

A new method to estimate risk and return of non-traded assets from cash flows: The case of private equity funds*

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Abstract

We develop a new GMM-style methodology with good small-sample properties to assess the abnormal performance and risk exposure of a non-traded asset from a panel of cash flow data. We apply this method to a sample of 958 mature private equity funds spanning 24 years. Our methodology uses actual cash flow data and not intermediary self-reported net asset values. In addition, it does not require a distributional assumption for returns. For venture capital funds, we find a high market beta and underperformance before and after fees. For buyout funds, we find a relatively low market beta and mixed evidence for outperformance. Larger venture capital funds have higher returns due to higher risk exposures and not due to higher alphas. We also find that net asset values significantly overstate fund market values for the subset of mature and inactive funds.

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We develop a new GMM-style methodology with good small-sample properties to assess the abnormal performance and risk exposure of a non-traded asset from a panel of cash flow data. We apply this method to a sample of 958 mature private equity funds spanning 24 years. Our methodology uses actual cash flow data and not intermediary self-reported net asset values. In addition, it does not require a distributional assumption for returns. For venture capital funds, we find a high market beta and underperformance before and after fees. For buyout funds, we find a relatively low market beta and mixed evidence for outperformance. Larger venture capital funds have higher returns due to higher risk exposures and not due to higher alphas. We also find that net asset values significantly overstate fund market values for the subset of mature and inactive funds.

JEL classification: C51; G12; G23

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1 Introduction

The estimation of risk exposure (beta) and abnormal performance (alpha) is at the heart of financial economics. Since Jensen's (1968) time-series regression approach to determine the alpha and beta of a mutual fund, a large literature has been dedicated to refining measures of risk and return (see Cochrane, 2005a, for an overview). However, the case of a non-traded asset for which we only observe cash flows has received little attention. For example, private equity fund investors give away cash at different points in time and receive dividends at other points in time during the finite life of the fund. In this paper, we propose a methodology to measure risk and abnormal return in such a context and apply it to a sample of private equity funds.

The econometrician observes, for instance, that a fund invested 100 at the end of year 1, distributed 200 at the end of year 3 and a final dividend of 100 at the end of year 5. Then the fund is liquidated. Given the absence of return data, time-series estimation of alpha and beta is not possible in this case. However, if one assumes that the alpha and beta are the same across funds, a natural way is to find the alpha and beta that provide the best fit of the cross-section of cash flows. We show that this boils down to finding the alpha and beta that get the net present values (NPV) of (portfolios of) funds closest to zero, where the NPV of the cash flows is calculated using a given asset pricing model. We also show that this can be written as a GMM estimation and that this method is asymptotically consistent.

Importantly, our method does not require an assumption for the probability density function (pdf) of one-period returns. This is a key contribution because it is basically impossible to estimate this pdf when an asset is not traded. At best, one can compute the pdf using the cross-section of geometric average returns such as internal rate of returns (IRRs). However, the distribution of IRRs is not the same as the pdf of the underlying one-period returns. In fact, several pdfs of one-period returns can be consistent with a given pdf of IRRs. In addition, in many asset classes (e.g. venture capital, buyout), the one-period return distribution is probably nonstandard given that the distribution of IRRs displays a substantial cluster at -100% and fat right tail. A contributing factor may be

that these one-period returns are generated by active fund managers. They choose when to invest and divest. This cash flow endogeneity leads to complex return distributions.¹

Turning to our GMM approach, we analytically show in a simple setting, and confirm by simulations, that the goal function (the sum of the squared NPVs) is not globally convex. We then propose two solutions. The first solution consists in minimizing the distance between the log of the present value of the investments and the log of the present value of the dividends. Although asymptotic consistency is preserved, this log-transformation generates a nonlinearity bias in small samples. We show that this is a downward bias for alpha. A second solution consists in minimizing the distance between (i) the ratio of the present values of the investments and the present value of the dividends and (ii) one. This specification also generates a nonlinearity bias in small samples, but in this case, it is an upward bias for alpha. The two solutions thus provide (estimates of) lower and upper bounds around the true value. In simulations, we find that the difference between these two alphas is small and that the log transformation provides the smallest small-sample bias.

In addition, we (i) form portfolios of funds in order to reduce idiosyncratic volatility and (ii) do so by inception year. Intuitively, portfolios with different inception years are subject to different market returns, and this identifies beta. Alpha is identified from the restriction that the expected net present values of all liquidated portfolios equal zero.

Importantly, our approach is validated by a number of simulations. We generate cash flows assuming a market model with given alpha and beta, and different non-standard one-period return distributions. In each case, and despite the fact that we do not use information on the true return distribution, we obtain alpha and beta estimates that are very close to the true values (within one basis point per month for alpha and 0.01 for beta).

We apply our new method to a trillion-dollar asset class: private equity funds. These funds are financial intermediaries that are typically classified as venture capital focused or buyout focused. They are not publicly traded and investors in these funds observe only a stream of cash flows for about 10 years. Hence, as mentioned above, standard estimation techniques cannot be applied to measure risk and abnormal return.

¹Note then, that by avoiding distributional assumptions, we can accommodate any type of cash flow endogeneity. We show this formally in appendix 1.

In addition to our methodology, we can contribute to the private equity literature in two ways. First, we provide an estimate of the cost of capital. This is an essential figure in practice. For example, venture capitalists need an estimate of the cost of capital to take their many investment decisions. Second, we provide an estimate of abnormal return. An ongoing debate in this literature is whether these financial intermediaries (private equity funds) add value. A number of papers argue that once a company is in the hand of private equity funds its free cash flow increases (e.g. Kaplan, 1989).² A number of reasons have been proposed for such a result. Kaplan and Stromberg (2003), for instance, show that private equity funds write sophisticated contracts, which induce the right incentives for the companies they finance. Yet, private equity funds may pay too much for the companies they buy. In public to private transactions, for example, the premium (over market value) paid by private equity funds averages as much as 40% in both the US and the UK (e.g. Kaplan, 1989, Renneboog, Simons and Wright, 2007). Hence, despite the fact that they may write the right contracts and generate extra cash flows, they may underperform public equity before fees.³ Another related question is that the fees they charge may be too large. They may outperform before fees but underperform after fees. In this case, they would add value but investors would be paying too much for this financial intermediation. Our data and methodology will bring some unique evidence on this important debate. We are the first paper to decompose private equity returns into what comes from systematic risk and alpha, and to show this decomposition both before fees and after fees.

Our dataset contains the cash flows (dividends and investments) of 958 mature private equity funds between 1980 and 2003. Mature funds are those that are more than 10 years old (the typical fund duration). For funds that are not reported as liquidated, we predict their final market value using an econometric model. This model relates the realized market value to fund characteristics at each age and is estimated using the subsample of liquidated funds.

²Evidence comes from buyout funds. There is no direct evidence we know of for venture capital.

³The cash flows that are provided to us are net-of-fees. We simulate a typical fee structure on these cash flows to obtain cash flows gross-of-fees. To our knowledge, there are no studies which have the exact fees charged by a fund. Existing papers also use simulations to quantify the fees (e.g. Metrick and Yasuda, 2009).

We find that, after fees, venture capital funds have a market beta of 3.21. Given the equity premium over our sample period, the cost of capital for venture capital (after fees) is 31% per annum (p.a.) with the CAPM. Before fees, the market beta is higher (3.36) because fees increase with performance. Buyout funds have a significantly different risk profile than venture capital funds. After fees, their market beta is 0.33. Before fees, their beta is 0.56. Buyout funds thus appear to have a risk below that of public equity, making a cost of capital of 8% p.a. according to the CAPM.

A casual look at the data supports this result. In Figure 1, we observe that many venture capital funds paid large dividends mainly in the late 1990s, precisely when the stock market had previously experienced large returns. In 2001-2003, when stock markets experienced lower returns, dividends from venture capital funds have been rare. Such a pattern is consistent with a high beta for venture capital. For buyout funds, we do not observe a strong dependence on market returns. The dividends of buyout funds have been remarkably steady throughout our time period. We discuss in the text potential explanations for such a result, with the caveat that the buyout sample is much smaller than the venture capital sample.

For venture capital, we find strong negative abnormal performance. The alpha is equal to -15% p.a. with the CAPM. Before fees, the underperformance is reduced to -11%, but it is still statistically significant. We also consider a three-factor Fama-French model and show that venture capital returns resemble those of small growth stocks. Relative to these small growth stocks, there is still underperformance of VC, but less so given that small growth stocks perform poorly too.

For buyout funds, abnormal performance is slightly positive both before and after fees, and for both the CAPM and the three-factor model. However, it is mostly statistically insignificant.

These results suggest that venture capital funds generate little (if any) added value. In addition, given their large fees, investors obtain negative alphas. This result may be surprising, but notice that learning about alpha and beta is not trivial. In fact, our paper is one of the first to estimate alphas and betas of private equity funds. Over our sample period,

investors had less data than we have, especially so in the 1980-1993 period, when they had to take their investment decisions (funds last for ten years). Notice also that investors may have observed absolute returns but these do not appear alarming (15% IRR on average). It is the risk correction that makes the alpha negative for venture capital funds and some casual evidence indicates that investors underestimated the risk of venture capital funds.⁴ Also, for buyout funds, where we do not observe underperformance, the beta is similar to that of public equity, hence the potential underestimation of beta may not have been an issue.

Our econometric model for final market values predicts that the value of non-liquidated funds beyond the typical liquidation age (10 years) is only around 30% of the self-reported Net Asset Value (NAV). In contrast, for funds that are liquidated, market values are close to NAV. This substantial discrepancy comes from the fact that the non-liquidated funds have not distributed dividends for a long time (more than 3 years) and have not updated their NAV for a long time (more than 2 years); two characteristics that are significantly associated with poorer subsequent cash flows according to our econometric model.

This paper shows that a cross-section of cash flow streams is sufficient to consistently estimate risk and return at the cost of assuming a common parametric structure for the cross-section of alphas and betas. For example, we first allow alpha and beta to be a function of focus (venture capital, buyout). Subsequently, we make the alpha and beta a function of both fund size and focus, thereby having a different alpha and beta for each fund. This specification allows us to shed light on the finding that large funds have a higher total return than small funds (Kaplan and Schoar, 2005, Ljungqvist, Richardson, and Wolfenzon, 2007, and Hochberg, Ljungqvist and Vissing-Jorgensen, 2008). We find that alpha is not related to size but beta is significantly and positively related to size. The higher return of large

⁴Our data provider is the historical provider of benchmarks for the private equity industry. Individual investors did not have sufficient data to estimate the risk and return of venture capital themselves (until maybe recently). They have thus relied on Thomson Venture Economics for estimates of return and risk. TVE have been providing benchmarks for a long time, but reported a beta for the first time only in 2001 (to our knowledge). In their "2001 Investment Benchmarks Report" they report a beta for venture capital of 0.84. Other firms providing data for asset allocation decisions (e.g. Ibbotson Associates) used estimates from Chen, Baierl and Kaplan (2002), who report a correlation between venture capital and public equity close to zero (4%). These low betas are due to the use of quarterly Net Asset Values that the funds self report. Because of NAV smoothing, betas appear to be low.

funds is thus due to higher risk exposure and not higher abnormal performance.

We also conduct a large number of robustness tests. We observe that results remain essentially unchanged for venture capital funds, while the results for buyout funds are somewhat less stable. Still, across all specifications and setups, buyout funds have low betas and abnormal performance close to zero. The robustness results also confirm that our estimator has good small-sample properties.

The rest of the paper proceeds as follows. In Section 2, we discuss related literature. Section 3 provides a simple example to generate intuition for our method. Section 4 contains a description of the GMM approach in a general setting and presents a simulation study to assess the small-sample properties. Section 5 describes the private equity industry, our data, and the model for final market values. Section 6 presents the empirical results and robustness checks. Section 7 concludes.

2 Related Literature

Cochrane (2005b) and Korteweg and Sorensen (2008) assess the alpha and beta of US venture-backed companies. They observe valuations of projects at each financing round.⁵ They can therefore compute a return. If one observes a return for each investment, a standard nonlinear least squares approach can be used. However, their data have two important features: (i) missing financing rounds and (ii) sample selection bias (only companies that perform well get a new financing round). Consequently, they need to assume a parametric structure for both the return distribution and the selection equation (e.g. assume lognormally distributed returns).

Our method is designed for cases when representative cash flow streams are observed, hence cases where (i) no return can be computed without restrictive assumptions⁶ and (ii)

⁵Venture Capital (VC) funds invest in distinct projects in so-called rounds. A return from round n to $n + 1$ is observed only if i) project valuation post-money at round n is observed, ii) there is a subsequent valuation round $n + 1$ which happens only if investments do well enough, and iii) project valuation pre-money at round $n + 1$ is observed. Cochrane (2005b) reports that a return could not be computed in 58% of the cases and a subsequent round (item 2) is not observed in 23% of the cases. Korteweg and Sorensen (2008) use an improved sample and cannot compute a return in 36% of the cases.

