Long Run Growth of Financial Data Technology

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Abstract

In most sectors, technological progress boosts efficiency. But "big data" financial technology and the associated data-intensive trading strategies have been blamed for market inefficiency. A key cause for concern is that better data processing technology might induce traders to extract others' information, rather than produce information themselves. We adopt the lens of long-run growth to understand how improvements in financial technology shape information choices, trading strategies and market efficiency, as measured by price informativeness and market liquidity. Our main insight is that unbiased technological change can explain a market-wide shift in data collection and trading strategies. The efficiency results that follow upend common wisdom. They offer a new take on what makes prices informative and whether trades typically deemed liquidity-providing actually make markets more resilient, in the long run.

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In most sectors, technological progress boosts efficiency. But in finance, more efficient data processing and the new data-intensive trading strategies it has spawned have been blamed for market volatility, illiquidity and inefficiency. One reason financial technology is suspect is that its rise has been accompanied by a shift in the nature of financial analysis and trading. Instead of "kicking the tires" of a firm, investigating its business model or forecasting its profitability, many traders today engage in statistical arbitrage: They search for "dumb money," or mine order flow data and develop algorithms to profit from patterns in others' trades. Why might investors choose one strategy versus the other and why are these incentives to process each type of data changing over time? Answering these questions requires a model. Just like past investment rates are unliable forecasts for economies in transition, empirically extrapolating past financial trends is dubious in the midst of a technological transformation.

To make sense of current and future long-run trends requires a growth model of structural change in the financial economy. Since much of the technological progress is in the realm of data processing, we use an information choice model to explore how unbiased technological progress changes what data investors choose to process, what investment strategies they adopt, and how the changing strategies alter financial market efficiency and real economic outcomes. Structural change in the financial sector arises because improvements in data processing trigger a shift in the type of data investors process. Instead of processing data about firm fundamentals, firms choose to processing more and more data about other investors' demand. Each data choice gives rise to an optimal trading strategy. The resulting shift in strategies resembles an abandonment of value investing and a rise in a strategy that is part statistical arbitrage, part retail market making, and part strategies designed to extract what others know. Just like the shift from agriculture to industry, some of our data-processing shift takes place because growing efficiency interacts with decreasing returns. But unlike physical production, information leaks out through equilibrium prices, producing externalities, and a region of endogenous increasing returns, that do not arise in standard growth models.

The consequences of this shift in strategy upend some common thinking. Contrary to popular wisdom, the abandonment of fundamentals-based investing does not necessarily compromise financial market efficiency. Efficiency, as measured by price informativeness, continues to rise, even as fundamental data gathering falls. Our results can inform measurement. They lend support to the common practice of using price informativeness to proxy for total information processing. But they call into question the interpretation that price informativeness is a measure information acquired specifically about firm fundamentals. Our second surprise is that the price impact of an uninformative trade (liquidity) stagnates. Even though demand data allows investors to identify uninformed trades, and even though investors use this information to "make markets" for demand-driven trades, market-wide liquidity may not improve.

There are many aspects to the financial technology revolution and many details of modern trading strategies that our analysis misses. But before developing a new framework that casts aside decades of accumulated knowledge, it is useful to first ask what existing tools can explain, if only to better identify where new thinking is needed. The most obvious and simplest tool for thinking about choices related to information and their equilibrium effects is the noisy rational expectations framework. To this framework, we add three ingredients. First, we add a continuous choice between firm fundamental information and investor demand information. We model data processing in a way that draws on the information processing

literatures in macroeconomics and finance.[1] But the idea that processing data on demand might trade off with fundamental information processing is central to modern concerns, is essential for our main results, and is new to this literature. Second, we add long-run technological progress. It is straightforward to grow the feasible signal set. But doing so points this tool in a new direction, to answer a different set of questions. Third, we use long-lived assets, as in Wang (1993), because ignoring the truth, that equity is a long-lived claim, fundamentally changes our results. The long-lived asset assumption is essential for our long-run balanced growth path, the stagnation of liquidity, and the model's modest predicted decline in the equity premium. A static model would reverse all three results.

The key to our results is understanding what makes each type of data valuable. Fundamental data is always valuable. It allows investors to predict future dividends and future prices. Demand data contains no information about any future cash flows. It has value because it enables an investor to trade against demand shocks – sometimes referred to as searching for "dumb money." By buying when demand shocks are low and selling when demand shocks are high, an investor can systematically buy low, sell high and profit. This is the sense in which the demand-data trading strategy looks like market making for uninformed retail investors. Demand-data processors stand ready to trade against – make markets for – uninformed orders. The mathematics of the model suggest another, complementary interpretation of the rise in demand-based strategies. Demand shocks are the noise in prices. Knowing something about this noise allows investors to remove that known component and reduce the noise in prices. Since prices summarize what other investors know, removing price noise is a way of extracting others' fundamental information. Seen in this way, the demand-driven trading strategy shares some of the hallmarks of automated trading strategies, largely based on order flow data, that are also designed to extract the information of other market participants.

Our main results in Section 2 describe the evolution of data processing in three phases. *Phase one: technology is poor and fundamental data processing dominates.* In this phase, fundamental data is preferred because demand data has little value. To see why, suppose no investors have any fundamental information. In such an environment, all trades are uninformed. No signals are necessary to distinguish informed and uninformed trades. As technology progresses and more trades are information-driven, it becomes more valuable to identify and trade against the remaining non-informational trades. *Phase two: moderate technology generates increasing returns to demand data processing.* Most physical production as well as most information choices in financial markets exhibit decreasing returns, also called strategic substitutability. Returns decrease because acquiring the same information as others, leads one to buy the same assets as others, and the assets others buy are expensive. Our increasing returns come from an externality specific to data: Information leaks through the equilibrium price. When more investors process demand data, they extract more fundamental information from equilibrium prices, and trade on that information. More trading on fundamental information, even if extracted, makes the price more informative, which encourages more demand data processing, to enable more information extraction from the equilibrium price. *Phase three: high technology restores balanced growth in data processing.* As technology progresses, both types of data become more abundant. In the high-technology limit, they grow in fixed proportion to each other. When information is abundant, the natural substitutability force in asset markets strengthens and

[1] See e.g., Caplin, Leahy, and Matejka (2016), Maćkowiak and Wiederholt (2012), Nimark (2008), Briggs, Caplin, Martin, and Tonetti (2017), Kacperczyk, Nosal, and Stevens (2015), Barlevy and Veronesi (2000), Goldstein, Ozdenoren, and Yuan (2013), Blanchard, L'Huillier, and Lorenzoni (2013), Fajgelbaum, Schaal, and Taschereau-Dumouchel (2017), Angeletos and La'O (2014), Atkeson and Lucas (1992), Chien, Cole, and Lustig (2016), Basetto and Galli (2017), Abis (2017).

overtakes complementarity. Information production in this region comes to resemble physical production.

A key force behind many of our results, including balanced growth, is rising *future information risk*. It exists only when assets are long-lived. No matter how much data processing one does, there are some events, not yet conceived of, that can only be learned about in the future. If tomorrow, we learn something about one of these events, that knowledge will affect tomorrow's asset price. Since part of the payoff of an asset purchased today is its price tomorrow, events that will be learned about tomorrow, but are not knowable today, make an asset purchased today riskier. This is future information risk. If more data will be processed tomorrow, then tomorrow's price will respond more to that information, raising future information risk. Data processing today reduces uncertainty about future dividends. Expected data processing tomorrow increases risk today. This idea, that long-run growth in information may create as much risk as it resolves, is the source of balanced growth, stagnating liquidity, and modest long-run changes in equity premia. These basic economic forces – decreasing returns, increasing returns, and future information risk – appear whether technology is unbiased, or biased, demand is persistent or not, and for most standard formulations of data constraints.

The consequences of this shift in data analysis and trading strategies involve competing forces. We identify these forces theoretically. However, to know which force is likely to dominate, we need to put some plausible numbers to the model. Section 3 calibrates the model to financial market data so that we can explore the growth transition path and its consequences for market efficiency numerically.

The market efficiency results upend some common wisdom. First, even as demand analysis crowds out fundamental analysis and reduces the discovery of information about the future asset value, price informativeness continues to rise. The reason is that demand information allows demand traders to extract fundamental information from prices. That makes the demand traders, and thus the average trader, better informed about future asset fundamentals. When the average trader is better informed, prices are more informative. According to this commonly-used measure, market efficiency continues to improve as technology progresses.

Second, even though demand traders systematically take the opposite side of uninformed trades, the rise of demand trading does not enhance market liquidity. This is surprising because taking the opposite side of uninformed trades is often referred to as "providing liquidity." This is one of the strongest arguments that proponents of activities such as high-frequency trading use to defend their methods. But if by providing liquidity, we really mean reducing the price impact of an uninformed trade, the rise of demand trading may not accomplish that. The problem is not demand trading today, but the expectation of future informed trading of any kind – fundamental or demand – creating *future information risk*. So future data processing raises the risk of investing in assets today. More risk per share of asset today is what causes the sale of one share of the asset to have a larger effect on the price. Finally, the rise in demand-driven trading strategies, while it arises concurrently with worrying market trends, is not causing those trends. The rise in return uncertainty, and the stagnation of liquidity, emerge as concurrent trends with financial data technology as their common cause.

Finally, Section 4 shows why these trends in data processing are also relevant for the real economy. This last section sketches two extensions of the model. One argues that, if firm managers are compensated with equity, better price informativeness improves their incentives to exert optimal effort. The second extension shows how the same forces that underlie market liquidity also reduce the cost of equity issuance

for a firm that wants to raise capital for real investment. Thus more liquid markets should also promote efficient real investment and long-run economic growth. If both channels are active, our model suggests that growing data processing efficiency is probably a net plus for the real economy.

Contribution to the existing literature Our model combines features from a few disparate literatures. Long run trends in finance are featured in Asriyan and Vanasco (2014), Biais, Foucault, and Moinas (2015), Glode, Green, and Lowery (2012), and Lowery and Landvoigt (2016), who model growth in fundamental analysis or an increase in its speed. Davila and Parlatore (2016) explore a decline in trading costs. The idea of long-run growth in information processing is supported by the rise in price informativeness documented by Bai, Philippon, and Savov (2016).

A small, growing literature examines demand information in equilibrium models. In Yang and Ganguli (2009), agents can choose whether or not to purchase a fixed bundle of fundamental and demand information. In Yang and Zhu (2016) and Manzano and Vives (2010), the precision of fundamental and demand information is exogenous. Babus and Parlatore (2015) examine intermediaries who observe the demands of their customers. Our demand signals also resemble Angeletos and La'O (2014)'s sentiment signals about other firms' production, Banerjee and Green (2015)'s signals about motives for trade, the signaling by He (2009)'s intermediaries, and the noise in government's market interventions in Brunnermeier, Sockin, and Xiong (2017). But none of these papers examines the choice that is central to this paper: The choice of whether to process more about asset payoffs or to analyze more demand data. Without that trade-off, these papers cannot explore how trading strategies change as productivity improves. Furthermore, this paper adds a long-lived asset in a style of model that has traditionally been static,[2] because assets are not static and assuming they are reverses many of our results.

One interpretation of our demand information is that it is what high-frequency traders learn by observing order flow. Like high-frequency traders, our traders use data on asset demand to distinguish information from uninformed trades, and they stand ready to trade against uninformed order flow. While our model has no high frequency, making this a loose interpretation, our model does contribute a perspective on this broad class of strategies. As such, it complements work by Du and Zhu (2017), Crouzet, Dew-Becker, and Nathanson (2016) on the theory side, as well as empirical work, such as Hendershott, Jones, and Menkveld (2011), which measures how fundamental and algorithmic trading affects liquidity.

Another, more theoretical, interpretation of demand signals is that they make a public signal, the price, less conditionally correlated. The choice between private, correlated or public information in strategic settings arises in work by Myatt and Wallace (2012), Chahrour (2014) and Amador and Weill (2010), among others.

1 Model

To explore growth and structural change in the financial economy, we use a noisy rational expectations model with three key ingredients: a choice between fundamental and demand data, long-lived assets, and unbiased technological progress in data processing. A key question is how to model structural change. The types of activities, the way in which investors earn profits has changed. A hallmark of that change is the rise

[2]Exceptions include 2- and 3-period models, such as Cespa and Vives (2012).

in information extraction from demand. In practice, demand-based trading takes many forms. Demand-based trading might take the form of high-frequency trading, where the information of an imminent trade is used to trade before the new price is realized. It could be mining tweets or Facebook posts to gauge sentiment. Extraction could take the form of "partnering," a practice where brokers sell their demand information (order flow) to hedge funds, who systematically trade against, what are presumed to be uninformed traders.[3] Finally, it may mean looking at price trends, often referred to as technical analysis, in order to discern what information others may be trading on. All of these practices have in common that they are not uncovering original information about the future payoff of an asset. Instead, they are using public information, in conjunction with private analysis, to profit from what others already know (or don't know). We capture this general strategy, while abstracting from many of its details, by allowing investors to observe a signal about the non-informational trades of other traders. This demand signal allows our traders to profit in three ways. 1) They can identify and then trade against uninformed order flow; 2) they can remove noise from the equilibrium price to uncover more of what others know; or 3) they can exploit the mean-reversion of demand shocks to buy before price rises and sell before it falls. These three strategies have an equivalent representation in the model and collectively cover many of the ways in which modern investment strategies profit from information technology.

Static models have been very useful in this literature to explain many forces and trade-offs in a simple and transparent way. However, when the assumption of one-period-lived assets reverses the prediction of the more realistic dynamic model, the static assumption is no longer appropriate. That is the case here. Long-run growth means not only more data processing today, but even more tomorrow. In many instances, the increase today and the further increase tomorrow have competing effects. That competition is a central theme of the paper. Without the long-lived asset assumption, the long-run balanced growth, stagnating liquidity and flat equity premium results would all be overturned.

Finally, technological progress takes the form of allowing investors access to a larger set of feasible signals, over time. While there are many possible frameworks that one might use to investigate financial growth, this ends up being a useful lens, because it explains many facts about the evolution of financial analysis, can forecast future changes that empirical extrapolation alone would miss, and offers surprising, logical insights about the financial and real consequences of the structural change.

1.1 Setup

Investors At the start of each date t, a measure-one continuum of overlapping generations investors is born. Each investor i born at date t has constant absolute risk aversion utility over total, end of period t consumption $\tilde{c}_{it}$:

$$U(\tilde{c}_{it}) = -e^{-\rho \tilde{c}_{it}} \tag{1}$$

where ρ is absolute risk aversion. We adopt the convention of using tildes to indicate t-subscripted variables that are not in the agents' information set when they make time-t investment decisions.

[3]Market evidence suggests that hedge funds value the opportunity to trade against the uninformed, as noted by Goldstein in a 2009 Reuters article: "Right now, ETrade sends about 40% of its customer trades to Citadels market-maker division ... Indeed, the deal is so potentially lucrative for Citadel that the hedge fund is willing to make an upfront $100 million cash payment to the financially-strapped online broker."

Each investor is endowed with an exogenous income that is e_{it} units of consumption goods. Investors can use their income to buy risky assets at the start of the period. But they cannot trade shares of or any asset contingent on this income.

There is a single tradeable asset.[4] Its supply is one unit per capita. It is a claim to an infinite stream of dividend payments $\{d_t\}$:

$$\tilde{d}_t = \mu + Gd_{t-1} + \tilde{y}_t. \tag{2}$$

where μ and $G < 1$ are known parameters. The innovation $\tilde{y}_t \sim N(0, \tau_0^{-1})$ is revealed and $\tilde{d}_t$ is paid out at the end of each period t. $\tilde{d}_t$ and d_{t-1} both refer to dividends, only d_{t-1} is already realized at time t, while $\tilde{d}_t$ has not due to innovation $\tilde{y}_t$.

An investor born at date t, sells his assets at price p_{t+1} to the $t+1$ generation of investors, collects dividends $\tilde{d}_t$ per share, combines that with the endowment that is left $(e_{it} - q_{it}p_t)$, times the rate of time preference $r > 1$, and consumes all those resources. Thus the cohort-t investor's budget constraint is

$$\tilde{c}_{it} = r(e_{it} - q_{it}p_t) + q_{it}(p_{t+1} + \tilde{d}_t) \tag{3}$$

where q_{it} is the shares of the risky asset that investor i purchases at time t and $\tilde{d}_t$ are the dividends paid out at the start of period $t+1$. Since we do not prohibit $c_t < 0$, all pledges to pay income for risky assets are riskless.

Hedgers The economy is also populated by a unit measure of hedgers in each period. Hedgers are endowed with non-financial income risk that is correlated with the asset payoff.[5] To hedge that risky income, each hedger buys $\tilde{h}_{jt} = \tilde{\epsilon}_{hjt} - \tilde{x}_t$ shares of the asset, where $\tilde{x}_t \sim N(0, \tau_x^{-1})$ is the common component of hedging demand and $\tilde{\epsilon}_{hjt} \sim N(0, \tau_h^{-1})$ is the idiosyncratic component. $\tilde{\epsilon}_{hjt}$ is independent across hedgers and independent of all the other shocks in the economy. For information to have value, prices must not perfectly aggregate asset payoff information. This aggregate hedging demand is our source of noise in prices. Equivalently, $\tilde{x}_t$ could also be interpreted as sentiment. For now, we assume that $\tilde{x}_t$ is independent over time. We discuss the possibility of autocorrelated $\tilde{x}_t$ in Section 2.4.

Information Choice If we want to examine how the nature of financial analysis has changed over time, we need to have at least two types of analysis to choose between. Financial analysis in this model means signal acquisition. Our constraint on acquisition could represent the limited research time for uncovering new information. But it could also represent the time required to process and compute optimal trades based on information that is readily available from public sources.

Investors choose how much information to acquire or process about the next-period dividend innovation $\tilde{y}_t$, and also about the hedgers' demand shocks, $\tilde{x}_t$. We call $\eta_{fit} = \tilde{y}_t + \tilde{\epsilon}_{fit}$ a fundamental signal and $\eta_{xit} = \tilde{x}_t + \tilde{\epsilon}_{xit}$ a demand signal. What investors are choosing is the precision of these signals. In other words, if the signal errors are distributed $\tilde{\epsilon}_{fit} \sim N(0, \Omega_{fit}^{-1})$ and $\tilde{\epsilon}_{xit} \sim N(0, \Omega_{xit}^{-1})$, then the precisions Ω_{fit}

[4] We describe a market with a single risky asset because our main effects do not require multiple assets. However, we have some results for the generalized, multi-asset setting.

[5] In previous versions of this paper, we micro-founded hedging demand, spelling out the endowment shocks that would rationalize this trading behavior. These foundations involved additional complexity, obfuscated key effects, and offered no additional economic insight.

and Ω_{xit} are choice variables for investor i.

The constraint that investors face when choosing information is

$$\Omega_{fit}^2 + \chi_x \Omega_{xit}^2 \leq K_t. \tag{4}$$

This represents the idea that getting more and more precise information about a given variable is tougher and tougher. But acquiring information about a different variable is a separate task, whose shadow cost is additive.

The main force in the model is technological progress in information analysis. Specifically, we assume that K_t is a deterministic, increasing process.

Information sets and equilibrium First, we recursively define two information sets. The first is all the variables that are known at the end of period $t-1$ to agent i. This information is $\{\mathcal{I}_{t-1}, y_{t-1}, d_{t-1}, x_{t-1}\} \equiv \mathcal{I}_{t-1}^+$. This is what investors know when they choose what signals to acquire. The second information set is $\{\mathcal{I}_{t-1}, y_{t-1}, d_{t-1}, x_{t-1}, \eta_{fit}, \eta_{xit}, p_t\} \equiv \mathcal{I}_{it}$. This includes the two signals the investor chooses to see, and the information contained in equilibrium prices. This is the information set the investor has when they make investment decisions. The time 0 information set includes the entire sequence of information capacity: $\mathcal{I}_0 \equiv \mathcal{I}_{i0}\ \forall i \supset \{K_t\}_{t=0}^{\infty}$.

An equilibrium is a sequence of information choices $\{\Omega_{fit}\}$, $\{\Omega_{xit}\}$ and portfolio choices $\{q_{it}\}$ by investors such that

1. Investors choose signal precisions Ω_{fit} and Ω_{xit} to maximize $E[ln(E[U(\tilde{c}_{it})|\mathcal{I}_{it}])|\mathcal{I}_{t-1}^+]$, where U is defined in (1), taking the choices of other agents as given.[6] This choice is subject to (4), $\Omega_{fit} \geq 0$ and $\Omega_{xit} \geq 0$.

2. Investors choose their risky asset investment q_{it} to maximize $E[U(\tilde{c}_{it})|\eta_{fit}, \eta_{xit}, p_t]$, taking the asset price and the actions of other agents as given, subject to the budget constraint (3).

3. At each date t, the risky asset price clears the market:

$$\int_i q_{it} di = 1 + \int_j h_{jt} dj \qquad \forall t. \tag{5}$$

1.2 Solving the Model

There are four main steps to solve the model. *Step 1: Solve for the optimal portfolios, given information sets.* Each investor i at date t chooses a number of shares q_{it} of the risky asset to maximize expected utility (1), subject to the budget constraint (3). The first-order condition of that problem is

$$q_{it} = \frac{E[p_{t+1} + \tilde{d}_t|\mathcal{I}_{it}] - rp_t}{\rho Var[p_{t+1} + \tilde{d}_t|\mathcal{I}_{it}]} \tag{6}$$

When using the term "investor," we do not include hedgers.

[6] E ln E preferences deliver a simple expression for the objective that is linear in signal precision. It is commonly used in information choice models (Kacperczyk, Nosal, and Stevens, 2015), (Crouzet, Dew-Becker, and Nathanson, 2016). The same trade-offs arise with expected utility. Results available on request.

Step 2: Clear the asset market. Let $\bar{\mathcal{I}}_t$ denote the information set of the average investor. Given the optimal investment choice, we can impose market clearing (5) and obtain a price function that is linear in past dividends d_{t-1}, the t-period dividend innovation $\tilde{y}_t$, and the aggregate component of the hedging shocks $\tilde{x}_t$:

$$p_t = A_t + Bd_{t-1} + C_t\tilde{y}_t + D_t\tilde{x}_t \tag{7}$$

Where A_t governs the equity premium, B is the time-invariant effect of past dividends, C_t governs the information content of prices about current dividend innovations (price informativeness) and D_t regulates the amount of demand noise in prices:

$$A_t = \frac{1}{r}\left[A_{t+1} + \frac{r\mu}{r-G} - \rho Var[p_{t+1} + \tilde{d}_t|\bar{\mathcal{I}}_t]\right]. \tag{8}$$

$$B = \frac{G}{r-G} \tag{9}$$

$$C_t = \frac{1}{r-G}\left(1 - \tau_0 Var[\tilde{y}_t|\bar{\mathcal{I}}_t]\right) \tag{10}$$

$$rD_t = -\rho Var[p_{t+1} + \tilde{d}_t|\bar{\mathcal{I}}_t] + \frac{r}{r-G}Var[\tilde{y}_t|\bar{\mathcal{I}}_t]\frac{C_t}{D_t}\tau_x \tag{11}$$

where Ω_{pit} is the precision of the information about $\tilde{d}_t$, extracted jointly from prices and demand signals, and

$$Var[\tilde{y}_t|\bar{\mathcal{I}}_t] = (\tau_0 + \Omega_{fit} + \Omega_{pit})^{-1} \tag{12}$$

is the posterior uncertainty about next-period dividend innovations and the resulting uncertainty about asset returns is proportional to

$$Var[p_{t+1} + \tilde{d}_t|\bar{\mathcal{I}}_t] = C_{t+1}^2\tau_0^{-1} + D_{t+1}^2\tau_x^{-1} + (1+B)^2 Var[\tilde{y}_t|\bar{\mathcal{I}}_t]. \tag{13}$$

Step 3: Compute ex-ante expected utility. When choosing information to observe, investors do not know what signal realizations will be, nor do they know what the equilibrium price will be. The relevant information set for this information choice is $\mathcal{I}_{t-1}^+$.

