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Optimal Financial Naïveté

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When agents first become active investors in financial markets, they are relatively inexperienced. Much of the literature focuses on the incentives of presumably sophisticated informed agents to produce information, and not on the naïve agents. However, unsophisticated agents are important aspects of financial markets and worth analyzing further. In this paper, we provide a theoretical perspective that addresses the issue of how many naïve traders would one expect in a financial market where policy makers try to educate the naïve agents. We show that such policy trades off the effects of naïve trades on price efficiency versus the losses incurred by these traders in financial markets. The optimal proportion of naïve agents varies with the value of information, the noise in private signals, and the inherent sensitivity of corporate investment to prices. We also show that the policy tool of encouraging insider trading can deter naïve investors and thus improve corporate governance and the efficacy of corporate investment.

1 Introduction

Recent years have seen a shift away from the notion that financial markets are populated exclusively by rational agents. For example, while early empirical studies by Black, Jensen, and Scholes (1972) and Fama and MacBeth (1973) suggest support for the capital asset pricing model (Sharpe (1964), Lintner (1965), and Mossin (1966)), more recently, Fama and French (1992) find that the relation between return and market beta is not significant. This calls into question the basic notion of a link between risk and expected returns.

There has been a lot of research supporting the notion that characteristics other than risk matter for expected returns. Fama and French (1992) find that size and the book/market ratio strongly predict future returns. Fama and French (1993) provide evidence that a three-factor model based on factors formed on the size and book-market characteristics explains average returns. But Daniel and Titman (1997) argue that, after controlling for size and book/market ratios, returns are not strongly related to betas calculated based on the Fama and French (1993) factors. More recently, Daniel and Titman (2006) argue that the book/market effect is driven by that part of the book/market ratio not related to accounting fundamentals. The part of this ratio that is related to fundamentals does not appear to forecast returns, thus raising issues about rational explanation for size and book/market effects. The momentum effect of Jegadeesh and Titman (1993) has also proved hard to explain.

All in all, it is reasonable to assert that the evidence on the predictability of returns

from book/market ratios at least partially supports non-risk-based (i.e., behavioral) explanations. Indeed, Barberis, Shleifer, and Vishny (1998) suggest that naïve extrapolation from past growth causes stock prices to overreact and reverse, resulting in return predictability from fundamental/price ratios. Similarly, Daniel, Hirshleifer, and Subrahmanyam (2001) suggest that overconfidence induces overreaction, and that extreme book/market ratios represent overreactions to extreme private signals which are later corrected. While Fama (1998) provides a contrary view to the notion that asset prices are driven by irrational traders, note that irrationals may persist in the market even if they do not influence market prices (Hirshleifer, Subrahmanyam, and Titman, 2006). Further, the work of Barber and Odean (2001), Brennan (1995), and Odean (1998, 1999) suggests that individual investors trade excessively due to overconfidence, have subpar investment performance, and are also susceptible to aspects such as loss aversion, indicating imperfect rationality.

Though, in recent years, it has been accepted by many scholars that some investors may indeed be financially unsophisticated, what has not been studied is whether there are sufficient economic incentives on the part of other agents to induce these agents to become rational. One way in which agents may learn to be rational is to simply learn from their trading. However, Gervais and Odean (2001) and Subrahmanyam (2009) argue that such activity is hampered due to a self-attribution bias. So the question is whether there is a role for direct financial education which may allow these agents to quickly reach a rational outcome.

Successful trading demands not only talent, but also some degree of sophistication. For example, one must know about aspects like the uselessness of already-public information, the importance of risk, and behavioral biases such as overconfidence and loss aversion. Being an experienced agent in financial markets thus is advantageous. Now, one aspect of financial markets is that the agents who are in the best position to educate the unsophisticated are themselves traders on their own account. These agents consist, for example, of financial institutions such as investment banks, traditional commercial banks, and pension funds. The investment success of such institutions depends on their financial sophistication relative to other agents, and therefore, these institutions do not have appropriate incentives to educate the unsophisticated traders (Subrahmanyam, 2009).¹ While education may of course be provided by entities other than those that trade professionally in financial markets (e.g., publishers of newsletters and bond ratings), even these agents may possibly be tempted to first exploit their knowledge by trading on their own account.

The preceding arguments indicate that the best education about financial investing is likely to be conferred by agents who are not experienced investors but have somewhat of a benevolent motive for financial education (e.g., policymakers). What is the equilibrium level of financial education in this scenario? Another related question is whether there are policy tools other than education that would allow regulators to prevent unsophisticated investors from trading in financial markets and thereby

¹In fact, Carlin (2009) suggests that intermediaries may add pricing complexity (e.g., hard to understand fees) to artificially differentiate between basically homogeneous financial products that are marketed to unsophisticated investors.

protecting their wealth, given the evidence that, on average, they earn negative abnormal returns (Odean, 1999).

To address these questions, we first build a model of a policymaker who possesses a technology that allows the correction of the biases of individual investors and/or directs them away from useless information sources.² The technology is disseminated in accordance with an exogenous cost function.³ The cost is incurred per agent, so that the total cost is assumed to be increasing in the proportion of investors educated to be rational. Within our setting, we show in our basic model that the optimal proportion of sophisticated agents trades off the cost of education versus the pricing efficiency gain from having more sophisticated agents in the market. Further, in an extension of the model, the liquidity created at the expense of naïve agents forms a wedge between what is optimal for individual investors and what is optimal for the policy makers providing the education.

