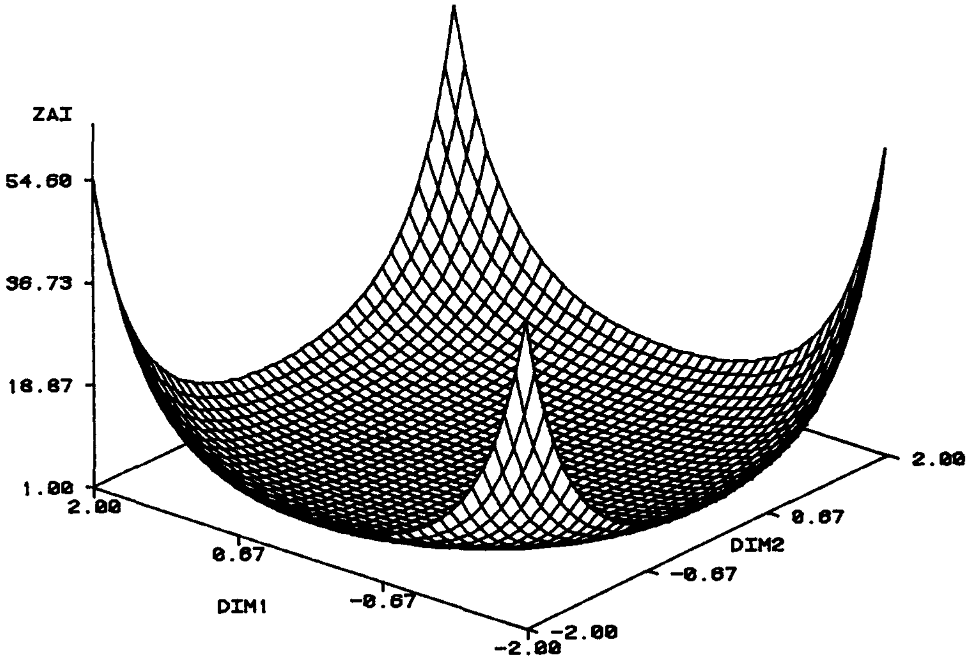

FIGURE 2.
Preference density, ZAI, as a function of departure from an anti-ideal point at (0, 0).

where α_{hi} is an unknown parameter representing the influence of dimension h on preference in group i.

For the logit ideal point model we have

$$\log \frac{\bar{f}_{ijk}}{\bar{f}_{ikj}} = d_{ij}^* - d_{ik}^* + (\log \varepsilon_{ij} - \log \varepsilon_{ik}). \tag{5}$$

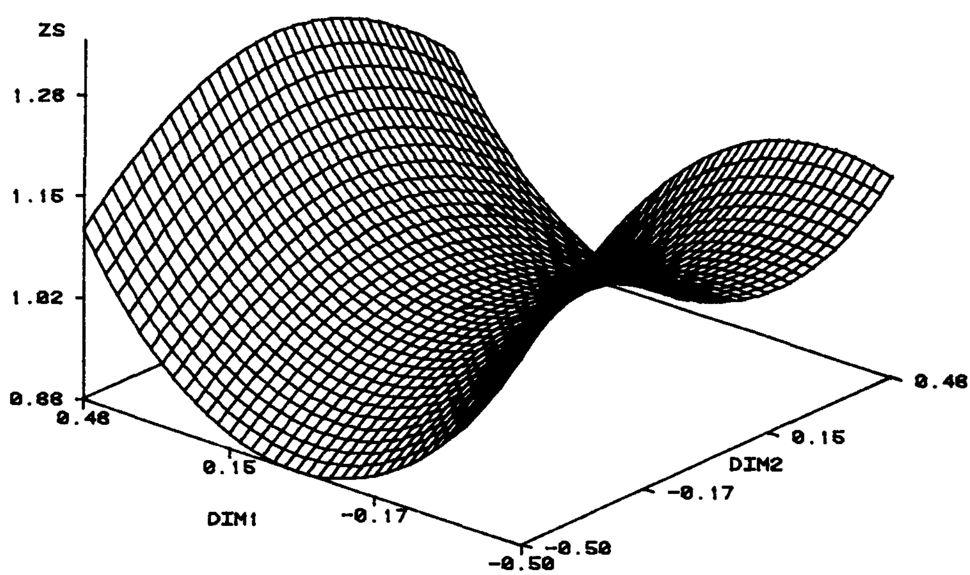

FIGURE 3.
Preference density ZS, as a function of departure from a saddle point at (0, 0).

