

Financial Exaggeration and the Allocation of Capital*

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ABSTRACT

Principals often face financial exaggeration when they allocate scarce resources. This is because agents usually have considerable discretion over their financial reporting policies and also have a conflict of interest when they compete for resources. Because exaggeration leads to lower quality information, the principal may indeed make a suboptimal allocation. However, as we show in this paper, in many cases the principal's expected payoff is actually higher with exaggeration, despite the loss of information caused by misreporting. This is because of a complementarity that exists between the agents' effort provision and their ability to exaggerate. As we demonstrate, it is often suboptimal for a principal to curb misreporting or monitor outcomes, even if it is costless to do so. Our results may explain why exaggeration is ubiquitous and provide implications for settings in which allocation decisions are prevalent: venture capital markets, loans, allocation in internal capital markets, and other business agreements.

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“Entrepreneurs tend to exaggerate. They exaggerate the success of their business when talking to startup investors. They exaggerate the market potential of their products to find distribution partners. They exaggerate the soundness of their strategy to recruit employees.”

–*Asheesh Advani, Founder of CircleLending and CEO of Covestor*¹

1 Introduction

Financial exaggeration is ubiquitous when agents compete for scarce resources (e.g., venture capital, bank loans, internal capital markets). This is because the state of the art in financial reporting grants ample discretion to forecasters and every agent has an incentive to put his best foot forward when trying to persuade a principal to choose his opportunity.² Because financial reporting depends on a complicated mix of judgement, gut instincts, biases, incentives, and possibly hubris, a principal who is tasked with making an allocation decision is often uncertain whether the intrinsic value of any proposal actually matches what is reported. Regrettably, this uncertainty might lead the principal to select a less desirable project, even if the principal is a rational Bayesian.

Decades of study of agency theory would suggest that financial exaggeration should (at least weakly) destroy the principal’s surplus because of decreased access to useful information. It turns out, as we show in this paper, that this is not necessarily the case. We analyze how financial exaggeration arises in equilibrium, how it interacts with the agents’ effort provision, and its effect on payoffs. We show that financial exaggeration is a complement to effort provision, and that it may enhance the quality of investments and increase the principal’s payoff.

What explains this departure? The answer rests on how the economics of capital allocation contrast with standard moral hazard models. In the typical agency problem, the agent’s productive effort is not observed by the principal, but the project’s outcome is observable and verifiable. In this case, the principal is always better off with more accurate information. When agents fight for scarce resources, the information structure is the mirror image: the principal observes each project’s probability of success (i.e. the agent’s effort) before she selects a project, but the project’s

¹See <http://www.entrepreneur.com/howto/startyourbiz.html> or <http://www.entrepreneur.com/article/197224>.

²For example, the discounted cash flow (DCF) method leaves its exact structure and its inputs unspecified: the length of the forecast period, the discount factor, the terminal growth rate, and the forecasted inputs, to name a few.

value is realized only *after* she chooses to pursue it and may be difficult to verify. Given this fundamental contrast, we arrive at the opposite result: the principal is often better off when she has less information about the project's outcome at the time she decides.

In our main model, a principal has a scarce, indivisible resource that she would like to allocate to one of two symmetric agents. Each agent develops a project of uncertain value, which requires the principal's resource as an essential input. At the beginning of the game, each agent chooses the quality of his production technology. A higher quality is more costly for the agent, but increases the chance that his project is highly valuable. Next, each agent chooses a financial reporting policy, which determines how the outcome of his project development is communicated to the principal. The policy may involve exaggeration, in which case the report overstates the project's value. Following this, each agent privately observes the outcome of his work and generates a report for the principal. The principal observes each agent's production technology, financial reporting policy, and the reports, and allocates the resource based on rational expectations. The principal's goal is to select the most valuable project, but each agent's goal is merely to attract the resource to his project, irrespective of its eventual value for the principal.

In equilibrium, each agent selects a symmetric quality, generating valuable projects with the same probability. Subsequently, the choice of whether and how much to exaggerate depends on an important tradeoff: exaggeration increases the probability that the principal receives a good report, but it also makes the principal treat a good report with greater skepticism (as a Bayesian). If the marginal cost of quality is low, agents select high quality technologies, so that projects are likely to be valuable. In this case, agents are more concerned with avoiding the principal's skepticism than with concealing bad outcomes, and the agents select reporting policies that perfectly reveal project value. However, if the marginal cost of quality is high, the agents choose lower quality technologies. Here, concealing a bad outcome becomes a more significant concern for the agents, and the degree of exaggeration is determined by a mixed strategy.