Overall, the optimal amount of education partially, but not fully, moves the agents towards rationality. That is, agents are not fully educated about their bias or lack of sophistication. The preceding result arises because in financial markets, educating unsophisticated traders is costly, naïve traders are necessary to provide liquidity to

²The Federal Government conducts a number of programs to enhance financial literacy. See, for example, <http://www.treas.gov/offices/domestic-finance/financial-institution/fin-education/resources/alpha-name.html>. Many of these programs are conducted at various venues around the country. Our assumption that the educational cost is increasing in the mass of educated agents is justifiable under the notion that the educational cost is incurred per venue.

³In our model agents cannot become sophisticated on their own. Indeed, the decision to make agents sophisticated is not governed by the unsophisticated but by the policy makers. This is contrast to the literature on information acquisition (e.g., Grossman and Stiglitz, 1980) wherein uninformed agents can choose to become informed on their own at a cost.

financial markets. Therefore, even benevolent policy makers do not have an incentive to remove the inexperience of the unsophisticated. This leads to an interior optimum for the level of financial sophistication in society.⁴

We show that under reasonable conditions, the greater the variance of information, the greater is the resource allocational benefit of informed trading and the smaller is the incentive of the regulator to educate the unsophisticated agents. Conversely, the greater is the variance of noise, the greater is the efficiency loss due to noise traders and the greater is the incentive to educate the irrational agents.

In an adaptation of our basic setting, we consider the policy tool of encouraging insider trading as a disincentive for unsophisticated investors to trade in financial markets. We show that allowing insider trading can discourage the entry of unsophisticated investors and reduce the noisiness in financial market prices. This may actually improve the efficacy of corporate investment. We also argue that deterring unsophisticated investors can allow for better corporate governance by allowing for better monitoring of wasteful rent seeking by management. Thus, insider trading has the hitherto unrecognized benefit that it deters naïve traders from entering, which makes prices more efficient, and allows better monitoring of management. These benefits of insider trading go beyond the direct benefit of more efficient stock prices that is afforded by allowing insiders to trade on private information.

⁴The assumption here is that many retail investors are not sophisticated enough to use statistical techniques such as regression analyses and the like to discover potentially useful sources of information. See, for example, Benartzi and Thaler (2001), Lo, Repin, and Steenbarger (2005), or Hong, Stein, and Yu (2007) for evidence regarding investor naïveté about financial markets.

This paper is organized as follows. Section 2 presents the basic model. Section 3 analyzes the equilibrium where agents are educated by policymakers not to trade on useless signals. Section 4 analyzes the implications of permitting insider trading. Section 5 concludes. Proofs appear in the Appendix.

2 The Economic Setting

We consider a model of a financial market where the security pays off an amount $F \equiv \theta + \epsilon$. The variable θ has an ex ante mean of μ_θ , while ϵ has a mean of zero.

2.1 Investors

Informed agents observe θ exactly. Risk-neutral, competitive uninformed agents set the price, and naïve agents observe a mean-zero random variable η , which they mistakenly attribute to be fundamental information. Thus, from the perspective of naïve agents, the final payoff on the asset is $\eta + \epsilon$. Both the informed and the irrational traders have negative exponential utility with coefficient R . All random variables are normally distributed and mutually independent. The mass of informed agents is unity, whereas the mass of unsophisticated agents is denoted as n , with $n \in [0, 1]$. Therefore, the mass of unsophisticated agents can be interpreted as the proportion of agents who mistakenly trade on a signal, among all of those agents in the population who potentially could do so. We will variously term this class of agents as naïve, unsophisticated, or irrational. Throughout the paper, the quantity v_S denotes the

variance of the random variable S .

Standard arguments indicate that the demands of the informed and naïve agents are given by

$$x = \frac{\theta - P}{Rv_\epsilon}, \quad (1)$$

$$y = \frac{\eta - P}{Rv_\epsilon}. \quad (2)$$

The total demand of the informed and rational agents is $x + ny$, and the pricing rule used by the risk-neutral market makers is

$$P = E(\theta | x + ny).$$

Note that given normality, the price will be a linear function of θ and η . Let us denote this pricing function as $P = D + a\theta + b\eta$, where D , a , and b are constants.

2.2 Corporate Investment

The risky security is the single security issued by the manager of a firm. The manager has control over μ_θ , and the cost function associated with this control is denoted

$$K(\mu_\theta) = 0.5g\mu_\theta^2.$$

We aim to formally explore how unsophisticated investors influence corporate investment through their effects on equilibrium prices.

The manager makes the choice of μ_θ at date 0, i.e., prior to the trade in the firm's shares at date 1. The corporate investment is made from the firm's initial cash

balance of B , which is observed only by the manager. Any remaining cash is paid out to shareholders at date 0 as a dividend.