⁶A return can be computed if and only if the cash flow stream consists only of one investment and one dividend. When there are intermediary cash flows, one can only compute a return at the cost of making

there are no significant sample selection biases.⁷ Point (i) means that we cannot use standard nonlinear least squares. Point (ii) means that we can avoid making any distributional assumptions which is an important feature and one of our main contributions.

The advantage of the dataset of Cochrane-Korteweg-Sorensen (CKS) compared to the one we use in the empirical section is that they have more disaggregated information, which (i) may lead to more precise estimates of risk and return and (ii) allow for an analysis of risk and return as a function of portfolio company characteristics. Further, it is important to note that, in the empirical section, we do not measure exactly the same object as CKS. For example, CKS would measure the return of Google from its valuation at the first round of venture financing until the IPO date. In contrast, we observe what investors paid and received from the investment in Google.⁸

In terms of empirical results, the beta for venture capital reported by Korteweg and Sorensen (2008) is close to our estimate, while Cochrane's estimate for beta is a bit lower (1.9). The alpha, however, is negative in our case, both before fees and after fees. Cochrane (2005b) and Korteweg and Sorensen (2008), in contrast, report large positive before-fee alphas.

Our paper is also related to Kaplan and Schoar (2005) and Phalippou and Gottschalg (2009) who benchmark private equity funds to the S&P 500 index, effectively assuming a CAPM with beta equal to one. Our results suggest that the natural benchmark for venture capital is much higher. In addition, we show via an econometric model that NAVs reported by mature funds (beyond their 10th anniversary) are exaggerated but not worthless; hence, we predict fund market values that are in between the Kaplan-Schoar assumption (market value of non-liquidated mature funds equals reported final NAV) and the Phalippou-Gottschalg assumption (market value of non-liquidated mature funds equals zero).⁹

a so-called re-investment assumption. An internal rate of return is such a return. Because our method does not require the observation of a return, we do not require assumptions on how paid-out dividends are reinvested by the investor.

⁷The data we use in the empirical section include the full cash flow history of a large number of funds. Hence, there are no (a priori) significant sample selection biases.

⁸An extreme example is the eBay IPO. Benchmark Partners return in eBay was 20 times the investment at the IPO. This is what CKS would observe. However, investors received the eBay stocks 6 months after the IPO, when the price had increased by more than 3000% making their stake worth 700 times the investment.

⁹Also related is the work of Jones and Rhodes-Kropf (2004), who estimate the risk and return of private equity funds from the time series of returns constructed from NAVs.

Another study proposing a risk adjustment for buyout investments is that of Ljungqvist and Richardson (2003). They match buyout investments to similar publicly traded companies, assume a certain leverage and propose a beta close to unity. Finally, Moskowitz and Vissing-Jorgensen (2002) document returns obtained by entrepreneurs (mainly family businesses). The asset class they study is distinct from ours (although both are called "private equity") but with similar characteristics (illiquid, skewed return distribution). Like them, we find puzzlingly low returns.

3 Estimating Risk and Abnormal Return: A simple example

We first show how our method works in a simple example. We assume a CAPM economy with two funds, no idiosyncratic shocks, a risk-free rate set to zero, and the true beta set to 1.5. Each fund consists of two separate projects costing 100 each and lasting for two years. Hence, the dividends are $100(1 + 1.5R_{m,t})(1 + 1.5R_{m,t+1})$. Assuming some market returns, the observed cash flows are as follows:

Year	Market ret.	Fund 1: Cash flows (year end)	Fund 2: Cash flows (year end)
0	—	-100	0
1	20%	-100	-100
2	15%	159	-100
3	5%	132	132
4	-10%	0	91
5	30%	0	0

If the econometrician knew all of the above, she would write the four project pricing errors, which are denoted $e_{ij}(t)$ for project j of fund i

$$\begin{aligned}
e_{11}(0) &= \frac{159}{(1 + \beta R_{m,1})(1 + \beta R_{m,2})} - 100 & (1) \\
e_{12}(1) &= \frac{132}{(1 + \beta R_{m,2})(1 + \beta R_{m,3})} - 100 \\
e_{21}(1) &= \frac{132}{(1 + \beta R_{m,2})(1 + \beta R_{m,3})} - 100 \\
e_{22}(2) &= \frac{91}{(1 + \beta R_{m,3})(1 + \beta R_{m,4})} - 100
\end{aligned}$$

and would find that $\beta = 1.5$ solves this system of equations.

The objective of this paper is to design an econometric approach that generates an estimate of beta as precise as possible when the econometrician does not observe the correspondence between each dollar invested and their payoff. The econometrician observes only the cash flows. In other words, the econometrician cannot distinguish between the above situation and, for example, a situation in which half of the first project would have been liquidated at date 2 and half would have been liquidated at date 3. Such a situation is common, especially with non-traded assets like private equity.

A simple and natural solution is to calculate the net present value (NPV) of each cash flow stream. The econometrician can write

$$\frac{159}{(1 + \beta R_{m,1})(1 + \beta R_{m,2})} + \frac{132}{(1 + \beta R_{m,1})(1 + \beta R_{m,2})(1 + \beta R_{m,3})} = 100 + \frac{100}{(1 + \beta R_{m,1})} \quad (2)$$

for the first fund and, for the second fund,

$$\frac{132}{(1 + \beta R_{m,2})(1 + \beta R_{m,3})} + \frac{91}{(1 + \beta R_{m,2})(1 + \beta R_{m,3})(1 + \beta R_{m,4})} = 100 + \frac{100}{(1 + \beta R_{m,2})} \quad (3)$$

Both equations have the present value of dividends on the left-hand-side, and the present value of the investments on the right-hand-side, where the discounting is done with the pricing model. Solving these two equations leads to the correct answer of $\beta = 1.5$. This is because these two NPVs are simply a weighted sum of the project-level pricing errors, i.e.

(2) and (3) can be rewritten as

$$e_{11}(0) + \frac{e_{12}(1)}{(1 + \beta R_{m,1})} = 0, \quad e_{21}(1) + \frac{e_{22}(2)}{(1 + \beta R_{m,2})} = 0 \quad (4)$$

Hence, the econometrician finds the right answer even though she did not know the correspondence between each dollar invested and their payoff. This result is generalized in the next section with a large number of investments, multiple risk factors, idiosyncratic shocks and mispricing (α).

An important point illustrated with this example is that having investments starting at different dates is key for identification. Both equations (2) and (3) above are a third-order polynomial, hence they may have multiple real solutions. In the above example, multiple real solutions exist only if the true beta is below -4. For instance, if the true beta is -5, the other solutions are $\beta = -18.8$ and $\beta = -12.8$ for fund 1 and -25.5 and 7.2 for fund 2. Yet, there is a unique solution if we solve the system of equations. Mathematically, the coefficients in the polynomial depend on the realized market returns so that all solutions (except the ‘correct’ value) depend on the realized market returns. Hence, as long as the market returns that each fund faces are different, there should be a unique solution. This makes intuitive sense. It is by observing cash flow amounts in different market environments that one can learn about systematic risk exposure. Although we do not have a formal proof for uniqueness of the solution, we always find a unique optimum in the examples we took, the many Monte Carlo simulations we run (section 4.6) and in the empirical application (section 6).

In addition, let us note that we do not make any assumptions on cash flow exogeneity. This will be shown analytically and via Monte Carlo simulations below but we can already see this here and gain some intuition. To this end, assume that cash flows are endogenous in the sense that investments are exited only if the market reaches a certain cumulated performance; say 20%. The new cash flow pattern is then:

Year	Market ret.	Fund 1: Cash flows (year end)	Fund 2: Cash flows (year end)
0	–	-100	0
1	20%	130-100=30	-100
2	15%	0	-100
3	5%	132	132
4	-10%	0	0
5	30%	0	132

Solving for beta with these cash flows again leads to a unique solution of $\beta = 1.5$, the true value. The intuition for this result is that delaying a cash flow by one period simply means that the project return grows at the cost of capital for one more period, hence it does not affect the estimate. The key underlying assumption here is that, as long as a fund has not paid out, the amount invested grows at the cost of capital, which is a typical assumption in asset pricing (see below, assumption 1). This example also shows that we do not need any assumption on how dividends are invested by the investor after receiving the dividend. Whether the investor re-invest dividends in the S&P 500 or in Treasury bills does not affect our calculations. This is reassuring since what we want to measure is the "intrinsic" private equity risk, and not that of an investment strategy that would mix private equity with another asset class.

The general case is developed in the next section. Things get a bit more complicated. As always when estimating systematic risk exposure, the issue stems from the existence of idiosyncratic shocks. In a nutshell, when there are idiosyncratic shocks the dividends will not be exactly 159, 132, 132 and 91 and the econometrician will have to solve an overidentified system.

4 Estimating Risk and Abnormal Return: General case

In this section, we formally derive our GMM-style methodology to estimate risk and return of a non-traded asset for which we only observe cash flows. We first set our two assumptions. Next, in section 4.2, we derive the simple GMM approach. Section 4.3 shows that,

although consistent, the simple GMM approach generates a small-sample bias. In section 4.4, we therefore introduce two improved GMM-style approaches. These approaches preserve statistical consistency, minimize the small-sample bias and offer bounds for the true parameter values. Section 4.5 discusses how to best group funds into portfolios and how to calculate standard errors. Finally, section 4.6 shows Monte Carlo simulations to illustrate our theoretical claims and select the best approach.

4.1 Assumptions

We now turn to a general framework. The main changes compared to the above example are that we i) allow for a mispricing parameter α , ii) introduce idiosyncratic shocks for each private equity project, and iii) allow for a larger cross-section of funds.

There are two assumptions we make. The first assumption is standard in the performance measurement literature.

Assumption 1: The latent return $R_{ij,t}$ on a dollar invested in project j of fund i in period t is generated by a linear factor model with idiosyncratic shocks. For example, in case of a CAPM (or one-factor market model) we assume

$$R_{ij,t} = r_{f,t} + \alpha_i + \beta_i r_{m,t} + \varepsilon_{ij,t} \quad (5)$$

where $r_{f,t}$ is the risk-free rate, $r_{m,t}$ is the market return in excess of the risk-free rate, $\varepsilon_{ij,t}$ and $r_{m,s}$ are independent for all t and s , $\varepsilon_{ij,t}$ and $\varepsilon_{ij,s}$ are independent if $t \neq s$, and $E[\varepsilon_{ij,t}] = 0$. This assumption implies that project returns and associated cash flows do not predict future stock market returns.¹⁰ In section 4.5 we discuss the assumption we make on the correlation of $\varepsilon_{ij,t}$ across projects for the calculation of standard errors. But to obtain a consistent estimate of risk and abnormal performance, we can keep this correlation unspecified.

Assumption 2: Some cross-sectional restrictions are placed on the α_i and β_i parameters.

An example of such an assumption is to assume that all funds have the same alpha

¹⁰To justify this assumption, we run a regression of monthly stock market returns on aggregate lagged dividend yields from private equity funds. For our sample, we do not find evidence that PE dividend yields predict stock market returns.

and beta. This is what Cochrane (2005b) and Korteweg and Sorensen (2008) assume and what we assume too for the main empirical part. Intuitively, since we observe only a cross-section of funds, we need to impose some cross-sectional restrictions to prevent having an underidentified system. Assuming parameters to be equal for all funds is, however, just one extreme case of such an assumption. All we need is some restrictions to ensure identification. For example, we can specify that $\beta_i = b_0 + b_{size} * \ln(\text{fund size}_i)$. In this case, virtually all funds have a different beta. The pair of parameters (b_0, b_{size}) however is the same for all funds.

Importantly, assumption 1 does not mean that we make an assumption about the reinvestment of cash flows. All we need is the assumption that, as long as an investment has not paid out, its value grows by $1 + r_f + \alpha_i + \beta_i r_{m,t} + \varepsilon_{ij,t}$ each period. This is appropriate for private equity funds, for example, because they directly pass through any dividends to their investors.¹¹ Note that we show direct evidence of this by simulations below. We simulate investments and dividends without any reinvestment assumptions, and when applying our approach we (basically) find the correct alpha and beta.

4.2 Derivation

We now develop the approach more formally. The derivations are done with a one-factor market index model for simplicity. A generalization to multi-factor pricing models is trivial as long as the factors are traded assets in order to measure abnormal performance with α .

Without loss of generality, we derive the moment conditions for portfolios of project investments. In the empirical section, we have fund-level data, with each fund typically having multiple project investments, and will form portfolios of funds. To facilitate the exposition, we thus label the portfolios of investments "fund-of-funds" or "FoF".

