

Time Varying Term Premia and Traditional Hypotheses about the Term Structure

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ABSTRACT

Empirical evidence of time varying term premia in bond returns is frequently interpreted as evidence against the Expectations Hypothesis. This paper shows that the Expectations Hypothesis can actually imply time varying term premia if the time frame for which the Expectations Hypothesis holds differs from the return measurement period. Furthermore, many of the properties of these term premia are consistent with those of observed term premia. These results are important because they imply that the case against the Expectations Hypothesis is weaker than claimed in the empirical literature.

MANY RECENT STUDIES HAVE presented empirical evidence that term premia in bond returns are time varying. For example, Fama and Bliss (1987) document that term premia in Treasury bond returns vary reliably through time and can be forecast using the information in forward rates. Similar evidence of variation through time in term premia is given by Shiller (1979), Startz (1982), Shiller, Campbell, and Schoenholtz (1983), Fama (1984), Mankiw (1986), Shiller (1986), Campbell (1987), Engel, Lilien, and Robins (1987), Shiller and McCulloch (1987), Froot (1989), Simon (1989), and others.

These empirical results have been interpreted in the literature as strong evidence against the Expectations Hypothesis (EH).¹ The reason for this interpretation is the widely held view that the EH is consistent only with term premia that are constant through time.²

In this paper, we show that the EH *need not* imply constant term premia. Specifically, we present an example in which an important form of the EH implies nonzero term premia that vary randomly through their dependence on the volatility of the term structure. We show that the variation in these term premia can be significant relative to the variation in expected returns for longer maturity bonds. Furthermore, we show that these term premia are directly related to the forward-spot rate differential. The key insight of this example is that the empirical implications of the EH depend critically on the length of the period over which the EH is assumed to hold. If this interval is shorter than the period over which bond returns are measured, aggregation over time can induce rich patterns of variation in observable term premia.

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¹ For example, see Shiller, Campbell, and Schoenholtz (1983) and Shiller and McCulloch (1987).

² See Campbell (1986), p. 184. Also see Shiller and McCulloch (1987).

This example is important because it demonstrates that the mounting empirical evidence of time varying term premia cannot be interpreted as clear-cut evidence against all versions of the EH—particularly versions that are of the greatest interest from a theoretical perspective. Tests of the EH must address the temporal aggregation issue in order to lead to unambiguous inferences about the validity of traditional hypotheses about the term structure of interest rates.

The remainder of the paper is organized as follows. Section I briefly reviews the EH and discusses the continuous time framework in which the example is developed. Section II derives an explicit expression for the term premia in discretely observed bond returns implied by the Local Expectations Hypothesis described by Cox, Ingersoll, and Ross (1981). The analytical properties of these term premia are examined in Section III. Section IV summarizes the results and makes concluding remarks.

I. The Continuous Time Framework

In developing this example, we follow Cox, Ingersoll, and Ross (CIR) (1981) in assuming that the EH holds instantaneously. We then derive the implications of the EH for the properties of discretely observed discount bond returns. This approach is particularly relevant because the majority of recent theoretical advances in term structure theory have been developed in a continuous time setting.³ In addition, this approach has the advantage of illustrating clearly how aggregation over time can affect the empirical implications of the EH.⁴

Let $P(Y,t,T)$ be the current (time t) price of a unit discount bond that matures at time T , where Y is a vector of state variables that summarizes the current state of the economy. We assume that the instantaneous return on a discount bond can be represented by the following stochastic differential equation:

$$\frac{dP(Y,t,T)}{P(Y,t,T)} = \alpha(Y,t,T)dt + \delta'(Y,t,T)dZ, \quad (1)$$

where $\alpha(Y,t,T)$ and $\delta'(Y,t,T)\delta(Y,t,T)$ represent the instantaneous expected return and variance of returns, respectively, and Z is an n -dimensional Wiener process. CIR (1981) show that, in continuous time, the EH is actually a set of several different propositions about the term structure which can be distinguished by their implications for the instantaneous expected return term $\alpha(Y,t,T)$. Of these different versions of the EH, CIR show that only the Local Expectations Hypothesis (L-EH) is consistent with the absence of arbitrage in a continuous time setting. Accordingly, we focus on the L-EH in examining the implications

³ For example, see Vasicek (1977), Richard (1978), Dothan (1978), Brennan and Schwartz (1979), CIR (1981, 1985), Campbell (1986), and Longstaff (1989b).