We abstract from agency problems and assume that the manager acts in the interest of the date 0 shareholders. The manager chooses μ_θ so as to maximize the expected date 1 stock price plus the date 0 dividend, i.e.,

$$E(P) - K(\mu_\theta). \quad (3)$$

The idea we here is that more efficient prices are more sensitive to the investment level and thus provide stronger incentives for managers to invest.⁵

Let μ_θ^c represent the market's conjecture about the mean of θ . We can then rewrite the pricing function P as

$$P = \mu_\theta^c + a(\theta - \mu_\theta^c) + b\eta.$$

The manager takes the market's conjecture about the mean as given and maximizes (3), i.e., he solves

$$\max_{\mu_\theta} a\mu_\theta - 0.5g\mu_\theta^2.$$

It is evident that the first order condition to this problem yields

$$\mu_\theta^* = \frac{a}{g}. \quad (4)$$

where the superscript * denotes an optimal value. Thus, (4) indicates that the ex ante mean of θ is linearly related to the coefficients of these variables in the date 1

⁵See Fishman and Hagerty (1989) and Subrahmanyam (1998) for similar models of the link between informational efficiency and corporate resource allocation.

price. The total value of the shares at date 0, denoted by V , is given after replacing μ_θ^* by μ_θ^c and adding the result to B , the initial cash balance of the firm. This yields

$$V = B + \mu_\theta^* - \frac{1}{2}g\mu_\theta^{*2} = B + \frac{a^2}{2g}. \quad (5)$$

Note that the firm's value is related monotonically to the square of the loading of the price on the information variable θ , and this loading is related to the extent of the information about θ that is revealed by the market price. Thus, more efficient prices lead to more efficient investment and raise security values.

2.3 Equilibrium Without Regulatory Intervention

Now, observing $x + ny$ is equivalent to observing the variable $\tau \equiv \theta + n\eta$, and so the price P is given by $P = E(\theta|\tau)$. It is fairly clear that the equilibrium price P in this setting is given by

$$P = \mu_\theta^c + \frac{v_\theta}{v_\theta + n^2v_\eta}[\theta + n\eta - \mu_\theta^c].$$

so that

$$\begin{aligned} a &= \frac{v_\theta}{v_\theta + n^2v_\eta}, \\ b &= na. \end{aligned}$$

This completes the equilibrium without allowing for policy intervention.

Note that the loading on the information variable is inversely related to n as well as v_η , indicating that prices are less sensitive to information when irrational trading activity is greater. It can easily be shown that the precision of the final value F

conditional on the market price is given by

$$[\text{var}(F|P)]^{-1} = \frac{v_\theta + n^2 v_\eta}{n^2 v_\eta},$$

which is also inversely related to n as well as v_η . Thus, unsophisticated trading adds noisiness to prices. This is why corporate resource allocation is adversely affected when irrational activity is greater.

3 Regulatory Intervention

3.1 Resource Allocation

Now, we propose that the naïve investors may be educated by a policymaker to not trade on noise. That is, we assume that the regulatory authority can educate an endogenous proportion of agents not to trade on the useless variable η . As discussed in the introduction, we assume that the total cost to educate $1 - n$ agents is $c(1 - n)^2$,⁶ i.e., it is increasing and convex in the mass of sophisticated investors.

We propose that the policymaker is not only concerned about the cost of education, but also about price efficiency, that, in turn, influences corporate investment and the value of the firm.⁷ Thus, the regulator maximizes

$$V - c(1 - n)^2 = B + \frac{a^2}{2g} - cn^2 = B + \frac{v_\theta^2}{2g(v_\theta + n^2 v_\eta)^2} - c(1 - n)^2.$$

⁶Recall that n is the proportion of *uneducated* naïve agents.

⁷We have assumed that the policymaker is not concerned with the actual trading losses of the unsophisticated investors. The model can be modified to account for this, but the qualitative results do not change. In any case, our assumption can be rationalized by the notion that the losses of the irrational agents translate to gains for the informed, who use them to pay for information collection, which is a useful activity.

Maximizing the above objective function yields the optimal value of n . It follows that the optimal value satisfies the following equation:

$$2c(1 - n) = \frac{2nv_\theta^2v_\eta}{(v_\theta + n^2v_\eta)^3}.$$

Further, using the implicit function theorem leads to the following proposition.

- Proposition 1**
1. *The optimal proportion of sophisticated agents is decreasing in the variance of information, v_θ , so long as v_θ exceeds an exogenous upper bound.*
 2. *The optimal proportion of sophisticated agents is increasing in the variance of noise, v_η , so long as v_θ is sufficiently large and v_η is sufficiently small.*

The intuition for the proposition is straightforward. The more the variance of information, the greater is the efficiency benefit of informed trading and the smaller is the incentive of the regulator to educate the unsophisticated agents. Conversely, the greater is the variance of noise, the greater is the efficiency loss due to noise traders and the greater is the incentive to educate the irrational agents.

Figures 1 and 2 plot the optimal mass of unsophisticated investors as a function of v_θ and v_η respectively. The parameter values used are $v_\theta = 1$, $v_\eta = 4$, $g = 1$, and $c = 0.2$. For low values of v_θ , almost everyone is educated. But for high values of v_θ , the proportion of educated agents drops to 90%. Similarly, for low values of v_η , the proportion of educated agents is about 90%, and increases sharply as v_η takes on progressively higher values.

3.2 Liquidity Costs

One potential benefit of unsophisticated agents is that they add liquidity to financial markets. Indeed, Black (1986, p. 532) states the following:

The more noise trading there is, the more liquid the market will be. . . But noise trading actually puts noise into the prices. The price of a stock reflects both the information that information traders trade on and the noise that noise traders trade on. . . What's needed for a liquid market causes prices to be less efficient.

The liquidity benefits of unsophisticated agents were not internalized in the previous section. To analyze such benefits, let us consider a class of risk-neutral traders who trades in a manner independent of price. These are agents who trade for reasons of immediacy and are thus liquidity demanders. Their total trade is in the amount of z . It is a reasonable policy goal that financial market policy should attempt to mitigate the trading costs of agents who trade for unforeseen liquidity reasons. Motivated by this observation, we will now analyze how the equilibrium is altered when the regulatory authority internalizes the losses of these liquidity traders.

