

## Macroeconomic-Driven Prepayment Risk and the Valuation of Mortgage-Backed Securities

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**Abstract.** We develop a three-factor no-arbitrage model for valuing mortgage-backed securities in which we solve for the implied prepayment function from the cross-section of market prices. This model closely fits the cross-section of mortgage-backed security prices without needing to specify an econometric prepayment model. We find that implied prepayments are generally higher than actual prepayments, providing direct evidence of significant macroeconomic-driven prepayment risk premiums in mortgage-backed security prices. We also find evidence that mortgage-backed security prices were significantly affected by Fannie Mae credit risk and the Federal Reserve's quantitative easing programs. (*JEL* G12, G13, G21)

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A mortgage-backed security is a securitized claim to the principal and interest payments generated by a pool of mortgage loans. Mortgage-backed securities have traditionally been issued either by agencies such as Fannie Mae, Freddie Mac, and Ginnie Mae, or by private issuers. As of the end of 2016, the total notional amount of agency mortgage-backed securities outstanding was \$7.545 trillion, making this market one of the largest sectors of the global fixed income markets.<sup>1</sup> Agency mortgage-backed securities have the attractive feature that the timely payment of principal and interest is backed by either an implicit or explicit government guarantee. Thus, the primary focus in agency mortgage-backed security valuation is on the timing of prepayments.

This paper advocates and implements a no-arbitrage approach to the valuation of mortgage-backed securities. Specifically, we propose a model that provides internally consistent valuation across the entire cross-section of mortgage-backed securities. Our strategy closely parallels that of standard affine term structure models which provide no-arbitrage valuation of bonds across all maturities. We solve for an implied risk-neutral prepayment function using the entire cross-section of mortgage-backed security prices. A key advantage of this approach is that by studying the implied prepayment function, we can identify the factors that the market views as important drivers of prepayment risk as well as the risk premiums associated with those factors. Thereby, we avoid modeling actual prepayment behavior via an econometric model, a daunting task by any measure. To account for the liquidity of mortgage-backed securities and perceived credit risk of the agency guaranteeing them, we allow for the possibility that mortgage cash flows may be discounted at a different rate than Treasuries. We apply our model to a broad cross-section and time series of actively traded mortgage-backed securities issued by Fannie Mae.

A number of important results emerge from the analysis. First, we find that our no-arbitrage model fits the cross-section of mortgage-backed security prices surprisingly well. The median root-mean-square error (RMSE) across the

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<sup>1</sup>See [www.sifma.org/research/statistics.aspx](http://www.sifma.org/research/statistics.aspx).

entire coupon stack is 25.7 cents per \$100 notional, which is on the same order of magnitude as the bid-ask spread for mortgage-backed securities. This accuracy compares well to previous generations of valuation models for mortgage-backed securities. This is achieved using only a simple two-factor implied prepayment model instead of a formal econometric prepayment model, which often includes many explanatory variables. Our results indicate that the pricing of mortgage-backed securities in the market may be much more rational than is commonly believed among market practitioners.

Second, we find that implied risk-neutral prepayments behave very differently from actual prepayments. Furthermore, implied prepayment rates are not simply scaled versions of empirical prepayment rates. The average implied prepayment rate across all mortgage-backed securities in our sample is 25.13% per year. In contrast, the corresponding average empirical prepayment rate is 20.96%. The difference between the implied and empirical prepayment rates provides direct evidence that the market incorporates significant prepayment-related risk premiums into the prices of mortgage-backed securities.

Third, we find that implied prepayments are driven not only by interest rates, but also two additional macroeconomic risk factors—turnover and rate response. The turnover rate reflects prepayments occurring for exogenous reasons unrelated to interest rates, but possibly correlated with macroeconomic fluctuations. Examples include adverse income shocks or unemployment resulting in a move or a foreclosure, negative shocks to housing values resulting in underwater borrowers strategically defaulting on non-recourse loans, or homeowners with appreciated property taking cash-out mortgages to extract home equity. The rate response factor represents the time variation in the sensitivity of prepayments to mortgage refinancing incentives. For example, borrowers may be less able to refinance into a lower mortgage rate after declines in housing prices, during recessions in which borrowers' income or credit may have been impaired, or during periods in which mortgage lending standards are tightened. Intuitively, declines in the rate response factor can be viewed as a marketwide form of burnout (in contrast to the security-specific type of burnout often incorporated into econo-

metric prepayment models).

Fourth, we study the determinants of the prepayment risk premium by decomposing it into the risk premiums associated with the turnover and rate response factors. We find that the turnover factor carries a significant positive premium throughout the entire sample period, consistent with the systematic nature of turnover risk. The risk premium for the rate response factor is also positive on average, but temporarily takes on negative values during the refi waves of 2001–2005. This result raises the possibility that a borrower’s ability to refinance during the refi waves may have been influenced by housing values in addition to standard income and credit considerations.

Fifth, we find that cash flows from mortgage-backed securities are discounted at a rate 65.5 basis points (bps) higher on average than are cash flows from Treasuries. This spread varies significantly through time and is strongly correlated with the credit spread between Fannie Mae debt and Treasuries. Furthermore, the spread is significantly related to supply-related factors such as Federal Reserve purchases of mortgage-backed securities during its quantitative easing programs and the volume of mortgage settlement fails among primary dealers. These results provide direct evidence that agency credit/liquidity spreads influence the pricing of mortgage-backed securities.

Sixth and finally, we apply the fitted model to a number of interest-only/principal-only securities as an out-of-sample test of the framework. We find that the model closely matches the market prices of these securities.

## **1. Related Literature**

Because agency mortgage-backed securities guarantee the timely payment of principal and interest, there is no direct borrower-related credit risk—a default is simply a prepayment from the investor’s perspective. Instead, the primary sources of risk are interest rate changes, agency credit spreads, and the timing of pre-

payments. The valuation of mortgage-backed securities, however, is challenging because the reasons for terminating and prepaying a mortgage may depend on factors besides interest rates such as housing prices, employment status, or family size. For reviews of the literature, see Kau and Keenan (1995), Capone (2001), Hayre (2001), Wallace (2005), and Fabozzi (2016).

The first generation of pricing models was pioneered by Dunn and McConnell (1981a, 1981b) and extended by Brennan and Schwartz (1985). This framework approaches the valuation of mortgage-backed securities from the perspective of contingent claims theory. In particular, this approach models mortgage prepayments as the result of a borrower attempting to maximize the value of an implicit interest rate option. Dunn and Spatt (2005) and Stanton and Wallace (1998) extend the approach to model the prepayment decision as the result of minimizing lifetime mortgage costs in the presence of refinancing costs. The models in these papers imply an upper bound on mortgage prices that is often violated empirically, as demonstrated by Stanton (1995) and Boudoukh et al. (1997). Later papers add frictions to allow for higher mortgage prices and consider the value of the prepayment option jointly with the option to default. Important contributions are Titman and Torous (1989), Kau et al. (1992), Kau and Slawson (2002), Downing, Stanton, and Wallace (2005), Longstaff (2005), and many others. An important drawback of this modeling approach is that actual mortgage cash flows and mortgage-backed security prices often diverge significantly from those implied by these types of models.

The second generation of mortgage-backed security pricing models takes a more empirical approach. Typically, these models begin with a detailed econometric model of the historical behavior of prepayments, including elements such as geography, seasoning, burnout, seasonality, and other macroeconomic factors. Key examples of this approach include Schwartz and Torous (1989, 1992, 1993), Richard and Roll (1989), and Deng, Quigley, and Van Order (2000). In this framework, interest rate paths are simulated (under the risk-neutral probability measure) and the econometric prepayment model (estimated under the actual probability measure) is applied to specify the cash flows along each interest rate

path. However, prepayments in these models are driven exclusively by interest rate changes, thus there is no scope for a separate prepayment risk premium.<sup>2</sup> In addition, market participants do not agree about which econometric prepayment model to use in projecting prepayments. Carlin, Longstaff, and Matoba (2014) show that there is major disagreement between dealers about forecasted prepayment rates. Forecasting actual prepayment rates is a difficult task that is fraught with many challenges and difficulties. Furthermore, these models often give prices that diverge significantly from market prices, and can only be reconciled by introducing option-adjusted spreads into the framework.

Option-adjusted spreads are often more volatile than the underlying mortgage-backed security prices. This is shown in Figure 1, which plots the time series of option-adjusted spreads for FNMA 4.50%, 5.50%, and 6.50% mortgage-backed securities as given by the sequence of pricing models used by a major Wall Street mortgage dealer. As shown, the dealer changed its model frequently during the 2007–2015 period, primarily because the prior version of the model was failing to capture current market prices. The plot shows that changes in the model are often associated with large discontinuities in the time series of the option-adjusted spread that can be on the order of 50 bps or higher. This behavior in the option-adjusted spread, even when holding the dealer fixed, provides a motivation for basing empirical analysis on mortgage-backed security prices directly, rather than on option-adjusted spreads.

Several recent papers modify the basic econometric prepayment framework by allowing the model to depend on parameters implied from market prices. Specifically, a number of these papers allow the prepayment rate given by the econometric model to be scaled by a multiplier implied from the option-adjusted spreads of interest-only/principal-only securities. This approach is known as

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<sup>2</sup>That prepayment in these models is exclusively driven by interest rates does not imply that previous researchers were unaware that additional factors may be important. Rather, it illustrates the difficulty of incorporating additional macroeconomic factors into these types of second generation models. We are grateful to the referee for this insight.

the implied-prepayment or break-even-prepayment model. Examples of this approach include Cheyette (1996), Chen (1996), Chan (1998), and Chaudhary (2006). A key advantage of this framework is that it allows for the possibility of a separate prepayment risk premium since the implied or risk-neutral prepayment rate need not equal the actual prepayment rate. This framework, however, has the drawback that a separate calibration is required for each pair of interest-only/principal-only securities—the implied multiplier is different for each pair of securities. Thus, this approach cannot provide consistent no-arbitrage pricing across the cross-section of mortgage-backed securities with varying coupon rates (the coupon stack). Furthermore, this framework is still tied to a specific econometric prepayment model. Levin and Davidson (2005) allow for two multipliers in scaling the components of their econometric prepayment model that they designate as turnover and refi risk. They also provide an example of how their model can be applied to the cross-section of mortgage coupon rates. Thus, their paper has some similarities to ours. Their approach is based on option-adjusted spreads, does not impose the no-arbitrage restriction, and depends on a specific econometric prepayment model.<sup>3</sup>

Although we primarily focus on developing a no-arbitrage valuation framework for mortgage-backed securities, some of our results have parallels in the recent literature on whether the expected returns of mortgage securities include prepayment risk premiums. For example, Gabaix, Krishnamurthy, and Vigneron (2007) study the interest-only strips market and document that their option-adjusted spreads covary with the moneyness of the market, consistent with a prepayment risk premium and the existence of specialized mortgage-backed security investors. An interesting paper by Boyarchenko, Fuster, and Lucca (2016) calibrates the break-even-prepayment model to the option-adjusted spreads of individual pairs of interest-only/principal-only strips. They find evidence of prepayment risk premiums in mortgage-backed securities. In particular, they

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<sup>3</sup>Other important contributions to this literature include Linetsky (2004), Goncharov (2006), Gorovoy and Linetsky (2007), Malkhozov et al. (2014), Hanson (2014), and Song and Zhu (2016).

find that prepayment risk premiums explain the cross-sectional smile in option-adjusted spreads and infer that the time variation in the implied option-adjusted spreads is due to a non-prepayment-related factor. Diep, Eisfeldt, and Richardson (2016) study Treasury-hedged mortgage-backed security returns and also find evidence of time-varying prepayment risk premiums. Furthermore, these prepayment risk premiums change signs over time in response to the relative supply of discount and premium mortgage-backed securities in the market.

Our paper provides a complementary perspective to this literature in several ways. First, Gabaix, Krishnamurthy, and Vigneron (2007), Boyarchenko, Fuster, and Lucca (2016), and Diep, Eisfeldt, and Richardson (2016) focus on the risk premiums in the expected returns of individual securities. In contrast, we focus on the risk premiums pertaining to the marketwide factors driving mortgage prepayments. We are able to measure these factor risk premiums by comparing risk-neutral prepayment rates with empirical prepayment rates. By focusing on marketwide factor risk premiums, however, our approach does not allow us to study directly the cross-sectional structure of risk premiums in expected returns.<sup>4</sup> Second, Gabaix, Krishnamurthy, and Vigneron (2007), and Boyarchenko, Fuster, and Lucca (2016) study prepayment risk premiums through the lens of option-adjusted spreads. In contrast, our approach does not require the estimation of a formal econometric prepayment model. An implication of this, however, is that the implied prepayment model needs to be simple enough to be identified from the cross-section of TBAs in the market.<sup>5</sup> Fortunately, the results suggest that even a simple specification such as ours is able to capture the pricing of TBAs, IOs, and POs fairly accurately. We note, however, that our one-factor model of the U.S. Treasury term structure is limited in its ability to hedge the interest-

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<sup>4</sup>To do so, for example, would require measuring the conditional default probability of Fannie Mae under both the actual and risk-neutral measures. We do not have sufficient observations to measure the actual conditional default probability of Fannie Mae.

<sup>5</sup>For example, we are limited in our ability to incorporate complex patterns of seasoning, burnout, and geographical concentration, etc., into the implied prepayment model. We are grateful to the referee for this observation.

rate risk of mortgage-backed securities. A multifactor model similar to that used by Boyarchenko, Fuster, and Lucca (2016) might be more appropriate for this purpose.

## 2. U.S. Agency Mortgage-Backed Securities

Agency mortgage-backed securities are issued by Fannie Mae (FNMA), Freddie Mac (FHLMC), or Ginnie Mae (GNMA).<sup>6</sup> Fannie Mae and Freddie Mac are government-sponsored enterprises (GSEs), whereas Ginnie Mae is a wholly owned government corporation. The U.S. agency mortgage-backed securities market is among the largest and most liquid bond markets worldwide. Furthermore, more than 70% of the \$9.8 trillion U.S. home mortgage market serves as collateral for agency mortgage-backed securities. Immediately prior to the financial crisis of 2007–2008, private financial institutions accounted for more than 50% of U.S. mortgage-backed security issuance. Since the crisis, however, “private label” issuance has declined dramatically and now represents less than 4% of total mortgage-related issuance. In contrast, agency mortgage-backed security issuance has grown rapidly; the total notional size of the agency mortgage-backed security market increased 58% from 2006 to 2015.<sup>7</sup> In this section, we review the key features of agency mortgage-backed securities.

### 2.1 Credit quality

In exchange for monthly fees, the agencies guarantee the timely payment of mortgage interest and principal. The guarantee protects investors from defaults on the underlying mortgages since delinquent mortgages must be purchased out of the trust at par by the issuer. This means that a default appears as a prepayment

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<sup>6</sup>Fannie Mae, Freddie Mac, and Ginnie Mae refer to the Federal National Mortgage Association, the Federal Home Loan Mortgage Corporation, and the Government National Mortgage Association, respectively.

<sup>7</sup>See Federal Reserve Board Flow of Funds Table L.217 and [www.sifma.org/research/statistics.aspx](http://www.sifma.org/research/statistics.aspx).

from an investor’s perspective. Because GNMA securities carry the full faith and credit guarantee of the United States, their credit quality should be the same as that of U.S. Treasuries. FNMA and FHLMC securities carry a credit guarantee from the issuing GSE rather than the United States. Historically, the GSE guarantee was viewed as an “implicit” government guarantee because investors believed that the government would back the agencies in times of stress. This view was validated in September 2008 when the government placed FNMA and FHLMC in conservatorship and provided them with unlimited access to collateralized funding. Both FNMA and FHLMC are supervised and regulated by the Federal Housing Finance Agency.<sup>8</sup>

## **2.2 Mortgage-backed security cash flows**

In this paper, we focus on agency mortgage-backed securities backed by pools of fixed-rate mortgages. A fixed-rate mortgage is structured so that the borrower is obligated to make the same payment each month, consisting of interest and principal. In general, fixed-rate mortgages can be prepaid at any time without penalty. Each month, therefore, a pool of mortgages generates cash flows consisting of scheduled interest, scheduled principal, and possibly prepaid principal. A pass-through mortgage-backed security distributes to investors the principal and interest payments from the underlying mortgage loans, less guaranty and servicing fees. Because the guaranty and servicing fees are based on the outstanding balance, these fees decline over the life of the mortgage.<sup>9</sup>

Mortgage servicers collect and aggregate payments from the underlying mortgage loans and pass the payments to the mortgage-backed security trust.

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<sup>8</sup>We note, however, that the current FNMA single family prospectus explicitly states that its certificates are not guaranteed by the United States and do not constitute a debt or obligation of the United States. Furthermore, the prospectus raises the possibility that if FNMA were to emerge from, and then later reenter, conservatorship, there is no assurance that the subsequent receiver or conservator would not repudiate the current guaranty.

<sup>9</sup>See [www.fanniemae.com](http://www.fanniemae.com), [www.freddiemac.com](http://www.freddiemac.com), and [www.ginniemae.gov](http://www.ginniemae.gov) for more information about agency securitization programs.

Mortgage payments are due on the first of the month (with a grace period determined by state law). Investors, however, receive the payments after a delay of 14, 19, or 24 days, depending on the mortgage-backed security program. If a loan becomes delinquent, servicers advance scheduled principal and interest until either the loan becomes current or is bought out of the trust at par. Servicers retain a monthly fee based on a percentage of the outstanding mortgage balance at the beginning of the month. This fee is often referred to as a “servicing strip” because the cash flows resemble an interest-only strip. In the FNMA, FHLMC, and GNMA II programs, mortgages with different gross coupons can be pooled together as long as the net coupon (gross coupon minus servicing and guaranty fees) is identical among all the loans in the mortgage pool. In the GNMA I program, the gross coupon is always 50 bps higher than the net coupon.

### **2.3 Agency mortgage-backed security trading**

Agency mortgage-backed securities trade on either a to-be-announced (TBA) basis or a specified-pool basis. The TBA market is a highly liquid forward market and accounts for 90% of all mortgage-backed security trading. From 2007 to 2014, the daily trading volume of U.S. agency mortgage-backed securities averaged \$276 billion, which compares well with the \$525 billion daily trading volume for U.S. Treasuries. Typically, pass-throughs are traded as specified pools if they command a premium over TBAs or if they are ineligible for TBA delivery.<sup>10</sup>

Similar to Treasury futures, a buyer of a TBA agrees to the trade without knowing the exact pools that will be delivered. Instead, the buyer and seller agree to six parameters: price, par amount, settlement date, agency program, mortgage type, and coupon. TBA trades generally settle to a monthly schedule set by the Securities Industry and Financial Markets Association (SIFMA). Nearly all TBA trades occur with settlement dates less than or equal to three months forward.

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<sup>10</sup>Trading volume data comes from FINRA TRACE <https://www.finra.org/industry/trace/structure-product-activity-reports-and-tables>. See Vickrey and Wright (2013) for a discussion of the TBA market. Also see Carlin, Longstaff, and Matoba (2014). See Gao, Schultz, and Song (2017) for a discussion of the specified pool market.

Two days prior to the settlement date of the trade, the seller notifies the buyer of the exact pools that will be delivered (the 48-hour rule). The pools are then exchanged for the cash payment on the settlement date.

Market participants generally adhere to standards referred to as the “Good Delivery Guidelines” maintained by SIFMA. These guidelines specify the eligible collateral for a TBA trade and various operational guidelines such as the number of bonds per million dollars notional of a trade, the allowable variation in the delivery amount, and the costs of failing to deliver. TBA trades may also be executed with stipulations such as production year, weighted average maturity (WAM), weighted average loan age (WALA), FICO score, loan-to-value ratio, or geographic distribution. A stipulated TBA trade, however, would likely occur at a price higher than an unstipulated TBA (if the stipulations provide favorable prepayment characteristics).

## **2.4 Quantitative easing programs**

Table 1 provides a listing of the major events in the agency mortgage-backed securities market during the study period. Among the most significant of these events are the Federal Reserve’s quantitative easing programs, commonly known as QE I, QE II, and QE III. The first program, QE I, was announced on November 25, 2008 and directed the purchase of up to \$500 billion of agency mortgage-backed securities and \$100 billion of GSE debt. The stated goal of QE I was to reduce the cost and increase the availability of credit for the purchase of houses. QE I was expanded on March 18, 2009 to allow additional purchases of up to \$750 billion of agency mortgage-backed securities and \$100 billion of agency debt. The QE II program was announced on November 3, 2010 and authorized the purchase of up to \$600 billion of longer-term Treasury securities. The QE III program was announced on September 13, 2012 and directed the purchase of up to \$40 billion per month of agency mortgage-backed securities and \$45 billion per month of Treasury securities. These programs had large effects on the supply of mortgage-backed securities in the market.<sup>11</sup>

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<sup>11</sup>For a detailed discussion of the effects of the quantitative easing programs,

### 3. Data

The primary data for the study consist of monthly prices (observed at the end of each month) from the TBA market for FNMA mortgage-backed securities with varying coupons. The sample period is January 1998 to September 2014. The data are obtained from a proprietary data set compiled by a major Wall Street mortgage-backed security dealer. However, we have cross validated the proprietary data with prices publicly available in the Bloomberg system and found the two sources to be very similar. To insure that we include only prices for actively traded mortgage-backed securities, we limit the data set to mortgage-backed securities with coupon rates that are within 300 bps of the current coupon mortgage rate.<sup>12</sup> Furthermore, we only include prices for pools that trade as general collateral in the TBA market—we do not include prices for any mortgage-backed security that trades with a pay-up in the specified pools market.<sup>13</sup> The data set also includes 1-month and 3-month horizon conditional prepayment rate (CPR) information for each coupon.

Table 2 presents summary statistics for the data. As shown, the sample

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see Gagnon, Raskin, Remache, and Sack (2011), Krishnamurthy and Vissing-Jorgensen (2011, 2013), Christensen and Rudebusch (2012), Thornton (2014), and Christensen and Gillan (2016).