If financial exaggeration takes place, the principal may be persuaded to allocate the resource to a lower value project. The principal could eliminate this inefficiency by requiring that the agents report truthfully (i.e. monitoring). However, this turns out to be suboptimal *ex ante* because

it induces the agents to choose less productive technologies. A complementarity exists between investment quality and exaggeration. Indeed, higher investment quality reduces the principal's skepticism, so that each agent can exaggerate more without undermining the credibility of his good report. With a higher marginal cost of quality, the ability to exaggerate is more valuable. Thus, the incentive to invest is stronger when exaggeration is allowed. As such, we show that it is better for the principal to cope with some *ex post* inefficiency in order to increase the probability that she allocates capital to a high value project. Furthermore, because the principal is indifferent between monitoring and not when the marginal cost of quality is low, we show that it is weakly suboptimal for the principal to require truthful reporting, even if doing so is costless.

Following this, we consider an extension in which the conflict of interest between the parties is not as severe. Here, each agent still prefers that the principal invest in his project, but if the principal does so, the payoff to the principal and agent are identical. In this case, the equilibrium financial reporting strategies perfectly reveal project values. Also, compared to our main model with full monitoring, there are higher levels of quality and better expected payoffs to the principal. However, the comparison with the main model when exaggeration is allowed is equivocal. If the cost of quality is sufficiently high, dealing with strongly biased agents and coping with exaggeration leads to higher investment quality and superior expected payoffs to the principal.

Subsequently, we analyze what happens when the agents are *ex ante* asymmetric: one agent's marginal cost of quality is higher than the other's. This situation naturally arises within an internal capital market in which one division is weaker than another. We find that, whether or not the principal monitors reports, equilibrium investment in quality is asymmetric, whereby the stronger agent invests more in quality. Two interesting findings emerge from this analysis. First, consistent with the complementarity described above, when exaggeration is allowed the agent with the higher quality technology exaggerates his project's value more than his competitor.³ Thus, the agent with the more productive technology is less likely to communicate that he has generated a low value project. Second, as in the case of symmetry, the complementarity between exaggeration and quality increases both agents investments, but the effect is stronger for the weaker agent. The

³Technically, this is because the stronger agent's distribution function in the mixed strategy first-order stochastically dominates the weaker agent.

relative increase in quality (compared to full monitoring) is higher for the weaker agent. While it is the stronger agent who exaggerates more, by committing not to monitor the principal motivates the weaker agent to step up to the plate.

Finally, we consider the effect of uninformative reporting on our results. In essence, this is a robustness-check to evaluate whether changing the timing of the game alters our primary result that less precise information leads to greater investment in quality and a higher payoff to the principal. In our main model, the agents both choose whether to exaggerate before they observe the outcomes of their efforts. We relax this and allow each agent to choose a reporting strategy after observing his project's value. Not surprisingly, the agents have a weakly dominant strategy to always report that their project is highly valuable. Given this, the principal rationally ignores the agents' reports and makes the allocation decision based only on the agents' production technologies. Anticipating this, however, both agents compete more on quality, which makes the principal better off compared to full monitoring and strategic exaggeration. This holds whenever the agents are sufficiently similar *ex ante*, making the results in the paper robust to different timing assumptions in the model.

The rest of the paper is organized as follows. We provide a brief literature review in Section 2. We then pose our base model in Section 3 and solve it in Section 4. In Section 5, we reconsider our results with a less severe conflict of interest. In Section 6, we consider equilibrium outcomes when agents are *ex ante* asymmetric. We consider uninformative reporting in Section 7 and evaluate the robustness of our results to changes in the timing of the model. Section 8 concludes. All mathematical proofs are in the the appendix.