A straightforward modification of the analysis in the previous subsection indicates that the price in the presence of liquidity traders is given by:

$$P = \mu_\theta^c + \frac{v_\theta}{v_\theta + n^2 v_\eta + R^2 v_\epsilon^2 v_z} [\theta + n\eta + Rv_\epsilon z - \mu_\theta^c].$$

We define

$$a \equiv \frac{v_\theta}{v_\theta + n^2 v_\eta + R^2 v_\epsilon^2 v_z}.$$

Note that the loading on θ , i.e., a , is negatively affected by z as well as n and v_η , since both the liquidity traders and the irrational agents add noisiness to prices.

Now, the expected trading costs incurred by the liquidity traders are given by

$$-E[(F - P)z] = Rav_\epsilon v_z.$$

As indicated earlier, we assume that one of the regulatory authority's goals is to minimize the trading costs of the liquidity traders. Thus, the regulatory authority maximizes:

$$\begin{aligned} & V - c(1 - n)^2 - Rav_\epsilon v_z \\ = & B + \frac{a^2}{2g} - c(1 - n)^2 - Rav_\epsilon v_z \\ = & B + \frac{v_\theta^2}{2g(v_\theta + n^2 v_\eta + R^2 v_\epsilon^2 v_z)^2} - c(1 - n)^2 - \frac{Rv_\epsilon v_z v_\theta}{v_\theta + n^2 v_\eta + R^2 v_\epsilon^2 v_z}. \end{aligned}$$

Solving for the equilibrium n is algebraically complex and closed-form solutions are not possible. Before presenting a proposition, we examine a numerical example. Thus, consider the parameter values $v_\theta = 1$, $v_\eta = 4$, $v_z = 0.5$, $g = 1$, $R = 0.5$, $v_\epsilon = 0.5$, and $c = 0.2$. The mass n of naïve agents is 4.9% without consideration of the liquidity traders and 6.2% when liquidity traders are considered. Thus, the liquidity benefits conveyed by naïve traders create a disincentive for naïve investors to be educated by the regulatory authority. We can prove the following proposition.

Proposition 2 *So long as v_θ exceeds an exogenous upper bound, the optimal proportion of sophisticated agents is decreasing in the variance of liquidity trading, v_z , in the limit as $g \rightarrow 0$.*

The intuition for the above proposition is that when the liquidity benefit is dominant (i.e., g is small), an increase in the amount of liquidity trading increases the liquidity benefit and thus decreases the proportion of agents who are allowed to become sophisticated by the policy-maker.

While we are able to prove the proposition analytically in the limit for v_θ sufficiently large, we have verified that the comparative static result of the proposition holds for other parameter spaces as well. Figure 3 plots the optimal mass of unsophisticated investors (n) as a function of v_z , considering the base parameter values used in the numerical example just before Proposition 2. The figure shows how liquidity considerations cause disincentives to educate the unsophisticated agents. Thus, when $v_z = 0.1$, 95% of agents are educated, whereas when v_z rises to 1, the mass of educated agents drops to 91%. Thus, under liquidity considerations, policymakers' incentives to educate agents are diminished.

4 The Effect of Allowing Insider Trading on Equilibrium Financial Sophistication

The previous section took explored the notion that regulators may not have incentives to fully educate unsophisticated agents in financial markets. Aside from paternalistic

education, another form of regulation is to control the amount of informed trading in the financial markets. This control can be accomplished by using the policy tool of regulating insider trading.

Elsewhere, the effect of insider trading on markets has been explored extensively, and existing papers on the social desirability of insider trading arrive at varied conclusions. For example, Manne (1966) and Carlton and Fischel (1983) provide arguments in favor of insider trading, Ausubel (1991), Bhattacharya and Spiegel (1991), and Fishman and Hagerty (1992) argue against the practice, and Leland (1992) concludes that the effect of insider trading on social welfare is ambiguous. These authors generally trade off the liquidity costs of insider trading versus its benefits from enhanced price efficiency.

In this section, we explore another potential benefit to insider trading: that it can deter unsophisticated agents from entering and trading in financial markets. This deterrence can improve price efficiency, and in turn, corporate resource allocation. We also propose that this deterrence can improve the quality of corporate governance.

4.1 The Impact of Insider Trading on Naïve Investing and Equilibrium Resource Allocation

First, we modify the model of the previous section slightly for notational convenience. Specifically, we assume that the payoff on the asset is $F \equiv \epsilon_1 + \epsilon_2$ and suppose that informed agents know the variable ϵ_1 . Both ϵ_1 and ϵ_2 are normal random variables with a common variance v_ϵ . We assume that ϵ_1 have an ex ante mean of μ_ϵ (while ϵ_2

has zero mean).

We assume that the naïve traders know the variable η and think the final payoff is $\eta + \epsilon_1 + \epsilon_2$. Importantly, note here that the naïve agents understand that there are informed agents in the market. Their optimization is performed under the mistaken notion that they have information over and above that of the other informed agents.

We denote the mass of informed agents is s and the mass of naïve traders as n . We assume that both s and n are bounded such that $s \in [0, 1]$ and $n \in [0, 1]$. Thus, the masses may be viewed as proportions of each type of agent in the market. Some of the informed agents may be insiders who may be precluded from trading by regulatory actions (e.g., the prohibition of insider trading). Each noise trader pays an entry cost of c to acquire and trade on η . Later, this assumption allows us to endogenize the mass of irrational agents.