Note that

$$d^*_{ij} - d^*_{ik} = \sum_{h=1}^{H} \beta_{hi}[(X^2_{hj} - 2X_{hj}X_{hi} + X^2_{hi}) - (X^2_{hk} - 2X_{hk}X_{hi} + X^2_{hi})]$$

$$= \sum_{h=1}^{H} \beta_{hi}[X^2_{hj} - X^2_{hk} - 2X_{hi}(X_{hj} - X_{hk})]$$

$$= \sum_{h=1}^{H} \beta_{hi1}(X_{hj} - X_{hk}) + \beta_{hi2}(X^2_{hj} - X^2_{hk}), \tag{6}$$

where β_{hi1} is a parameter associated with the difference in known scale values for alternatives on axis h ($\beta_{hi1} = -2\beta_{hi}X_{hi}$), and β_{hi2} is a parameter associated with the difference in the squares of known scale values on axis h ($\beta_{hi2} = \beta_{hi}$). These parameters, and consequently the ideal points, can be estimated from sample values for the relative frequency, f_{ijk}, from the multiple regression equation:

$$\log \frac{f_{ijk}}{f_{ikj}} = \sum_{h=1}^{H} \beta_{hi1}(X_{hj} - X_{hk}) + \beta_{hi2}(X^2_{hj} - X^2_{hk}) + u_{ijk}, \tag{7}$$

where u_{ijk} is a stochastic disturbance term which represents the combined influences of specification error ε_{ij} and sampling error for the departure of f_{ijk} from $\bar{f}_{ijk}$. In general, the error variance-covariance matrix will be nonspherical. Such a situation calls for the generalized least squares estimation procedures presented in the next section.

For the logit vector model we have

$$\log \frac{f_{ijk}}{f_{ikj}} + \sum_{h=1}^{H} \alpha_{hi}(X_{hj} - X_{hk}) + u_{ijk} \tag{8}$$

Hence, if β_{hi2} in (7) approaches zero, then the ideal point approaches plus or minus infinity and β_{hi1} approaches α_{hi}. The model in (8) is hierarchically nested within the model in (7), making comparative testing very straightforward.

For each homogeneous group i, the logit ideal point model in (7) implies a multiple regression through the origin with two independent variables for each dimension and $m_i(m_i - 1)$ dependent measures where m_i is the number of choice alternatives considered by homogeneous group i. Any standard multiple regression routine which allows one to specify a model without an intercept term will provide the required OLS estimates of β_{hi1} and β_{hi2}. The auxiliary information usually available with a standard statistical package will not be useful however. The squared multiple correlation computed without an intercept is like a congruence coefficient rather than what one might normally expect. The degrees of freedom for error and regression are not properly counted. As a consequence the F value calculated is also incorrect. All of these effects are due to the fact that half of the observations are merely the negatives of the other half. For the ideal point model there are $2H$ degrees of freedom associated with regression and $(m_i(m_i - 1)/2) - 2H - 1$ degrees of freedom associated with error. This suggests a much simplified OLS procedure. If one includes only the half of the logits in (7) which are unique (i.e. $m_i(m_i - 1)/2$ logits) along with the corresponding independent variables and an intercept term, then the multiple regression routine will produce the proper OLS estimates of β_{hi1} and β_{hi2}, the proper squared multiple correlation, degrees of freedom, mean squares and F statistic. It is possible to prove the equivalence of estimates from these two multiple regression approaches. (N.b., the appendix in Nakanishi & Cooper, 1982, has an analogous proof.) If one becomes concerned about possible multicollinearity from including linear and squared terms in a multiple regression model, one can use deviation scores for the original coordinates of the choice alternatives to reduce the potential problem.

