Deflation Risk

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We study the nature of deflation risk by extracting the objective distribution of inflation from the market prices of inflation swaps and options. We find that the market expects inflation to average about 2.5% over the next 30 years. Despite this, the market places substantial weight on deflation scenarios in which prices significantly decline over extended horizons. The market prices the economic tail risk of deflation similarly to other types of tail risks, such as corporate default or catastrophic insurance losses. We find that deflation risk is strongly negatively correlated with outcomes in the financial markets and with consumer confidence. (*JEL* C22, C58, G12, E31, E44)

Deflation has played a central role in the worst economic meltdowns experienced in U.S. history. Key examples include the deflations associated with the Panic of 1837, the Long Depression of 1873–1896, and the Great Depression of the 1930s. In light of this, it is not surprising that deflation is now one of the most-feared risks facing participants in the financial markets. In recent years, the financial press has increasingly raised concerns about a global deflationary spiral by using terms like "nightmare scenario" or "looming disaster" to describe the growing threat.[1] Furthermore, addressing the risk of deflation was one of the primary motivations behind a number of actions, such

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[1] For example, see Hannon and Blackstone (2015), "Eurozone Faces Renewed Deflation Threat as Consumer Prices Fall," www.wsj.com/articles/eurozone-faces-renewed-deflation-threat-1443604122, Coy (2012), "Five

as unconventional monetary policy and quantitative easing programs, taken by the Federal Reserve in recent years.[2]

Despite the severe potential effects of deflation, relatively little is known about how large the risk of deflation actually is, or about the economic and financial factors that contribute to deflation risk. The primary reason for this simply may be that deflation risk traditionally has been difficult to measure. For example, as shown by Ang, Bekaert, and Wei (2007) and others, econometric models based on the time series of historical inflation perform poorly even in estimating the first moment of inflation. In addition, while surveys of inflation tend to do better, these surveys are limited to forecasts of expected inflation over shorter horizons and provide little or no information about the tail probability of deflation.

This paper presents a simple market-based approach for measuring deflation risk. This approach allows us to solve directly for the market's assessment of the probability of deflation for horizons of up to 30 years by using the prices of inflation swaps and options. In doing this, we use a standard maximum likelihood approach familiar from the affine term-structure literature. A key advantage of this approach is that we recover the entire distribution of inflation rather than just the first moment or expected inflation. This is important since this allows us to measure the probability of tail events, such as deflation.

We find that the inflation risk premium significantly varies over time and can be both positive and negative in value. On average, the inflation risk premium is on the order of 10 to 20 basis points. We also find that the market expects inflation of roughly 2.00% to 2.50% for horizons ranging from 5 to 30 years. Note that the horizons of up to 30 years for these market-implied forecasts far exceed those of existing survey-based forecasts.

We solve for the probability of deflation over horizons ranging up to 30 years directly from the distribution of inflation. The empirical results are very striking. We find that the market places a significant amount of weight on the probability that deflation occurs over extended horizons. Furthermore, the market-implied probability of deflation can be substantially higher than that estimated by policy makers. For example, in a speech on August 27, 2010, Federal Reserve Chairman Ben S. Bernanke stated that, "Falling into deflation is not a significant risk for the United States at this time."[3] On the same date, the market-implied probability of deflation was 28.28% for a two-year horizon, 20.61% for a three-year horizon, and 11.04% for a five-year horizon.

Charts that Show that Deflation is a Growing Threat," www.businessweek.com/articles/1012-06-05/five-charts-that-show-deflation-is-a-growing-threat, Lange (2011), "Nightmare Scenario: U.S. Deflation Risks Rising," www.reuters.com/article/2011/10/16/us-economy-deflation-idUSTRE79P7FV201110126, and Bennington (2010), "Deflation: Looming Disaster for Banks," www.cnbc.com/id/39648704/.

[2] For example, see Bernanke (2012), "Monetary Policy since the Onset of the Crisis," www.federalreserve.gov/newsevents/speech/bernanke20120831a.htm.

[3] See Bernanke (2010), "The Economic Outlook and Monetary Policy," www.federalreserve.gov/newsevents/speech/bernanke20100827a.htm.

These probabilities are clearly not negligible. On average, the market-implied probability of deflation during the sample period was 13.80% for a two-year horizon, 5.68% for a five-year horizon, and 1.39% for a ten-year horizon. The risk of deflation, however, significantly varies and these probabilities have at times been substantially larger than the averages. In particular, large increases in the probability of deflation often coincide with major events in the financial markets, such as the ratings downgrades of Spain in 2010 or the downgrade of U.S. Treasury debt by Standard and Poors in August 2011.

Deflation is clearly an economic tail risk, and changes in deflation risk may reflect the market's fears of a meltdown scenario.[4] Thus, a natural next step is to examine whether deflation risk is related to other serious types of tail risk in the financial markets or in the macroeconomy in general. Focusing first on the pricing of deflation risk, we find that the ratio of the risk-neutral probability of deflation to the actual probability of deflation ranges between roughly two for a five-year horizon, to seven or more for ten-year and longer horizons. These ratios of risk-neutral to objective probabilities are similar to those for other types of tail risk. For example, Froot (2001) finds that the ratio of the price of catastrophic reinsurance to expected losses is on the order of two to seven. Driessen (2005), Berndt et al. (2005), Giesecke et al. (2011) estimate that the ratio of the price of expected losses on corporate bonds to actual expected losses is also about two. These findings are also consistent with models with rare consumption disasters, such as that pioneered by Rietz (1988) and further developed by Longstaff and Piazzesi (2004), Barro (2006), and Gourio (2008), which were explicitly engineered to produce high risk-neutral probabilities for rare consumption disasters, such as the Great Depression. Recently, Barro argued that this class of models can account for the equity premium when calibrated to the 20th century experience of developed economies. Gabaix (2012) and Wachter (2013) have extended these models to incorporate a time-varying intensity of consumption disasters. This extension delivers bond and stock market return predictability similar to what is observed in the data.

Next, we consider the relation between deflation risk and specific types of financial and macroeconomic tail risks described in the literature. In particular, we consider a number of measures of systemic financial risk, collateral revaluation risk, sovereign default risk, and business-cycle risk and investigate whether these are linked to deflation risk. We find a strong negative relation between deflation risk and the stock market for all horizons. Similarly, we find a significant negative correlation between deflation risk and consumer confidence. These results are intuitive and support the view that the risk of macroeconomic shocks that lead to deflation is closely related to tail risks in financial markets. Thus, option prices are highly informative about the

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[4] Note that we are interpreting tail risk as including more than just event risk or jump risk. Event or jump risks are adverse economic events that occur relatively suddenly. In contrast, tail risk also can include extreme scenarios with severe economic consequences that may unfold over extended periods.

probability the market imputes to these rare disaster states, arguably more informative than quantity data (see, for example, recent work by Backus, Chernov, and Martin (2011) who use equity options). Our findings imply that market participants primarily expect deflation in the U.S. in these disaster states. This is consistent with U.S. historical experience in which depressions/deflationary spirals are associated with major collapses in the financial system.

Finally, we also compute the probabilities of inflation exceeding various thresholds. The results indicate that the market views the probability of inflation exceeding 4% over a five-year or ten-year horizon as a credible threat.

From a broader perspective, the approach and results of this paper have a number of other important economic implications. First, these results illustrate that the information contained in the market prices of derivatives, such as inflation swaps and options, provides a powerful lens for studying macroeconomic variables. In a literal sense, these types of markets may make it possible to study the effects of changes in fiscal and monetary policy in real time, potentially opening a window on high-frequency macroeconomic analysis conducted on intraday time scales.

Second, our results have implications for Treasury debt management. In particular, whenever the Treasury issues Treasury Inflation-Protected Securities (TIPS), the Treasury essentially writes an at-the-money deflation put and packages it together with a standard inflation-linked bond. The returns on writing these deflation puts are potentially large because of the substantial risk premium associated with deflation tail risk. If the Treasury is better suited to bear deflation tail risk than the marginal investor in the market for inflation protection, then providing a deflation put provides an extra source of revenue for the Treasury that is nondistortionary. There are good reasons to think that the Treasury is better equipped to bear deflation risk, not in the least because the Treasury and the Federal Reserve jointly control the price level.

This paper contributes to the extensive literature on estimating the properties of inflation. Important papers on estimating inflation risk premia and expected inflation include Hamilton (1985), Barr and Campbell (1997), Evans (1998, 2003), Campbell and Viceira (2001), Bardong and Lehnert (2004), Buraschi and Jiltsov (2005), Ang, Bekaert, and Wei (2007, 2008), Adrian and Wu (2009), Bekaert and Wang (2010), Chen, Liu, and Cheng (2010), Christensen, Lopez, and Rudebusch (2010, 2016), Gürkaynak, Sack, and Wright (2010), Pflueger and Viceira (2011a, 2011b), Gürkaynak and Wright (2012), Chernov and Mueller (2012), Haubrich, Pennacchi, and Ritchken (2012), Faust and Wright (2013), Grishchenko and Huang (2012), and many others. Key papers on deflation include Hamilton (1992), Fisher (1933), Cecchetti (1992), Atkeson and Kehoe (2004), Kilian and Manganelli (2007), and Campbell, Sunderam, and Viceira (2013).

An important recent paper by Kitsul and Wright (2013) parallels ours in several ways. In their paper, Kitsul and Wright use a standard econometric

model to estimate the actual density of inflation, but then use inflation options prices to solve for the risk-neutral density of inflation. While focusing primarily on the properties of risk-neutral probabilities, they also contrast the actual and risk-neutral densities and report summary statistics for their ratios. Our paper differs from theirs both in terms of its scope and its fundamental approach. In particular, Kitsul and Wright use an econometric model to estimate the actual density of inflation out to a horizon of ten years, but do not provide explicit results about inflation risk premia or expected inflation. In contrast, we estimate the actual density of inflation out to a horizon of 30 years along with corresponding inflation risk premia and expected inflation estimates. At a more fundamental level, Kitsul and Wright's estimates of the actual density are based entirely on historical data. In contrast, our estimates of the actual density are based on the forward-looking information embedded in the term structure and cross-section of inflation swaps and options. While both approaches have merit, the use of inflation derivatives prices to infer the actual distribution of inflation differentiates this paper from the previous literature, while complementing and extending the work of Kitsul and Wright.

1. Deflation in U.S. History

The literature on deflation in the United States is far too extensive for us to review in this paper. Key references on the history of deflation in the United States include North (1961), Friedman and Schwartz (1963), and Atack and Passell (1994). We simply observe that deflation was a relatively frequent event during the 19th century, but has diminished in frequency since then. Bordo and Filardo (2005) report that the frequency of an annual deflation rate was 42.4% from 1801 to 1879, 23.5% from 1880 to 1913, 30.6% from 1914 to 1949, 5.0% from 1950 to 1969, and 0% from 1970 to 2002. The financial crisis of 2008–2009 was accompanied by the first deflationary episode in the United States since 1955. Figure 1 presents a time-series plot of historical annual inflation rates.

Economic historians have identified a number of major deflationary episodes. Key examples include the crisis of 1815–1821 in which agricultural prices fell by nearly 50%. The banking-related Panic of 1837 was followed by six years of deflation in which prices fell by nearly 30%. The post-Civil-War greenback period experienced a number of severe deflations and the 1873–1896 period has been called the Long Depression. This period experienced massive amounts of corporate bond defaults and Friedman and Schwartz (1963) estimate that the price level declined by 1.7% per year from 1875 to 1896. The United States suffered a severe deflationary spiral during the early stages of the Great Depression from 1929 to 1933 as prices rapidly fell by more than 40%.

Although Atkeson and Kehoe (2004), Bordo and Filardo (2005), and others show that not all deflations have been associated with severe declines in economic output, a common thread throughout U.S. history is that deflationary
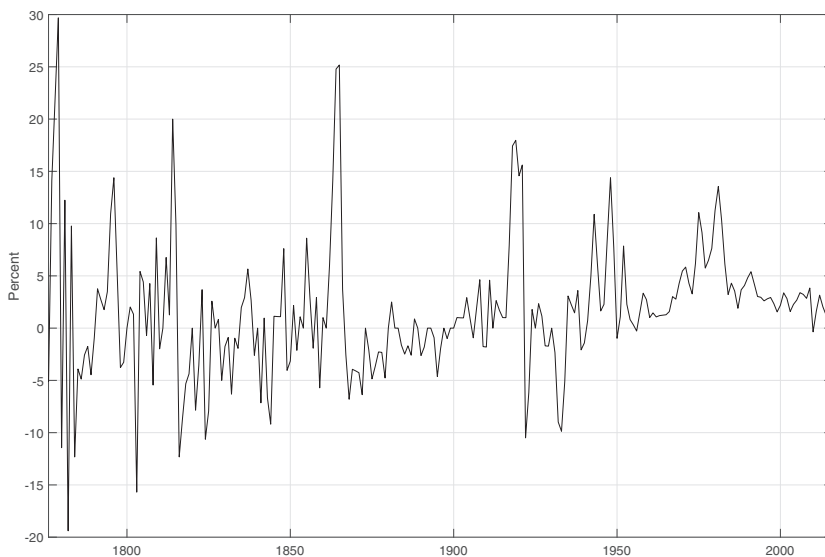
Figure 1
Historical annual inflation rates
This figure plots the time series of annual inflation rates from 1775 to 2015. Data are from Ken Rogoff at https://www.aeaweb.org/aer/data/may2013/P2013_4410_data.zip.

episodes are typically associated with turbulence or crises in the financial system.