Now, each informed agent submits an order:

$$x = \frac{\epsilon_1 - P}{Rv_\epsilon},$$

and each noise trader submits an order

$$y = \frac{\eta - P}{2Rv_\epsilon}.$$

The 2 in the denominator of y appears because the naïve agents explicitly consider that they are dealing with two unknowns: ϵ_1 and ϵ_2 . We assume that they are not sophisticated enough to condition the variance on the market price; however, this assumption is of little material relevance.

Now, the market maker sees the order flow $sx + ny$, so that the price is

$$\begin{aligned} P &= E(\epsilon_1 | sx + ny) \\ &= \mu_\epsilon^c + \frac{2sv_\epsilon}{4s^2v_\epsilon + n^2v_\eta} [2s\epsilon_1 + n\eta - \mu_\epsilon^c]. \end{aligned}$$

Note from the above expression that the loading of the price on the information variable ϵ_1 is monotonically increasing in the mass of informed agents s , but the loading on η is not monotonic in the mass of naïve investors, n . While the former result is intuitive, the latter result is not so. Basically, increasing n can increase or decrease the covariance of η with price. The increase happens simply because an increase in n implies an increase in net demand, which is increasing in η . But an increase in n also increases the variance of irrational demand, which is a nuisance variable for the price setting agents. This tends to decrease the loading of the price on η .

4.1.1 Resource Allocation

Now, let

$$a' \equiv \frac{4s^2v_\epsilon}{4s^2v_\epsilon + n^2v_\eta}. \quad (6)$$

Note that a' is the loading of the price on the informational variable ϵ_1 . Suppose the manager needs to make an investment decision before obtaining the signal ϵ_1 and chooses μ_ϵ to maximize

$$E(P) - K(\mu_\epsilon),$$

where

$$K(\mu_\epsilon) = 0.5g\mu_\epsilon^2.$$

Similar to the previous section, we then have that

$$\mu_\epsilon = \frac{a'}{g},$$

and it follows that the ex ante value of the firm is

$$V = B + \frac{a'^2}{2g}. \quad (7)$$

4.1.2 Endogenous Entry by the Unsophisticated Agents

Let

$$P^* \equiv \frac{2sv_\epsilon}{4s^2v_\epsilon + n^2v_\eta} [2s\epsilon_1 + n\eta].$$

P^* is then the stochastic part of P . Now, the perceived wealth of the irrationals is given by

$$\frac{\eta - P^*}{2Rv_\epsilon} [\eta + \epsilon_1^* + \epsilon_2 - P^*], \quad (8)$$

where ϵ_1^* is the demeaned version of ϵ_1 . The appendix shows that the perceived certainty equivalent of the irrationals, denoted CE_n for a given value of n is given by

$$CE_n = (2R)^{-1} \ln[Det],$$

where

$$Det = \frac{J}{K}, \quad (9)$$

and where, in turn,

$$\begin{aligned} J &= n^4v_\eta^2(2v_\epsilon + v_\eta) - 4n^3sv_\epsilon v_\eta^2 + 4n^2s^2v_\epsilon v_\eta(3v_\epsilon + 2v_\eta) - 16ns^3v_\epsilon^2v_\eta + 8s^4v_\epsilon^2(3v_\epsilon + 2v_\eta), \\ K &= 2v_\epsilon(n^2v_\eta + 4s^2v_\epsilon)^2. \end{aligned}$$

The appendix proves the following proposition:

Proposition 3 *The perceived expected utility of naïve traders is higher in the absence of insiders ($s = 0$) than in the presence of insiders ($s > 0$).*

We now turn to the issue of equilibria with endogenous entry of the unsophisticated investors. While it is always the case that unsophisticated traders perceive themselves as better off without any informed agents in the market (viz. Proposition 3 above), it turns out that the perceived expected utility of these agents is not monotonically decreasing in their own mass n (this can be verified by differentiating Det above with respect to n). The reason is that an increase in n has two opposing effects from the perspective of the unsophisticated agents. While on the one hand the presence of more such agents can decrease their perceived expected profits, on the other hand, it also tends to move prices farther away from their information (since from their perspective, the market erroneously disregards their information signal). The latter effect may actually cause their perceived expected utility to increase in n , which may cause a corner solution where the equilibrium n equals its maximum value of unity.⁸

Note that corner solutions will not exist for a given value of s provided that the perceived certainty equivalent of the unsophisticated agents is monotonically decreas-

⁸While unsophisticated investors perceive themselves as having useful information in our framework, they earn expected disutility in our setting since in actuality their information is useless. This is consistent with the work of Odean (1998, 1999) who indicate that unsophisticated investors seem to actively trade stocks even if they underperform on average. Our setting involves a single round so that the persistence of these agents is not relevant. In a repeated, dynamic, setting, the persistence of these agents can be justified by assuming that they directly derive satisfaction from trading (as a consumption good). Indeed, Kumar (2008) indicates that individual investors may trade simply because they obtain utility from stock market investing as a form of gambling.

ing in n for $n \in [0, 1]$ and the cost c of entering the financial market is sufficiently high. In these instances, the equilibrium level of naïve trading is that value of n when $CE_n = c$, where, as before, CE_n denotes the perceived certainty equivalent of the expected utility of naïve traders when n naïve traders trade in the market.