Generalized Least Squares Estimation

Since the error covariance matrix will, in general, be nonspherical, we propose the following generalized least squares (GLS) estimation procedures for the model in (7). The proportion of individuals in group i who choose alternative j over alternative k, f_{ijk}, may be thought of as being generated by a multinomial sampling process. If we let the expected value of f_{ijk} be $\bar{f}_{ijk}$, then the variance of f_{ijk} is $\bar{f}_{ijk}(1-\bar{f}_{ijk})/n_i$, where n_i is the sample size of group i. For a large sample size (say $n_i > 100$), f_{ijk} is approximately normally distributed. It is known that the asymptotic distribution of the sampling error term, $\log(f_{ijk}/1-f_{ijk}) - \log(\bar{f}_{ijk}/1-\bar{f}_{ijk})$, is normal with zero mean and variance equal to

$$\sigma_{ijk}^2 = 1/n_i \bar{f}_{ijk}(1-\bar{f}_{ijk}) \tag{9}$$

[Rao, 1973, p. 385].

It is clear that σ_{ijk}^2 is a minimum for $\bar{f}_{ijk} = .5$ and approaches infinity as $\bar{f}_{ijk}$ approaches zero or one. Thus σ_{ijk}^2 is nonspherical over different pairs (j, k) as we asserted in calling for GLS procedures for estimating the parameters in (5) and (7).

Note that the error term, u_{ijk}, is a sum of two error components, specification error, $\log \varepsilon_{ij} - \log \varepsilon_{ik}$, and sampling error, $\log(f_{ijk}/(1-f_{ijk})) - \log(\bar{f}_{ijk}/(1-\bar{f}_{ijk}))$. Assuming that sampling errors are uncorrelated with specification errors, we have

$$E(u_{ijk}) = 0, \tag{10}$$

$$\text{Var}(u_{ijk}) = \sigma_{\varepsilon_i}^2 + \sigma_{ijk}^2. \tag{11}$$

Since σ_{ijk}^2 is estimable by replacing $\bar{f}_{ijk}$ in (9) with f_{ijk}, and since the expected sum of squares of OLS errors, u_{ijk}, is equal to $\sigma_{\varepsilon_i}^2$ plus σ_{ijk}^2 both summed over distinct pairs (jk), one may estimate $\sigma_{\varepsilon_i}^2$ by

$$\hat{\sigma}_{\varepsilon_i}^2 = \frac{\sum_{(j,k)} (\hat{u}_{ijk})^2 - \sum_{(j,k)} \hat{\sigma}_{ijk}^2}{\frac{m_i(m_i-1)}{2}}. \tag{12}$$

Thus we need only premultiply both the dependent variable,

$$\log \frac{f_{ijk}}{(1-f_{ijk})},$$

and the independent variables, X_{hj}, X_{hj}^2, X_{hk}, X_{hk}^2 $(h = 1, 2, \ldots, H)$ by

$$\left[\hat{\sigma}_{\varepsilon_i}^2 + \frac{1}{n_i f_{ijk}(1-f_{ijk})}\right]^{-1/2}$$

and re-estimate parameters in an ordinary multiple regression routine to obtain the generalized least squares estimates.

The estimates of mean squared error (MSE) for the multiple regression model and the sampling error variances must be nonnegative. But the difference between the estimate of the MSE and the mean of the estimates of the sampling error variances (i.e., the estimate of the specification error variance in (12)) could be negative. In such a case the estimate of the specification error variance should be set to zero. This makes the GLS procedures already outlined properly redress the heteroscedasticity of the original sampling error variances.

As with the OLS results, the GLS results can be obtained in two ways. With the complete set of $m_i(m_i - 1)$ logits in each homogeneous group, a multiple regression through the origin is performed on the reweighted dependent and independent variables. The simpler procedure involves using half of the reweighted logits, the corresponding half of the reweighted explanatory variables and an intercept term which is not reweighted. The squared multiple correlation (R^2) from the OLS step should not be directly compared to the R^2 from the GLS procedure since the dependent variables differ because of this reweighting. But the other statistical tests are applicable. A significant overall F test indicates the parameter vector is statistically different from the null vector. When only sampling error variance is considered (i.e., when $\hat{\sigma}^2_{\varepsilon_i}$ is zero or is set to zero), the logit vector model is hierarchically nested within the logit ideal point model. Standard testing procedures will indicate if the extra parameter per dimension is statistically worthwhile. When both sampling error and specification error are present, the logit vector model is no longer nested within the logit ideal point model. This is because the dependent measures of these two models would be differentially reweighted when different specification error variances are estimated.