After we substitute the optimal portfolio choice (6) and the equilibrium price rule (7) into utility (1), and take logs and then the beginning of time-t expectation ($-E[ln(E[\exp(\rho c_{it})|\eta_{fit}, \eta_{xit}, p_t])|\mathcal{I}_{t-1}^+]$), we get a time-1 expected utility expression that is similar to most CARA-normal models: $\rho\, r\, e_{it} + \rho E[q_{it}(E[p_{t+1} + \tilde{d}_t|\mathcal{I}_{it}] - p_t r)|\mathcal{I}_{t-1}^+] - \frac{\rho^2}{2}E[q_{it}^2 Var[p_{t+1} + \tilde{d}_t|\bar{\mathcal{I}}_t]^{-1}|\mathcal{I}_{t-1}^+]$. Appendix A shows that the agent's choice variables Ω_{fit} and Ω_{xit} show up only through the conditional precision of payoffs, $Var[p_{t+1} + \tilde{d}_t|\bar{\mathcal{I}}_t]^{-1}$. The reason for this is that the first-moment terms in asset demand – $E[p_{t+1} + \tilde{d}_t|\mathcal{I}_{it}]$ and p_t – have ex-ante expected values that do not depend on the precision of any given investor's information choices. In other words, choosing to get more data of either type does not, by itself, lead one to believe that payoffs or prices will be particularly high or low. So, information choices amount to minimizing the payoff variance $Var[p_{t+1} + \tilde{d}_t|\bar{\mathcal{I}}_t]$, subject to the data constraint. The payoff variance, in turn, has a bunch of terms the investor takes as

given, plus a term that depends on dividend variance, $Var[\tilde{y}_t|\bar{\mathcal{I}}_t]$. Equation (12) shows that $Var[\tilde{y}_t|\bar{\mathcal{I}}_t]$ depends on the sum of fundamental precision Ω_{fit} and price information Ω_{pit}. Price information precision is $\Omega_{pit} = (C_t/D_t)^2(\tau_x + \Omega_{xit})$, which is linear in Ω_{xit}. Thus expected utility is a function of the sum of Ω_{fit} and $(C_t/D_t)^2\Omega_{xit}$.

Thus, optimal information choices maximize the weighted sum of fundamental and demand precisions:

$$\max_{\Omega_{fit},\Omega_{xit}} \ \Omega_{fit} + \left(\frac{C_t}{D_t}\right)^2 \Omega_{xit} \tag{14}$$

$$\text{s.t. } \Omega_{fit}^2 + \chi_x \Omega_{xit}^2 \leq K_t, \quad \Omega_{fit} \geq 0, \text{ and } \Omega_{xit} \geq 0.$$

Step 4: Solve for information choices. The first order conditions yield

$$\Omega_{xit} = \frac{1}{\chi_x}\left(\frac{C_t}{D_t}\right)^2 \Omega_{fit} \tag{15}$$

This solution implies that information choices as symmetric. Therefore, in what follows, we drop the i subscript to denote an agent's data processing choice. Moreover, the information set of the average investor is the same as information set of each investor, $\bar{\mathcal{I}}_t = \mathcal{I}_{it} = \mathcal{I}_t$.

The information choices are a function of pricing coefficients, like C and D, which are in turn functions of information choices. To determine the evolution of analysis and its effect on asset markets, we need to compute a fixed point to a highly non-linear set of equations. After substituting in the first order conditions for Ω_{ft} and Ω_{xt}, we can write the problem as two non-linear equations in two unknowns.

1.3 Interpreting Demand Data Trading

Why are demand signals useful? They don't predict future dividends or future prices. They only provide information about current demand. The reason that information is valuable is that it tells the investor something about the difference between price and expected asset value. One can see this by looking at the signal extracted from prices. Price is a noisy signal about dividends. To extract the price signal, we subtract the expected value of all the terms besides the dividend, and divide by the dividend coefficient C_t. The resulting signal extracted from prices is

$$\frac{\left(p_t - A_t - Bd_{t-1} - D_t E[\tilde{x}_t|\bar{\mathcal{I}}_t]\right)}{C_t} = \tilde{y}_t + \underbrace{\frac{D_t}{C_t}(\tilde{x}_t - E[\tilde{x}_t|\bar{\mathcal{I}}_t])}_{signal\ noise}. \tag{16}$$

Notice how demand shocks $\tilde{x}_t$ are the noise in the price signal. So information about this demand reduce noises in the price signal. In this way, the demand signal can be used to better extract others' dividend information from the price. This is the sense in which demand analysis is information extraction.

Of course, real demand traders are not taking their orders, and then inverting an equilibrium pricing model to infer future dividends. But another way to interpret the demand trading strategy is that it is identifying non-information trades to trade against. In equation (16), notice that when $\tilde{x}_t$ is high, hedging traders are mostly sales. Since $(D_t/C_t) < 0$, high $\tilde{x}_t$ makes the expected dividend minus price high, which leads those with demand information to buy. Thus, demand trading amounts to finding the non-informational trades and systematically taking the opposite side. This trading strategy of trading

against uninformed trades is commonly referred to as trading against "dumb money." An alternative way of interpreting the choice between fundamental data and demand data is that agents are choosing between decoding private or public signals. Fundamental signals have noise that is independent across agents. These are private. But demand data, although its noise is independent, is used in conjunction with the price, a public signal. The resulting inference about the shock $\tilde{y}_t$, conditional on the price and the $\tilde{x}_t$ signal, is conditionally correlated across agents, like a public signal would be.

The key to the main results that follow is that reducing the noise in $\tilde{x}_t$ reduces price noise variance in proportion to $(D_t/C_t)^2$. Put conversely, increasing precision of information about $\tilde{x}_t$ (the reciprocal of variance) increases the precision of dividend information, in proportion to $(C_t/D_t)^2$. What causes the long-run shifts is that the marginal rate of substitution of demand signals for fundamental signals, $(C_t/D_t)^2$, changes as technology grows.

If we interpret demand trading as finding dumb money, it is easy to see why it becomes more valuable over time. If there is very little information, everyone is "dumb," and finding dumb money is pointless. But when informed traders become sufficiently informed, distinguishing dumb from smart money, before taking the other side of a trade, becomes essential.

1.4 Measuring Financial Market Efficiency

To study the effects of financial technology on market efficiency, we assess efficiency in two ways. One measure of efficiency is price informativeness. The asset price is informative about the unknown future dividend innovation $\tilde{y}_t$. The coefficient C_t on the dividend innovation $\tilde{y}_t$ in the equilibrium price equation (7) measures price informativeness. C_t governs the extent to which price reacts to a dividend innovation. It corresponds to the price informativeness measure of Bai, Philippon, and Savov (2016).

The other measure of market efficiency is liquidity. Liquidity is the price impact of an uninformed (hedging) trade. That impact is the price coefficient D_t. Note that D_t is negative because a high endowment of risk correlated with dividends makes an investor less willing to hold risky assets; the reduced demand lowers the price. So, a more negative D_t represents a higher price impact and a less liquid market. Increasing (less negative) D_t is an improvement in liquidity.

2 Main Results: A Secular Shift in Financial Analysis

Our main objective is to understand how technological progress in information (data) processing affects financial analysis choices, trading strategies, and market efficiency. In this section, we focus mainly on the question: How does the decision to analyze fundamental or demand information change as technology improves? The information choice results illuminate why the effects of technology on market efficiency are mixed. Along the way, we learn why both types of information can improve price informativeness and also why both can create payoff risk and thereby impair market liquidity.

We begin by exploring what happens in the neighborhood near no information processing, $K \approx 0$. We show that all investors prefer to acquire only fundamental information in this region. Thus, at the start of the growth trajectory, investors primarily investigate firm fundamentals. Next, we prove that an increase in aggregate information processing increases the value of demand information, relative to fundamental information. Fundamental information has diminishing relative returns. But in some regions,

demand information has increasing returns. What does this mean for the evolution of analysis? The economy starts out doing fundamental analysis and then rapidly shifts to demand analysis. We explore this mechanism, as well as its long run limit, in the following propositions.

2.1 Analysis Choices when Information Is Scarce

In order to understand why investors with little information capacity use it all on fundamental information, we start by thinking about what makes each type of information valuable. Fundamental information is valuable because it informs an investor about whether the asset is likely to have a high dividend payoff tomorrow. Since prices are linked to current dividends, this also predicts a high asset price tomorrow and thus a high return. Knowing this allows the investor to buy more of the asset in times when its return will be high and less when return is likely to be low.

In contrast, demand information is not directly relevant to future payoff or future price. But one can still profit from trading on demand. An investor who knows that hedging demands are high will systematically profit by selling the asset because high hedging demands will make the price higher than the fundamental value, on average. In other words, demand signals allow one to trade against dumb money The next result proves that if the price has very little information embedded in it, because information is scarce (K_t is low), then getting demand data to extract price information is not very valuable. In other words, if all trades are "dumb," then identifying the uninformed trades has no value.

Result 1 ***When information is scarce, demand analysis has zero marginal value:***
As $K_t \to 0$, for any future path of prices (A_{t+j}, B_{t+j}, C_{t+j} and D_{t+1}, $\forall j > 0$), $dU_1/d\Omega_{xt} \to 0$.

The proof (in Appendix B) establishes two key claims: 1) that when $K \approx 0$, there is no information in the price: $C_t = 0$ and 2) that the marginal rate of substitution of demand information for fundamental information is proportional to $(C_t/D_t)^2$. Thus, when the price contains no information about future dividends ($C_t = 0$), then analyzing demand is has no marginal value $(C_t/D_t)^2 = 0$. Demand data is only valuable in conjunction with the current price p_t because it allows one to extract more information from price. Demand data trading when $K_t = 0$ is like removing noise from a signal that has no information content. Put differently, when there is no fundamental information, the price perfectly reveals hedging trades. There is no need to process data on hedging if it can be perfectly inferred from the price.

This results explains why analysts focus on fundamentals when financial analysis productivity is low. In contrast, when prices are highly informative, demand information is like gold because it allows one to identify exactly the price fluctuations that are not informative and are therefore profitable to trade on. The next results explain why demand analysis increases with productivity growth and why it may eventually start to crowd out fundamental analysis.

As financial technology grows, demand analysis takes off. The concern with the deleterious effects of financial technology on market efficiency stemmed from the concern that technology will deter the research and discovery of new fundamental information. This concern is not unwarranted. Not only does more fundamental information encourage extraction of information from demand, but once demand analysis starts, it feeds on itself.

The next result shows that, as long as price information is low or demand analysis is not too large, both types of analysis increase the ratio of the information content C to the noise D. This increases the marginal value of demand information, relative to fundamental information. Thus, fundamental analysis complements demand information and demand information complements itself.

Result 2 ***Complementarity in demand analysis:***
If $\Omega_{xt} < \tau_0 + \Omega_{ft}$ *and either*

1. C_t/D_t *is smaller in absolute value than* $(2\,Var[p_{t+1} + \tilde{d}_t|\mathcal{I}_t])^{-1}$*, or*
2. $Var[p_{t+1} + \tilde{d}_t|\mathcal{I}_t] < \sqrt{3}$

then $\frac{\partial (C_t/D_t)^2}{\partial \Omega_{ft}} > 0$ *and* $\frac{\partial (C_t/D_t)^2}{\partial \Omega_{xt}} \geq 0$.

Unlike fundamental analysis, the rise in demand analysis can increase the value of further demand analysis. For fundamental information, the increase in $|C_t/D_t|$ makes additional fundamental information less valuable. This result resembles the strategic substitutability in information identified by Grossman and Stiglitz (1980), in a model with a different information structure. But for demand information, the effect is the opposite. More precise average demand information (higher Ω_{xt}) can increase $(C_t/D_t)^2$, which is the marginal rate of substitution of demand information for fundamental information. The rise in the relative value of demand data is what makes investors shift data analysis from fundamental to demand when others do more demand analysis. That is complementarity.[7]

Complementarity comes from a rise in the price signal-to-noise ratio. From (10), we know that C_t is proportional to $1 - \tau_0 Var[\tilde{y}_t|\mathcal{I}_t]$. As either type of information precision (Ω_{ft} or Ω_{xt}) improves, the uncertainty about next period's dividend innovation $Var[\tilde{y}_t|\mathcal{I}_t]$ declines, and C_t increases. D_t is the coefficient on noise $\tilde{x}_t$. The price impact of uninformative trades $|D_t|$ may also increase with information, as we explain below. But conditions (1) and (2) guarantee that $|D_t|$ does not rise at a rate faster than C_t so that the ratio $C_t/|D_t|$, which is the signal-to-noise ratio of prices, and the marginal value of demand precision, increases with more information.

Intuitively, higher signal-to-noise (more informative) prices encourage demand trading because the value of demand analysis comes from the ability to better extract the signal from prices. In this model (as in most information processing problems), it is easier to clear up relatively clear signals than very noisy ones. So the aggregate level of demand analysis improves the signal clarity of prices, which makes demand analysis more valuable.[8]

2.2 Market Efficiency and Future Information Risk

To understand how the value of information changes, we consider marginal changes in fundamental and demand analysis. To do so, we endow investors with fixed amount of data which they cannot change, and let them do the portfolio choice. Then we consider exogenous changes in the data endowment.

[7] With a linear information constraint, or a simple cost function for K_t, the same intuition holds. With linearity, there is a secular shift to demand information acquisition once $\frac{C_t}{D_t}$ falls below -1. After that, the equilibrium level of the two types of information will be such that investors remain indifferent.

[8] When we consider a marginal change in analysis choice in the infinite future (a change in the steady state), the results are similar, but with more complex necessary conditions.

Since the investors cannot change the information allocation decision, this exercise does not directly map to our full model. However, it allows us to isolate the forces involved in the spillover from information allocation to the asset market equilibrium.[9]

We begin by exploring the effect on each price coefficient (C_t, D_t) separately. Then, we turn to the question of how analysis affects the ratio $(C/D)^2$, which governs the marginal rate of substitution between demand and fundamental analysis. Taken together, these results paint a picture of technological progress having mixed effects on market efficiency. The proofs are in Appendix B.

Result 3 ***Both fundamental and demand analysis increase price informativeness.*** *If $r - G > 0$ and $(\tau_x + \Omega_{xt})$ is sufficiently small, then $\partial C_t/\partial \Omega_{ft} > 0$ and $\partial C_t/\partial \Omega_{xt} > 0$.*

The more information investors have, the more information is reflected in the risky asset price. While the idea that dividend (fundamental) information improves price informativeness is unsurprising, the question of whether demand speculation improves or reduces price informativeness is not obvious. It turns out that they increase the information content because by selling the asset when the price is high for non-fundamental reasons and buying when the price is erroneously low, they make it easier to extract information from prices. Better informed traders who learn both from independent signals and from prices, therefore have better information, take more aggressive positions which in turn, cause the price to reveal even more information.

Liquidity here is the impact a non-informational trade has on price. A liquid market is one where one can buy or sell large quantities, in a way that is not correlated with dividends, without moving price by much. The next two results together show that information today and information tomorrow have opposite effects on today's liquidity. These opposite results are why it was important to use a dynamic model to think about the long run effects on increasing information technology.

Result 4 ***If demand is not too volatile, then both fundamental and demand analysis improve concurrent liquidity.*** *If $\tau_x > \rho r/(r - G)$, then $\partial D_t/\partial \Omega_{ft} > 0$ and $\partial D_t/\partial \Omega_{xt} > 0$.*

The contemporaneous effect is that both types of analysis can increase liquidity. The rationale is that both types of traders trade against non-informational trades and mitigate their price impact. Demand data investors profit by identifying and trading against non-informational trades. Non-informational trades that are clearly identifiable, will find eager counterparties, and will have little price impact. Fundamental traders buy when the price is low, relative to their fundamental information. This is exactly the same states where hedgers are selling. By taking the other side of the hedging trade, both types of traders mitigate hedgers' price impact. Lower price impact is higher liquidity.

Why would this result be reversed if demand was volatile (τ_x low)? A low τ_x means that prices are very noisy. When information improves, noise trades can be mis-attributed to agents having fundamental information. This mis-attribution causes prices to move more. In other words, the presence of informed traders makes others more hesitant to trade against hedging trades, increasing their price impact. Both components of this contemporaneous effect are present in static models as well.

[9] Consider a step cost function for each type of information processing, with jumps at equilibrium values of Ω_f and Ω_x for fundamental and demand analysis, respectively. The marginal cost of information acquisition is zero below the jump threshold and infinite above. This analysis is equivalent to marginal changes in the jump threshold of the cost function.

Another way of understanding liquidity is to think about it as a change in the quantity of risk per share. More information of either type today makes the dividend less risky – lower conditional variance – and helps to forecast tomorrow's price. If one share of the asset involves bearing little risk, then market investors don't need much price concession to induce them to hold a little extra risk. When one share is riskier, then inducing the market to buy one more share requires them to take on lots of risk, which requires a large price concession. This effect shows up in (11), the formula for D_t, which depends negatively on $Var[p_{t+1} + \tilde{d}_t|\mathcal{I}_t]^{-1}$, the variance of the asset payoff. Assets with more uncertain payoffs have more negative D_t, which means selling or buying a share has more price impact. This risk-based interpretation helps explain the next result about how future information affects today's liquidity.

Result 5 ***More future information reduces liquidity today.*** *If $|C_{t+1}/D_{t+1}|$ is sufficiently large, then $\partial D_t/\partial Var[\tilde{y}_{t+1}|\mathcal{I}_{(t+1)}] < 0$.*

The reason that future information can reduce liquidity is because it makes future price p_{t+1} more sensitive to future information and thus harder to forecast today. If tomorrow, many investors will trade on precise $(t+1)$ information, then tomorrow's price will be very sensitive to tomorrow's dividend information y_{t+1} and tomorrow's demand information x_{t+1}. In other words, both C_{t+1} and D_{t+1} will be high. But investors today do not know what will be learned tomorrow. Therefore, tomorrow's analysis makes tomorrow's price (p_{t+1}) more sensitive to shocks that today's investors are uninformed about. Because tomorrow's price is a component of the payoff to the asset purchased at date t, today's investors face high asset payoff risk $(Var[p_{t+1} + \tilde{d}_t|\mathcal{I}_t])$. This is what we call *future information risk*. Invoking the logic above, a riskier asset has a less liquid market. We can see this relationship in the formula for D_t (eq 11) where $Var[p_{t+1} + \tilde{d}_t|\mathcal{I}_t]$ shows up in the first term. Thus, future information reduces today's liquidity.

At this point, the assumption that assets are long-lived becomes essential. In a repeated static model, payoffs are exogenous. Without dynamics, information learned tomorrow cannot affect payoff risk today. Thus, the contribution of using a long-lived asset model to think about information choice is all the results that depend on future information risk.

We can see the relationship between tomorrow's price coefficients and future information risk in the formula for the variance of the asset payoff:

$$Var[p_{t+1} + \tilde{d}_t|\mathcal{I}_t] = C_{t+1}^2\tau_0^{-1} + D_{t+1}^2\tau_x^{-1} + (1+B)^2 Var[\tilde{y}_t|\mathcal{I}_t] \tag{17}$$

We know that time-t information increases period-t information content C_t. Similarly, time $t+1$ information increases C_{t+1}. Future information may increase or decrease D_{t+1}. But as long as C_{t+1}/D_{t+1} is large enough, the net effect of $t+1$ information is to increase $C_{t+1}^2\tau_0^{-1} + D_{t+1}^2\tau_x^{-1}$. Since future information cannot affect today's dividend uncertainty $Var[\tilde{y}_t|\mathcal{I}_t]$, the net effect of future information is to raise today's payoff variance. What this means economically is that tomorrow's prices will be more responsive to tomorrow's fundamental and demand shocks. That is what makes the price more uncertain today.

In our dynamic model, information improves today and improves again tomorrow. That means the static effect and dynamic effect are competing.[10] The net effect of the two is sometimes positive, sometimes negative. But it is never as clear-cut as what a static information model would suggest. What we learn is

[10]This variance argument is similar to part of an argument made for information complementarity in Cai (2016), an information choice model with only fundamental information.

that information technology efficiency and liquidity are not synonymous. If fact, because it makes prices more informative, financial technology can also make markets function in a less liquid way.

2.3 Analysis and Price in the Long-Run

The result that demand analysis feeds on itself suggests that in the long run, demand analysis will crowd out fundamental analysis. But that does not happen. When demand precision (Ω_{xt}) is high, the necessary conditions for Proposition 2 break down. The next result tells us that, in the long run as information becomes abundant, growth in fundamental and demand analysis becomes balanced. For this result, the long-lived asset assumption is crucial.

Result 6 High-Information Limit *As $K_t \to \infty$, both analysis choices Ω_{ft} and Ω_{xt} tend to ∞ such that*

(a) Ω_{ft}/Ω_{xt} does not converge to 0;

(b) Ω_{ft}/Ω_{xt} does not converge to ∞; and

(c) if τ_0 is sufficiently large, there exists an equilibrium where Ω_{ft}/Ω_{xt} converges to finite, positive constant.

See Appendix B for the proof and an expression (90) for the lower bound on τ_0.

It is not surprising that fundamental analysis will not out-strip demand analysis (part (a)). We know that more fundamental analysis lowers the value of additional fundamental analysis and raises the value of demand analysis. This is the force that prompts demand analysis to explode at lower levels of information K.

But what force restrains the growth of demand analysis? It's the same force that keeps liquidity in check: information today, competing with the risk of future information that will be learned tomorrow. The first order condition tells us that the ratio of fundamental and demand analysis is proportional to the squared signal-to-noise ratio, $(C_t/D_t)^2$. If this ratio converges to a constant, the two types of analysis remain in fixed proportion. Recall from Result 5 that information acquired tomorrow reduces D_t. That is, D_t becomes more negative, but larger in absolute value. As data observed today becomes more abundant, price informativeness (C_t) grows and liquidity improves – D_t falls in absolute value. As data processing grows, the upward force of current information and downward force of future information bring $(C_t/D_t)^2$ to rest, as a constant, finite limit. In the Appendix, Lemma 4 explores this limit. It shows formally that $(C_t/D_t)^2$ is bounded above by the inverse of future information risk. When assets are not long-lived, their payoffs are exogenous, future information risk is zero, and $(C_t/D_t)^2$ can grow without bound. Without a long-lived asset, the limit on $(C_t/D_t)^2$ is infinite. The growth path would not be balanced.

2.4 Persistent Demand or Information about Future Events

A key to many of our results is that the growth of financial technology creates more and more future information risk. This is the risk that arises because shocks that affect tomorrow's prices are not learnable today. This raises the question: What if information about future dividend or demand shocks were available today? Similarly, what if demand shocks were persistent so that demand signals today had future relevance? Would future information processing still increase risk?

Yes, as long as there is still some uncertainty and thus something to be learned in the future, future information will still create risk for returns today. Tomorrow's price would depend on the new information, learned tomorrow about shocks that will materialize in $t+2$ or $t+3$. That new information observed in $t+1$ will affect $t+1$ prices. That new future information, only released in $t+1$ cannot be known at time t. This future information becomes a new source of unlearnable risk. The general point is this: New information is constantly arriving; it creates risk. The risk is that before the information arrives, one does not know it and can not know it, no matter how much analysis is done. And whether it is about tomorrow, the next day or the far future, this information, yet to arrive, will affect future prices in an uncertain way. When information processing technology is poor, the poorly-processed information has little price effect. Thus future information poses little risk. When information processing improves, the risk of unknown future information grows.

Of course, if demand were persistent, then signals about $\tilde{x}_t$ would be payoff relevant. The $\tilde{x}_t$ signal would be informative about $\tilde{x}_{t+1}$, which affects the price p_{t+1} and thus the payoff of a time t risky asset. Learning directly about asset future asset payoffs is fundamentally different than learning about demand shocks that only affect the interpretation of the current price. In such a model, agents would attempt to distinguish the persistent and transitory components of demand. The persistent, payoff-relevant component would play the role of dividend information in this model. The transitory component of demand would play the role of the i.i.d. $\tilde{x}_t$ shock in this setting.