2. Inflation Swap and Options Markets

In this section, we begin by reviewing the inflation swap market. We then provide a brief introduction to the inflation options market.

2.1 Inflation swaps

As discussed by Fleckenstein, Longstaff, and Lustig (2014), U.S. inflation swaps were first introduced when the Treasury began auctioning TIPS in 1997, and are increasingly popular among institutional investment managers. Pond and Mirani (2011) estimate the notional size of the inflation swap market to be on the order of hundreds of billions.

Recent research by Fleming and Sporn (2012) concludes that "the inflation swap market appears reasonably liquid and transparent despite the market's over-the-counter nature and modest activity." They estimate that realized bid-ask spreads for customers in the inflation swap market are on the order of three basis points. Conversations with inflation swap traders confirm that these

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instruments are fairly liquid with typical bid-ask spreads consistent with those reported by Fleming and Sporn.[5]

In this paper, we focus on the most widely used type of inflation swap, which is designated a zero-coupon swap. This swap is executed between two counterparties at time zero and has only one cash flow, which occurs at the maturity date of the swap. For example, imagine that at time zero, the ten-year zero-coupon inflation swap rate is 300 basis points. As is standard with swaps, there are no cash flows at time zero when the swap is executed. At the maturity date of the swap in ten years, the counterparties to the inflation swap exchange a cash flow of $(1+.0300)^{10} - I_T$, where I_T is the relative change in the price level between now and the maturity date of the swap. The timing and index lag construction of the inflation index used in an inflation swap are chosen to match precisely the definitions applied to TIPS issues.

The zero-coupon inflation swap rate data used in this study are collected from the Bloomberg system. Although inflation swap data for most maturities are available beginning in 2004, the empirical results in this paper are based on the shorter period over which we also have inflation options data. To provide a more complete historical perspective, however, we report the summary statistics for inflation swap rates over the entire period for which we have data. Table 1 provides summary statistics for inflation swaps with maturities ranging from 1 to 55 years for the July 23, 2004 to October 28, 2015 period.

As shown, average inflation swap rates range from 1.660% for one-year inflation swaps, to a high of 2.872% for 55-year inflation swaps. The volatility of inflation swap rates is generally declining in the maturity of the contracts. The dampened volatility of long-horizon inflation swap rates suggests that the market may view expected inflation as being mean-reverting in nature. Table 1 also shows evidence of deflationary concerns during the sample period. For example, the one-year swap rate reached a minimum of −4.545% during the height of the 2008 financial crisis amid serious fears about the U.S. economy sliding into a full-fledged depression/deflation scenario.

2.2 Inflation options

The inflation options market had its inception in 2002 with the introduction of caps and floors on the realized inflation rate. While trading in inflation options was initially muted, the market gained considerable momentum as the financial crisis emerged and total interbank trading volume reached $100 billion. [6] While the inflation options market is not yet as liquid as, for instance,

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[5] Since inflation swaps are not as liquid as Treasury securities, this raises the issue of whether liquidity effects could play a role in our analysis. We note, however, that Fleckenstein, Longstaff, and Lustig (2014) find that Treasury bonds are priced at a premium relative to TIPS. In contrast, they find no such relation for corporate bonds. Since they use the same set of inflation swap prices in conducting both analyses, they conclude that inflation swap prices may be less influenced by liquidity effects than Treasuries and TIPS.

[6] For a discussion of the inflation derivatives markets, see Jarrow and Yildirim (2003), Mercurio (2005), Kerkhof (2005), and James (2010).

Table 1
Summary statistics for inflation swap rates

Swap maturity	Mean	SD	Minimum	Median	Maximum	N
1	1.660	1.211	−4.545	1.744	3.802	2,939
2	1.861	0.952	−3.605	1.889	3.460	2,939
3	2.020	0.754	−2.047	2.036	3.351	2,939
4	2.147	0.652	−1.228	2.166	3.342	2,939
5	2.251	0.532	−0.570	2.293	3.310	2,939
6	2.332	0.468	−0.080	2.388	3.310	2,939
7	2.401	0.416	0.402	2.470	3.229	2,939
8	2.459	0.378	0.640	2.534	3.195	2,939
9	2.507	0.345	0.904	2.576	3.135	2,939
10	2.553	0.319	1.146	2.627	3.145	2,939
12	2.610	0.305	1.280	2.680	3.160	2,939
15	2.678	0.307	1.161	2.746	3.300	2,939
20	2.737	0.318	1.070	2.816	3.360	2,939
25	2.776	0.329	1.211	2.848	3.390	2,939
30	2.819	0.335	1.455	2.876	3.500	2,939
35	2.685	0.283	1.890	2.763	3.119	1,349
40	2.693	0.335	1.455	2.787	3.377	1,975
45	2.734	0.294	1.996	2.823	3.305	1,349
50	2.785	0.285	1.465	2.789	3.500	1,975
55	2.872	0.144	2.386	2.851	3.287	1,349

This table reports summary statistics for the inflation swap rates for the indicated maturities. Swap maturity is expressed in years. Inflation swap rates are expressed as percentages. The sample consists of daily observations for the period from July 23, 2004 to October 28, 2015.

the stock index options market, the market is sufficiently liquid that active quotations for inflation cap and floor prices have been readily available in the market since 2009 for a wide range of strikes.

In Europe and the United Kingdom, insurance companies are among the most active participants in the inflation derivatives market. In particular, much of the demand in 10- and 30-year 0% floors is due to pension funds trying to protect long inflation swaps positions. In contrast, insurance companies and financial institutions that need to hedge inflation risk are the most active participants on the demand side in the U.S. inflation options market.

The most actively traded inflation options are year-on-year and zero-coupon inflation options. Year-on-year inflation options are caps and floors that pay the difference between a strike rate and annual inflation on an annual basis. Zero-coupon options, in contrast, pay only one cash flow at the expiration date of the contract based on the cumulative inflation from inception to the expiration date. To illustrate, assume that the realized annualized inflation rate over the next ten years was 2%. A ten-year zero-coupon cap struck at 1% would pay a cash flow of $\max(0, 1.0200^{10} - 1.0100^{10})$ at its expiration date. In this paper, we focus on zero-coupon inflation options since their cash flows parallel those of zero-coupon inflation swaps.

As with inflation swaps, we collect inflation cap and floor data from the Bloomberg system. Data are available for the period from October 5, 2009 to October 28, 2015 for strikes ranging from negative 3% to 6% generally in increments of 50 basis points. We check the quality of the data by insuring that

Table 2
Summary statistics for inflation caps and floors

Option maturity	Average floor value by strike								Average cap value by strike								Ave.	N
	−3	−2	−1	0	1	2	3	4	−1	0	1	2	3	4	5	6		
1	1	4	8	18	41	91	165	290	236	152	83	33	11	6	3	2	21.5	1,554
2	4	9	15	28	62	150	305	544	523	349	190	75	28	13	7	5	21.6	1,544
3	2	12	19	33	73	187	411	755	833	571	322	134	51	22	13	8	23.2	1,545
5	3	18	27	42	94	242	581	1,066	1,456	1,044	623	270	94	47	26	16	22.2	1,545
7	4	18	28	48	106	282	712	1,309	2,031	1,504	953	455	167	86	48	30	22.3	1,545
10	4	17	27	54	128	344	887	1,587	2,751	2,109	1,432	760	339	138	83	48	22.2	1,569
12	4	15	25	53	131	368	979	1,727	3,117	2,419	1,706	930	418	188	95	72	22.6	1,544
15	4	14	24	52	134	395	1,109	1,948	3,532	2,789	2,063	1,147	520	234	119	67	22.4	1,545
20	4	14	24	50	135	426	1,310	2,290	3,979	3,269	2,550	1,433	639	275	154	87	22.2	1,545
30	5	12	23	52	144	505	1,785	3,179	5,498	4,463	3,445	2,000	888	416	235	141	20.3	1,545

This table reports the average values for inflation caps and floors for the indicated maturities and strikes. The average values are expressed in terms of cents per $100 notional. Option maturity is expressed in years. Ave. denotes the average number of caps and floors available each day, from which the risk-neutral density of inflation is estimated. N denotes the number of days for which the risk-neutral density of inflation is estimated. The sample consists of daily observations for the period from October 5, 2009 to October 28, 2015.

the cap and floor prices included satisfy standard option pricing bounds, such as those described in Merton (1973), including put-call parity, monotonicity, intrinsic value lower bounds, strike price monotonicity, slope, and convexity relations. To provide some perspective on the data, Table 2 provides summary statistics for call and put prices for selected strikes.

As illustrated, inflation cap and floor prices are quoted in basis points, or equivalently, as cents per $100 notional. Interestingly, inflation option prices are not always monotonically increasing in maturity. This may seem counterintuitive given standard option pricing theory, but it is important to recognize that the inflation rate is a macroeconomic variable rather than the price of a traded asset.[7] For most maturities, we have about 20 to 25 separate cap and floor prices with strikes varying from −3% to 6% from which to estimate the density of inflation.

3. Modeling Inflation

In this section, we present the continuous time model used to describe the dynamics of inflation under both the objective and risk-neutral measures. We also describe the application of the model to the valuation of inflation swaps and options.

3.1 Inflation model

We begin with a few key items of notation. For notational simplicity, we assume that all inflation contracts are valued as of time zero and that the initial price

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[7] We observe that similar nonmonotonic behavior occurs with interest rate options such as interest rate caps, floors, and swaptions; see Longstaff, Santa-Clara, and Schwartz (2001).

level at time zero is normalized to one.[8] Furthermore, time-zero values of state variables are unsubscripted. Let I_t denote the relative change in the price level from time zero to time t.

Under the objective or actual measure P, the dynamics of the price level are given by

$$dI = X \ I \ dt + \sqrt{V} \ I \ dZ_I, \tag{1}$$

$$dX = \kappa \ (Y - X) \ dt + \eta \ dZ_X, \tag{2}$$

$$dY = (\mu - \xi \ Y) \ dt + s \ dZ_Y, \tag{3}$$

$$dV = (\delta - \psi \ V) \ dt + \sigma \ \sqrt{V} \ dZ_V. \tag{4}$$

In this specification, X_t represents the instantaneous expected inflation rate. The state variable Y_t represents the long-run trend in expected inflation towards which the process X_t reverts. The variable V_t denotes the stochastic variance of the inflation process. The processes Z_I, Z_X, Z_Y, and Z_V are standard Brownian motions and are assumed to be uncorrelated with each other. This affine specification has parallels to both the Heston (1993) stochastic volatility option pricing model and the long-run risk model of Bansal and Yaron (2004), and allows for a wide range of possible time-series properties for realized inflation.[9]

Under the risk-neutral valuation measure Q, the dynamics of the price level are given by

$$dI = X \ I \ dt + \sqrt{V} \ I \ dZ_I, \tag{5}$$

$$dX = \lambda \ (Y - X) \ dt + \eta \ dZ_X, \tag{6}$$

$$dY = (\alpha - \beta \ Y) \ dt + s \ dZ_Y, \tag{7}$$

$$dV = (\theta - \phi \ V) \ dt + \sigma \ \sqrt{V} \ dZ_V, \tag{8}$$

where the parameters that now appear in the drift terms in the system of equations allow for the possibility that the market incorporates time-varying inflation-related risk premia into asset prices. In particular, the model allows the risk-neutral distributions of I_t, X_t, Y_t, and V_t to differ from the corresponding

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[8] Since the initial price level equals one, we will further simplify notation by not showing the dependence of valuation expressions on the initial price level I_0.

[9] We note that more general specifications for the drift terms in the dynamics of I, X, Y, and V are possible. We adopt this more parsimonious specification since it reduces the (already large) number of parameters that need to be estimated while still allowing the model to fit the term structure of inflation swaps closely.

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distributions under the actual dynamics.[10] Thus, the model permits a fairly general structure for inflation risk premia.[11]

Finally, let r_t denote the nominal instantaneous riskless interest rate. We can express this rate as $r_t = R_t + X_t$ where R_t is the real riskless interest rate and X_t is expected inflation. For tractability, we also assume that R_t is uncorrelated with the other state variables I_t, X_t, Y_t, and V_t.