<sup>12</sup>Ideally, we would like to have a larger cross-section of coupon rates from which to estimate the model. We note, however, that the results are very similar when we use a more restrictive filter on the coupon rates included in the sample. See the discussion in Section 10.3.

<sup>13</sup>As discussed by Song and Zhu (2016), participants in the TBA market have incentives to deliver the cheapest collateral at settlement. This has little effect on our results, however, since we focus exclusively on the broad cohort of securities that are currently cheapest to deliver and do not carry a pay-up premium. Furthermore, a buyer in the TBA market can always stipulate delivery of the currently cheapest-to-deliver securities without having to pay a premium. A review of the quote sheets provided by a number of major dealers suggests that individual mortgage-backed securities can begin trading with a pay-up as small as 0.50 to 2.00 32nds. This places an upper bound on how much variation there can be in the values of the securities in the cohort of securities deliverable in the TBA market.

includes mortgage-backed securities with coupons ranging from 2.50% to 9.50%. Of course, not all coupons are actively traded throughout the entire sample period. The higher coupon mortgage-backed securities appear during the early part of the sample period when mortgage rates were considerably higher, and vice versa for the lower coupon mortgage-backed securities.

We also collect data for a wide variety of macroeconomic, mortgage market, and financial variables that will be used in the analysis throughout the paper. The appendix provides a description of each of these variables and the sources of the data. Finally, we collect historical data on Treasury constant maturity rates from the Federal Reserve H.15 release. We use a standard cubic spline approach to bootstrap the prices of zero-coupon bonds  $D(t)$  for maturities ranging up to 30 years for each month during the sample period (the methodology is described in the appendix).

#### 4. Valuation Framework

In valuing mortgage-backed securities, we use a reduced-form framework in which an instantaneous prepayment process  $p_t$  plays the central role. Specifically,  $p_t$  is the fraction of the remaining notional balance of the underlying mortgage pool that is prepaid each instant. Thus,  $p_t$  can be viewed as a prepayment intensity or hazard rate. Our approach will be to solve for the implied value of  $p_t$  and its dynamics from the cross-section and time series of prices of mortgage-backed securities with different mortgage rates.

For expositional clarity, we assume for the present that mortgage cash flows are paid continuously and that the fixed mortgage rate  $m$  on the mortgages in the underlying pool is the same as the coupon rate on the mortgage-backed security. Let  $c$  denote the payment on a mortgage with an initial principal balance of one. Since the present value of the mortgage equals one at inception,

$$1 = c \int_0^T e^{-mt} dt, \quad (1)$$

$$= (c/m)(1 - e^{-mT}), \quad (2)$$

and the mortgage payment  $c$  is,

$$c = \frac{m}{1 - e^{-mT}}. \quad (3)$$

The mortgage payment  $c$  includes both interest and scheduled principal. Let  $I_t$  denote the principal balance of the mortgage at time  $t$ . The change in the principal balance is just the difference between the interest on the mortgage balance and the mortgage payment,

$$dI_t = (mI_t - c) dt. \quad (4)$$

Solving this first-order differential equation subject to the initial condition implies

$$I_t = \frac{1 - e^{-m(T-t)}}{1 - e^{-mT}}. \quad (5)$$

Now, consider a mortgage-backed security where the individual mortgages in the underlying pool are all  $T$ -year fixed-rate mortgages. Without loss of generality, we normalize the initial notional balance of the pool to be one. We denote the remaining notional balance of the underlying pool at time  $t$  as  $N_t$ , which, given the definition of  $p_t$ , can be expressed as

$$N_t = \exp\left(-\int_0^t p_s ds\right). \quad (6)$$

In turn, the remaining principal balance of the underlying pool is given by  $N_t I_t$ . It is important to distinguish between the remaining notional amount and the

principal balance since mortgage payments are based on the original notional amount of the mortgages while prepayment cash flows are based on the remaining principal balance. The product  $N_t I_t$  reflects both the effect of prepayments and, through Equation (4), the effect of scheduled principal payments on  $I_t$ .

Finally, let  $F(m, T)$  denote the value of a mortgage-backed security where the underlying mortgages have a mortgage rate of  $m$  and maturity of  $T$ . The value of the mortgage-backed security at time zero is formally given by

$$F(m, T) = E^Q \left[ \int_0^T \exp \left( - \int_0^t r_s + w_s ds \right) N_t (c + p_t I_t) dt \right], \quad (7)$$

where  $E^Q[\cdot]$  denotes expectation under the risk neutral probability measure and  $r_t$  is the riskless interest rate. Following Duffie and Singleton (1997, 1999), Longstaff, Mithal, and Neis (2005), and many others,  $w_t$  plays the role of a credit/liquidity spread. The rationale for including  $w_t$  in the model is to allow for the possibility that cash flows from agency mortgages may be discounted a higher rate than Treasury cash flows, either because the credit of the agency may not be as strong, or because agency mortgages may be less liquid than Treasuries.

## 5. Prepayment Function

To complete the valuation framework for mortgage-backed securities, we need to specify the prepayment process  $p_t$ . Before doing this, however, it is useful to first consider some of the stylized facts about actual prepayment rates.

To illustrate the relation between prepayments and refinancing incentives, Figure 2 plots the prepayment rates for FNMA mortgage-backed securities as a function of the refinancing incentives for these securities. As shown, there is a strong relation between the prepayment rate and the refinancing incentive. When the coupon rate on the mortgage is lower than the current market rate,

the borrower has no incentive to refinance. When the coupon rate is higher than the current market rate, the borrower may be able to reduce his mortgage costs by refinancing. Interestingly, the relation between prepayment rates and the refinancing incentive has the appearance of a piecewise linear function similar to that of a call option payoff.

In particular, when the prepayment option is out of the money, the relation is flat, although generally not zero. In fact, the prepayment rates for these mortgage-backed securities can be as high as 10% to 20%, because borrowers often prepay mortgages for reasons other than to reduce mortgage costs. For example, borrowers often prepay mortgages even when the market rate is higher than their mortgage rate for exogenous reasons such as a retirement or a career-related move. Also, borrowers may refinance into a higher mortgage rate to extract home equity after an increase in housing prices. During the recent financial crisis, a major source of exogenous prepayments has been the high rate of foreclosures throughout the United States. A foreclosure results in the pass through of the entire remaining mortgage balance to the holders of an agency-guaranteed mortgage-backed security. Thus, foreclosures trigger prepayments for agency mortgage-backed securities.

When the prepayment option is in the money, the relation is generally increasing, but spreads out as the price increases. A closer inspection of the data, however, indicates that the relation is actually close to linear at a point in time, but that the slope of the relation varies over time. Thus, the unconditional relation appears spread out. To illustrate this, Figure 3 plots the prepayment rate and refinancing incentive relation for selected dates during the sample period. As shown, the prepayment functions display varying slopes over time.

Motivated by these stylized facts, we use a simple generic specification of the implied prepayment function that allows for both exogenous and rate-related prepayments. Specifically, we model the prepayment function as

$$p_t = x_t + y_t \max(0, m - a - b r_t(10)), \quad (8)$$

where  $r_t(10)$  is the 10-year Treasury rate. In this specification,  $x_t$  denotes the exogenous hazard rate at which mortgages are prepaid in the absence of refinancing incentives. Intuitively,  $x_t$  captures all the non-interest-rate-related background factors that lead to prepayments. For example, when a borrower defaults and the mortgage is foreclosed, investors receive repayment of principal since agency mortgage-backed securities are guaranteed against default. Similarly, when a mortgage loan is put-back to its originators, investors receive their principal.<sup>14</sup> Thus, we will refer to  $x_t$  simply as the turnover rate.  $x_t$  is a hazard rate rather than a prepayment rate. The value of  $x_t$ , however, can be easily mapped into an annualized prepayment rate using the expression  $1 - e^{-x_t}$ , and two values are generally close to each other. Thus, with little loss of generality, we can simply think of the units of  $x_t$  as being expressed in terms of a prepayment rate. The exact relationship is shown in the appendix.

The refinancing incentive is determined by the difference between the mortgage rate  $m$  and the implied rate at which mortgages can be refinanced. We allow this implied rate to be a general affine function  $a + b r_t(10)$  of the 10-year Treasury rate  $r_t(10)$ , rather than constraining it to be a specific short-term or long-term rate. We use the 10-year Treasury rate since it is strongly correlated with mortgage rates—the correlation between the 10-year Treasury rate and the FNMA primary mortgage rate during the sample period is 0.9825. This suggests that representing the market mortgage rate as a linear function of the 10-year rate provides a realistic approximation. The values of  $a$  and  $b$  will be estimated from the data.<sup>15</sup>

The term  $y_t$  that multiplies the refinancing incentive term  $\max(0, m - a - b r_t(10))$  in Equation (8) measures how sensitive borrowers are to refinancing incentives. For example, borrowers whose home values were less than

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<sup>14</sup>We are grateful to the referee for this observation.

<sup>15</sup> $m$  will typically be higher than  $r_t(10)$  when a loan is originated,  $m$  remains fixed, whereas  $r_t(10)$  varies over time. Because of this, the moneyness of the mortgage can become negative, and, therefore, the maximum operator is always relevant.

their mortgage balances would typically have a very low propensity to refinance, or equivalently, a low value of  $y_t$ . After the introduction of the Home Affordable Refinancing Program (HARP) in 2009, however, this set of borrowers might have been much more likely to refinance given the same level of refinancing incentive. Thus, changes in home values might be one source of the time variation in the rate response factor. Similarly, the propensity to refinance could also vary with the required loan-to-value underwriting standards in the mortgage market. Given the role that  $y_t$  plays in the prepayment function, we denote it as the rate response factor. Since  $y_t$  is a multiplier for the refinancing incentive term, it is not expressed in any specific units. However, the product of  $y_t$  and the refinancing incentive represents a hazard rate, which, in turn, can be mapped into a prepayment rate as in the discussion above about the turnover rate.

Finally, it is important to acknowledge that our specification of the implied prepayment function in Equation (8) is among the simplest possible. In particular, our simple specification does not explicitly include many of the features that researchers and practitioners incorporate into formal econometric prepayment models such as seasoning, burnout, nonlinear dependence of refinancing activity on the refinancing incentive, housing values, macroeconomic conditions, etc. There are three reasons we have intentionally chosen one of the simplest possible specifications of the implied repayment function rather than mimicking state-of-the-art econometric prepayment models.

First, the implied prepayment function represents prepayments under the risk-neutral probability measure—not under the actual or econometric probability measure. If mortgage-backed security prices incorporate prepayment risk premiums, then the implied prepayment function could be very different from the actual or econometric prepayment function. For this reason, our approach will be to begin with the most basic risk-neutral specification, and then evaluate whether more complex features such as those used in state-of-the-art econometric prepayment models are necessary in modeling mortgage-backed security prices accurately.

Second, the reduced-form nature of the implied prepayment function allows for the possibility that the state variables  $x_t$  and  $y_t$  may play a similar role in modeling risk-neutral prepayments that features such as seasoning, burnout, etc. play in econometric modeling. In particular, time variation in the turnover factor may reflect changes in macroeconomic conditions. Similarly, the rate response factor can be viewed as a generalized form of burnout. For example, a decrease in the implied value of  $y_t$  may reflect a decline in the general willingness or ability of borrowers to refinance mortgages into lower rates in a way that parallels the usual security-specific notion of burnout. Note, however, that since  $y_t$  is a marketwide factor impacting all mortgage-backed securities, it clearly cannot capture seasoning and burnout in the usual cross-sectional sense.

Third, by choosing such a simple specification for the implied prepayment function, we are biasing the results against the model. If it turns out, however, that even with this simple implied prepayment specification, the model is able to capture the cross-section of mortgage-backed securities accurately, then this would provide strong support for the usefulness and viability of these types of implied prepayment models.

## 6. Estimation Methodology

In this framework, the value of a mortgage-backed security is a function of the three state variables:  $w_t$ ,  $x_t$ , and  $y_t$  (in addition to the interest rate). To complete the specification of the model, we assume that the dynamics of the state variables are given by the following system of stochastic differential equations under the risk-neutral pricing probability measure,

$$dw = (\alpha_w - \beta_w w) dt + \sigma_w dZ_w, \quad (9)$$

$$dx = (\alpha_x - \beta_x x) dt + \sigma_x \sqrt{x} dZ_x, \quad (10)$$

$$dy = (\alpha_y - \beta_y y) dt + \sigma_y \sqrt{y} dZ_y. \quad (11)$$

The credit/liquidity spread  $w_t$  follows a mean-reverting process that can take on both positive and negative values. The spread parallels the specification used by Duffie and Singleton (1997, 1999), Longstaff, Mithal, and Neis (2005), and many others. The state variables  $x_t$  and  $y_t$  driving prepayments both follow mean-reverting square-root processes, ensuring that prepayment rates are always nonnegative. This specification of dynamics places this model within the familiar affine framework widely used throughout the financial literature.

To model the evolution of the riskless rate, we assume that  $r_t$  follows the single-factor Hull and White (1990) model

$$dr = (\alpha_r(t) - \beta_r r) dt + \sigma_r dZ_r, \quad (12)$$

where  $\alpha_r(t)$  is a deterministic function of time, and  $\beta_r$  and  $\sigma_r$  are positive constants. The function  $\alpha_r(t)$  allows for an exact fit to the Treasury term structure on a given date. The 10-year rate  $r_t(10)$  that determines the refinancing incentive is an affine function of the short rate  $r_t$ . The interest rate model could easily be relaxed to allow for a more general multifactor specification.<sup>16</sup>

We allow for correlation between the state variables. Specifically, we assume that  $dZ_r$  is correlated with  $dZ_x$  and  $dZ_y$ , and that  $dZ_x$  and  $dZ_y$  are correlated with each other. We denote the correlation of  $dZ_r$  with  $dZ_x$  as  $\rho_{r,x}dt$ , the correlation of  $dZ_r$  with  $dZ_y$  as  $\rho_{r,y}dt$ , and the correlation of  $dZ_x$  with  $dZ_y$  as  $\rho_{x,y}dt$ .<sup>17</sup>

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<sup>16</sup>Clearly, the single-factor Hull White (1990) model has limitations relative to a more general multifactor model. For example, a multifactor model would likely perform better in terms of hedging the interest rate risk of mortgage-backed securities (see Gupta and Subrahmanyam 2005; Driessen, Klaassen, and Melenberg 2003). Some practitioners use multifactor models in their MBS valuation frameworks. We are grateful to the referees for these observations.

<sup>17</sup>As is common in the literature, we assume that  $dZ_w$  is uncorrelated with the other state variables. This standard assumption likely has little effect on the results. For example, see Duffie and Singleton (1997), Longstaff, Mithal, and Neis (2005), Pan and Singleton (2008), Longstaff et al. (2011), and Ang and Longstaff (2013).

As discussed in the appendix, the parameters for the riskless rate are estimated separately from the mortgage model. For each date, we solve for  $\beta_r$  and  $\sigma_r$  parameters to minimize the relative pricing error over the swaption volatility surface. Specifically, we fit these parameters to the 15 European swaptions with expirations of 1, 2, 3, 4, and 5 years, and tenors of 5, 7, and 10 years. Given the estimates of  $\beta_r$  and  $\sigma_r$ , the function  $\alpha_r(t)$  is determined by fitting the model to the Treasury yield curve.

The estimation of the mortgage model can be viewed as consisting of three steps. First, we select an initial parameter vector  $\theta$ , where  $\theta = \{a, b, \alpha_w, \alpha_x, \alpha_y, \beta_w, \beta_x, \beta_y, \sigma_w, \sigma_x, \sigma_y, \rho_{r,x}, \rho_{r,y}, \rho_{x,y}\}$ . Second, conditional on  $\theta$  and for each month  $t$  during the sample period, we solve for the values of  $w_t$ ,  $x_t$ , and  $y_t$  that best fit the model to the prices of the cross-section of mortgage-backed securities with different coupon rates (the coupon stack) by minimizing the RMSE.  $w_t$ ,  $x_t$ , and  $y_t$  are separately identifiable because each has different effect on mortgage-backed security prices. Specifically, the effect of an increase in  $w_t$  is to increase the discount rate on all mortgage-backed security cash flows, which has the effect of lowering the prices of all mortgage-backed securities.<sup>18</sup> In contrast, an increase in the turnover rate  $x_t$  has the effect of increasing the prepayment rate for all mortgage-backed securities. In turn, an increase in the prepayment rate increases the values of discount mortgage-backed securities while decreasing the values of premium mortgage-backed securities. Furthermore, an increase in  $y_t$  has the effect of increasing the prepayment rate for premium mortgage-backed securities, but has no impact on the prepayment rate or prices of discount mortgage-backed securities. Since the nonlinear structure of the prepayment function makes it difficult to express the price of mortgage-backed securities in closed-form, we use simulation to solve for the model-based mortgage-backed security values. Third, we iterate over alternative values of the parameter vector  $\theta$  until we find the vector that results in the lowest global RMSE. Table 3 reports the parameter

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<sup>18</sup>Since an increase in  $w_t$  affects all mortgage-backed securities, its effect differs from that of an option-adjusted spread, which is security specific and constant through time.

values obtained from the estimation along with their asymptotic standard errors. The outputs are the parameter values and the time series of state variables. The details of the mortgage model estimation process are described in the appendix.<sup>19</sup>

## 7. Implied Prepayment Factors

In this section, we discuss the empirical results and their implications. First, we examine how well the model is able to fit the market prices of mortgage-backed securities. We then study the properties of the three state variables of the model: the credit/liquidity spread  $w_t$ , the turnover rate  $x_t$ , and the rate response factor  $y_t$ .

### 7.1 Fitting mortgage-backed security prices

The coupon stack for each month in the sample period typically includes between 6 to 10 mortgage-backed securities with varying coupon rates at 50 bp increments. The estimation algorithm solves for the values of the three state variables  $w_t$ ,  $x_t$ , and  $y_t$  that best fit the model to the coupon stack. Since there are more prices than state variables, it is clear that there will be residual differences between model values and market values. To quantify the magnitude of these differences, we compute the RMSE for each month in the sample period.

Figure 4 plots the time series of the RMSEs. As shown, the model fits the mortgage-backed security prices extremely well. For much of the sample period, the RMSEs range from about 5 to 30 cents for mortgage-backed security prices quoted in terms of a \$100 notional position. This range compares well with the

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<sup>19</sup>The appendix also reports the results from a number of robustness checks including the inclusion of burnout and seasoning features in the model, the use of the swap curve as the discounting curve, the restriction of the set of TBAs used in the estimation to the five with coupons closest to the current coupon rate, and an analysis of the relation between fitting errors and TBA characteristics such as WALA.

bid-ask spreads of actively traded mortgage-backed securities, which discussions with traders indicate are typically on the order of three to four ticks, or 32nds of a point. Once the financial crisis begins in 2008, however, the RMSEs tend to become larger in value. Intuitively, this may simply be the result of the massive shocks that the housing and mortgage markets experienced during the financial crisis, as well as a lack of liquidity and risk capital in the markets to arbitrage mispricing among mortgage-backed securities. The median RMSE for the pre-crisis period is 21.5 cents.<sup>20</sup> The median RMSE for the entire sample period is 25.7 cents.<sup>21</sup>

## 7.2 Mortgage-backed security pricing factors

The estimation algorithm solves for the implied values of the three factors driving mortgage-backed securities prices for each month during the sample period: the credit/liquidity spread, the turnover rate, and the rate response factor. Table 4 provides summary statistics for the implied values of these factors. These pricing factors are discussed individually below.

## 7.3 Credit/liquidity spread

Table 4 shows that the mean value of the credit/liquidity spread is about 65.5 bps with a standard deviation of 43.1 bps. This mean value is in relatively close agreement with the average spread on FNMA debt issues during the sample period. For example, the average spread of FNMA 10-year debt over Treasuries during the January 2000 to September 2014 period is 49.8 bps. We will study

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<sup>20</sup>To provide additional perspective, we also compute the RMSE under the static assumption that the future prepayment rate for each mortgage-backed security equals its current 3-month prepayment rate. In this estimation, however, we again solve for the implied credit/liquidity spread. The resultant RMSE is 242 cents. We are grateful to the referee for suggesting this comparison.

<sup>21</sup>The median RMSE for discount and premium mortgage-backed securities is 18.0 and 26.8 cents, respectively.

the link between the implied spread and FNMA credit spreads in more depth shortly.

Figure 5 plots the time series of the implied credit/liquidity spread values over the sample period, along with the spread for FNMA agency debt. As shown, the majority of the implied spreads are positive. In particular, 184, or 91.5% of the 201 estimates are positive. That some of the implied spreads are negative, however, hints that the implied spreads may be reflecting more than the credit risk of FNMA bonds, particularly since FNMA credit spreads are uniformly positive throughout the 2000–2014 period.

This latter observation is reinforced by comparing the spread values shown in Figure 5 with the key events in the timeline given in Table 1. For example, the large decline in the spread beginning in April 2009 coincides with the large expansion of the QE I program to purchase an additional \$750 billion of mortgage-backed securities. The large downward spike around September 2012 coincides with the announcement of the QE III program to purchase \$40 billion of agency mortgage-backed securities per month. Thus, these observations hint that the massive purchases of mortgage-backed securities during QE I and QE III may have had an effect via new production and existing collateral being removed from the market. The potential effect is two-fold: a direct decrease in supply would increase prices and decrease spreads, an indirect effect on liquidity would increase spreads. It appears that the first effect dominates the second.

On the other hand, Figure 5 also shows that the implied spreads appear to be related to key events that may impact the credit risk of FNMA. For example, the spread attains its largest values during the Lehman crisis period of Fall 2008. However, after FNMA and FHLMC are placed into conservatorship and their credit risk is essentially defeased, the implied spread quickly returns to pre-crisis levels, and subsequently actually reaches historical lows.