3 Consequences of the Shift in Data Processing: Market Efficiency

Our main results revealed that low-tech investors process fundamental data. As financial technology develops, demand data analysis takes off and feeds on itself; and eventually, with advanced technology, both types of data processing grow proportionately. These results raise auxiliary questions: How does this trend affect financial market outcomes? Data processing is not directly observable. What testable predictions are consistent with this theory? Since equilibrium effects inevitably involve multiple forces moving in opposite directions, it is useful to quantify the model, in order to have some understanding of which effect is likely to dominate.

The equilibrium effects we focus on are price informativeness and liquidity. A common concern is that, as financial technology improves, the extraction of information from demand will crowd out original research, and in so doing, will reduce the informativeness of market prices. On the flip side, if technology allows investors to identify uninformed trades and take the other side of those trades, such activity is thought to improve market liquidity. Finally, some argue that if data is much more abundant, then risk and risk premia must fall an price volatility must rise. Since we have not observed a large decline in the risk premium, the financial sector must not be processing data or using it in the way we describe. While each argument has some grain of truth, countervailing equilibrium effects mean that none of these conjectures is correct.

We begin by revisiting the forces that make demand information more valuable over time, this time, assigning a magnitude to the effect. Then, we explore why the change from information production to extraction does not harm price informativeness. Next, we use our numerical model to tease out the reasons for stagnating market liquidity, despite a surge in activity that looks like liquidity provision. Finally, we ask whether the model contradicts the long-run trends in equity premia and price volatility and explore

the possibility of biased technological change.

3.1 Calibration

Our calibration strategy is to estimate our equilibrium price equation on recent asset price and dividend data. By choosing model parameters that match the pricing coefficients, we ensure that we have the right average price, average dividend, volatility and dividend-price covariance at the simulation end point. What we do not calibrate to is the evolution of these moments over time. The time path of price and price coefficients are over-identifying moments that we can use to evaluate model performance.

First, we describe the data used for model calibration. Next, we describe moments of the data and model that we match to identify model parameters. Most of these moments comes from estimating a version of our price equation (7) and choosing parameters to match the price coefficients in the model with the data. In the next section, we report the results.

Data We use two datasets that both come from CRSP. The first is the standard S&P 500 market capitalization index based on the US stock market's 500 largest companies.[11] The dataset consists of: the value-weighted price level of the index p_t, and the value-weighted return $(p_t + d_t)/p_{t-1}$, where d_t is dividends. Both are reported at a monthly frequency for the period 2000-2015.

Given returns and prices, we impute dividends per share as

$$d_t = \left(\frac{p_t + d_t}{p_{t-1}} - \frac{p_t}{p_{t-1}} \right) p_{t-1}.$$

Both the price series and the dividend series are seasonally adjusted and exponentially detrended. As prices are given in index form, they must be scaled to dividends in a meaningful way. The annualized dividend per share is computed for each series by summing dividends in 12 month windows. Then, in the same 12-month window, prices are adjusted to match this yearly dividend-price ratio.

Finally, because the price variable described above is really an index, and this index is an average of prices, the volatility of the average will likely underestimate the true volatility of representative stock prices. In order to find an estimate for price volatility at the asset level, we construct a monthly time series of the average S&P constituent stock price for the period 2000-2015. Compustat gives us the S&P constituent tickets. From CRSP, we extract each company's stock price for each month.

Moments Using the price data and implied dividend series, we estimate the dividend AR(1) process (2) and the linear price equation (7). We let $\tilde{y}_t$ and Dx_t be regression residuals. We estimate $A = 16.03$, $C = 7.865$ and $D = -5.7$. We can then map these estimates into the underlying model parameters G, τ_x^{-1}, τ_0^{-1}, μ and χ_x, using the model solutions (8), (9), (10) and (11), as well as

$$Var[p_t] = (C_t^2 + \frac{B^2}{1 - G^2})\tau_0^{-1} + D_t^2 \tau_x^{-1}.$$

[11] As a robustness check, we redo the calibration using a broader index: a composite of the NYSE, AMEX and Nasdaq. This is a market capitalization index based on a larger cross-section of the market - consisting of over 8000 companies (as of 2015). The results are similar. Moment estimates are within about 20% of each other. This is close enough that the simulations differ imperceptibly. Results are available upon request.

Of course, in the model, A_t, C_t and D_t take on different values at different dates t. So we need to choose a theoretical date t at which to calibrate. Since our model requires solution by backwards induction, we choose the last date T. Given $t+1$ parameters, we can solve the model and find t parameters. Therefore, we use the empirical price coefficient estimates to tell us the model coefficients at the end of our simulation, denoted A_T, C_T and D_T. We use steady state solutions of the model to map these estimated coefficients back into model parameters.[12] Note that B is constant because it is a simple function of fixed parameters.

Table 1: Parameters

G	μ	τ_0^{-1}	τ_x^{-1}	χ_x	r	ρ
0.937	0.415	0.245	0.551	0.686	1.03	0.10

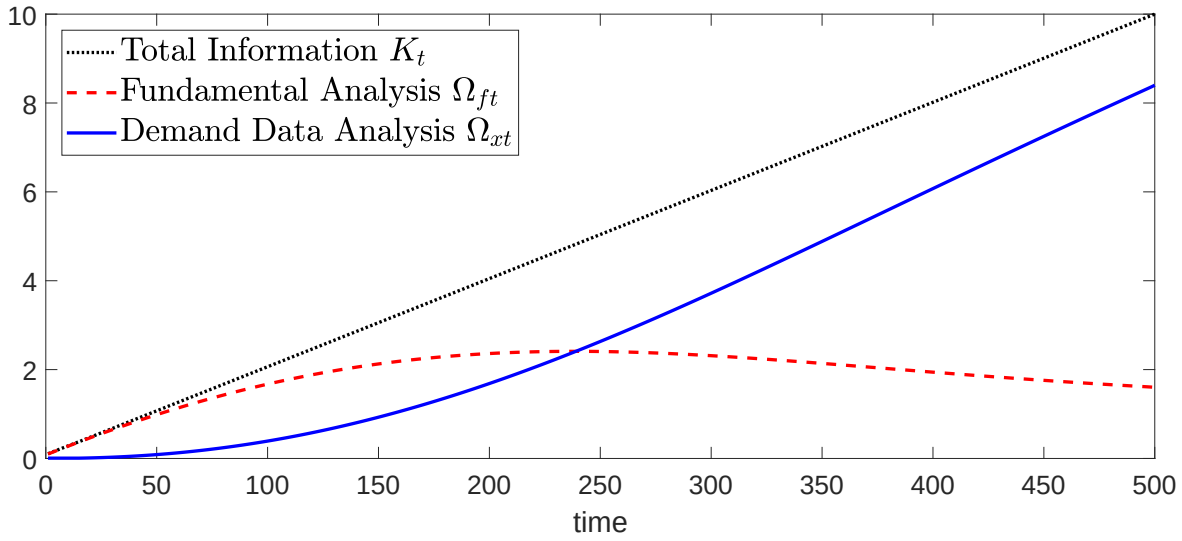
The first five parameters in Table 1 are calibrated to match the model and data values of the five equations above. This is an exactly identified system. The riskless rate is set to match a 3% net return. The last parameter is risk aversion. Risk aversion clearly matters for the level of the risky asset price. But it is tough to identify. The reason for the difficulty is that if we change risk aversion, and then re-calibrate the mean, persistence and variance parameters to match price coefficients and variance at the new risk aversion level, the predictions of the model are remarkably stable. Roughly, doubling variance and halving risk aversion mostly just redefines units of risk. Therefore, we use the risk aversion $\rho = 0.10$ in what follows and explore other values to show that the results do not depend on this choice. This ρ implies a relative risk aversion that is 0.65, not particularly high. In the appendix, we show an example of an alternative parameterization with even lower risk aversion, show how the other parameters change, and show that it yields similar results. We explore variations in other parameters as well.

Computation The one thing that changes at each date is the total information capacity K_t. We start the routine with $K_T = 10$.[13] In each period prior to that, we reduce K_t by 0.02. So if the last period is denoted T, then $K_{T-1} = 9.99$ and $K_{T-2} = 9.98$. We simulate the model in this fashion for 500 periods.

We solve the model by choosing a final date T and using our estimated price function parameters to initialize the backwards induction algorithm. We use the A_T, B_T, C_T and D_T from the data and our calibrated parameters to solve backwards for A_t, B_t, C_t and D_t, $t = T-1, \cdots, 1$. Knowing time-t price coefficients, we can solve for optimal information choices Ω_{ft} and Ω_{xt}. Then, we use the time-t solutions and our model solution to get $t-1$ information choices and price coefficients, and so forth. At each date, we are using a function minimization routine that finds the zeros of a non-linear equation in $\frac{C_t}{D_t}$.

Multiple Equilibria The non-linear equation in $\frac{C_t}{D_t}$ that characterizes the solution can have multiple solutions. It turns out, that for the parameter values we explore, this equation has only one real root.

Figure 1: Evolution of fundamental analysis and demand analysis. What is driving the change over time is an increase in total information processing K. Fundamental information is the choice variable Ω_{ft}, scaled by fundamental variance τ_0^{-1}. Demand information is the choice variable Ω_{xt}, scaled by non-fundamental demand variance τ_x^{-1}.



3.2 Quantifying the Transition from Fundamental to Demand Analysis

Figure 1 shows that demand analysis is scarce initially. Consistent with Result 1, we see that when information processing ability is limited, almost all of that ability is allocated to processing fundamental information. But once fundamental information is sufficiently abundant, demand analysis takes off. Not only does demand processing surge, but it increases by so much that, the amount of fundamental information declines, even though the total ability to process information has improved. Once it takes off, demand trading quickly comes to dominate fundamentals-based trading.

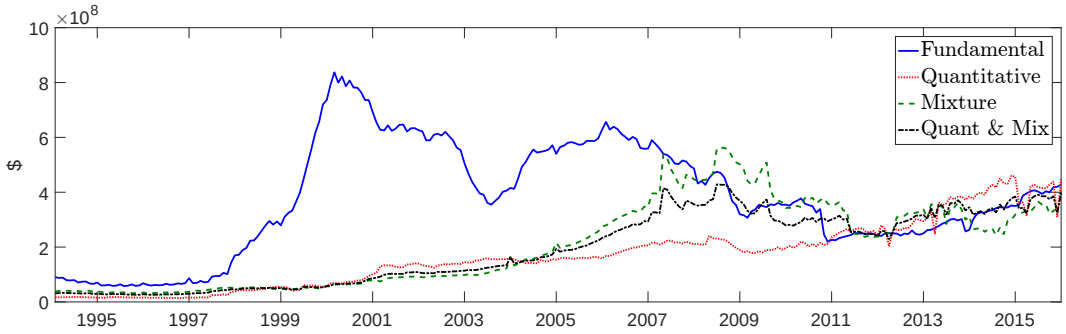
Exploring alternative parameter values reveals that this result is quite robust. Ω_{xt} consistently surpasses Ω_{ft} once C_t/D_t crosses $\sqrt{\chi_x}$. There are parameters for which C_t/D_t never exceeds $\sqrt{\chi_x}$, but even in those cases, Ω_{xt} increases faster, while Ω_{ft} is concave. Thus, over time, the growth of fundamental analysis is slowing down.

Related trends in data The shift from fundamental to demand analysis in our model should show up empirically as a change in investment strategies. Indeed, there is some evidence that funds have shifted their strategy over time, in a way that is plausibly consistent with our predictions. In the TASS database, many hedge funds report that their fund has a "fundamental", "mixture," or "quantitative" strategy. Figure 2 illustrates the evolutions of assets under management, by fund, and in total, for these different styles of funds. While other trends are also apparent, one clear trend is that fundamental analysis is waning in recent years, in favor of strategies based on market data. This shift in reported style suggests a

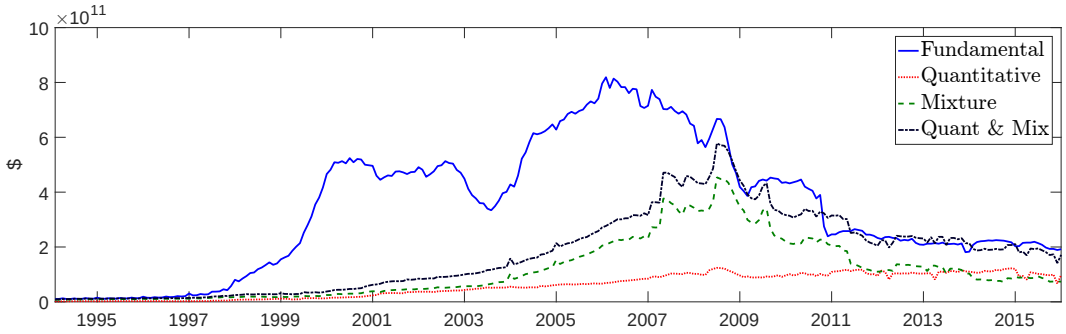
[12]Steady state solutions means solutions to a model where we believe that forever after that information would remain constant $K_{t+1} = K_t$ and price would have stable coefficients, $A_{t+1} = A_t$, $C_{t+1} = C_t$ and $D_{t+1} = D_t$.

[13]We checked the robustness of alternative K_T values and found that it makes no difference to our conclusions. For example, when we used $K_T = 5$, we found that the results look as if we'd simulated the results with $K_T = 10$ and truncated the time series plot where K_t reaches 5. The other calibrated parameters are identical when we vary K, except for χ_x, which moves approximately proportionately with K_T. For example, for $K_t = 5$, χ_x falls by about one-half, from 0.68 to 0.31.

Figure 2: **Hedge Funds are Shifting Away from Fundamental Analysis.**
Source: Lipper TASS. Data is monthly from 1994-2015. Database reports on 17,534 live and defunct funds.



(a) Assets Under Management per Fund



(b) Total Assets Under management

transformation in the way information technologies are used in finance.

Another quite different indicator that points to the growing importance of demand data comes from the frequency of web searches. Google trends reports the frequency of searches that involve specific search terms. Figure 3 shows that from 2004 to 2016, the frequency of searches for information about "order flow" has risen roughly 3-fold. This is not an overall increase in attention to asset market information. In contrast, the frequency of searches for information about "fundamental analysis" fell by about one-half over the same time period.

The demand data analysis in the model resembles a combination of current statistical arbitrage, retail market making, and order flow trading, all strategies we see surging in practice. Much of the trade against order flow takes the form of algorithmic trading. This happens for a couple of reasons. First, while firm fundamentals are slow-moving, demand can reverse rapidly. Therefore, mechanisms that allow traders to trade quickly are more valuable for fast-moving demand based strategies. Second, while fundamental information is more likely to be textual, partly qualitative, and varied in nature, demand is more consistently data-oriented and therefore more amenable to algorithmic analysis. Hendershott, Jones, and Menkveld (2011) measure algorithmic trading and find that it has increased, but it increased most rapidly during the period between the start of 2001 and the end of 2005. During this six-year window, average trade size fell and algorithmic trading increased, about seven-fold (Figure 4). This rapid transition is another feature of the data our model can explain.

Figure 3: **Google trends:** Fraction of Google searches involving "order flow" or "fundamental analysis." Source: Google trends. Data is the weekly fraction of searches involving these search terms. Series is normalized to make the highest data point equal to 100.

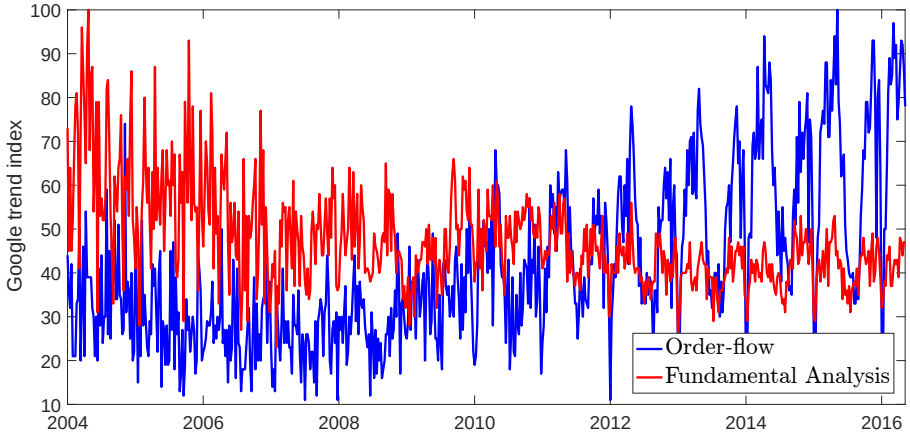
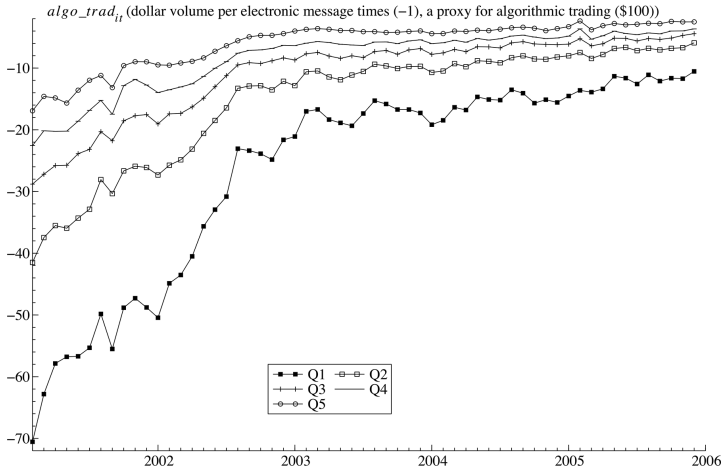


Figure 4: **Algorithmic Trading Growth** 2001-2006. Source: Hendershott, Jones, and Menkveld (2011). Their proxy for algorithmic trading is the dollar volume of trade per electronic message. The rise is more pronounced for largest market cap (Q1) stocks. Q1-Q5 are the 5 quintiles of NYSE stocks, ordered by size (market capitalization).



3.3 Price Informativeness

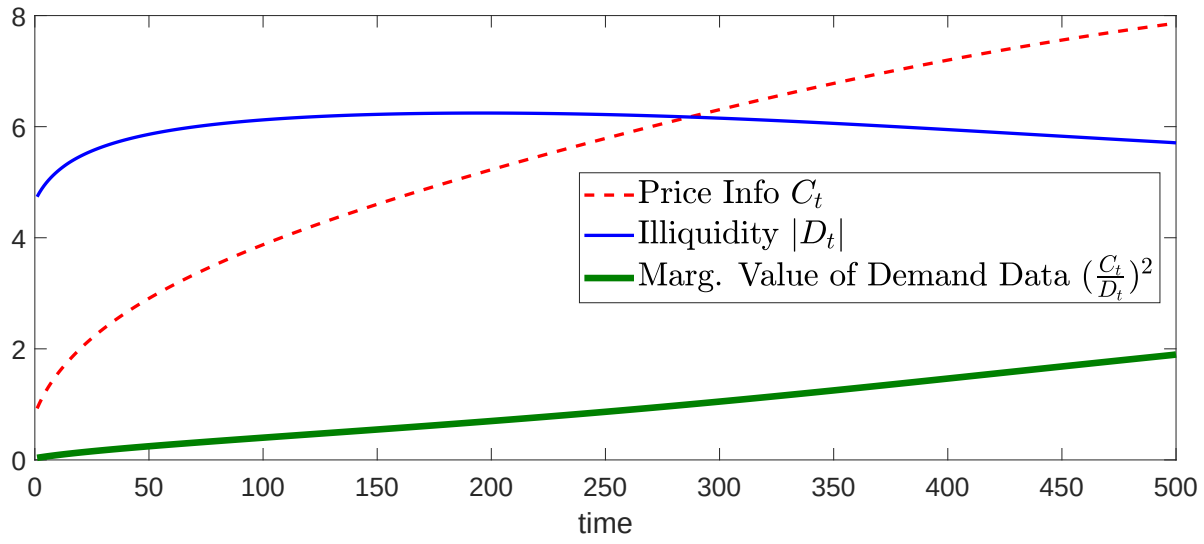
Price informativeness measures financial market efficiency in the sense that efficient prices aggregate all the information known to market participants about future firm fundamentals. Informative prices are important because they can inform firm managers' investment decisions and make equity compensation a useful incentive tool by aligning firm value and equity compensation. Finally, informative prices allocate new capital to the most productive firms.

Prices are informative if a change in future dividends is reflected in the price. Our equilibrium price solution (7) reveals that this marginal price impact $dp_t/d\tilde{y}_t$ is C_t. As the productivity of financial analysis rises, and more information is acquired and processed, the informativeness of the price (C_t) rises. Both fundamental analysis and demand analysis have the same objective, to help investors better discern the true value of the asset. Thus both raise price informativeness.

The solid line labeled C_t in Figure 5 confirms that as financial analysis becomes more productive,

informativeness rises. The effect of a one-unit change in the dividend innovation, which is about 2 standard deviations, increases the price by between 0 and 8 units. Since the average price level is about 80, this 2 standard deviation shock to dividends produces a negligible price change for very low levels of technology and a 10% price rise when financial technology becomes more advanced.

Figure 5: **Price Informativeness (C_t) Rises and Price Impact of Trades ($|D_t|$) Stagnates.** C_t is the impact of future dividend innovations on price. ($|D_t|$) is the price impact of a one-unit uninformed trade. This illiquidity measure is flat, despite the rise of market-making (demand driven) trades. $(C_t/D_t)^2$ tells us the marginal value of demand information, relative to fundamental information. The x-axis is time.



Related trends in data Bai, Philippon, and Savov (2016) measure a long-run rise in equity price informativeness. They measure price informativeness using a coefficient from a regression of future earnings (at 1-year, 3-year and 5-year horizons) on the current ratio of market value to book value. Over the period 1960-2010, they find a 60% rise in three-year price informativeness and an 80% rise in five year price informativeness, both of which are highly statistically significant.

Our claim is not that our model explains all of this phenomenon, or that we can match the timing or magnitude of the increase. We only wish to suggest that our predictions are not at odds with other long-run trends in financial markets. This is a model with only one risky asset, with no frictions, no habits or low-frequency risks. It is deliberately kept simple, in order to explore the workings of a new mechanism governing long run shifts in trading strategies. In reality, there are many assets and many sectors, which each go through the transition from fundamental to demand research at different times. The rise in financial technology does not prompt more analysis of all assets at all times. Because of complementarity in demand analysis, more information drives up the price of one or a few assets, leaving others unstudied. Thus, the average rise in price informativeness is overstated by the 1-asset model.[14]

[14]Our framework could come closer to the data with multiple assets. What we cannot do with multiple assets is characterize the long run growth path that is central to this paper's results. Because of the complementarity in demand processing, many such growth paths may exist. Therefore, we stick with our transparent one-asset model, with clear predictions, at the expense of being able only to make qualitative comparisons with data.

3.4 Price Impact of Trades (Liquidity)

Market liquidity is an important object of study in finance (Hasbrouck, 2007). Liquidity is particularly important in the debate on financial technology because it is one of the most common arguments in defense of demand based trading strategies. The claim is that traders who identify uninformed demand and offer to take the other side of those orders provide market liquidity.

A common metric of market liquidity is the sensitivity of an asset's price to a buy or sell order. If a buy order causes a large increase in the asset price and conversely a sell order causes a large fall, then buying and selling this asset is costly. In such a market, trading strategies that require frequent or large trades would have a harder time generating a profit. In our model, price impact is the impact of a one-unit hedging trade ($dp_t/d(-\tilde{x}_t)$).[15] The linear price solution (7) reveals that price impact is $dp_t/d(-\tilde{x}_t) = -D_t$.

Looking at the dashed line in Figure 5, we see that the price impact of hedging trades, $-D_t$, rises in the early periods when only Ω_{ft} is increasing and then declines as information becomes more abundant. But what is striking about this result is that the changes are quite small. A hedging trade that is the size of 1% of all outstanding asset shares would increase the price by $0.05 - 0.06$ units. Since the average price is 80, this amounts to a $0.6\% - 0.7\%$ (60 - 70 basis point) increase in the price. Exploring different parameters, we see that the dynamics of market liquidity can vary. But what is consistent is that the changes are small compared to the change in price informativeness.