2 Literature Review

Decision-makers are commonly faced with the predicament that we study in this paper: they need to rely on people for information who themselves are affected by this information. Since the choice of information can be manipulated, this can be considered a game of persuasion in which a principal rightfully needs to view the information that is conveyed with skepticism (Milgrom and Roberts, 1986). The importance of persuasion is now well-appreciated. Indeed, as McCloskey and Klammer (1995) state persuasion is the “third part of economic talk”, accounting for one quarter of

US GDP.⁴ Our contribution in this paper is to show that the exaggeration used to persuade the principal turns out to improve his expected welfare. As such, our paper is related but distinct from Kamenica and Gentzkow (2011), who derive conditions under which a Sender of information can manipulate their signal for self-serving purposes and succeed on getting a Bayesian Receiver to act in the Sender's interest. In their setting, the Sender solves an optimal information transmission problem and successfully persuades a Bayesian Receiver. But, the Receiver is not better off. In fact, the Receiver is generally indifferent because all of the gains from information production are appropriated by the Sender. In contrast, in our model, the Receiver is indeed better off and in many cases should optimally abet the use of exaggeration.

Our work also contributes to a long line of papers that imply that people may be better off ignoring information or by committing not to collect it. Starting with Hirshleifer (1971), welfare may be lower when information is made publicly available. Bikhchandani, Hirshleifer, and Welch (1992) show that public disclosures can engender inefficient informational cascades when they lead agents to ignore the value of their private information. Teoh (1997) studies the provision of public goods and shows that public disclosures may exacerbate the underinvestment and free-riding problems associated with team production. Burguet and Vives (2000) model a series of short-lived agents who may exert costly effort to learn about a common random variable. Since agents fail to account for the information their actions reveal to later generations, the release of public information may decrease welfare since agents invest less in acquiring private information. Morris and Shin (2002) study a beauty contest game and show that the release of public information may induce agents to ignore their private signals and inefficiently herd. Angeletos and Pavan (2007) generalize Morris and Shins (2002) analysis and characterize the conditions under which public disclosure destroys economic surplus, given that agent actions may be substitutes or complements. Finally, Amador and Weill (2012) consider a continuous-time model to explore how public disclosures inefficiently slow learning.

As noted in the Introduction, the persuasion that we consider in this paper applies to internal capital markets. Starting with Williamson (1975) and Donaldson (1984) and then later with Stein

⁴According to McCloskey and Klamer (1995) the three parts of economic talk are issuing orders, conveying information, and persuasion.

(1997), it has long been recognized that agents who run divisions within firms compete for resources as top management seeks to allocate them to the best opportunities.⁵ As Stein (1997) points out, “there is a tendency for these agents to overstate their investment prospects”. In turn, this may lead to efficiency losses as in Scharfstein and Stein (2000) who analyze divisional rent-seeking. Our paper contributes to this literature by taking a step back and considering the level of competition that takes place in the first place. As we show, allowing exaggeration actually heightens the intensity of competition. So, if the CEO of a firm were to fully monitor his divisional managers, which may be reasonable in some cases, his allocation decisions would be more accurate and efficient, but his own personal payoff would be lower. Further, if the CEO were faced with the problem of getting a weaker division to not give up because of their relative inferiority, allowing exaggeration may help to improve effort provision.

Finally, our paper also adds to a large literature on agency conflicts in venture capital markets, where entrepreneurs compete for capital. Starting with Sahlman (1990), it has been long appreciated that there exist many agency conflicts associated with venture capital investments. These may be managed strategically with sequential investments (Gompers, 1995), syndication (Lerner, 1994), or convertible securities. However, this literature has focused on conflicts that arise after an investment has been made. Also, the conflict typically involves two parties: the investor and the entrepreneur. Our paper adds to this literature because we address the conflicts that may arise *before* investment is made. We analyze how a venture capitalist optimally chooses among multiple opportunities. As we show, surprisingly, in some cases a venture capitalist may wish to commit to forego some future control in order to increase the competition from potential suitors.

3 Base Model

Consider that a principal has a scarce resource that she wishes to allocate to one of two agents. The resource could be a bank loan, a venture capital investment, internal capital within a firm, or a commitment to enter into a business relationship (e.g., supplier contract or labor contract). Each agent invests in a production technology that yields a positive NPV project. As such, absent any

⁵As such, the investment of the divisions is strongly related to the cash flows generated by the rest of the firm (Lamont, 1996; Shin and Stulz, 1996).