Numerical Illustrations

This section contains two illustrations of the logit models for external analysis of preferences using data previously analyzed by Bock and Jones [1968]. The first illustration involves employees' preference for alternative plans for increased compensation. A vector model should represent such data. The second illustration involves preferences for different concentrations of salt and sugar in the brine of samples of canned green beans. Such data should be represented by an ideal point model. In both illustrations the "proper" model was identified by the testing procedures developed in the previous sections.

Illustration 1

Bock and Jones [1968] analyzed salaried employees' preferences for nine alternative plans for an increase in compensation. The data, originally collected by Dr. J. Stacy Adams from 143 males, was used to illustrate analysis of partially balanced incompleted paired comparison (PBIPC) designs. The alternative plans varied in amount of additional annual salary, additional annual vacation days and additional percentage contribution to retirement fund (cf. Bock & Jones 1968, p. 178). Rather than compute affective values for the plans from the paired comparisons, our analysis attempts to relate the preference judgments to the three dimensions involved in the design of the plans. The 27 angular deviates reported by Bock and Jones [1968, p. 180] were converted back into proportions from the PBIPC design. The OLS results reported in Table 1 were used to select the appropriate model. While both the logit vector and logit ideal point models give a good account of the preference judgments, the logit ideal point model is not a significant improvement over the logit vector model. Table 2 presents the GLS results for the logit vector model. The R^2 of .86 is the proportion of linearly accountable variance in the logits, not in the original choice proportions. Since the scale of measurement differs for salary, vacation time and retirement benefits, the t-values reported provide a good indication of the relative importance of each aspect of the compensation plans. As one would expect, all the additional benefits contribute positively to preferences, and salary increments were the most valued component. Figure 4 presents the plots of the logits versus the predicted values and the logits versus the residuals. The plot of logits versus the predicted values seems approximately linear, with perhaps a slight departure downward at both extremes. The plot of logits versus the residuals shows no particular pattern. The logit vector model seems to be an apt representation for these data.

Table 1

Comparing Logit Preference Models for Compensation Plans from Bock and Jones--OLS Results.

Logit Ideal Point Model			Logit Vector Model			Difference	
R^2	F	p	R^2	F	p	F	p
.88 (.85)†	24.7 (6,20)††	<.001	.83 (.80)†	36.7 (3,23)††	<.001	3.0 (3,20)††	NS

†Adjusted R^2.
††Degrees of freedom.

Illustration 2

Bock and Jones [1968, p. 141, ff.] reanalyzed judgments from Buck and Weckel [1956] to determine the optimal concentrations of salt and sugar in the brine of canned green beans. While the original study involved twenty-five samples, each containing one combination of five levels of salt and sugar (0, 1, 2, 3, 4 grams/milliliter of brine), Bock and Jones dealt with only the 16 samples involving the nonzero concentrations. Twenty-five different subjects judged each pair in each order. Bock and Jones found a large order effect indicating preference for the first sample presented to each subject. Even for a pooled analysis the error Chi-square rejected the model.

Again we use OLS procedures to select the appropriate model and use GLS procedures to refine our parameter estimates. The regression model contains an intercept term and a dummy variable indicating the order of presentation (giving a one to the first order and a zero to the second order). The parameter value for the intercept compensates for the missing half of the logits for the second order, while the sum of the parameter value for the intercept and the parameter value for the dummy variable compensates for the missing half of the logits for the first order. So both the logit vector model and the logit ideal point model have an additional parameter. Table 3 presents the test results comparing vector and ideal point models. Although both models produce R^2 values which are significantly greater than zero, the general level is lower than in the previous examples. The logit ideal point models is a significant improvement over the logit vector model. The significance of the order effect within the logit ideal point model is tested by

Table 2

Logit Vector Model for Compensation Plans from Bock and Jones--GLS Results.

