Interest Rate Volatility and Bond Prices

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The risk of a default-free bond stems from two major sources—interest rate shifts and changes in bond market volatility. The first type of risk is well known. The second is less familiar, although it can represent a major component of the total risk of a fixed-income portfolio.

The past 15 years have included some of the most volatile periods fixed-income portfolio managers have experienced this century. One-month Treasury-bill yields have been higher than 16% and lower than 4%. As a result, fixed-income managers have become acutely aware of interest rate risk and their exposure to it.

Managing interest rate risk requires measuring it first. Duration analysis has become an important tool, allowing portfolio managers to measure the sensitivity of their portfolios to changes in the level of interest rates. But duration analysis has a number of serious drawbacks. Standard duration analysis, for instance, allows for only parallel shifts in the term structure. Thus portfolio managers may remain exposed to substantial risk arising from nonparallel shifts. Furthermore, because duration and convexity analyses focus on the risk of changes in the *level* of interest rates, they ignore other types of relevant risks, including changes in the frequency of large movements in interest rates.

This paper shows that the price risk inherent in a default-free bond has two major sources—the risk of changes in the level of interest rates and the risk of changes in the volatility of interest rates. The volatility risk of a bond can be an important component of its total risk, but few managers currently hedge fixed-income portfolios against volatility risk.

Historical Volatility Risk

To demonstrate that interest rate volatility has a significant effect on bond prices, we first examine the historical relation between volatility and yields. In doing this, we let ΔY_T represent the monthly change in the T-maturity Treasury yield; Δr the monthly change in the one-month Treasury-bill yield; and ΔV the change in the volatility of interest rates.[1]

Now consider the following regression equation:

$$\Delta Y_T = \alpha + \beta \Delta r + \gamma \Delta V + \epsilon.$$

In this regression, β measures how sensitive changes in T-maturity Treasury yields are to changes in the short-term interest rate r. If shifts in the yield curve were always parallel, there would be a one-to-one relation between ΔY_T and Δr, and the slope parameter β would equal one. The regression parameter γ is a measure of how sensitive changes in yields are to changes in interest rate volatility.[2] Again, if term structure shifts were always parallel, changes in volatility would not be useful in explaining yield movements, after controlling for the effects of a change in r, and γ would equal zero.

Table I gives the results of this regression, estimated using Treasury yield data, for the 1964–89 period. Note that the parameter β is always *less* than one, and gets smaller as we go farther out along the yield curve. Table I implies, for example, that if the one-month T-bill yield increased by 100 basis points, the one-year yield would increase by only 52.2 basis points, while the five-year yield would increase by only 25.4 basis points. This is clear evidence that shifts in the yield curve are dramatically different from parallel shifts—a fundamental assumption of standard duration and convexity analyses.

Table I also shows that, even after considering the effects of changes in the level of the yield curve as measured by r, there is a significant relation between yields and interest rate volatility for all maturities. This is direct evidence that prices of Treasury bonds are affected by both the level of interest rates and the volatility of interest rates.

Interestingly, Table I indicates that the relation between yields and volatility is negative. This means that an increase in the uncertainty about interest rate changes leads to a decrease in bond yields. This makes sense, because it means that investors are willing to pay more for securities that allow them to lock in a long-term, guaranteed rate of return when the uncertainty about future money-market yields increases.

Table I Regression of Changes in Short-Term Interest Rate and Changes in Interest Rate Volatility on Changes in Yields to Maturity, 1964 to 1989

$$\Delta Y_T = \alpha + \beta \Delta r + \gamma \Delta V + \epsilon$$

Maturity	α	β	γ	t_α	t_β	t_γ	R^2
3 Months	.000	.666	−.730	.34	22.51	−2.51	.630
6 Months	.000	.562	−.991	.33	17.21	−3.08	.504
9 Months	.000	.514	−.788	.29	14.65	−2.29	.422
1 Year	.000	.522	−1.141	.31	15.34	−3.41	.451
2 Years	.000	.377	−.962	.34	11.92	−3.09	.336
3 Years	.000	.328	−.777	.41	11.16	−2.69	.305
4 Years	.000	.278	−.731	.41	9.37	−2.50	.239
5 Years	.000	.254	−.820	.46	9.40	−3.01	.246