Now, differentiating the right-hand side of (9) with respect to n yields that

$$\partial Det/\partial n = -\frac{2nv_\eta(n^4v_\eta^2 + 2n^3sv_\epsilon v_\eta - 16s^4v_\epsilon^2)}{(n^2v_\eta + 4s^2v_\epsilon)^3}.$$

Interestingly, the derivative of Det with respect to s is simply the above expression multiplied by the ratio (s/n) . In an interior equilibrium, from the implicit function theorem,

$$\frac{dn}{ds} = -\frac{\partial Det/\partial s}{\partial Det/\partial n} = \frac{n}{s}.$$

From Equation (7), it can be seen that the total value of the firm is monotonically related to a' . The total derivative of a' with respect to s is given by

$$\frac{da'}{ds} = \frac{\partial a'}{\partial s} + \frac{\partial a'}{\partial n} \frac{dn}{ds}.$$

From the definition of a' in (6) and the expression for dn/ds above, it can easily be seen that $da'/ds = 0$.

Now, assume that the policy maker can reduce s from s_H to s_L (i.e., that the difference $s_H - s_L$ represents insiders that are precluded by regulatory fiat). The following proposition then follows from the preceding discussion.

Proposition 4 *1. If only interior solutions exist for both s_H and s_L , then changing the mass of informed agents s , causes a corresponding change in n but has*

no effect on corporate resource allocation.

- 2. Assume that only interior solutions exist for s_H . Then if $CE_{n=1} > CE_{n \in [0,1]}$ at $s = s_L$, decreasing s causes an increase in the mass of unsophisticated investors and causes a reduction in the value of the firm's shares due to a reduction in the efficacy of resource allocation.*

Decreasing s tends to increase n and these phenomena have opposite effects on price efficiency. They exactly offset each other in interior equilibria. In a move to a corner equilibrium, the mass of irrationals hits its maximum, which causes the efficacy of resource allocation to deteriorate.

Consider the case where all parameters are unity except that the cost of entry for noise traders is $c = .0077$. Then, suppose that $s_L = 0.1$ and $s_H = 0.9$. When insider trading is allowed, so that $s = 1$, the optimal $n = 0.5$, and V is 0.4429. However, when s drops to 0.1 (i.e., when some informed agents, being insiders, are precluded from trading), n reaches a corner solution at 1, and V drops to 0.0007. This confirms that allowing insider trading improves price efficiency by deterring noise traders.

4.2 Implications for Corporate Governance

While not explicitly modeled in the previous section, the deterrence of naïve traders may also have implications for corporate governance. It is reasonable to suppose that only will naïve investors be more prone to trading on useless signals, they also may not have the sophistication to monitor management for governance-related excesses. For

example, management may be prone to dissipating free cash flow on perks, or paying itself hidden compensation in the form of practices like spring-loading and backdating options (Lie, 2005). Concealed arrangements, consisting of deferred compensation, post-retirement income guarantees, and stock option packages, are not only difficult to value but likely difficult to understand.⁹ Thus, naïve investors are likely to face significant challenges in deciphering compensation packages, and other forms of value dissipation, and may also be unable to remove pliant boards of directors that are reluctant to investigate management too deeply.

The previous arguments suggest that the deterrence of naïve investors by permitting insider trading may have the added benefit of allowing for better monitoring of management. For example, suppose that a portion B_E of the cash balance B of the firm may be used by management for perks and hidden compensation.¹⁰ Also, suppose that this activity is detectable only if the proportion of sophisticated investors, i.e., $s_L/(s_L + n)$ exceeds a threshold ρ . This can be justified by saying that naïve investors do not have the sophistication to allow for the replacement of pliant members of the board of directors that are beholden to management. It can then easily be seen that in the numerical example above, so long as $9.09\% < \rho < 20\%$ (i.e., $0.1/1.1 < \rho < 0.1/0.5$), the hidden compensation is detected and prevented only when

⁹A press release dated July 6, 2006 from Reuters notes that more than 50 companies' option granting practices are being investigated. A list of companies under examination for options scandals may be found at <http://online.wsj.com/public/resources/documents/info-optionsscore06-full.html>. Other recent articles have focused on how details of compensation packages are difficult to decipher. Core, Guay, and Larker (2007) is one of many related studies that focus on the role of media in bringing the levels and types of executive compensation to the public's attention.

¹⁰We assume that the exogenous parameters are such that the quantity $B - B_E$ is still sufficient to cover the investment needs of the firm.

insider trading is allowed. This indicates how allowing insider trading, by deterring trading by unsophisticated investors, may for improved monitoring of management.

5 Conclusion

When agents first become active investors, they are relatively inexperienced and likely lack financial sophistication. Indeed, many models, such as those based on Kyle (1985) or Grossman and Stiglitz (1980), simply assume the existence of unsophisticated agents, and instead focus on the incentives of presumably sophisticated informed agents to produce information. However, recent research in finance (e.g., Odean, 1998, 1999) indicates that unsophisticated agents are important aspects of financial markets. Hence, they are worth analyzing further.

In this paper, we provide a theoretical perspective that addresses the issue of how many naïve traders would one expect in a financial market where policy makers try to educate the naïve agents. We show that such policy trades off the effects of naïve trades on price efficiency and liquidity versus the losses incurred by these traders in financial markets. The optimal proportion of naïve agents varies with the value of information, the noise in private signals, and the inherent sensitivity of corporate investment to prices. We also show that the policy tool of encouraging insider trading can deter naïve investors and actually improve corporate governance as well as the efficacy of corporate investment.