R^2	F	p	Variable	Weights	t Value
.86 (.84)†	46.3 (3,23)††	<.001	Salary	.0105	7.4
			Vacation	.3135	3.8
			Retirement	.8944	5.4

†Adjusted R^2.
††Degrees of freedom.

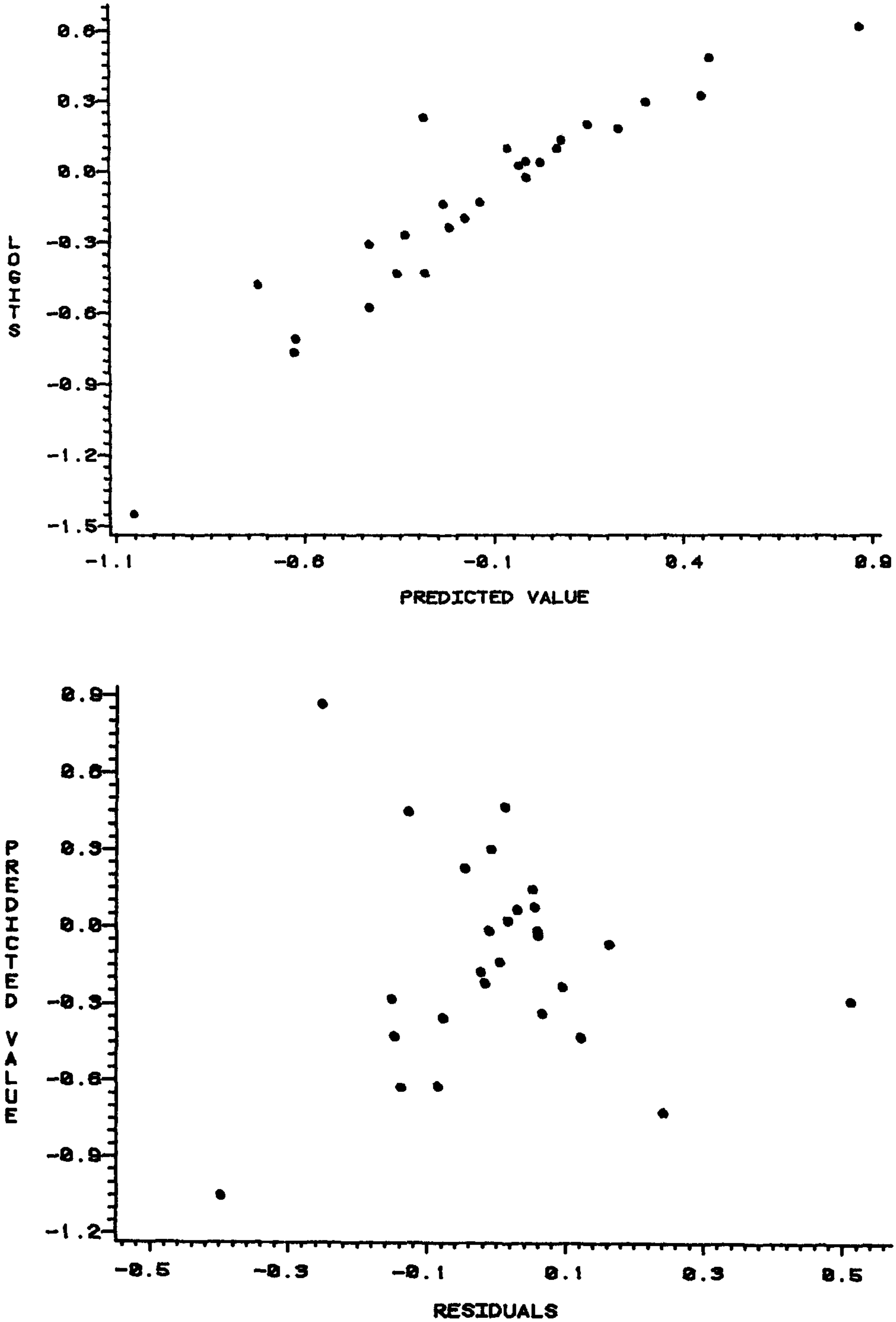

FIGURE 4.
Aptness of the logit vector model for preferences for the compensation plans from Bock and Jones.