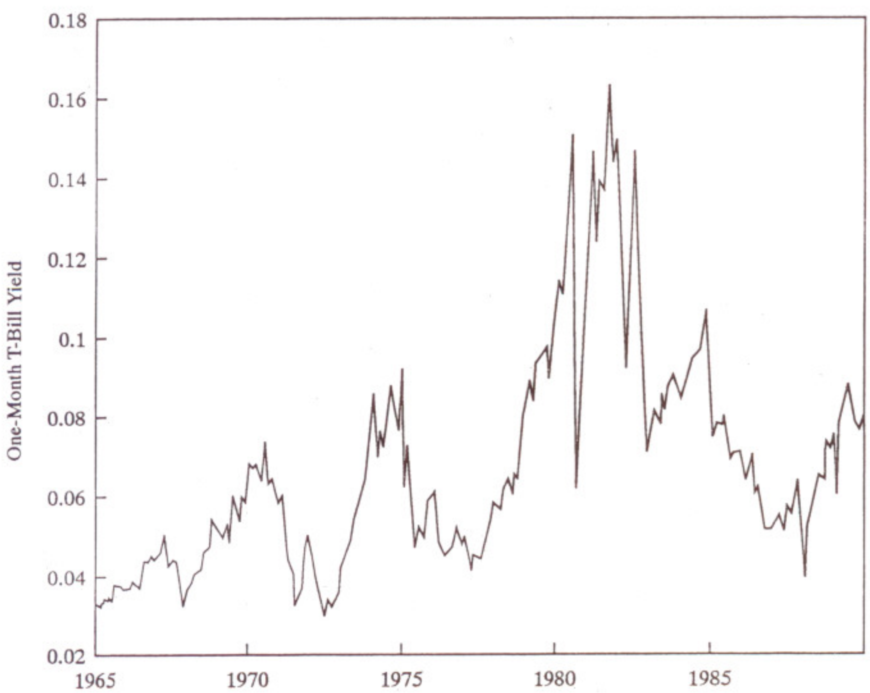
This negative relationship, however, does not grow in line with the maturity of the bond. In particular, γ is −0.730 for three-month yields, increases to a maximum of −1.141 for one-year yields, and then decreases for longer maturities. Table I shows that three-month and four-year yields are almost equally sensitive to changes in volatility.

These results demonstrate that the price risk of default-free bonds has two important dimensions—the risk of changes in interest rates and the risk of changes in interest rate volatility. To give a historical perspective of how variable these two sources of risk are, Figure A plots the one-month Treasury-bill yield during the 1964–89 period, and Figure B plots the volatility of interest rates over the same period. Note that the one-month rate and the volatility measure move together but are not perfectly correlated. Furthermore, while interest rates experience dramatic changes, the *level* of interest rate volatility is even more variable.

The Longstaff-Schwartz Model

To manage the price risk of a fixed-income portfolio, a manager clearly needs to be able to estimate the sensitivity of the portfolio to changes in interest rate volatility. In developing tools for measuring volatility risk, we draw upon a recent paper by Longstaff and Schwartz that develops a simple general-equilibrium,

Figure A One-Month Treasury-Bill Rate



Glossary

▶***Closed-Form Solution:***
An explicit formula that involves only simple and easily programmable mathematical expressions.

▶***Convexity:***
A measure of the nonlinear effect on bond prices of changes in the level of the yield curve.

▶***Duration:***
A measure of the percentage sensitivity of a bond price to changes in the level of the yield curve.

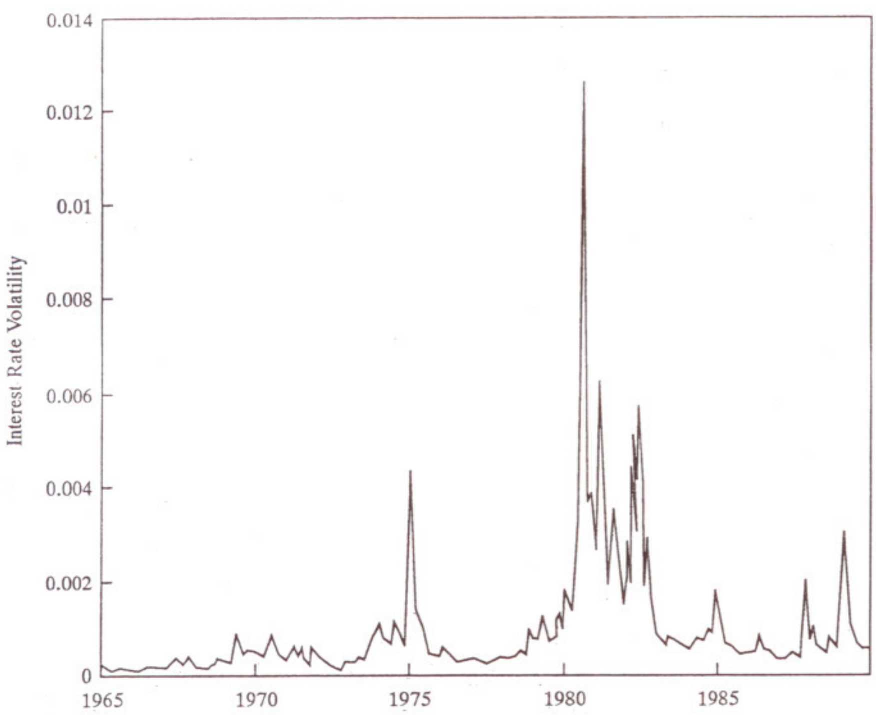
▶***Interest Rate Volatility:***
The variance of changes in the level of the yield curve.