We believe that our work is a first consideration of the policy issues surrounding

naïve investors and our work suggests a fertile agenda for future research. For example, in our paper, we have modeled education as directing unsophisticated investors away from useless information sources. There are many alternative ways of modeling this notion. In particular, naïve investors may be educated about biases such as loss aversion and overconfidence, and also about the perils of using flawed heuristics such as allocating $1/M$ of one's wealth to each of M risky assets with very different risk profiles (Benartzi and Thaler, 2001). The equilibria surrounding such alternative models of education would be of considerable interest. These issues are left for future research.

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Appendix

Proof of Proposition 1: The first order condition is

$$G \equiv \frac{2nv_{\theta}^2v_{\eta}}{(v_{\theta} + n^2v_{\eta})^3} + 2c(1 - n) = 0.$$

From the above, we have that

$$\partial G / \partial v_{\theta} = \frac{2nv_{\theta}v_{\eta}(n^2v_{\eta} - 4v_{\theta})}{(v_{\theta} + n^2v_{\eta})^4}.$$

Similarly, we have that

$$\partial G / \partial n = \frac{2v_{\theta}^2v_{\eta}(v_{\theta} - 5n^2v_{\eta}) - 2c(v_{\theta} + n^2v_{\eta})^4}{(v_{\theta} + n^2v_{\eta})^4}.$$

All this implies that

$$\frac{dn}{dv_{\theta}} = \frac{2nv_{\theta}v_{\eta}(4v_{\theta} - n^2v_{\eta})}{2v_{\theta}^2v_{\eta}(v_{\theta} - 5n^2v_{\eta}) - 2c(v_{\theta} + n^2v_{\eta})^4}.$$

The above expression will be positive so long as v_{θ} is large enough (given that $n \in [0, 1]$).

We also have that

$$\partial G / \partial v_{\eta} = \frac{2nv_{\theta}^2(v_{\theta} - 2n^2v_{\eta})}{(v_{\theta} + n^2v_{\eta})^4},$$

so that

$$\frac{dn}{dv_{\eta}} = -\frac{2nv_{\theta}(v_{\theta} - 2n^2v_{\eta})}{2v_{\theta}^2v_{\eta}(v_{\theta} - 5n^2v_{\eta}) - 2c(v_{\theta} + n^2v_{\eta})^4}.$$

This derivative will be negative so long as v_{θ} is large enough and v_{η} is small enough. ||

Proof of Proposition 2: The regulatory authority maximizes

$$\frac{a^2}{2g} - c(1 - n)^2 - Rav_{\epsilon}v_z.$$

where

$$a = \frac{v_\theta}{v_\theta + n^2 v_\eta + R^2 v_\epsilon^2 v_z}$$

The first order condition that solves for the optimal n is

$$H \equiv \frac{2a(da/dn)}{2g} + 2c(1 - n) - R(da/dn)v_\epsilon v_z = 0.$$

Using the implicit function theorem on H and letting g become vanishingly small yields the following result:

$$\lim_{g \rightarrow 0} [dn/dv_z] = \frac{v_\theta + R^2 v_\epsilon^2 v_z - 5n^2 v_\eta}{3nR^2 v_\epsilon^2}.$$

Examination of the above derivative immediately yields Proposition 2.||

Proof of Proposition 3: We begin by stating the following lemma, which is a standard result on multivariate normal random variables (see, for example, Brown and Jennings, 1989).

Lemma 1 *Let $Q(\chi)$ be a quadratic function of the random vector χ : $Q(\chi) = C + B'\chi - \chi' A \chi$, where $\chi \sim N(\mu, \Sigma)$, and A is a square, symmetric matrix whose dimension is the same as that of χ . We then have*

$$E[\exp(Q(\chi))] = |\Sigma|^{-\frac{1}{2}} |2A + \Sigma^{-1}|^{-\frac{1}{2}} \times \exp\left(C + B'\mu + \mu' A \mu + \frac{1}{2}(B' - 2\mu' A')(2A + \Sigma^{-1})^{-1}(B - 2A\mu)\right). \quad (10)$$

The ex ante utility of the agents is derived by an application of Lemma 1. Define $\lambda = [\epsilon_1^* \ \epsilon_2 \ \eta]$ and let Σ denote the variance matrix for this vector. Then, we can

construct the square, symmetric matrix A such that $RW = \lambda' A \lambda$, where W is given by (8). Noting then that the ex ante expected utility is given by $EU = E[-\exp(-RW)]$, we can apply Lemma 1 with $\mu = 0$, $C = 0$, and $B = 0$. The agent's ex ante utility thus becomes

$$EU = E[-\exp(-\lambda A \lambda')] = -|\Sigma|^{-\frac{1}{2}} |2A + \Sigma^{-1}|^{-\frac{1}{2}} = -|2A\Sigma + I|^{-\frac{1}{2}}.$$

We denote the determinant $|2A\Sigma + I|$ as Det . Note that the expected utility is monotonically increasing in this determinant. Applying this lemma to (8), it can easily be shown that Det calculates to the right-hand side of (9). We also find that the differential Det when $s > 0$ and $s = 0$ is given by

$$-\frac{2s[n^3 v_\eta^2 + n^2 s v_\epsilon v_\eta + 4n s^2 v_\epsilon v_\eta + 2s^3 v_\epsilon^2]}{(n^2 v_\eta + 4s^2 v_\epsilon)^2}$$

which is negative. This proves the proposition. ||

Proof of Proposition 3: The first part of the proposition is proved in the main text. The second part follows from the numerical example below the proposition's statement.||