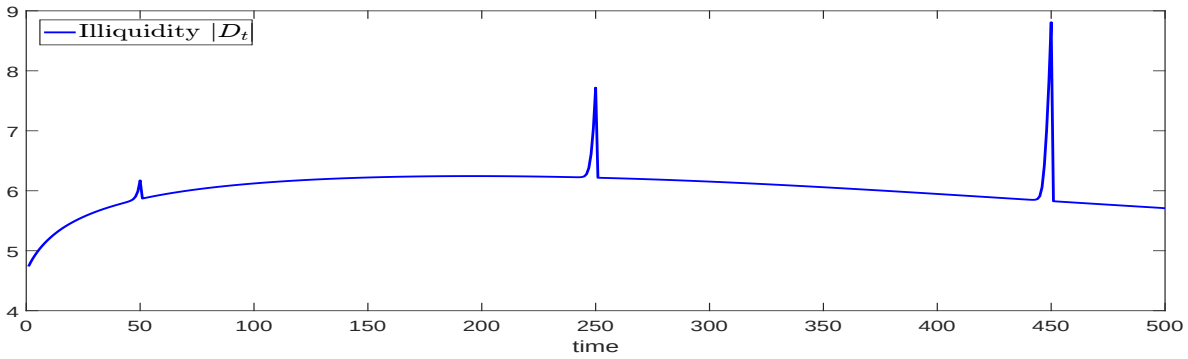
Flat liquidity is a result of two competing forces. Recall from Section 2 that the liquidity of a risky asset is determined by the riskiness (uncertainty) of its payoff. Purchases or sales of assets with more uncertain payoffs have larger price effects. Result 4 tells us that more information today reduces uncertainty about dividends $\tilde{d}_t$, which in turn reduces the price impact of non-fundamental trades, improving liquidity. But Result 5 tells us that if information technology is advanced tomorrow, then tomorrow's shocks will have a large effect on tomorrow's price, which makes today's payoff risky and today's liquidity low. The reason liquidity changes so little is that the static force $(r/(r-G))Var[\tilde{y}_t|\mathcal{I}_t](C_t/D_t)$ and the dynamic force $-\rho Var[p_{t+1} + \tilde{d}_t|\mathcal{I}_t]$ are nearly cancelling each other out.[16]

Related trends in data Many empirical researchers have found little in the way of long-run trends in market liquidity. Studying liquidity over the last century, Jones (2002) finds lots of cyclical variation, but little trend in bid-ask spreads. Recent work by Koijen and Yogo (2016) however, measures a large fall in the price impact of institutional traders. This may not be inconsistent with our results for two reasons. First, our liquidity measure is the price impact of a non-informational trade. That is not the same as the price impact of an institutional trader who will often be trading on information. Second, in many cases, the way institutional traders have reduced their price impact is to find uninformed demand to trade against. To the extent that reduced price impact reflects more market making and less direct trading on information, this reduced impact is consistent with our long-run demand analysis trend.

[15] We consider a hedging trade because the alternative is considering an information-based trade. The impact of an information-based trade would reflect the fundamental (future dividend) which must have moved to change the information. That question of how much a change in the fundamental changes price is one we already explored. That is price informativeness.

[16] A version of this effect can arise in a dynamic model with only fundamental analysis (see Cai (2016)).

Figure 6: **Liquidity Fragility Grows.** Figure plots $|D_t|$ where there are three one-time, zero-probability, unanticipated uncertainty shocks. Each shock is a change in expected precision $\tau_{0,t+1}$ to $1/2 \cdot \tau_0$, which is then not realized at $t+1$.



Fragile Liquidity Although liquidity remains mostly flat as technology improves, liquidity becomes more fragile, meaning that it is more sensitive to changes in model parameters. For example, suppose agents face a one-time increase in uncertainty. Specifically, investors find out that the variance of next period's dividend innovations $\tau_{0,t+1}^{-1}$ will be doubled, only for one period, and never again. The actual high variance shock is never realized, making this a pure belief shock. Figure 6 shows that when information technology is poor, uncertainty has little effect on liquidity. But when information technology is very productive, the same change in uncertainty results in a dramatic fall in market liquidity.

Liquidity is fragile in response to other shocks as well. A similar exercise where the cost of demand processing (χ_x) surges for one period produces similar outcomes. See appendix for detailed results.

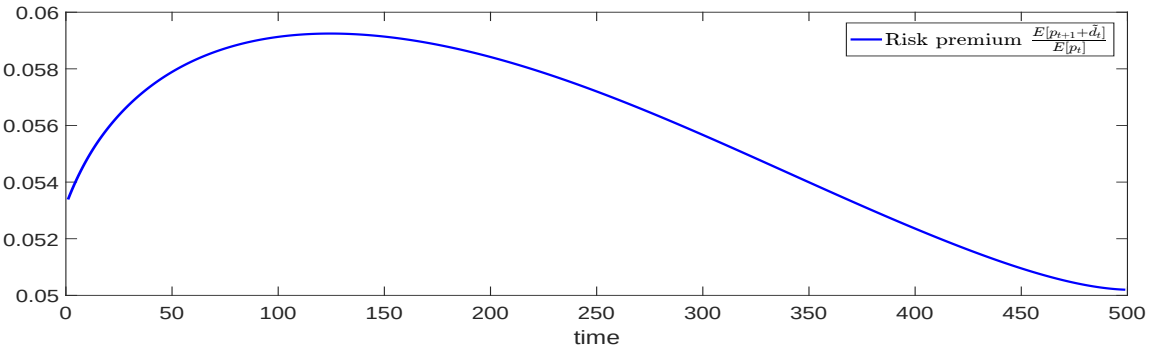
3.5 Trends in the Equity Premium and Return Volatility

Our focus is on how technological change affects trading strategies and market efficiency, not on asset price movements. But it is useful to know whether this mechanism is at odds with long-run trends in the equity premium and in price volatility, or not. The idea that information reduces risk, which lowers the return on risky assets is an old one. However, exploring the magnitude of that decline in our setting offers some insight about the magnitude of the information trend in the model. If the model's equity premium needed to fall by some outrageous amount, in order to see any effect on price informativeness or liquidity, it would diminish the relevance of our mechanism.

Instead, Figure 7 shows that the decline in the risk premium predicted by the model is quite modest. The risk premium falls from a maximum of around 6% to 5% by the end. Of course, replicating the level of the risk premium is not a success. That is nearly a by-product of calibrating the model to match the end-of-sample price regression coefficients in (7). This calibration approach implies that the model matches the price-dividend ratio, and by extension, comes close to matching the equity premium. However, the decline in the premium is related to the growth in information processing.

As for equity premium trends in data, Jones (2002) documents that the equity premium is 1% lower in the 2000's than it was in the early 1900's. Our results are also a similar magnitude to those of Lettau, Ludvigson, and Wachter (2008) who report that the price-dividend ratio rose from 3 to 4 in the late 20th century. They estimate a structural asset pricing model with regime switches in volatility and conclude that the equity premium shifted down by 1.5%. Taken together, these findings tell us that the amount

Figure 7: **Technological Progress Reduces the Risk Premium Modestly.** The risk premium is $1 + \frac{\bar{d}}{A+B\bar{d}} - r$, where $\bar{d} = \mu/(1-G)$ is the average dividend payment. The x-axis is time.



of information needed to explain the declining equity premium is consistent with the amount needed to explain growing price informativeness and flat liquidity. This is an over-identifying moment that lends support to our modeling and calibration approach.

When we examine the volatility of returns, we find no significant time trend, in either the model or the data. We simulate prices from the model, inflate them with a CPI price index, and then construct a return series that is a log difference in price $(ln(p_{t+1} + d_{t+1}) - ln(p_t)S)$. We then compare this model return to an empirical return series, derived from the monthly S& P 500 price index (1980-2015). For both model and data, we calculate a volatility for each month, using the relevant return series to estimate a GARCH model.[17] When we regress this GARCH-implied return variance σ_t on time (monthly), the average variance is 0.005, but the coefficient on time is zero out to 5 digits. Because of the large number of observations that we can simulate, this number is statistically different from zero. But its economic magnitude is trivial. Variance of the S& P 500 returns has a time trend that is 0.00001. That coefficient is statistically indistinguishable from zero. In short, return variance in both the model and the data seems to be stable over time.

3.6 Unbalanced Technological Change

We have modeled technological progress that increases the potential precision of fundamental or demand information equally. But it is quite possible that technological progress has not been balanced. The concern is that the productivity of demand analysis has grown faster than fundamental analysis, because fundamental information tends to be more textual or qualitative. To explore this possibility, we take an extreme view of the imbalance and consider a world where the only efficiency growth is in demand data processing. The truth is likely somewhere between this unbalanced growth model and the balanced growth model we analyzed before.

When only demand data processing improves, a few things change (Figure 8). First, fundamental information analysis falls monotonically, rather than rising and then falling. This is simply because when demand analysis becomes more productive, it makes fundamental information processing strictly less attractive. Also, the marginal value of demand data $(C_t/|D_t|)$ is mostly flat. In contrast, with balanced

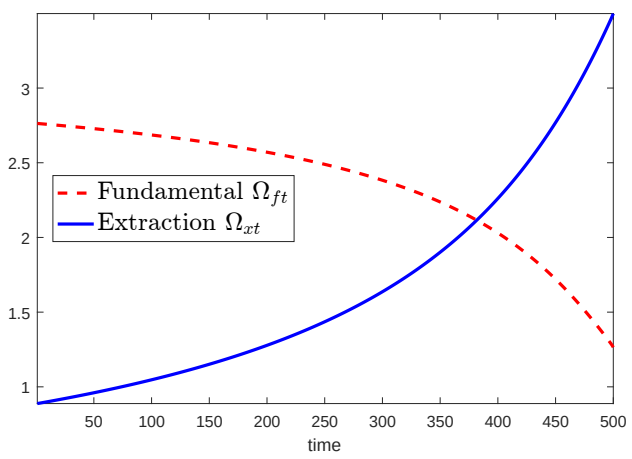
[17]The dividends are imputed, as described in the calibration section. The equation we estimate is a TGARCH(1,1), which takes the form, $\sigma^2_{t|t-1} = \omega + (\alpha + \gamma \cdot 1_{(r_{t-1}<0)})r^2_{t-1} + \beta\sigma^2_{t-1|t-2}$. It allows for positive and negative returns to affect volatility differently. We estimated the coefficients by maximum likelihood.

growth, it was steadily increasing. The trajectory of $C_t/|D_t|$ is flatter: While both types of information processing make prices clearer signals, fundamental information processing, which was more prevalent in the previous exercise, improved signal quality by more.

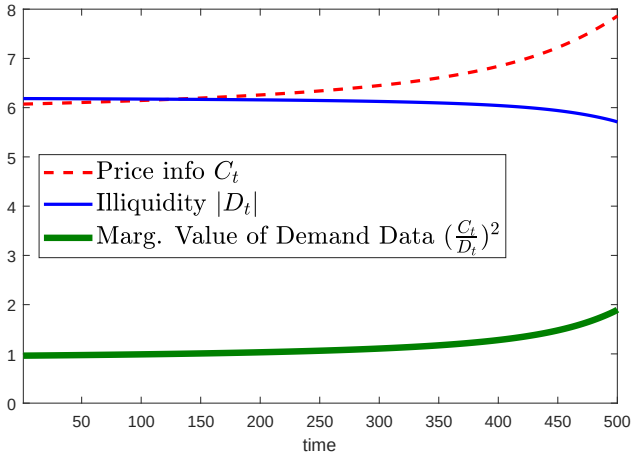
What is surprising is that $C_t/|D_t|$ does not fall. Even C_t alone does not fall. Even though the discovery of new information about future dividends Ω_{ft} falls precipitously, dividend information is still more heavily weighted (C_t) and more clearly reflected ($C_t/|D_t|$) in prices. Demand traders are adept at inferring what others know from prices. This inference makes them well-informed about $\tilde{y}_t$, albeit indirectly. If many traders have precise knowledge of demand, the average trader ends up being well-informed about $\tilde{y}_t$, even if less research on $\tilde{y}_t$ was done by the market. The net result of less research but more learning through prices is an increase in total information. This shows up in prices as a higher price impact C_t of changes in dividend innovations.

In short, our main conclusions are unaltered. Liquidity is still flat. Market efficiency does not plummet, by either measure, even through demand analysis crowds out fundamental analysis. The unbalanced change simply affects the rate at which market efficiency evolves.

Figure 8: **Unbalanced Technological Progress: χ_x falls.** Information choices (left) and market efficiency (right) with progress only in demand analysis. C_t is the impact of future dividend innovations on price. $(-D_t)$ is the price impact of a one-unit uninformed trade. $(C_t/D_t)^2$ tells us the marginal rate of transformation of demand and fundamental information. The x-axis is time. This version of the model reduces χ_x over time, without changing K.



(a) Information Acquisition



(b) Price Coefficients

4 Real Economic Effects

We have argued that the growth in financial technology has transformed the financial sector and affected financial market efficiency in unexpected ways. But why should we care about financial market efficiency? What are the consequences for real economic activity? There are many possible linkages between the financial and real sectors. In this section, we illustrate two possible channels through which changes in informativeness and price impact can alter the efficiency of real business investment.

4.1 Manager Incentive Effects

The key friction in the first spillover model is that the manager's effort choice is unobserved by equity investors. The manager exerts costly effort only because he is compensated with equity. The manager only has an incentive to exert effort if the value of his equity is responsive to his effort. Because of this, the efficiency of the manager's effort choice depends on asset price informativeness.

Of course, this friction reflects the fact that the wage is not an unconstrained optimal contract. The optimal compensation for the manager is to pay him for effort directly or make him hold all equity in the firm. We do not model the reasons why this contract is not feasible because it would distract from our main point. Our stylized sketch of a model is designed to show how commonly-used compensation contracts that tie wages to firm equity prices (e.g., options packages) also tie price informativeness to optimal effort.

Time is discrete and infinite. There is a single firm whose profits $\tilde{d}_t$ depend on a firm manager's labor choice l_t. Specifically, $\tilde{d}_t = g(l_t) + \tilde{y}_t$, where g is increasing and concave and $\tilde{y}_t \sim N(0, \tau_0^{-1})$ is unknown at t. Because effort is unobserved, the manager's pay w_t is tied to the equity price p_t of the firm: $w_t = \bar{w} + p_t$. However, effort is costly. We normalize the units of effort so that a unit of effort corresponds to a unit of utility cost. Insider trading laws prevent the manager from participating in the equity market. Thus the manager's objective is

$$U_m(l_t) = \bar{w} + p_t - l_t \tag{18}$$

The firm pays out all its profits as dividends each period to its shareholders. Firm equity purchased at time t is a claim to the present discounted stream of future profits $\{\tilde{d}_t, \tilde{d}_{t+1} \ldots\}$.

The preferences, endowments, budget constraint and information choice sets of investors are the same as before. Demand data signals are defined as before. Fundamental analysis now generates signals of the form $\eta_{fit} = g(l_t) + \tilde{y}_t + \tilde{\epsilon}_{fit}$, where the signal noise is $\tilde{\epsilon}_{fit} \sim N(0, \Omega_{ft})$. Investors choose the precision Ω_{ft} of this signal, as well as their demand signal Ω_{xt}. Equilibrium is defined as before, with the additional condition that the manager effort decision maximizes (18).

Solution As before, the asset market equilibrium has a linear equilibrium price:

$$p_t = A_t + C_t(g(l_t) + \tilde{y}_t) + D_t\tilde{x}_t \tag{19}$$

Notice that since dividends are not persistent, d_{t-1} is no longer relevant for the t price

The firm manager chooses his effort to maximize (18). The first order condition is $C_t g'(l_t) = 1$, which yields an equilibrium effort level $l_t = (g')^{-1}(1/C_t)$. Notice that the socially optimal level would set the marginal utility cost of effort equal to the marginal product $g'(l_t) = 1$. When C_t is below one, managers under-provide effort, relative to the social optimum because their stock compensation moves less than one-to-one with the true value of their firm.

Similar to before, the equilibrium level of price informativeness C is

$$C_t = \frac{1}{r}\left(1 - \tau_0 Var[g(l_t) + \tilde{y}_t|\mathcal{I}_t]\right). \tag{20}$$

Thus, as more information is analyzed, dividend uncertainty ($Var[g(l_t)+\tilde{y}_t|\mathcal{I}_t]$) falls, C_t rises and managers are better incentivized to exert optimal effort. While the model is stylized and the solution presented here

is only a sketch, it is designed to clarify why trends in financial analysis matter for the real economy.

The most obvious limitation of the model is its single asset. One might wonder whether the effect would disappear if the asset's return was largely determined by aggregate risk, which is out of the manager's control. However, if there were many assets, one would want to rewrite the compensation contract so that the manager gets rewarded for high firm-specific returns. This would look like benchmarked performance pay. If the contract focused on firm-specific performance, the resulting model would look similar to the single asset case here.

In short, this mechanism suggests that recent financial sector trends boost real economic efficiency. More data analysis – of either type – improves price informativeness, and thereby improves incentives. But this is only one possible mechanism, offering one possible conclusion. The next example offers an alternative line of thought.

4.2 Equity Issuance Cost

The second real spillover highlights a downside of financial technology growth. More information technology creates future information risk, which raises the risk of holding equity, raising the equity premium, and making capital more costly for firms. This enormously simplified mechanism is meant as a stand-in for a more nuanced relationship such as that in Bigio (2015).

Suppose that a firm has a profitable investment opportunity and wants to issue new equity to raise capital for that investment. For every dollar of capital invested, the firm can produce an infinite stream of dividends d_t. Dividends follow the same stochastic process as described in the original model. However, the firm needs funds to invest and raises those funds by issuing equity. The firm chooses a number of shares $\bar{x}$ to maximize the total revenue raised (maximize output). Each share sells at price p, which is determined by the investment market equilibrium, minus an investment or issuance cost:

$$E[\bar{x}p - c(\bar{x})|\mathcal{I}_f]$$

The firm makes its choice conditional on the same prior information that all the investors have. But does not condition on p. It does not take price as given. Rather, the firm chooses $\bar{x}$, taking into account its impact on the equilibrium price. The change in issuance is permanent and unanticipated. The rest of the model is the same as the dynamic model in section 1.

Solution Given the new asset supply $\bar{x}$, the asset market solution and information choice solution to the problem are the same as before. But how the firm chooses $\bar{x}$ depends on how new issuance affects the asset price. When the firm issues new equity, all asset market participants are aware that new shares are coming online. Equity issuance permanently changes the known supply of the asset $\bar{x}$. Supply $\bar{x}$ enters the asset price in only one place in the equilibrium pricing formula, through A_t. Recall from (8) that

$$A_t = \frac{1}{r}\left[A_{t+1} + \frac{r\mu}{r-G} - \rho Var[p_{t+1} + \tilde{d}_t|\mathcal{I}_t]\bar{x}\right]. \tag{21}$$

Taking A_{t+1} as given for the moment, $dA_t/d\bar{x} = -\rho Var[p_{t+1} + \tilde{d}_t|\mathcal{I}_t]/r$.[18] In other words, the impact of a

[18] In principle, a change in issuance $\bar{x}$ could change payoff variance, $Var[p_{t+1}+\tilde{d}_t|\mathcal{I}_t]$. However, in this setting, the conditional variance does not change because information choices do not change. Information does not change because the marginal rate

one-period change in asset supply depends on the conditional variance (the uncertainty about) the future asset payoff, $p_{t+1} + \tilde{d}_t$. Recall from the discussion of price impact of trades in Section 3.4 that in a dynamic model, more information analysis reduces dividend uncertainty but can result in more uncertainty about future prices. These two effects largely offset each other.

Figure 9: **Payoff Risk and The Cost of Raising Capital**. The left panel shows payoff risk, which is $Var[p_{t+1} + \tilde{d}_t | \mathcal{I}_t]$. The right panel shows the absolute price impact of a one-unit change in issuance, normalized by the average level of dividends.

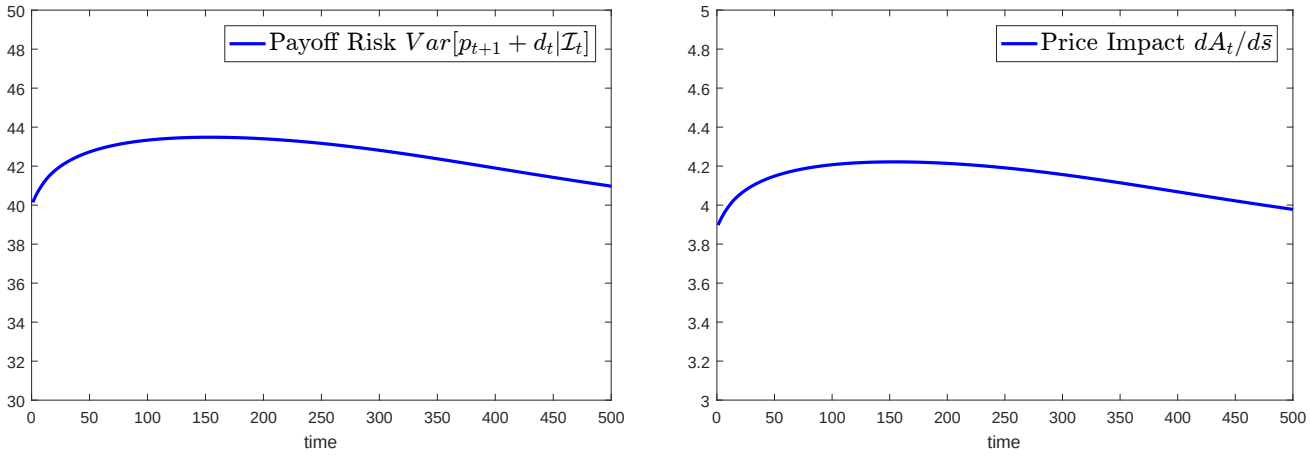


Figure 9 plots the modest increase and decrease in payoff risk from these competing effects on the price impact of issuing new equity. To give the units of the price impact some meaning, the issuance cost is scaled by the average dividend payment so that it can be interpreted as the change in the price-dividend ratio from a one-unit change in equity supply. Thus a one-unit increase in issuance reduces the asset price by an amount equal to 4 months of dividends, on average.

We learn that technological progress in information analysis – of either type – initially makes asset payoffs slightly more uncertain, which makes it more costly to issue new equity. When we now take into account that the increase in asset supply is permanent, the effect of issuance is amplified, relative to the one-period (fixed A_{t+1}) case. But when analysis becomes sufficiently productive, issuance costs decrease again, as the risk-reducing power of more precise information dominates.

Again, one key limitation of the model is its single asset. With multiple assets, one firm's issuance is a tiny change in the aggregate risk supply. But the change in the supply of firm-specific risk looks similar to this problem. If one were to evaluate this mechanism quantitatively, the magnitude would depend on how much the newly issued equity loads on idiosyncratic risk versus aggregate risk.

5 Conclusion

Technological progress is the driving force behind most models of long-run economic growth. Yet it is surprisingly absent in models of the financial economy. We explore the consequences of a simple deterministic increase in the productivity of information processing in the financial sector. While studies have documented an increase in price informativeness (Bai, Philippon, and Savov, 2016), we know of no theories

of transformation of fundamental and demand information depends on $(C_t/D_t)^2$, which is not dependent on $\bar{x}$. If there were multiple assets, issuance would affect information choices, as in Begenau, Farboodi, and Veldkamp (2017).

that explore the consequences of such changes for market equilibrium or efficiency.

We find that when the financial sector becomes more efficient at processing information, it changes the incentives to acquire information about future dividends (fundamentals) versus demand (non fundamental shocks to price). Thus a simple rise in information processing productivity can explain a transformation of financial analysis from a sector that primarily investigates the fundamental profitability of firms to a sector that does a little fundamental analysis but mostly concentrates on acquiring and processing client demand. This is consistent with suggestive evidence that the nature of financial analysis and associated trading strategies have changed.