To examine the properties of the implied spread in more detail, we regress monthly changes in the implied spread on a number of explanatory variables reflecting changes in the credit risk and liquidity of the mortgage-backed securities

market. First, we include monthly changes in the yield spread between FNMA notes and Treasury notes with similar maturities. The intuition for including this spread is that if FNMA's cost of debt capital were to increase relative to that of the Treasury, then the value of the FNMA guarantee should decline, resulting in lower mortgage-backed security prices, or equivalently, higher implied spreads.

Second, we include three measures relating to the supply of mortgage-backed securities in the market. The first of these is the amount of mortgage-backed securities purchased by the Federal Reserve via its quantitative easing programs. The scale of these purchases represented a large fraction of the total available supply of mortgage-backed securities in the market and, therefore, could potentially have a sizable effect on the liquidity of these securities. The second is the total amount of settlement fails of mortgage-backed securities by primary dealers. Settlement fails occur when dealers face challenges in obtaining enough mortgage-backed security collateral to settle trades, and are a reflection of tight supply in the market.<sup>22</sup> The third is the net issuance of mortgage-backed securities. This measure reflects the change in the supply of mortgage-backed securities available in the financial markets.

Third, we include the change in primary dealers' holdings of mortgage-backed securities as reported by the Federal Reserve Bank of New York. The intuition for this measure is that when primary dealers increase their inventories, we would expect that the liquidity of the mortgage-backed securities market would improve, leading to a decline in the implied spread. Finally, we include the first two lagged values of the change in the credit/liquidity spread to control for the time series properties of this variable. Details of the variables used in this regression are provided in the appendix.

Table 5 presents the regression results. As illustrated, the change in the FNMA credit spread is strongly related to the change in the credit/liquidity spread implied from the prices of mortgage-backed securities. The regression coefficient is positive and highly significant with a  $t$ -statistic of 3.44. Although

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<sup>22</sup>We are grateful to the referee for suggesting this explanatory variable.

this result is very intuitive, to our knowledge, this is the first direct evidence that the credit risk of the agency guaranteeing the timely payment of principal and interest is related to the pricing of mortgage-backed securities. The regression coefficient of roughly 0.74 indicates that while the implied spread is closely related to the spread on FNMA debt, the relation is not one-to-one and that there are other drivers of the implied spread.

In particular, Table 5 shows that the supply-related variables have significant effects on the credit/liquidity spread, consistent with a liquidity interpretation of this variable. For example, the coefficient for Federal Reserve purchases is negative with a  $t$ -statistic of  $-2.08$ . Intuitively, this suggests that as the large purchases by the Federal Reserve crowded out other market participants, the resultant scarcity of mortgage-backed securities led to an increase in their prices. This effect is also consistent with the significant negative coefficient for settlement fails, which suggests that mortgage-backed securities increase in value when the supply of mortgage-backed security collateral is tight in the market.

Given the strong empirical relation between the credit/liquidity rate and the FNMA credit spread, a simple estimate of the size of the liquidity component in mortgage-backed securities can be obtained by subtracting the FNMA credit spread from the credit/liquidity spread. This difference or liquidity spread is also plotted in Figure 5. As shown, during the pre-crisis period, the liquidity spread is positive with an average value of around 23.2 bps. After the crisis of 2008, the liquidity spread declines to near zero with downward spikes coinciding with the initiation and extension of the QE I program. The initiation of the QE III program with its massive purchases of agency mortgage-backed securities coincides with the large negative spike in the liquidity spread. Discussions with industry sources suggest that as the Federal Reserve's purchases of agency mortgage-backed securities began to crowd other players out of the market, the difficulty of finding tradeable collateral made existing supplies of mortgage-backed securities trade at a premium. The liquidity estimates shown in Figure 5 are consistent with this view and with the regression results in Table 5. Furthermore, our results provide support for previous research that finds

links between QE activity and mortgage-backed security prices including Krishnamurthy and Vissing-Jorgensen (2011, 2013), the Treasury Market Practices Group (2012), Kandrak (2013), Song and Zhu (2016), and Boyarchenko, Fuster, and Lucca (2016).

#### 7.4 Turnover rate

Table 4 reports summary statistics for the implied turnover rate. The implied turnover rate is based on the risk-neutral probability measure since its value is inferred from the prices of mortgage-backed securities. Because prepayment rates are directly observable, however, the turnover rate under the actual or empirical probability measure can be directly estimated from the data. The details of the estimation procedure are given in the appendix.<sup>23</sup> As part of our analysis, we will contrast the properties of the empirical and implied turnover rates and examine their implications for risk premiums.

Figure 6 plots the time series of the implied turnover rate and the empirical turnover rate. As illustrated, virtually all of the implied turnover rates are higher than the realized turnover rates. Some of the largest values of the implied turnover rate occur during 2003 and 2005. Similarly, some of the largest spikes in realized turnover occur in 2003 and in 2004. Industry sources suggest that a sizable fraction of this turnover may have been motivated by borrowers attempting to “cash out” some of the equity in their homes resulting from the rapid increase in housing values. Thus, the increase in turnover rates during this period could partially reflect a shift towards consumption-related incentives for refinancing. Similarly, the spike in the implied turnover rate during the early stages of the financial crisis may reflect expectations of higher mortgage defaults

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<sup>23</sup>We estimate the empirical turnover rate and rate response factor each month during the sample period from realized 1-month CPRs using a nonlinear regression framework. In this approach, we use the CPRs for the exact same set of mortgage-backed securities that we use in estimating the implied turnover and rate response factors.

and foreclosures.<sup>24</sup>

To explore this further, we regress quarterly changes in both the empirical and implied turnover rates on variables that reflect the state of the macroeconomy, risk premiums in the fixed income and other markets, and the level of distress in the mortgage markets. As macroeconomic measures, we include the lagged growth rate in U.S. personal consumption expenditures, the lagged change in the Conference Board Consumer Confidence Index, and the lagged change in the unemployment rate. As measures of risk premiums, we include the change in the BBB corporate credit spread over Treasuries, the change in the Treasury two-year to 10-year term structure slope, and the change in the VIX index. Finally, to capture the level of distress in the mortgage markets, we include the lagged change in the mortgage foreclosure rate, and the doubly lagged change in the mortgage delinquency rate (both from the Mortgage Bankers Association National Delinquency Survey). We include these distress variables in these lagged forms since it is likely they would affect turnover with a delay. We also include the lagged changes in both the empirical and implied turnover rates as controls in the regression. The appendix provide details for each of the variables used in the regression.

Table 6 reports the results from the regressions. Focusing first on the regression for changes in the actual turnover rate, the results show that turnover is significantly positively related to consumption growth. A 1% increase in consumption maps into an increase in the turnover rate of 0.89%. This is consistent with anecdotal evidence that turnover increased during the mid-2000s as homeowners with increased equity in their homes used cash-out refinancings to fund high consumption.<sup>25</sup> On the other hand, the turnover rate is significantly posi-

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<sup>24</sup>Although a number of the empirical turnover rates take values close to zero, only two of the empirical turnover rates are actually zero. These two zero values occur in months in which the CPRs for discount mortgage-backed securities are reported as identically zero.

<sup>25</sup>The time series of consumption is measured with noise. Thus, our results should be viewed as suggestive rather than definitive. To examine the robustness of the results, we reestimated the regression in Table 6 separately for the first

tively related to changes in unemployment. In particular, a 1% increase in the unemployment rate maps into an increase in the turnover rate of 4.02%. This is consistent with the interpretation that involuntary turnover increases during economic downturns as borrowers face adverse shocks and distress-related prepayments increase (via foreclosures, employment-related moves, etc.). Lagged foreclosures are significantly negatively related to actual turnover. The reason for the negative sign of the relation is that foreclosures may actually have the effect of resolving uncertainty about future turnover. Thus, holding fixed delinquency rates, higher foreclosures during the current period reduce the overhang of distressed mortgages, resulting in lower future turnover rates. Finally, Table 6 shows that actual turnover is significantly related to the change in the BBB corporate credit spread, although the sign of the relation is negative.

Turning our attention now to the regression for changes in implied turnover, we see that implied turnover behaves very differently from actual turnover. In particular, the risk premium measures appear to be key drivers of the implied turnover rate. To see this, note that the most significant variable in the regression is the change in the BBB corporate credit spread. The positive sign for this coefficient indicates that increases in the credit spread are associated with higher implied turnover values. In addition, the coefficient for changes in the VIX index is also positive and significant, indicating that implied turnover tends to increase with market volatility. In contrast, neither of the two mortgage market distress variables are significant. Of the macroeconomic measures, only consumption growth is significant. Finally, finding that the coefficients for consumption and the corporate credit spread have opposite signs than in the empirical turnover regression highlights that the behavior of implied prepayments can be very different from that of empirical prepayments.

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and second halves of the sample period. For the first half, the coefficient for consumption growth is 1.5293 with a  $t$ -statistic of 3.47. For the second half, the coefficient for consumption growth is 1.4690 with a  $t$ -statistic of 1.76. Thus, the coefficient estimates appear similar across the two halves of the sample period. We are grateful to the referee for raising this issue.

In summary, the relation between actual turnover rates and macroeconomic factors such as consumption, unemployment, and foreclosures in the mortgage markets suggests that turnover risk may be very systematic in nature. If so, it would not be surprising if turnover risk were to carry a large risk premium. This possibility is strengthened by finding that changes in the implied turnover rate are more strongly correlated with financial market returns than with macroeconomic fundamentals. We will explore this issue in depth later in the paper.

## 7.5 Rate response factor

Table 4 also reports summary statistics for the implied rate response factor. As before, the empirical rate response factor is also estimated directly from observed prepayment data.

Figure 7 plots the time series of the implied and empirical rate response factors. As shown, the implied and empirical rate response factors display considerable time series variation and generally track each other closely. Some of the higher values of the empirical rate response factor occur during the 2001–2005 period when refinancings reached historically high levels. The implied rate response factor attains its highest values during the financial crisis. More recent increases in the rate response factor coincide with the rapid expansion of the Home Affordability Refinancing Program (HARP) in which investors with home values below their mortgage balance were allowed to refinance their homes.

As in the previous section, we regress quarterly changes in the empirical and implied rate response factors on a number of explanatory variables. In particular, we include the same set of macroeconomic variables and risk premium measures used in the previous regression discussed above. In addition, we include two measures that reflect the level of frictions that borrowers may face in obtaining mortgage credit in the market. The first measure is the average loan-to-value ratio for newly originated FNMA mortgages. Changes in this ratio reflect variation in loan underwriting standards. For example, a decrease in the loan-to-value ratio may indicate that lenders require higher downpayments in order for borrowers

to obtain mortgage credit. The second measure is the housing credit availability index reported by the Housing Finance Policy Center. The appendix provides details for each of the variables used in the regression.

Table 7 reports the results from the regressions. As before, we begin with the results for the empirical rate response factor. As shown, only the lagged change in the empirical rate response is significant in the regression. In particular, none of the macroeconomic, risk premiums, or financial frictions variables are significant. This result suggests that changes in the empirical rate response factor may be driven more by idiosyncratic influences and are less systematic in nature than is the case for changes in turnover.

Focusing next on the implied rate response factor, we again find that it behaves differently from the empirical rate response factor. For example, the lagged change in the loan-to-value ratio is positive and significant in the regression. Thus, the implied rate response factor increases as mortgage underwriting guidelines are relaxed. Furthermore, the risk premium measures are again the most significant variables in the regression. In particular, the change in the BBB corporate credit spread is positive and highly statistically significant with a  $t$ -statistic of 8.71. In addition, the coefficient for the VIX is significantly negative.<sup>26</sup> Again, these results suggest that implied rate response factor incorporates a significant risk premium component. This issue is explored in the next section below.

## 8. Prepayment Risk Premium

In this section, we examine whether the market prices of mortgage-backed securities incorporate a risk premium for prepayment risk. Since we model prepayment risk as an explicit function of the turnover rate and the rate response factor, our

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<sup>26</sup>We also estimate this regression including changes in the prepayment rate disagreement index of Carlin, Longstaff, and Matoba (2014). This variable was not significant.

framework also allows us to break down the total prepayment risk premium further into the components related to the turnover rate and the rate response factor. Mortgage-backed securities may also incorporate premiums for interest rate risk and agency credit risk. Rather than focusing on these well-known and extensively researched types of risk premiums, however, we exclusively focus on the prepayment risk premium. It is important to observe that our approach measures the marketwide risk premiums associated with the factors driving mortgage prepayments. This approach contrasts with that of recent papers such as Boyarchenko, Fuster, and Lucca (2016) and Diep, Eisfeldt, and Richardson (2016) that focus on the risk premiums incorporated into the expected returns of individual mortgage-backed securities. Thus, our paper provides a marketwide perspective on prepayment risk premiums that is complementary to the results of these other papers.

### **8.1 Is there a prepayment risk premium?**

To address the issue of whether there is a prepayment risk premium, we follow the standard approach of comparing values estimated under the risk-neutral probability measure with those estimated under the actual or empirical probability measure. Because the implied prepayment function is estimated directly from the market prices of mortgage-backed securities, it represents the prepayment function under the risk-neutral probability measure. In contrast, prepayments under the actual or empirical probability measure are directly observable. Our estimates of marketwide prepayment factor risk premiums are expressed in terms of hazard rates rather than in terms of the expected returns of individual securities.

It is important to observe that since the implied prepayment rate is based on the risk-neutral probability measure, the implied prepayment rate need not equal the empirical prepayment rate. This follows from Jarrow, Lando, and Yu (2005) who show that if hazard rates or intensities are sensitive to shocks that carry risk premiums (e.g., such as macroeconomic factors), then their values can differ between the risk-neutral and actual probability measures. This is analogous

to what occurs in reduced-form credit models in which the risk-neutral default probability or hazard rate need not equal the actual default probability. A key difference, however, is that the actual probability of default is extremely difficult to measure given how rare default events are. Thus, it is very challenging to estimate the difference between risk-neutral and actual default probabilities.<sup>27</sup> In contrast, empirical prepayment rates are directly observed and differences between the prepayment rate under the risk-neutral and empirical probability measures are easily identified.

The implied prepayment rate for each mortgage-backed security is given by simply substituting its weighted average coupon rate into the fitted prepayment hazard rate function and solving for the corresponding prepayment rate. Observe that in doing this, we are solving for the instantaneous implied prepayment rate which can be directly compared to the 1-month realized CPR for the mortgage-backed security.<sup>28</sup>

The upper panel of Figure 8 plots the time series of the monthly averages for both the implied and empirical prepayment rates. These monthly averages are calculated as the simple average of the prepayment rates for all coupons for each month. The lower panel plots the time series of the prepayment risk premium, which is computed as the difference between the implied and realized prepayment rates. As shown in the upper panel, the implied and realized prepayment rates tracked each other closely up until the middle of 2006. Through most of the financial crisis, however, implied prepayments were much higher than empirical prepayments. This is particularly evident in the lower panel which shows that the prepayment risk premium attained large values during late 2008 and early 2009.

Table 8 presents summary statistics for the implied prepayment rates, the

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<sup>27</sup>For example, see Huang and Huang (2012) and Giesecke et al. (2011).

<sup>28</sup>To solve for the risk premium over longer horizons, we would need also need to solve for the parameters of the  $w_t$ ,  $x_t$ , and  $y_t$  processes under the objective probability measure.

empirical prepayment rates, and the prepayment risk premium. The summary statistics in Table 8 are computed using the time series of the monthly averages. As shown, the average implied prepayment rate across the entire sample of mortgage-backed securities is 25.130%. In contrast, the average empirical prepayment rate for the same sample of mortgage-backed securities is 20.956%. Thus, the implied prepayment rate is clearly different from the actual prepayment rate. The average difference between the implied and actual prepayment rates is 4.174%. The hypothesis that this difference is zero is strongly rejected by the data. These results provide direct confirmation that there is a substantial prepayment risk premium incorporated into mortgage-backed security prices. This direct evidence of prepayment risk premiums in the mortgage-backed securities market corroborates the evidence of prepayment risk premiums in the option-adjusted spreads of interest-only/principal-only securities reported by Gabaix, Krishnamurthy, and Vigneron (2007) and Boyarchenko, Fuster, and Lucca (2016).<sup>29</sup>

The result that the implied prepayment rate is substantially higher than the empirical prepayment rate has important implications for the pricing of mortgage-backed securities, particularly for the literature that relies on econometric models of prepayment. The prices of mortgages with coupon rates below the current market rate are increasing in the prepayment rate while the opposite is true for the prices of mortgages with coupon rates above the current market rate. Thus, the positive prepayment risk premium implies that discount mortgages will have higher values than implied by empirical prepayment functions, while the reverse will be the case for premium mortgages. These results are broadly consistent with the empirical evidence provided in Duarte, Longstaff, and Yu (2007). To provide more insight into the nature of the prepayment risk

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<sup>29</sup>As an alternative way of corroborating the existence of prepayment risk premiums, we regress the monthly excess returns on the Bloomberg 30-Year MBS Return Index on the first two lagged values of the prepayment risk premium. The coefficient for the first lagged value is positive and has significant forecast power for excess returns ( $t$ -statistic of 1.94). We are grateful to the referee for suggesting this test.

premium, it is useful to break it down into its components. In the following sections, we examine the turnover and rate response risk premiums separately.

## 8.2 Turnover risk premium

Similar to the previous section, can solve for the prepayment rate that is due exclusively to turnover by comparing the prepayment rate given by the hazard rate function to the prepayment rate obtained by setting  $x_t = 0$  in the hazard rate function (details provided in the appendix). For clarity, we will designate this as the turnover prepayment rate to distinguish it from the turnover rate (which is a hazard rate rather than a prepayment rate). We can then identify the turnover risk premium by comparing the implied and empirical turnover prepayment rates.

Figure 9 plots the time series of the implied and empirical turnover-related prepayment rates along with the turnover risk premium, which is calculated as the difference. As shown, the turnover risk premium is generally positive throughout the sample period. The turnover risk premium, however, attains some of its largest values during the refinancing waves of the 2001–2005 period (total refinancing volume during this period was many times higher than the average during the prior ten years). The turnover risk premium also attains high values during the financial crisis. Recall that a positive turnover risk premium has the effect of increasing the values of discount mortgage-backed securities while decreasing the values of premium mortgage-backed securities.

Table 8 presents summary statistics for the implied and empirical turnover prepayment rates, and the turnover risk premium. The average implied turnover prepayment rate is roughly twice as large as its empirical counterpart. In particular, the average implied turnover prepayment rate is 7.378%, while the average empirical turnover prepayment rate is 3.575%. Thus, the average turnover risk premium is 3.803% for the sample period. This value is highly statistically significant.

Recall from the previous section that the average prepayment risk premium is 4.174% on average. Thus, the average turnover risk premium of 3.803% represents 91.11% of the entire average prepayment risk, making it the primary component. Given the earlier evidence that turnover risk is related to broad trends in the economy, these results suggest that much of the prepayment risk premium in mortgage-backed securities can be linked to the effects of non-interest-rate-related macroeconomic fluctuations on prepayment behavior.

### 8.3 Rate response risk premium

Following the approach in the discussion above, we identify the rate response prepayment risk premium as the difference between the prepayment rates due specifically to the implied and empirical rate response factors. The upper panel in Figure 10 plots the time series of empirical and implied rate response prepayment rates; the lower panel plots the rate response risk premium.

As shown in the Figure 10, the empirical and implied rate response prepayment rates track each other closely during the 1998–2000 period, and both reach a level of about 30% by the end of 2000. Beginning in 2001, however, both the empirical and implied rate response prepayment rates start to decline, although the implied prepayment rate clearly declines more rapidly than the empirical prepayment rate. Because of this pattern, the rate response risk premium tends to be negative during the 2001–2005 period. With the arrival of the financial crisis in 2007–2008, the implied rate response prepayment rate increases rapidly and attains its highest levels. In contrast, the empirical rate response prepayment rate declines to very low levels similar to those during the 2000–2001 downturn. Thus, the rate response risk premium takes on very large positive values during the early stages of the financial crisis. In fact, during this period, the rate response risk premium is the dominant component of the total prepayment risk premium since the turnover risk premium is close to zero during the 2007–2008 period. With the inception of the HARP program in March 2009, the empirical and implied rate response prepayment rates quickly converge and track each other

closely throughout the remainder of the sample period. This suggests that the HARP program and other similar interventions may have removed much of the systematic risk in the ability of borrowers to respond to refinancing incentives.

Table 8 also presents summary statistics for the implied and empirical rate response prepayment rates along with the risk premium. The average implied rate response prepayment rate of 17.752% is slightly higher than the average empirical rate response prepayment rate of 17.381%. The average rate response risk premium is positive with a value of 0.371%. A closer look at the data, however, indicates that the rate response risk premium is generally significantly positive, with the one exception of the 2001–2005 period. Excluding this period, the average rate response risk premium is 3.41% which closely compares with the overall average turnover risk premium of 3.80%.

There are good reasons to believe, however, that the 2001–2005 period may have been an unusual period during which the normal covariance between rate response and consumption may have changed signs. As one example, housing values increased dramatically during this 5-year period and many homeowners refinanced into higher balance loans (and even higher interest rate loans) in order to cash out equity and increase their consumption. For example, annual cash-out refis averaged \$23.3 billion from 1993 to 2000, but increased more than 350% to \$82.9 billion in 2001. Similarly, annual cash-out refis exceeded \$100 billion from 2002 to 2005. During normal times, a borrower's credit and employment/income situation would be major determinants of their ability to refinance a mortgage. During this period of rapidly increasing housing values, however, borrowers were often able to refinance primarily on the strength of their home equity rather than the usual credit/income criteria. Thus, it is possible that the typical positive rate response risk premium reflects the covariance between consumption and the macroeconomic factors that affect borrowers' credit scores, employment, and household income. In contrast, the negative rate response risk premium during this period may represent compensation for a different set of risks (related to housing values) that temporarily drove refinancing activity during this period.

