sion of merits and considerations in choice of approach along with several large scale illustrations.

An extensive catalog of production-inventory models appears in Chapter 6. The models are, for the most part, adequately motivated, discussed, and sequenced, beginning with the simple lot size problem and working up through probabilistic, dynamic models with oligopolistic interdependence of sales between firms. There is also a brief report of econometric results regarding aggregative inventory behavior. Similar cataloging and development is accorded the firm's capacity expansion problem. A nice development of queuing models and a discussion of PERT also appear in this chapter.

Chapter 7 is entitled "Models of Resource Allocation and Planning in Educational Institutions and Systems." The chapter illustrates how OR models and approaches of economics can be applied in nonmarket contexts. The models range from linear input-output structures of the national economy, with special emphasis on educational sectors (including students and teachers at various levels of educational attainment, attrition and graduation, the complementarity and substitution between teaching and research), to the standard assignment model for pairing teachers with classes. Dynamic models for calculation of resource requirements to meet any prespecified university throughput are presented. This chapter may be the strongest in the book with regard to adequacy of examples, discussion of the limitations of models, and implications of results. The book concludes with application of decomposition techniques to certain problems of regional economics and some suggestive remarks on OR and complex social systems.

Clearly the authors set themselves an ambitious and laudable task. No prerequisites are explicit, but the reader would be aided by a prior familiarity with linear programming, Kuhn-Tucker theory, and notions of probability. While the book may not be appropriate for a basic course in OR, it could be successfully used where such fundamentals have been studied previously or are to be bypassed in favor of learning about available OR techniques (as distinct from mastering the techniques) and/or studying certain applications of the OR/economics approach to modeling. The authors have achieved notational consistency, although they eschew a completely integrated treatment of the topics and models in favor of clear documentation of sources. Thus the reader can find what tools, techniques, and models are available and where to learn about each in more detail. (Actually, the materials developed in the book are intended to be more representative than exhaustive, however.) There are very few small worked examples, but more large scale illustrations of results. While there are no exercises, there are numerous unproved assertions that the reader can verify for himself to check his understanding.

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LEON S. LASDON, *Optimization Theory for Large Systems*, Macmillan Co., New York, N.Y., 1970, 523 pages, $14.95.

THIS IS THE first book devoted exclusively to the burgeoning field of large-scale mathematical programming. In the author's own words, he attempts "to collect together the best of this literature and unify it . . . in a straight-forward

and logical manner from a small set of basic ideas and principles." The time is surely ripe for such an effort, as anyone can attest who has tried to teach a course on the subject or pick his way through the jumbled literature.

I would like to applaud at the outset two conspicuous qualities that alone ought to assure widespread popularity of the book as a graduate-level text and reference work for professionals. These qualities are clarity of exposition and balance of emphasis between models, theory, algorithms, and computational experience. The style of writing is smooth, liberal and adept use is made of graphical illustrations and numerical examples, and enough patience has been exercised to make the discussion of each topic accurate and self-contained. The meticulously balanced emphasis is especially appropriate for this particular subject field.

On the broader question of how the author fared in meeting his stated objectives, I would say that he succeeded reasonably well considering that his is the first book-length attempt. I do have some subjective reservations concerning selection of material, choice of unifying principles and organization, but these can for the most part be compensated for if desired by discretion in using the book.

Weighing in at just over 100 pages, Chapter 1 accomplishes more than its minimal task of reviewing the required background material. It is self-contained and complete enough to be a slim but teachable supplementary text in its own right on the basic theory and computational methods of linear and nonlinear programming.

Chapter 2 discusses the genesis of structured large-scale problems, including those arising from the influence of time, uncertainty, interactions between subsystems, and the side effects of certain types of linear approximation. No mention is made of one of the most prolific sources, namely problems involving combinatorial features such as routes or schedules. The basic ideas of DANTZIG-WOLFE decomposition are also introduced, perhaps a little prematurely, with unnecessary confusion added by the choice of the continuous-time linear optimal control problem (with its systems of differential equations) as the vehicle for this introduction. A digression into the dual decomposition algorithm of ABADIE AND WILLIAMS also seems curiously out of place.

Chapter 3 is devoted to Dantzig-Wolfe decomposition for linear programs. The treatment is classical and thorough. BELL's unpublished procedure for using the DANTZIG-FORD-FULKERSON primal-dual algorithm to solve the Dantzig-Wolfe master problem is also explained at length. Thus three alternative treatments of the Dantzig-Wolfe master problem for linear programs are given: the original one based on the primal simplex method, the Abadie-Williams scheme based on the dual method, and the primal-dual scheme just mentioned. As the closing discussion comparing these three treatments recognizes, the practicality of the latter two is open to serious question. The resurrection of these two variants in the present book therefore seems largely an exercise of archival value.

The monumentally important idea of column generation in linear programming by subproblem optimization is taken up in Chapter 4. It is amply illustrated in several contexts, but a somewhat redundant series of sections is included on generalized LP, grid linearization, and a convex separable application of the Dantzig-Wolfe approach. These sections might better have been compressed into a single nonlinear generalization of Dantzig-Wolfe decomposition at the end of the

previous chapter. An unfortunate omission was the elegant and historic application by Ford and Fulkerson of column generation to the computation of maximal multi-commodity network flows.