Many feared that this technological transformation was harming market efficiency, while others argued that markets are more liquid/efficient than ever before. The concern was that the decline of fundamental analysis would compromise price informativeness. We do not find that to be the case. Although fundamental analysis declines, price informativeness continues to rise. The reason is that even if many traders are extracting others' information, this still makes the average trader better informed and the price more informative. But the benefits of the technological transformation may also be overstated. The promise that traders standing ready to take the other side of uninformed traders would improve market liquidity is only half the story. What this narrative misses is that more informed traders in the future make prices react more strongly to new information, which makes future asset values riskier. This increase in risk makes traders move market prices by more and pushes market liquidity back down. The net effect could go either way and is likely to be small.

Of course, there are many other features one might want to add to this model to speak to other related trends in financial markets. One might make fundamental changes more persistent than demand innovations so that different styles of trade were associated with different trading volumes. Another possibility is to explore regions in this model where the equilibrium does not exist and use the non-existence as the basis for a theory of market breakdowns or freezes. Another extension might ask where demand signals come from. In practice, people observe demand data because they intermediate trades. Thus, the value of the demand information might form the basis for a new theory of intermediation. In such a world, more trading might well generate more information for intermediaries and faster or stronger responses of markets to changes in market conditions. Finally, one might regard this theory as a prescriptive theory of optimal investment, compare it to investment practice, and compute expected losses from sub-optimal information and portfolio choices. For example, a common practice now is to blend fundamental and demand trading by first selecting good fundamental investment opportunities and then using demand information to time the trade. One could construct such a strategy in this model, compare it to the optimal blend of trading strategies, and see if the optimal strategy performs better on market data.

While this project with its one simple driving force leaves many questions unanswered, it also provides a tractable foundation on which to build, to continue exploring how and why asset markets are evolving, as financial technology improves.

References

Abis, S. (2017): "Market Timing in Bayesian Portfolio Optimization," Columbia GSB working paper.

Amador, M., and P.-O. Weill (2010): "Learning from prices: Public communication and welfare," *Journal of Political Economy*, forthcoming.

Angeletos, M., and J. La'O (2014): "Sentiments," *Econometrica*, 81(2), 739–779.

Asriyan, V., and V. Vanasco (2014): "Informed Intermediation over the Cycle," Stanford University Working Paper.

Atkeson, A., and R. Lucas (1992): "On Efficient Distribution with Private Information," *The Review of Economic Studies*, 59 (3), 427–453.

Babus, A., and C. Parlatore (2015): "Strategic Fragmented Markets," Working Paper New York University Stern school of Business.

Bai, J., T. Philippon, and A. Savov (2016): "Have Financial Markets Become More Informative?," *Journal of Financial Economics*, 122 35), 625–654.

Bali, T. G., S. J. Brown, and M. O. Caglayan (2014): "Macroeconomic Risk and Hedge Fund Returns," *Journal of Financial Economics*, 114(1), 1–19.

Banerjee, S., and B. Green (2015): "Signal or noise? Uncertainty and learning about whether other traders are informed," *Journal of Financial Economics*, 117 (2), 398–423.

Barlevy, G., and P. Veronesi (2000): "Information Acquisition in Financial Markets," *Review of Economic Studies*, 67, 79–90.

Basetto, M., and C. Galli (2017): "Is Inflation Default? The Role of Information in Debt Crises," Chicago Federal Reserve Bank working paper.

Begenau, J., M. Farboodi, and L. Veldkamp (2017): "Big Data in Finance and the Growth of Large Firms," work in progress.

Biais, B., T. Foucault, and S. Moinas (2015): "Equilibrium Fast Trading," *Journal of Financial Economics*, 116, 292–313.

Bigio, S. (2015): "Endogenous Liquidity and the Business Cycle," *American Economic Review*, 105(6), 1883–1927.

Blanchard, O., J.-P. L'Huillier, and G. Lorenzoni (2013): "News, Noise, and Fluctuations: An Empirical Exploration," *American Economic Review*, 103(7), 3045–3070.

Briggs, J., A. Caplin, D. Martin, and C. Tonetti (2017): "Due Diligence: Job Search with Rationally Inattentive Workers," NYU working paper.

Brunnermeier, M., M. Sockin, and W. Xiong (2017): "China's Model of Managing the Financial System," Working Paper, Princeton University.

Cai, Z. (2016): "Dynamic Information Complementarity in Information Acquisition," Ph.D. Working Paper, University of Minnesota.

Caplin, A., J. Leahy, and F. Matejka (2016): "Rational Inattention and Inference from Market Share Data," NYU Working Paper.

Cespa, G., and X. Vives (2012): "Dynamic Trading and Asset Prices: Keynes vs. Hayek," *Review of Economic Studies*, 79 (2), 539–580.

Chahrour, R. (2014): "Public Communication and Information Acquisition," *American Economic Journal: Macroeconomics*, 6(3), 73–101.

Chien, Y., H. Cole, and H. Lustig (2016): "Implications of Heterogeneity in Preferences, Beliefs, and Asset Trading Technologies in an Endowment Economy," *Review of Economic Dynamics*, 20, 215–239.

Crouzet, N., I. Dew-Becker, and C. Nathanson (2016): "A Model of Multi-Frequency Trade," Northwestern University Working Paper.

Davila, E., and C. Parlatore (2016): "Trading Costs and Informational Efficiency," NYU Working Paper.

Du, S., and H. Zhu (2017): "What Is the Optimal Trading Frequency in Financial Markets?," *Review of Financial Studies*, forthcoming.

Edelman, D., W. Fund, and D. A. Hsieh (2013): "Exploring Uncharted Territories of the Hedge Fund Industry: Empirical Characteristics of Mega Hedge Fund Firms," *Journal of Financial Economics*, 109(3), 734–758.

Fajgelbaum, P., E. Schaal, and M. Taschereau-Dumouchel (2017): "Uncertainty Traps," *Quarterly Journal of Economics*, Forthcoming.

Glode, V., R. Green, and R. Lowery (2012): "Financial Expertise as an Arms Race," *Journal of Finance*, 67, 1723–1759.

Goldstein, I., E. Ozdenoren, and K. Yuan (2013): "Trading frenzies and their impact on real investment," *Journal of Financial Economics*, 109(2), 566–82.

Grossman, S., and J. Stiglitz (1980): "On the impossibility of informationally efficient markets," *American Economic Review*, 70(3), 393–408.

Hasbrouck, J. (2007): *Empirical Market Microstructure*. Oxford University Press, first edn.

He, Z. (2009): "The Sale of Multiple Assets with Private Information," *Review of Financial Studies*, 22, 4787–4820.

Hendershott, T., C. Jones, and A. Menkveld (2011): "Does Algorithmic Trading Improve Liquidity?," *Journal of Finance*, 66, 1–34.

Jones, C. (2002): "A Century of Stock Market Liquidity and Trading Costs," Columbia University Working Paper.

Kacperczyk, M., J. Nosal, and L. Stevens (2015): "Investor Sophistication and Capital Income Inequality," Imperial College Working Paper.

Koijen, R., and M. Yogo (2016): "An Equilibrium Model of Institutional Demand and Asset Prices," NYU Working Paper.

Lettau, M., S. Ludvigson, and J. Wachter (2008): "The Declining Equity Premium: What Role Does Macroeconomic Risk Play?," *Review of Financial Studies*, 21 (4), 1653–1687.

Lowery, R., and T. Landvoigt (2016): "Financial Industry Dynamics," in *2016 Meeting Papers*, no. 1248. Society for Economic Dynamics.

Maćkowiak, B., and M. Wiederholt (2012): "Business cycle dynamics under rational inattention," Working Paper, Goethe University Frankfurt.

Manzano, C., and X. Vives (2010): "Public and Private Learning from Prices, Strategic Substitutability and Complementarity, and Equilibrium Multiplicity," CEPR Discussion Papers 7949, C.E.P.R. Discussion Papers.

Myatt, D., and C. Wallace (2012): "Endogenous Information Acquisition in Coordination Games," *Review of Economic Studies*, 79, 340–374.

Nimark, K. (2008): "Monetary Policy with Signal Extraction from the Bond Market," *Journal of Monetary Economics*, 55(8), 1389–1400.

Wang, J. (1993): "A Model of Intertemporal Asset Prices Under Asymmetric Information," *Review of Economic Studies*, 60, 249–282.

Yang, L., and J. Ganguli (2009): "Complementarities, Multiplicity, and Supply Information," *Journal of the European Economic Association*, 7(1), 90–115.

Yang, L., and H. Zhu (2016): "Back-Running: Seeking and Hiding Fundamental Information in Order Flows," .

A Model Solution Details

A.1 Bayesian Updating

To form the conditional expectation, $E[f_{it}|\mathcal{I}_{it}]$, we need to use Bayes' law. But first, we need to know what signal investors extract from price, given their observed endowment exposure h_t and their demand signal η_x. We can rearrange the the linear price equation (7) to write a function of the price is the dividend innovation plus mean zero noise: $\eta_{pit} = \tilde{y}_t + (D_t/C_t)(\tilde{x}_t - E[\tilde{x}_t|\eta_{xit}])$, where the price signal and the signal precision are

$$\eta_{pit} \equiv (p_t - A_t - Bd_{t-1} - D_t\, E[x|\eta_{xit}])/C_t \tag{22}$$

$$\Omega_{pt} \equiv (C_t/D_t)^2(\tau_x + \Omega_{xt}) \tag{23}$$

For the simple case of an investor who learned nothing about demand ($E[x] = 0$) the information contained in prices is $(p_t - A_t - Bd_t)/C_t$, which is equal to $\tilde{y}_t + D_t/C_t\tilde{x}_t$. Since $\tilde{x}_t$ is a mean-zero random variable, this is an unbiased signal of the asset dividend innovation $\tilde{y}_t$. The variance of the signal noise is $Var[D/Cx] = (D/C)^2\tau_x^{-1}$. The price signal precision Ω_{pt} is the inverse of this variance.

But conditional on η_{xit}, $\tilde{x}_t$ is typically not a mean-zero random variable. Instead, investors use Bayes' law to combine their prior that $\tilde{x}_t = 0$, with precision τ_x with their demand signals: η_{xit} with precision Ω_{xit}. The posterior mean and variance are

$$E[x|\eta_{xit}] = \frac{\Omega_{xit}\eta_{xit}}{\tau_x + \Omega_{xit}} \tag{24}$$

$$V[x|\eta_{xit}] = \frac{1}{\tau_x + \Omega_{xit}} \tag{25}$$

Since that is equal to $\tilde{y}_t + D_t/C_t(\tilde{x}_t - E[\tilde{x}_t|\eta_{xit}])$, the variance of price signal noise is $(D_t/C_t)^2 Var[\tilde{x}_t|\eta_{xit}]$. In other words, the precision of the price signal for agent i (and therefore for every agent since we are looking at symmetric information choice equilibria) is $\Omega_{pit} \equiv (C_t/D_t)^2(\tau_x + \Omega_{xit})$.

Now, we can use Bayes' law for normal variables again to form beliefs about the asset payoff. We combine the prior μ, the price/demand information η_{pit}, and the fundamental signal η_{fit} into a posterior mean and variance:

$$E[\tilde{y}_t|\mathcal{I}_{it}] = (\tau_0 + \Omega_{pit} + \Omega_{fit})^{-1}(\tau_0\mu + \Omega_{pit}\eta_{pit} + \Omega_{fit}\eta_{fit}) \tag{26}$$

$$V[\tilde{y}_t|\mathcal{I}_{it}] = (\tau_0 + \Omega_{pit} + \Omega_{fit})^{-1} \tag{27}$$

Average expectations and precisions: Next, we integrate over investors i to get the average conditional expectations. Begin by considering average price information. The price informativeness is $\Omega_{pit} \equiv (C_t/D_t)^2(\tau_x + \Omega_{xit})$. In principle, this can vary across investors. But since all are ex-ante identical, they make identical information decisions. Thus, $\Omega_{pit} = \Omega_{pt}$ for all investors i. Since this precision is identical for all investors, we drop the i subscript in what follows. But the realized price signal still differs because signal realizations are heterogeneous. Since the signal precisions are the same for all agents, we can just integrate over signals to get the average signal: $\int \eta_{pit}di = (1/C_t)(p_t - A_t - Bd_{t-1}) - (D_t/C_t)Var(\tilde{x}_t|\mathcal{I}_t)\Omega_{xt}\tilde{x}_t$. Since $\Omega_{pt}^{-1} = (D/C)^2 Var(x|\mathcal{I})$, we can rewrite this as

$$\int \eta_{pi}di = \frac{1}{C}(p_t - A_t - Bd_{t-1}) - \frac{C_t}{D_t}\Omega_{pt}^{-1}\Omega_{xt}\tilde{x}_t \tag{28}$$

Next, let's define some conditional variance / precision terms that simplify notation. The first term, Ω_t, is the precision of future price plus dividend (the asset payoff). It comes from taking the variance of the pricing equation (7). It turns out that the variance Ω_t^{-1} can be decomposed into a sum of two terms. The first, $\hat{V}$, is the variance of the dividend innovation. This variance depends on information choices Ω_{ft} and Ω_{xt}. The other term Z_t depends on future information choices through $t+1$ price coefficients.

$$\hat{V}_t \equiv Var(\tilde{y}_t|\mathcal{I}_t) = (\tau_0 + \Omega_f + \Omega_{pt})^{-1} = (\tau_0 + \Omega_f + (C/D)^2(\tau_x + \Omega_{xt}))^{-1} \tag{29}$$

$$\Omega_t^{-1} \equiv Var[p_{t+1} + \tilde{d}_t|\mathcal{I}_t] = C_{t+1}^2\tau_0^{-1} + D_{t+1}^2\tau_x^{-1} + (1+B_t)^2\hat{V} \tag{30}$$

$$Z_t = \frac{\rho}{r}(r-G)(C_{t+1}^2\tau_0^{-1} + D_{t+1}^2\tau_x^{-1}) \tag{31}$$

$$\Omega_t^{-1} = \frac{r}{\rho(r-G)}Z_t + (\frac{r}{r-G})^2\hat{V} \tag{32}$$

The last equation (32) shows the relationship between Ω, $\hat{V}$ and Z_t. This decomposition is helpful because we will repeatedly take derivatives where we take future choices (Z_t) as given and vary current information choices ($\hat{V}$).

Next, we can compute the average expectations

$$\int E[\tilde{y}_t|\bar{\mathcal{I}}_t]\,di = \hat{V}_t\left[\Omega_{ft}\tilde{y}_t + \Omega_{pt}\left(\frac{1}{C}(p_t - A_t - Bd_{t-1}) - \frac{C_t}{D_t}\Omega_{pt}^{-1}\Omega_{xt}\tilde{x}_t\right)\right] \tag{33}$$

$$= \hat{V}\left[\Omega_{ft}\tilde{y}_t + \Omega_{pt}\frac{1}{C}(p_t - A_t - Bd_{t-1}) - \frac{C_t}{D_t}\Omega_{xt}\tilde{x}_t\right] \tag{34}$$

$$\int E[p_{t+1} + \tilde{d}_t|\bar{\mathcal{I}}_t]\,di = A_t + (1+B)E[\tilde{d}_t|\bar{\mathcal{I}}_t] = A_t + (1+B)\left(\mu + Gd_{t-1} + E[\tilde{y}_t|\bar{\mathcal{I}}_t]\right). \tag{35}$$

A.2 Solving for Equilibrium Prices

The price conjecture is

$$p_t = A_t + B_t d_t + C_t\tilde{y}_t + D_t\tilde{x}_t \tag{36}$$

We will solve for the prices for general supply of asset, $\bar{x}$, although in the main text it is normalized to one unit.

The price coefficients solve the system of recursive equations

$$A_t = \frac{1}{r}\left[A_{t+1} + (1+B_{t+1})\mu - \rho\bar{x}\left(C_{t+1}^2\tau_0^{-1} + D_{t+1}^2\tau_x^{-1} + (1+B_t)^2\Big(\tau_0 + \big(\frac{K}{\chi_f\big(1+\frac{\chi_f}{\chi_x}\xi^4\big)}\big)^{\frac{1}{2}} + \xi^2\big(\tau_x + \frac{\xi^2\chi_f}{\chi_x}\big(\frac{K}{\chi_f\big(1+\frac{\chi_f}{\chi_x}\xi^4\big)}\big)^{\frac{1}{2}}\big)\Big)^{-1}\right)\right]$$

$$B_t = \frac{1}{r}(1+B_{t+1})G$$

$$C_t = \frac{1}{r-G}\left(1 - \tau_0\Big(\tau_0 + \big(\frac{K}{\chi_f\big(1+\frac{\chi_f}{\chi_x}\xi^4\big)}\big)^{\frac{1}{2}} + \xi^2\big(\tau_x + \frac{\xi^2\chi_f}{\chi_x}\big(\frac{K}{\chi_f\big(1+\frac{\chi_f}{\chi_x}\xi^4\big)}\big)^{\frac{1}{2}}\big)\Big)^{-1}\right)$$

$$D_t = \Big(\frac{\tau_x}{r-G}\xi - \frac{\rho r}{(r-G)^2}\Big)\Big(\tau_0 + \big(\frac{K}{\chi_f\big(1+\frac{\chi_f}{\chi_x}\xi^4\big)}\big)^{\frac{1}{2}} + \xi^4\big(\tau_x + \frac{\xi^2\chi_f}{\chi_x}\big(\frac{K}{\chi_f\big(1+\frac{\chi_f}{\chi_x}\xi^4\big)}\big)^{\frac{1}{2}}\big)\Big)^{-1} - \frac{\rho}{r}\Big(C_{t+1}^2\tau_0^{-1} + D_{t+1}^2\tau_x^{-1}\Big)$$

where $\xi = \frac{C_t}{D_t}$ denotes date-t signal to noise ratio which is the solution to equation (57). The steady state pricing coefficients are the fixed point of the above system.

The sequence of pricing coefficients is known at every date. The signals η_{fit} and η_{xit} are the same as before, except that their precisions Ω_{ft} and Ω_{xt} may change over time if that is the solution to the information choice problem.

The conditional expectation and variance of $\tilde{y}_t$ (26) and (27) are the same, except that the Ω_{pt} term gets a t subscript now because $\Omega_{pt} \equiv (C_t/D_t)^2(\tau_x + \Omega_{xt})$. Likewise the mean and variance of $\tilde{x}_t$ (24) and (25) are the same with a time-subscripted Ω_{xt}. Thus, the average signals are the same with t-subscripts:

$$\int \eta_{pi}di = \frac{1}{C_t}(p_t - A_t - B_t d_t) - \frac{D_t}{C_t}Var(x_t|\mathcal{I}_t)\Omega_{xt}\tilde{x}_t \tag{37}$$

Since $\Omega_{pt}^{-1} = (D_t/C_t)^2 Var(x_t|\mathcal{I}_t)$, we can rewrite this as

$$\int \eta_{pi}di = \frac{1}{C_t}(p_t - A_t - B_t d_t) - \frac{C_t}{D_t}\Omega_{pt}^{-1}\Omega_{xt}\tilde{x}_t \tag{38}$$

Solving for non-stationary equilibrium prices To solve for equilibrium prices, start from the portfolio first-order condition for investors (6) and equate total demand with total supply. The total risky asset demand (excluding hedging shocks) is

$$\int q_{it}di = \frac{1}{\rho}\Omega_t\left[A_{t+1} + (1+B_{t+1})\left(\mu + Gd_t + \hat{V}_t\left[\Omega_{ft}\tilde{y}_t + \Omega_{pt}\frac{1}{C_t}(p_t - A_t - B_t d_t) - \frac{C_t}{D_t}\Omega_{xt}\tilde{x}_t\right]\right) - p_t r\right]. \tag{39}$$

The market clearing condition equates the expression above to the residual asset supply $\bar{x} + \tilde{x}_t$. The model assumes the asset supply is 1. We use the notation $\bar{x}$ here for more generality because then we can apply the result to the model with issuance costs where asset supply is a choice variable. Rearranging the market clearing condition (just multiplying through by $\rho\Omega_t^{-1}$ and bringing p terms to the left) yields

$$[r - (1 + B_{t+1})\hat{V}_t\Omega_{pt}\frac{1}{C_t}]p_t = -\rho\Omega_t^{-1}(\bar{x} + \tilde{x}_t) + A_{t+1} \tag{40}$$

$$+(1 + B_{t+1})(\mu + Gd_t) + (1 + B_{t+1})\hat{V}_t\Omega_{ft}\tilde{y}_t - (1 + B_{t+1})\hat{V}_t\Omega_{pt}\frac{1}{C_t}(A_t + B_t d_t) - (1 + B_{t+1})\frac{C_t}{D_t}\hat{V}_t\Omega_{xt}\tilde{x}_t$$

Solving for p and matching coefficients yields

$$A_t = [r - (1 + B_{t+1})\hat{V}_t\Omega_{pt}\frac{1}{C_t}]^{-1}[A_{t+1} + (1 + B_{t+1})\mu - \rho\Omega_t^{-1}\bar{x} - (1 + B_{t+1})\hat{V}_t\Omega_{pt}\frac{1}{C_t}A_t] \tag{41}$$

Multiplying both sides by the inverse term:

$$rA_t - (1 + B_{t+1})\hat{V}_t\Omega_{pt}\frac{1}{C_t}A_t = A_{t+1} + (1 + B_{t+1})\mu - \rho\Omega_t^{-1}\bar{x} - (1 + B_{t+1})\hat{V}_t\Omega_{pt}\frac{1}{C_t}A_t \tag{42}$$

and canceling the $1 + B$ term on both sides leaves

$$A_t = \frac{1}{r}\left[A_{t+1} + (1 + B_{t+1})\mu - \rho\Omega_t^{-1}\bar{x}\right] \tag{43}$$

Note that for the main model $\bar{x} = 1$, so in the main text, equation (8) is with $\bar{x}$ set to 1.

Matching coefficients on d_t yields:

$$B_t = [r - (1 + B_{t+1})\hat{V}_t\Omega_{pt}\frac{1}{C_t}]^{-1}\left[(1 + B_{t+1})G - (1 + B_{t+1})\hat{V}_t\Omega_{pt}\frac{B_t}{C_t}\right] \tag{44}$$

Multiplying on both sides by the inverse term

$$rB_t - (1 + B_{t+1})\hat{V}_t\Omega_{pt}\frac{1}{C_t}B_t = (1 + B_{t+1})G - (1 + B_{t+1})\hat{V}_t\Omega_{pt}\frac{B_t}{C_t} \tag{45}$$

and canceling the last term on both sides yields

$$B_t = \frac{1}{r}(1 + B_{t+1})G \tag{46}$$

As long as r and G don't vary over time, it seems that a stationary solution for B at least exists. That stationary solution would be (9).