⁴ The temporal aggregation issue has also been studied by Grossman, Melino, and Shiller (1987) and Longstaff (1989a). The focus of these papers, however, is on asset pricing models rather than the term structure.

of the EH for observable term premia.⁵ We note that the L-EH plays an important role in many models of contingent claim prices including Merton (1973), Brennan and Schwartz (1977), Dothan (1978), Ramaswamy and Sundaresan (1985), and Bailey (1987).

As described by CIR (1981), the L-EH implies that all bonds have the same expected return over the next (shortest possible) holding period. Thus, the L-EH implies

$$\alpha(Y,t,T) = r(Y,t), \quad (2)$$

where $r(Y,t)$ is the instantaneous risk-free rate of interest. Since (2) provides a complete characterization of the expected returns on discount bonds, we need only specify the dynamics of the state variables in order to derive the partial differential equation defining bond prices in the L-EH. For the purposes of this paper, we follow CIR (1985) and assume that Y is a scalar and that there is a change of variables which allows us to treat the current risk-free rate as the relevant state variable in determining discount bond prices. In addition, we assume that the risk-free rate follows the well-known CIR (1985) square root process

$$dr = \kappa(\mu - r)dt + \sigma\sqrt{r}dZ, \quad (3)$$

where κ , μ , and σ^2 are parameters and Z is now a scalar process. With these assumptions, we can derive explicit closed-form expressions for discount bond prices and then directly examine the properties of term premia implied by the L-EH. Following CIR (1981) and Fama (1984), we define the term premium to be the expected return on a bond held from t to s (where $t \leq s \leq T$) minus the expected return from rolling over a series of instantaneously maturing bonds during the same holding period (returns are continuously compounded).

II. Term Premia and the Local Expectations Hypothesis

In order to obtain expressions for the term premia implied by the L-EH, we first need to derive the corresponding closed-form expressions for bond values. Applying Itô's Lemma to $P(r,t,T)$, taking expectations, and using the L-EH relation in (2) results in the following partial differential equation for the price of a discount bond when the L-EH holds:

$$\frac{\sigma^2 r}{2} P_{rr} + \kappa(\mu - r)P_r - rP + P_t = 0, \quad (4)$$

subject to the maturity condition $P(r,T,T) = 1$. Applying a standard separation of variables approach to (4) results in the following closed-form expression for

⁵ However, Campbell (1986) argues that the differences between the different versions of the EH are of second order importance when term structure volatility is low. Although we focus on the L-EH, the major results of this paper can also be obtained if instantaneous term premia are given by the Return to Maturity EH described by CIR (1981).

the discount bond price:

$$P(r,t,T) = A(t,T)\exp(-B(t,T)r), \tag{5}$$

where

$$A(t,T) = \exp\left(\frac{\kappa\mu(\kappa + \phi_1)(T - t)}{\sigma^2}\right)\left(\frac{1 - \phi_2}{1 - \phi_2\exp(\phi_1(T - t))}\right)^{2\kappa\mu/\sigma^2}$$

$$B(t,T) = \frac{\phi_1 - \kappa}{\sigma^2} - \frac{2\phi_1}{\sigma^2(1 - \phi_2\exp(\phi_1(T - t)))},$$

$$\phi_1 = \sqrt{\kappa^2 + 2\sigma^2},$$

$$\phi_2 = \frac{\kappa + \phi_1}{\kappa - \phi_1}.$$

It is straightforward to show that this bond price is equivalent to that obtained if the market price of risk in the CIR (1985) equilibrium discount bond price is zero. This follows because CIR (1981) show that the L-EH is consistent with general equilibrium when preferences are logarithmic and returns from physical capital are uncorrelated with shifts in the state variable. Thus, if the state variable cannot be hedged, the market price of risk in the CIR equilibrium model is zero and the L-EH and CIR models coincide.⁶