Figure 1: At $t = 0$, each agent chooses the quality of their production technology, q_i . At $t = 1$, each agent chooses the amount they exaggerate their financial performance. Then, at $t = 2$, each agent realizes their type τ_i and makes a report to the principal $s_i \in \{G, B\}$. Finally, at $t = 3$, the principal allocates her capital to one of the projects.

further information, the principal prefers to make an investment rather than sit out of the market. However, the principal's goal is to invest in the better project, whereas each agent strictly wishes to attract the resource towards his own opportunity.

Figure 1 illustrates the timing of the game. At $t = 0$, each agent $i \in \{a, b\}$ simultaneously chooses the quality of his production technology, which we denote as $q_i \in [0, 1]$. Given this, agent i 's eventual project will have a high value ($\tau_i = H$) with probability q_i and low value ($\tau_i = L$) with probability $1 - q_i$. Examples of investment in q_i might be the intensity of an agent's R&D efforts or the skill of his management team. Investment in q_i is associated with a convex cost

$$C(q_i) = \frac{q_i^2}{\rho},$$

and is public information.⁶

At $t = 1$, each agent simultaneously chooses a reporting policy, which determines how financial projections are communicated to the principal.⁷ Each agent can claim that the value of his project is high, $s_i = H$, or low $s_i = L$. Each agent always reports $s_i = H$ when he has a valuable project, ($s_i = H$ whenever $\tau_i = H$), but he may misreport when $\tau_i = L$. To capture this, suppose that agent i chooses the probability $\theta_i \in [0, 1]$ with which a low value project is reported as high value. As such,

$$Pr(s_i = H | \tau_i = H) = 1 \text{ and } Pr(s_i = H | \tau_i = L) = \theta_i.$$

If $\theta_i = 0$, then agent i always reports truthfully. The greater the value of θ_i , the higher the

⁶For now, the agents are ex ante identical. In Section 6, we consider what happens when $\rho_a > \rho_b$.

⁷In Section 7, we reconsider the timing of the game and allow the agents to observe outcomes before choosing a reporting strategy. There we show that the our results are, if anything, magnified.

probability that an agent exaggerates the prospects of a low-type opportunity. We refer to the variable θ_i as the level of financial exaggeration for agent i .

While we focus on financial exaggeration per se in this setting, θ has two potential alternative interpretations. First, θ might parameterize the effort that agents employ to stress test their financial projections to identify bad projects. For example, if the agent performs thorough sensitivity analysis and can confidently report that the project is high-type, then $\theta = 0$ (i.e., there is no possibility that the project could turn out to be a low-type). If the agent employs no effort, then $\theta = 1$ and projects could turn out to be low-type, even if they are reported as high-types. Thus, $\Pr(s_i = L | \tau_i = L) = 1 - \theta$, could be interpreted as the intensity or scrutiny directed by the agent toward uncovering a low quality project. The second interpretation of θ might be the degree to which financial information is obfuscated. In an alternative model in which a principal has a cost of sorting out financial projections, $\theta = 0$ makes it easy to contrast high- and low-type projects, whereas $\theta = 1$ makes this impossible and an intermediate value of θ obfuscates project quality in a way that induces the principal to sometimes make mistakes. The outcome of our analysis that follows could be applied equally well to either of these alternative interpretations.

At $t = 2$ each agent privately observes the value of his project $\tau_i \in \{H, L\}$ and makes a report $s_i \in \{H, L\}$ consistent with his financial reporting strategy. Finally, at $t = 3$, the principal allocates her capital and subsequently learns the outcome. If she invests in a high value project, she receives a payoff of one. Otherwise, her payoff is zero. Given this, the principal's expected payoff of making an investment with each agent is equal to the posterior probability that $\tau_i = H$. It is therefore sequentially rational for her to invest in the project that she believes is more likely to be $\tau_i = H$. If she holds the same beliefs about each project, then we assume that she randomizes fairly between them. Once the investment is made, the true type of the project is revealed and the payoffs are realized. For now, we consider agents who seek only to attract capital to their opportunity: each agent receives one if the principal invests in his project, regardless of its value and zero otherwise. In Section 5, we consider an agents with preferences that are more closely aligned with the principal.