Next, collecting all the terms in $\tilde{y}_t$

$$C_t = [r - (1 + B_{t+1})\hat{V}_t\Omega_{pt}\frac{1}{C_t}]^{-1}(1 + B_{t+1})\hat{V}_t\Omega_{ft} \tag{47}$$

multiplying both sides by the first term inverse yields $rC_t - (1 + B_{t+1})\hat{V}_t\Omega_{pt} = (1 + B_{t+1})\hat{V}_t\Omega_{ft}$. Then dividing through by r and collecting terms in $\hat{V}(1 + B_{t+1})$ yields $C_t = (1/r)(1 + B_{t+1})\hat{V}_t(\Omega_{pt} + \Omega_{ft})$. Next, using the fact that $\hat{V}^{-1} = \tau_0 + \Omega_{pt} + \Omega_f$, we get $C_t = 1/r(1 + B_{t+1})(1 - \tau_0\hat{V}_t)$. Of course the $\hat{V}$ term has C_t and D_t in it. If we use the stationary solution for B (if r and G don't vary) then we can simplify to get

$$C_t = \frac{1}{r - G}(1 - \tau_0\hat{V}_t). \tag{48}$$

Lemma 1 *If $\Omega_{ft} > 0$, then $C_t > 0$.*

Proof: Using equation (48), it suffices to show that $1/(r - G) > 0$ and $(1 - \tau_0\hat{V}_t) > 0$. From the setup, we assumed that $r > 1$ and $G < 1$. By transitivity, $r > G$ and $r - G > 0$. For the second term, we need to prove equivalently that $\tau_0\hat{V}_t < 1$ and thus that $\tau_0 < \hat{V}_t^{-1}$. Recall from (29) that $\hat{V}^{-1} = \tau_0 + \Omega_{ft} + \Omega_{pt}$. Since Ω_{ft} and Ω_{pt} are defined as precisions, they must be non-negative. Furthermore, we supposed that $\Omega_{ft} > 0$. Thus, $\tau_0 < \hat{V}_t^{-1}$, which completes the proof. □

Finally, we collect terms in $\tilde{x}_t$.

$$D_t = [r - (1 + B_{t+1})\hat{V}_t\Omega_{pt}\frac{1}{C_t}]^{-1}[-\rho\Omega_t^{-1} - (1 + B_{t+1})\frac{C_t}{D_t}\hat{V}_t\Omega_{xt}] \tag{49}$$

multiply by the inverse term, and the use $\Omega_{pt} = (C_t/D_t)^2(\tau_x + \Omega_{xt})$ to get

$$rD_t - (1+B_{t+1})\hat{V}_t\frac{C_t}{D_t}(\tau_x + \Omega_{xt}) = -\rho\Omega_t^{-1} - (1+B_{t+1})\frac{C_t}{D_t}\hat{V}_t\Omega_{xt} \tag{50}$$

Then, adding $(1+B)C/D\hat{V}\Omega_x$ to both sides, and substituting in B (stationary solution), we get

$$D_t - \frac{1}{r-G}\hat{V}_t\tau_x\frac{C_t}{D_t} = -\frac{\rho}{r}\Omega_t^{-1} \tag{51}$$

Of course, D_t still shows up quadratically, and also in $\hat{V}_t$. The future coefficient values C_{t+1} and D_{t+1} show up in Ω_t.

Lemma 2 $D_t < 0$

Proof: Start from equation (50) and substitute in (29). Since we will often treat the signal-to-noise ratio in prices as a single variable, we define

$$\xi \equiv \frac{C_t}{D_t} \tag{52}$$

Moreover, let $\alpha \equiv \frac{\rho r}{r-G}$. Simplify to get:

$$\xi^3(Z_t\tau_x + Z_t\Omega_x) + \xi^2(\Omega_x) + \xi(\alpha + Z_t\tau_0 + Z_t\Omega_f) + \Omega_f = 0 \tag{53}$$

Then, use the budget constraint to express the first order conditions as (15). One can solve for both Ω_x and Ω_f in terms of ξ:

$$\Omega_f = \Big(\frac{K}{\chi_f\big(1+\frac{\chi_f}{\chi_x}\xi^4\big)}\Big)^{\frac{1}{2}} \tag{54}$$

$$\Omega_x = \Big(\frac{K}{\chi_x}\big(1 - \frac{1}{1+\frac{\chi_f}{\chi_x}\xi^4}\big)\Big)^{\frac{1}{2}} = \Big(\frac{K\frac{\chi_f}{\chi_x}}{\chi_x\big(1+\frac{\chi_f}{\chi_x}\xi^4\big)}\Big)^{\frac{1}{2}}\xi^2 = \frac{\xi^2\chi_f}{\chi_x}\Big(\frac{K}{\chi_f\big(1+\frac{\chi_f}{\chi_x}\xi^4\big)}\Big)^{\frac{1}{2}} \tag{55}$$

Now I can substitute both of these into equation (53), which fully determines ξ, in terms of exogenous variables.

$$\xi\Big(\xi^2 Z_t\tau_x + \alpha + Z_t\tau_0\Big) + \xi^2\Omega_x(1+\xi Z_t) + \Omega_f(1+\xi Z_t) = 0 \tag{56}$$

First note that

$$\Omega_f + \xi^2\Omega_x = -\frac{\xi(\xi^2 Z_t\tau_x + \alpha + Z_t\tau_0)}{(1+\xi Z_t)}$$

where the left hand side is the objective function. So we know the maximized value of objective function solely as a function of $\xi = \frac{C}{D}$. Keep in mind that since we already imposed an optimality condition, this latter equation holds only at the optimum.

Substituting in for Ω_{ft} and Ω_{xt} from (54) and (55) yields an equation that implicitly defines ξ as a function of primitives, K and future equilibrium objects, embedded in Z_t.

$$\begin{aligned}
&\xi\Big(\xi^2 Z_t\tau_x + \alpha + Z_t\tau_0\Big) + (1+\xi Z_t)(1+\frac{\chi_f}{\chi_x}\xi^4)\Big(\frac{K}{\chi_f\big(1+\frac{\chi_f}{\chi_x}\xi^4\big)}\Big)^{\frac{1}{2}} = 0 \\
&\xi^3 Z_t\tau_x + \xi(\alpha + Z_t\tau_0) + (1+\xi Z_t)(\frac{K}{\chi_f})^{\frac{1}{2}}(1+\frac{\chi_f}{\chi_x}\xi^4)^{\frac{1}{2}} = 0
\end{aligned} \tag{57}$$

The left hand side must equal zero for the economy to be in equilibrium. However, all the coefficients $K, \chi_f, \chi_x, \tau_0, \tau_x$ are assumed to be positive. Furthermore, Z_t is a variance. Inspection of (31) reveals that it must be strictly positive. Thus, the only way that the equilibrium condition can possibly be equal to zero is if $\xi < 0$. Recall that $\xi = C_t/D_t$. The previous lemma proved that $C_t > 0$. Therefore, it must be that $D_t < 0$.

A.3 Solving Information Choices

Details of Step 3: Compute ex-ante expected utility. Note that the expected excess return $(E[p_{t+1} + \tilde{d}_t|\mathcal{I}_{it}] - p_t r)$ depends on fundamental and supply signals, and prices, all of which are unknown at time $t = 0$. Because asset prices are linear functions

of normally distributed shocks, $E[p_{t+1}+\tilde{d}_t|\mathcal{I}_{it}]-p_t r$, is normally distributed as well. Thus, $(E[p_{t+1}+\tilde{d}_t|\mathcal{I}_{it}]-p_t r)\Omega(E[p_{t+1}+\tilde{d}_t|\mathcal{I}_{it}]-p_t r)$ is a non-central χ^2-distributed variable. Computing its mean yields the expression in the text.

Details of Step 4:

Solve for fundamental information choices. Note that in expected utility (14), the choice variables Ω_{ft} and Ω_{xt} enter only through the posterior variance Ω^{-1} and through $V[E[p_{t+1}+\tilde{d}_t|\mathcal{I}_{it}]-p_t r|\mathcal{I}^+_{t-1}]=V[p_{t+1}+\tilde{d}_t-p_t r|\mathcal{I}^+_{t-1}]-\Omega_t^{-1}$. Since there is a continuum of investors, and since $V[p_{t+1}+\tilde{d}_t-p_t r|\mathcal{I}^+_{t-1}]$ and $E[E[p_{t+1}+\tilde{d}_t|\mathcal{I}_{it}]-p_t r|\mathcal{I}^+_{t-1}]$ depend only on $t-1$ variables, parameters and on aggregate information choices, each investor takes them as given. If the objective is to maximize an increasing function of Ω, then information choices must maximize Ω as well.

Internet Appendix: Not for Publication

B Proofs

The next lemma proves the following: If no one has information about future dividends, then no one's trade is based on information about future dividends, thus the price cannot contain information about future dividends. Since C_t is the price coefficient on future dividend information, $C_t = 0$ means that the price is uninformative. In short, price cannot reflect information that no one knows.

Lemma 3 ***When information is scarce, price is uninformative****: As $K_t \to 0$, for any future path of prices (A_{t+j}, B_{t+j}, C_{t+j} and D_{t+1}, $\forall j > 0$), the unique solution for the price coefficient C_t is $C_t = 0$.*

Proof: Step 1: As $\Omega_{ft} \to 0$, prove C_t is always a solution.

Start with the equation for D_t (11). Substitute in for Ω using (32) and $1 + B = r/(r - G)$ and rewrite it as

$$D_t = \frac{1}{r-G}\hat{V}_t\left[\tau_x \frac{C_t}{D_t} - \frac{\rho r}{(r-G)} - Z_t \hat{V}_t^{-1}\right] \tag{58}$$

Then, express C_t from (48) as $C_t = 1/(r-G)\hat{V}_t(\hat{V}_t^{-1} - \tau_0)$ and divide C_t by D_t, cancelling the $\hat{V}_t/(r-G)$ term in each to get

$$\frac{C_t}{D_t} = \frac{\hat{V}_t^{-1} - \tau_0}{\tau_x \frac{C_t}{D_t} - \frac{\rho r}{(r-G)} - Z_t \hat{V}_t^{-1}} \tag{59}$$

If we substitute in $\hat{V}_t^{-1} = \tau_0 + \Omega_{pt} + \Omega_{ft}$ from (29) and then set $\Omega_{ft} = 0$, we get

$$\frac{C_t}{D_t} = \frac{\Omega_{pt}}{\tau_x \frac{C_t}{D_t} - \frac{\rho r}{(r-G)} - Z_t(\tau_0 + \Omega_{pt})} \tag{60}$$

Then, we use the solution for price information precision $\Omega_{pt} = (C/D)^2(\tau_x + \Omega_x)$ and multiply both sides by the denominator of the fraction to get

$$\frac{C_t}{D_t}\left[\tau_x \frac{C_t}{D_t} - \frac{\rho r}{(r-G)} - Z_t(\tau_0 + \left(\frac{C_t}{D_t}\right)^2(\tau_x + \Omega_x))\right] = \left(\frac{C_t}{D_t}\right)^2(\tau_x + \Omega_x) \tag{61}$$

We can see right away that since both sides are multiplied by C/D, as $\Omega_{ft} \to 0$, for any given future price coefficients C_{t+1} and D_{t+1}, $C = 0$ is always a solution.

Step 2: prove uniqueness.

Next, we investigate what other solutions are possible by dividing both sides by C/D:

$$\tau_x \frac{C_t}{D_t} - \frac{\rho r}{(r-G)} - Z_t(\tau_0 + \left(\frac{C_t}{D_t}\right)^2(\tau_x + \Omega_x)) - \left(\frac{C_t}{D_t}\right)(\tau_x + \Omega_x) = 0 \tag{62}$$

This is a quadratic equation in C/D. Using the quadratic formula, we find

$$\frac{C_t}{D_t} = \frac{\Omega_{xt} \pm \sqrt{\Omega_{xt}^2 - 4Z_t(\tau_x + \Omega_{xt})(\rho r/(r-G) + \tau_0 Z_t)}}{-2Z_t(\tau_x + \Omega_{xt})} \tag{63}$$

If we now take the limit as $\Omega_{xt} \to 0$, the term inside the square root becomes negative, as long as $r - G > 0$. Thus, there are no additional real roots when $\Omega_{xt} = 0$.

Similarly, if Ω_x is not sufficiently large, there are no real roots of (63), which proves that: As $\Omega_{ft} \to 0$, if we take C_{t+1} and D_{t+1} as given, and Ω_{xt} is sufficiently small, then the unique solution for the price coefficient C is $C = 0$. □

Proof of Result 1 From lemma 3, we know that as $C_t = 0$. From the first order condition for information (15), we see that the marginal utility of demand information relative to fundamental information (marginal rate of substitution) is a positive constant times $(C_t/D_t)^2$. If $C_t = 0$, then $\partial U_{it}/\partial \Omega_{xit}$ is a positive constant time zero, which is zero.

Proof of Result 3

Claim: If $r - G > 0$ and $(\tau_x + \Omega_{xt})$ is sufficiently small, then $\partial C_t/\partial \Omega_{ft} > 0$ and $\partial C_t/\partial \Omega_{xt} > 0$.

From (48), $C_t = \frac{1}{r-G}(1 - \tau_0 \hat{V}_t)$.

From (29), $\hat{V}_t$ is defined as

$$\hat{V} = [\tau_0 + \Omega_{ft} + \left(\frac{C_t}{D_t}\right)^2 (\tau_x + \Omega_{xt})]^{-1} \tag{64}$$

Notice that C_t shows up twice, once on the left side and once in $\hat{V}$. Therefore, we use the implicit function theorem to differentiate. If we define $F \equiv C_t - \frac{1}{r-G}(1 - \tau_0 \hat{V})$, then $\partial F/\partial C_t = 1 + \frac{1}{r-G}\tau_0 \partial \hat{V}/\partial C_t$. Since τ_x and Ω_{xt} are both precisions, both are positive. Therefore, $\partial \hat{V}^{-1}/\partial C_t = 2C_t/D_t^2(\tau_x + \Omega_{xt})$. This is positive, since we know that $C_t > 0$. That implies that the derivative of the inverse is $\partial \hat{V}/\partial C_t = -\hat{V}^2 2C_t/D_t^2(\tau_x + \Omega_{xt})$, which is negative. The $\partial F/\partial C_t$ term is therefore one plus a negative term. The result is positive, as long as the negative term is sufficiently small: $\frac{2}{r-G}\tau_0 \hat{V}^2 C_t/D_t^2(\tau_x + \Omega_{xt}) < 1$. We can express this as an upper bound on $\tau_x + \Omega_{xt}$ by rearranging the inequality to read: $(\tau_x + \Omega_{xt}) < 1/2(r-G)\tau_0^{-2}\hat{V}^{-2}D_t^2/C_t$.

Next, we see that $\partial \hat{V}^{-1}/\partial \Omega_{ft} = 1$. Thus, $\partial \hat{V}/\partial \Omega_{ft} < 0$. Since $\partial F/\partial \hat{V} > 0$, this guarantees that $\partial F/\partial \Omega_{ft} < 0$.

Likewise, $\partial \hat{V}^{-1}/\partial \Omega_{xt} = (C_t/D_t)^2$. Since the square is always positive, $\partial \hat{V}/\partial \Omega_{xt} < 0$. Since $\partial F/\partial \hat{V} > 0$, this guarantees that $\partial F/\partial \Omega_{xt} < 0$.

Finally, the implicit function theorem states that $\partial C_t/\partial \Omega_{ft} = -(\partial F/\partial \Omega_{ft})/(\partial F/\partial C_t)$. Since the numerator is positive, the denominator is negative and there is a minus sign in front, $\partial C_t/\partial \Omega_{ft} > 0$. Likewise, $\partial C_t/\partial \Omega_{xt} = -(\partial F/\partial \Omega_{xt})/(\partial F/\partial C_t)$. Since the numerator is positive, the denominator is negative and there is a minus sign in front, $\partial C_t/\partial \Omega_{xt} > 0$. □

Proof of Result 4, part 1

Claim: If $\tau_x > \rho r/(r-G)$ and $D_t < 0$, then $\partial D_t/\partial \Omega_{ft} > 0$.

Proof:

From market clearing:

$$D_t = [r - (1+B)\hat{V} + \Omega_p \frac{1}{C}]^{-1}[-\rho \Omega_t^{-1} - (1+B)\frac{C}{D}\hat{V}\Omega_x] \tag{65}$$

Use $\Omega_p = (\frac{C}{D})^2(\Omega_x + \tau_x)$ to get $D_t r - (1+B)\hat{V}_t \frac{C}{D}(\tau_x) = -\rho \Omega_t^{-1}$. Then, use the stationary solution for $B: 1 + B = \frac{r}{r-G}$:

$$D_t - \frac{1}{r-G}\hat{V}_t \frac{C}{D}\tau_x = -\frac{\rho}{r}\Omega_t^{-1} \tag{66}$$

Then use (32) to substitute in for Ω_t^{-1}:

$$D_t = -\frac{1}{r-G}Z_t - \frac{r\rho}{(r-G)^2}\hat{V} + \frac{1}{r-G}\hat{V}_t \frac{C_t}{D_t}\tau_x \tag{67}$$

In the above, the RHS, less the last term, is the loading on X_{t+1}, and the last term represents price feedback. We then define $F \equiv$ L.H.S. of (67) − R.H.S. of (67). So that we can apply the implicit function theorem as $\partial D_t/\partial \Omega_f = -\frac{\partial F}{\partial \Omega_f}/\frac{\partial F}{\partial D_t}$. We begin by working out the denominator.

$$\frac{\partial F}{\partial D_t} = 1 + 0 + \frac{r\rho}{(r-G)^2}\frac{\partial \hat{V}}{\partial D_t} - \frac{1}{r-G}\frac{\partial \hat{V} + \frac{C_t}{D_t}}{\partial D_t}\tau_x \tag{68}$$

$$\frac{\partial \hat{V}}{\partial D_t} = \frac{\partial \hat{V}}{\partial \hat{V}^{-1}}\frac{\partial \hat{V}^{-1}}{\partial D_t} = -\hat{V}^2[-\frac{2C_t^2}{D_t^3}(\tau_x + \Omega_x)] = 2\frac{C^2}{D^3}\hat{V}_t^3(\tau_x + \Omega_x) \tag{69}$$

$$\frac{\partial \hat{V}\frac{C_t}{D_t}}{\partial D_t} = \frac{C_t}{D_t}\frac{\partial \hat{V}_t}{\partial D_t} + \hat{V}(-\frac{C}{D^2}) \tag{70}$$

$$= \frac{C}{D^2}\hat{V}[2\frac{C_t}{D_t}(\tau_x + \Omega_x) - 1] \tag{71}$$

$$\frac{\partial F}{\partial D_t} = 1 + \frac{r\rho}{(r-G)^2} \cdot 2\frac{C^2}{D^3}\hat{V}_t^3(\tau_x + \Omega_x) - \frac{\tau_x}{r-G}\frac{C}{D^2}\hat{V}_t[2\frac{C_t}{D_t}(\tau_x + \Omega_x) - 1] \tag{72}$$

$$\frac{\partial F}{\partial \Omega_f} = 0 - 0 + \frac{r\rho}{(r-G)^2}\frac{\partial \hat{V}}{\partial \Omega_t} - \frac{1}{r-G}\frac{C_t}{D_t}\tau_x\frac{\partial \hat{V}}{\partial \Omega_t} \tag{73}$$

Recall the definition $\hat{V}_t \equiv [\tau_0 + \Omega_{ft} + \frac{C_t}{D_t}^2(\tau_x + \Omega_x)]^{-1}$. Differentiating $\hat{V}$, we get

$$\frac{\partial \hat{V}}{\partial \Omega_f} = \frac{\partial \hat{V}_t}{\partial \hat{V}_t^{-1}} \cdot \frac{\partial \hat{V}_t^{-1}}{\partial \Omega_f} = -\hat{V}_t^2\frac{\partial \hat{V}_t^{-1}}{\partial \Omega_f} = -\hat{V}_t^2 \tag{74}$$

substituting this in to (73) yields

$$\frac{\partial F}{\partial \Omega_f} = \frac{1}{r-G}\hat{V}_t^2[\frac{C_t}{D_t}\tau_x - \frac{r\rho}{r-G}] \tag{75}$$

Substituting in the derivative of $\hat{V}$, we get

$$\frac{\partial D_t}{\partial \Omega_f} = -\frac{\frac{1}{r-G}\hat{V}_t^2[\frac{C_t}{D_t}\tau_x - \frac{r\rho}{r-G}]}{1\frac{2r\rho}{(r-G)^2}\frac{C^2}{D^3}\hat{V}_t^2(\tau_x + \Omega_x) - \frac{\tau_x}{r-G}\frac{C}{D^2}\hat{V}_t[2\frac{C}{\rho}(\tau_x + \Omega_x) - 1]} \tag{76}$$

Observe that if $\frac{C_t}{D_t} < 0$, and $r > G$, then the numerator is positive (including the leading negative sign).
The denominator is positive if the following expression is positive:

$$\frac{r-G}{\frac{C}{D^2}\hat{V}} + 2\rho\frac{r}{r-G}\frac{C_t}{D_t}\hat{V}_t(\tau_x + \Omega_x) - \tau_x\hat{V}_t[\frac{2C}{D}(\tau_x + \Omega_x - 1)] > 0 \tag{77}$$

This is equivalent to

$$\frac{r-G}{\hat{V}_t}\frac{D^2}{C} + 2\hat{V}_t\frac{C_t}{D_t}(\tau_x + \Omega_x)[\frac{r\rho}{r-G} - \tau_x] + \tau_x\hat{V}_t > 0. \tag{78}$$

Lemma 2 proves that $D < 0$. That makes the middle term potentially negative. However, if $[\frac{r\rho}{r-G} - \tau_x] < 0$ as well, the product of this and D is positive. Thus the middle term is positive. That inequality can be rearranged as $\tau_x > \frac{r\rho}{r-G}$. Since the rest of the terms are squares and precisions, the rest of the expression is positive as well.

Thus if $\tau_x > \frac{r\rho}{r-G}$, then $\frac{\partial D_t}{\partial \Omega_t} > 0$. □

Proof of Result 4, part 2

If $\tau_x > \rho r/(r-G)$ and $D_t < 0$, then $\partial D_t/\partial \Omega_{xt} > 0$.

Proof: Begin with the implicit function theorem: $\partial D_t/\partial \Omega_x = -\frac{\partial F}{\partial \Omega_x}/\frac{\partial F}{\partial D_t}$. The previous proof already proved that if $\tau_x > \frac{r\rho}{r-G}$, the denominator is positive. All that remains is to sign the numerator.

$$\frac{\partial F}{\partial \Omega_x} = 0 + 0 + \frac{r\rho}{(r-G)^2}\frac{\partial \hat{V}}{\partial \Omega_x} - \frac{1}{r-G}\frac{C_t}{D_t}\tau_x\frac{\partial \hat{V}}{\partial \Omega_x}$$

where $\partial\hat{V}/\partial\Omega_x = -\hat{V}^2(C^2)/(D^2)$. Substituting the partial of $\hat{V}$ into the partial of F yields

$$\frac{\partial F}{\partial \Omega_x} = \hat{V}^2 \frac{C^2}{D^2}(-\frac{r\rho}{(r-G)^2} + \frac{1}{r-G}\frac{C_t}{D_t}\tau_x).$$

Combining terms,

$$\frac{\partial D_t}{\partial \Omega_x} = -\frac{\hat{V}^2 \frac{C^2}{D^2}(-\frac{r\rho}{(r-G)^2} + \frac{1}{r-G}\frac{C_t}{D_t}\tau_x)}{\frac{\partial F}{\partial D_t}}$$

We know from lemmas 1 and 2 that $\frac{C_t}{D_t} < 0$. Since $r > G$, by assumption, $\partial F/\partial\Omega_x$ is negative (i.e., the $\frac{C^2}{D^2}$ factor does not change the sign). Applying the implicit function theorem tells us that $\partial D_t/\partial\Omega_{xt} > 0$. □

Proof of Result 2

The strategy for proving this result is to apply the implicit function theorem to the price coefficients that come from coefficient matching in the market-clearing equation. After equating supply and demand and matching all the coefficients on $\tilde{x}_t$, we arrive at (11). Rearranging that equation gives us the expression for C_t/D_t in (59). If we subtract the right side of (59) from the left, we are left with an expression that is equal to zero in equilibrium, which we'll name F:

$$F = \frac{C_t}{D_t} - \frac{\hat{V}_t^{-1} - \tau_0}{\tau_x \frac{C_t}{D_t} - \frac{\rho r}{(r-G)} - Z_t \hat{V}_t^{-1}}$$

We compute $\frac{\partial C/D}{\partial \Omega_x} = -\left(\frac{\partial F}{\partial C/D}\right)^{-1}\frac{\partial F}{\partial \Omega_x}$ and $\frac{\partial C/D}{\partial \Omega_f} = -\left(\frac{\partial F}{\partial C/D}\right)^{-1}\frac{\partial F}{\partial \Omega_f}$. In particular, we have:

$$\begin{aligned}
\frac{\partial F}{\partial C/D} &= 1 - \left(2\frac{C_t}{D_t}(\tau_x + \Omega_x)\right)\left(\tau_x\frac{C_t}{D_t} - \frac{\rho r}{r-G} - Z_t\hat{V}^{-1}\right)^{-1} \\
&\quad + (\hat{V}^{-1} - \tau_0)\left(\tau_x\frac{C_t}{D_t} - \frac{\rho r}{r-G} - Z_t\hat{V}^{-1}\right)^{-2}\left(\tau_x - Z_t\left(2\frac{C_t}{D_t}(\tau_x + \Omega_x)\right)\right) \\
&= 1 - \left(\tau_x\frac{C_t}{D_t} - \frac{\rho r}{r-G} - Z_t\hat{V}^{-1}\right)^{-2} \\
&\quad \left[\left(2\frac{C_t}{D_t}(\tau_x + \Omega_x)\right)\left(\tau_x\frac{C_t}{D_t} - \frac{\rho r}{r-G} - Z_t\hat{V}^{-1}\right) - (\hat{V}^{-1} - \tau_0)\left(\tau_x - Z_t\left(2\frac{C_t}{D_t}(\tau_x + \Omega_x)\right)\right)\right]
\end{aligned}$$

$$\begin{aligned}
\frac{\partial F}{\partial \Omega_f} &= -(1)\left(\tau_x\frac{C_t}{D_t} - \frac{\rho r}{r-G} - Z_t\hat{V}^{-1}\right)^{-1} + (\hat{V}^{-1} - \tau_0)\left(\tau_x\frac{C_t}{D_t} - \frac{\rho r}{r-G} - Z_t\hat{V}^{-1}\right)^{-2}(-Z_t) \\
&= -\left(\tau_x\frac{C_t}{D_t} - \frac{\rho r}{r-G} - Z_t\hat{V}^{-1}\right)^{-2}\left[\left(\tau_x\frac{C_t}{D_t} - \frac{\rho r}{r-G} - Z_t\hat{V}^{-1}\right) + Z_t(\hat{V}^{-1} - \tau_0)\right]
\end{aligned}$$

We notice that $\frac{\partial F}{\partial \Omega_x} = \left(\frac{C_t}{D_t}\right)^2\frac{\partial F}{\partial \Omega_f}$ since

$$\frac{\partial F}{\partial \Omega_x} = \frac{\partial F}{\partial \hat{V}^{-1}}\frac{\partial \hat{V}^{-1}}{\partial \Omega_x} = \frac{\partial F}{\partial \hat{V}^{-1}}\left(\frac{C_t}{D_t}\right)^2\frac{\partial \hat{V}^{-1}}{\partial \Omega_f} = \left(\frac{C_t}{D_t}\right)^2\frac{\partial F}{\partial \Omega_f}$$

.