▶***Macaulay Duration:***
The simplest and most common approach to measuring bond duration, the Macaulay duration measure assumes that all yield curve shifts are parallel.

▶***Volatility Risk:***
The relation between bond prices and interest rate volatility makes fixed-income portfolios susceptible to changes in volatility.

▶***Yield Sensitivity:***
The relation between yields and the level of interest rate volatility.

Figure B Variance of Changes in One-Month Treasury-Bill Rate



term-structure model that explicitly captures the effect on bond prices of changes in interest rate volatility.[3]

The Longstaff and Schwartz model starts from fundamental economic considerations about investment opportunities, investors' risk preferences, technological change, the nature of financial markets, and investment uncertainty in the economy. It solves for the market price of interest rate risk that determines the required expected rate of return on securities with interest-rate-sensitive prices. Once the required expected rate of return is known, the equilibrium prices for default-free bonds can be determined simply by discounting their cash flows at the appropriate discount rate. Longstaff and Schwartz show that the prices of default-free bonds depend on two variables in addition to their maturity—the value of the short-term interest rate, r, and the level of interest-rate volatility, V.[4]

Measuring Volatility Risk

Since the Longstaff-Schwartz model results in simple, closed-form solutions for the value of bonds in terms of the variables r and V, it is easy to use the model to compute the price effect of a change in interest rate volatility. Table II shows the percentage price change resulting from a 25-basis-point change in the standard deviation of interest rate changes for bonds with varying maturities and coupon rates, holding r constant. In computing these price changes, we used parameter values for the Longstaff-Schwartz model that allowed the model to match the one-month, one-year, two-year, three-year, four-year and 30-year Treasury yields as of January 15th, 1992, as well as imply long-run average values for r and the standard deviation of changes in r of 0.05 and 0.0225, respectively.[5]

As Table II shows, percentage changes in bond values increase with maturity. For example, a 25-basis-point increase in volatility increases a six-month, zero-coupon bond price by 0.19%, a one-year, zero-coupon bond price by 0.49%, and a 30-year, zero-coupon bond price by 1.31%. Similar results hold for the other coupon bonds. The table also shows that the percentage change in bond price is smaller, the larger the coupon rate of the bond. Observe that the percentage changes increase rapidly out to about five years, and then level off.

Table III shows the effects of a 25-basis-point increase in the standard deviation of interest rate volatility on bond prices. For the zero-coupon bond, the price ef-

Table II Percentage Change in Price for Bonds with Par Value of 100 Resulting from 25-Basis-Point Change in Annualized Standard Deviation of Changes in Short-Term Rate (from 0.0275 to 0.0300)

	Coupon Rate			
Maturity	*0%*	*5%*	*10%*	*15%*
6 Months	0.19	0.19	0.19	0.19
1 Year	0.49	0.49	0.48	0.47
2 Years	0.93	0.90	0.87	0.85
3 Years	1.14	1.09	1.04	1.01
4 Years	1.24	1.17	1.12	1.08
5 Years	1.28	1.20	1.15	1.11
10 Years	1.31	1.21	1.17	1.15
15 Years	1.31	1.20	1.18	1.16
20 Years	1.31	1.20	1.18	1.17
25 Years	1.31	1.19	1.18	1.18
30 Years	1.31	1.19	1.18	1.18

Table III Price Change for Bonds with Par Value of 100 Resulting from a 25-Basis-Point Change in Annualized Standard Deviation of Changes in Short-Term Rate (from 0.0275 to 0.0300)