Each FoF i invests an amount T_{ij} in project j at date t_{ij} . There is a total of n_i projects for FoF i and a liquidation dividend D_{ij} is paid at date d_{ij} for each project. For each FoF

¹¹See, for example, Lerner and Schoar (2004, page 7): "The general partners (the private equity fund's managers) invest the capital raised from limited partners, typically large institutional and individual investors, in entrepreneurial or restructuring funds. After the firms go public or are sold, the proceeds (whether in the form of equity or cash) are divided between the limited and general partners, leading to a close alignment of the incentives of the two parties."

i , the cash flows occur between the inception date t_{0i} and the liquidation date L_i . The dividend of the project j at date d_{ij} is given by

$$D_{ij} = T_{ij} \prod_{t=t_{ij}+1}^{d_{ij}} (1 + r_{f,t} + \alpha + \beta r_{m,t} + \varepsilon_{ij,t}) \quad (6)$$

We then divide by $\prod_{t=t_{ij}+1}^{d_{ij}} (1 + r_{f,t} + \alpha + \beta r_{m,t})$ and take expectations on both sides of equation (6). From assumption 1, $r_{m,s}$ and $\varepsilon_{ij,t}$ are independent for all t and s and the expectations of the cross-products of the form $\varepsilon_{ij,t}\varepsilon_{ij,s}$ are equal to zero (as well as higher-order cross-products) for $t \neq s$, and we obtain

$$E_{t_{ij}} \left[\frac{D_{ij}}{\prod_{t=t_{ij}+1}^{d_{ij}} (1 + r_{f,t} + \alpha + \beta r_{m,t})} - T_{ij} \right] = 0 \quad (7)$$

This equation simply says that the expected pricing error of a project equals zero if one uses the correct pricing model, conditional upon the information set at time t_{ij} . This is like equation (1) in the example above. Appendix 1 shows that equation (7) holds irrespective of the timing of the dividend d_{ij} (endogeneity). Hence, our method allows for the possibility of exit timing (e.g. early exit if good performance).

As explained in the previous section (equation (2)), the econometrician observes only a stream of cash flows. We thus need to discount all cash flows from time t_{ij} to the inception date of FoF i (t_{0i}). We thus divide equation (7) on both sides by $\prod_{s=t_{0i}+1}^{t_{ij}} (1 + r_{f,s} + \alpha + \beta r_{m,s})$ to obtain

$$E \left[\frac{1}{\prod_{s=t_{0i}+1}^{t_{ij}} (1 + r_{f,s} + \alpha + \beta r_{m,s})} \left(\frac{D_{ij}}{\prod_{t=t_{ij}+1}^{d_{ij}} (1 + r_{f,t} + \alpha + \beta r_{m,t})} - T_{ij} \right) \right] = 0 \quad (8)$$

Note that the discount factor $\prod_{s=t_{0i}+1}^{t_{ij}} (1 + r_{f,s} + \alpha + \beta r_{m,s})$ enters the expectation as a result of assumption 1: if the pricing model is correctly specified then, by definition, pricing

errors are not predictable by lagged discount factors. Importantly, multiplying the pricing error by this discount factor does not imply any assumption on how the investor reinvests dividends after being paid out or how the investor invests his cash before it is taken by the fund at t_{ij} . This discount factor simply weights the pricing error. We one could have used any other discount factor (observable at date t_{ij}) to derive the same moment conditions only with different weights. Our particular choice of the discount factor $\prod_{s=t_{0i}+1}^{t_{ij}} (1+r_{f,s}+\alpha+\beta r_{m,s})$ makes sure that we only need fund-level data to estimate the parameters.

Equation (8) can be written more conveniently as

$$E \left[\frac{D_{ij}}{\prod_{t=t_{0i}+1}^{t_{ij}} (1+r_{f,t}+\alpha+\beta r_{m,t})} \right] = E \left[\frac{T_{ij}}{\prod_{s=t_{0i}+1}^{t_{ij}} (1+r_{f,s}+\alpha+\beta r_{m,s})} \right] \quad (9)$$

The moment condition (9) is the basis of our estimation methodology. We have one moment condition per FoF, so having multiple FoFs generates an overidentified system.

Next, we construct the sample equivalent of the expectations in equation (9) by averaging across projects within a FoF. The left hand side of (9) is estimated by

$$\overline{PV}^{D_i}(\alpha, \beta) = \frac{1}{n_i} \sum_{j=1}^{n_i} \left[\frac{D_{ij}}{\prod_{t=t_{0i}+1}^{t_{ij}} (1+r_{f,t}+\alpha+\beta r_{m,t})} \right] \quad (10)$$

which is simply the present value of all dividends of the FoF (scaled by $1/n_i$). The right hand side of (9) is then estimated by

$$\overline{PV}^{T_i}(\alpha, \beta) = \frac{1}{n_i} \sum_{j=1}^{n_i} \left[\frac{T_{ij}}{\prod_{s=t_{0i}+1}^{t_{ij}} (1+r_{f,s}+\alpha+\beta r_{m,s})} \right] \quad (11)$$

which is the present value of all investments given the discount rate $(1+r_{f,t}+\alpha+\beta r_{m,t})$. Note that the expressions (10) and (11) can be calculated even if we do not know the correspondence between each dollar invested and its payoff. The econometrician needs to only observe the cash flow stream and compute the present values of both the investments

and the dividends to the FoF inception date using the cost of capital.

We then have N moment conditions, one for each FoF, so that we can construct a GMM estimator if $N \geq 2$ (in case of the market model). Mathematically, the first-step GMM estimator with identity weighting matrix is the solution of the following optimization

$$\min_{\alpha, \beta} \sum_{i=1}^N [\overline{PV}^{D_i}(\alpha, \beta) - \overline{PV}^{T_i}(\alpha, \beta)]^2 \quad (12)$$

We have just derived a generalization of what we did in the simple example above. As the number of projects n_i per FoF tends to infinity, the averages \overline{PV}^{D_i} and \overline{PV}^{T_i} converge to the expectations in equation (9).¹² Therefore, the parameters estimated from the GMM optimization (12) are consistent under standard GMM regularity conditions. Our asymptotics are thus ‘cross-sectional’, in the sense that we let the number of projects (or funds) in a FoF go to infinity. We label this method the ‘Net Present Value (NPV) approach’.¹³

4.3 Small-sample bias in the simple GMM approach

We have just derived a consistent estimator of risk and return based on cash flow data. Although computing an NPV is a natural approach, in practice, the goal function is not globally convex. Hence, the solution may diverge in small/finite samples. This is because one way to get equation (8) to hold is to have $\prod_{s=t_{0i}+1}^{t_{ij}} (1 + r_{f,s} + \alpha + \beta r_{m,s})$ going to infinity (by letting $\alpha \rightarrow \infty$). In this subsection, we provide intuition for this bias using a simplified example. Deriving the bias in closed-form proved to be unfeasible and we rely on a Monte Carlo simulation to assess it in a realistic setting (section 4.6).

In this simplified example, we assume that all projects in a FoF have a takedown equal

¹²To make sure that the averages in equation (12) converge, we need restrictions on the size of the projects. A sufficient condition is that the amounts invested are drawn from a probability distribution with a finite mean.

¹³Our approach is related to the GMM estimation of the pricing kernel based on the Euler equation (Cochrane, 2005a). As shown in Cochrane (2005a, equation 8.3), the parameters a and b in the CAPM pricing kernel ($a + bR_{m,t}$) can be found by imposing that the risk-free asset and market return are priced correctly. Then, one can write an Euler equation for the cash flows of a fund, and test whether this equation holds. However, this does not lead to direct estimates of the abnormal performance α and risk β . β is usually estimated by $Cov(R, R_m)/V(R_m)$, which is not feasible here because we do not have return time-series. Hence, in contrast to the Euler equation approach, our method renders direct estimates of α and β .

to 1, a duration of one period, $r_f = 0$ and N FoFs which are active in different time periods. The average realized dividend of FoF i can then be written as

$$\frac{1}{n_i} \sum_{j=1}^{n_i} D_{ij} = \frac{1}{n_i} \sum_{j=1}^{n_i} [T_{ij}(1 + \alpha + \beta r_{m,t(i)} + \varepsilon_{ij})] = (1 + \alpha + \beta r_{m,t(i)} + \bar{\varepsilon}_i) \quad (13)$$

where $\bar{\varepsilon}_i = \frac{1}{n_i} \sum_{j=1}^{n_i} \varepsilon_{ij}$ and where $r_{m,t(i)}$ is the one-period market return.

The term that causes the bias ($\prod_{s=t_0+1}^{t_{ij}} (1 + r_{f,s} + \alpha + \beta r_{m,s})$) is present because we discount all the cash flows back to the inception date of the FoF. Again, as explained in the example, this is because the econometrician does not observe the correspondence between each dollar invested and their payoff; thus, she takes the NPV of the whole cash flow series.

To capture the bias in a nutshell, we discount each project to one period before its starting date. Let α and β be the true parameter values, and $\tilde{\alpha}$ and $\tilde{\beta}$ be the parameters over which we optimize. The optimization becomes (using equation (13))

$$\begin{aligned} &= \min_{\tilde{\alpha}, \tilde{\beta}} \sum_{i=1}^N \left[\frac{(1 + \alpha + \beta r_{m,t(i)} + \bar{\varepsilon}_i)}{(1 + \tilde{\alpha} + \tilde{\beta} r_{m,t(i)-1})(1 + \tilde{\alpha} + \tilde{\beta} r_{m,t(i)})} - \frac{1}{(1 + \tilde{\alpha} + \tilde{\beta} r_{m,t(i)-1})} \right]^2 \quad (14) \\ &= \min_{\tilde{\alpha}, \tilde{\beta}} \sum_{i=1}^N \left[\frac{(\alpha - \tilde{\alpha}) + (\beta - \tilde{\beta}) r_{m,t(i)} + \bar{\varepsilon}_i}{(1 + \tilde{\alpha} + \tilde{\beta} r_{m,t(i)-1})(1 + \tilde{\alpha} + \tilde{\beta} r_{m,t(i)})} \right]^2 \end{aligned}$$

This shows that the ‘pricing error’ $(\alpha - \tilde{\alpha}) + (\beta - \tilde{\beta}) r_{m,t(i)} + \bar{\varepsilon}_i$ is divided by $(1 + \tilde{\alpha} + \tilde{\beta} r_{m,t(i)-1})(1 + \tilde{\alpha} + \tilde{\beta} r_{m,t(i)})$. Asymptotically, $\bar{\varepsilon}_i$ tends to zero and $\tilde{\alpha}$ and $\tilde{\beta}$ are consistent estimators of α and β . In a small sample, however, minimizing these ‘discounted pricing errors’ generates a tendency to enlarge the (positive) term $(1 + \tilde{\alpha} + \tilde{\beta} r_{m,t(i)-1})(1 + \tilde{\alpha} + \tilde{\beta} r_{m,t(i)})$; thus, in particular, an upward bias for $\tilde{\alpha}$. Moreover, the goal function in (14) tends to zero as $\tilde{\alpha} \rightarrow \infty$, since the denominator in (14) has a cubic dependence on $\tilde{\alpha}$ and the numerator depends linearly on $\tilde{\alpha}$. Hence, the goal function is not globally convex. This is shown in Figure 2 with the goal function we obtain from the Monte Carlo simulation described in section 4.6.

4.4 An improved GMM-style approach

The small-sample bias that we document above comes from multiplying the pricing error by $1/\prod_{s=t_{0i}+1}^{t_{ij}} (1+r_{f,s}+\alpha+\beta r_{m,s})$ in equation (8). To reduce the effect of this multiplicative term, we take relative differences between the value of dividends and the value of investments, instead of absolute differences. Relative differences can be calculated using either a ratio or a difference in log. Either approach leads to a statistically consistent estimator (see appendix 2 for a formal proof) but generates other small-sample biases that come from the nonlinearity of the ratio or log transformation. These biases will be evaluated numerically in section 4.6.

4.4.1 Public Market Equivalent approach (PME)

This method minimizes the distance between unity and the ratio of the present values of dividends and investments (known as Public Market Equivalent, see Ljungqvist and Richardson, 2003, and Kaplan and Schoar, 2005):

$$\min_{\alpha, \beta} \sum_{i=1}^N \left[\frac{\overline{PV}^{D_i}(\alpha, \beta)}{\overline{PV}^{T_i}(\alpha, \beta)} - 1 \right]^2 \quad (15)$$

Asymptotically, this approach is consistent as it is based on the same moment condition as the NPV method above (equation (9); see also appendix 2).