Then:

$$\frac{\partial C/D}{\partial \Omega_f} = \frac{\left(\tau_x\frac{C_t}{D_t} - \frac{\rho r}{r-G} - Z_t\hat{V}^{-1}\right) + Z_t(\hat{V}^{-1} - \tau_0)}{\left(\tau_x\frac{C_t}{D_t} - \frac{\rho r}{r-G} - Z_t\hat{V}^{-1}\right)^2 - \left[\left(2\frac{C_t}{D_t}(\tau_x + \Omega_x)\right)\left(\tau_x\frac{C_t}{D_t} - \frac{\rho r}{r-G} - Z_t\hat{V}^{-1}\right) - (\hat{V}^{-1} - \tau_0)\left(\tau_x - Z_t\left(2\frac{C_t}{D_t}(\tau_x + \Omega_x)\right)\right)\right]} \tag{79}$$

Result 2, part 1: If $C/D \leq 0$, $\Omega_x < \tau_0 + \Omega_f$ and $C/D > -Z_t/2$, then $\frac{\partial C/D}{\partial \Omega_f} < 0$ and $\frac{\partial C/D}{\partial \Omega_x} \leq 0$

The numerator of (79) is

$$\left(\tau_x\frac{C_t}{D_t} - \frac{\rho r}{r-G} - Z_t\hat{V}^{-1}\right) + Z_t(\hat{V}^{-1} - \tau_0) = \tau_x\frac{C_t}{D_t} - \frac{\rho r}{r-G} - Z_t\tau_0 < 0$$

The inequality holds since we've proven that $C_t/D_t < 0$ and $r > G$.

In the denominator, however, not all the terms are negative. The denominator of (79), divided by by $\left(\tau_x \frac{C_t}{D_t} - \frac{\rho r}{r-G} - Z_t \hat{V}^{-1}\right) + Z_t(\hat{V}^{-1} - \tau_0)$ is:

$$\left(\tau_x \frac{C_t}{D_t} - \frac{\rho r}{r-G} - Z_t \hat{V}^{-1}\right) - \left(2\frac{C_t}{D_t}(\tau_x + \Omega_x)\right) + (\hat{V}^{-1} - \tau_0)\left(\tau_x - Z_t\left(2\frac{C_t}{D_t}(\tau_x + \Omega_x)\right)\right)\left(\tau_x \frac{C_t}{D_t} - \frac{\rho r}{r-G} - Z_t \hat{V}^{-1}\right)^{-1} \tag{80}$$

The only positive term is $-2\frac{C_t}{D_t}\Omega_x$. Then, is it easy to see that if C/D is sufficiently close to zero, then $-2\frac{C_t}{D_t}\Omega_x < \frac{\rho r}{r-G} + Z_t(\tau_0 + \Omega_f)$, so (80) is negative.

Thus, the numerator is negative and if C/D is sufficiently close to zero the denominator is positive, so $\frac{\partial C/D}{\partial \Omega_f} < 0$ and $\frac{\partial C/D}{\partial \Omega_x} = \left(\frac{C_t}{D_t}\right)^2 \frac{\partial C/D}{\partial \Omega_f} < 0$ if $C/D < 0$ and $\frac{\partial C/D}{\partial \Omega_x} = 0$ if $C/D = 0$. □

Proof of Result 2, part 2 *Claim: If $C/D \leq 0$, and $C/D < -\frac{2Z_t^{-1}}{3}$, then $\frac{\partial C/D}{\partial \Omega_f} < 0$ and $\frac{\partial C/D}{\partial \Omega_x} \leq 0$*

To see this, we analyze if under these new condition inequality (80) holds. We have:

$$\begin{aligned} & -\frac{\rho r}{r-G} - Z_t(\tau_0 + \Omega_f) - 2\frac{C_t}{D_t}\Omega_x - 3Z_t\left(\frac{C_t}{D_t}\right)^2(\tau_x + \Omega_x) \\ = \; & -\frac{\rho r}{r-G} - Z_t(\Omega_x) - \frac{C_t}{D_t}\Omega_x\left(2 - 3Z_t\frac{C_t}{D_t}\right) - 3Z_t\left(\frac{C_t}{D_t}\right)^2\tau_x \end{aligned}$$

So if $C/D < -\frac{2Z_t^{-1}}{3}$, we can prove the above claim:

$$\begin{aligned} = \; & -\frac{\rho r}{r-G} - Z_t(\Omega_x) - \frac{C_t}{D_t}\Omega_x\left(2 - 3Z_t\frac{C_t}{D_t}\right) - 3Z_t\left(\frac{C_t}{D_t}\right)^2\tau_x \\ < \; & -\frac{\rho r}{r-G} - Z_t(\Omega_x) - 3Z_t\left(\frac{C_t}{D_t}\right)^2\tau_x \\ < \; & 0 \end{aligned}$$

Now, combining the two previous claims, we have that if $\Omega_x < \tau_0 + \Omega_f$ and $Z_t > \frac{1}{\sqrt{3}}$, then $\frac{\partial C/D}{\partial \Omega_f} < 0$ and $\frac{\partial C/D}{\partial \Omega_x} \leq 0$. The condition $Z_t > \frac{1}{\sqrt{3}}$ implies that $\frac{-Z_t}{2} < -\frac{2Z_t^{-1}}{3}$ so with claims 3, 4 and 5 we have guaranteed the result for the entire support of C/D and thus proved result 2.

Proof of Result 6a: Ω_{ft}/Ω_{xt} does not converge to 0

If Ω_{ft}/Ω_{xt} converges to ∞, then by the first order condition, it must be that $\xi \to \infty$. It is sufficient to show that $\xi \to \infty$ violates equation (57). Rearrange (57) to get

$$\left[\xi Z_t\left(\xi^2\tau_x + (\frac{K}{\chi_f})^{\frac{1}{2}}(1 + \frac{\chi_f}{\chi_x}\xi^4)^{\frac{1}{2}} + \tau_0\right) + \xi\alpha\right] + (\frac{K}{\chi_f})^{\frac{1}{2}}(1 + \frac{\chi_f}{\chi_x}\xi^4)^{\frac{1}{2}} = 0 \tag{81}$$

The term in square brackets is negative and the one outside is positive. Assume $\xi \to \infty$. If Z_t does not go to zero, then the negative term grows faster and the equality cannot hold. So it must be that $Z_t \to 0$. Using equation (31) of the draft, that requires that both $C_{t+1} \to 0$ and $D_{t+1} \to 0$. In order for C_{t+1} to go to zero, $\hat{V} \to \tau_0^{-1}$. But since $\xi \to \infty$, from equation (29) in the main draft, $\hat{V} \to 0$, which is a contradiction.

Proof of Result 6b: As $K \to \infty$, Ω_{ft}/Ω_{xt} does not converge to ∞

If Ω_{ft}/Ω_{xt} did converge to ∞ as $K \to \infty$, then by the first-order condition (15), it would have to be that $\xi \to 0$. So it suffices to show that $\Omega_{ft}/\Omega_{xt} = \infty$ is inconsistent with $\xi = 0$, in equilibrium.

Start from the equilibrium condition (56), which must be zero in equilibrium. If $\xi \to 0$, then the first term goes to zero. The proof of lemma 4 proves, along the way, that $(1 + \xi Z_t) > 0$. (Otherwise, (56) can never be zero because it is always negative.) Thus the second term $\Omega_{xt}\xi^2(1 + \xi Z_t)$ must be non-negative.

The third term $\Omega_{ft}(1 + \xi Z_t)$ also converges to ∞ because $\Omega_{ft} \to \infty$ and $(1 + \xi Z_t) > 0$. How do we know that $\Omega_{ft} \to \infty$? In principle, Ω_{ft}/Ω_{xt} could become infinite either because Ω_{ft} became infinite or because Ω_{xt} goes to zero. But if Ω_{xt} goes to

zero and Ω_{ft} is finite, then the information processing constraint (3), which requires that the weighted sum of Ω_{ft} and Ω_{xt} be K cannot be satisfied as $K \to \infty$.

Since one term of (56) becomes large and positive and the other two are non-negative in the limit, the sum of these three terms cannot equal zero. Therefore, $\Omega_{ft}/\Omega_{xt} \to \infty$ cannot be an equilibrium.

Proof of Result 6c: there exists an equilibrium where Ω_{ft}/Ω_{xt} converges to a constant.

By the first order condition (15), we know that Ω_{ft}/Ω_{xt} converges to a constant, if and only if ξ converges to a constant. Thus, it suffices to show that there exists a constant ξ that is consistent with equilibrium, in the high-K limit.

Suppose ξ and Z_t are constant in the high-K limit. In equation (57) as $K \to \infty$, the last term goes to infinity, unless $\xi \to \frac{1}{Z_t}$. If the last term goes to infinity and the others remain finite, this cannot be an equilibrium because equilibrium requires that the left side of (57) is zero. Therefore, it must be that $\xi \to \frac{-1}{Z_t}$. The question that remains is whether ξ and Z_t are finite constants, or whether one explodes and the other converges to zero, in the high-K limit.

Suppose $\xi = -\frac{1}{Z_t}$, which is constant ($\xi = \bar{\xi}$). Then $Z_t = \bar{Z}$ is constant too. The rest of the proof checks to see if such a proposed constant- $\bar{\xi}$ solution is consistent with equilibrium. We do this by showing that ξ does not explode on contract as K increases. In other words, for $\xi = \frac{-1}{Z_t}$ to be stable and thus the ratio of fundamental to technical analysis to be stable, we need that $\partial\xi/\partial K \to 0$, in other words, ξ and therefore Ω_{ft}/Ω_{xt} converges to a constant as $K \to \infty$.

Step 1: Derive $d\xi/dK$: Start from the equilibrium condition for ξ (57) and apply the implicit function theorem:

$$\Big(3Z_t\tau_x\xi^2 + A + Z_t\tau_0\Big)d\xi + \frac{1}{2}(\frac{1}{K\chi_f})^{\frac{1}{2}}(1+\xi Z_t)(1+\frac{\chi_f}{\chi_x}\xi^4)^{\frac{1}{2}}dK$$
$$+\left[\frac{1}{2}(\frac{K}{\chi_f})^{\frac{1}{2}}(1+\xi Z_t)(1+\frac{\chi_f}{\chi_x}\xi^4)^{-\frac{1}{2}}(4\frac{\chi_f}{\chi_x}\xi^3) + Z_t(\frac{K}{\chi_f})^{\frac{1}{2}}(1+\frac{\chi_f}{\chi_x}\xi^4)^{\frac{1}{2}}\right]d\xi = 0$$

So we have

$$\frac{d\xi}{dK} = \frac{1}{2}(\frac{1}{K\chi_f})^{\frac{1}{2}}\frac{-(1+\xi Z_t)(1+\frac{\chi_f}{\chi_x}\xi^4)^{\frac{1}{2}}}{3Z_t\tau_x\xi^2 + A + Z_t\tau_0 + 2\frac{\chi_f}{\chi_x}(\frac{K}{\chi_f})^{\frac{1}{2}}(1+\xi Z_t)(1+\frac{\chi_f}{\chi_x}\xi^4)^{-\frac{1}{2}}\xi^3 + Z_t(\frac{K}{\chi_f})^{\frac{1}{2}}(1+\frac{\chi_f}{\chi_x}\xi^4)^{\frac{1}{2}}}$$

Use equation 57 to write the numerator as

$$(1+\xi Z_t)(1+\frac{\chi_f}{\chi_x}\xi^4)^{\frac{1}{2}} = -(\frac{\chi_f}{K})^{\frac{1}{2}}\xi(\xi^2 Z_t\tau_x + A + Z_t\tau_0) \tag{82}$$

Now use this to rewrite $\frac{d\xi}{dK}$ as

$$\frac{d\xi}{dK} = \frac{1}{2K}\frac{1}{\frac{3Z_t\tau_x\xi^2+A+Z_t\tau_0}{\xi(\xi^2 Z_t\tau_x+A+Z_t\tau_0)} - 2\frac{\chi_f}{\chi_x}(1+\frac{\chi_f}{\chi_x}\xi^4)^{-1}\xi^3 - \frac{Z_t}{(1+\xi Z_t)}} \tag{83}$$

Step 2: Show that $d\xi/dK \to 0$ as $K \to \infty$, as long as $X(\cdot) \not\to 0$

As $K \to \infty$, it is clear that $1/2K \to 0$. As long as the term that multiplies $1/2K$ stays finite, the product will converge to zero. Since the numerator is just 1, the second term will be finite, as long as the denominator does not go to zero. Define

$$X(\xi, Z_t) = \frac{3Z_t\tau_x\xi^2 + A + Z_t\tau_0}{\xi(\xi^2 Z_t\tau_x + A + Z_t\tau_0)} - 2\frac{\chi_f}{\chi_x}(1+\frac{\chi_f}{\chi_x}\xi^4)^{-1}\xi^3 - \frac{Z_t}{(1+\xi Z_t)} \tag{84}$$

which is the denominator of the second fraction on the rhs of equation (83). Then if $X \not\to 0$, $1/X$ is finite, then $1/2K * 1/X$ goes to zero as K gets large. Thus, we get that $\partial\xi/\partial K \to 0$ as $K \to \infty$.

Step 3: $X(\cdot) \not\to 0$.

To complete the proof, we need to show that $\bar{\xi} = -\frac{1}{\bar{Z}}$ which satisfies the equilirium condition (89) as $K \to \infty$, does not cause $X(\cdot) = 0$. We can check this directly: in equation (84), if $\xi = -\frac{1}{Z_t}$, the denominator of the last term becomes zero; so last term becomes infinite. The only term in (84) with opposite sign is the middle term, which is finite if $\xi = \frac{C}{D}$ is finite (the running assumption). If the last term of X tends to infinity and the only term of opposite sign is finite, the sum cannot be 0. Thus, for $\bar{\xi} = -\frac{1}{\bar{Z}}$, which is the limit attained in the limit as $K \to \infty$, we have that $X(\bar{\xi}) \neq 0$.

Step 4: As $K \to \infty$, if (90) holds, the real, finite-ξ solution exists.

From equations (29-32), as $K \to \infty$ at least one of the two information choices goes to ∞, so with finite, non-zero $\frac{C}{D}$:

$$\lim_{K\to\infty} \hat{V} = 0 \tag{85}$$

$$\lim_{K\to\infty} \Omega_t^{-1} = \frac{r}{\rho(r-G)} Z_t = D_{t+1}^2(\xi_{t+1}^2 \tau_0^{-1} + \tau_x^{-1}) \tag{86}$$

$$\lim_{K\to\infty} D_t = -\frac{\rho}{r}\Omega_t^{-1} = -\frac{1}{(r-G)} Z_t \tag{87}$$

A word of interpretation here: Equation (32), which defines Ω^{-1} is the total future payoff risk. As $\hat{V} \to 0$, it means the predictable part of this variance goes away as information capacity gets large. Z_t, which is the unpredictable part, remains and governs liquidity, D_t.

Next, solve (86) for D_{t+1}, backdate the solution 1 period, to get an expression for D_t, and equate it to the expression for D_t in (87). This implies that $\lim_{K\to\infty} D = \bar{D}$ is constant and equal to both of the following expressions

$$\bar{D}^2 = \frac{-rZ_t}{\rho(r-G)\bar{\xi}(\bar{\xi}^2\tau_0^{-1} + \tau_x^{-1})} = \frac{Z_t}{(r-G)^2\bar{\xi}^2} \tag{88}$$

We can cancel Z_t on both sides, which delivers a quadratic equation in one unknown in $\bar{\xi}$:

$$\bar{\xi}^2\tau_0^{-1} + \frac{r(r-G)}{\rho}\bar{\xi} + \tau_x^{-1} = 0. \tag{89}$$

In order for $\bar{\xi}$ to exist equation (89) requires that the expression inside the square root term of the quadratic formula (often written as $(b^2 - 4ac)$) not be negative. This imposes the parametric restriction

$$\left(\frac{r(r-G)}{\rho}\right)^2 - 4\tau_0^{-1}\tau_x^{-1} \geq 0. \tag{90}$$

Rearranging this to put τ_0 on the left delivers $\tau_0 \geq \underline{\tau}$, where $\underline{\tau} = 4\tau_x^{-1}\rho^2(r(r-G))^{-2}$. If we instead rearrange this to put τ_x on the left delivers $\tau_x \geq \underline{\tau}$, where $\underline{\tau} = 4\tau_0^{-1}\rho^2(r(r-G))^{-2}$.

Lemma 4 ***Balanced growth path depends on future information risk and long-lived assets.*** $|D_t| \geq \frac{\rho(r-G)}{r} C_t \left(C_{t+1}^2\tau_0^{-1} + D_{t+1}^2\tau_x^{-1}\right)$, *with strict inequality if $K > 0$.*

Proof. Use equation (57) to write

$$(1+\xi Z_t)(1+\frac{\chi_f}{\chi_x}\xi^4)^{\frac{1}{2}} = -(\frac{\chi_f}{K})^{\frac{1}{2}}\xi(\xi^2 Z_t\tau_x + \alpha + Z_t\tau_0) \tag{91}$$

Since we've proven that $\xi \leq 0$ (lemma 2). And we know from lemma 1 that if $K > 0$, then $C_t > 0$ so that $\xi < 0$ with strict inequality. The other terms on the right side are strictly positive squares or positive constants, with a negative sign in front. Thus, the right hand side of the equation (82) is positive. On the left, since $(1 + \frac{\chi_f}{\chi_x}\xi^4)^{\frac{1}{2}}$ is a square root, and therefore positive, this implies that $(1 + \xi Z_t)$ must be positive as well for the equality to hold. $(1 + \xi Z_t) > 0$ implies that $Z_t < -1/\xi$ Substitute for Z_t to get the result. This result puts a bound on how liquid the price can be. The liquidity is bounded by the product of price informativeness and un-learnable, future risk. ■

To get from this result to balanced growth requires the following steps: The result says that $C_t/|D_t| < \left(\rho\left((r-G)/r\right)(C_{t+1}^2\tau_0^{-1} + D_{t+1}^2\tau_x^{-}\right.$ The first term is just fixed parameters. The second term, $(C_{t+1}^2\tau_0^{-1} + D_{t+1}^2\tau_x^{-1})$ is the variance of the part of tomorrow's price that depends on future shocks, x_{t+1} and y_{t+1}. This is the future information risk. It converges to a large, positive number as K grows. When information is abundant, high future information risk pushes $C_t/|D_t|$ down, toward a constant.

In contrast, if demand analysis were to keep growing faster than fundamental analysis (Ω_{ft}/Ω_{xt} were to fall to zero), by the first order condition (15), it means that $(C_t/D_t)^2$ keeps rising to infinity. But if $(C_t/D_t)^2$ is converging to infinity, then at some point, it must violate the inequality above because the right side of the inequality is decreasing over time. Thus, demand analysis cannot grow faster than fundamental analysis forever.

The only solution that reconciles the first order condition, with the equilibrium price coefficients, is one where $(\Omega_{ft}/\Omega_{xt})$ stabilizes and converges to a constant. If fundamental analysis grows proportionately with demand analysis, the rise in the amount of fundamental analysis makes prices more informative about dividends: C_t increases. Proportional growth in fundamental and demand analysis allows C_t to keep up with the rise in D_t, described above. Therefore, as information technology grows ($K \to \infty$), a stable C_t/D_t rationalizes information choices (Ω_{xt}, Ω_{ft}) that grow proportionately, so that Ω_{xt}/Ω_{ft} converges to a constant.

C Robustness of Numerical Results

We want to investigate the effect of changing parameters on the predictions of the numerical model. First, we show how re-calibrating the model with different risk aversion affects the values of other calibrated parameters. Then we show how changes in risk aversion and other parameters have modest effects on results. We consider changes to the exogenous, yet important parameters of time preference, risk aversion and terminal capacity, first. Then, we consider altering endogenous, calibrated parameters of dividend innovation variance, hedging innovation variance and relative cost of demand information.

Lower risk aversion The steady state coefficients with low risk aversion $\rho = 0.05$ are We find $A_T = 16.03$, $C_T = 7.865$ and $D_T = -3.0$. A_T and C_T are unchanged, while D_T changed from $= -5.7$, for high risk aversion to 3.0. Table 2 shows the original calibration and a lower-risk aversion calibration to highlight how the other parameters adjust when risk aversion changes.