3.2 Valuing inflation swaps

From the earlier discussion, an inflation swap pays a single cash flow of $I_T - F$ at maturity date T, where F is the inflation swap price set at initiation of the contract at time zero. Note that $F = (1+f)^T$ where f is the inflation swap rate. Appendix A shows that the inflation swap price can be expressed in closed form as

$$F(X,Y,T) = \exp(-A(T) - B(T)X - C(T)Y), \tag{9}$$

where

$$
\begin{aligned}
A(T) = {} & \frac{\alpha\lambda}{\beta-\lambda}\left(\frac{1}{\beta}\left(T - \frac{1}{\beta}(1-e^{-\beta T})\right) - \frac{1}{\lambda}\left(T - \frac{1}{\lambda}(1-e^{-\lambda T})\right)\right) \\
& + \frac{s^2\lambda^2}{2(\lambda-\beta)^2}\left(\frac{1}{\beta^2}\left(T - \frac{2}{\beta}(1-e^{-\beta T}) + \frac{1}{2\beta}(1-e^{-2\beta T})\right)\right. \\
& - \frac{2}{\beta\lambda}\left(T - \frac{1}{\beta}(1-e^{-\beta T}) - \frac{1}{\lambda}(1-e^{-\lambda T}) + \frac{1}{\beta+\lambda}(1-e^{-(\beta+\lambda)T})\right) \\
& + \left.\frac{1}{\lambda^2}\left(T - \frac{2}{\lambda}(1-e^{-\lambda T}) + \frac{1}{2\lambda}(1-e^{-2\lambda T})\right)\right) \\
& + \frac{\eta^2}{2\lambda^2}\left(T - \frac{2}{\lambda}(1-e^{-\lambda T}) + \frac{1}{2\lambda}(1-e^{-2\lambda T})\right),
\end{aligned} \tag{10}
$$

$$B(T) = \frac{-(1-e^{-\lambda T})}{\lambda}, \tag{11}$$

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[10] Note that our implicit assumption that the functional form of the drift of I_t is the same under both the P and Q measures does not imply that the expected value of I will be the same under both measures. This is because the dynamics and future path of X will vary across measures.

[11] Earlier versions of the paper used alternative specifications for the inflation process. For example, in an earlier version, we used a more general specification that allowed jumps and stochastic volatility. The results from this alternative specification were very similar to those we obtain using this specification. In contrast, a specification in which inflation volatility was constant gave similar results for expected inflation and inflation risk premia, but gave slightly higher deflation probabilities for longer horizons.

$$C(T) = \frac{\lambda}{\beta - \lambda}\left(\frac{1}{\beta}(1 - e^{-\beta T}) - \frac{1}{\lambda}(1 - e^{-\lambda T})\right). \tag{12}$$

3.3 Valuing inflation options

Let $C(X, Y, V, T; K)$ denote the time-zero value of a European inflation cap or call option with strike K and expiration date T. The payoff on this option at its expiration date is $\max(0, I_T - (1 + K)^T)$.

From the dynamics of I_t above,

$$I_T = I_0 \exp(w_T + u_T), \tag{13}$$

where

$$w_T = \int_0^T X_t \, dt, \tag{14}$$

$$u_T = -\frac{1}{2}\int_0^T V_t \, dt + \int_0^T \sqrt{V_t} \, dZ_I. \tag{15}$$

Appendix B shows that w_T is normally distributed with mean $\ln F(X, Y, T) - G(T)/2$ and variance $G(T)$ under the risk-neutral measure. Following Heston (1993), the density of u_T is given by inverting the characteristic function,

$$h(u_T) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\zeta u_T} \exp(L(T) + M(T)V + i\zeta) \, d\zeta. \tag{16}$$

Given the density functions for w_T and u_T, the value of a European inflation cap can be expressed as

$$C(X, Y, V, T; K) = D(T)\left[\int_{-\infty}^{\infty} F(X, Y, T)N(a)\right.$$

$$\left. - (1 + K)^T \, N(a - \sqrt{G(T)}) \, h(u_T) \, du_T\right], \tag{17}$$

where

$$a = \frac{u_T - T\ln(1 + K) + \ln F(X, Y, T) + G(T)/2}{\sqrt{G(T)}}, \tag{18}$$

$N(\cdot)$ is the standard normal distribution function, and $D(T)$ is the price of a riskless discount bond with maturity T. A similar representation holds for the value of an inflation floor or put option $P(X, Y, V, T; K)$ with payoff at expiration date T of $\max(0, (1 + K)^T - I_T)$.

4. Model Estimation

In estimating the distribution of inflation, we use a simple maximum likelihood approach that closely parallels standard techniques used in the term structure

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estimation literature. In particular, following Duffie and Singleton (1997), Duffee (2002), and many others, we solve for the values of X_t, Y_t, and V_t from specific inflation swap and option prices, and then jointly estimate the parameters of both the risk-neutral and objective dynamics for these variables using maximum likelihood. In this sense, we use a completely standard approach, which is based entirely on existing methodology extensively applied in the literature.

Each daily observation in the data set includes inflation swap data for 15 maturities ranging from 1 to 30 years, and 200 to 250 inflation call and put prices for varying strikes and ten different maturities also ranging from 1 to 30 years. To identify X_t, Y_t, and V_t, we assume that the 2-year and 30-year inflation swap rates and the variance of $\ln I_T$ implied from the cross-section of 3-year inflation options are measured without error. Given a parameter vector Θ, the state variables are given by inverting the pricing expressions for these inflation swaps and options (Appendix C provides details of the estimation procedure). Let J denote the Jacobian of the linear mapping from the two inflation swap rates and the implied variance into X_t, Y_t, and V_t. At time t, we can now solve for the inflation swap rate or inflation option price implied by the model for any maturity and strike from the values of X_t, Y_t, and V_t and the parameter vector Θ.

Let ϵ_t denote the vector of differences between the market and model values of the inflation swaps for the individual maturities (excluding the 2-year and 30-year maturities). We assume that ϵ_t is conditionally multivariate normally distributed with mean vector zero and a diagonal covariance matrix Σ with main diagonal values v_j (where the subscripts represent the maturities of the corresponding inflation swaps).

Rather than focusing on the differences between model and market values for individual options (which would literally involve estimating hundreds of additional parameters), we adopt a slightly simpler approach. For each maturity, we solve for the implied variance of the risk neutral density of $\ln I_t$ from the prices of all options with that maturity. Briefly, this is done by using an Edgeworth expansion for the risk-neutral density and then solving for the implied cumulants of the distribution (Appendix C provides the details). We then define e_t as the vector of differences between the market implied volatility and the model implied volatility for the various maturities (excluding the options with three-year maturities). Analogously to inflation swaps, we assume that e_t is conditionally normally distributed with mean vector zero and a diagonal covariance matrix Ψ with main diagonal values d_k (where the subscripts denote the maturities of the corresponding options). We also assume that ϵ_t and e_t are uncorrelated.

Given these assumptions, the log of the joint likelihood function LLK_t of the 2- and 30-year inflation swap rates, the average 3-year option prices, and

the vectors $\epsilon_{t+\Delta t}$ and $e_{t+\Delta t}$, conditional on the data is given by

$$= -(22/2)\ln(2\pi) + \ln k + \ln|J_{t+\Delta t}|$$

$$-\frac{1}{2}\ln|\Sigma| - \frac{1}{2}\epsilon'_{t+\Delta t}\,\Sigma^{-1}\,\epsilon_{t+\Delta t} - \frac{1}{2}\ln|\Psi| - \frac{1}{2}e'_{t+\Delta t}\,\Psi^{-1}\,e_{t+\Delta t}$$

$$-\ln\left(2\pi\sigma_X\sigma_Y\sqrt{1-\rho_{XY}^2}\right) - \frac{1}{2(1-\rho_{XY}^2)}\left[\left(\frac{X_{t+\Delta t}-\mu_{X_t}}{\sigma_X}\right)^2\right.$$

$$\left.-2\rho_{XY}\left(\frac{X_{t+\Delta}-\mu_{X_t}}{\sigma_X}\right)\left(\frac{Y_{t+\Delta t}-\mu_{Y_t}}{\sigma_Y}\right) + \left(\frac{Y_{t+\Delta t}-\mu_{Y_t}}{\sigma_Y}\right)^2\right]$$

$$-k(V_{t+\Delta t}+V_t e^{-\psi\Delta}) + \frac{1}{2}q(\ln V_{t+\Delta t} - \ln V_t + \psi\Delta t)$$

$$+\ln I_q\left(2k\sqrt{V_{t+\Delta t}V_t e^{-\psi\Delta t}}\right) \tag{19}$$

where $I_q(\cdot)$ is the modified Bessel function (see Abramowitz and Stegun (1970)),

$$k = \frac{2\psi}{\sigma^2(1-e^{-\psi\Delta t})}, \tag{20}$$

$$q = 2\delta/\sigma^2 - 1, \tag{21}$$

and where the conditional moments μ_{X_t}, μ_{Y_t}, σ_X^2, σ_Y^2, and $\rho_{XY} = \sigma_{XY}/\sqrt{\sigma_X^2\sigma_Y^2}$ of $X_{t+\Delta t}$ and $Y_{t+\Delta t}$ are given in Appendix D along with the conditional density of $V_{t+\Delta t}$. The total log likelihood function is given by summing LLK_t over all values of t.[12]

We maximize the log likelihood function over the 35-dimensional parameter vector $\Theta = \{\kappa, \eta, \mu, \xi, s, \delta, \psi, \sigma, \lambda, \alpha, \beta, \theta, \phi, v_1, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{12}, v_{15}, v_{20}, v_{25}, d_1, d_2, d_5, d_7, d_{10}, d_{12}, d_{15}, d_{20}, d_{30}\}$ using a standard quasi-Newton algorithm with a finite difference gradient. As a robustness check that the algorithm achieves the global maximum, we repeat the estimation using a variety of different starting values for the parameter vector. Table 3 reports the maximum likelihood estimates of the parameters and their asymptotic standard errors. The fitting errors from the estimation for the inflation swaps are all relatively small with the typical standard deviation ranging from roughly six to ten basis points, depending on maturity.[13]

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[12] Note that we do not need to condition on the realized value of inflation in this estimation. This is because the state variables X_t, Y_t, and V_t fully characterize the distribution of future inflation due to the joint Markovian nature of the dynamics in Equations (1) through (4).

[13] It is important to acknowledge, however, that we are estimating the model using only six years of time-series data. Thus, our results will likely not be as precise as they would be if we had access to a significantly longer time series.

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Table 3
Maximum likelihood estimation of the inflation swap model

Parameter	Value	SE
λ	0.627234	0.0009125
α	0.000620	0.0000005
β	0.002199	0.0000193
θ	0.000001	0.0000001
ϕ	0.002631	0.0001462
κ	0.407045	0.6043220
μ	0.036186	0.0015381
ξ	1.417810	0.0572805
δ	0.000343	0.0000854
ψ	1.003313	0.3711752
η	0.012586	0.0000019
s	0.006325	0.0000023
σ	0.034017	0.0017279
v_1	0.001734	0.0000895
v_3	0.000607	0.0000255
v_4	0.000808	0.0000338
v_5	0.000857	0.0000358
v_6	0.000836	0.0000147
v_7	0.000762	0.0001063
v_8	0.000675	0.0000294
v_9	0.000617	0.0000339
v_{10}	0.000589	0.0000196
v_{12}	0.000475	0.0000187
v_{15}	0.000400	0.0000148
v_{20}	0.000365	0.0000087
v_{25}	0.000348	0.0000083
d_1	0.008444	0.0003013
d_2	0.002279	0.0000773
d_5	0.007584	0.0004527
d_7	0.011567	0.0006942
d_{10}	0.014759	0.0005072
d_{12}	0.013893	0.0003362
d_{15}	0.011802	0.0006983
d_{20}	0.010575	0.0003832
d_{30}	0.026595	0.0011522

This table reports the maximum likelihood estimates of the parameters of the inflation swap model along with their asymptotic standard errors. The model is estimated using daily inflation swap prices for the period from October 5, 2009 to October 28, 2015.

We test for the existence of risk premia by examining whether the five parameters κ, μ, ξ, δ, and ψ that appear in the objective dynamics are equal to the corresponding parameters λ, α, β, θ, and ϕ in the risk-neutral dynamics. Specifically, we test whether $\kappa = \lambda$, $\mu = \alpha$, $\xi = \beta$, $\delta = \theta$, and $\psi = \phi$. Finding that there are significant differences between the objective and risk-neutral parameters would indicate the presence of a risk premium embedded in inflation derivatives prices. Although not shown, the tests solidly reject the hypothesis of equality for all of the pairs of coefficients, with the exception of κ and λ (the t-statistic for the hypothesis $\kappa = \lambda$ is -0.30). Thus, the results indicate that significant risk premia are incorporated into the inflation swap and option prices.[14]

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[14] We also conduct model validation tests in which we examine whether the conditional distributions of inflation swap rates one month forward are consistent with the distributions implied by the fitted model. These chi-square

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5. Distribution of Inflation

As a preliminary to the analysis of deflation risk, we first present the empirical results for inflation risk premia and expected inflation.

5.1 Inflation densities

To provide some perspective on the nature of the inflation density under the objective measure, Figure 2 plots the time series of inflation densities for several horizons. As shown, there is considerable variation in the shape of the inflation distribution for the shorter horizons. In contrast, the distribution of inflation at the ten-year horizon is more stable over time.

5.2 Inflation risk premia

We measure the inflation risk premium as the difference between the fitted inflation swap and expected inflation rates. This is essentially the way in which many market participants think about inflation risk premia. When the inflation swap rate is higher than expected inflation, the inflation risk premium is positive, and vice versa. There is no compelling theoretical reason why the inflation risk premium could not be negative in sign. In this case, the risk premium might be viewed as a deflation risk premium.

Table 4 presents summary statistics for the average inflation risk premia for horizons ranging from 1 year to 30 years.[15] Figure 3 plots the time series of inflation risk premia for several horizons. As shown, the average values of the risk premia are relatively small. For most maturities, the average risk premium is in the range of 20 to 25 basis points.