There are, however, other possible reasons this period may have been an unusual one for risk premiums. For example, Lustig and Van Nieuwerburgh (2005) argue that the ratio of housing collateral to human wealth is an important determinant of risk premiums. Their figure 6 shows that 2002 was associated with a 70-year high in the housing collateral ratio. Given the close link between housing values and the potential ability to refinance, it is possible that their results may help explain the decline in the rate response risk premium during this period. Similarly, the 2002 to 2005 period experienced a dramatic decline in the spread between BBB corporate yields and Treasury yields. This decline may also have been associated with a reduction in credit-related risk premiums, which in turn could have impacted the risk premium for the credit-availability-related rate response factor.

Numerous other examples of risk premiums change signs over time. For example, Fleckenstein, Longstaff, and Lustig (2017) show that the inflation risk premium changed signs during the 2010–2015 period. Vedolin (2013) shows that volatility risk premiums for individual stocks can be both positive and negative, and change signs frequently. The Federal Reserve’s term structure model has implied negative term premiums throughout much of 2016.<sup>30</sup>

Finally, another important implication of these results is that rate-response-related refinancing activity is an important driver of mortgage-backed security pricing. On average, rate-response-related prepayments represent 79.27% of empirical prepayments and 69.45% of implied prepayments. This can easily be seen by comparing the empirical and implied turnover prepayment rates in Figure 9 with the empirical and implied rate response prepayment rates in Figure 10. In particular, the turnover prepayment rates seldom exceed 20% during the sample period, while the rate response prepayment rates often exceed 20% during the refi wave, financial crisis, and HARP periods. Thus, even though the rate response risk premium may be small on average, rate-response-related refinancing activity is the primary factor driving total prepayments. This means that rate-response-

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<sup>30</sup>See <https://www.bloomberg.com/news/articles/2016-05-19/not-much-worries-bond-traders-as-term-premium-falls-to-1962-lows> .

related refinancing activity is of first order importance both empirically as well as in the risk-neutral world in which mortgage-backed securities are priced.

## 9. Pricing IO/PO Securities

To test the robustness of our model, we value the interest-only (IO) and principal-only (PO) classes (“strips”) of a selection of Fannie Mae stripped mortgage-backed securities (SMBS). An interest-only (IO) strip receives 100% of the interest and 0% of the principal from the pass-throughs backing the security, and a principal-only (PO) strip receives 0% of the interest and 100% of the principal. The market prices of IO and PO securities are highly sensitive to prepayment expectations, and these securities allow for a demanding out-of-sample test of our model. Traditional mortgage valuation models have difficulty pricing IO and PO strips, and our model performs significantly better, even though we make no adjustments for the specified nature of IO/PO collateral and the lower liquidity of the IO/PO market. In this section, we provide a brief overview of the IO/PO market, describe the data, and discuss the estimation and the results.

### 9.1 IO/PO markets

IOs and POs can be created as part of any collateralized mortgage obligation (CMO) deal. However, the most liquid sector of the IO/PO market are the IOs and POs created from SMBS deals. The reason SMBS are more liquid than CMOs is two fold. First, each SMBS deal has an exchange option. This option allows someone that holds both the IO and PO (i.e, the IO/PO “combo”) to exchange the combo for a pass-through security for a small fee. The pass-through can then be sold in the specified pool market or the TBA market if the collateral does not trade at premium to TBA. Second, each SMBS deal is very large, typically several billion dollars of notional.<sup>31</sup>

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<sup>31</sup>For an overview of the IO/PO market and the risks of IO and PO securities,

Even though SMBS is liquid compared to IOs and POs from CMOs, there are many reasons to believe that SMBS is still much less liquid than TBAs. First, the trading volume for the IO/PO market is tiny compared to that of the TBA market. TRACE data, obtained from a large MBS dealer, indicates that the daily trading volume of FNMA IO/POs was only 0.20% of the FNMA TBA volume from 2011 to 2017. Second, Chaudhary (2006) discusses how SMBS becomes less liquid as it seasons, and the value of exchange option may deteriorate as IO or PO strips get locked up in CMOs. Third, the funding markets of IO/POs are different than TBAs. The TBA market has a corresponding dollar roll market where MBS often trades “special,” and similar to specialness in the Treasury repo market, this increases prices (see Song and Zhu (2016)). However, IOs and POs are financed in the MBS repo market where financing rates and haircuts are generally higher.

There is also reason to believe that even IOs and POs from the same SMBS are likely to have different liquidity. First, IOs have much greater price volatility (in percentage terms), and are subject to greater haircuts and holding costs. Second, POs have favorable accounting treatment for banks and they do not necessarily need to be marked-to-market. Finally, there is a greater supply of IOs in the market than POs. Each time an MBS pass-through is created, an IO strip, called a mortgage servicing right (MSR), is created, and a portion of the IO strip can be sold as an agency-guaranteed IO security.

Another dimension along which SMBS differ from TBAs is that each SMBS has unique collateral characteristics that often provide valuable prepayment behavior. Mortgage strip pricing reports from MBS dealers show that SMBS combo pay-ups can be as much as \$2 to \$3 per \$100 notional. This means that the collateral backing the IO/PO combo has superior prepayment behavior that commands a premium over TBA, even after accounting for the liquidity discount.

## 9.2 IO/PO data

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see Hayre (2001), Chaudhary (2006), and Fabozzi (2016).

We obtained end-of-day marks for IOs and POs from various SMBS trusts from two major Wall Street dealers. The sample begins in 2004 and we end our sample on December 31, 2009 because the quality of the marks deteriorate in later years. We found that the price of IO/PO combos were marked at constant spread to TBAs beginning in 2008 and by 2010 they were marked at constant spreads for months at a time. This leads us to question the quality of the marks. We focus on the 5.00%, 5.50%, and 6.00% coupons because Chaudhary (2006) indicates that these were the most liquid coupons in 2006 and they are traded throughout the sample period.

Table 9 shows the summary statistics for the SMBS trusts. These deals tend to be very large—the average deal size is \$2.6 billion notional. Each deal has different collateral characteristics. For example, each SMBS deal corresponds to a different vintage of mortgages and the underlying mortgage coupons, loan sizes, loan-to-value ratios, and geographic distributions are different for each SMBS. These characteristics translate into to different prepayment speeds and different prices for the IO/PO combo relative to TBA. Even though these are the most liquid SMBS securities, there is significant disagreement in the end-of-month dealer marks—the RMSE between the two dealer marks range from 0.31 to 2.86 dollars for the IO class of the SMBS deals. Surprisingly, FNS 397, which is the second largest deal and should be the most liquid, has the largest RMSE between marks.

### **9.3 Estimation and results**

Our model is estimated from the most liquid sector of the MBS market—the TBA market. Using the fitted model, we calculate the prices of the interest-only and principal-only portions of each TBA coupon. We then compare the model’s IO/PO prices for the 5.00%, 5.50%, and 6.00% coupon TBAs to 5.00%, 5.50%, and 6.00% coupon IO and PO classes for the SMBS deals shown in Table 9. This means that our IO/PO prices incorporate the same credit/liquidity spread and prepayment assumptions as the TBA market. To accurately value SMBS IOs

and POs, our model would need to be extended to account the different liquidity and prepayment characteristics of SMBS. It is not clear how our prices should compare to SMBS because it is unclear what the joint effect is of the different liquidity and prepayment characteristics.

Despite this, our model performs well in tracking the IO and PO prices for the SMBS pools. For example, the average correlation between our prices and the SMBS marks is 89.6%. Figure 11 plots the time series of market values and fitted values for the IOs and POs. As shown, the fitted values track the market values very closely. The RMSE between our IO prices and the closest SMBS IO strip is \$1.73, \$2.17, and \$2.56, for the 5.00%, 5.50%, and 6.00% coupons, respectively. This compares favorably to the RMSE between the dealer marks. The option-adjusted spreads from dealer prepayment models for both the IO and the PO classes of these SMBS ranged from  $-1,000$  to  $2,200$  bps over the sample period, even though these models adjust the prepayment forecast given the collateral characteristics for each SMBS trust. Hence, our model drastically improves on the traditional approach.

## 10. Conclusion

We present a new three-factor no-arbitrage model for the valuation of mortgage-backed securities. Rather than using an empirical prepayment function, our approach solves for the implied prepayment function used by the market in pricing mortgage-backed securities. By studying the properties of the implied prepayment function, our goal is to shed light on the key drivers of prepayment risk as perceived by the market.

We show that this modeling framework is very successful in capturing the cross-sectional structure of mortgage-backed security prices. This result is important since it suggests that the standard approach of calibrating mortgage models to each mortgage-backed security separately using option-adjusted spreads may not be necessary.

We also find that implied prepayments can be very different from actual prepayments. This provides direct evidence that mortgage-backed securities incorporate significant prepayment risk premiums. Furthermore, the results indicate that macroeconomic factors play a large role in driving prepayment risk. In particular, we find that prepayment risk is driven not only by changes in interest rates, but also by changes in turnover and rate response factors. We find that these factors are related to macroeconomic fundamentals and are also associated with significant risk premiums.

Finally, we provide the first direct evidence that mortgage-backed security prices are also driven by changes in the credit risk of the agency guaranteeing the timely payment of principal and interest as well as by changes in the liquidity of these securities. These results are consistent with findings for other markets.

Our results suggest a number of possible directions for future research. Although the simple implied prepayment model we use performs well, it may be worthwhile to explore whether alternative specifications that include formal models of seasoning and burnout might enhance the performance further. In this paper, we have focused primarily on the pricing of TBAs. An interesting direction for future research might be the extension of the framework to the specified pools market or to broader categories of IOs, POs, and CMOs. Future work could also focus on identifying how much of the risk premium in the returns of mortgage-backed securities is due to agency credit and how much is due to actual prepayment risk. A framework such as ours that explicitly incorporates a credit/liquidity spread could provide a useful starting point in this analysis.

## References

- Ang, A., and F. A. Longstaff. 2013. Systemic sovereign credit risk: Lessons from the U.S. and Europe. *Journal of Monetary Economics* 60:493–510.
- Brigo, D., and F. Mercurio. 2006. *Interest rate models - theory and practice: with smile, inflation and credit, 2nd Edition*. New York: Springer.
- Boudoukh, J., M. Richardson, R. Stanton, and R. F. Whitelaw. 1997. Pricing mortgage-backed securities in a multifactor interest rate environment: A multivariate density estimation approach. *Review of Financial Studies* 10:405–46.
- Boyarchenko, N., A. Fuster, and D. O. Lucca. 2016. Understanding mortgage spreads. Staff Report, Federal Reserve Bank of New York.
- Brennan, M. J., and E. S. Schwartz. 1985. Determinants of GNMA mortgage prices. *Real Estate Economics* 13:209–28.
- Capone, C. A. 2001. Introduction to the special issue on mortgage modeling. *Journal of Real Estate Finance and Economics* 23:131–37.
- Carlin, B. I., F. A. Longstaff, and K. Matoba. 2014. Disagreement and asset prices. *Journal of Financial Economics* 114:226–38.
- Chan, Y. K. 1998. Valuation of inverse IOs and other mortgage derivatives. *Salomon Smith Barney, Global Fixed Income Research* (August):1–12.
- Chaudhary, S. 2006. The trust IO/PO market. *RMBS Trading Strategy, Bank of America* (October 23):1–29.
- Chen, S. 1996. Understanding option-adjusted spreads: The implied prepayments hypothesis. *Journal of Portfolio Management* 23 (Summer):104–13.
- Cheyette, O. 1996. Implied prepayments. *Journal of Portfolio Management* 23 (Fall):107–15.
- Christensen, J., and J. M. Gillan. 2016. Does quantitative easing affect market

- liquidity? Working Paper, Federal Reserve Bank of San Francisco.
- Christensen, J., and G. D. Rudebusch. 2012. The response of interest rates to U.S. and U.K. quantitative easing. *Economic Journal* 122:F385–F414.
- Corb, H. 2012. *Interest rate swaps and other derivatives*. New York: Columbia University Press.
- Deng, Y., J. M. Quigley, and R. v. Order. 2000. Mortgage terminations, heterogeneity and the exercise of mortgage options. *Econometrica* 68:275–307.
- Diep, P., A. L. Eisfeldt, and S. Richardson. 2016. Prepayment risk and expected MBS returns. Working Paper, UCLA.
- Downing, C., R. Stanton, and N. Wallace. 2005. An empirical test of a two-factor mortgage valuation model: How much do house prices matter? *Real Estate Economics* 33:681–710.
- Driessen, J., P. Klaassen, and B. Melenberg. 2003. The performance of multi-factor term structure models for pricing and hedging caps and wptions. *Journal of Financial and Quantitative Analysis* 38:635–72.
- Duarte, J., F. A. Longstaff, and F. Yu. 2007. Risk and return in fixed-income arbitrage: Nickels in front of a steamroller? *Review of Financial Studies* 20:769–811.
- Duffie, D., and K. J. Singleton. 1997. An econometric model of the term structure of interest-rate swap yields. *Journal of Finance* 52:1287–1321.
- . 1999. Modeling term structures of defaultable bonds. *Review of Financial Studies* 12:687–720.
- Dunn, B. R., K. B. Dunn, F. J. Fabozzi, and R. Sella. 2016. Hedging agency mortgage-related securities. In *The handbook of mortgage-backed securities, 7th Edition*. Ed. F. J. Fabozzi. New York: Oxford University Press.
- Dunn, K. B., and J. J. McConnell. 1981a. A comparison of alternative models

- for pricing GNMA mortgage-backed securities. *Journal of Finance* 36:471–84.
- . 1981b. Valuation of GNMA mortgage-backed securities. *Journal of Finance* 36:599–616.
- Dunn, K. B., and C. S. Spatt. 2005. The effect of refinancing costs and market imperfections on the optimal call strategy and the pricing of debt contracts. *Real Estate Economics* 33:595–617.
- Fabozzi, F. J. 2016. *The handbook of mortgage-backed securities, 7th Edition*. Ed. F. J. Fabozzi. Oxford: Oxford University Press.
- Fleckenstein, M., F. A. Longstaff, and H. Lustig. 2017. Deflation risk. *Review of Financial Studies* 30:2719–60.
- Gabaix, X., A. Krishnamurthy, and O. Vigneron. 2007. Limits of arbitrage: Theory and evidence from the mortgage-backed securities market. *Journal of Finance* 62:557–95.
- Gagnon, J., M. Raskin, J. Remache and B. Sack. 2011. The financial market effects of the Federal Reserve’s large-scale asset purchases. *International Journal of Central Banking* 7:3–43.
- Gao, P., P. Schultz, and Z. Song. 2017. Liquidity in a market for unique assets: Specified pools and to-be-announced trading in the mortgage-backed securities market. *Journal of Finance* 72:1119–70.
- Giesecke, K., F. A. Longstaff, S. Schaefer and I. Strebulaev. 2011. Corporate bond default risk: A 150-year perspective. *Journal of Financial Economics* 102:233–50.
- Goncharov, Y. 2006. An intensity-based approach to the valuation of mortgage contracts and computation of the endogenous mortgage rate. *International Journal of Theoretical and Applied Finance* 9:889–914.
- Gorovoy, V., and V. Linetsky. 2007. Intensity-based valuation of residential mortgages: An analytically tractable model. *Mathematical Finance* 17:541–73.

- Gupta, A., and M. G. Subrahmanyam. 2005. Pricing and hedging interest rate derivatives: Evidence from cap-floor markets. *Journal of Banking and Finance* 29:701–33.
- Hanson, S. G. 2014. Mortgage convexity. *Journal of Financial Economics* 113:270–99.
- Hayre, L. 2001. *Salomon Smith Barney guide to mortgage-backed and asset-backed securities*. New York: John Wiley & Sons.
- Ho, T. S. Y. 1992. Key rate durations: Measures of interest rate risks. *Journal of Fixed Income* 2:29–44.
- Huang, J.-Z., and M. Huang. 2012. How much of the corporate-Treasury yield spread is due to credit risk? *Review of Asset Pricing Studies* 2:153–202.
- Hull, J. C. 2015. *Options, futures, and other derivatives, 9th Edition*. Boston: Pearson.
- Hull, J. C., and A. White. 1990. Pricing interest-rate derivative securities. *Review of Financial Studies* 3:573–92.
- Jamshidian, F. 1989. An exact bond option pricing formula. *Journal of Finance* 44:205–09.
- Jarrow, R. A., D. Lando, and F. Yu. 2005. Default risk and diversification: Theory and empirical implications. *Mathematical Finance* 15:1–26.
- Kaelo, P., and M. Ali. 2006. Some variants of the controlled random search algorithm for global optimization. *Journal of Optimization Theory and Applications* 130:253–64.
- Kandrak, J. 2013. Have Federal Reserve MBS purchases affected market functioning? *Economic Letters* 121:188–91.
- Kau, J. B., and D. C. Keenan. 1995. An overview of the option-theoretic pricing of mortgages. *Journal of Housing Research* 6:217–44.

- Kau, J. B., D. C. Keenan, W. J. Muller III, and J. F. Epperson. 1992. A generalized valuation model for fixed-rate residential mortgages. *Journal of Money, Credit and Banking* 24:279–99.
- Kau, J. B., and V. C. Slawson. 2002. Frictions, heterogeneity and optimality in mortgage modeling. *Journal of Real Estate Finance and Economics* 24:239–60.
- Krishnamurthy, A., and A. Vissing-Jorgensen. 2011. The effects of quantitative easing on interest rates: Channels and implications for policy. *Brookings Papers on Economic Activity*, Fall.
- . 2013. The ins and outs of large scale asset purchases, Kansas City Federal Reserve Symposium on Global Dimensions of Unconventional Monetary Policy.
- Levin, A., and A. Davidson. 2005. Prepayment risk and option-adjusted valuation of MBS. *Journal of Portfolio Management* 31:73–85.
- Linetsky, V. 2004. The spectral decomposition of the option value. *International Journal of Theoretical and Applied Finance* 7:337–84.
- Longstaff, F. A. 2005. Borrower credit and the valuation of mortgage-backed securities. *Real Estate Economics* 33:619–61.
- Longstaff, F. A., S. Mithal, and E. Neis. 2005. Corporate yield spreads: Default risk or liquidity? New evidence from the credit default swap market. *Journal of Finance* 60:2213–53.
- Longstaff, F. A., J. Pan, L. H. Pedersen, and K. J. Singleton. 2011. How sovereign is sovereign credit risk? *American Economic Journal: Macroeconomics* 3:75–103.
- Lustig, H., and S. V. Nieuwerburgh. 2005. Housing collateral, consumption insurance, and risk premia: An empirical perspective. *Journal of Finance* 60:1167–1219.
- Malkhozov, A., P. Mueller, A. Vedolin, and G. Venter. 2014. Mortgage risk and

the yield curve. Working Paper, London School of Economics.

Newey, W. K., and K. D. West. 1987. A Simple, positive, semi-definite heteroskedasticity and autocorrelation consistent covariance matrix. *Econometrica* 55:703–8.

Pan, J., and K. J. Singleton. 2008. Default and recovery implicit in the term structure of sovereign CDS spreads. *Journal of Finance* 63:2345–84.

Richard, S. F., and R. Roll. 1989. Prepayments on fixed-rate mortgage-backed securities. *Journal of Portfolio Management* 15:73–82.

Schwartz, E. S., and W. N. Torous. 1989. Prepayment and the valuation of mortgage-backed securities, *Journal of Finance* 44:375–92.

———. 1992. Prepayment, default, and the valuation of mortgage pass-through securities. *Journal of Business* 65:221–39.

———. 1993. Mortgage prepayment and default decisions: A Poisson regression approach. *Real Estate Economics* 21:431–49.

Song, Z., and H. Zhu. 2016. Mortgage dollar roll. Working Paper, MIT.

Stanton, R. 1995. Rational prepayment and the valuation of mortgage-backed securities. *Review of Financial Studies* 8:677–708.

Stanton, R., and N. Wallace. 1998. Mortgage choice: What’s the point? *Real Estate Economics* 26:173–205.

Titman, S., and W. N. Torous. 1989. Valuing commercial mortgages: An empirical investigation of the contingent-claims approach to pricing risky debt. *Journal of Finance* 44:345–73.

Treasury Market Practices Group. 2012. Margining in agency MBS trading, White Paper.

Thornton, D. L. 2014. QE: Is there a portfolio balance effect? *Federal Reserve Bank of St. Louis Review* 96:55–72.

Vedolin, A. 2013. Uncertainty and leveraged Lucas trees: The cross section of equilibrium volatility risk premia. Working Paper, London School of Economics.

Vickery, J., and J. Wright. 2013. TBA trading and liquidity in the agency MBS market. *Economic Policy Review* 19:1–18.

Wallace, N. 2005. Innovations in mortgage modeling: An introduction. *Real Estate Economics* 33:587–593.

## **Appendix**

### **A1. Data Sources**

Table A1 presents the definitions and data sources for the variables used in the study.

### **A2. Fannie Mae Mortgage-Backed Security Cash Flows**

The pricing data are from the to-be-announced (TBA) market for 30-year Fannie Mae (FNMA) mortgage-backed securities (MBS). Before describing how we estimate the model, we consider the timing of cash flows generated by a FNMA TBA trade.

TBA trades settle in accordance with a monthly schedule set by the Securities Industry and Financial Markets Association (SIFMA). Thirty-year FNMA MBS falls into SIFMA's class A, which typically settles during the second week of the month. Because we select prices at the end of each month, the settlement date corresponding to these observations is around the second week of the following month (the exact settlement dates can be found on Bloomberg). On the notification date, 2 days prior to settlement, the buyer is notified of the exact pools to be delivered. On the settlement date, the buyer transfers a payment to the seller, which consists of the agreed on price (which we observe at month end) plus accrued interest on the face value of the pools identified on the notification date (the variance permitted on TBA trades is plus or minus 0.01% of the dollar amount of the transaction agreed to by the parties). On the record date, the last day of the month, Fedwire records the buyer as the new holder of the security. On the fifth or sixth business day of the next month, the pool factors (the ratio of the current balance to the original balance) are released. The pool factor determines the new face value of the mortgage after accounting for scheduled principal payments and prepayments from the previous month. Then, on the payment date later that month, the scheduled principal payments, interest payments, and prepayments, less servicing and guaranty fees are passed to the holder of the security. For FNMA MBS, the payment date is the 25th of the

month. If the 25th day happens to fall on a bond market holiday or a weekend, the payment is made on the following business day. A time line for the timing of payments for a hypothetical TBA trade is shown in Table A2.