In order to solve for the term premium expected from holding a discount bond with maturity date T during the period from t to s , we apply Itô's Lemma to $P(r,t,T)$ to obtain the dynamics for the discount bond price:

$$\frac{dP(r,t,T)}{P(r,t,T)} = rdt - \sigma\sqrt{r}B(t,T)dZ. \tag{6}$$

Next, we apply Itô's Lemma to the natural logarithm of $P(r,t,T)$ to obtain the stochastic differential equation for the instantaneous return on the bond. The term premium is then found by subtracting r from the drift term in the expression for instantaneous returns, aggregating temporally by integrating the resulting expression from t to s , and taking the appropriate expectation.⁷ The term premia in bond returns implied by the L-EH are given by

$$\beta_1(t,s,T) + \beta_2(t,s,T)r, \tag{7}$$

where

$$\beta_1(t,s,T) = \frac{\mu\sigma^2}{2} \int_t^s (e^{\kappa(t-u)} - 1)B^2(u,T)du,$$

$$\beta_2(t,s,T) = -\frac{\sigma^2}{2} \int_t^s e^{\kappa(t-u)}B^2(u,T)du.$$

⁶ Note that the market price of interest-rate risk in CIR (1985) is proportional to the covariance of changes in the state variable with returns on physical investment. If this covariance is zero, then the market price of interest-rate risk is also zero.

⁷ The expectation is taken with respect to the information currently available to the market which includes the risk-free rate r . The resulting expression is simplified by applying Fubini's Theorem, which allows us to represent the expectation of the temporally aggregated return as an integral of an expectation. The conditional expectation of the future interest rate is given in CIR (1985).

III. Properties of the Term Premia

One of the most important features of the L-EH term premia in (7) is their dependence on the current level of the risk-free interest rate. Thus, the L-EH implies time varying term premia in this example. The intuition for this result is that term premia in bond returns measured over discrete periods of time depend not only on the instantaneous premium in returns (which is zero in the L-EH) but also on the variance of the bond's return. Consequently, the dependence of this variance on the risk-free rate is inherited by the term premia.⁸ The relation between term premia and term structure volatility implied by the L-EH is consistent with a number of recent empirical studies such as Campbell (1987), Engel, Lilien, and Robins (1987), and Simon (1989), who find evidence that excess returns on bonds are related to term structure volatility.

The first term in (7) reflects the constant portion of the term premia and is due to the mean reversion of the risk-free rate. Intuitively, this term arises because mean reversion implies a long-run steady state distribution of term structure volatility that is independent of the current value of r . Thus, even if $r = 0$, term premia need not be zero. Note that, as the mean reversion parameter $\kappa \rightarrow 0$, $\beta_1(t,s,T) \rightarrow 0$. The second term $\beta_2(t,s,T)r$ is the time varying portion of the term premia. The coefficient $\beta_2(t,s,T)$ reflects the average sensitivity of term structure volatility to changes in the risk-free rate over the return measurement period.

The expression for the term premia illustrates clearly how the empirical implications of the EH depend on the length of the period over which returns are measured. As $s \rightarrow t$, the term premia in (7) approach zero; as $s \rightarrow T$, the term premia approach

$$\frac{\mu}{\kappa}(1 - e^{-\kappa(T-t)}) - \mu(T - t) - \ln A(t,T) + \left(B(t,T) + \frac{(e^{-\kappa(T-t)} - 1)}{\kappa} \right) r. \quad (8)$$

Differentiation shows that term premia are decreasing functions of μ and σ^2 . However, the relation between κ and the term premia is indeterminate. The intuition for this is that the effect of κ on the term premia depends on whether the risk-free rate is above or below its long-term mean μ ; if $r > \mu$, then an increase in the speed of adjustment parameter κ can lower the average volatility of the term structure over the next $s - t$ periods, and vice versa.

From (7), the relation between term premia and the level of interest rates is negative. This property is consistent with the behavior of term premia for longer maturity bonds as shown by Fama and Bliss (1987) and Froot (1989).⁹ Since r can be arbitrarily large in the CIR (1985) setting, the magnitude of term premia in the L-EH for discrete holding periods is also unbounded. However, simple calculations using parameter values that imply a mean and standard deviation

⁸ In showing that the L-EH can imply time varying term premia, we have assumed that the risk-free rate follows the CIR (1985) square root model. However, the same result obtains for any stochastic term structure model that implies time varying term structure volatility. Examples of these types of term structure models include Dothan (1978), Richard (1978), Brennan and Schwartz (1979), and Longstaff (1989b).

⁹ For example, see Froot (1989), footnote 29.