These inflation risk premia estimates are broadly consistent with previous estimates obtained using alternative approaches and different data sets by other researchers. For example, Haubrich, Pennacchi, and Ritchken (2012) estimate the inflation risk premium to be slightly negative for horizons under 29 months, 17 basis points for a 5-year horizon, and 45 basis points for a 10-year horizon. Buraschi and Jiltsov (2005) estimate the inflation risk premium to be 15 basis points for a 1-month horizon, 40 basis points for a 5-year horizon, and 70 basis points for a 10-year horizon. Campbell and Viceira (2001) estimate the 10-year inflation risk premium to be 35 basis points for a 3-month horizon and 110 basis points for a 10-year horizon. Chernov and Mueller (2012) estimate the average 10-year inflation premium to be 67 basis points. Ang, Bekaert, and Wei (2008) estimate the 5-year inflation risk premium to be 115 basis points.

Table 4 also shows that the inflation risk premia vary through time and take on both positive and negative values. Finding that inflation risk premia

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tests cannot reject the hypothesis for any horizon that the probability integral transform is uniformly distributed. These validation tests provide support for the model specification. We are grateful to the referee for suggesting these model validation tests.

[15] Note that the risk premium is for the horizon from time zero to T, not from time $T-1$ to T.
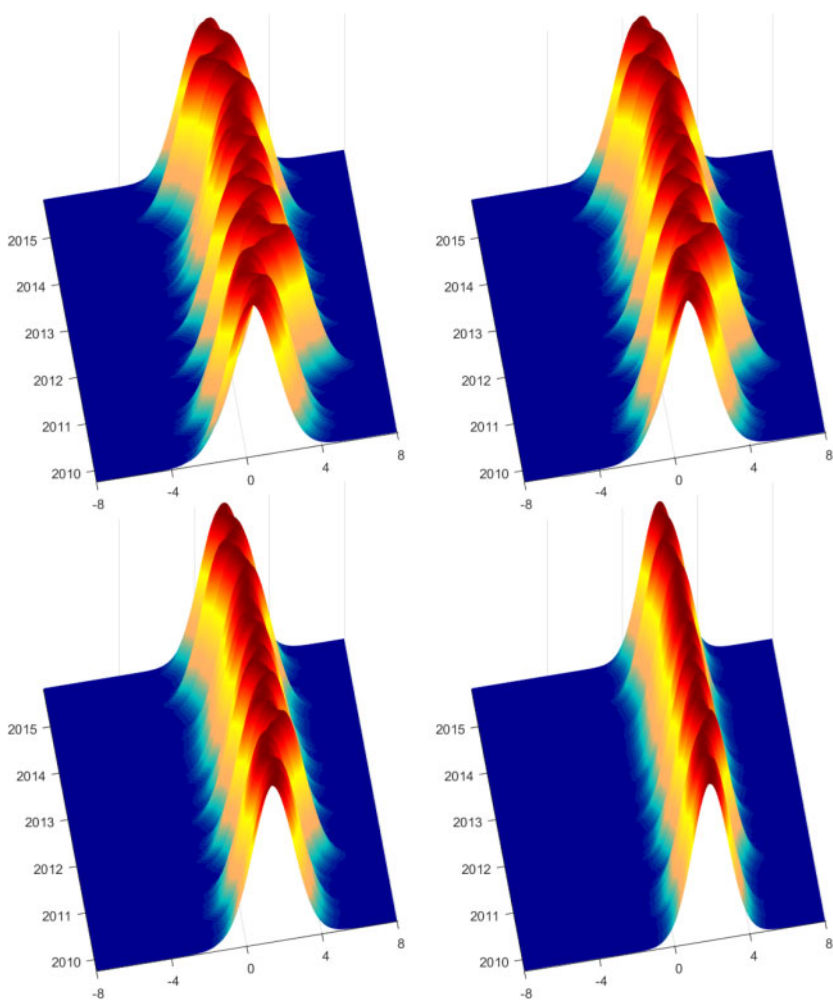
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Figure 2
Inflation densities
This figure plots the time series of inflation densities for horizons of one year (upper left), two years (upper right), five years (lower left), and ten years (lower right).

change sign through time is an intriguing result. In addition, the fact that all of the estimated risk premia take negative values at some point during the sample period is consistent with the findings of Campbell, Shiller, and Viceira (2009), Bekaert and Wang (2010), Campbell, Sunderam, and Viceira (2013), and others. Intuitively, one way to think about why the risk premium could change signs is in terms of the link between inflation and the macroeconomy. For example, when inflation risk is perceived to be countercyclical, the market price of inflation risk should be positive. This is the regime in which aggregate

Table 4
Summary statistics for inflation risk premia

Horizon	Mean	SE of the Mean	SD	Minimum	Median	Maximum	N
1	12.08	0.43	6.05	−1.57	11.11	27.24	1,569
2	17.69	0.74	10.51	−7.03	16.66	43.46	1,569
3	20.10	0.98	13.82	−12.68	19.25	54.46	1,569
4	20.99	1.15	16.28	−17.50	20.37	61.47	1,569
5	21.22	1.28	18.15	−23.89	21.03	66.05	1,569
6	21.23	1.38	19.59	−29.42	21.58	69.19	1,569
7	21.23	1.46	20.74	−33.89	21.88	71.47	1,569
8	21.30	1.53	21.66	−37.47	22.42	73.26	1,569
9	21.47	1.58	22.41	−40.30	22.82	74.73	1,569
10	21.72	1.63	23.04	−42.53	23.59	76.01	1,569
12	22.42	1.70	24.01	−45.68	24.77	78.18	1,569
15	23.60	1.77	25.01	−48.55	26.58	80.73	1,569
20	24.74	1.84	26.00	−51.48	28.22	83.08	1,569
25	23.86	1.88	26.55	−54.61	27.70	82.78	1,569
30	20.45	1.90	26.86	−59.35	24.33	79.62	1,569

This table reports summary statistics for the estimated annualized inflation risk premia for the indicated horizons. Horizon is expressed in years. The inflation risk premia are measured in basis points. The standard error of the mean is given by the Newey-West estimator (seven lags). The inflation risk premia are estimated using the period from October 5, 2009 to October 28, 2015.
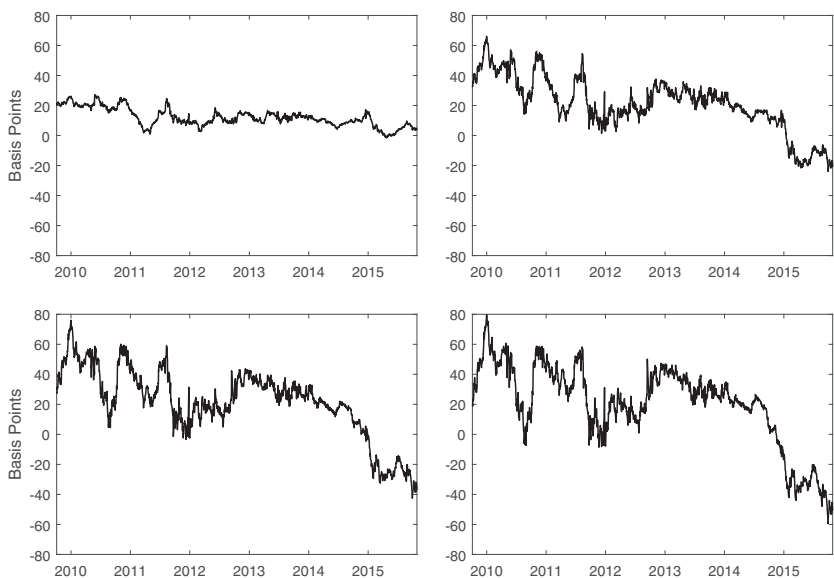

Figure 3
Inflation risk premia
This figure plots the time series of inflation risk premia for horizons of 1 year (upper left), 5 years (upper right), 10 years (lower left), and 30 years (lower right).

supply shocks (e.g., oil shocks) account for most of the variation in output: High inflation coincides with low output growth. Thus, investors pay an insurance premium when buying inflation protection in the market.

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Table 5
Summary statistics for expected inflation

Horizon	Mean	SE of the Mean	SD	Minimum	Median	Maximum	N
1	1.292	0.041	0.586	−0.479	1.350	2.880	1,569
2	1.476	0.030	0.430	0.486	1.530	2.687	1,569
3	1.628	0.025	0.356	0.801	1.679	2.609	1,569
4	1.759	0.021	0.305	1.023	1.802	2.600	1,569
5	1.881	0.019	0.268	1.198	1.919	2.617	1,569
6	1.983	0.017	0.238	1.370	2.017	2.619	1,569
7	2.069	0.015	0.210	1.528	2.096	2.621	1,569
8	2.137	0.013	0.182	1.659	2.152	2.629	1,569
9	2.193	0.011	0.163	1.761	2.205	2.626	1,569
10	2.242	0.010	0.147	1.840	2.250	2.645	1,569
12	2.302	0.008	0.121	1.973	2.314	2.646	1,569
15	2.365	0.007	0.105	2.087	2.381	2.642	1,569
20	2.407	0.006	0.086	2.147	2.427	2.602	1,569
25	2.439	0.005	0.067	2.165	2.451	2.605	1,569
30	2.494	0.004	0.052	2.371	2.501	2.641	1,569

This table reports summary statistics for the expected inflation rate for the indicated horizons. Horizon is expressed in years. Expected inflation rates are expressed as percentages. The standard error of the mean is given by the Newey-West estimator (seven lags). The sample consists of daily observations for the period from October 5, 2009 to October 28, 2015.

The same line of reasoning implies that the sign of the inflation risk premium could be negative when inflation risk was perceived to have become pro-cyclical. In that regime, inflation innovations tend to be negative when the average investor's marginal utility is high. Hence, a nominal bond provides insurance against bad states of the world, whereas a real bond does not. In this case, investors receive an insurance premium for buying inflation hedges. When aggregate demand shocks account for most of the variation in output growth, then we would expect to see pro-cyclical inflation: High inflation coincides with high output growth.

The variation in the sign of the inflation risk premium also raises a number of interesting questions for optimal monetary policy. Since the Treasury seems constrained to issue mostly nominal bonds, it may prefer lower inflation risk premia. Higher inflation risk premia may increase the costs of government debt financing, funded by distortionary taxes. In particular, when inflation risk premia are negative, issuing nominal bonds could be appealing to the Treasury. However, since the level of expected inflation is related to the size of the inflation risk premium, the government may have the ability to influence the risk premium to some degree by how it targets inflation.

5.3 Expected inflation

To solve for the expected inflation rate for each horizon, we use the actual (not fitted) inflation swap rates observed in the market and adjust them by the inflation risk premium implied by the fitted model. Table 5 presents summary statistics for the expected inflation rates for the various horizons.

The results indicate that the average term structure of inflation expectations is monotonically increasing during the 2009–2015 sample period. The average
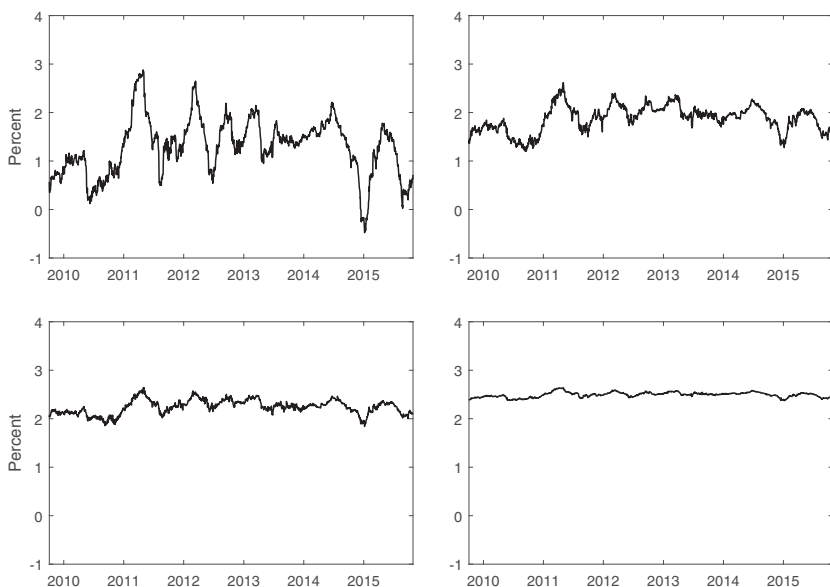
Figure 4
Expected inflation
This figure plots the time series of expected inflation for horizons of 1 year (upper left), 5 years (upper right), 10 years (lower left), and 30 years (lower right).

1-year expected inflation rate is 1.29%, and the average 30-year expected inflation rate is 2.49%. The table also shows time variation in expected inflation, although the variation is surprisingly small for longer horizons. In particular, the standard deviation of expected inflation ranges from 59 basis points for the 1-year horizon to about 5 basis points for the 30-year horizon. To illustrate the time variation in expected inflation more clearly, in Figure 4, we plot the expected inflation estimates for the one-, two-, five-, and ten-year horizons.

It is also interesting to contrast these market-implied forecasts of inflation with forecasts provided by major inflation surveys. As discussed by Ang, Bekaert, and Wei (2007), these surveys of inflation tend to be more accurate than those based on standard econometric models and are widely used by market practitioners. Furthermore, these inflation surveys have also been incorporated into a number of important academic studies of inflation, such as Fama and Gibbons (1984), Chernov and Mueller (2012), and Haubrich, Pennacchi, and Ritchken (2012).