### A3. Adjustment for Fees

Fixed-rate mortgage pools consist entirely of fixed-rate loans, but the underlying loans may bear different fixed rates of interest. Interest on a fixed-rate pool is set on the issue date of the related certificates, and it is equal to the interest rates less the fee percentages for each loan in the pool. The fee percentage is the sum of the servicing fee and the guaranty fee for that loan. Fixed-rate loans in a single pool have interest rates that are within a 2% range (sometimes a wider range may be allowed). However, the pass-through rate of each loan in fixed rate pool is the same. Therefore, the pass-through rate will not change if prepayments occur.

Consider the cash flow generated by a pass-through of an individual fixed rate mortgage. Prior to either prepayment or default, the owner of the pass-through receives the constant cash flow  $c$  (consisting of both interest and scheduled principal) generated by the mortgage loan less the servicing and guaranty fees, which are a percentage of the principal balance  $I_t$ . Denote the servicing and guaranty fees by the constant  $g$ . Then the cash flow generated by the pass-through security,  $c_t^{PT}$ , in absence of prepayment, is

$$c_t^{PT} = c - gI_t. \quad (A1)$$

Therefore, the value of a FNMA MBS, after accounting for fees, is given by

$$F(m, T) = E^Q \left[ \int_0^T \exp \left( - \int_0^t r_s + w_s ds \right) N_t (c + (p_t - g) I_t) dt \right], \quad (A2)$$

where

$$p_t = x_t + y_t \max(0, m - a - b r_t(10)). \quad (A3)$$

#### A4. Valuation

Because of the independence of  $w_t$  from the other stochastic processes, we can write

$$F(m, T) = \int_0^T S(t) E^Q \left[ \exp \left( - \int_0^t r_s ds \right) N_t (c + (p_t - g) I_t) \right] dt, \quad (A4)$$

where

$$S(t) = E^Q \left[ \exp \left( - \int_0^t w_s ds \right) \right]. \quad (A5)$$

The expression for  $S(t)$  is given by

$$S(t) = A(t) \exp(-w_0 B(t)), \quad (A6)$$

where

$$A(t) = \exp \left( \left( \frac{\sigma_w^2}{2} - \frac{\alpha_w}{\beta_w} \right) t + \left( \frac{\alpha_w}{\beta_w^2} - \frac{\sigma_w^2}{\beta_w^3} \right) (1 - \exp(-\beta_w t)) + \frac{\sigma_w^2}{4\beta_w^3} (1 - \exp(-\beta_w t)) \right), \quad (A7)$$

$$B(t) = \frac{1}{\beta_w} (1 - \exp(-\beta_w t)). \quad (A8)$$

We assume that the interest rate follows the Hull and White (1990) model,

$$dr = (\alpha_r(t) - \beta_r r) dt + \sigma_r dZ_r, \quad (A9)$$

where  $\beta_r$  and  $\sigma_r$  are positive constants and the deterministic function  $\alpha_r(t)$  is chosen to match the Treasury term structure exactly. Given the market discount function  $D(t)$  and values for  $\beta_r$  and  $\sigma_r$ ,

$$\alpha_r(t) = -\frac{\partial^2 \ln D(t)}{\partial t^2} - \beta_r \frac{\partial \ln D(t)}{\partial t} + \frac{\sigma_r^2}{2\beta_r} (1 - \exp(-\beta_r t)). \quad (\text{A10})$$

At time  $t$  (where  $t$  is measured in years), the 10-year interest rate  $r_t(10)$  is an affine function of the short rate  $r_t$ ,

$$r_t(10) = \frac{1}{10}(-A_r(t) + B_r r_t), \quad (\text{A11})$$

where

$$A_r(t) = \ln \frac{D(t+10)}{D(t)} - B_r \frac{\partial \ln D(t)}{\partial t} - \frac{\sigma^2}{4a} (1 - \exp(-2\beta_r t)) B_r^2, \quad (\text{A12})$$

$$B_r = \frac{1 - e^{-10\beta_r}}{\beta_r}. \quad (\text{A13})$$

Therefore, we can rewrite the prepayment intensity in terms of  $r_t$ ,

$$p_t = x_t + y_t \max(0, m - a - b(-A_r(t) + B_r r_t)/10). \quad (\text{A14})$$

## A5. Adjustments For Discrete Cash Flows

In this section, we adjust the mortgage valuation formula to account for the actual cash flows from a TBA trade. Let

$$\tilde{T} \equiv \{0, t_s, t_1, \dots, t_K\}, \quad (\text{A15})$$

be the set of points in time related to the payments associated with a mortgage-backed security with  $K$  months until maturity. The valuation date, or trade date, is  $t = 0$ . The MBS settles at  $t = t_s$ , and the MBS investor receives payments on dates  $t_1$  through  $t_K$ . Since the settlement dates are fixed by SIFMA, the amount of time from the valuation date  $t = 0$  through the settlement date  $t = t_s$  varies

depending on the trade date. Also, because of holidays and weekends, each time step after settlement, that is,  $\Delta t_i \equiv t_{i+1} - t_i$ ,  $i = 1, \dots, K - 1$ , may vary by a couple of days. Let  $CF_i$  be the cash flow received by the mortgage investor at time  $t_i$ . In our framework, the value of the mortgage is

$$F(m, K) = \frac{1}{S(t_s)} \sum_{i=1}^K S(t_i) E^Q[ \exp(-\int_{t_s}^{t_i} r_s ds) CF_i ]. \quad (A16)$$

To determine the cash flow  $CF_i$  at  $t_i$ , we can apply standard mortgage cash flow formulas (recall that in continuous time, the cash flow is  $N_t [c + (p_t - g) I_t]$ ). Following Hayre (2001), for each dollar of a mortgage in month  $i$ ,

$$\text{Monthly payment} = \text{PAY}_i = \frac{m/12}{1 - (1 + m/12)^{-K}}, \quad (A17)$$

$$\text{Balance (end of month)} = \text{BAL}_i = \frac{1 - (1 + m/12)^{-K+i}}{1 - (1 + m/12)^{-K}}, \quad (A18)$$

$$\text{Principal portion of payment} = \text{PRIN}_i = \text{PAY}_i \times (1 + m/12)^{-K-1+i}, \quad (A19)$$

$$\text{Interest portion of payment} = \text{INT}_i = \text{PAY}_i - \text{PRIN}_i. \quad (A20)$$

Let

$$\tilde{T}_{CF} \equiv \{\tau_0, \tau_1, \dots, \tau_K\}, \quad (A21)$$

be the set of points in time relevant to determine the monthly cash flows of the MBS. This set corresponds to month-ends. For the example in Table A2,  $\tau_0$  is March 31st, the month end before the settlement date, and  $\tau_1$  is April 30th. It is possible that  $\tau_0$  is either before or after the trade date depending on whether the trade date and settlement date occur in the same month. However, because the data are observed at each month end, the elements of  $\tilde{T}$  and  $\tilde{T}_{CF}$  are ordered as

$$\tau_0 = 0 < t_s < \tau_1 < t_1 < \tau_2 < t_2 < \dots < \tau_K < t_K, \quad (A22)$$

as shown in the example in Table A2.

Recall that  $N_t$  represents the fraction of the mortgage pool that has not yet prepaid (i.e., a survival factor). To keep track of monthly prepayments, we calculate the single monthly mortality (SMM), a common object in mortgage modeling. SMM is fraction of the pool's outstanding balance at the beginning of the month that is prepaid during the month. Hence,

$$\text{SMM}_i = \frac{N_{\tau_{i-1}} - N_{\tau_i}}{N_{\tau_{i-1}}}. \quad (\text{A23})$$

Therefore, the prepayments in a given month  $i$ ,  $\text{PP}_i$ , can be written as

$$\text{PP}_i = (\text{BAL}_{i-1} \times N_{\tau_{i-1}} - \text{PRIN}_i \times N_{\tau_{i-1}}) \times \text{SMM}_i. \quad (\text{A24})$$

The cash flow  $\text{CF}_i$  received by the investor at  $t_i$  reflects the payments (scheduled and prepaid) at  $\tau_i$  from the underlying mortgage loans, less servicing and guaranty fees  $g$ . Therefore,

$$\text{CF}_i = \text{PRIN}_i \times N_{\tau_{i-1}} + \text{PP}_i + \frac{m-g}{m} \times \text{INT}_i \times N_{\tau_{i-1}}. \quad (\text{A25})$$

Given paths of  $r_t$ ,  $x_t$ , and  $y_t$ , we calculate a path of  $p_t$ . After integration and exponentiation, we calculate  $N_t$  for the relevant time points. Then, the standard mortgage formulas provide the cash flows.

## A6. Discount Function

We collect historical data on nominal-constant maturity Treasury rates from the Federal Reserve's H.15 statistical release. Then, we use a standard cubic spline to bootstrap the prices of zero-coupon bonds  $D(t)$  for the relevant time points for up to 30 years. For a discussion of this methodology, see Longstaff, Mithal, and Neis (2005).

## A7. Estimation of Interest Rate Dynamics

As discussed above, in the dynamics for the riskless rate in the Hull and White (1990) model,

$$dr = (\alpha_r(t) - \beta_r r) dt + \sigma_r dZ_r, \quad (A26)$$

the deterministic function  $\alpha_r(t)$  is chosen to match the Treasury term structure exactly. Therefore, estimation of the model involves finding the values of  $\beta_r$  and  $\sigma_r$  that best fit a set of market instruments on a given date.

Pass-through mortgage-backed securities are most sensitive to changes in intermediate-term yields (e.g., see Ho 1992 or Dunn et al. 2016). Moreover, the refinancing incentive in our model is a function of the 10-year Treasury yield. As such, we fit the interest rate model to the intermediate sector of the volatility surface. Because European swaptions are among the most liquid options on interest rates, we estimate the interest rate volatility using the swaption volatility surface. Specifically, we select the set of 1-, 2-, 3-, 4-, and 5-year into 5-, 7-, and 10-year at-the-money-forward European receivers swaptions, giving a total of 15 instruments. We then calculate the prices of at-the-money-forward receivers swaptions referencing the Treasury curve ( $D(t_i)$  is from the Treasury curve). The price of a  $T$  into  $t_n - T$  receivers swaption with payment dates  $t_1, t_2, \dots, t_n$  and normal volatility  $\sigma_N$  is

$$P = \sigma_N \sqrt{\frac{T}{2\pi}} \sum_{i=1}^n D(t_i). \quad (A27)$$

Corb (2012) provides an extensive discussion of the normal swaption model.

To calculate the prices of the swaptions in the Hull and White (1990) model, we apply the Jamshidian (1989) decomposition, allowing us to write the swaption price as a weighed sum of zero-coupon bond options, for which there are analytical formulas (see Brigo and Mercurio 2006 or Hull 2015 for a textbook treatment of this approach).

Finally, we solve for values of  $\sigma_r$  and  $\beta_r$  on a given date that minimize the sum of squared percentage price error:

$$\sum_{i=1}^{15} \left[ \frac{P_{\text{market}}(i) - P_{\text{model}}(i)}{P_{\text{market}}(i)} \right]^2, \quad (\text{A28})$$

of the 15 swaptions using the Levenberg-Marquardt algorithm. The use of the sum of squared percentage price errors as an objective function follows standard practice. The model fits the Black volatility surface of the 15 swaptions with a median RMSE of 38 bps. Over the sample period, the median Black volatility is 20.64%.

## A8. Estimation Methodology

The estimation of the model can be viewed as consisting of three steps.

1. We select an initial parameter vector  $\theta$ , where  $\theta = \{a, b, \alpha_w, \alpha_x, \alpha_y, \beta_w, \beta_x, \beta_y, \sigma_w, \sigma_x, \sigma_y, \rho_{r,x}, \rho_{r,y}, \rho_{x,y}\}$ .
2. Conditional on  $\theta$  and for each month  $t$  during the sample period, we solve for the values of  $w_t$ ,  $x_t$ , and  $y_t$  that best fit the model to the prices for the cross-section of mortgage-backed securities with different coupon rates (the coupon stack) by minimizing the RMSE.

Since the nonlinear structure of the prepayment function makes it difficult to express the price of mortgage-backed securities in closed-form, we use simulation to solve for the model-based mortgage-backed security values. The simulation step solves

$$F(m, K) = \frac{1}{S(t_s)} \sum_{i=1}^K S(t_i) E^Q \left[ \exp\left(-\int_{t_s}^{t_i} r_s ds\right) CF_i \right], \quad (\text{A29})$$

for given values of  $w_t$ ,  $x_t$ , and  $y_t$ . Since  $S(t)$  has a closed-form solution, we can focus on the expectation in the equation above. We simulate paths of  $r_t$ ,  $x_t$ , and  $y_t$  with monthly time steps, and then compute a path of  $p_t$  since

$$p_t = x_t + y_t \max(0, m - a - b r_t(10)). \quad (A30)$$

From  $p_t$ , we compute the survival factors  $N(t)$  for each month, and then compute the mortgage cash flows  $CF_i$ . Along each path, we also compute the discount factor to apply to each cash flow. The average discounted cash flows over all paths provides the estimate of the expectation. Given the value of the expectation, we then solve for the mortgage price.

The RMSE of the simulated prices and the market prices provides the objective function for the optimization. We use the controlled random search (CRS) algorithm of Kaelo and Ali (2006) to solve for the initial values of  $w_t$ ,  $x_t$ , and  $y_t$ , which minimize the RMSE for each date  $t$ .

3. We apply the CRS algorithm to the parameter vector  $\theta$  to find the vector that results in the lowest global RMSE. The outputs are the parameter values and the time series of state variables.

### **A9. Identifying the Empirical Turnover and Rate Response Factors**

In identifying the empirical turnover and rate response factors, we use the one-month conditional prepayment rates (CPRs) for the same set of mortgage-backed securities used to estimate the implied turnover and rate response factors. To illustrate how this is done, let  $CPR_{it}$  denote the 1-month realized CPR observed at time  $t$  for the  $i$ th mortgage-backed security, where  $i = 1, 2, \dots, n_t$ , and where  $n_t$  is the number of individual mortgage-backed securities in the sample at time  $t$ . For a given  $t$ , we estimate the empirical turnover and rate response factors, denoted as  $\hat{x}_t$  and  $\hat{y}_t$ , from the following cross-sectional nonlinear regression,

$$CPR_{it} = 1 - \exp(-\hat{x}_t - \hat{y}_t \max(0, m - a - b r_t(10))) + \epsilon_{it}, \quad (A31)$$

where  $\epsilon_{it}$  denotes an i.i.d. normally distributed residual. The exponential term in this expression results from the mapping of the hazard rate function into the

conditional prepayment rate. This nonlinear regression is estimated separately for each date  $t$  in the sample using the CPRs for the  $n_t$  individual mortgage-backed securities in the sample on date  $t$ . We solve for the best fitting values of  $\hat{x}_t$  and  $\hat{y}_t$  by minimizing the sum of squared residuals using a standard genetic algorithm numerical optimizer. We repeat this process using different sets of starting values to ensure that we achieve the global minimum. Given the relatively small values of  $n_t$  in the sample, we make a minor concession to the data and impose the lower bound constraints  $\hat{x}_t \geq -\ln(1 - \min_i CPR_{it})$  and  $\hat{y}_t \geq 0$  in the estimation to guard against the effects of outliers in the data. These lower bounds ensure that estimated empirical prepayment rate remains positive.

### A10. Decomposing the Prepayment Rate into Components

To make our approach to decomposing the prepayment rate into its turnover and rate response prepayment components more intuitive, we introduce the following notation:

$$\text{CPR} = 1 - \exp(-x_t - y_t \max(0, m - a - b r_t(10))), \quad (\text{A32})$$

$$\text{CPR}_x = 1 - \exp(-x_t), \quad (\text{A33})$$

$$\text{CPR}_y = 1 - \exp(-y_t \max(0, m - a - b r_t(10))), \quad (\text{A34})$$

where  $\text{CPR}_x$  and  $\text{CPR}_y$  denote the CPR values resulting from setting  $y_t$  and  $x_t$ , respectively, equal to zero. From these expressions, it directly follows that,

$$1 - \text{CPR} = (1 - \text{CPR}_x) (1 - \text{CPR}_y), \quad (\text{A35})$$

which implies

$$\text{CPR} = \text{CPR}_x + \text{CPR}_y - \text{CPR}_x \text{CPR}_y. \quad (\text{A36})$$

The cross-product term in the above expression is typically very small. To decompose the CPR into turnover and rate response components, we simply dis-

tribute the cross-product term based on the values of  $CPR_x$  and  $CPR_y$ . Thus, the turnover prepayment rate is

$$CPR_x - \frac{CPR_x}{CPR_x + CPR_y} CPR_x CPR_y. \quad (A37)$$

Similarly, the rate response prepayment rate is

$$CPR_y - \frac{CPR_y}{CPR_x + CPR_y} CPR_x CPR_y. \quad (A38)$$

### A11. Burnout and Seasoning

In this section, we discuss how the implied prepayment model can be extended to allow for burnout and seasoning effects. Burnout refers to the fact that the longer a pool of mortgages is exposed to refinancing incentives, the less responsive the pool is to subsequent refinancing incentives. Burnout can be modeled as a cap  $h$  on the prepayment incentive so that

$$p_t = x_t + y_t \min(\max(0, m - a - b r_t(10)), h). \quad (A39)$$

Seasoning refers to the increase in prepayment speeds with the age of the pool. Mortgage pools can season with respect to both refinancing and turnover. Typically, seasoning is modeled as a ramp up to a steady-state level. For example, the Public Securities Association (PSA) standard prepayment model assumes a 30-month ramp-up period. In our model, we can incorporate burnout, turnover seasoning, and refinancing seasoning with the specification

$$p_t = \hat{x}_t + \hat{y}_t \min(\max(0, m - a - b r_t(10)), h), \quad (A40)$$

where

$$\hat{x}_t = \min\left(\frac{\text{WALA}_t}{30}, 1\right) x_t, \quad (A41)$$

$$\hat{y}_t = \min\left(\frac{\text{WALA}_t}{30}, 1\right) y_t, \quad (A42)$$

and  $\text{WALA}_t$  is the weighted average loan age in months for the mortgage pool at time  $t$ .

## A12. Additional Robustness Results

As discussed earlier, our results are based on what is perhaps the simplest possible specification of the implied prepayment function. As a robustness check, we examine how the results are affected when the implied prepayment function is modified to include seasoning and burnout effects. Recall that seasoning and burnout appear to be important features of empirical prepayment rates. Thus, it is possible that incorporating these features into the implied prepayment function may improve the ability of the model to explain mortgage-backed security prices. To capture seasoning effects, we use the standard prepayment model convention of the Public Securities Association (PSA) that prepayment rates increase or ramp up linearly over the first 30 months of the life of the mortgage loans underlying the mortgage-backed securities. Specifically, we replace the values of  $x_t$  and  $y_t$  in the implied prepayment function with the terms  $\min(1, \text{WALA}/30)x_t$  and  $\min(1, \text{WALA}/30)y_t$ , where WALA denotes the weighted average life of the loans in months. To capture burnout effects, we replace the maximum operator in the implied prepayment specification in Equation (8) with the expression  $\min(\max(0, m - a - b r_t(10)), 0.024)$ . This specification implies that the refinancing incentive increases linearly from zero to 240 bps points in the money, but then becomes constant for mortgage-backed securities that are more than 240 bps in the money. Thus, this nonlinear specification implies that deep-in-the-money mortgage-backed securities exhibit burnout behavior and do not prepay at higher rates. The burnout threshold of 240 bps is determined by solving for the value that best matches the 3-month empirical prepayment rates for the mortgage-backed securities in the sample. We acknowledge, however, that since burnout

is a function of the entire history of a mortgage-backed security, our approach of conditioning on the current refinancing incentive—necessitated by the limited cross-section of TBAs available in the sample—will likely not fully capture potential burnout effects.

We reestimate the model using this extended implied prepayment function. The model actually does worse when seasoning and burnout are incorporated into the implied prepayment function. Specifically, the median RMSE for the model increases from 25.7 cents to 26.5 cents when seasoning and burnout are included. The values of  $x_t$  and  $y_t$  we obtain using this implied prepayment specification are similar to those reported earlier, although slightly more volatile. In particular, the average values of  $x_t$  and  $y_t$  from this specification are 0.0972 and 11.334, respectively, which are close to those in Table 4.

As another robustness check, we also reestimate the model with the Hull and White (1990) model fitted to the swap curve rather than the Treasury curve. In doing this, we also recalibrate the Hull and White model to match the same set of swaption volatilities using the same procedure as before.

The results from this exercise are very similar to those reported previously. The model fits the data slightly worse when the swap curve is used to discount cash flows—the median RMSE increases from 25.7 cents to 26.9 cents. The estimated values of  $x_t$  and  $y_t$  are virtually identical to those obtained previously using the Treasury curve to discount cash flows. Not surprisingly, the only discernible effect of using the swap curve is that the estimates of the credit/liquidity factor  $w_t$  are lower by the average swap spread during the sample period. In particular, the average value of  $w_t$  decreases from 65.5 bps to 24.4 bps.

To evaluate the effect on the empirical results, we reestimate the regression in Table 5 using the values of  $w_t$  obtained when the swap curve is used. The regression results are very similar to those before. In particular, the FNMA spread and settlement fails variables continue to be significant with the same sign and similar regression coefficients as before.