Table 3

Comparing Logit Preference Models for Cut Green Beans from Bock and Jones--OLS Results.

Logit Ideal Point Model			Logit Vector Model			Difference	
R^2	F	p	R^2	F	p	F	p
.40 (.38)†	30.8 (5,234)††	<.001	.24 (.23)†	25.3 (3,236)††	<.001	29.8 (2,234)††	<.001

†Adjusted R^2.
††Degrees of freedom.

comparing it to a model which included the interactions of the order effect dummy variable with the four variables of the logit ideal point model. Table 4 presents the results of this test. Adding the interaction terms leads to a significant increase in the fit of the logit ideal point model.

Table 7 presents the GLS test results and parameter estimate for the logit ideal point model with interaction terms. It should be noted that the dummy variables are not rescaled when obtaining the GLS estimates. To obtain the estimates of the ideal coordinates for the second order one divides the parameter value for the salt variable by minus two times the parameter value for the salt-squared variable, and then divides the parameter value for the sugar variable by minus two times the parameter value for the sugar-squared variable. For the ideal coordinates corresponding to the first order one needs two sums. The first sum adds together the parameter value for the salt variable with the parameter value for the interaction of the order effect and the salt variable. The second sum adds together the parameter value for the salt-squared variable with the parameter value for the interaction of the salt-squared variable with the order effect. The first sum is divided by minus two times the second sum to estimate the ideal coordinate for the salt dimension for the first order of presentation. A parallel set of sums is formed for the sugar dimension to estimate the ideal coordinate for the sugar concentration. A consensus or joint estimate

Table 4

Test for Significance of Order Effects--OLS Results.

Logit Ideal Point Model With Interactions			Differences	
R^2	F	p	F	p
.44 (.42)†	20.1 (9,230)††	<.001	4.5 (4,230)††	<.005

†Adjusted R^2.
††Degrees of freedom.

of ideal coordinates for both orders of presentation comes from a model which includes the order effect dummy variables, but excludes the interaction terms. The bottom of Table 5 presents the ideal coordinates estimated from the two orders, the joint model, and the optimal values obtained from a quadratic response surface model estimated by Bock and Jones [1968, p. 192, ff.]. The order differences seem to have the greatest impact on the ideal coordinate for salt concentration. The ideal coordinates listed for the joint model came from the GLS results which excluded the interaction terms. This joint model gives results reasonably similar to the quadratic response surface model from Bock and Jones.

The plot of the logits versus the predicted logits in Figure 5(a) shows a linear relation with more scatter than we have witnessed in the earlier example. The plot of the predicted logits versus the residuals in Figure 5(b) shows no systematic pattern. Even though the fit of the model to these data is modest when heuristically compared to the R^2 values of the other example, the model seems apt.

Table 5

Order Effects Model for Canned Green Beans--GLS Results.

Logit Ideal Point Model

R^2	F	p
.49 (.47)†	24.5 (9,230)††	<.001

Variable	Parameter Estimate	t-Value
Intercept	- .155	-3.33
Salt	.771	3.69
Sugar	.300	1.46
Salt squared	- .102	-2.52
Sugar squared	- .077	-1.93
Order Effect	.299	4.52
Salt*Order	1.277	4.31
Sugar*Order	.261	.90
Salt squared*Order	- .234	-4.08
Sugar squared*Order	- .051	- .89

Ideal Point Estimates

	Dimension I (Salt)	Dimension II (Sugar)
Order I	3.05	2.19
Order II	3.78	1.94
Joint Model	3.22	2.11
Bock & Jones	3.27	2.39

†Adjusted R^2.
††Degrees of freedom.