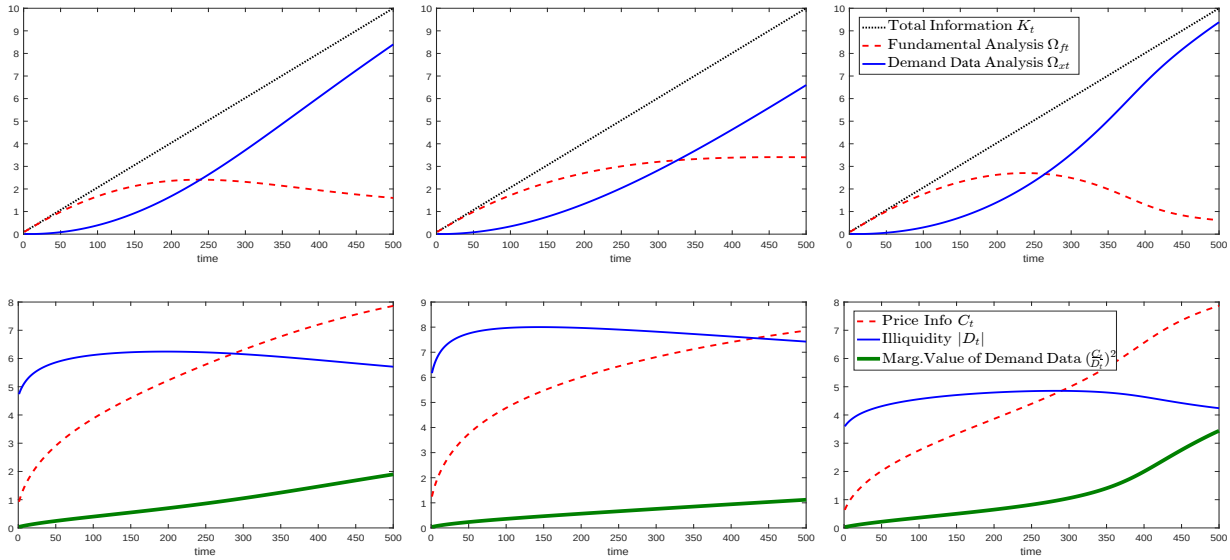
Table 2: Parameters

	low risk av	high risk av
G	0.9365	0.9365
μ	0.235	0.4153
τ_0^{-1}	0.2575	0.2445
τ_x^{-1}	1.9850	0.5514
χ_x	10.6625	0.6863
r	1.03	1.03
ρ	0.05	0.1

Similarly, after re-calibrating, risk aversion makes only a minor difference. With $\rho = 0.05$, demand analysis still outstrips fundamental analysis between periods 4 and 5. But if falls slightly more slowly. The ending value of Ω_{ft} is 1.8, instead of 1.6.

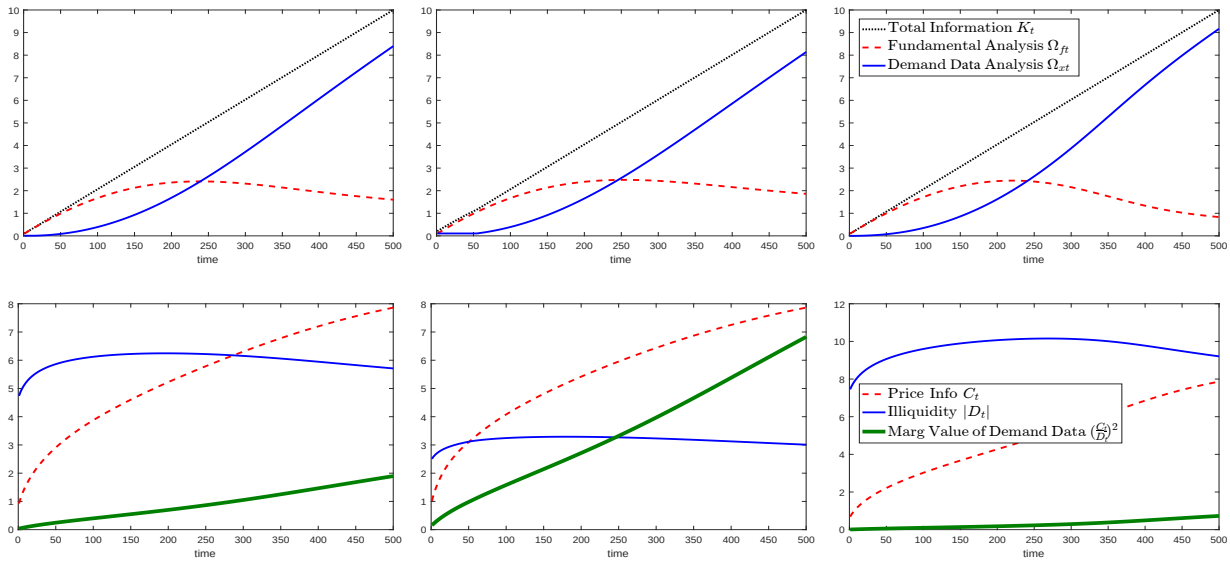
Changes to fixed parameters We consider lower/higher time preference, risk aversion and terminal capacity. Whenever a parameter is changed, all other parameters are re-calibrated to match that new value and the numerical model is simulated again.

Figure 10: Results with different rates of time preference. The first row is information acquisition, the second row is capacity allocation and the third row are the price coefficients. Column 1 is the baseline calibration used in the paper, corresponding to $r = 1.03$. Column 2 displays the path with $r = 1.01$ and column 3 with $r = 1.05$.



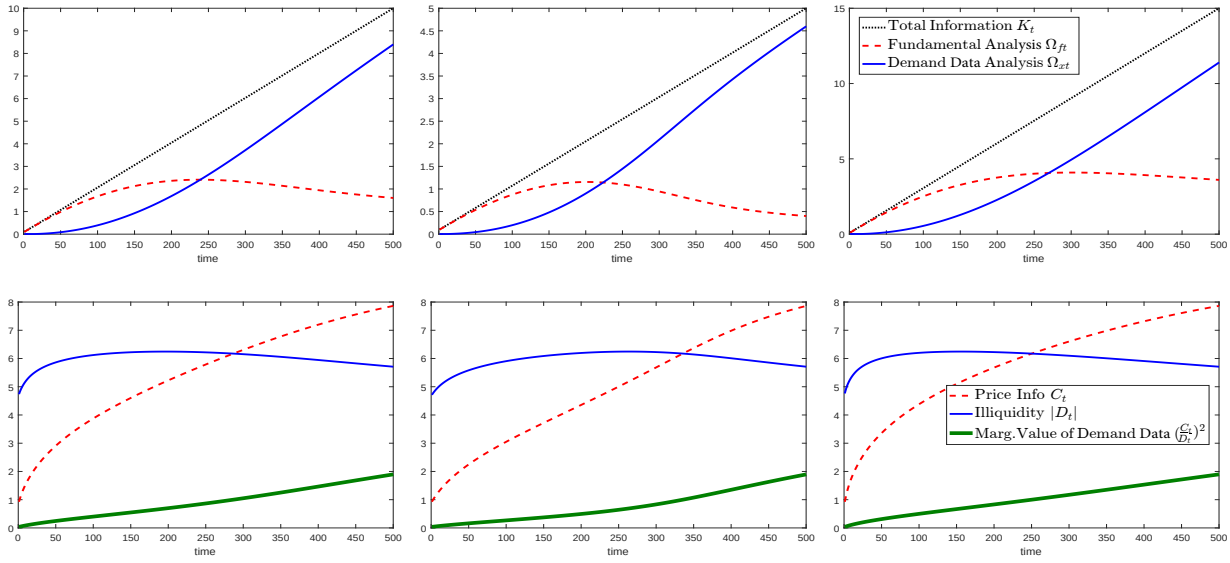
(a) Baseline $r = 1.03$ (b) $r = 1.01$ (c) $r = 1.05$

Figure 11: Results with different risk premia. The first row is capacity allocation and the second row is the price coefficients. Column 1 is the baseline calibration used in the paper, corresponding to $\rho = 0.1$. Column 2 displays the path with $\rho = 0.05$ and column 3 with $\rho = 0.2$.



(a) Baseline $\rho = 0.1$ (b) $\rho = 0.05$ (c) $\rho = 0.2$

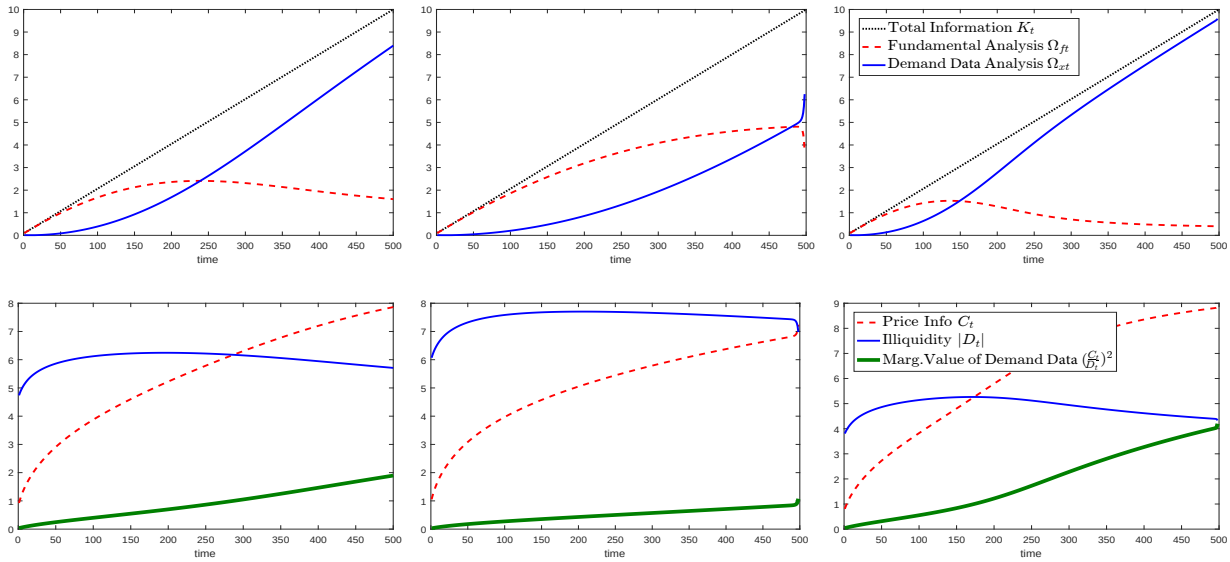
Figure 12: Results with different terminal capacities. The first row is capacity allocation and the second row is the price coefficients. Column 1 is the baseline calibration used in the paper, corresponding to $K_T = 10$. Column 2 displays the path with $K_T = 5$ and column 3 with $K_T = 15$.



(a) Baseline $K_T = 10$ (b) $K_T = 5$ (c) $K_T = 15$

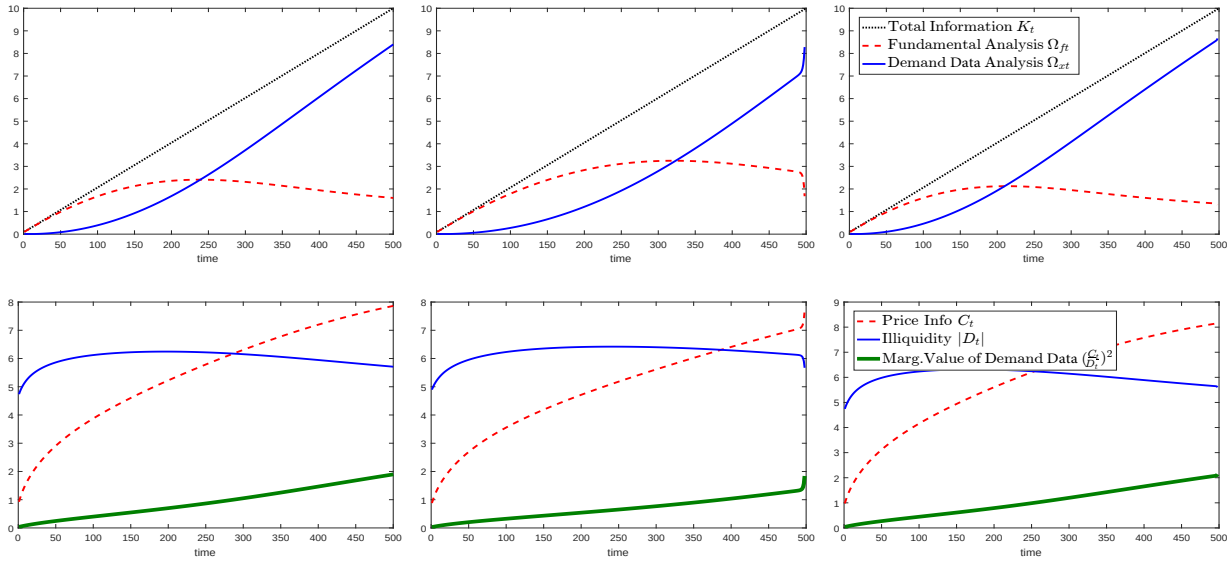
Changes to calibrated parameters We consider lower/higher dividend shock variance, hedging shock variance and relative cost of demand information. As these parameters are determined jointly by the calibration, we cannot simply change them and re-calibrate as above. Rather, we calibrate to the baseline then change the parameter of interest for the experiment and then recover the model's terminal values associated with that new parameter of interest. It is important to note that we do not re-calibrate the other parameters when we make changes here.

Figure 13: Results with different terminal values of τ_0. The first row is capacity allocation and the second row is the price coefficients. Column 1 is the baseline calibration used in the paper. Column 2 displays the path for a lower τ_0 and column 3 for a higher τ_0.



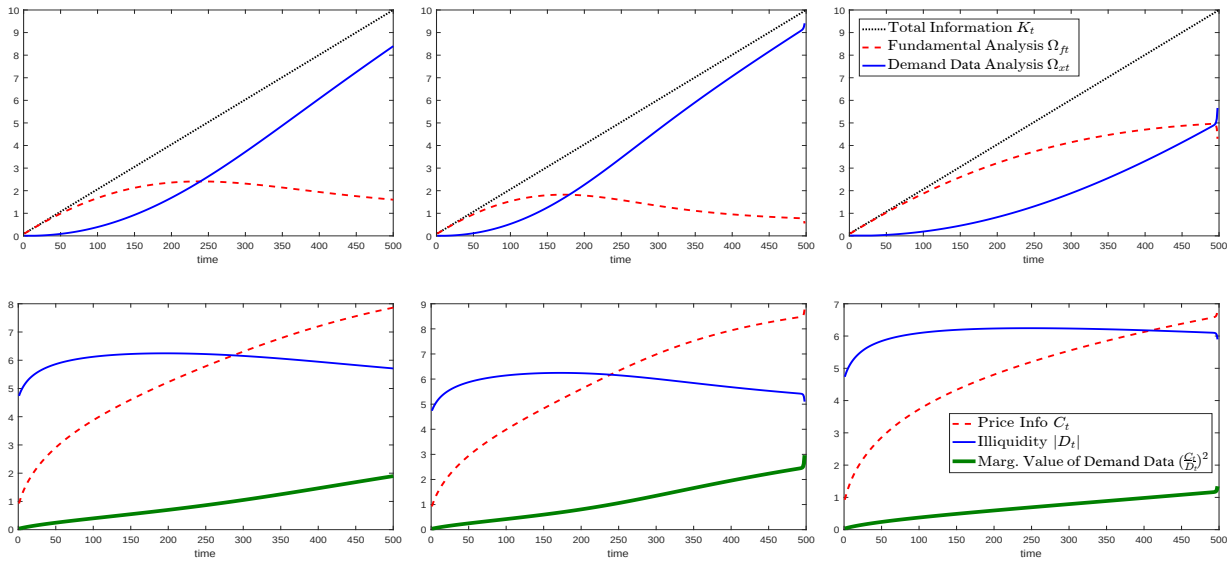
(a) Baseline τ_0 (b) $\tau_0^* = 0.8\tau_0$ (c) $\tau_0^* = 1.2\tau_0$

Figure 14: Results with different terminal values of τ_x. The first row is capacity allocation and the second row is the price coefficients. Column 1 is the baseline calibration used in the paper. Column 2 displays the path for a lower τ_x and column 3 for a higher τ_x.



(a) Baseline τ_x (b) $\tau_x^* = 0.8\tau_x$ (c) $\tau_x^* = 1.2\tau_x$

Figure 15: Unbalanced growth model under different terminal values of χ_x. The first row is capacity allocation and the second row is the price coefficients. Column 1 is the baseline calibration used in the paper. Column 2 displays the path for a lower χ_x and column 3 for a higher χ_x.

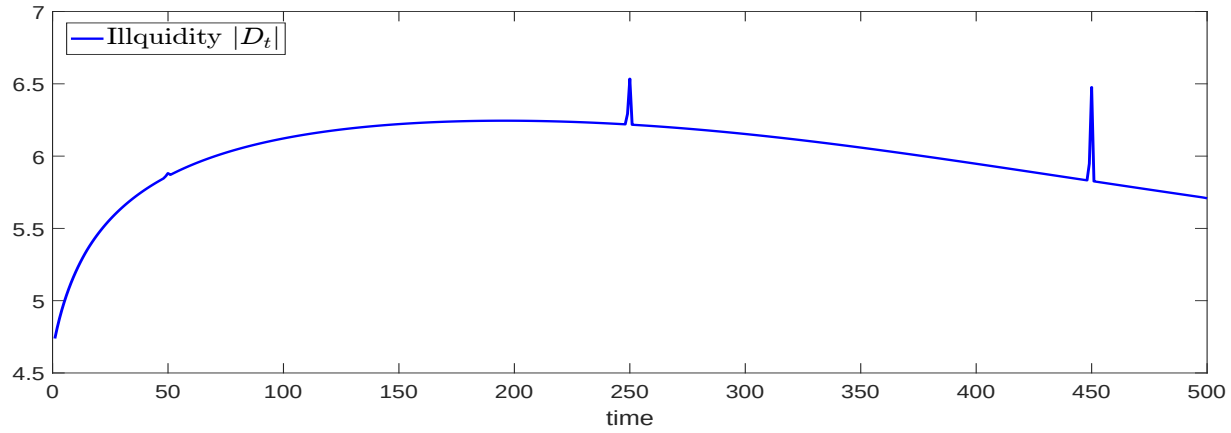


(a) Baseline χ_x (b) $\chi_x^* = 0.5\chi_x$ (c) $\chi_x^* = 2\chi_x$

Liquidity Fragility

The main text shows that liquidity becomes more sensitive to one-period changes in future expected dividend variance. A similar degree of liquidity fragility arises from changes in the cost χ_x of demand data processing. Figure 16 shows the reaction of liquidity $|D_t|$ to a one-time, unexpected doubling of χ_x. Notice that an equal sized movement in χ_x has a small effect when technology is low and a larger effect as financial technology progresses.

Figure 16: Fragility of Liquidity: Cost of Demand Data Processing



D Data Appendix

Asset price and return data for calibration Calibrating the numerical model requires some price and dividend series that accurately represents the market as a whole. However, it is not clear what the best method for defining this representative asset it. One option is to pick some historically representative stock, such as General Electric, or Apple, but even though these stocks may be the best available representative, that does not mean that they capture the market as a whole. Another option is to take an index, such as the S&P500, as a representative of the market. While using an index may capture more about the market, its realizations in levels are not representative of actual prices or dividends, but rather just a tracking mechanism of the evolution of the market. Aware of the deficiencies in both approaches, we choose the added information of the S&P500 index and live with the difficulty of normalizing prices and dividends to better fit a representative asset.

We use CRSP's monthly S&P500 data from 2000-2015 to calibrate the steady-state of our model. Cleaning and normalizing the data takes several steps:

1. Impute dividends. In order to impute a dividend series for the market as a whole, we use the price, return including dividends and return excluding dividends series.

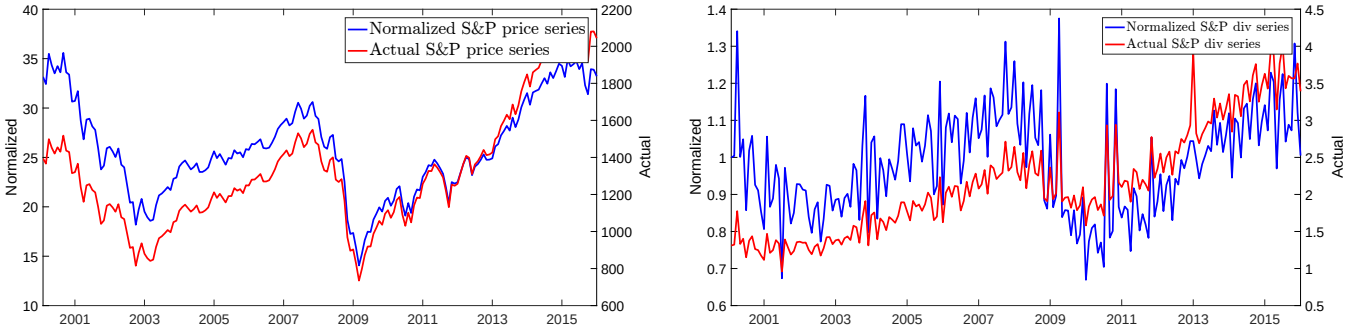
$$d_t = p_t\Big(\frac{p_{t+1}+d_t}{p_t} - \frac{p_{t+1}}{p_t}\Big)$$

2. Clean up data. We log de-trend and deseasonalize the price and dividend series and then normalize the dividend series to 1.

3. Normalize data. In order to match the price series to dividends in a meaningful way, we take price-dividend (PD) ratios from CRSP for all S&P500 members and calculate an annual market cap.-weighted PD ratio. Then, prices are normalized year-by-year to match that observed PD ratio.

It turns out that this normalization process loses little of the dynamics of the index series, while also being far more accurate in terms of the describing the level relationship between prices and dividends for a representative asset of the market. Figure 18(a) displays the normalized price series with the actual price-index series. Figure 18(b) displays the normalized dividend series with the imputed dividend series described above.

Figure 17: Comparison of normalized series with actual series. Source: CRSP



(a) Price comparison

(b) Dividend comparison

Hedge Fund Data: Lipper TASS Database The figure showing the shift over time in investment strategies is based on hedge fund data from Lipper. Lipper TASS provides performance data on over 7,500 actively reporting hedge funds and funds of Hedge Funds and also provides historical performance data on over 11,000 graveyard funds that have liquidated or stopped reporting. In addition to performance data, data are also available on certain fund characteristics, such as investment approach, management fees, redemption periods, minimum investment amounts and geographical focus. This database is accessible from Wharton Research Data Services (WRDS).

Though the database provides a comprehensive window into the hedge fund industry, data reporting standards are low. There is a large portion of the industry (representing about 42% of assets) that simply do not report anything (Edelman, Fund, and Hsieh, 2013). Reporting funds regularly report only performing assets (Bali, Brown, and Caglayan, 2014). While any empirical analysis must be considered with caution, some interesting stylized facts about the current state and evolution of the hedge fund industry do exist in these data.

All hedge fund data is monthly and come from Lipper TASS. In total, the database reports on 17,534 live and defunct funds. Data are from 1994-2015, as no data was kept on defunct funds before 1994. A significant portion of this total consists of the same fund reported in different currency and thus are not representative of independent fund strategies (Bali, Brown, and Caglayan, 2014). Therefore, we limit the sample to only U.S.-based hedge funds and remove funds of funds. This limits the sample size to 10,305 funds. As the focus is to gain insight into the division between fundamental and quantitative strategy in the market, We further limit the sample to the 7093 funds who explicitly possess these characteristics, described below. Firms are born and die regularly throughout the sample. There are never more than 3000 existing, qualifying funds at any point in time. By the end of 2015, there were just over 1000 qualifying funds.

Lipper TASS records data on each fund's investment strategies. In total, there are 18 different classifications and most of these classifications have qualities of both fundamental and quantitative analysis. An example of a strategy that could be considered both, "Macro: Active Trading strategies utilize active trading methods, typically with high frequency position turnover or leverage; these may employ components of both Discretionary and Systematic Macro strategies." However, 4 strategy classifications explicitly denote fund strategy as being fundamental or quantitative. They are:

- Fundamental: This denotes that the fund's strategy is explicitly based on fundamental analysis.
- Discretionary: This denotes that the fund's strategy is based upon the discretion of the fund's manager(s).
- Technical: This denotes that the fund deploys a technical strategy.
- Systematic Quant: This denotes that funds deploy technical/algorithmic strategy.

Using these classifications, it is possible to divide hedge fund strategy into three broad groups:

- Fundamental: Those funds whose strategy is classified as fundamental and/or discretionary, and *not* technical and/or sytematic quant.
- Quantitative: Those funds whose strategy is classified as technical and/or systematic quant, and *not* technical and/or sytematic quant.
- Mixture: Those funds whose strategy is classified as having at least one of fundamental or discretionary and at least one of technical or systematic quant.

From 2000-2015, the assets under management (AUM) has systematically shifted away from fundamental firms to firms that deploy some sort of quantitative analysis in their investment approach. In mid-2000, the assets under management per fundamental firm was roughly 8 times the size of that in a quantitative or mixture firm, but this had equalized by 2011, representing a true shift away from fundamental analysis and towards quantitative analysis in the hedge fund industry.