4 Equilibrium Characterization

4.1 Monitoring Benchmark

We begin by analyzing a benchmark in which financial reporting must be truthful, i.e., $\theta_i = \theta_j = 0$. In the absence of exaggeration, the principal is perfectly informed about the true value of each project when she invests. Thus, when each agent selects his production technology, he anticipates that $\Pr(\tau_i = H) = q_i$ and $\Pr(\tau_i = L) = 1 - q_i$. Thus, for fixed investment levels (q_a, q_b) , the expected payoffs are

$$u_a(q_a, q_b) = q_a(1 - q_b) + \frac{1}{2}(q_a q_b + (1 - q_a)(1 - q_b)) = \frac{1}{2}(1 + q_a - q_b) - \frac{q_a^2}{\rho}$$

$$u_p(q_a, q_b) = 1 - (1 - q_a)(1 - q_b).$$

Agent i 's project receives the scarce resource with probability $\frac{1}{2}$ when the realized types of both opportunities are the same, and with probability 1 when the project is high quality and his competitor's is low. By inspection, the marginal benefit of improving quality for either agent is independent of the other agent's choice and each agent's investment level satisfies the first order condition: $1/2 = 2q_i/\rho$. Therefore, at $t = 0$, each agent has a dominant strategy to choose $q_i = \rho/4$ and each agent is equally likely to receive the resource. The associated expected payoffs are

$$u_i = \frac{1}{2} - \frac{\rho}{16} \quad \text{and} \quad u_p = 1 - (1 - \frac{\rho}{4})^2 = \frac{\rho}{2} - \frac{\rho^2}{16}. \quad (1)$$

In what follows, we will compare these quantities to when financial exaggeration is allowed.

4.2 Equilibria with Financial Exaggeration

We solve for the Perfect Bayesian Equilibria of the game and proceed by backward induction. We first characterize the equilibrium choices of θ_i for each agent given the quality investments (q_i, q_j) made at $t = 0$. Following that, we analyze the symmetric Nash equilibrium in quality (q^*, q^*) .

4.2.1 Strategic Financial Exaggeration

If agent i chooses investment level q_i at $t = 0$ and exaggeration θ_i at $t = 1$, then the probability that a project is reported as high-type ($s_i = H$) is

$$r_i = q_i + \theta_i(1 - q_i).$$

This probability is clearly increasing in θ_i . At the same time, the principal's updated belief that a project with $s_i = H$ is in fact $\tau_i = H$ is computed from Bayes' rule:

$$g_i = \frac{q_i}{q_i + \theta_i(1 - q_i)} = \frac{q_i}{r_i}.$$

An agent's choice of θ_i involves a simple tradeoff. Increasing θ_i assigns $s_i = H$ to projects more often and increases the overall probability that the principal gets a good report, but causes the principal to view good signals with greater skepticism. Thus, the decision to exaggerate performance involves a tradeoff between the probability that a project "looks good" and *how good* it actually looks.

If θ_i and θ_j are such that $g_i > g_j$, then the principal will invest in project i whenever both projects are reported to be high-types. In this case, therefore, agent i receives the resource whenever he gets a good realization, regardless of the other agents draw, and will also receive the resource with probability $\frac{1}{2}$ when both agents report $s_i = L$. Therefore, following the investment decisions (q_i, q_j) at $t = 0$, agent i 's expected payoff as a function of the exaggeration levels (θ_i, θ_j) is:

$$u_i(\theta_i, \theta_j | q_i, q_j) = \begin{cases} r_i(1 - r_j) + \frac{1}{2}(1 - r_i)(1 - r_j) & \text{if } g_i < g_j \\ r_i + \frac{1}{2}(1 - r_i)(1 - r_j) & \text{if } g_i > g_j \\ r_i(1 - \frac{1}{2}r_j) + \frac{1}{2}(1 - r_i)(1 - r_j) & \text{if } g_i = g_j. \end{cases} \quad (2)$$

We make two observations about best responses. First, given g_j , agent i always prefers to select a θ_i for which g_i is marginally higher than g_j , rather than for which $g_i = g_j$. This means that an equilibrium involving $g_i = g_j$ is possible only at $g_i = g_j = 1$ (which corresponds to the no exaggeration case of $\theta_i = \theta_j = 0$) at which point it is no longer possible to increase g_i . Second, notice that among all financial reporting policies for which $g_i < g_j$, agent i prefers $g_i = q_i$ (which corresponds to the uninformative policy $\theta_i = 1$).⁸ Taken together, these observations suggest that the equilibrium of the exaggeration stage is either a fully-revealing pure strategy equilibrium $\theta_i = \theta_j = 0$, or a mixed strategy equilibrium.⁹

⁸If $g_i < g_j$ then project i will be selected for certain if and only if his reported type is high and j 's is low; otherwise project i will either not be chosen, or the principal will randomize. By choosing an uninformative reporting policy, agent i assures that its reported type will be high, and therefore guarantees that its project will be selected if the other agent's reported type is low. Therefore, if an asymmetric pure strategy equilibrium exists, then one agent must play an uninformative strategy. However, because of an open set issue, the best response to an uninformative policy is undefined.