To gain insight into the small-sample properties, we go back to the simplified example above (using equation (13)). In this case, the estimation can be rewritten as

$$\begin{aligned} &= \min_{\tilde{\alpha}, \tilde{\beta}} \sum_{i=1}^N \left[\frac{(1 + \alpha + \beta r_{m,t(i)} + \bar{\varepsilon}_i)}{(1 + \tilde{\alpha} + \tilde{\beta} r_{m,t(i)-1})(1 + \tilde{\alpha} + \tilde{\beta} r_{m,t(i)})} \Big/ \frac{1}{(1 + \tilde{\alpha} + \tilde{\beta} r_{m,t(i)-1})} - 1 \right]^2 \quad (16) \\ &= \min_{\tilde{\alpha}, \tilde{\beta}} \sum_{i=1}^N \left[\frac{(\alpha - \tilde{\alpha}) + (\beta - \tilde{\beta}) r_{m,t(i)} + \bar{\varepsilon}_i}{(1 + \tilde{\alpha} + \tilde{\beta} r_{m,t(i)})} \right]^2 \end{aligned}$$

The ‘extra’ discounting to period $t(i) - 1$ thus fully drops out in this case. Yet, the PME method will still generate an upward small-sample bias for $\tilde{\alpha}$ since the pricing error is divided by $(1 + \tilde{\alpha} + \tilde{\beta} r_{m,t(i)})$. However, relative to the NPV-estimator, this bias will be

smaller because there is "less discounting" than in equation (14). Specifically, since both the denominator and numerator depend linearly on $\tilde{\alpha}$, the goal function does not tend to zero as $\tilde{\alpha} \rightarrow \infty$ so that there is a unique optimum for this estimator. This is confirmed in section 4.6 in a more realistic setting.

An important condition for this method to be implemented is to observe dividends and investments separately, which is the case with the data we use in the empirical section. If we had net cash flows, it would not be feasible.¹⁴

4.4.2 Natural Logarithm of PME (Log-PME) approach

As mentioned above, another approach is to take the log of both sides of equation (9). Estimation is then performed as follows

$$\min_{\alpha, \beta} \sum_{i=1}^N [\ln(\overline{PV}^{D_i}(\alpha, \beta)) - \ln(\overline{PV}^{T_i}(\alpha, \beta))]^2 \quad (17)$$

In our simplified example, the estimator is the result of the following optimization

$$\min_{\tilde{\alpha}, \tilde{\beta}} \sum_{i=1}^N [\ln(1 + \alpha + \beta r_{m,t(i)} + \bar{\varepsilon}_i) - \ln(1 + \tilde{\alpha} + \tilde{\beta} r_{m,t(i)})]^2 \quad (18)$$

As in the PME case above, any 'extra' discounting drops out of the optimization. Since $E[\ln(1 + \alpha + \beta r_{m,t(i)} + \bar{\varepsilon}_i)] < E[\ln(1 + \alpha + \beta r_{m,t(i)} + \varepsilon_i)]$, equation (18) still generates a small-sample bias. This bias disappears asymptotically since $\bar{\varepsilon}_i$ tends to zero as $n_i \rightarrow \infty$. In small samples, however, this creates a tendency to lower the term $(1 + \tilde{\alpha} + \tilde{\beta} r_{m,t(i)})$, leading to a downward bias for $\tilde{\alpha}$. Like the PME method, this method does not suffer from numerical problems. This is confirmed by our simulations and empirical results. We always find a globally convex goal function. For example, Figure 3 shows the goal function we obtain in the main simulation (section 4.6).

Note that the log-PME and PME methods generate opposite small-sample biases for $\tilde{\alpha}$. This can be used to obtain lower and upper bounds for the estimate of α and to gauge

¹⁴In our data, monthly dividends ('distributions') and investments ('takedowns') are not netted. This feature is not necessary for the NPV method, but it is necessary for the improved GMM methods.

the extent to which small-sample biases are at work. We use this in section 6 to evaluate our empirical results.

4.5 Portfolio formation and inference

In section 3, we argued that it is important to create portfolio of funds (FoFs) that have as little overlap in time as possible to allow for cross-sectional identification of β . This can also be seen in our derivations above. For example, equation (18) shows that β is identified if $r_{m,t(i)}$ is different across FoFs, i.e. if FoFs are active during different time periods. This indicates that we should form FoFs based on the date of the investment. With fund-level data, it means that we group funds based on their starting year (called vintage year).

In addition, we noted in the previous subsection that the bias is a direct function of the variance of $\bar{\varepsilon}_i$. This suggests to group funds into fund-of-funds. However, reducing the number of FoFs implies that the time overlap between the FoFs becomes larger. When choosing the number of funds-of-funds, we thus face a trade off in terms of precision and bias, which we discuss in the next sub-section.

To obtain standard errors, we use a cross-sectional bootstrap technique. We resample the funds with replacement within each FoF, and then re-estimate alpha and beta. Repeating the process 1,000 times yields the bootstrap distribution of alpha and beta.¹⁵ Intuitively, it is conceivable that projects within a fund are correlated, as funds may specialize in certain sectors or invest in related projects. By resampling at the fund level, we thus assume that the idiosyncratic shocks of projects are perfectly correlated within a given fund but idiosyncratic shocks to projects are uncorrelated between funds.

The assumption on the independence of project idiosyncratic shocks between funds (within and across FoFs) is not needed for consistency but is required for our inference. To test whether it is a reasonable assumption, we propose two tests. First, we perform a block bootstrapping. We group funds into four blocks (2x2 sort on EU/US focus and fund size) within each vintage year. Within each block, the shocks are thus assumed to be perfectly

¹⁵As discussed by Horowitz (2001), in some cases it is beneficial to ‘re-center’ the moment conditions when performing the bootstrap analysis. We find similar standard errors when we re-center the moment conditions. We also find that a bootstrap bias-correction for GMM hardly changes the estimates (Horowitz, 2001).

correlated. This enables to gauge the validity of the independence assumption *within* vintage years. Second, we compute a pricing error for each of the 14 moment conditions (for the 14 vintage years we have in our data). This enables to gauge the validity of the independence assumption *between* vintage years. We conjecture that if there is significant cross-vintage-year dependence, pricing errors will be autocorrelated. Empirical results of these tests support our assumptions and are discussed in section 5.1.

4.6 Small-sample properties: A Monte Carlo Simulation

As discussed above, our GMM-style methodology generates asymptotically consistent estimates of α and β but we expect small-sample biases. To measure these biases, we run a Monte Carlo experiment.

We simulate project-level cash flows and aggregate these to obtain fund-level cash flow streams of investments and dividends. This matches the type of data we use in the empirical application below. Next, we apply our GMM methods to these simulated fund-level data. We aim to mimic the size and characteristics of our main dataset (venture capital funds). At the beginning of year = 1980,...,1993, 50 funds are started. They all invest \$1 per project and start 3 projects from year 1 to 5. Hence, a fund has 15 projects in total.¹⁶ The quarterly growth in the (latent) value of project j of fund i follows a simple market model

$$\frac{V_{ij,t+1}}{V_{ij,t}} = 1 + \alpha + r_f + \beta(R_{m,t+1} - r_f) + \varepsilon_{ij,t+1} \quad (19)$$

where $R_{m,t+1}$ is i.i.d. shifted lognormal over time, i.e. $R_{m,t+1} = e^x - c$, where x is normally distributed with mean μ_m and variance σ_m^2 , and c is a constant. Similarly, $\varepsilon_{ij,t}$ is i.i.d. shifted lognormal across projects and over time. We use a shifted lognormal distribution in order to make sure that the project return is bounded below by -100% . Given the large volatility of project returns, using a normal distribution would generate returns below -100% with nonnegligible probability. In appendix 3 we describe in detail how we calibrate the parameters of the shifted lognormal distributions. In short, for the market return we match S&P 500 data and for the idiosyncratic volatility we match Cochrane's (2005b)

¹⁶This number matches the venture capital sample that we describe below.

estimate (leading to an annual idiosyncratic volatility of 108% per year). Risk-free rate is set to 4% p.a., "true" α is set to zero and "true" β is set to one.

Dividends are endogenous. They follow the process assumed by Cochrane (2005b). That is, the probability that a project exits at time t is given by the following logistic function $\frac{1}{1+e^{-a(\ln(V_{i,j,t})-b)}}$. Hence, the project is more likely to exit as it reaches higher values. In addition, a project is more likely to exit if it reaches a low value. The probability of exiting is given by $\frac{k-V_t}{k}$, with $V_t < k$. The parameter values are taken from Cochrane (2005b): $a = 1, b = 3.8, k = 0.25$. Finally, if a project is still alive after 5 years, it is liquidated and a dividend equals to its value is paid.¹⁷

Following the argument in the previous section, we group all the funds with the same vintage year into a FoF. We thus have 14 moment conditions (one for each vintage year).

To begin with, we use the NPV method. Consistent with what we argued above, we find that the NPV method delivers goal functions that are not globally convex (figure 2). Consequently, we observe several estimates for α diverging towards infinity (non-tabulated).

In Table 1, we display results for the two "improved GMM methods" we proposed above: the PME approach and log-PME approach. Panel A of Table 1 shows that the PME approach generates a small upward bias for alpha (2 basis points per month) and that the log-PME performs even better: the bias is slightly negative (due to the convexity effect discussed in section 4.4.2) but negligible (1 basis point per month). Both the PME and log-PME approaches estimate beta without any bias.¹⁸

As mentioned above, the bias of our estimates depends on the size of idiosyncratic shocks. Panel A therefore presents results with different levels of idiosyncratic volatility. In addition to the benchmark volatility given above, we show results with a relatively 'low' idiosyncratic volatility (25% per annum)¹⁹ and very high idiosyncratic volatility (twice that

¹⁷Note that we do not make any assumptions on how dividends are reinvested after being paid out to the investor.

¹⁸In unreported results we have analyzed a setup with less than 14 moment conditions, thus grouping funds across vintage years. We find that this generates significant biases in both alpha and beta. As explained above, by grouping vintage years we lose information on the relation between the performance of a single vintage year and the associated market returns, which is important to pin down the beta.

¹⁹This corresponds to the Ang, Hodrick, Xing and Zhang (2006) estimate for the highest idiosyncratic volatility quintile of US stocks. Their Table 6 provides total volatility estimates across quintile portfolios. We correct these total volatilities for market volatility to obtain idiosyncratic volatility.

of the benchmark.)

Results show that, as suggested above, our estimators are precise when idiosyncratic volatility is relatively low. Both methods provide unbiased estimates with high precision. Turning to the very high idiosyncratic volatility case, we see that the differences between the two methods are much larger. The PME method generates a bias of 28 basis points per month for alpha, while the log-PME approach still produces a reasonably small bias of -3 basis points per month.

In order to assess the sensitivity of the simulation results to the exit rule used, we redo the simulation using an alternative exit rule. The probability of exit is simply a function of contemporaneous market return. All projects exit if the market return is higher than its 95th percentile. Hence, expected duration is 20 quarters (i.e. 5 years). In this case, exit only depends on market-wide returns and not individual project returns. Project exit is thus highly clustered. Intuitively, this reduces the information in the hand of the econometrician. Yet, Panel B shows that we obtain satisfactory results. The standard deviation and inter-quartile range of the estimates increase, but remains small. The average bias remains basically the same for the log-PME method and increases slightly for the PME method.

Finally, we provide numerical evidence of the statistical consistency of our two GMM-style methods by increasing the number of projects per fund from 15 to 50 and then from 50 to 100. Panel C shows that the estimators converge towards the true value with increased precision.

5 Data

In this section, we describe the data and discuss our treatment of funds that are not officially liquidated (report positive NAV at the end of our sample period).

5.1 Data source

Data on both private equity fund cash flows (net of fees) and quarterly NAVs are from Thomson Venture Economics. We have separate observations for investments and dividends. This dataset is the most comprehensive source of financial performance of both US and

European private equity funds and has been used in previous studies (e.g., Kaplan and Schoar, 2005). It covers an estimated 66% of both venture capital funds and buyout funds (Phalippou and Gottschalg, 2009).

We consider all funds (with size over \$5 million) raised between 1980 and 1993 as they have reached their normal liquidation age (10 years) at the end of our sample time period (2003). As discussed above, we construct venture capital fund-of-funds and buyout fund-of-funds based on vintage years. We exclude vintage years with less than 10 funds; this excludes buyout funds raised between 1980 and 1983 but does not affect venture capital funds.

Descriptive statistics are reported in Table 2. We have 958 funds, of which 686 have a Venture Capital (VC) objective and 272 have a buyout (BO) objective. In total, we have 25,800 cash flows. Our descriptive statistics are similar to what has been reported in the literature.