	Coupon Rate			
Maturity	*0%*	*5%*	*10%*	*15%*
6 Months	.189	.194	.198	.203
1 Year	.470	.486	.503	.519
2 Years	.827	.882	.936	.991
3 Years	.944	1.045	1.145	1.246
4 Years	.947	1.095	1.243	1.391
5 Years	.905	1.099	1.293	1.487
10 Years	.622	1.003	1.384	1.764
15 Years	.417	.924	1.430	1.936
20 Years	.279	.870	1.461	2.052
25 Years	.186	.835	1.481	2.129
30 Years	.126	.811	1.496	2.181

fect is 0.189 for a maturity of six months, increases to 0.947 for a maturity of four years, and then decreases for longer maturities. Similarly, the price effect for the 5% coupon bond reaches a maximum of 1.099 for a maturity of five years. In contrast, the price effects for the 10% and 15% bonds increase with maturity. Thus volatility sensitivity is hump-shaped for low-coupon bonds but monotonically increasing for higher-coupon bonds. Figure C plots the data from Table III.

Because changes in volatility of the magnitudes shown in Tables II and III are not uncommon, these results show that volatility can have significant effects on bond prices, even when the *level* of the term structure is held constant. To give a better sense of the relative sizes of these price effects, Table IV presents the corresponding changes in yield to maturity for the same bonds shown in Tables II and III.

Figure C Price Change in Bond with Par Value of 100 Resulting from 25-Basis-Point Change in Standard Deviation of Interest Rate Changes

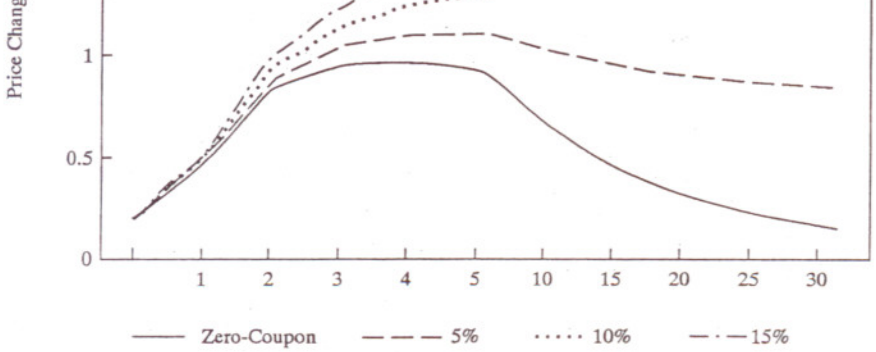


Table IV shows that the yields on intermediate-maturity bonds experience the largest impact from volatility changes. For example, a 25-basis-point increase in volatility decreases the yield on a six-month, zero-coupon bond by 38.6 basis points, the yield on a one-year, zero-coupon bond by 49.3 basis points, and the yield on a 30-year, zero-coupon bond by only 4.4 basis points. Similar results hold for the other three coupon rates. Note also that the higher the coupon rate, the higher the sensitivity of yields to changes in volatility. The negative relation between yields and volatility, as well as the hump at the one-year maturity, are in close agreement with the actual properties of yields shown in Table I.

Volatility Risk and Duration

To compare the volatility sensitivity of bonds to their yield sensitivity, Table V gives the simple Macaulay durations for the bonds. It is easily seen by comparing Tables V and II that the volatility sensitivity of a bond bears little relation to its duration. For example, the duration or yield sensitivity is proportional to bond maturity. But Table II shows that the volatility sensitivity of a zero-coupon bond levels off rapidly with increasing maturity. Furthermore, the volatility sensitivity of a bond is much less affected by the coupon rate than is the duration of the bond.

Finally, it is easy to show that the volatility sensitivity of a bond is quite different from its convexity. This is because convexity increases gradually for shorter durations but more rapidly for longer durations.[6] The opposite is true for volatility sensitivity.