We obtain inflation expectations from three surveys: the University of Michigan Survey of Consumers, the Philadelphia Federal Reserve Bank Survey of Professional Forecasters (SPF), and the Livingston Survey. The sample period for the forecasts matches that for the inflation swap and options data in the study. Appendix E provides background information about the surveys and describes how the surveys are conducted.

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Table 6
Comparison of survey forecasts with market-implied forecasts

Forecast horizon	Survey	Frequency	Survey forecast	Market-implied forecast	N
1 year	SPF	Quarterly	1.95	1.32	25
	Livingston	Semiannual	3.68	1.10	12
	Michigan	Monthly	3.10	1.31	72
5 years	SPF	Quarterly	2.15	1.90	25
	Michigan	Monthly	2.82	1.89	72
10 years	SPF	Quarterly	2.29	2.26	25
	Livingston	Semiannual	2.40	2.20	12

This table reports the average values of the survey forecasts for the indicated forecast horizon along with the corresponding average of the market-implied expected inflation rate for the same horizon. Inflation forecasts are expressed as percentages. The sample period is October 2009 to October 2015.

Table 6 reports the average values of the various surveys during the sample period and the corresponding average values of the market-implied forecasts. These averages are computed using the prior month-end values for the months in which surveys are released. Thus, monthly averages are compared with monthly averages, quarterly averages with quarterly averages, etc. As shown, the average market-implied forecasts of inflation tend to be a little lower than the survey averages for shorter horizons. The market-implied forecasts, however, closely parallel those from the surveys for longer horizons. While it would be interesting to compare the relative accuracy of the market-implied and survey forecasts, our sample is clearly too short to do this rigorously.

5.4 Inflation volatility

In the model, the volatility of inflation has two components. The first is the volatility due to the variation in expected inflation X_t. The second is the volatility resulting from unexpected inflation and is driven by the state variable V_t.

Figure 5 plots the time series of the estimates of V_t obtained from the estimation procedure. As shown, the volatility of unexpected inflation significantly varies through time. Near the beginning of the sample period, V_t takes values in the range of 2% to 3% per year. Later in the sample, however, the values of V_t decline significantly and are well under 1%. Note that since the dynamics of X_t are Gaussian, the volatility due to expected inflation is constant.

Table 7 reports summary statistics for inflation volatility along with the average fractions of the total variance that are due to the expected inflation component. As shown, the average volatilities are relatively uniform across horizons with values on the order of 1.50%. In contrast, the average fractions are monotonically increasing in horizon, with values ranging from about 15% for the 1-year horizon to more than 95% for the 30-year horizon. This pattern is intuitive since we would anticipate the effects of unexpected volatility to dampen out over long horizons. In contrast, the effects of variation in expected inflation would likely be more persistent.
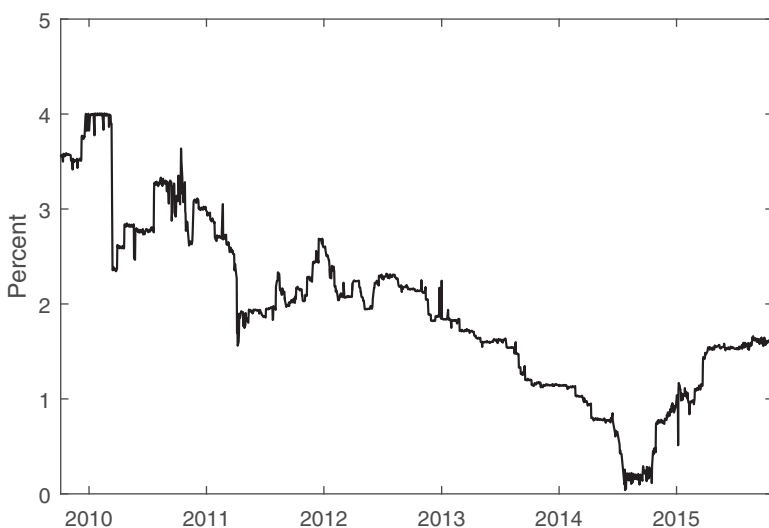
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Figure 5
Inflation volatility
This figure plots the time series of the square root of the estimated stochastic variance V_t of inflation.

Table 7
Summary statistics for inflation volatility

Horizon	Average volatility	Fraction due to expected inflation
1	1.422	14.68
2	1.556	27.60
3	1.396	37.10
5	1.548	51.08
7	1.590	61.82
10	1.648	73.66
12	1.619	79.21
15	1.585	85.02
20	1.524	90.64
30	1.449	95.39

This table reports the average volatility of the mean percentage inflation rate for the indicated horizons. The table also reports the average percentage of total inflation variance that is due to variation in expected inflation (the remaining variance is due to unexpected inflation) for the indicated horizons. The sample period is October 2009 to October 2015.

6. Deflation Risk

We turn now to the central issue of measuring the risk of deflation implied by market prices and studying the properties of deflation risk. First, we present descriptive statistics for the implied deflation risk. We then examine how the market prices the tail risk of deflation and contrast the results with those found in other markets.

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Table 8
Summary statistics for deflation probabilities

Horizon	Mean	SE of the Mean	SD	Minimum	Median	Maximum	*N*
1	18.759	0.720	10.155	2.003	15.739	47.757	1,545
2	13.799	0.514	7.261	1.801	11.621	35.412	1,545
3	10.216	0.367	5.177	1.581	8.670	26.081	1,545
5	5.684	0.188	2.661	1.132	4.919	14.023	1,545
7	3.210	0.099	1.400	0.757	2.815	7.614	1,545
10	1.389	0.039	0.558	0.385	1.236	3.138	1,545
12	0.802	0.022	0.309	0.238	0.719	1.769	1,545
15	0.356	0.009	0.131	0.114	0.321	0.764	1,545
20	0.094	0.002	0.033	0.032	0.085	0.196	1,545
30	0.007	0.001	0.002	0.003	0.006	0.014	1,545

This table reports summary statistics for the probability of the average inflation rate being below zero for the indicated horizons. Horizon is expressed in years. Probabilities are expressed as percentages. The standard error of the mean is given by the Newey-West estimator (seven lags). The sample consists of daily observations for the period from October 5, 2009 to October 28, 2015.
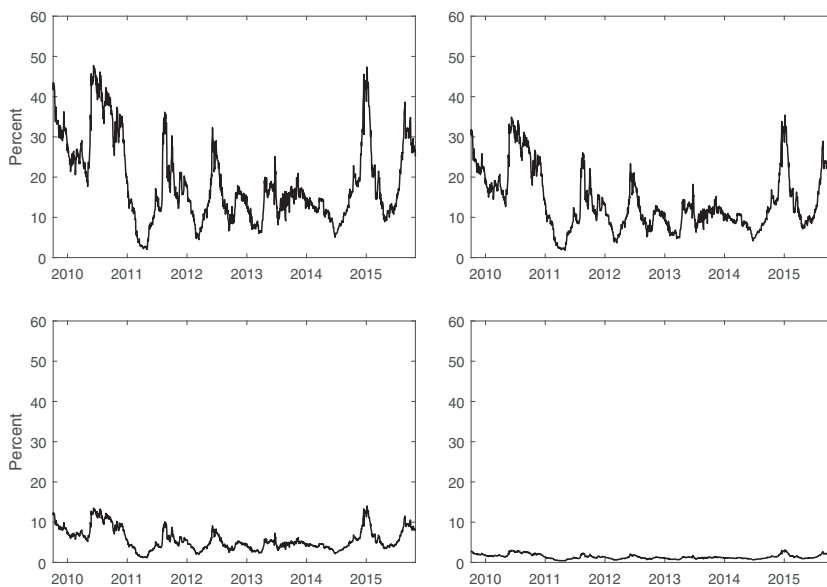

Figure 6
Deflation probabilities
This figure plots the time series of deflation probabilities for horizons of one year (upper left), two years (upper right), five years (lower left), and ten years (lower right).

6.1 How large is the risk of deflation?

Having solved for the inflation distribution, we can directly compute the probability that the average realized inflation rate over a specific horizon is less than zero, which represents the risk of deflation. Table 8 provides summary statistics for the estimated probabilities of deflation over the various horizons. To provide some additional perspective, Figure 6 graphs the time series of deflation probabilities over one-, two-, five-, and ten-year horizons.

As shown, the market places a surprisingly large weight on the possibility that deflation may occur over extended horizons. In particular, the average probability that the realized inflation rate will be less than or equal to zero is 18.76% for a 1-year horizon, 13.80% for a 2-year horizon, 5.68% for a 5-year horizon, and 1.39% for a 10-year horizon.

What is perhaps more striking is that the probability of deflation, although fairly persistent, significantly varies over time and reaches relatively high levels during the sample period. For example, the probability of deflation reaches a value of 47.76% for a 1-year horizon, 35.41% for a 2-year horizon, and 26.08% for a 5-year horizon. At other times, the market assesses the probability of deflation at any horizon to be less than 1%. This variation in the probability of deflation is due not only to changes in expected inflation, but also to changes in the volatility of inflation.

These probabilities are broadly consistent with the historical record on deflation in the United States. For example, based on the historical inflation rates from 1800 to 2012, the United States has experienced deflation over a 1-year horizon 65 times, which represents a frequency of 30.5%. Considering only nonoverlapping periods, the United States has experienced a 2-year deflation 41 times, a 5-year deflation 19 times, a 10-year deflation 11 times, and a 30-year deflation three times. These translate into frequencies of 24.0%, 14.4%, 10.6%, and 3.1%, respectively.[16]

Figure 6 also shows that the deflation probabilities for the shorter horizons have occasional jumps upward. These jumps tend to occur around major financial events, such as those associated with the European debt crisis. For example, the Eurozone experienced major turmoil during April and May 2010 as concerns about the ongoing solvency of Portugal, Italy, Ireland, Greece, and Spain became more urgent, and a number of bailout plans were instated. Spain's debt was first downgraded by Fitch on May 29, 2010. The one-year deflation probabilities increase dramatically during this period. In addition, the one-year deflation probability spikes again in early August of 2011, coinciding with the downgrade of U.S. Treasury debt by Standard and Poors. We will explore the link between deflation risk and major financial risk more formally later in the paper.

To provide some additional perspective, Figure 7 plots the time series of risk-neutral deflation probabilities for the same horizons as shown in Figure 6. A comparison of Figures 6 and 7 indicates that the ratio of the two deflation probabilities significantly varies over time.

6.2 Pricing deflation tail risk
Although we have solved for the inflation risk premium embedded in inflation swaps earlier in the paper, it is also interesting to examine how the market

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[16] Historical inflation rates are tabulated by Sahr (2016), http://liberalarts.oregonstate.edu/sites/liberalarts.oregonstate.edu/files/polisci/faculty-research/sahr/inflation-conversion/pdf/infcf17742016.pdf. More recent inflation rates are reported by the Bureau of Labor Statistics.
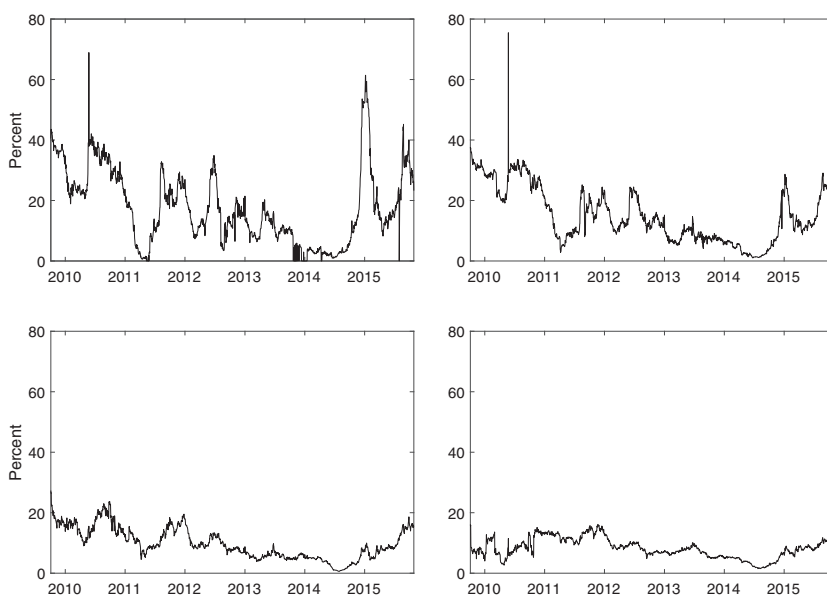
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Figure 7
Risk-neutral deflation probabilities
This figure plots the time series of risk-neutral deflation probabilities for horizons of one year (upper left), two years (upper right), five years (lower left), and ten years (lower right).

prices the risk that the tail event of a deflation occurs. This analysis can provide insights into how financial market participants view the risk of events that may happen infrequently, but which may have catastrophic implications.

A number of these types of tail risks have been previously studied in the literature. For example, researchers have investigated the pricing of catastrophic insurance losses, such as those caused by hurricanes or earthquakes. Froot (2001) finds that the ratio of insurance premia to expected losses in the market for catastrophic reinsurance ranges from about two to seven or more during the 1989 to 1998 period. Lane and Mahul (2008) estimate that the pricing of catastrophic risk in a sample of 250 catastrophe bonds is about 2.69 times the actual expected loss over the long term. Garmaise and Moskowitz (2009) and Ibragimov, Jaffee, and Walden (2011) offer both empirical and theoretical evidence that the extreme left tail catastrophic risk can be significantly priced in the market.