5.2 Estimating Final Market Values

Table 2 shows that two thirds of the funds report a positive NAV at the end of our sample period despite having passed their tenth anniversary. Existing work either treats these final NAVs as a final cash flow (Kaplan and Schoar, 2005) or writes them off (Phalippou and Gottschalg, 2009). One of the problems faced in the literature and which partly explains these simple choices is that the conversion of NAVs into a market value necessitates an estimate of systematic risk.

This section describes how we model the final market value of these non-liquidated funds. We take the fully liquidated funds at different ages, compute their realized market value (MV) as the net present value of subsequent cash flows where we discount with the pricing model estimated by our GMM method. Then, for each age $a=10,11,12$, and 13, we separately estimate the following model

$$\ln(1 + MV_{a,i}(\alpha, \beta)) = b_{a0} + b'_{a1}X_{a,i} + \varepsilon_{a,i} \quad (20)$$

The vector of explanatory variables $X_{a,i}$ includes $\ln(1 + NAV)$, the log of fund size, the

log of the time elapsed since the last dividend distribution, the log of the time elapsed since the last NAV update, and fund’s performance multiple excluding NAV (sum of capital distributed divided by sum of capital invested); where all variables are computed at age a .

Results from the regression (20) are shown in Table 3 - Panel A. We find that a 1% increase in NAV leads to slightly less than 1% increase in market value and that this elasticity decreases with age. Large funds and better performing funds have higher market values, hence more conservative accounting valuations. Funds that have not paid a dividend for a long time or not updated their NAVs for a long time have lower market values.

Next, we use equation (20) to predict final market values for the non-liquidated funds.²⁰ Results of the prediction are shown in Table 3 - Panel B.²¹ The ratio of total predicted market values to total reported NAVs is between 21% (age 12) and 37% (age 10). These low ratios are mainly due to the fact that non-liquidated funds have not paid any dividends for about 3.5 years (versus 1 year for liquidated funds) and non-liquidated funds have not updated their NAVs for 2.5 years (versus 6 months for liquidated funds). We thus provide evidence that NAVs of old and inactive funds largely overstate the true market value.

The results described above require a joint estimation setup. To run regression (20) for the fully liquidated funds, we need $MV_{a,i}(\alpha, \beta)$ which depends on the discount rate and hence on α and β . In turn, to estimate α and β with GMM, we need the predicted values of $MV_{a,i}(\alpha, \beta)$ for the non-liquidated funds. We thus simultaneously estimate equation (20) and α and β from the GMM equation (17).

6 Risk and Return Estimates

In this section, we first report the estimates of risk and abnormal return of private equity funds. Next, we allow α and β to be a function of fund characteristics. Finally, we present several robustness checks.

²⁰Beyond the 13th anniversary, we observe very few funds with positive NAV that are subsequently liquidated. For funds older than 13 years, we predict the market value at the end of their 13th anniversary using the coefficients from the age 13 regression. This is why most of the funds in the prediction sample are in the age 13 category (N=434).

²¹The results described above are those obtained when estimating a one-factor market model. We find similar coefficients for the three-factor model.

6.1 Benchmark Results

Following results in section 4.6, we create one portfolio (fund-of-funds, FoF) per vintage year and use the log-PME approach. We weight the moment conditions by the number of funds in the FoF.

Table 4 - Panel A shows results after fees. We find that venture capital funds have a particularly large market beta (3.21). This entails a large cost of capital. Taking a naive estimate for the equity premium of 8% (the average over our time period) and a risk free rate at 5%, the cost of capital is $5\% + 3.21 * 8\% \approx 31\%$, according to the CAPM. The abnormal return (alpha) is significantly negative at -1.25% per month, thus about -15% annual. The large negative abnormal performance of venture capital is a direct result of the large beta. Using a back-of-the-envelop calculation, venture capital funds have an average IRR of 15%, a cost of capital of 31%, hence abnormal return is about -16%.

We also consider the three-factor Fama-French model and find that venture capital returns resemble those of small growth stocks. Relative to these small growth stocks, there is still underperformance of VC, but much less so given that small growth stocks perform poorly too. The alpha decreases substantially to -0.69% per month or -8% annual.²²

Buyout funds have a very different risk profile. We find a low market beta (0.33). Consequently, the cost of capital is relatively low at $5\% + 0.33 * 8\% \approx 8\%$, and there is a positive but not statistically significant alpha. According to the three-factor model of Fama-French, the alpha is even lower and very close to zero (1.5% annually).

So far, all results reported are net of fees, since all cash flows in our sample are net of fees. To assess the impact of fees, we add some simulated fees to the original cash flows. We assume a standard and simple fee structure consisting of a 2% management fee (on committed capital), 20% carry with an 8% hurdle rate (see Metrick and Yasuda, 2009). The results in Table 4-Panel B show that fees affect both betas and alpha. Adding

²²Note that with the three-factor Fama-French model, the precision of the estimates decrease. Given that we perform a cross-sectional estimation, the parameter estimates are correlated to some extent and the correlations between these parameter estimates are higher for the three-factor model. This makes it harder to precisely pin down the different risk exposures. Still, the exposures to SMB and HML make intuitive sense. Venture capital funds resemble small growth stocks and buyout funds tend to co-move more with large stocks. Although these exposures are economically substantial, they are not always statistically significant.

back fees increases the annualized alpha by 3% to 4%. Hence, a substantial part of the underperformance of VC funds is due to fees. Table 4-Panel B also shows an interesting effect of fees on the betas. Given the hurdle rate, fees are nonlinear and this increases the beta when we add fees. The increase in beta is 0.15 for VC funds and slightly larger (0.23) for BO funds. This means that the fee bill is actually lower than if one simply measures the difference between after fee performance and before fee performance. For example, Phalippou and Gottschalg (2009) assess fees to be 6% annual, but implicitly maintain the assumption that beta is one.

As mentioned in section 4.5, standard errors are derived under the assumption that the idiosyncratic shocks are perfectly correlated within a fund, but independent between funds (both within and between FoFs). We have proposed two tests to gauge the validity of this assumption. First, we repeat all the inference in Table 4 with block bootstrapping instead of simple bootstrapping. As mentioned in section 4.5, we group funds within each FoF by geographical focus and size, thereby assuming that funds of similar size and geographical focus have perfectly correlated pricing errors. The standard errors obtained in this way are extremely similar (non-tabulated). Second, we compute the autocorrelation of FoF pricing errors, which we report underneath each specification in Table 4. We find that the autocorrelation is negative in most specifications and never significantly positive.²³ These results are a strong indication that the independence assumption we make for inference is reasonable on this dataset.

6.2 Reality check

To gain confidence in our beta estimates, we investigate the time series of ‘dividend yields’ for venture capital funds and buyout funds. The dividend yield in year t for fund i is defined as the sum of all dividends paid over this year divided by fund size. To obtain an aggregate dividend yield, we take the average across all funds that are in their divestment phase, i.e. fund age is between 4 and 10 years. Figure 1 shows the resulting series. On the same graph,

²³In unreported results, we find that the autocorrelation of the IRR per vintage year is positive. This suggests that, once we correct for the overlapping market return exposure of different vintage years, the residual returns are not correlated.

we plot the 5-year moving annual average of the S&P 500 returns. The idea is that if the stock-market does well during 5 years - which is the average duration of an investment - then a high (low) beta asset will distribute larger (smaller) dividends in the following year. On the figure, it is apparent that the first pick of the stock-market in 1995 and the rally of 1998-1999 goes hand-in-hand with a huge spike in dividend yield for venture capital funds. When the stock market went down the following three years, so did the dividends. Our high estimate for the venture capital beta reflects these features of the data. Interestingly, the same figure shows that buyout fund dividends are smoother across years. In addition, the venture capital dividend yield fluctuates widely with a minimum of 5% and maximum of 80%. In contrast, the dividend yield for buyout funds fluctuates between 5% and 25% per year and appears relatively flat. Figure 1 is thus consistent with our empirical estimates of risk. More formally, when regressing this dividend yield on the lagged annual market return we obtain a positive and significant coefficient for venture capital and an insignificant coefficient for buyout (non-tabulated).

Yet, one may still wonder how a leveraged buyout investment can have a low beta. Kaplan and Stein (1990) find that in highly leveraged transactions, a sharp increase in debt coincides with a "surprisingly small" increase in equity beta. Hence, increased leverage may have a countervailing effect on the asset beta. In addition, a related puzzle appears to be present for stocks. E.g. Korteweg (2005) talks about a "leverage puzzle" as he, like other researchers, finds no positive relation between leverage and expected stock returns.

An important caveat here is that our estimates for buyout funds are less precise than for venture capital funds. In addition, while the venture capital beta estimate is very robust to changes in sample selection and method, this is less so for the buyout sample (section 5.3). This is probably due to the smaller sample for buyout funds. Also, compared to venture capital funds, buyout funds are more heterogeneous in terms of size, geographic focus and investment type.

6.3 Economic interpretation

As mentioned in the introduction, an ongoing debate in this literature is whether private equity funds add value. To answer this question, one needs to know the alpha before fees. For venture capital, it is negative according to both the CAPM and the three-factor model of Fama-French. However, as explained above, the Fama-French model judges VC performance more positively. Our results can be interpreted as saying that compared to the average small-growth publicly-traded stocks, VC added-value is slightly negative. But, compared to the average publicly-traded stocks, VC added-value is quite negative (-10% annually). A classic interpretation of this negative before fees alpha would be that the price paid by VC funds to acquire assets is too high. One reason could be that VC funds have underestimated the systematic risk of their investments and thus used too low a discount rate. Another reason could be that there is too much money chasing too few opportunities (Gompers and Lerner, 2000). This over-capacity could be created by a number of investors that are not performance maximizing. In VC, particularly in Europe (where performance is lowest), large amounts are invested by government sponsored bodies to stimulate local economies.²⁴

Obviously, if alphas are negative before fees, they get only worse after fees. This raises the question of why investors accept to pay 4% per year for negative alphas. First, as mentioned above, there may be some side benefits of investing in venture capital. Second, our paper shows that measuring risk is not trivial and is only possible when a substantial set of funds are liquidated or close to liquidation. Over our sample period, investors had less data than we have, especially so in the early years. Little data and high idiosyncratic volatility make learning difficult. Investors may observe absolute returns but widely used proxies do not seem alarming (e.g. 15% IRR on average). It is the risk correction that makes the alpha negative for venture capital funds. One may expect that, in the future, alphas move towards zero as some investors may learn. Our estimate of beta, however, may

²⁴For example, the European Investment Fund (EIF), sponsored by the European Union, "now accounts for about 10% of the early-stage venture capital market in the EU, with a portfolio of EUR 2.4bn invested in about 180 funds from across the EU and the Candidate Countries." See <http://www.eif.org/about/news/quarterly-newsletter-issue-1.htm>

be a reasonable estimate looking forward as it has been observed with public equity that betas tends to be more stable (than alphas) over time.

6.4 Return and fund characteristics

The literature has shown that some fund characteristics are related to returns and it is thus important to incorporate these regularities in our estimations to increase precision. At the same time, we shed light on the nature of these regularities. For example, Kaplan and Schoar (2005) find that fund returns (measured by Public Market Equivalent or IRR) are positively related to the fund size. Our framework allows us to investigate whether this effect is due to higher abnormal performance or higher risk exposures. We make alpha and beta a function of fund size

$$\alpha = a_0 + a_{size} * \ln(\text{fund size}), \quad \beta = b_0 + b_{size} * \ln(\text{fund size}) \quad (21)$$

Next, we form size-sorted portfolios (i.e. FoFs) for each vintage year. This allows us to pin down the effect of size from the cross-section of moment conditions. If we would use the 14 vintage-year portfolios, size effects would only be identified to the extent that funds with different vintage year have different size. We thus form 2 equal portfolios per vintage year - one with large funds and one with small funds.

We show results in Table 5. We first include size in the alpha specification only (spec 1) and confirm that the performance is positively and significantly related to size. Like Kaplan and Schoar (2005), we find that this result is highly significant for venture capital funds (Panel A) and weaker for buyout funds (Panel B). Next, we allow beta to depend on size as well (spec 2). For venture capital funds, the size effect in alpha becomes insignificant while the size effect in beta is positive and significant. Hence, the documented positive relation between fund size and performance in venture capital can be attributed to a higher level of systematic risk rather than abnormal performance.

For both buyout funds, we also find that beta is significantly increasing with fund size. Interesting, we find that the alpha of buyout funds is negatively related to fund size. Small buyout funds have a lower beta and a higher abnormal performance.

We repeat the same exercise with a dummy capturing whether a fund is a first-time fund or not (capturing experience of the firm) and fund focus (US / Europe). We do not find a significant effect for firm experience (specs 3 and 4). However, fund focus is significant (specs 5 and 6). US-focused venture capital funds also have a higher beta but this is not enough to explain why they outperform their European counterparts. We find that US funds have higher alphas, not only in venture capital but also in buyout.