To evaluate the sensitivity of the results to the filters we used in creating the data set, we reestimated the model using only the five TBAs with coupons closest to the current coupon mortgage rate. The liquidity of TBAs is generally lower for the mortgage-backed securities with coupons that are farthest from the current coupon mortgage rate. Thus, this approach eliminates some of less liquid TBAs from the estimation since their prices are more likely to be measured with error. Despite the reduction in the number of TBAs used in the estimation, however, the estimates of the credit/liquidity spread, the turnover factor, and the rate response factor are very similar to those we obtain using the entire data set.

As a final robustness check on the model specification, we also regressed the pricing errors from the individual mortgage-backed securities on their price, price squared, WAC, and WALA. Although not shown, the results imply that there is little apparent relation between the pricing errors and these measures. The exception is the WALA of the TBA, which is significantly positively related to the pricing errors. This suggests that one possible direction for extending the simple implied prepayment specification used in this paper might be to incorporate the age of the loan into the model. Intuitively, this would parallel the seasoning and burnout features often incorporated into empirical prepayment models.

**Table A1**  
**Data definitions and sources**

Data	Frequency	Description and source
FNMA MBS Prices	Monthly	Proprietary data set provided by a major Wall Street MBS dealer. Data cross-validated with Bloomberg data.
FNMA CPR Data	Monthly	One-month and three-month CPR prepayment rate data collected and provided by eMBS Inc.
Treasury CMT Data	Monthly	Constant maturity Treasury rates from Federal Reserve H.15 Selected Interest Rates Release.
Discount Function D(T)	Monthly	Discount function out to 30 years bootstrapped from Treasury CMT rates using standard cubic spline interpolation algorithm as described in Longstaff, Mithal, and Neis (2005).
Interest Rate Volatility	Monthly	Basis point volatility for 1-, 2-, 3-, 4-, and 5-year into 5-, 7-, and 10-year swaptions. Proprietary data set provided by major Wall Street MBS dealer.
FNMA Agency Credit Spread	Monthly	Ten-year FNMA cash flow spread (Z spread) to the Treasury curve. Proprietary data set provided by a major Wall Street MBS dealer.
Primary Dealers' MBS Holdings	Weekly	Federal Reserve Bank of New York: <a href="http://www.newyorkfed.org/markets/gsds/search.html">http://www.newyorkfed.org/markets/gsds/search.html</a> .
Net MBS Issuance	Monthly	Net MBS issuance in \$ millions provided by eMBS Inc.
Federal Reserve MBS Purchases	Weekly	Board of Governors of the Federal Reserve System, Mortgage-Backed Securities Held by the Federal Reserve: All Maturities [MBST], retrieved from FRED, Federal Reserve Bank of St Louis, <a href="https://research.stlouisfed.org/fred2/series/MBST">https://research.stlouisfed.org/fred2/series/MBST</a> . Weekly data aggregated to monthly and quarterly frequency.
Consumption Expenditures	Monthly	Seasonally adjusted at annual rates. US Bureau of Economic Analysis, Personal Consumption Expenditures [PCE], retrieved from FRED, Federal Reserve Bank of St. Louis, <a href="https://research.stlouisfed.org/fred2/series/PCE">https://research.stlouisfed.org/fred2/series/PCE</a> .
Unemployment Rate	Monthly	Seasonally adjusted unemployment rate provided by Bureau of Labor Statistics, <a href="http://data.bls.gov/timeseries/LNS14000000">http://data.bls.gov/timeseries/LNS14000000</a> .
Consumer Confidence Index	Monthly	The Conference Board. Provided by Bloomberg (CONCCONF Index).
Delinquency Rate	Quarterly	Mortgage Bankers Association National Delinquency Survey, provided by Bloomberg (DLQTDLQT Index).
Foreclosure Rate	Quarterly	Mortgage Bankers Association National Delinquency Survey, provided by Bloomberg (DLQTFORE Index).

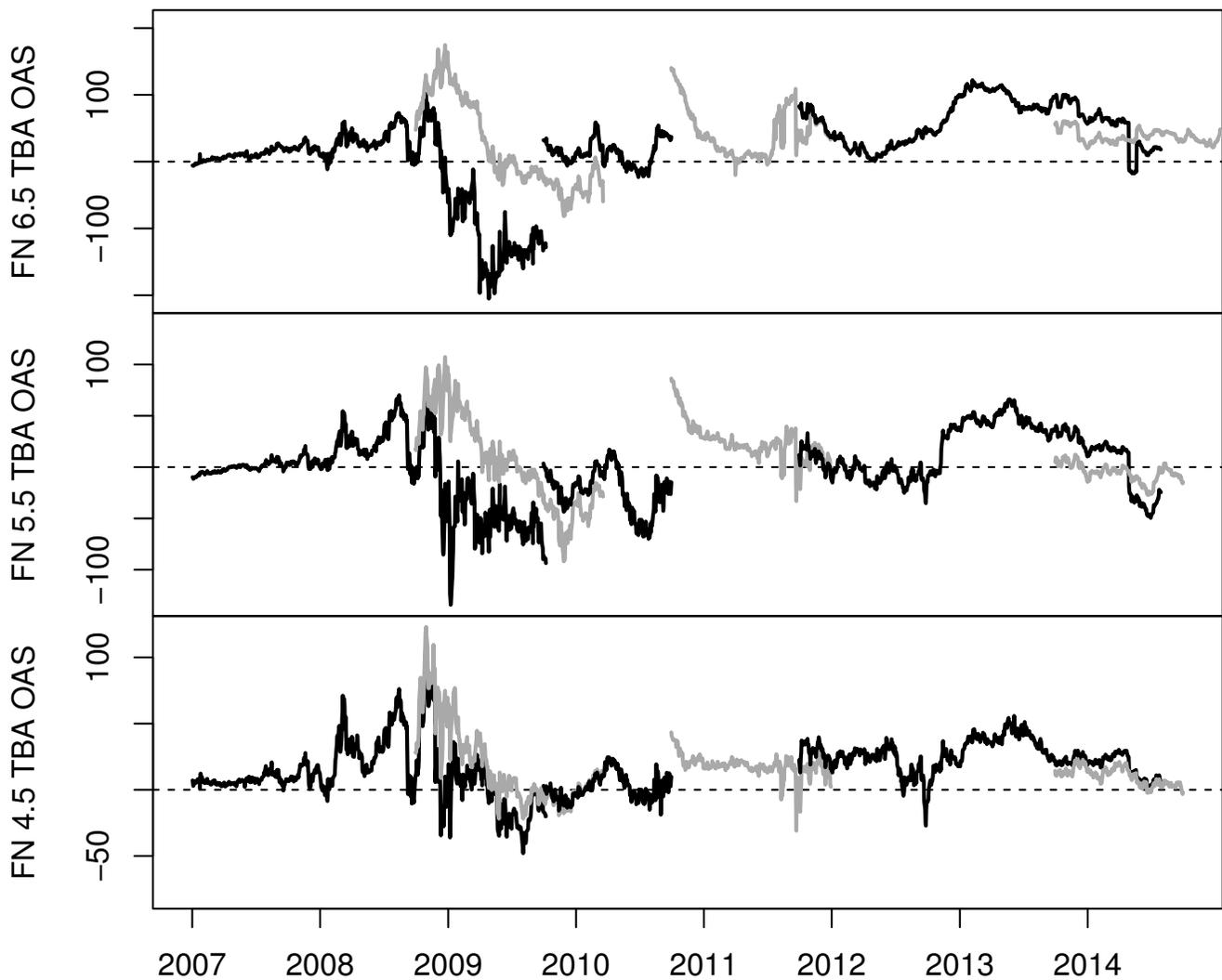
**Table A1 Continued**

Data	Frequency	Description and source
Credit Availability Index	Quarterly	Housing Finance Policy Center Index. Indicates the difficulty of getting a mortgage in the United States. The index calculates the percentage of owner-occupied purchase loans that are likely to default. <a href="http://www.urban.org/policy-centers/housing-finance-policy-center/projects/housing-credit-availability-index">http://www.urban.org/policy-centers/housing-finance-policy-center/projects/housing-credit-availability-index</a> .
Primary Dealers' MBS Fails	Weekly	Total MBS settlement fails with primary dealers, retrieved from Federal Reserve Bank, <a href="https://www.newyorkfed.org/markets/gsds/search.html">https://www.newyorkfed.org/markets/gsds/search.html</a> .
BBB Credit Spreads	Monthly	Five-year BBB fitted par spread to Treasuries assuming 40% recovery. Proprietary data provided by a major Wall Street MBS dealer.
2- to 10-Year Treasury Slope	Monthly	Federal Reserve Bank of St. Louis, 10-year Treasury constant maturity minus 2-year constant maturity [T10Y2Y], retrieved from FRED, <a href="https://fred.stlouisfed.org/series/T10Y2Y">https://fred.stlouisfed.org/series/T10Y2Y</a> .
LTV of New FNMA MBS	Monthly	Balance-weighted average LTV of Fannie Mae fixed-rate 30-year MBS by origination month, Fannie Mae single-family loan performance data, <a href="http://www.fanniemae.com/portal/funding-the-market/portal/loan-performance-data.html">http://www.fanniemae.com/portal/funding-the-market/portal/loan-performance-data.html</a> .
VIX Volatility Index	Monthly	Chicago Board Options Exchange Volatility Index (VIX) provided by Bloomberg (VIX Index).
FNMA MBS OAS	Daily	Fannie Mae MBS option-adjusted spreads from various prepayment models. Data provided by a major Wall Street MBS dealer.

**Table A2****Cash flow time line for a hypothetical 30-Year FNMA TBA trade**

Date	Event	Time	Note
March 31	Trade date	0	Trade parameters: issuer, maturity, coupon, face value, price, settlement date.
April 6	Factor date		Pool factors are released by FNMA.
April 12	48-hour day		The buyer is notified of the pools the seller will deliver to settle the TBA trade.
April 14	Settlement date	$t_s$	The buyer wires the payment to the seller.
April 30	Record date	$\tau_1$	Fedwire records the buyer as the new holder of record.
May 7	Factor date		Pool factors are released by FNMA, reflecting April prepayments.
May 25	Payment date	$t_1$	The buyer receives the first payment from the MBS.
⋮	⋮		⋮
May 31	Month end	$\tau_2$	Payment at $t_2$ reflects prepayments over $\tau_1$ to $\tau_2$ .
⋮	⋮		⋮
June 26	Payment date	$t_2$	The buyer receives the second payment from the MBS.
⋮	⋮		⋮
June 31	Month end	$\tau_3$	Payment at $t_3$ reflects prepayments over $\tau_2$ to $\tau_3$ .
⋮	⋮		⋮
July 25	Payment date	$t_3$	The buyer receives the third payment from the MBS.
⋮	⋮		⋮

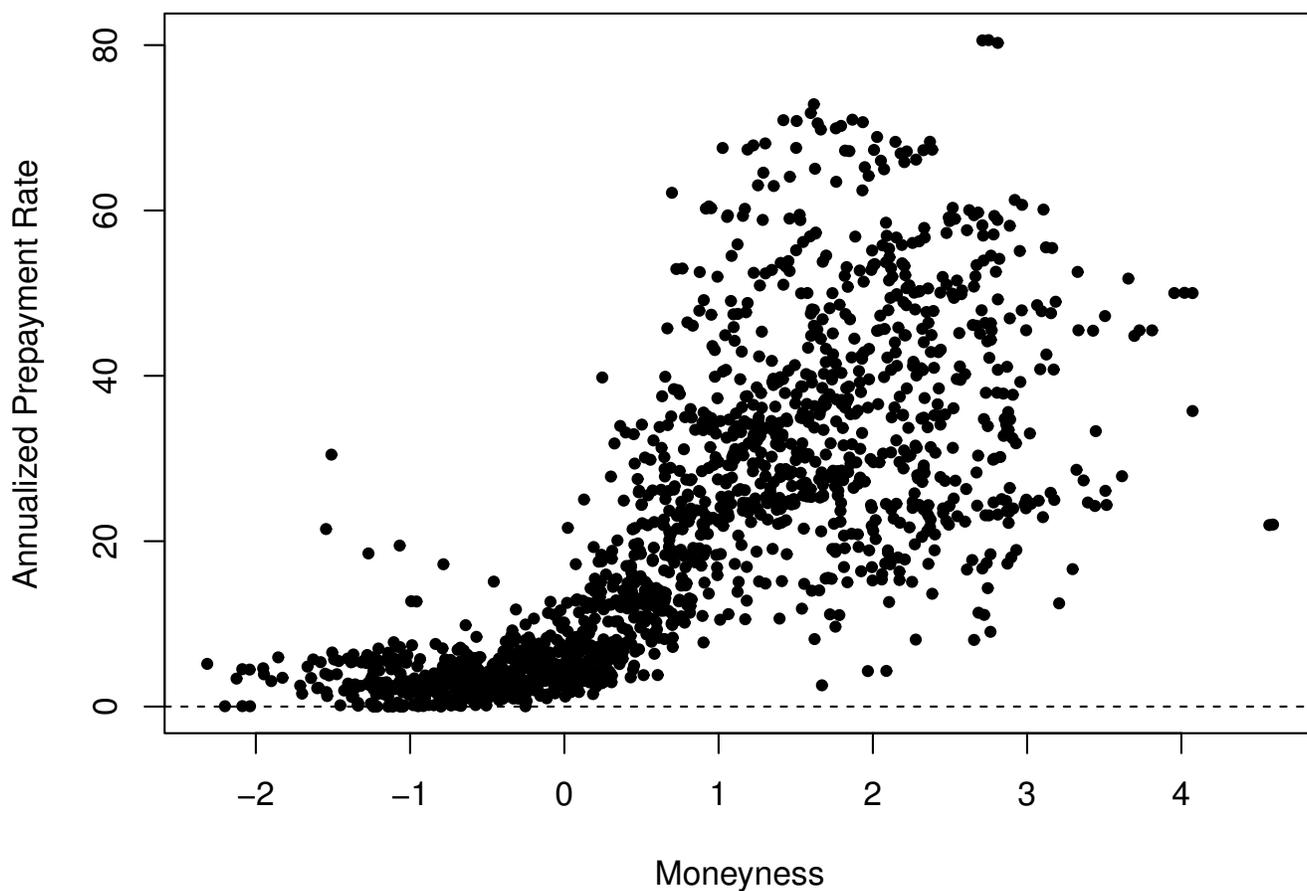
This table shows the key events and cash flows surrounding a 30-year FNMA TBA trade executed on March 31.



**Figure 1**

**Effects of prepayment model changes on option-adjusted spreads**

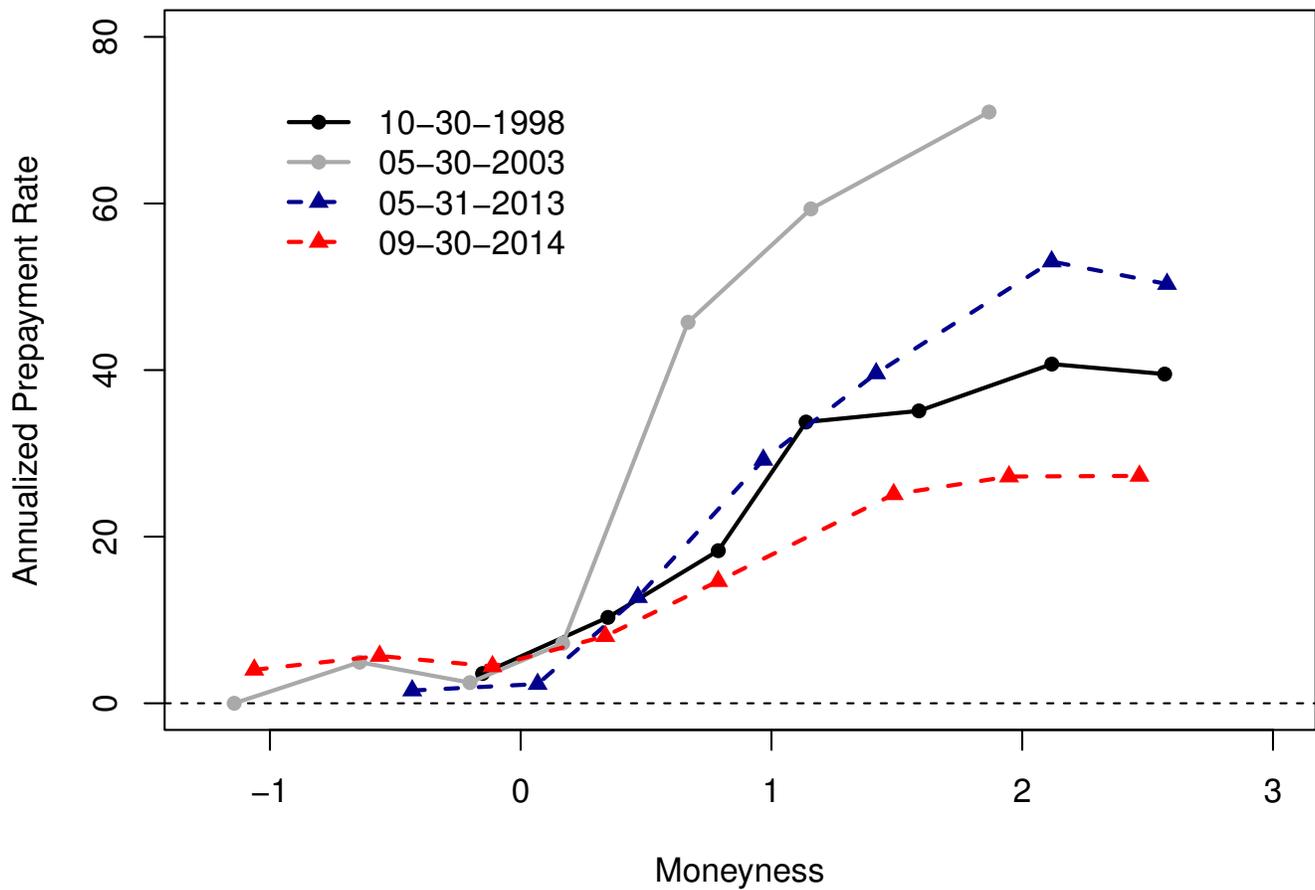
This figure shows the option-adjusted spread (OAS) in basis points for FNMA 4.50%, 5.50%, and 6.50% mortgage-backed securities implied by the series of prepayment models used by a specific major Wall Street dealer. Each line, alternating black and gray, represents a different version of the dealer's prepayment model. During the time period illustrated, the dealer used six different versions of its prepayment model. The option-adjusted spread is highly model dependent, and updates to the prepayment model can lead to large differences in the option-adjusted spread.



**Figure 2**

**Prepayment rates for FNMA mortgage-backed securities**

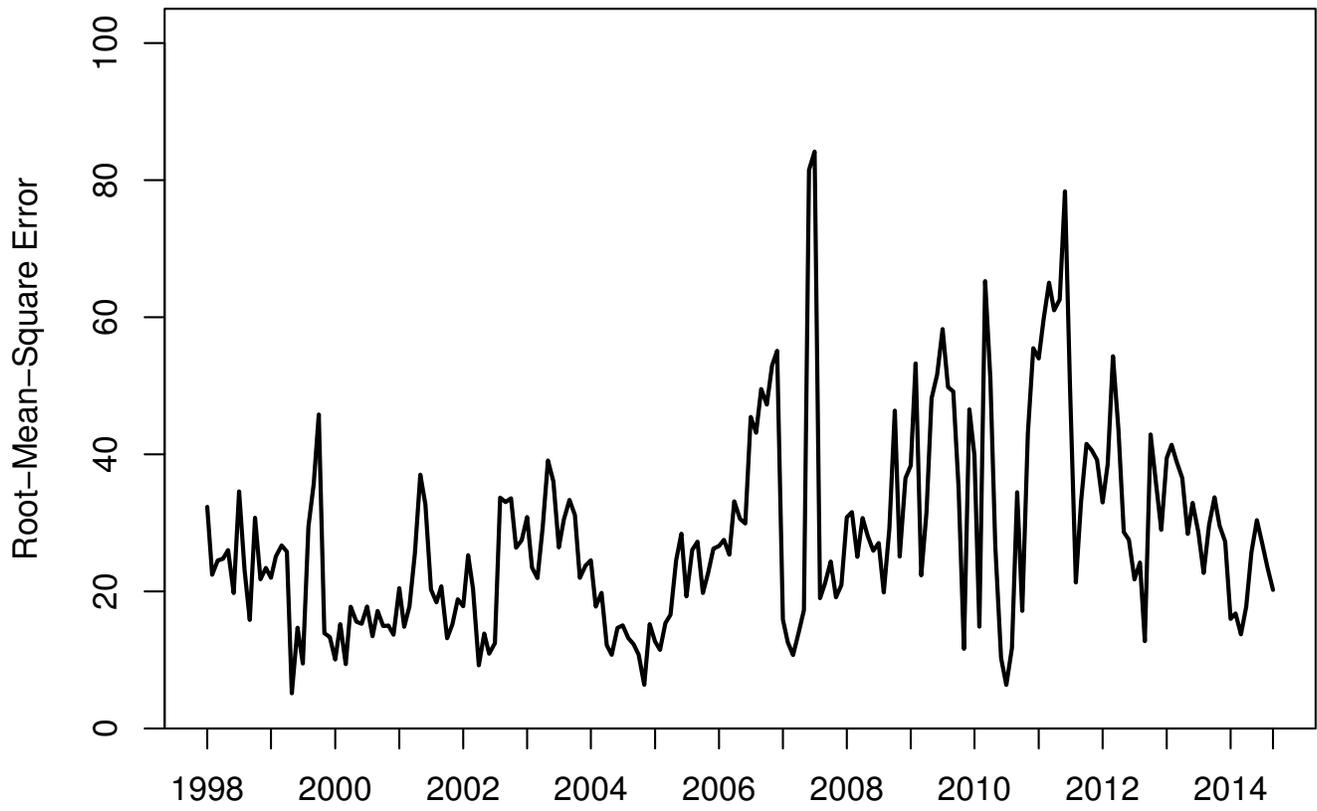
This figure plots the 3-month prepayment rates for FNMA mortgage-backed securities against the moneyiness of the mortgage-backed securities. Moneyiness is expressed in percentage points. The prepayment rates are expressed as annualized percentages of the outstanding principal balance of the mortgage-backed security. The data consist of monthly observations for all liquid coupons over the January 1998 to September 2014 sample period.