⁹The mixed strategy equilibrium requires that agents select policies without perfectly anticipating the other agent's choices, and that adjusting one's policy is infeasible after $t = 1$.

The following proposition characterizes the structure of the sub-game at $t = 1$ for any combination of initial investment in quality, $q_a \geq q_b$.

Proposition 1. *Suppose that $q_a + q_b \geq 1$. Then, the unique Nash equilibrium is $g_a = g_b = 1$ with payoffs to agent i of*

$$u_i(q_i, q_j) = \frac{1}{2}(1 + q_i - q_j) - \frac{q_i^2}{\rho}. \quad (3)$$

If $q_a + q_b < 1$, a mixed strategy equilibrium exists in which there is financial exaggeration. In this case, the expected payoffs to the agents are

$$u_i(q_i, q_j) = \frac{q_i}{q_i + q_j} - \frac{q_i^2}{\rho}. \quad (4)$$

Also, the higher quality technology leads to more exaggeration, θ_a first-order stochastically dominates θ_b .

According to Proposition 1, if $q_a + q_b \geq 1$, then neither agent exaggerates the prospects of their project (i.e., $\theta_a = \theta_b = 0$). Otherwise, financial exaggeration is part of the equilibrium. This inequality can be appreciated as follows. Suppose that agent b always tells the truth and consider agent a 's best response. If $\tau_b = L$ and $\tau_a = H$, then there is no gain to exaggeration. If agent a reports truthfully, he will get allocated the resource anyway. If $\tau_b = H$ and $\tau_a = L$, exaggeration still does not yield a payoff. Because the principal forms a rational posterior belief, she will award the resource to the agent with the highest probability of having a high-type project. In this case, the probability that agent a has a high-type project is strictly less than one. The only time that exaggeration has a potential benefit or cost is when $\tau_a = \tau_b$. Suppose that $\tau_a = \tau_b = L$, exaggeration has a positive payoff to agent a . Given that agent b tells the truth, if the principal has the belief that agent a has a positive probability of having a high-type project, she will allocate the resource to agent a . However, if $\tau_a = \tau_b = H$, then exaggeration hurts agent a . In this case, $\theta_a > 0$ causes the principal to be skeptical and allocate the resource to agent b .

The probability that $\tau_a = \tau_b = H$ is $q_a q_b$ and the probability that $\tau_a = \tau_b = L$ is $(1 - q_a)(1 - q_b)$. If $q_a + q_b \geq 1$, then $q_a \geq 1 - q_b$ and $q_b \geq 1 - q_a$. This implies, in turn, that $q_a q_b \geq (1 - q_a)(1 - q_b)$. Therefore, $q_a + q_b \geq 1$ implies that it is more likely for the agents to both get a high-type outcome

than for them to both get a low-type. In this case, the expected cost of exaggeration is higher than its benefit, and therefore agent a also reports truthfully. Intuitively, when good outcomes are likely, both agents would rather avoid the skepticism of the principal. In contrast, when $q_a + q_b < 1$, $q_a q_b < (1 - q_a)(1 - q_b)$, and the expected benefit of exaggeration is higher than its cost. In such case, bad outcomes are more likely and it is better for each agent to exaggerate and make the project look more attractive.

Also according to Proposition 1, the agent with a higher quality production technology exaggerates more-aggressively. This implies that there is a complementarity between investment in a production technology and financial exaggeration. The more an agent invests in his production technology, the more optimistic the principal is about his project's quality. Thus, the agent can exaggerate to a larger degree, without compromising the principal's perception of project's quality. Exaggeration is valuable to the agent, because it reduces the probability with which the principal observes a bad report. Thus, the agent with the more productive technology exaggerates more in equilibrium, and expects a higher equilibrium payoff.