6.5 Robustness

In this subsection, we investigate the robustness of our results. We first show results with a different treatment of NAVs. Next, we show results for different samples and different methodological choices.

6.5.1 NAV treatment

We begin by re-estimating abnormal return and risk with i) final NAVs treated as fair market value (as in Kaplan and Schoar, 2005) and with ii) writing final NAVs off (as in Phalippou and Gottschalg, 2009). Table 6 shows the results. VC beta decreases from 3.45 (final NAV treated as correct) to 2.98 (final NAV written off) in the CAPM specification. The estimate we find in the main analysis is between these two values (3.21 for the CAPM specification). Similar results are observed for buyout funds. Interestingly, the effect on abnormal performance is minimal. This is because writing off NAVs decreases both beta and raw performance. Consequently, the effect on alpha is minimal. For venture capital, alpha changes from -1.08% per month to -1.27% per month in the CAPM specification. The change in alpha is larger for buyout funds but not significant.

6.5.2 Change in empirical design

Table 7 shows estimates of alpha and beta (CAPM model) for different samples and different methodological choices. On the left hand side, we show results for our benchmark case where the moment conditions are weighted by the number of funds in the FoF. On the right hand side, we show results when moment conditions are value weighted. We use the log-PME

method for all estimations except for one case where we use the PME method. The default results (those shown in Table 4) are also included (alpha is -1.24% and beta is 3.21 for VC and alpha is 0.49% and beta is 0.33 for BO) in the first line for convenience.

The first result is that value weighting the moment conditions does not substantially change the estimated risk and abnormal performance for venture capital, while for buyout funds the estimated beta is typically larger in case of value weighting.

The second result is that varying the number of FoFs has little impact on the estimates. For each vintage year, we sort funds by size and create either 2, 3 or 4 portfolios. Irrespective of how many portfolios we create, we find similar alpha and beta estimates. A partial exception is for BOs for which beta changes from 0.57 to 0.28 when moving from 2 to 3 FoFs per vintage year. This may be due to the lower number of buyout funds which means that as the number of portfolios increases the number of funds in each portfolio quickly decreases towards unity. This is also an indication that our estimates of BO fund risk and abnormal performance are not as precise as they are for VC funds.

Our third result is that estimates obtained by the PME method are similar to those obtained by the log-PME method. Importantly, this shows that the PME and log-PME estimates are hardly affected by small-sample biases. As we saw above, the PME method introduces an upward bias in alpha whereas the log-PME method introduces a small negative bias. If our sample is large enough then the estimates provided by the two methods are similar indicating that small-sample biases are minimal. For VC, alpha is -1.20% with the PME method and -1.24% with the log-PME method. These values are respectively -1.22% and -1.15% if we value weight the moment conditions.

Consistent with the above results, the estimates are less stable for BO funds. Alpha is 0.97% with the PME method and 0.49% with the log-PME method (0.46% and 0.06% respectively if we value weight the moment conditions). Combined with the confidence interval we obtain for the benchmark estimate of the buyout beta (Table 4), it seems likely that the true buyout beta is between zero and one, which is an interesting result but, unfortunately, it is difficult to give a more precise point estimate for BO funds. Again, this may be due to some uncaptured heterogeneity (omitted factors) across BO funds or the

smaller sample size for buyouts.

Our fourth result is that changing the time period does not significantly change estimates. Again, this is especially true for VC funds. Given the nature of our data, we provide a sense of the impact of the sample time period (funds raised between 1980 and 1993) by adding and removing one vintage year.

Our fifth result is that the sub-sample of liquidated funds has higher alphas but the increase is not substantial. This is an important result because if our model to convert NAVs is misspecified then our estimates would be biased. Since it is difficult to know whether a model is properly specified or not, the fact that the estimates are similar on a sub-sample without NAVs is reassuring.

Finally, we run our estimations using different benchmark factor portfolios. We begin by using a different market portfolio for the non-US focused funds. In the above analysis we have used one market portfolio for all funds. Implicitly we have assumed that financial markets are integrated. We now assume that financial market are perfectly segmented and thus use non-US stock indices for non-US focused funds (with returns in US dollars to be consistent with the cash flow currency). The indices come from the website of Ken French. We use either the Europe index or the UK index (as most non-US funds are UK-based). For venture capital, the beta increases from 3.21 (benchmark case) to 3.76 with the Europe index and 3.57 with the UK index. The alpha, however, increases slightly compared to the benchmark case. For buyout funds, a similar result is obtained. Next, we use the NASDAQ for venture capital funds. We find a lower beta (1.55 instead of 3.21) and a higher alpha (-0.55% versus -1.24%). This result indicates that VC funds performance is more closely related to that of the NASDAQ than that of the S&P 500. This means that part of the large beta we find for the VC funds can be attributed to the fact that VC investments resemble NASDAQ stocks, which, themselves, have a high beta. This is confirmed when we use the small-growth portfolio of Fama-French. There, we also obtain a similar beta as the one obtained with the NASDAQ (1.70) but the alpha with respect to the small-growth portfolio is positive. The small growth portfolio has had historically a very low performance which is difficult to explain. This result shows that although they co-move closely with small-growth

stocks, VC funds have a better performance than small-growth stocks. This result is similar to what Cochrane (2005b) finds with venture capital projects.

In sum, we find that the VC results are robust and not subject to small-sample biases. In contrast, the buyout sample is smaller and estimates are somewhat less stable across specifications.

7 Conclusion

We develop a new econometric methodology to estimate the risk and return of an asset using cash flow data. We apply it to a sample of private equity funds. The GMM-type methodology we develop is based on moment conditions that state that expected discounted dividends should equal expected discounted investments, where the discounting is done using a factor pricing model of which the parameters are to be estimated. An advantage of our approach is that it does not require an assumption about the return distribution. This is an appealing feature as the return distribution is not directly observable given the lack of a time-series of market values. The method is statistically consistent and we show how to optimize the small-sample performance by constructing appropriate moment conditions and portfolios. A simulation study shows that the small-sample properties are satisfactory.

We find that venture capital funds have a high CAPM-beta, while buyout funds have a much lower CAPM-beta. Venture capital funds have a significantly negative alpha. Buyout funds have a slightly positive alpha, but statistically insignificant. Our model indicates that the net asset values reported by funds that are inactive and mature are highly upward biased estimates of their market value. The flexibility of our GMM model also enables us to study the interaction between the characteristics of the funds and their alpha and beta. We find that larger funds have higher level of systematic risk. In terms of abnormal performance, we do not find a significant difference in venture capital but small buyout funds outperform.

Our method can be used for other limited life non-traded private partnerships (e.g. mezzanine debt funds and some real estate funds) and for corporate investments in case the CFO observes a stream of cash flows from a division/project but no market values.

Appendix 1: Endogenous timing of dividends

In this appendix we show that the moment condition in (7) is not affected by endogenous timing of dividends. To this end, consider a two-period setup²⁵, where the project may exit after one period or two periods, and the timing of exit may depend on lagged market returns and idiosyncratic shocks (for example, the project may exit early if the first-period return is high); hence, $t_{ij} = 0$ and d_{ij} equals one or two. Filling in the expression for the dividend (equation (6)), we can write the pricing error in equation (7) as

$$\begin{aligned}
 & E \left[\frac{T_{ij} \prod_{t=1}^{d_{ij}} (1 + r_{f,t} + \alpha + \beta r_{m,t} + \varepsilon_{ij,t})}{\prod_{t=1}^{d_{ij}} (1 + r_{f,t} + \alpha + \beta r_{m,t})} - T_{ij} \right] \\
 = & E \left[1_{d_{ij}=1} \frac{T_{ij}(1 + r_{f,1} + \alpha + \beta r_{m,1} + \varepsilon_{ij,1})}{(1 + r_{f,1} + \alpha + \beta r_{m,1})} + 1_{d_{ij}=2} \frac{T_{ij} \prod_{t=1}^2 (1 + r_{f,t} + \alpha + \beta r_{m,t} + \varepsilon_{ij,t})}{\prod_{t=1}^2 (1 + r_{f,t} + \alpha + \beta r_{m,t})} - T_{ij} \right]
 \end{aligned} \tag{22}$$

where $1_{d_{ij}=1}$ is an indicator function denoting exit at time 1. Note that $1_{d_{ij}=2} = 1 - 1_{d_{ij}=1}$. We then use the law of iterated expectations, where we condition upon the information set at time 1 (denoted F_1), to write the expression in (22) as

$$E \left[\begin{aligned} & 1_{d_{ij}=1} \frac{T_{ij}(1+r_{f,1}+\alpha+\beta r_{m,1}+\varepsilon_{ij,1})}{(1+r_{f,1}+\alpha+\beta r_{m,1})} \\ & + (1 - 1_{d_{ij}=1}) \frac{T_{ij}(1+r_{f,1}+\alpha+\beta r_{m,1}+\varepsilon_{ij,1})}{(1+r_{f,1}+\alpha+\beta r_{m,1})} E \left[\frac{(1+r_{f,2}+\alpha+\beta r_{m,2}+\varepsilon_{ij,2})}{(1+r_{f,2}+\alpha+\beta r_{m,2})} \mid F_1 \right] \end{aligned} - T_{ij} \right] \tag{23}$$

From assumption 1, we have $E \left[\frac{(1+r_{f,2}+\alpha+\beta r_{m,2}+\varepsilon_{ij,2})}{(1+r_{f,2}+\alpha+\beta r_{m,2})} \mid F_1 \right] = 1$, so that (23) becomes

$$E \left[\begin{aligned} & 1_{d_{ij}=1} \frac{T_{ij}(1+r_{f,1}+\alpha+\beta r_{m,1}+\varepsilon_{ij,1})}{(1+r_{f,1}+\alpha+\beta r_{m,1})} \\ & + (1 - 1_{d_{ij}=1}) \frac{T_{ij}(1+r_{f,1}+\alpha+\beta r_{m,1}+\varepsilon_{ij,1})}{(1+r_{f,1}+\alpha+\beta r_{m,1})} \end{aligned} - T_{ij} \right] \tag{24}$$

and again from assumption 1 it follows that this expression is equal to 0, hence equation (7) holds even if d_{ij} is endogenous.

²⁵The derivation can easily be extended to a situation with more than two periods.

Appendix 2: GMM with nonlinear functions

The PME and log-PME estimator are based on a nonlinear transformation of the moment condition (9). In this appendix, we derive the asymptotic consistency of these estimators. We follow the standard way of deriving asymptotic normality of the GMM estimator. Using general notations, the standard GMM moment condition can be written as

$$E(f(\theta_0, x_i)) = 0 \quad (25)$$

where $f(\cdot, \cdot)$ is a k -dimensional function. Let $g_N(\theta) = 1/N \sum_{i=1}^N f(\theta, x_i)$. The GMM estimator is $\hat{\theta}_N = \arg \min_{\theta} g_N(\theta)' W_N g_N(\theta)$, with W_N a weighting matrix. For our PME and log-PME estimators, we take a nonlinear function $h(\cdot)$ of the moment condition

$$h(E(f(\theta_0, x_i))) = h(0) \quad (26)$$

Normalizing $h(0) = 0$ (without loss of generality) our estimator is equal to $\hat{\theta}_N = \arg \min_{\theta} h(g_N(\theta))' W_N h(g_N(\theta))$. Applying the mean value theorem we have

$$h(g_N(\hat{\theta}_N)) = h(g_N(\theta_0)) + \frac{\partial h(g_N(\tilde{\theta}_N))'}{\partial \theta} (\hat{\theta}_N - \theta_0) \quad (27)$$

where $\tilde{\theta}_j$ is between $\theta_{0,j}$ and $\hat{\theta}_{N,j}$. Using the first order condition $\frac{\partial h(g_N(\hat{\theta}_N))}{\partial \theta'} W_N h(g_N(\hat{\theta}_N)) = 0$, and premultiplying the above equation by $\frac{\partial h(g_N(\hat{\theta}_N))}{\partial \theta'} W_N$ gives

$$0 = \frac{\partial h(g_N(\hat{\theta}_N))}{\partial \theta'} W_N h(g_N(\theta_0)) + \frac{\partial h(g_N(\hat{\theta}_N))}{\partial \theta'} W_N \frac{\partial h(g_N(\tilde{\theta}_N))'}{\partial \theta} (\hat{\theta}_N - \theta_0) \quad (28)$$

which can be rewritten as

$$\sqrt{N}(\hat{\theta}_N - \theta_0) = - \left(\frac{\partial h(g_N(\hat{\theta}_N))}{\partial \theta'} W_N \frac{\partial h(g_N(\tilde{\theta}_N))'}{\partial \theta} \right)^{-1} \frac{\partial h(g_N(\hat{\theta}_N))}{\partial \theta'} W_N \sqrt{N} h(g_N(\theta_0)) \quad (29)$$

If $h(\cdot)$ is continuous and continuously differentiable (which is the case for our PME and log-PME estimators), the delta method implies that $\sqrt{N}h(g_N(\theta_0))$ has an asymptotically

normal distribution and the premultiplying weighting matrices converge to their probability limits, so that the estimator is consistent and asymptotically normal (see Greene, 2003, p. 70).