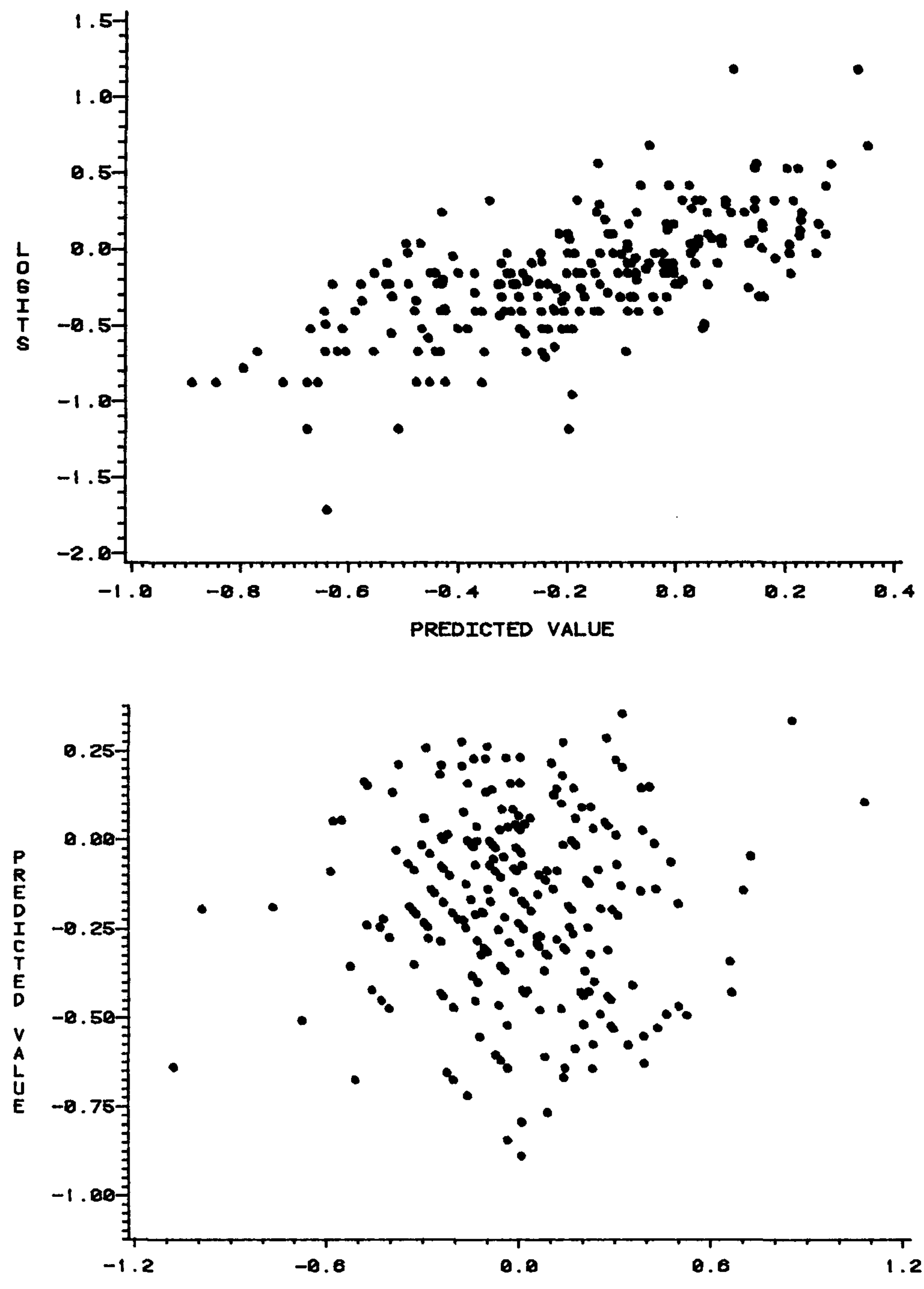

FIGURE 5.
Aptness of logit ideal point models for preferences for canned green beans from Bock and Jones.

Discussion

Debates on the suitability of vector models versus ideal point models for scaling preferences usually end by recognizing that neither one model nor the other is uniformly superior. Some domains are better represented by vector models and other domains are better represented by ideal point models. For external analysis of preferences, researchers select the information base to which preferences are related. Coombs (1975) and Davison (1976b) point out that vector models may be appropriate for relating preferences to one information base, while ideal point models may be appropriate for relating the same preferences to another information base. Under such circumstances it is very important to be able to distinguish between the two models.