**Figure 3**

**Prepayment rates for FNMA mortgage-backed securities for selected dates**

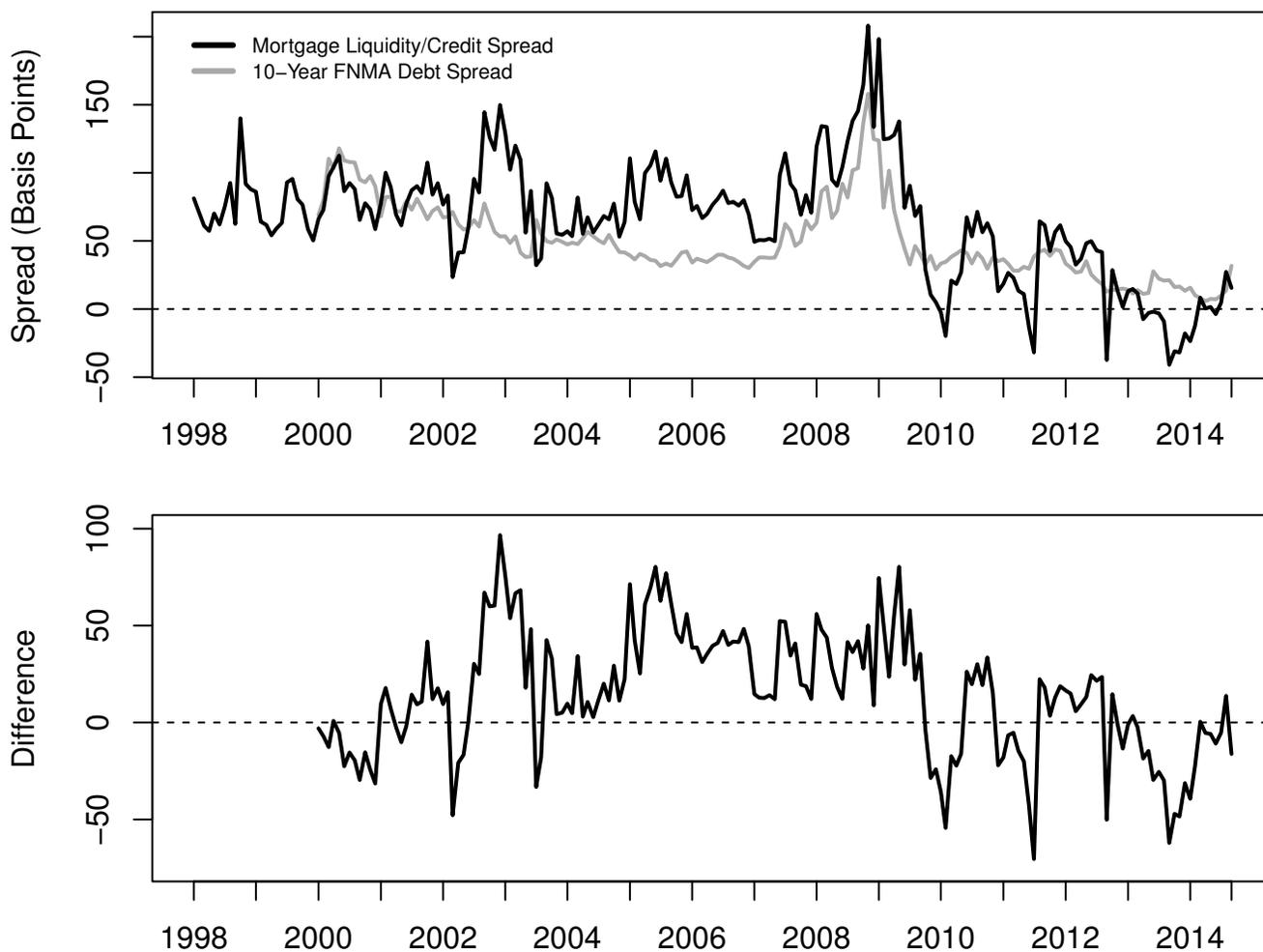
This figure plots the 3-month prepayment rates for FNMA mortgage-backed securities against the moneyness of the mortgage-backed securities for the indicated dates. Moneyness is expressed in percentage points. The prepayment rates are expressed as annualized percentages of the outstanding principal balance of the mortgage-backed security.



**Figure 4**

**Root-mean-square errors from fitting the model**

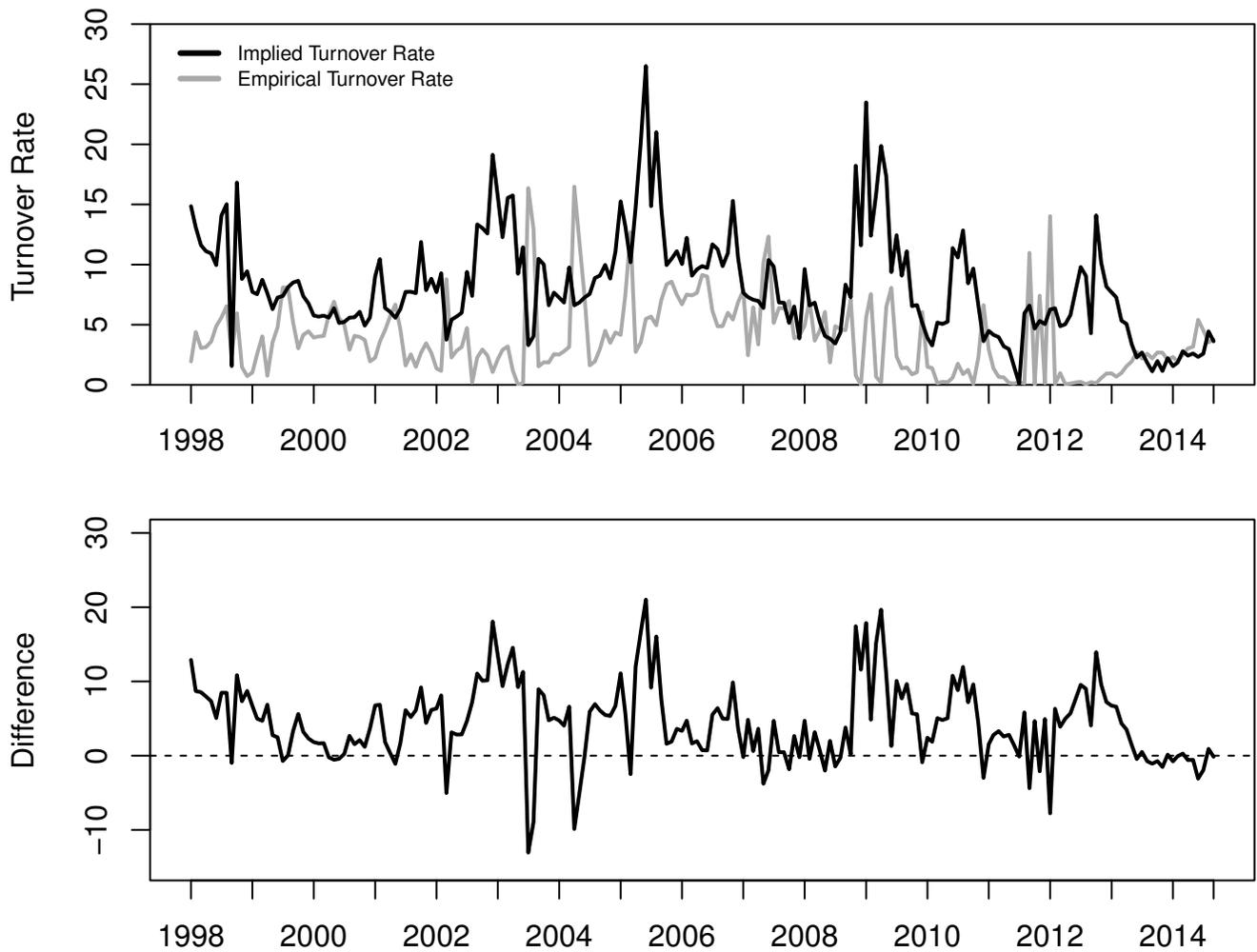
This figure plots the time series of root-mean-square errors from fitting the model to the cross-section of mortgage-backed security prices. The root-mean-square error is expressed as cents per \$100 notional position.



**Figure 5**

**Implied credit/liquidity spread, the credit spread for FNMA agency debt, and the liquidity spread**

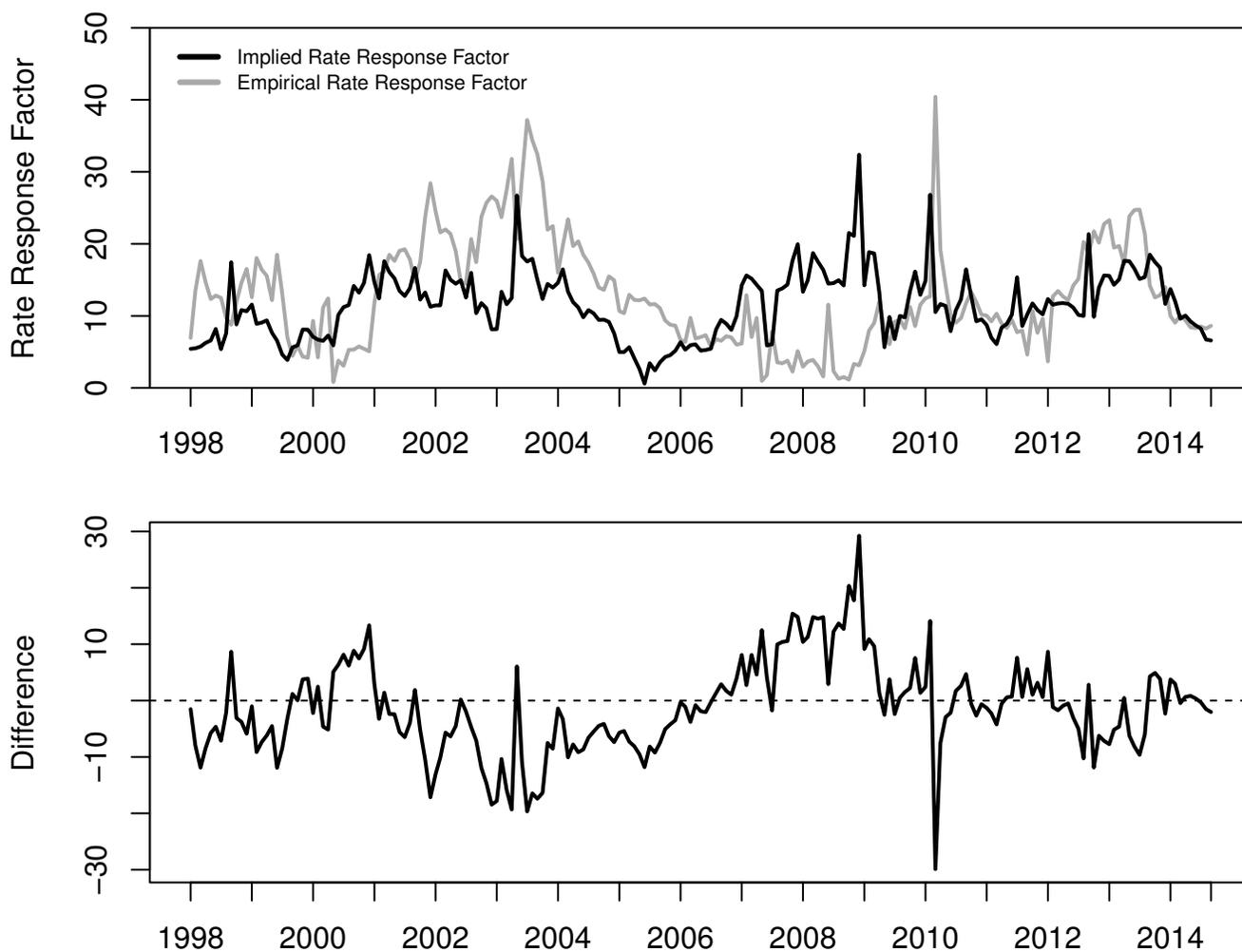
The upper panel plots the time series of the implied credit/liquidity spread as well as the credit spread for 10-year FNMA agency debt over the 10-year Treasury rate. The lower panel plots the difference between the spreads, which is designated the liquidity spread. The spreads are expressed in bps.



**Figure 6**

**Implied and empirical turnover rates**

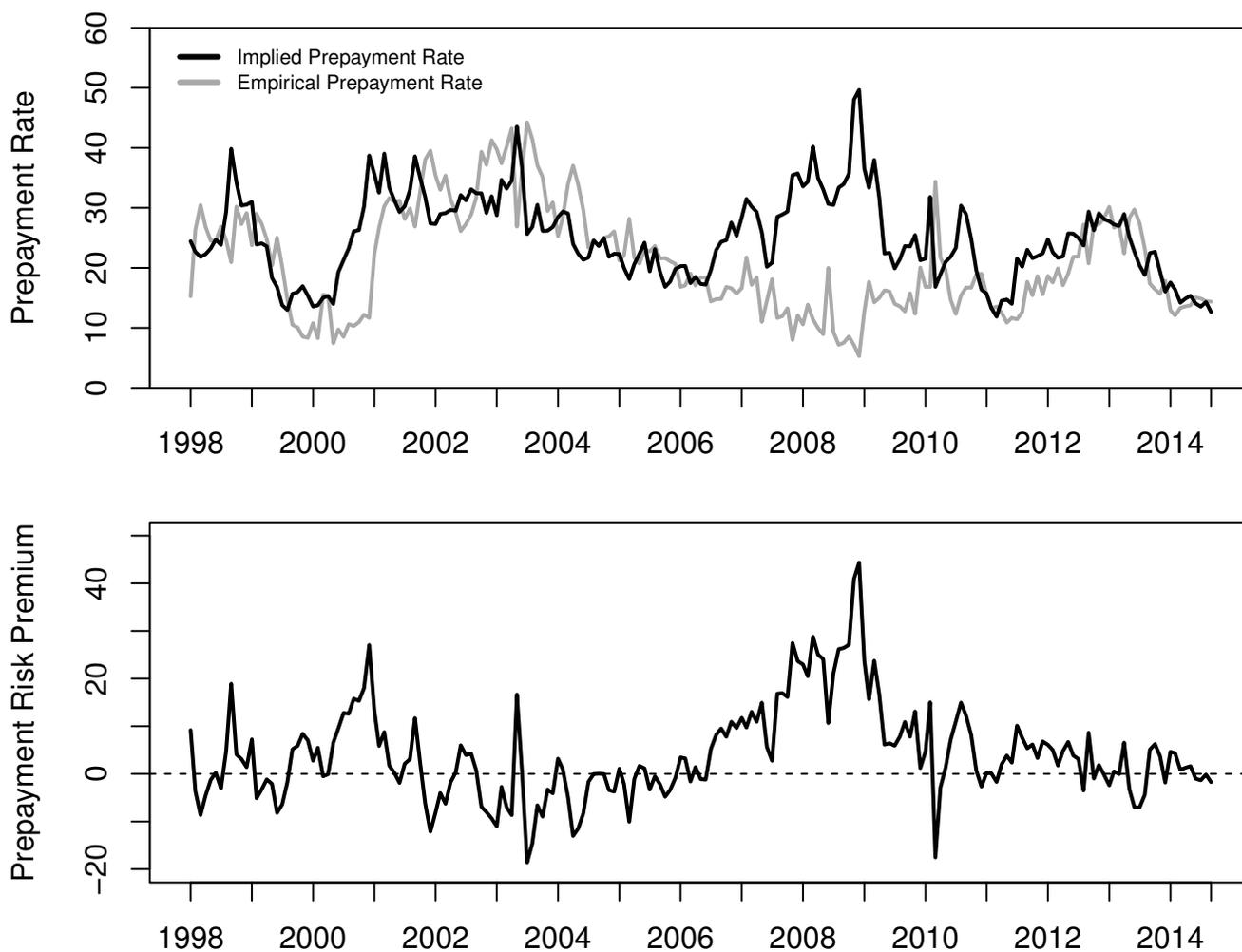
The upper panel plots the time series of the implied and empirical turnover rates. The lower panel plots the difference between the implied and empirical turnover rates. The turnover rates are expressed as annualized percentages of the outstanding principal balance of the mortgage-backed security.



**Figure 7**

**Implied and empirical rate response factors**

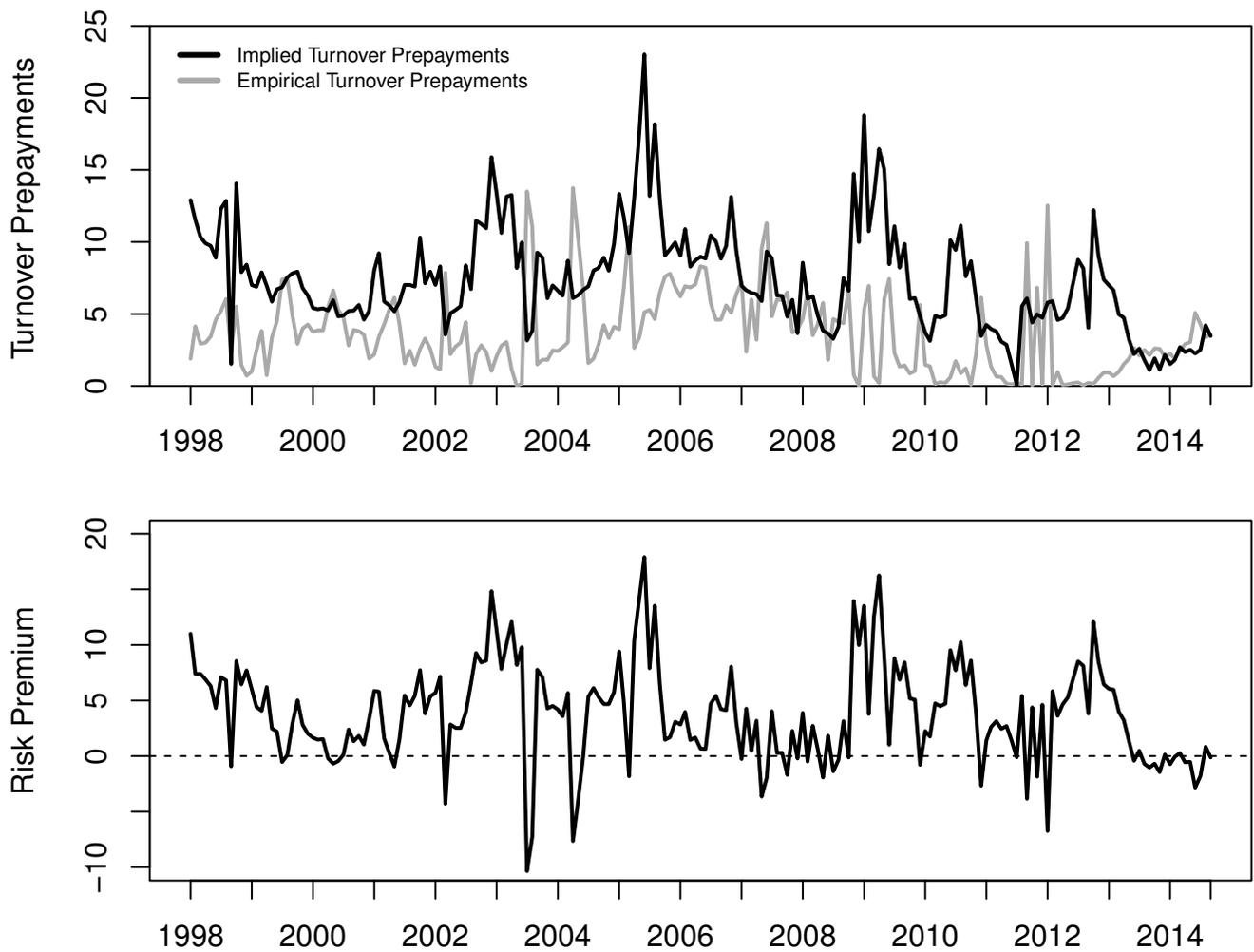
The upper panel plots the time series of the implied and empirical rate response factors. The lower panel plots the difference between the implied and empirical rate response factors. The rate response factors are multipliers measuring the sensitivity of the prepayment hazard rate to the refinancing incentive.