Appendix 3: Calibration of the shifted lognormal distribution

This appendix describes how we calibrate the parameters of the shifted lognormal distribution for the market return and idiosyncratic shocks. The shifted lognormal distribution of $e^x - c$, where $x \sim N(\mu, \sigma)$, has 3 parameters: μ , σ , and c .

For the market return, we set the minimum return c to -20% (the minimum is -20.9% in our sample). μ_m and σ_m are so that we match the average S&P 500 return and volatility over the 1980-2003 sample period. That is, μ_m and σ_m solve

$$\begin{aligned} E(R_m) &= e^{\mu_m + \frac{\sigma_m^2}{2}} - c_m \\ \text{Var}(R_m) &= (e^{\sigma_m^2} - 1)(e^{2\mu_m + \sigma_m^2}) \end{aligned} \quad (30)$$

For the idiosyncratic error $\varepsilon_{ij,t}$, c_ε is so that return is always above -100%:

$$\alpha + R_f + \beta * (c_m - R_f) + c_\varepsilon = -1 \quad (31)$$

μ_ε and σ_ε solve a system of two equations. First, like above, we have $E(\varepsilon_{ij,t}) = e^{\mu_\varepsilon + \frac{\sigma_\varepsilon^2}{2}} - c_\varepsilon = 0$. Second, we match Cochrane (2005b) estimate for the standard deviation of idiosyncratic shocks (86% per year in a log-CAPM setting).²⁶

²⁶ Cochrane (2005b) uses the following log-CAPM specification $\ln(\frac{V_{t+1}}{V_t}) = R_f + a + b(R_m - R_f) + \eta$, where η is normal with variance σ^2 . Doing a Taylor-expansion, we get $\frac{V_{t+1}}{V_t} = e^{R_f + a + b(R_m - R_f) + \eta} \cong (1 + R_f + a + b(R_m - R_f))e^{\eta - \sigma^2/2}$. Here we centralize η such that $E(e^{\eta - \sigma^2/2}) = 1$, and choose $\alpha = a + \sigma^2/2$ and $\beta = b$. For the CAPM we have $\varepsilon = \frac{V_{t+1}}{V_t} - (1 + R_f + \alpha + \beta(R_m - R_f))$. This gives $\varepsilon = \frac{V_{t+1}}{V_t} - (1 + R_f + \alpha + \beta(R_m - R_f)) = (1 + R_f + \alpha + \beta(R_m - R_f))(e^{\eta - \sigma^2/2} - 1)$.

Using the law of total variance and $E(\varepsilon|R_m) = 0$ (since $E(e^{\eta - \sigma^2/2}) = 1$), we have $\text{Var}(\varepsilon) = E[\text{Var}(\varepsilon|R_m)] = E\left[(1 + R_f + \alpha + \beta(R_m - R_f))^2 \text{Var}(e^{\eta - \sigma^2/2})|R_m\right] = E\left[(1 + R_f + \alpha + \beta(R_m - R_f))^2\right] (e^{\sigma^2} - 1)e^{\sigma^2}e^{-\sigma^2}$. Hence, μ_ε and σ_ε are so that $\text{Var}(\varepsilon) = (e^{\sigma_\varepsilon^2} - 1)(e^{2\mu_\varepsilon + \sigma_\varepsilon^2}) = E\left[(1 + R_f + \alpha + \beta(R_m - R_f))^2\right] (e^{\sigma^2} - 1)$, where σ is set to 0.86.

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Table 1: Monte Carlo Simulations

Each year (from 1980 to 1993), 50 funds are started. Each fund invests \$1 per project, starts 3 projects per year from year 1 to 5, making a total of 15 projects per fund. The project return follows a one-factor market model. Market returns and error terms are drawn from shifted-lognormal distributions. Market returns are matched to the empirical distribution of the S&P 500 (appendix 3). In the benchmark case, idiosyncratic volatility is matched to that of Cochrane (2005) (108% per year, see appendix 3 for details). Idiosyncratic volatility is set to 25% p.a. and 150% p.a. in the low and high volatility case respectively. This exercise is repeated 1000 times with true alpha set to zero and true beta set to one. Two estimation methods are executed (“PME” and “Log-PME”; see section 4). The mean, standard deviation, and inter-quartile range of the 1,000 estimated pair of parameters (alpha, beta; monthly frequency) are displayed. In Panels A and C, the project liquidation rule is that of Cochrane (2005). In Panel B, the project liquidation rule is: liquidates if market return that quarter is above 17% (its 95th percentile). In Panel C, the number of projects per year per fund is set to either 10 or 20 (instead of 3).

Panel A: Estimator as a function of idiosyncratic volatility (true alpha=0%, true beta=1)

Method	Low volatility		Benchmark		High volatility	
	PME	Log-PME	PME	Log-PME	PME	Log-PME
Mean Alpha	0.00%	0.00%	0.02%	-0.01%	0.28%	-0.03%
Std Alpha	0.03%	0.03%	0.18%	0.18%	0.88%	0.56%
Inter-Quartile	[-0.01% 0.02%]	[-0.01% 0.01%]	[-0.08% 0.12 %]	[-0.10% 0.08%]	[-0.17% 0.53%]	[-0.36% 0.24%]
Mean Beta	1.00	1.00	1.00	1.00	0.95	0.98
Std Beta	0.06	0.06	0.31	0.29	1.12	0.83
Inter-Quartile	[0.97 1.03]	[0.97 1.03]	[0.83 1.17]	[0.85 1.16]	[0.41 1.47]	[0.52 1.43]

Panel B: Change liquidation rule (true alpha=0%, true beta=1)

Method	PME	Log-PME
Mean Alpha	0.08%	-0.00%
Std Alpha	0.35%	0.26%
Inter-Quartile	[-0.10% 0.19%]	[-0.14% 0.11%]
Mean Beta	0.97	0.99
Std Beta	0.51	0.39
Inter-Quartile	[0.77 1.19]	[0.81 1.18]

Panel C: Large sample results (true alpha=0%, true beta=1)

Method	50 projects per fund		100 projects per fund	
	PME	Log-PME	PME	Log-PME
Mean Alpha	0.01%	-0.00%	0.00%	-0.00%
Std Alpha	0.10%	0.10%	0.07%	0.07%
Inter-Quartile	[-0.05% 0.06%]	[-0.05% 0.05%]	[-0.03% 0.04%]	[-0.04% 0.03%]
Mean Beta	1.00	1.00	1.00	1.00
Std Beta	0.16	0.16	0.12	0.12
Inter-Quartile	[0.92 1.09]	[0.92 1.09]	[0.94 1.06]	[0.94 1.06]

Table 2: Descriptive Statistics

This table shows descriptive statistics for our sample. We report: (i) the average and the median of the amount committed to funds in million of 2003 U.S. dollars (size); (ii) the total final Net Asset Value reported (December 2003), total capital distributed and total capital invested; (iii) the overall multiple (sum NAV + sum Distributed) / (sum Invested); (iv) the proportion of first time funds; (v) the proportion of non-US focused funds; (vi) the proportion of funds with positive final Net Asset Value; and (vii) the number of cash flows and the number of funds.

		All funds	Venture Capital	Buyout
Mean size	(\$ million)	170.40	90.86	371.02
Median size	(\$ million)	63.98	51.82	133.38
Sum NAV	(\$ billion)	27.93	8.08	19.87
Sum Distributed	(\$ billion)	209.69	81.40	128.69
Sum Invested	(\$ billion)	119.89	39.57	80.50
Multiple		1.98	2.26	1.85
First time funds		49%	46%	57%
Non-US funds		29%	22%	47%
Funds with positive final NAV		64%	63%	64%
Number of cash-flows		25,800	16,859	8,941
Number of funds		958	686	272

Table 3: Final Market Value

Panel A shows the relation between fund market value (MV) and fund characteristics for the sample of liquidated funds. Market Value (MV) at a given age is computed as the present value of the subsequently realized cash flows using the market model to discount. Fund characteristics include Net Asset Value (NAV), fund size, time elapsed since last dividend distribution (LastDiv) and since last NAV change (LastNAV), and Profitability Index (present value of dividends over present value of takedowns). *t*-statistics are reported below for each coefficient in italics. The estimation is done separately for each age (10 to 13 years old). As discussed in section 5.2 this regression is estimated simultaneously with the GMM estimation of alpha and beta. Panel B shows summary statistics of the liquidated sample and non-liquidated sample; including the predicted Market Values of non-liquidated funds using the estimated statistical model (Panel A).

Panel A: Market values as a function of fund characteristics – liquidated sample

	Dependent variable: ln (Market Value)			
	Age 10	Age 11	Age 12	Age 13
Constant	-0.06	-0.11	-0.42	-0.35
	<i>-0.28</i>	<i>-0.39</i>	<i>-1.55</i>	<i>-0.98</i>
ln(1+NAV)	***0.89	***0.84	***0.83	***0.73
	<i>19.30</i>	<i>16.50</i>	<i>16.32</i>	<i>11.68</i>
ln(Size)	0.09	*0.12	**0.13	*0.13
	<i>1.56</i>	<i>1.80</i>	<i>2.10</i>	<i>1.79</i>
ln>LastDiv)	** <i>-0.09</i>	* <i>-0.10</i>	<i>-0.03</i>	** <i>-0.16</i>
	<i>-2.00</i>	<i>-1.86</i>	<i>-0.56</i>	<i>-2.11</i>
ln>LastNAV)	*** <i>-0.25</i>	*** <i>-0.22</i>	*** <i>-0.28</i>	*** <i>-0.18</i>
	<i>-5.88</i>	<i>-4.26</i>	<i>-5.08</i>	<i>-2.55</i>
Profitability Index	-0.03	0.06	0.14	***0.36
	<i>-0.60</i>	<i>0.60</i>	<i>1.25</i>	<i>2.65</i>
Adj. R-square	0.68	0.66	0.70	0.65
N-observations	280	226	182	136

Panel B: Summary Statistics

		Liquidated funds				Non-liquidated funds			
		Age 10	Age 11	Age 12	Age 13	Age 10	Age 11	Age 12	Age 13+
NAV	- Mean	35.32	30.96	21.15	19.76	78.47	48.55	56.09	55.89
Size	- Mean	121.21	122.77	122.08	123.42	234.81	202.30	140.89	203.93
LastDiv	- Mean	14.00	12.75	11.97	11.40	37.09	36.58	57.46	41.03
LastNAV	- Mean	7.39	5.82	5.60	5.26	22.22	23.54	37.62	29.77
PI	- Mean	0.90	0.92	0.94	0.99	0.82	0.94	0.59	0.81
NAV/Size		0.23	0.16	0.09	0.07	0.33	0.24	0.40	0.27
MV/NAV		1.00	1.00	1.06	1.13				
(Predicted MV)/NAV						0.37	0.32	0.21	0.29
N_obs		280	226	182	136	79	50	50	434

Table 4: Risk and Abnormal Performance of Private Equity Funds

This table shows monthly abnormal performance (Alpha; in percentage) and risk loadings using either a one factor market model (S&P 500; specs 1 and 3) or the three-factor Fama-French model (specs 2 and 4). Panel A shows the results with the original (net-of-fee) cash flows. Panel B shows the results with the simulated gross-of-fee cash flows. Gross-of-fee cash flows are obtained by adding fees to the net-of-fees cash flows. Fees are assumed to be made of 2% management fee and 20% carry with an 8% hurdle rate (see text for details). Estimation is executed by GMM using the ‘log-PME’ approach with joint estimation of final market values (see text for details). Moment conditions are weighted by the number of funds. Standard errors are obtained by bootstrapping and are show between parentheses. Below each specification, the autocorrelation of the pricing errors across vintage years is reported with its corresponding standard error.