When $q_a + q_b < 1$, there are two cases to consider. Whenever $q_b \geq \frac{1}{2}(1 - q_a)$, each agent misreports with positive probability; if $q_b \leq \frac{1}{2}(1 - q_a)$, each agent utilizes an exaggerating policy with probability one. Both of these are described in detail in the proof of Proposition 1 in the appendix, but we describe them briefly here.

Case I: $q_b \leq \frac{1}{2}(1 - q_a)$. The lower quality agent chooses θ_b according to a mixed strategy. The support for θ_b is an interval contained within $(0, 1)$. Thus, the exaggeration policy at the lower-quality agent always inflates the types of some—but not all—of its low-type projects. The higher quality agent also chooses θ_a according to a mixed strategy. Financial exaggeration is more extreme with the higher-quality agent: its mixed strategy over levels of exaggeration first-order-stochastically dominates the mixed strategy of the lower quality agent. Furthermore, the higher-quality agent's mixed strategy has a mass point at $\theta_a = 1$, the completely uninformative policy.

Case II: $\frac{1}{2}(1 - q_a) \leq q_b \leq 1 - q_a$. Here, the lower-quality agent chooses a mixed strategy with a

mass point at $\theta_b = 0$, and support over an interval inside $(0, 1)$. The higher-quality agent's mixed strategy has mass points on both $\theta_a = 0$ and on $\theta_a = 1$, and support over an interval inside $(0, 1)$. As in Case I, the high quality agent's mixed strategy first order stochastically dominates the low quality agent's mixed strategy. The primary qualitative difference in the policies here as opposed to in Case I is that each agent sometimes implements a fully-informative reporting policy.

4.2.2 Investment Quality and Principal Payoff

Based on Proposition 1, the payoff to each agent depends on their relative investment in quality:

$$u_i(q_i, q_j) = \begin{cases} \frac{q_i}{q_i + q_j} - \frac{q_i^2}{\rho} & \text{if } q_i + q_j \leq 1 \\ \frac{1}{2}(1 + q_i - q_j) - \frac{q_i^2}{\rho} & \text{if } q_i + q_j > 1 \end{cases}$$

This payoff function is continuous everywhere and is differentiable everywhere except for possibly $q_i = q_j$. The non-differentiability arises when the investment levels change the equilibrium from one of full disclosure to one in which exaggeration is present. Despite this non-differentiability, characterizing the symmetric equilibria of the game is straightforward.

Proposition 2. *The symmetric Nash equilibrium of the investment game is as follows:*

- (i) *Suppose that $\rho > 2$. Then, $q_a = q_b = \frac{\rho}{4}$ at $t = 0$ and $\theta_a = \theta_b = 0$ at $t = 1$.*
- (ii) *Suppose that $\rho \leq 2$. Then, $q_a = q_b = \frac{\sqrt{2\rho}}{4}$ at $t = 0$ and the agents engage in financial exaggeration at $t = 1$.*

According to Proposition 2, the cost of investment affects the agents' tendency to exaggerate if given the opportunity to do so. When ρ is sufficiently high, $C(q_i)$ is lower and less convex, making it easier for the agents to invest in quality. In this case, the agents do not exaggerate because they find it better to avoid the principal's skepticism. Given symmetry in equilibrium, the inequality in Proposition 1 may be written as $q^* + q^* \geq 1$, or $q^* \geq \frac{1}{2}$. Substituting $q^* = \frac{\rho}{4}$ yields $\rho \geq 2$, which is true by assumption. Therefore, when the marginal cost of quality is sufficiently low, the principal can rely on the agents to tell the truth.

When ρ is lower, making $C(q_i)$ higher and more convex, the agents tend to exaggerate the performance of their projects. In this case, the agents are more constrained and invest less. Revisiting the inequality in Proposition 1, $q^* < \frac{1}{2}$, which means that exaggeration is part of the equilibrium. In this case, the agents are more concerned with concealing bad outcomes than with the principal's skepticism. However, as we show in the next corollary, investment by the agents is higher with exaggeration than without it. In essence, the ability to exaggerate relaxes the constraint imposed by the high cost function and introduces a complementarity between investment and misreporting.