The default of a corporate bond is also an example of an event that is relatively rare for a specific firm, but which would result in an extremely negative outcome for bondholders of the defaulting firm. The pricing of default risk has been considered in many recent papers. For example, Giesecke et al. (2011) study the pricing of corporate bond default risk and find that the ratio of corporate

credit spreads to their actuarial expected loss is 2.04 over a 150-year period. Similarly, Driessen (2005) and Berndt et al. (2005) estimate ratios using data for recent periods that range in value from about 1.80 to 2.80.

The recent literature on disaster risk models also has important implications for the pricing of tail risk. For example, see Rietz (1988), Longstaff and Piazzesi (2004), Barro (2006, 2009), Gabaix (2012), Backus, Chernov, and Martin (2011), Kelly and Jiang (2014), Farhi and Gabaix (2016), and many others. A particularly interesting example is found in Gabaix in which the actual probability of a disaster is 3.63%, while the risk-neutral probability of a disaster is 19.20%, implying a ratio of risk-neutral to actual probabilities of greater than five.

Following along the lines of this literature, we solve for the ratio of the risk-neutral probability of deflation to the objective probability of deflation. This ratio provides a simple measure of how the market prices the tail risk of deflation and has the advantage of being directly comparable to the ratios discussed above. To avoid computing ratios when the denominator is near zero, we only compute the ratio when the objective probability of deflation is in excess of 0.10%.

Before presenting the results, it may be useful to first review why there could be significant differences between tail probabilities under the objective and risk-neutral measures. First, the means of the two distributions may differ due to the inflation risk premium embedded in inflation derivative prices. Second, and perhaps most importantly, the variance of the inflation distribution can differ significantly across the two measures. The reason for this is that the speed at which expected inflation reverts back to its long-run mean differs across the two measures. The speed of mean reversion, however, can have a large impact on the variance of inflation over any finite horizon. For instance, from Table 3, the speeds of mean reversion for Y and V are substantially lower under the risk-neutral measure than under the actual measure. This results in the variance of inflation being significantly larger under the risk-neutral measure than under the actual measure, particularly for longer horizons. The combination of the differences in means and variances between the objective and risk-neutral distributions can translate into significant differences in both the left and right tail probabilities.

Table 9 presents summary statistics for the ratios across the various horizons. As shown, the mean and median ratios are close to one for the one-year through three-year horizons. For longer horizons, however, the mean and median ratios are much higher with values ranging from roughly two to seven for horizons from five to ten years. For longer horizons, the mean and median ratios are much higher. These large values are qualitatively similar to those for the different types of tail risk discussed above. These ratios suggest that the market is deeply concerned about economic tail risks that may be difficult to diversify or that may be strongly systematic in nature.

Table 9
Summary statistics for the pricing of deflation tail risk

Horizon	Mean ratio	Median ratio
1	0.906	0.983
2	1.135	1.118
3	1.018	1.009
5	1.901	1.773
7	3.091	2.615
10	6.964	5.960
12	11.031	9.483
15	22.278	19.790
20	42.372	35.746

This table reports the means and medians of the ratio of the probability of deflation under the risk-neutral Q measure divided by the probability of deflation under the actual P measure. The ratio is only computed for observations for which the probability under the P measure is greater than or equal to 0.10%. Horizon is expressed in years. The sample consists of daily observations for the period from October 5, 2009 to October 28, 2015.

7. What Drives Deflation Risk?

A key advantage of our approach is that by extracting the market's assessment of the objective probability of deflation, we can then examine the relation between these probabilities and other financial and macroeconomic factors. In particular, we can study the relation between the tail risk of deflation and other types of tail risk that may be present in the markets.

In doing this, we focus on four broad categories of tail risk that have been extensively discussed in the literature. Specifically, we consider the links between deflation risk and systemic risk to the financial system, collateral revaluation risk, sovereign default risk, and business-cycle risk.

The link between systemic risk in the financial system and major economic crises is well established in many important papers, including Bernanke (1983), Bernanke, Gertler, and Gilchrist (1996), and others. Systemic risk in the financial system is widely viewed as having played a central role in the recent global financial crisis and represents a motivating force behind major regulatory reforms, such as the Dodd-Frank Act. We use the five-year swap spread as a measure of the systemic credit and liquidity stresses on the financial system. As discussed by Duffie and Singleton (1997), Liu, Longstaff, and Mandell (2006), and others, the swap spread also reflects differences in the relative liquidity and credit risk of the financial sector and the Treasury. We obtain five-year swap spread data from the Bloomberg system.

We also considered a number of other measures of systemic risk, such as the spread between three-month Libor and the overnight index swap (OIS) rate, the average CDS spreads for both major U.S. and non-U.S. banks and financial firms, and the average CDS spread for investment grade bonds as measured by the CDX index. These measures, however, were highly correlated with swap spreads and provided little incremental information.

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Recent economic theory has emphasized the role that the value of collateral plays in propagating economic downturns. Key examples include Kiyotaki and Moore (1997), who show that declines in asset values can lead to contractions in the amount of credit available in the market, which, in turn, can lead to further rounds of declines in asset values. Bernanke and Gertler (1995) describe similar interactions between declines in the value of assets that serve as collateral and severe economic downturns. Collateral revaluation risk, or the risk of a broad decline in the market value of leveraged assets, played a major role in the Great Depression as the sharp declines in the values of stock and corporate bonds triggered waves of defaults among both speculators and banks. A similar mechanism was present in the recent financial crisis as sharp declines in real estate values led to massive defaults by "underwater" mortgagors. We explore the relation between deflation probabilities and stock market returns since they represent changes in the value of one of the largest potential sources of collateral in the macroeconomy. Specifically, we include the time series of returns on the value-weighted CRSP stock index.

Another major type of economic tail risk stems from the risk that a sovereign defaults on its debt. As documented by Reinhart and Rogoff (2009) and many others, sovereign defaults tend to be associated with severe economic crisis scenarios. As a measure of the tail risk of a major sovereign default, we include in the analysis the time series of sovereign CDS spreads on the U.S. Treasury. Ang and Longstaff (2013) show that the US CDS spread reflects variation in the valuation of major sources of tax revenue for the United States such as capital gains on stocks and bonds. This data is obtained from the Bloomberg system.

Finally, to capture the effect of traditional types of business-cycle risk or economic downturn risk, we also include a number of key macroeconomic variables that can be measured at a monthly frequency. In particular, we include the monthly percentage change in industrial production as reported by the Bureau of Economic Analysis, the monthly change in the national unemployment rate as reported by the Bureau of Labor Statistics, and the change in the Consumer Confidence Index reported by the University of Michigan. The link between the business cycle and its effects on output and employment are well established in the macroeconomic literature and forms the basis of many classical theories, such as the Phillips curve.

In examining the relation between these measures of tail risk and deflation risk, we use the simple approach of regressing monthly changes in deflation probabilities on monthly changes in these measures.[17] Note that while we estimate these regressions separately for each horizon, the deflation probabilities are driven by the same set of state variables: X_t, Y_t, and V_t. As a result, it is not surprising that the regression results are very similar across

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[17] In doing this, we are implicitly assuming that any estimation error in the deflation probabilities is uncorrelated with the explanatory variables in the regression. For example, see Hausman (2001).

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Table 10
Results from the regression of monthly changes in deflation probabilities on financial and macroeconomic variables

			t-statistics					
Horizon	Swap spread	Stock return	Trsy CDS	Conf	IP	Unem	Adj. R^2	N
1	1.42	−5.54**	−1.50	−2.06**	1.02	0.06	0.295	72
2	1.28	−5.87**	−1.52	−2.06**	1.08	0.08	0.306	72
3	1.36	−6.05**	−1.51	−2.05**	1.10	0.11	0.311	72
5	1.35	−6.21**	−1.49	−2.03**	1.11	0.14	0.314	72
7	1.35	−6.28**	−1.47	−2.02**	1.10	0.16	0.314	72
10	1.37	−6.31**	−1.45	−2.02**	1.08	0.18	0.315	72
12	1.38	−6.31**	−1.44	−2.01**	1.07	0.18	0.315	72
15	1.39	−6.32**	−1.42	−2.01**	1.06	0.19	0.315	72
20	1.40	−6.32**	−1.41	−2.01**	1.05	0.20	0.314	72
30	1.42	−6.31**	−1.40	−2.00*	1.02	0.20	0.314	72

This table reports the *t*-statistics and adjusted R^2s from the regression of monthly changes in the deflation probabilities for the indicated horizons on the monthly changes in the following variables: the five-year swap spread, the return on the CRSP value-weighted stock index, the five-year U.S. Treasury CDS spread, the Conference Board's Consumer Confidence Index (Conf), industrial production (IP, percentage change), and the unemployment rate (Unemp). The *t*-statistics are based on the Newey-West estimator of the covariance matrix (three lags). * and ** indicate significance at the 10% and 5% level, respectively. The sample consists of monthly observations for the period from October 2009 to October 2015.

the various horizons. Table 10 presents summary statistics for the regression results.

These results provide a number of intriguing insights into the nature of deflation risk. The most significant variable by far in the regressions is the return on the stock market. The strong negative relation (*t*-statistics on the order of six) between the variables implies that as the stock market increases, concerns about deflation diminish. This result is intuitive and consistent with the view that deflations are associated with severe adverse macroeconomic scenarios in which stock market valuations may be negatively impacted.

The results also show that deflation risk is significantly negatively related to consumer confidence. This is again consistent with the interpretation that investors associate deflation with very negative macroeconomic conditions. Finally, neither the swap spread nor the U.S. Treasury CDS spread is significant.

Although not reported, we also estimate the regressions with alternative specifications as robustness checks. For example, we include changes in the Libor-OIS spread, the CDX investment grade index, the VIX index, and the German CDS spread in the regressions. The key results from these regressions are very similar to those reported in Table 10.[18] In particular, both the stock market return and the change in consumer confidence remain significantly negatively related to changes in deflation risk, while none of these additional variables are significant. Finally, the overall adjusted R^2s from the regressions

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[18] In addition to reflecting systemic risk, measures, such as the swap spread and the Libor-OIS spread, also may be related to the level of liquidity in financial markets, potentially making their interpretation less clear. As it turns out, however, none of these variables is significant in the regressions. We are grateful to the referee for this observation.

Table 11
Summary statistics for inflation probabilities

Horizon	Mean	SE of the Mean	SD	Minimum	Median	Maximum	N
1	2.615	0.175	2.460	0.139	2.086	16.268	1,545
2	3.089	0.170	2.394	0.295	2.702	15.315	1,545
3	3.466	0.161	2.274	0.496	3.186	14.319	1,545
5	3.856	0.139	1.950	0.911	3.707	12.319	1,545
7	3.859	0.115	1.615	1.209	3.781	10.462	1,545
10	3.470	0.085	1.192	1.375	3.445	8.090	1,545
12	3.108	0.069	0.972	1.353	3.098	6.793	1,545
15	2.553	0.051	0.719	1.220	2.555	5.222	1,545
20	1.767	0.032	0.444	0.924	1.772	3.379	1,545
30	0.807	0.013	0.178	0.460	0.810	1.442	1,545

This table reports summary statistics for the probability of the average inflation rate being above four percent for the indicated horizons. Horizon is expressed in years. Probabilities are expressed as percentages. The standard error of the mean is given by the Newey-West estimator (seven lags). The sample consists of daily observations for the period from October 5, 2009 to October 28, 2015.

are surprisingly high. In particular, the adjusted R^2s for all horizons are on the order of 30%.

8. Inflation Risk

Although the focus of this paper is on deflation risk, it is straightforward to extend the analysis to other aspects of the distribution of inflation. As one last illustration of this, we compute the probabilities that the inflation rate exceeds 4% using the techniques described earlier. Table 11 reports summary statistics for these probabilities.

As shown, the market-implied probabilities of experiencing significant inflation are nontrivial. Specifically, the average probability of inflation exceeding four percent is in the range of about 2% to 4% for horizons ranging from 1 to 20 years. Figure 8 plots the time series of probabilities that inflation exceeds 4% for several horizons. These plots also show that the probability of inflation in the long-run is generally higher than in the short-run. This is the opposite of the situation for deflation risk, which tends to be higher in the short-run. In general, however, periods of high inflation uncertainty coincide with periods of both high deflation and inflation probabilities. This is intuitive since an increase in the volatility of inflation shifts probability mass to both tails.[19]

As we did earlier for deflation tail risk, we also can examine the pricing of inflation tail risk by computing the ratio of the probability of inflation under the risk-neutral measure to the corresponding probability under the actual measure. Table 12 provides summary statistics for the ratios.