Panel A: Net-of-fee				
	Venture Capital		Buyout	
	Spec 1	Spec 2	Spec 3	Spec 4
Alpha (% , monthly)	***-1.25 (0.05)	*-0.69 (0.38)	0.49 (0.37)	0.13 (0.44)
Beta_Market	***3.21 (0.22)	***2.57 (0.49)	0.33 (0.38)	**0.94 (0.44)
Beta_SMB		0.99 (0.78)		***-2.05 (0.79)
Beta_HML		-0.56 (0.42)		-0.16 (0.86)
Number obs.	686	686	272	272
Error autocorrelation	-0.16 (0.26)	-0.28 (0.26)	0.35 (0.30)	-0.08 (0.35)

Panel B: Gross-of-fee

	Venture Capital		Buyout	
	Spec 1	Spec 2	Spec 3	Spec 4
Alpha (% , monthly)	***-0.90 (0.07)	-0.32 (0.30)	**0.74 (0.38)	0.30 (0.57)
Beta_Market	***3.36 (0.20)	***2.69 (0.43)	0.56 (0.41)	**1.26 (0.52)
Beta_SMB		1.09 (0.69)		***-2.07 (0.75)
Beta_HML		-0.58 (0.36)		-0.01 (1.19)
Number obs.	686	686	272	272
Error autocorrelation	-0.11 (0.26)	-0.24 (0.26)	0.31 (0.32)	-0.14 (0.35)

Table 5: Risk, Abnormal Performance and Fund Characteristics

This table shows monthly abnormal performance (Alpha; in percentage) and risk loadings using a one factor market model. Alpha and beta are specified as a function of fund characteristics as follows: $\text{Alpha} = a_0 + a_{\text{fund_characteristic}} * \text{fund_characteristic}$ and $\text{Beta} = b_0 + b_{\text{fund_characteristic}} * \text{fund_characteristic}$.

Fund characteristics include logarithm of fund size, a dummy variable that is one if the fund is an experienced fund, and a dummy variable that is one if the fund is US focused. Panel A shows results for Venture Capital funds and Panel B shows results for Buyout funds. Estimation is executed by GMM using the ‘log-PME’ approach with joint estimation of final market values (see text for details). Moment conditions are weighted by the number of funds. Standard errors are obtained by bootstrapping and are shown between parentheses. Below each specification, the autocorrelation of the pricing errors across vintage years is reported with its corresponding standard error.

Panel A: Venture Capital Funds

	Spec 1	Spec 2	Spec 3	Spec 4	Spec 5	Spec 6
Alpha (% , monthly)						
a_0	***-2.14 (0.22)	***-1.61 (0.42)	***-1.35 (0.07)	***-1.33 (0.44)	***-1.14 (0.06)	***-1.11 (0.07)
a_{size}	***0.18 (0.05)	0.08 (0.09)				
$a_{\text{experience}}$			0.16 (0.11)	0.12 (0.44)		
$a_{\text{US_focused}}$					***0.59 (0.11)	*0.37 (0.22)
Beta_Market						
b_0	***2.70 (0.32)	0.15 (1.40)	***3.00 (0.27)	***2.70 (0.89)	***3.36 (0.19)	***3.55 (0.16)
b_{size}		*0.57 (0.31)				
$b_{\text{experience}}$				0.44 (0.92)		
$b_{\text{US_focused}}$						**1.31 (0.63)
Error autocorrelation	-0.24 (0.24)	-0.18 (0.22)	-0.10 (0.22)	-0.08 (0.21)	-0.04 (0.23)	-0.05 (0.23)

Panel B: Buyout Funds

	Spec 1	Spec 2	Spec 3	Spec 4	Spec 5	Spec 6
Alpha (% , monthly)						
a_0	-0.08 (0.52)	***2.63 (0.80)	0.12 (0.33)	0.10 (0.50)	*0.71 (0.39)	0.74 (0.45)
a_{size}	0.07 (0.08)	***-0.39 (0.11)				
$a_{experience}$			*0.37 (0.20)	0.42 (0.69)		
$a_{US_focused}$					*0.46 (0.18)	0.53 (0.73)
Beta_Market						
b_0	0.23 (0.37)	***-3.37 (0.70)	0.44 (0.41)	0.46 (0.64)	0.25 (0.41)	0.21 (0.47)
b_{size}		***0.64 (0.12)				
$b_{experience}$				-0.07 (0.81)		
$b_{US_focused}$						-0.08 (0.81)
Error autocorrelation	-0.01 (0.38)	-0.02 (0.38)	-0.03 (0.38)	-0.03 (0.38)	-0.01 (0.38)	-0.01 (0.38)

Table 6: Impact of final NAV on risk and abnormal return

This table is like Table 4. Instead of jointly estimating the final Net Asset Value (NAV) and the risk profile, it treats final NAVs either as market value or as worthless (written off).

	Final NAV as market value				Final NAV written off			
	Venture Capital		Buyout		Venture Capital		Buyout	
	Spec 1	Spec 2	Spec 1	Spec 2	Spec 1	Spec 2	Spec 1	Spec 2
Alpha (%)	***-1.08	** -0.60	0.37	0.19	***-1.27	-0.51	*0.62	0.13
	(0.06)	(0.24)	(0.32)	(0.44)	(0.06)	(0.38)	(0.38)	(1.24)
Beta	***3.45	***2.58	0.59	**1.06	***2.98	***2.27	0.14	0.75
	(0.19)	(0.35)	(0.39)	(0.46)	(0.28)	(0.47)	(0.39)	(0.79)
SMB		***1.35		***-1.57		*1.32		** -2.56
		(0.44)		(0.56)		(0.79)		(1.01)
HML		-0.34		-0.08		-0.64		0.40
		(0.33)		(0.82)		(0.43)		(1.68)

Table 7: Robustness Tests

This table shows monthly abnormal performance (Alpha; in percentage) and risk loadings using a one factor market model. Moment conditions are either weighted by the number of funds (N_funds weighted) or by their total size (Value weighted). Parameter estimates are shown for different number of fund-of-funds (FoFs), method (PME instead of log-PME), time periods, sub-sample (liquidated funds), and with different market portfolio proxies (benchmarks). Benchmarks include Nasdaq, and indices from Kenneth French's webpage (Small growth 5x5, Europe dollar return, United Kingdom dollar return; all value-weighted).

	N_funds Weighted Moments				Value Weighted Moments			
	VC		BO		VC		BO	
	Alpha	Beta	Alpha	Beta	Alpha	Beta	Alpha	Beta
Base estimation	-1.25	3.21	0.49	0.33	-1.22	3.13	0.06	0.75
Number FoFs per vint. year								
Default + 1 (2 FoFs)	-1.38	2.88	0.25	0.31	-1.24	3.12	-0.02	0.82
Default + 2 (3 FoFs)	-1.40	2.76	0.11	0.28	-1.23	3.18	0.03	0.73
Default + 3 (4 FoFs)	-1.38	2.65	-0.10	0.57	-1.23	3.17	-0.02	0.80
PME method	-1.20	3.19	0.97	-0.02	-1.15	3.04	0.46	0.41
Vintage cut (default is 1993)								
Default + 1 (1980-1994)	-1.24	3.22	0.63	0.16	-1.21	3.14	0.17	0.57
Default - 1 (1980-1992)	-1.26	3.15	0.47	0.38	-1.23	3.07	0.00	0.83
Liquidated funds	-1.15	3.15	0.52	0.40	-1.08	3.47	0.48	0.47
Other benchmarks								
S&P500 and \$-EU index	-0.94	3.76	0.59	0.34	-0.92	3.82	0.46	0.44
S&P500 and \$-UK index	-1.10	3.57	0.40	0.47	-1.09	3.61	-0.02	0.87
Nasdaq	-0.52	1.55			-0.47	1.47		
Small Growth	1.47	1.70			1.43	1.65		

Figure 1: Fund dividend yields (average of the next 12 months dividend yields of funds in their 4th to 10th year). Dividend yield is the sum of the dividends paid divided by fund size. S&P 500 returns are the 5 years cumulated returns, divided by 5. Time spans 1990 to 2003.

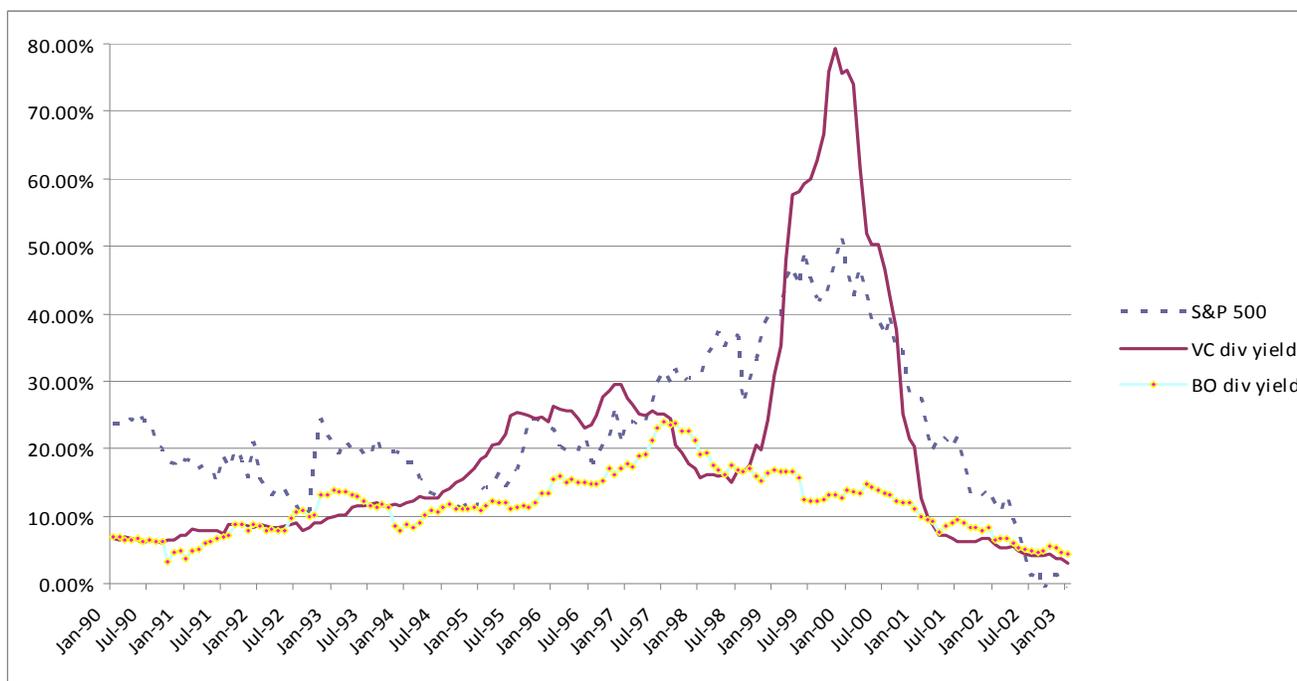


Figure 2: GMM goal function for the NPV method calculated using the simulated data (section 4.6).

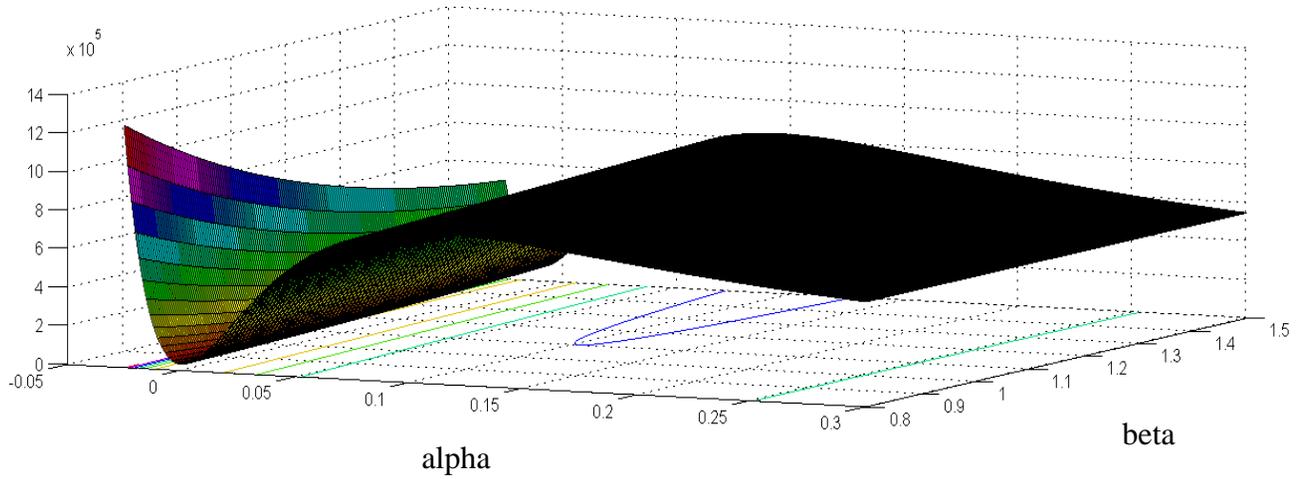


Figure 3: GMM goal function for the log-PME method calculated using the simulated data (section 4.6).