The numerical examples illustrate that hierarchical testing procedures can distinguish the logit vector model from the logit ideal point model for preferences. In the reanalysis of data from Bock and Jones [1968] on preferences for additional compensation, both the logit vector model and the logit ideal point model had statistically significant R^2 values. The hierarchical test indicated that the additional parameters associated with the logit ideal point model were not statistically worthwhile. The incremental nature of the benefits offered inclined us to expect a vector model would be appropriate. If there is an ideal point for additional salary, or additional vacation day, or additional retirement contribution, it is far beyond the range of benefits offered in this data set. So we feel the hierarchical testing procedures selected the proper model. In the reanalysis of data from Buck and Weckel [1956] we expected an ideal point representation should be appropriate. Bock and Jones [1968] had found an optimum within the range of the salt and sugar concentrations which were investigated. Again both of the logit models produced statistically significant R^2 values and the hierarchical tests also revealed the significance of the order effects. Similar procedures could be employed to test for the systematic structure of other sources of group differences.

Davison [1976b] reported that other formulations of the unfolding model seldom significantly outpredict vector models, even when unfolding models prove superior in cross-validation. Results to date make us hopeful that the logit vector and logit ideal point models will be more readily distinguishable.

REFERENCE NOTE

1. Carroll, J. D. and Chang, J. J., Relating preference data to multidimensional solutions via a generalization of Coomb's unfolding model. Paper presented at the meeting of the Psychometric Society, April, 1967. (Unpublished manuscript, Bell Laboratories, 1967.)

REFERENCES

Bennett, R. S. and Hays, W. L. Multidimensional unfolding: Determining the dimensionality of ranked preference data. *Psychometrika*, 1960, *25*, 27–43.

Bock, R. D. and Jones, L. V. *The Measurement and Prediction of Judgment and Choice*. San Francisco: Holden-Day, 1968.

Buck, P. A. and Weckel, K. G. Study of consumer preference of salt and sugar levels in canned green beans. *Food Technology*, 1956, *10*, 421–423.

Carroll, J. D. Individual differences and multidimensional scaling. In R. N. Shepard, A. K. Romney and S. Nerlove (Eds.) *Multidimensional Scaling: Theory and Applications in the Social Sciences. Volume I: Theory*. New York: Seminar Press, Inc., 1972, 105–155.

Carroll, J. D. Models and methods for multidimensional analysis of preferential choice (or other dominance) data. In E. D. Lantermann and H. Feger (Eds.) *Similarity and Choice: Papers in Honour of Clyde Coombs*. Bern: Hans Huber, 1980, 234–289.

Coombs, C. H. *A Theory of Data*. New York: Wiley, 1964.

Coombs, C. H. A note on the relation between the vector model and the unfolding model for preferences. *Psychometrika*, 1975, *40*, 115–116.

Davison, M. L. Fitting and testing Carroll's weighted unfolding model for preferences. *Psychometrika*, 1976, *41*, 233–247. (a)

Davison, M. L. External analysis of preferences. *Psychometrika*, 1976, *41*, 557–558. (b)

Nakanishi, M. and Cooper, L. G. Simplified estimation procedures for MCI models. *Marketing Science*, 1982, *1 Summer*, 314–322.

Rao, C. R. *Linear Statistical Inference and Its Applications, Second Edition*. New York: Wiley, 1973.

Shepard, R. N. Stimulus and response generalization: A stochastic model relating generalization to distance in a psychological space. *Psychometrika*, 1957, *22*, 325–345.

Srinivasan, V. and Shocker, A. D. Linear programming techniques for multidimensional analysis of preferences. *Psychometrika*, 1973, *38*, 337–369.

Yellott, J. I. The relationship between Luce's choice axiom, Thurstone's theory of comparative judgments, and the double exponential distribution. *Journal of Mathematical Psychology*, 1977, *15*, 109–144.

Manuscript received 7/2/82
First revision received 3/7/83
Final version received 7/28/83