Conclusion

We have shown that changes in the volatility of interest rates can have large effects on the prices and yields of bonds. These effects

Table IV Change in Yield to Maturity (measured in basis points) for Bonds with Par Value of 100 Resulting from a 25-Basis-Point Change in Annualized Standard Deviation of Changes in Short-Term Rate (from 0.0275 to 0.0300)

	Coupon Rate			
Maturity	*0%*	*5%*	*10%*	*15%*
6 Months	−38.6	−38.6	−38.6	−38.6
1 Year	−49.3	−50.3	−50.2	−50.1
2 Years	−46.3	−47.7	−47.7	−47.7
3 Years	−37.9	−39.5	−39.9	−40.2
4 Years	−30.8	−32.7	−33.4	−34.0
5 Years	−25.5	−27.6	−28.6	−29.4
10 Years	−13.1	−16.1	−17.6	−18.6
15 Years	−8.7	−12.4	−13.9	−14.8
20 Years	−6.5	−10.7	−12.1	−12.8
25 Years	−5.2	−9.9	−11.1	−11.6
30 Years	−4.4	−9.4	−10.4	−10.7

bear little or no relation to the duration or convexity of the bond. In fact, these effects have the greatest impact on the prices and yields of intermediate-term bonds.

The sensitivity of bond prices to changes in volatility has many important implications for fixed-income portfolio managers. Clearly, if volatility risk is not recognized and hedged, the portfolio manager may be exposed to significant losses in the event of a sudden change in the level of market uncertainty—an event that has occurred with increasing frequency in recent years. The Longstaff-Schwartz model discussed here gives fixed-income managers an important tool for quantifying and hedging their exposures to shifts in interest-rate volatility.

Table V Macaulay Durations

	Coupon Rate			
Maturity	*0%*	*5%*	*10%*	*15%*
6 Months	0.50	0.50	0.50	0.50
1 Year	1.00	0.99	0.98	0.97
2 Years	2.00	1.93	1.87	1.82
3 Years	3.00	2.82	2.68	2.58
4 Years	4.00	3.66	3.43	3.26
5 Years	5.00	4.46	4.12	3.88
10 Years	10.00	7.75	6.84	6.35
15 Years	15.00	10.03	8.71	8.10
20 Years	20.00	11.52	10.03	9.41
25 Years	25.00	12.44	10.97	10.39
30 Years	30.00	12.98	11.64	11.14

Footnotes

1. *The one-month Treasury bill yields used are from the data maintained by the Center for Research in Security Prices (CRSP) at the University of Chicago. These yields are based on the average of bid and ask prices for Treasury bills and are normalized to reflect a standard month of 30.4 days. The longer-maturity yields are from the data set used in E. F. Fama and R. Bliss, "The Information in Long-Maturity Forward Rates,"* American Economic Review *77 (1987), pp. 680–92.*
2. *The volatility of changes in the short-term interest rate is estimated using a simple GARCH (1,1) model. In this model, the volatility of changes in the short-term interest rate is assumed to be a linear function of its lagged value, the short-term interest rate, and the square of the last unexpected change in the short-term interest rate. See F. A. Longstaff and E. S. Schwartz, "Interest Rate Volatility and the Term Structure: A Two-Factor General Equilibrium Model,"* Journal of Finance *47 (1992), 1259–82.*
3. *The Longstaff and Schwartz model is a two-factor extension of the Cox, Ingersoll and Ross term structure model in which production returns in the economy are random and technological change occurs via changes in the mean and variance of production returns. See J. C. Cox, J. E. Ingersoll and S. A. Ross, "An Intertemporal General Equilibrium Model of Asset Prices,"* Econometrica *53 (1985), pp. 363–84. In the Longstaff and Schwartz model, the two factors are the short-term riskless interest rate and the volatility of the short-term interest rate. See Longstaff and Schwartz, "Interest Rate Volatility and the Term Structure,"* op. cit.
4. *In the Longstaff and Schwartz model, the present value of one dollar to be received T periods in the future can be expressed as $A(T)\exp[B(T)r + C(T)V]$, where $A(T)$, $B(T)$ and $C(T)$ are functions of T as well as six parameters α, β, γ, δ, η and ν that describe the dynamic evolution of the short-term interest rate over time. This means that yields to maturity are linear functions of r and V for a given T.*
5. *There are six parameters in the Longstaff and Schwartz model. By specifying the long-run average values of r and V, as well as the average yield on a consol bond, two of the six parameters can be solved for analytically. The remaining four are determined by setting actual yields for four different maturities equal to the closed-form expression implied by the model and then inverting the system of four equations to solve for the four parameters. This is similar to the procedure of inverting Black-Scholes option prices to solve for the implied volatility parameter. We solve the system of four equations using a numerical gridsearch algorithm that is easily implemented on a PC.*
6. *This is illustrated on page 78 of F. J. Fabozzi and T. D. Fabozzi,* Bond Markets, Analysis and Strategies *(Englewood Cliffs, NJ: Prentice Hall, 1989).*