Table I
Examples of Average Term Premia
and Term Premia Volatility in
One-Year Holding Period
Returns Implied by the Local
Expectations Hypothesis

Term premia statistics are expressed in basis points per annum. The values in the table are obtained by numerically integrating the expression for term premia given in the text. The parameter values used are $\kappa = 0.05$, $\mu = 0.05$, and $\sigma^2 = 0.002$. These parameter values imply a long-run average risk-free rate of 5% and an unconditional standard deviation for the risk-free rate of approximately 3% per annum.

Bond Maturity in Years	Mean Term Premium	Std. Dev. of Term Premium
5	-8.0	5.0
10	-27.3	16.8
15	-48.4	29.9
20	-66.9	41.2
25	-81.5	50.3
30	-92.4	57.0

for the risk-free rate of 0.05 and 0.03 respectively—shown in Table I—suggest that the average L-EH term premia for holding periods of one year are on the order of -8 to -90 basis points for bonds with maturities ranging from 5 to 30 years. These average term premia values are similar to those reported by Fama and Bliss (1987) for bonds with 2- to 5-year maturities.¹⁰ Differentiating the average term premia with respect to T shows that they flatten out at $T \rightarrow \infty$. Fama and Bliss (1987) find that the average term structure of term premia is flat for longer maturity bonds.

The variation in term premia can be significant in economic terms. For example, the variation in term premia can represent 5% to 20% of the variation in the expected returns of longer maturity bonds. The values given in Table I indicate that, for holding periods of one year, the unconditional standard deviation of term premia is on the order of 5 basis points for bonds with a maturity of 5 years, rises rapidly to about 17 basis points for maturities of 10 years, and can exceed 55 basis points for 30-year maturities. This pattern is also consistent with Froot (1989), who finds that the variability of term premia increases with the maturity of the bond.

It is also interesting to examine the relation between the term premia implied by the L-EH and the forward-spot rate differential. Fama (1984) argues that the

¹⁰ Table 2 of Fama and Bliss (1987) reports average term premia for one-year holding periods for 2- to 5-year maturity bonds for the 1964-1985 period. The average term premia range from -11 to -83 basis points.

slope coefficient in the regression of realized term premia on the forward-spot rate differential should be zero under the “pure expectations” hypothesis.¹¹ While this may be true in the discrete time versions of the EH discussed by Fama (1984), this slope coefficient need not be zero in the L-EH. To see this, recall that in continuous time the forward rate is given by $-P_T/P$. Thus, from (5), the forward-spot rate differential is

$$\kappa\mu B(t,T) - \left(\kappa B(t,T) + \frac{\sigma^2}{2} B^2(t,T) \right) r. \quad (9)$$

Noting that both the term premia and the forward-spot rate differential are linear functions of the current risk-free rate, it is straightforward to show that the slope coefficient in the regression of the term premium on the forward-spot rate differential is

$$\frac{-\beta_2(t,s,T)}{\kappa B(t,T) + \sigma^2 B^2(t,s,T)/2}, \quad (10)$$

which is positive—consistent with the empirical results of Fama (1984), Fama and Bliss (1987), and others. Calculations using parameter values similar to those in Table I indicate that the slope coefficient in (10) can range from about 0.10 for bonds with maturities of 5 years to over 0.50 for 30-year bonds.

IV. Conclusion

We have shown that the L-EH can imply the presence of time varying term premia in discretely observed bond returns. The reason for this is that term premia in observable returns depend on the volatility of the term structure. Thus, even if there is no instantaneous premium, term premia may appear in temporally aggregated bond returns. Furthermore, a number of the properties of these term premia appear consistent with empirical evidence about the behavior of observed term premia.

These results are important because they show that the case against the EH is weaker than claimed in the empirical literature—evidence of time varying term premia is not inconsistent with all forms of the EH. These results, of course, do not imply that the EH is correct. Rather, the implication of these results is that new approaches of testing term structure models are needed—approaches that will explicitly take into account the dependence of the models’ empirical implications on the sampling frequency of the data. As yet, no completely unambiguous test of the EH has appeared in the empirical literature.

¹¹ Fama (1984) shows that, when this slope coefficient is zero, the slope coefficient in the regression of the change in the spot rate on the forward-spot rate differential is one. Thus, if the former slope coefficient is nonzero, the latter slope coefficient will deviate from one. Empirical evidence that the latter slope coefficient differs from one has frequently been interpreted as evidence against the EH. See Shiller and McCulloch (1987).

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