Chapter 5 begins with a careful discussion of the concept of relaxation as a means of coping with large numbers of constraints. The concept is applied in describing a method by RITTER for solving block-angular linear programs with coupling variables *and* constraints. The remainder of the chapter gives an account of ROSEN's important primal partition programming method for block-angular linear programs with coupling constraints only. The account seems to me rather labored, though technically correct, because it is possible to give a cleaner derivation phrased in terms of the dual problem (a block-angular linear program with coupling *variables*) 'projected' onto the coupling variables and then solved by a 'piecewise' application of the simplex method that exploits the natural piecewise linearity of the objective function. Regrettably, the important concept of projection is never crisply defined in this book, although it is actually used on occasion (pp. 361, 373) under the alias "partitioning"—a loosely defined term used mostly in the set theoretic sense. The concept of a piecewise solution strategy is absent from the book.

Chapter 6 does a nice job describing the product form of the inverse, generalized upper bounding, and a few related topics.

Chapter 7 gives a complete if occasionally windy explanation of Rosen's convex partition programming method and BENDERS decomposition. The dual relation between Benders and Dantzig-Wolfe decomposition is spelled out in detail.

The subject of Chapter 8 is decomposition for decentralized nonlinear systems based on manipulation of prices, a topic sometimes known as Lagrangean decomposition. The foundation of this subject is nonlinear convex duality theory, to which the author rightly devotes a good deal of space. The dual problem adopted in STOER's minimax dual, and the use of graphical examples is particularly effective. Some especially interesting results are developed characterizing the directional derivatives of the dual objective function, with the help of DANSKIN's theory for max-min problems. Only a portion of the duality results developed are actually needed for the relatively brief discussion of gradient ascent and tangential approximation approaches to solving the dual problem (i.e., obtaining the correct 'prices' for the Langrangean decomposition). The latter approach is properly revealed as dually equivalent to Dantzig-Wolfe decomposition.

The final chapter is on decomposition in decentralized nonlinear systems via direct resource (right-hand-side) allocation, rather than via prices as in the previous chapter. The feasible-directions and tangential approximation approaches are described in detail. As the author does not address the question of convergence, it should perhaps be pointed out that the feasible-directions approach requires anti-jamming modifications if convergence is to be ensured in general. The methods described in this chapter should be regarded as mainly of speculative value, since they have not yet been put to any significant computational test.

The appendices are splendid summaries of important advanced material on convex analysis, though largely inessential to the body of the text.

In reading the book or using it as a course text, it might be more logical to cover the chapters in the order 1, 2, 6, 4, 3, 5, 7, 8, 9. This would take advantage of the

fact that compact inverse methods depend on nothing beyond the simplex method, and would introduce column-generation in its pure form before compounding it with inner linearization à la Dantzig-Wolfe decomposition. The material in Chapter 4 relating to Dantzig-Wolfe decomposition should be covered at the end of Chapter 3. Also, presenting the fundamentals of nonlinear duality immediately after Chapter 1 rather than waiting until Chapter 8 would eliminate lacunae now evident in the intervening chapters when aspects of nonlinear duality arise. For similar reasons, one should probably also transplant the easiest and most frequently used results from the appendices.

PROFESSOR LASDON deserves our thanks for making the field of large-scale optimization so much more accessible to student and researcher alike.

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Books Received

DENNIS J. AIGNER, *Basic Econometrics*, Prentice-Hall, Englewood Cliffs, N.J., 1971, 298 pages, $11.95.

T. W. ANDERSON, *The Statistical Analysis of Time Series*, John Wiley & Sons, New York, N.Y., 1971, 704 pages, $13.50.

P. ARMITAGE, *Statistical Methods in Medical Research*, John Wiley and Sons, New York, N.Y., 1971, 504 pages, $16.00.

A. V. BALAKRISHNAN, *Introduction to Optimization Theory in Hilbert Space*, Springer-Verlag, New York, N.Y., 1971, 153 pages, $4.40 (paper).

IVAN S. BANKI, *The Dictionary of Administration Supervision*, Systems Research, Los Angeles, Cal., 1971, 131 pages, $5.95.

ROBERT I. BENJAMIN, *Control of the Information System Development Cycle*, John Wiley & Sons, New York, N.Y., 1971, 94 pages, $7.95 (paper).

THOMAS L. BOULLION AND PATRICK L. ODELL, *Generalized Inverse Matrices*, John Wiley & Sons, New York, N.Y., 1971, 103 pages, $9.95.

GEORGE K. CHACKO, *Applied Statistics in Decision Making*, American Elsevier, New York, N.Y., 1971, 491 pages, $15.00.

I. M. DATZ, *Planning Tools for Ocean Transportation*, Cornell Maritime Press, Cambridge, Maryland, 1971, 168 pages, $10.00.

RONALD E. FRANK AND WILLIAM F. MASSY, *An Econometric Approach to a Marketing Decision Model*, MIT Press, Cambridge, Mass., 1971, 224 pages, $12.95.

J. DÖRR AND G. HOTZ, *Automatentheorie und Formale Sprachen*, Hochschultaschenbucher-Verlag, Bibliographisches Institut, Mannheim, West Germany, 1970, 505 pages, (paper).

RONALD A. HOWARD, *Dynamic Probabilisitc Systems, Volume 1: Markov Models*, John Wiley & Sons, 1971, 576 pages, $17.95.

WALTER ISARD AND THOMAS W. LANGFORD, *Regional Input-Output Study*, MIT Press, Cambridge, Mass., 1971, 228 pages, $10.00.

E. L. JACKS, *Associative Information Techniques*, American Elsevier, New York, N.Y., 1971, 224 pages, $15.50.

G. S. KOCH, JR., AND R. F. LINK, *Statistical Analysis of Geological Data, Volume 2*, John Wiley & Sons, New York, N.Y., 1971, 238 pages, $17.50.