As shown, inflation tail risk is significantly priced for all maturities. For many maturities, the mean and median ratios range from two to seven. This is is very consistent with the pricing of tail risk in other markets. In comparing

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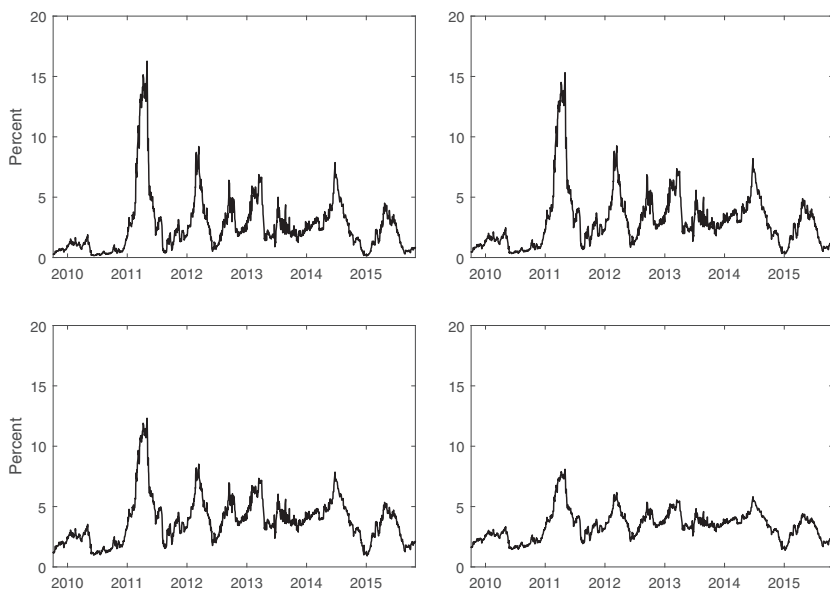[19] We are grateful to the referee for this observation.

Figure 8
Inflation probabilities
This figure plots the time series of probabilities that inflation is greater than or equal to 4% for horizons of one year (upper left), two years (upper right), five years (lower left), and ten years (lower right).

Table 12
Summary statistics for the pricing of inflation tail risk

Horizon	Mean ratio	Median ratio
1	3.776	1.689
2	3.884	2.062
3	2.963	1.684
5	3.515	2.832
7	3.974	3.589
10	4.967	4.732
12	5.538	5.423
15	6.704	6.680
20	9.078	9.235

This table reports the means and medians of the ratio of the probability of inflation greater than or equal to 4% under the risk-neutral Q measure to the corresponding probability under the actual P measure. The ratio is only computed for observations where the probability under the P measure is greater than or equal to 0.10%. Horizon is expressed in years. The sample consists of daily observations for the period from October 5, 2009 to October 28, 2015.

these ratios to those for deflation risk, we see that the tail risk of inflation is more aggressively priced for shorter horizons than is the tail risk of deflation. The opposite, however, is true for longer horizons where the mean and median ratios for deflation tail risk are many times higher than those for inflation risk. This is intuitive since the market may anticipate that the Federal Reserve is

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well equipped to manage situations of excessive inflation, making the risk of persistent high inflation relatively low. In contrast, it may be more difficult to avoid prolonged deflation, as evidenced by the case of Japan at the zero lower bound.[20] Nevertheless, Table 12 clearly shows that the market is also concerned about inflation risk, particularly in the short-term.

Consumption disaster models can replicate the key facts about nominal bond return predictability provided that inflation jumps in a disaster state (see Gabaix 2012). As a result of these inflation jumps in disaster states, the risk-neutral probability of a large inflation is much higher than the actual probability, and the nominal bond risk premium increases as a result. Our results point in the opposite direction. We actually find direct evidence from inflation derivatives that market participants are pricing in large deflation in disaster states, because the risk-neutral probability of a deflation is larger than the actual probability. This actually makes nominal bonds less risky because they provide a hedge against large consumption disasters.

9. Conclusion

We solve for the objective distribution of inflation by using the market prices of inflation swap and option contracts and study the nature of deflation risk. We find that the market-implied probabilities of deflation are substantial, even though the expected inflation rate is roughly 2.50% for horizons of up to 30 years. We show that deflation risk is priced by the market in a manner similar to that of other major types of tail risks, such as catastrophic insurance losses or corporate bond defaults. By embedding a deflation floor into newly issued TIPS, the Treasury insures bondholders against deflation. Our findings imply that the Treasury receives a generous insurance premium in return. In contrast, the market appears less concerned about long-run inflation tail risk.

In theory, economic tail risks, such as deflation, may be related to other financial and macroeconomic tail risks. We find that the tail risk of deflation is strongly negatively related to the level of the stock market and consumer confidence. These results support the view that the risk of economic shocks severe enough to result in deflation is fundamentally related to the risk of major shocks in the financial markets, both locally and globally.

Appendix A. Inflation Swap Rate

From Equations (1) and (5), the relative price index level at time T can be expressed as

$$I_T = I_0 \exp\left(\int_0^T X_s \, ds\right) \exp\left(-\frac{1}{2}\int_0^T V_s \, ds + \int_0^T \sqrt{V_s} dZ_I(s)\right). \tag{A1}$$

Without loss of generality, we normalize I_0 to one. The cash flow associated with a zero-coupon inflation swap at time T is simply $I_T - F(X,Y,V,T)$, where $F(X,Y,V,T)$ is the inflation swap

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[20] We are grateful to the referee for these insights.

price at the initiation of the contract at time zero. Since the present value of the inflation swap is zero at inception, we have,

$$E^Q\left[\exp\left(-\int_0^T r_s\, ds\right)(I_T - F(X,Y,V,T))\right] = 0, \tag{A2}$$

where the expectation is taken with respect to the risk-neutral measure (represented by a Q). Substituting in for r_t and I_T gives,

$$E^Q\left[\exp\left(-\int_0^T R_s\, ds\right)\right]$$

$$\left[E^Q\left[\exp\left(-\int_0^T X_s\, ds\right)\exp\left(\int_0^T X_s\, ds\right)\exp\left(-\frac{1}{2}\int_0^T V_s\, ds + \int_0^T \sqrt{V_s}\, dZ_I(s)\right)\right]\right]$$

$$-E^Q\left[\exp\left(-\int_0^T X_s\, ds\right)F(X,Y,V,T)\right]\Bigg] = 0, \tag{A3}$$

which implies

$$F(X,Y,V,T) = \frac{E^Q\left[\exp\left(-\frac{1}{2}\int_0^T V_s\, ds + \int_0^T \sqrt{V_s}\, dZ_I(s)\right)\right]}{E^Q\left[\exp\left(-\int_0^T X_s\, ds\right)\right]}, \tag{A4}$$

$$= \frac{1}{E^Q\left[\exp\left(-\int_0^T X_s\, ds\right)\right]}. \tag{A5}$$

Let $H(X,Y,\tau)$ denote the value of the expectation $E^Q[\exp(-\int_t^T X_s\, ds)]$, where $\tau = T - t$. Standard results imply that this expectation satisfies the partial differential equation

$$\tfrac{1}{2}\eta^2 H_{XX} + \tfrac{1}{2}s^2 H_{YY} + \lambda(Y - X)H_X + (\alpha - \beta Y)H_Y - XH = H_\tau, \tag{A6}$$

subject to the terminal condition $H(X,Y,0) = 1$. We conjecture a solution of the form $H(X,Y,\tau) = \exp(A(\tau) + B(\tau)X + C(\tau)Y)$. Taking derivatives of this expression and substituting into Equation $(A6)$ results in a system of three linear first-order ordinary differential equations for the horizon-dependent functions $A(\tau)$, $B(\tau)$, and $C(\tau)$,

$$B' + \lambda B = -1, \tag{A7}$$

$$C' + \beta C = \lambda B, \tag{A8}$$

$$A' = \tfrac{1}{2}\eta^2 B^2 + \tfrac{1}{2}s^2 C^2 + \alpha C. \tag{A9}$$

These three equations are readily solved by the use of an integrating factor and direct integration. We substitute these solutions into the expression for $H(X,Y,\tau)$, and then substitute $H(X,Y,\tau)$ into Equation $(A5)$. Finally, evaluating as of time zero ($\tau = T$) and recognizing that the inflation swap price does not depend on the value V, we obtain the expression for the inflation swap price in Equation (9).

Appendix B. Inflation Option Prices

Let $C(X,Y,V,T;K)$ denote the price at time zero of a European call option on the price level at time T with strike K. The cash flow at the option expiration date is $\max(0, I_T - (1+K)^T)$. The

present value of this cash flow can be expressed as

$$E^Q\left[\exp\left(-\int_0^T r_s\,ds\right)\max(0,I_T-(1+K)^T)\right], \tag{B1}$$

which can be written as

$$E^Q\left[\exp\left(-\int_0^T R_s\,ds\right)\right]E^Q\left[\exp\left(-\int_0^T X_s\,ds\right)\max(0,I_T-(1+K)^T)\right], \tag{B2}$$

after substituting in for r_t. Let $N(I,X,Y,V,\tau)$ denote the value of the expectation $E^Q[\exp(-\int_t^T X_s\,ds)\max(0,I_T-(1+K)^T)]$, where for clarity we show the explicit functional dependence of $N(I,X,Y,V,\tau)$ on I. The value of $N(I,X,Y,V,\tau)$ satisfies the following partial differential equation,

$$\tfrac{1}{2}VI^2N_{II}+\tfrac{1}{2}\eta^2N_{XX}+\tfrac{1}{2}s^2N_{YY}+\tfrac{1}{2}\sigma^2VN_{VV}+XIN_I$$

$$+\lambda(Y-X)N_X+(\alpha-\beta Y)N_Y+(\theta-\phi V)N_V-XN=N_\tau, \tag{B3}$$

subject to the terminal condition $N(I,X,Y,V,0)=\max(0,I_T-(1+K)^T)$. We conjecture that the solution is of the form

$$N(I,X,Y,V,\tau)=H(X,Y,V,\tau)\,W(I,X,Y,V,\tau). \tag{B4}$$

Substituting this expression into the partial differential equation in Equation (B3), recognizing that $H(X,Y,V,\tau)$ satisfies Equation (A6), and simplifying gives

$$\tfrac{1}{2}VI^2W_{II}+\tfrac{1}{2}\eta^2W_{XX}+\tfrac{1}{2}s^2W_{YY}+\tfrac{1}{2}\sigma^2VW_{VV}+XIW_I$$

$$+(\lambda(Y-X)+\eta^2B(\tau))W_X+(\alpha-\beta Y+s^2C(\tau))W_Y$$

$$+(\theta-\phi V)W_V=W_\tau. \tag{B5}$$

This implies that $W(I,X,Y,V,\tau)$ can be expressed as

$$W(I,X,Y,V,\tau)=E^{Q^*}[\max(0,I_T-(1+K)^T)], \tag{B6}$$

where the expectation is taken with respect to the density of I_T implied by the dynamics (we represent this measure by Q^*),

$$dI=X\,I\,dt+\sigma\,I\,dZ_I, \tag{B7}$$

$$dX=(\lambda\,(Y-X)+\eta^2B(\tau))\,dt+\eta\,dZ_X, \tag{B8}$$

$$dY=(\alpha-\beta\,Y+s^2C(\tau))\,dt+s\,dZ_Y. \tag{B9}$$

$$dV=(\theta-\phi)V\,dt+\sigma\sqrt{V}\,dZ_V, \tag{B10}$$

Since

$$D(T)=E^Q\left[\exp\left(-\int_0^T r_s\,ds\right)\right], \tag{B11}$$

$$=E^Q\left[\exp\left(-\int_0^T R_s\,ds\right)\right]H(X,Y,V,\tau), \tag{B12}$$

combining these results implies

$$C(X,Y,V,T;K)=D(T)\,E^{Q^*}[\max(0,I_T-(1+K)^T)]. \tag{B13}$$

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To derive inflation option prices, we next solve for the distribution of $\ln I_T$ under the Q^* measure. From Equation (A1), $\ln I_T$ can be expressed as

$$\ln I_T = w_T + u_T, \tag{B14}$$

where

$$w_T = \int_0^T X_t \, dt, \tag{B15}$$

$$u_T = -\frac{1}{2} \int_0^T V_t \, dt + \int_0^T \sqrt{V_t} \, dZ_I(t). \tag{B16}$$

w_T and u_T are independent given the structure of the model.

We first focus on the distribution of w_T and solve the stochastic differential equation in Equation (B9) for Y_u, which gives

$$Y_u = Y_0 e^{-\beta u} + \frac{\alpha}{\beta}(1 - e^{-\beta u}) + s^2 e^{-\beta u} \int_0^u e^{\beta r} C(T - r) \, dr$$

$$+ s e^{-\beta u} \int_0^u e^{\beta r} \, dZ_Y(r). \tag{B17}$$

Likewise, solving for X_t gives

$$X_t = X_0 e^{-\lambda t} + \lambda e^{-\lambda t} \int_0^t e^{\lambda u} Y_u \, du + \eta^2 e^{-\lambda t} \int_0^u e^{\lambda u} B(T - u) \, du$$

$$+ \eta \, e^{-\lambda t} \int_0^u e^{\lambda u} \, dZ_X(u). \tag{B18}$$

Substituting Equation (B17) into the above equation, interchanging the order of integration, and evaluating terms gives the following expression for X_t,

$$X_t = X_0 e^{-\lambda t} + \eta^2 e^{-\lambda t} \int_0^t e^{\lambda u} B(T - u) \, du$$

$$\lambda e^{-\lambda t} \int_0^t e^{\lambda u} \left(Y_0 e^{-\beta u} + \frac{\alpha}{\beta}(1 - e^{-\beta u}) + s^2 e^{-\beta u} \int_0^u e^{\beta r} C(T - r) \, cr \right) du$$

$$+ \frac{\lambda s}{\lambda - \beta} \int_0^t (e^{-\beta(t-u)} - e^{-\lambda(t-r)}) dZ_Y(r)$$

$$+ \eta e^{-\lambda t} \int_0^t e^{\lambda u} dZ_X(u), \tag{B19}$$

Taking the integral of X_t, interchanging the order of integration, and evaluating terms gives

$$\int_0^T X_t \, dt = \int_0^T \left(X_0 e^{-\lambda t} + \eta^2 e^{-\lambda t} \int_0^t e^{\lambda u} B(T - u) \, du \right.$$

$$+ \lambda e^{-\lambda t} \int_0^t e^{\lambda u}(Y_0 e^{-\beta u} + \frac{\alpha}{\beta}(1 - e^{-\beta u})$$

$$\left. + s^2 e^{-\beta u} \int_0^u e^{\beta r} C(T - r) \, dr) \, du \right) dt$$

$$+ \frac{\lambda s}{\lambda - \beta} \int_0^t \frac{1}{\beta} \left(1 - e^{-\beta(T-r)} \right) - \frac{1}{\lambda} \left(1 - e^{-\lambda(T-r)} \right) dZ_Y(r)$$

$$+ \frac{\eta}{\lambda} \int_0^T 1 - e^{-\lambda(T-u)} dZ_X(u), \tag{B20}$$

Thus, $\int_0^T X_t \, dt = w_T$ is a normally distributed random variable.