**Figure 8**

**Implied and empirical prepayment rates and the prepayment risk premium**

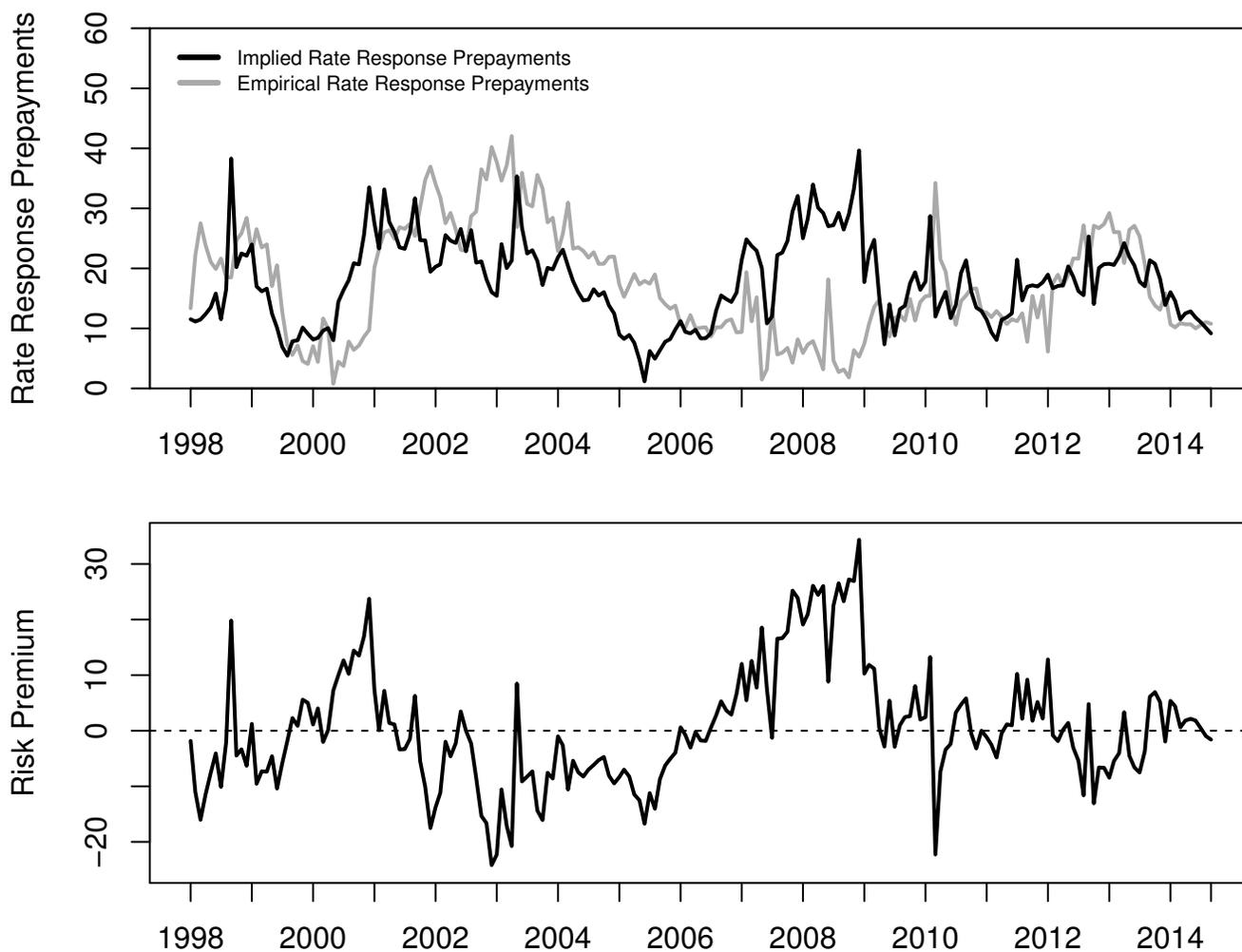
The upper panel plots the time series of the implied and empirical prepayment rates (both averaged across all coupon rates for each month). The lower panel plots the time series of the prepayment risk premium defined as the difference between the implied and empirical prepayment rates. The prepayment rates and the risk premium are expressed as annualized percentages of the outstanding principal balance of the mortgage-backed security.



**Figure 9**

**Implied and empirical turnover prepayment rates and the turnover risk premium**

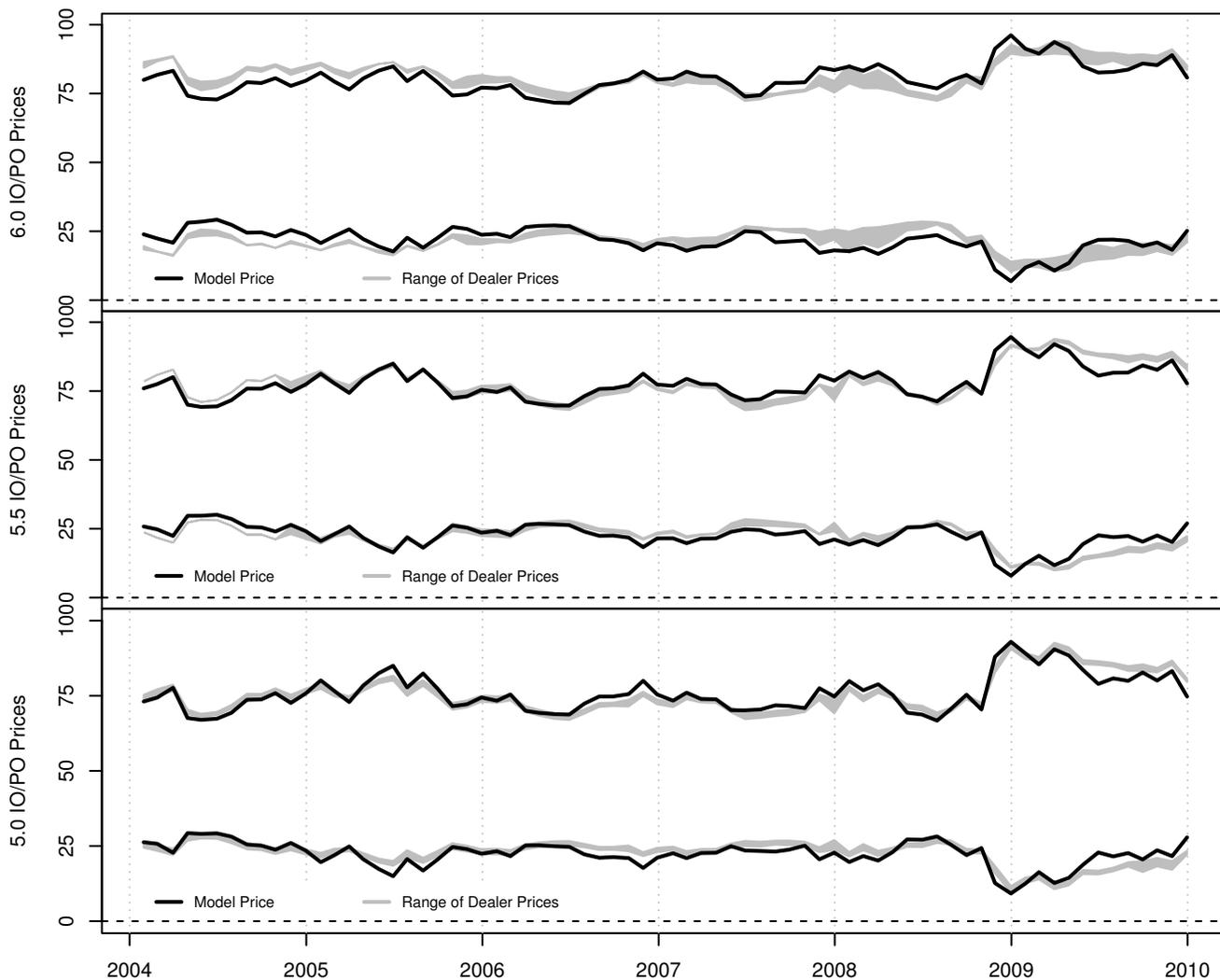
The upper panel plots the time series of the implied and empirical turnover prepayment rates. The lower panel plots the time series of the turnover risk premium defined as the difference between the implied and empirical turnover prepayment rates. The prepayment rates and the risk premium are expressed as annualized percentages of the outstanding principal balance of the mortgage-backed security.



**Figure 10**

**Implied and empirical rate response prepayment rates and the rate response risk premium**

The upper panel plots the time series of the implied and empirical rate response prepayment rates. The lower panel plots the time series of the rate response risk premium defined as the difference between the implied and empirical rate response prepayment rates. The prepayment rates and the risk premium are expressed as annualized percentages of the outstanding principal balance of the mortgage-backed security.



**Figure 11**

**Model prices and dealer marks for IO/PO strips**

This figure plots the IO and PO prices from the fitted model and compares them to dealer end-of-day marks for IO/PO strips. The range of dealer marks for the IO/PO strips is shown by the gray-shaded areas.

**Table 1**  
**Major events in the agency mortgage-backed securities market**

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2002	Sep–Dec	High levels of refinancing activity after Federal Reserve lowers interest rates.
2003	Jan–Jun	Refinancing activity continues and reaches historically high levels.
2005	Jan–Jun	Mortgage delinquency rates reach historically low levels.
2007	Jun–Jul	Two Bear Stearns MBS funds suffer large losses and are liquidated. S&P places 612 subprime CDOs on creditwatch.
2008	Mar	Financially distressed Bear Stearns avoids bankruptcy by being acquired by JP Morgan.
	Jul	Federal Reserve Bank of New York is authorized to lend to FNMA and FHLMC if need arises.
	Sep	FNMA and FHLMC are placed into conservatorship, Lehman Brothers defaults.
	Nov	Federal Reserve announces QE I program to purchase up to \$500 billion of agency MBS.
2009	Mar	Home Affordable Refinance Program and Stability Plan announced, making refinancing easier for high LTV loans.
	Mar	Federal Reserve expands QE I program to purchase up to an additional \$750 billion of agency MBS.
	Dec	Treasury lifts all caps on the amount of FNMA and FHLMC preferred stock it may hold.
2010	Mar	QE I purchases of agency MBS ends.
	Aug	FOMC agrees to keep Fed holdings of securities at constant levels by reinvesting cash flows in Treasuries.
	Nov	Federal Reserve announces QE II program to purchase up to \$600 billion of Treasuries.
2011	Jun	QE II purchases of Treasuries ends.
	Sep	Maturity Extension Program “Operation Twist” announced. Agency MBS cash flows to be reinvested in agency MBS.
2012	Sep	Federal Reserve announces QE III program, an open-ended program to purchase up to \$40 billion of agency MBS per month.
2013	Jun	Ben Bernanke announces “tapering” of QE programs, Dow drops 659 points.
2014	Oct	QE III purchases of agency MBS and Treasuries ends.

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Sources: <https://www.stlouisfed.org/financial-crisis/full-timeline>, <https://research.stlouisfed.org/publications/review/13/01/Fawley.pdf>, <http://www.federalreserve.gov/releases/chargeoff/delallsa.htm>.

**Table 2**  
**Summary statistics for FNMA mortgage-backed securities**

Coupon	Average moneyness	Average CPR	Minimum price	Average price	Maximum price	<i>N</i>
2.50	-0.416	4.794	90.566	96.557	103.219	37
3.00	-0.146	2.449	89.258	98.382	105.555	49
3.50	0.057	6.637	92.250	100.048	107.250	70
4.00	0.470	9.355	87.688	102.255	107.758	74
4.50	0.055	10.244	90.609	99.804	108.313	137
5.00	0.493	15.543	93.484	101.862	111.047	145
5.50	0.345	14.703	86.500	100.816	111.969	184
6.00	0.656	18.082	89.813	102.026	113.031	185
6.50	0.830	21.787	92.531	102.496	113.219	160
7.00	1.208	29.586	94.875	103.632	113.906	150
7.50	1.499	33.953	97.094	103.975	109.563	132
8.00	1.881	34.790	99.188	104.705	108.250	120
8.50	2.104	35.830	101.031	104.974	108.688	91
9.00	2.231	41.759	102.500	105.229	107.563	58
9.50	2.374	22.478	103.000	105.478	107.188	35

This table reports summary statistics for FNMA mortgage-backed securities with the indicated coupon rates. Average moneyness denotes the average difference between the coupon rate and the current coupon mortgage rate. Average CPR denotes the average 3-month conditional prepayment rate. *N* denotes the number of observations. The sample consists of monthly observations for the period from January 1998 to September 2014.

**Table 3**  
**Estimates of model parameters**

Parameter	Value	SE
$a$	0.01025	0.00142
$b$	0.86567	0.01485
$\alpha_w$	0.00006	0.00081
$\alpha_x$	0.00138	0.00098
$\alpha_y$	0.03885	0.04279
$\beta_w$	0.00834	0.01216
$\beta_x$	0.00978	0.01104
$\beta_y$	0.00234	0.01057
$\sigma_w$	0.00020	0.02982
$\sigma_x$	0.02281	0.00793
$\sigma_y$	0.08945	0.03720
$\rho_{r,x}$	-0.15430	0.14225
$\rho_{r,y}$	0.12657	0.21620
$\rho_{x,y}$	-0.04890	0.11873

This table reports the estimate of the model parameters along with their asymptotic standard errors.

**Table 4**  
**Summary statistics for the mortgage-backed security pricing factors**

Statistic	Spread	Turnover	Rate response
Mean	65.534	8.233	11.492
Minimum	-40.968	0.100	0.592
Median	68.327	7.543	11.462
Maximum	208.157	26.508	32.383
SD	43.090	4.365	4.751
Serial correlation	0.842	0.688	0.704
Number	201	201	201

This table reports summary statistics for the agency credit/liquidity spread (spread), the turnover rate (turnover), and the rate response factor (rate response). The factors are estimated from the cross-section of mortgage-backed security prices. Spread is expressed in basis points. Turnover is expressed as a percentage. Rate response is expressed as a multiplier for the refinancing incentive. The sample consists of monthly observations for the period from January 1998 to September 2014.

**Table 5**

**Results from the regression of monthly changes in the implied credit/liquidity spread on explanatory variables**

Variable	Coefficient	<i>t</i> -statistic
Intercept	5.2450	1.90
First lagged change in implied spread	-0.3667	-5.68*
Second lagged change in implied spread	-0.1434	-2.17*
Change in FNMA spread	0.7371	3.44*
Fed MBS purchases	-0.1163	-2.08*
MBS settlement fails	-0.0193	-2.26*
Lagged MBS settlement fails	0.0093	0.74
Net MBS issuance	-0.2779	-1.93
Change in dealer inventories	-0.0176	-0.13
Adj. $R^2$		0.2052
$N$		176

This table reports the results from the regression of the monthly change in the implied credit/liquidity spread (measured in basis points) on its first two lagged values, the change in the FNMA credit spread (measured in basis points), Federal Reserve purchases of mortgage-backed securities (in \$ billions), contemporaneous and lagged primary dealers' mortgage-backed security fails (in \$ billions), net mortgage-backed security issuance (in \$ billions), and the change in primary dealers' holdings of mortgage-backed securities (measured in \$ billions). All changes are monthly. The *t*-statistics are based on the Newey-West (1987) estimator of the covariance matrix (with four lags). \* denotes significance at the 5% level. The sample period is January 2000 to September 2014.

Table 6

Results from the regression of quarterly changes in the empirical and implied turnover rates on explanatory variables

Variable	Actual turnover rate		Implied turnover rate	
	Coefficient	<i>t</i> -statistic	Coefficient	<i>t</i> -statistic
Intercept	-0.8518	-2.19*	1.5006	2.25*
Lagged empirical turnover	-0.5179	-9.60*	0.1198	0.67
Lagged implied turnover	0.5779	0.07	-0.2888	-2.80*
Lagged change in consumption	0.8946	3.03*	-1.3014	-2.76*
Lagged change in consumer confidence	-0.0270	-1.24	-0.0353	-1.26
Lagged change in unemployment	4.0239	3.16*	-1.7677	-1.63
Change in credit spread	-0.0097	-3.02*	0.0070	3.21*
Change in term structure slope	-0.0218	-1.84	-0.0182	-1.49
Change in VIX	0.0721	0.87	0.1159	2.54*
Lagged delinquencies	1.9908	1.45	0.3445	0.30
Lagged foreclosures	-8.4475	-4.09*	1.2311	0.50
Adj. $R^2$		0.390		0.146
$N$		67		67

This table reports the results from regressions of the quarterly change in the turnover rate on the lagged change in the empirical turnover rate, the lagged change in the implied turnover rate, the lagged percentage change in personal consumption expenditures, the lagged change in The Conference Board Consumer Confidence Index, the lagged change in the unemployment rate, the change in 5-year BBB credit spreads over Treasuries (measured in basis points), the change in the Treasury 2- to 10-year slope (measured in basis points), the change in the VIX index, the doubly lagged change in the mortgage delinquency rate, and the lagged change in the foreclosure rate. The center panel presents the results for the regression in which the change in the empirical turnover rate is the dependent variable. The right panel presents the results for the regression in which the change in the implied turnover rate is the dependent variable. All variables are measured quarterly. The *t*-statistics are based on the Newey-West (1987) estimator of the covariance matrix (with three lags). \* denotes significance at the 5% level. The sample period is January 1998 to September 2014.

Table 7

Results from the regression of quarterly changes in the empirical and implied rate response factors on explanatory variables

Variable	Actual rate response		Implied rate response	
	Coefficient	<i>t</i> -statistic	Coefficient	<i>t</i> -statistic
Intercept	0.6771	0.46	-1.1468	-1.35
Lagged empirical rate response	-0.3750	-6.89*	-0.0271	-0.33
Lagged implied rate response	0.2501	1.15	-0.2491	-3.57*
Lagged change in consumption	-0.8365	-0.61	1.0449	1.58
Lagged change in consumer confidence	-0.0634	-0.83	0.0071	0.13
Lagged change in unemployment	-0.6027	-0.19	1.6600	1.41
Change in credit spread	-0.0029	0.66	0.0328	8.71*
Change in term structure slope	0.0322	0.08	0.0060	0.46
Change in VIX	-0.1971	0.20	-0.2234	-2.16*
Lagged change in LTV	-0.6099	0.26	0.9435	1.99*
Lagged change in credit availability	0.6785	0.60	-0.2598	-0.21
Adj. $R^2$		0.234		0.340
$N$		59		59

This table reports the results from regressions of the quarterly change in the rate response factor on the lagged change in the empirical rate response factor, the lagged change in the implied rate response factor, the lagged percentage change in personal consumption expenditures, the lagged change in the Conference Board Consumer Confidence Index, the lagged change in unemployment, the change in 5-year BBB credit spreads over Treasuries (measured in basis points), the change in the Treasury 2- to 10-year slope (measured in basis points), the change in the VIX index, the lagged change in the loan-to-value ratio for new FNMA mortgages, and the lagged change in the credit availability index. The center panel presents the results for the regression in which the change in the empirical rate response factor is the dependent variable. The right panel presents the results for the regression in which the change in the implied rate response factor is the dependent variable. All variables are measured quarterly. The *t*-statistics are based on the Newey-West (1987) estimator of the covariance matrix (with three lags). \* denotes significance at the 5% level. The sample period is January 1998 to September 2014.

**Table 8****Summary statistics for prepayment rates and prepayment risk premiums**

	Average	SD	Minimum	Median	Maximum	<i>N</i>
Implied prepayment rate	25.130	7.124	11.865	24.335	49.651	201
Empirical prepayment rate	20.956	8.717	5.289	19.056	44.250	201
Prepayment risk premium	4.174	9.839	-18.617	2.358	44.362	201
Implied turnover prepayment rate	7.378	3.641	0.100	6.890	23.023	201
Empirical turnover prepayment rate	3.575	2.774	0.000	3.005	13.795	201
Turnover risk premium	3.803	4.422	-10.353	3.884	17.905	201
Implied rate response prepayment rate	17.752	7.150	1.173	17.014	39.659	201
Empirical rate response prepayment rate	17.381	9.168	0.779	15.483	42.057	201
Rate response risk premium	0.371	10.554	-24.185	-0.986	34.370	201

This table reports summary statistics for the implied and empirical prepayment rates, the implied and empirical turnover prepayment rates, the implied and empirical rate response prepayment rates, and the corresponding risk premiums (defined as the difference between the implied and empirical prepayment rates). All variables are expressed as percentages. The sample consists of monthly observations for the period from January 1998 to September 2014.

**Table 9**  
**Summary statistics for FNMA stripped mortgage-backed securities**

Trust number	Trust size(bn)	Characteristics of the mortgage loans in each trust					1Q2010 3m CPR	Max pay-up	Dealer RMSE	N
		Vintage	WAC	ALOS	% LTV	% CA				
5.0% pass-through coupon										
FNS 337	1.55	4/2003	5.64	155,558	71.46	24.70	14.97	0-152	0.52	72
FNS 340	2.24	6/2003	5.45	151,534	69.34	24.20	13.70	0-152	0.41	72
FNS 360	2.50	6/2005	5.69	158,768	70.15	19.10	15.73	0-100	0.31	54
FNS 377	3.78	12/2005	5.45	167,246	69.41	17.10	17.67	0-100	0.35	38
FNS 397	4.00	4/2009	5.49	198,319	76.02	22.30	14.10	0-000	2.86	4
5.5% pass-through coupon										
FNS 346	2.00	8/2003	5.98	158,320	70.54	29.20	16.43	0-164	0.38	72
FNS 354	2.90	9/2004	5.94	170,414	72.67	20.50	18.13	0-150	0.31	62
FNS 363	2.05	9/2005	5.93	180,474	71.59	16.30	15.57	0-110	0.37	48
FNS 379	4.45	2/2007	6.10	198,830	71.75	13.40	26.30	0-080	0.53	32
FNS 399	2.15	8/2008	5.99	187,733	73.49	20.40	24.83	0-004	—	2
6.0% pass-through coupon										
FNS 293	0.51	9/1993	6.70	93,785	95.00	14.80	12.53	1-035	—	72
FNS 344	2.20	3/2003	6.54	119,071	75.55	25.90	16.70	0-165	0.71	72
FNS 370	2.75	3/2006	6.43	165,217	72.51	13.90	23.03	0-125	0.48	43
FNS 372	3.00	5/2006	6.47	162,630	72.32	11.30	22.80	0-110	0.49	41

This table reports summary statistics for FNMA stripped mortgage-backed securities (SMBS). Each SMBS is identified by a trust number and has two classes: an IO class and a PO class. For the mortgage loans backing each SMBS, vintage denotes the weighted-average origination month, WAC denotes the weighted average coupon in percentage points, ALOS denotes the average loan size at origination in dollars, LTV denotes the loan-to-value ratio in percent, and % CA denotes the percentage backed by homes in California. 1Q210 3m CPR denotes the conditional prepayment rate for the 1st quarter of 2010. Max pay-up denotes the price difference between the IO price plus the PO price and the TBA price, where the price is expressed in points and ticks (32nds) (the last digit represents eighths of a tick). N denotes the number of observations. Dealer RMSE denotes the root-mean-square error between two dealer's end-of-day marks for IO. The sample consists of end-of-month observations for the period from January 2004 to December 2009.