Figure 1: Mass of unsophisticated investors vs. the variance of information

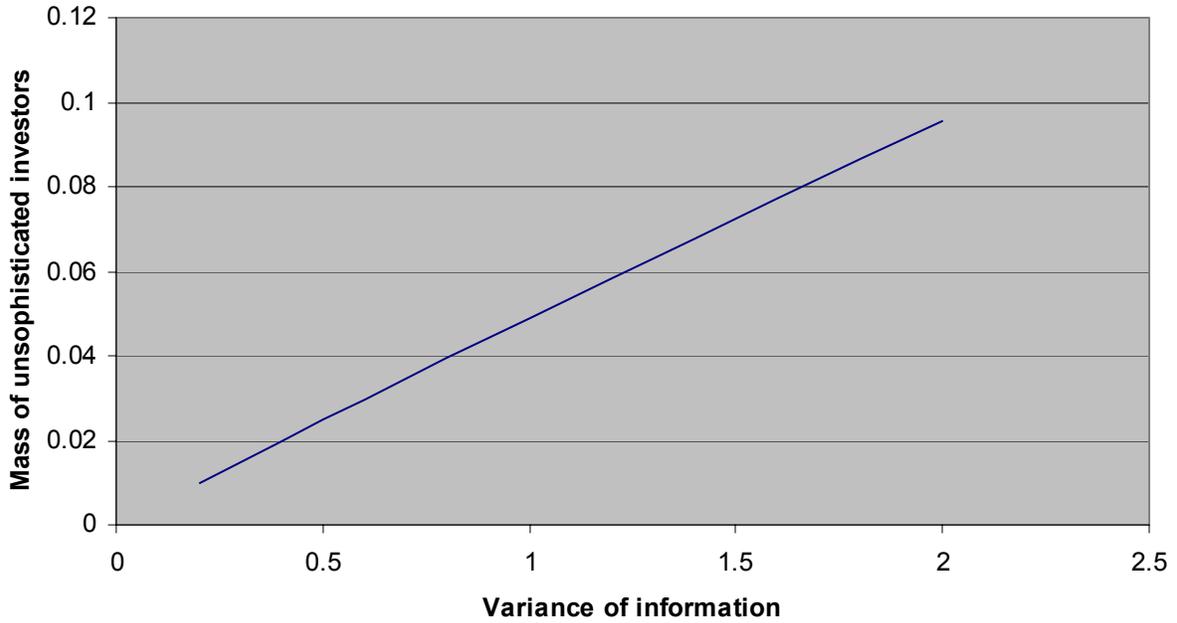


Figure 2: Mass of unsophisticated investors vs. the variance of noise

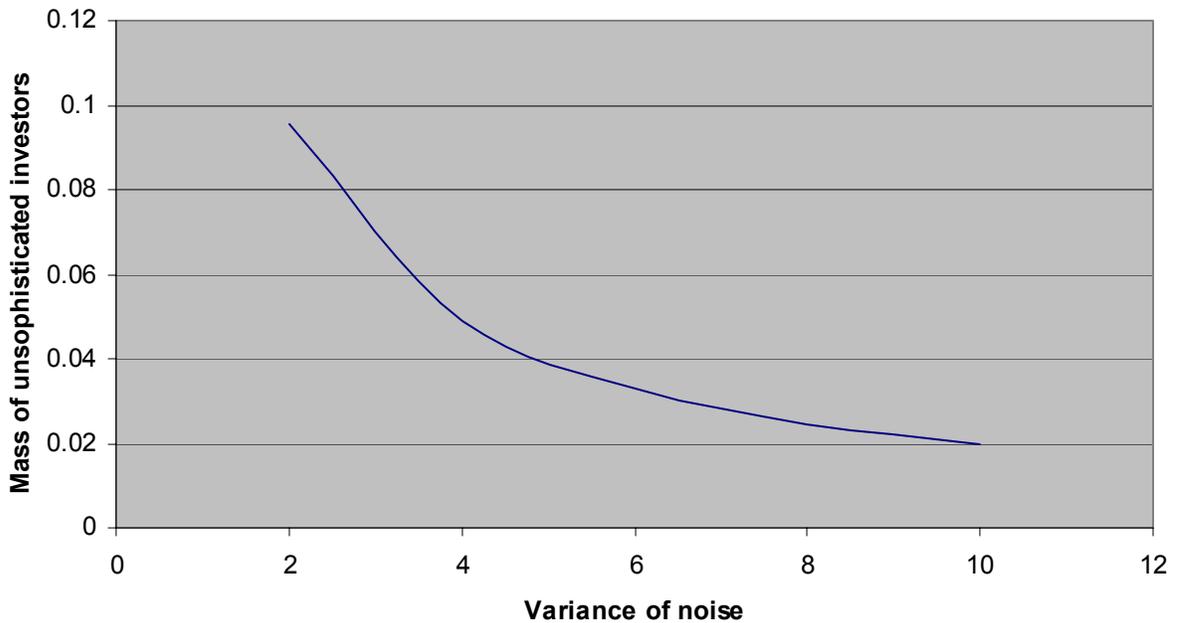


Figure 3: Mass of unsophisticated investors vs. the variance of liquidity trade