Under the Q^* measure, the expected value of the price level equals the inflation swap price F. This follows since the cash flow from an inflation swap at time T is $I_T - F$. Under the Q^* measure, however, the present value of this cash flow is given by $D(T)E^{Q^*}[I_T - F]$. Since the initial value of the inflation swap contract is zero, this implies $E^{Q^*}[I_T] = F$.

This result, in conjunction with the fact that $E^{Q^*}[e^{u_T}] = 1$, implies that w_T is normally distributed with mean

$$\ln F(X,Y,T) - \tfrac{1}{2}G(T), \tag{B21}$$

and variance $G(T)$

$$G(T) = \frac{s^2\lambda^2}{(\lambda-\beta)^2}\left(\frac{1}{\beta^2}\left(T - \frac{2}{\beta}(1-e^{-\beta T}) + \frac{1}{2\beta}(1-e^{-2\beta T})\right)\right.$$

$$- \frac{2}{\beta\lambda}\left(T - \frac{1}{\beta}(1-e^{-\beta T}) - \frac{1}{\lambda}(1-e^{-\lambda T}) + \frac{1}{\beta+\lambda}(1-e^{-(\beta+\lambda)T})\right)$$

$$+ \left.\frac{1}{\lambda^2}\left(T - \frac{2}{\lambda}(1-e^{-\lambda T}) + \frac{1}{2\lambda}(1-e^{-2\lambda T})\right)\right)$$

$$+ \frac{\eta^2}{\lambda^2}\left(T - \frac{2}{\lambda}(1-e^{-\lambda T}) + \frac{1}{2\lambda}(1-e^{-2\lambda T})\right). \tag{B22}$$

Turning now to the distribution of u_T, we observe that Equation (B16) implies the dynamics

$$du = -\tfrac{1}{2}V\,dt + \sqrt{V}\,dZ_I, \tag{B23}$$

$$dV = (\alpha - \beta V)dt + \sigma\sqrt{V}\,dZ_V, \tag{B24}$$

But this is a special case of the Heston (1993) model. Following his results, the characteristic function $E[e^{i\zeta u_T}]$ of u_T is given by

$$\exp(L(T) + M(T)V + i\zeta), \tag{B25}$$

where

$$L(T) = \frac{\alpha(\beta+\gamma)T}{\sigma^2} + \frac{2\alpha}{\sigma^2}\ln\left(\frac{1-k_0}{1-k_0 e^{\gamma T}}\right), \tag{B26}$$

$$M(T) = \frac{\beta-\gamma}{\sigma^2} + \frac{2\gamma}{\sigma^2(1-k_0 e^{\gamma T})}, \tag{B27}$$

and where

$$\gamma = \sqrt{\sigma^2(\zeta^2 + i\zeta) + \beta^2}, \tag{B28}$$

$$k_0 = (\beta+\gamma)/(\beta-\gamma), \tag{B29}$$

The density of u_T is given by inverting the characteristic function, resulting in the expression in Equation (16). The value of the inflation call option is given by evaluating the expectation in Equation (B13), where the joint density for w_T and u_T is the product of the normal density for w_T and the density of u_T. Substituting these densities into Equation (B13), and integrating with respect to w_T leads to the expression in Equation (17).

Appendix C. Details of the Estimation Procedure

At each date t, we have inflation swap rates for the following maturities: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 15, 20, 25, and 30 years. In addition, for each date t, we have roughly 20 to 25 inflation cap and floor prices for varying strikes ranging from -3% to 6% for the following maturities: 1, 2, 3, 5, 7, 10, 12, 15, 20, and 30 years. This means that for each date during the sample period, we have 15 inflation swap rates and roughly 200 to 250 option prices available from which to estimate the state variables X_t, Y_t, and V_t and the parameters of the model. Given the large volume of pricing data available at each date, it is helpful to first reduce the dimensionality of the estimation problem to retain computational tractability.

In doing this, for each date t and for each horizon T, we first solve for the implied variance of the risk neutral density of $\ln I_T$ from the 20 to 25 option prices available for that horizon. Specifically, for each horizon, we use a four-term Edgeworth expansion of the density of $\ln I_T$. This Edgeworth expansion is parameterized in terms of the cumulants of the distribution of $\ln I_T = w_T + u_T$. Since w_T and u_T are independent, the cumulants of the distribution of $\ln I_T$ are equal to the sum of the cumulants for the distributions of w_T and u_T. As shown above, w_T is normally distributed. Thus, given the properties of the normal distribution, the first and second cumulants of w_T are the mean and the variance of w_T, while all higher cumulants are zero. The cumulants of u_T can be obtained in closed form by repeatedly differentiating the log of the characteristic function of u_T

$$\exp(L(T) + M(T)V + i\zeta), \tag{C1}$$

with respect to the argument ζ and evaluating the derivatives at $\zeta = 0$. The first cumulant of u_T is the mean of u_T and is given by

$$-\frac{1}{2}\left(V - \frac{\alpha}{\beta}\right)\frac{1}{\beta}\left(1 - e^{-\beta T}\right) - \frac{1}{2}\frac{\alpha}{\beta}T. \tag{C2}$$

The second cumulant of u_T is the variance of u_T and is given by

$$c_1\left(-\frac{\alpha}{\beta^2}(1 - e^{-\beta T}) + \frac{\alpha}{\beta}T\right)$$

$$+c_2\left(-\frac{\alpha}{\beta}e^{-\beta T}T + \frac{\alpha}{\beta^2}(1 - e^{-\beta T})\right)$$

$$+c_3\left(-\frac{\alpha}{\beta^2}(e^{-\beta T} - e^{-2\beta T}) + \frac{\alpha}{2\beta^2}(1 - e^{-2\beta T})\right)$$

$$+\left(\frac{c_1}{\beta}(1 - e^{-\beta T}) + c_2 e^{-\beta T}T + \frac{c_3}{\beta}(e^{-\beta T} - e^{-2\beta T})\right)V, \tag{C3}$$

where

$$c_1 = 1 + \frac{\sigma^2}{4\beta^2}, \tag{C4}$$

$$c_2 = -\frac{\sigma^2}{2\beta^2}, \tag{C5}$$

$$c_3 = \frac{\sigma^2}{4\beta^2}, \tag{C6}$$

The third and fourth cumulants of u_T have very complex representations. For a wide range of realistic parameter values, however, we find that these higher order cumulants are orders of magnitude smaller than the first two cumulants. In fact, we find that the option prices obtained using the values of these higher cumulants are numerically indistinguishable from those obtained

by setting these cumulants to zero. For computational efficiency, we adopt this simplification since it results in no loss of accuracy.

Given the Edgeworth expansion of the risk-neutral density of $\ln I_T$, we can now obtain the value of a specific inflation option by integrating the expression in Equation (17) numerically. We then solve for the implied cumulants of the distribution of $\ln I_T$ that result in the best fit to the actual market prices of the inflation options in a root-mean-squared sense. That $E^{Q^*}[I_T]$ equals the inflation swap price places a constraint on the first cumulant of the distribution for $\ln I_T$ which we impose as part of the fitting procedure. The implied second cumulant obtained through this fitting procedure is also the implied variance of the distribution of $\ln I_T$. We solve for the implied variance of $\ln I_T$ for each date t and each horizon T in the sample. An inspection of the fitting errors indicates that the algorithm is able to fit inflation caps and floors equally well across strikes.

Following Duffie and Singleton (1997), Duffee (2002), and others, we use the following approach to identify the values of the state variables. Specifically, we assume that the 2-year and 30-year inflation swap rates are measured without error. Similarly, we assume that the variance of the risk-neutral density implied from the three-year options is measured without error. Given a parameter vector Θ, the logs of the 2-year and 30-year inflation swap prices

$$\ln F(X,Y,2) = -A(2) - B(2)X - C(2)Y, \tag{C7}$$

$$\ln F(X,Y,30) = -A(30) - B(30)X - C(30)Y, \tag{C8}$$

are linear in the state variables X and Y. Since $\ln I_T = w_T + u_T$, the above results imply that the variance of the risk-neutral density of $\ln I_3$ is the sum of $G(T)$ plus the expression in Equation (C3) (where both are evaluated at $T=3$) which is linear in V. Thus, Equations (C7), (C8), and (C3) constitute a linear system in X, Y, and V. Let J denote the Jacobian of the linear mapping from the inflation swaps and the implied variance into the state variables X, Y, and V.

Appendix D. Conditional Moments and the Density of V

Integrating the dynamics for X and Y under the P measure as given in Appendix A results in the following expressions for the conditional means and variances

$$\mu_{Y_t} = Y_t e^{-\xi \Delta t} + (\mu/\xi)(1 - e^{-\xi \Delta t}), \tag{D1}$$

$$\mu_{X_t} = X_t e^{-\kappa \Delta t} + (\mu/\xi)(1 - e^{-\kappa \Delta t})$$
$$+ \frac{\kappa}{\kappa - \xi}(Y_t - \mu/\xi)(e^{-\xi \Delta t} - e^{-\kappa \Delta t}), \tag{D2}$$

$$\sigma_Y^2 = \frac{s^2}{2\xi}(1 - e^{-2\xi \Delta t}), \tag{D3}$$

$$\sigma_X^2 = \frac{\kappa^2 s^2}{(\kappa - \xi)^2}\left(\frac{1}{2\xi}(1 - e^{-2\xi \Delta t}) - \frac{2}{\xi + \kappa}(1 - e^{-(\xi + \kappa)\Delta t})\right.$$
$$\left. + \frac{1}{2\kappa}(1 - e^{-2\kappa \Delta t})\right) + \frac{\eta^2}{2\kappa}(1 - e^{-2\kappa \Delta t}), \tag{D4}$$

$$\sigma_{XY} = \frac{\kappa s^2}{\kappa - \xi}\left(\frac{1}{2\xi}(1 - e^{-2\xi \Delta t}) - \frac{1}{\xi + \kappa}(1 - e^{-(\xi + \kappa)\Delta t})\right). \tag{D5}$$

The density of $V_{t+\Delta t}$ conditional on V_t is obtained directly from the expression for the density of the interest rate given in Cox, Ingersoll, and Ross (1985) Equation (18) (after substituting in the notation in Equation (4)).

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Appendix E. Inflation Surveys

The data from the University of Michigan Survey of Consumers consist of one- and five-year ahead inflation forecasts. The series is released at monthly frequency and reports the median expected price change over the next 12 months and the next five years, respectively. A detailed description of how the survey is conducted is available at http://www.sca.isr.umich.edu/documents php?c=i. In contrast to the participants in the Livingston survey and the Survey of Professional Forecasters, the participants in the University of Michigan Survey of Consumers are actual consumers (households), not professionals. The time between when the survey is conducted and the time survey results are released is up to three weeks. The University of Michigan reports that a review of the estimates of inflation expectations indicated that for comparisons over time, the median, rather than the mean, may be a more reliable measure of the central tendency of the response distribution due to the changing influence of extreme responses. Therefore, we use the median survey forecasts throughout our analysis.

The Philadelphia Federal Reserve Bank Survey of Professional Forecasters is conducted on a quarterly basis. The questionnaires are sent to the participants at the end of January, April, July, and October, and the survey results are published in the middle of February, May, August, and November, for the first, second, third, and fourth quarter, respectively. In contrast to the Livingston survey, participants in the SPF forecast changes in the quarterly average CPI-U levels. A detailed description of how the survey is conducted is available at http://www.philadelphiafed.org/research-and-data/real-time-center/survey-of-professionalfore casters/spf-documentation.pdf.

The Livingston survey is conducted twice a year, in June and in December, usually in the middle of the month. Participants include economists from industry, government, and academia. The surveys taken in June consist of two annual average CPI forecasts: for the current year, and for the following year. The December surveys include three annual average forecasts: for the current year, for the next year, and for the year after. The participants forecast the nonseasonally adjusted CPI level 6 and 12 months in the future. The survey also includes a forecast of the CPI ten years in the future. A detailed description of how the Livingston survey is conducted is available at: http://www.philadelphiafed.org/research-and-data/real-time-center/livingston-survey/livingston-documentation.pdf.

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