Corollary 1. *Suppose that $\rho \leq 2$. Then, in a symmetric equilibrium, there is more investment quality when exaggeration is allowed than in the fully-informative benchmark.*

This result can be appreciated by comparing the quality choices in Proposition 2 to those in the fully-revealing benchmark. Indeed, exaggeration leads to higher quality if

$$\frac{\sqrt{2\rho}}{4} \geq \frac{\rho}{4},$$

or $\rho \leq 2$. This implies that the ability to exaggerate allows for a complementarity between investment in project quality and future financial exaggeration. If the marginal cost of investment in quality is sufficiently high, then it might be preferable for a principal to allow agents to misreport. We explore this formally in the next proposition.

Proposition 3. *(Principal's Payoff) Suppose that $\rho < 2$. If no exaggeration is allowed, the payoff to the principal is*

$$u_p^{NE} = \frac{\rho}{2} - \frac{\rho^2}{16}. \quad (5)$$

If exaggeration is allowed, the payoff to the principal is

$$u_p^{EX} = \begin{cases} \frac{19}{12} \frac{\sqrt{2\rho}}{4} & \text{if } \rho \leq \frac{8}{9} \\ \frac{1}{12(\frac{\sqrt{2\rho}}{4})^2} (1 - 9\frac{\sqrt{2\rho}}{4} + 27(\frac{\sqrt{2\rho}}{4})^2 - 8(\frac{\sqrt{2\rho}}{4})^3) & \text{if } \frac{8}{9} \leq \rho \leq 2 \end{cases} \quad (6)$$

For all values of $\rho < 2$, $u_p^{EX} > u_p^{NE}$.

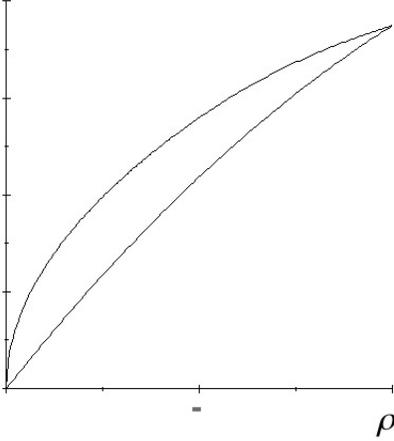


Figure 2: Two plots comparing the payoff to the principal when exaggeration is allowed (upper curve) and when full monitoring is employed (lower curve) for $\rho \in [0, 2]$. According to plot, payoff is strictly higher when exaggeration is allowed.

According to Proposition 3, the principal is strictly better off allowing the agents to exaggerate. The plot in Figure 2 shows this to be the case. In total, then, for any ρ , the principal is weakly better off not monitoring the agents. Indeed, for $\rho > 2$, there is truth-telling anyway and $u_p = \frac{\rho}{2} - \frac{\rho^2}{16}$. As such, it is never optimal for the principal to monitor the agents.

5 Preference Alignment

So far, we have only considered a setting in which there is a strong conflict of interest between the agents and the principal. The agent who gets chosen receives a fixed payoff, irrespective of whether the outcome is high or low. In what follows, we reconsider our analysis with a less severe conflict of interest. We now suppose that the agent receives a payoff if the principal accepts his project *and* it turns out to be a high type.

The set-up is unchanged from Section 3 except that an agent receives a payoff only when his project is funded and $\tau = H$. Given this, in the reporting stage, following investment levels (q_a, q_b)

with $q_a \geq q_b$, agent i 's expected payoff of choosing θ_i against θ_j is

$$u_i(\theta_i, \theta_j | q_i, q_j) = \begin{cases} q_i(1 - r_j) & \text{if } g_i < g_j \\ q_i & \text{if } g_i > g_j \\ q_i(1 - \frac{1}{2}r_j) & \text{if } g_i = g_j \end{cases}$$

This payoff function differs from the one in (2) because generating a bad signal leads to a zero payoff whether or not the project is accepted. Furthermore, if $g_i > g_j$, agent i gets a positive payoff when he has a high type project. If $g_i < g_j$, then agent i also needs his competitor to generate a bad report. Given this, it is clear from inspection that $g = 1$ is a weakly dominant strategy for each player. That is, if the agents only get paid if the project turns out to be a high-type ex post, then the agents will report honestly (i.e., $\theta_a = \theta_b = 0$).

Given honest reporting, calculating the equilibrium investment levels is straightforward. The expected payoff to each agent is

$$u_i(q_i, q_j) = q_i(1 - \frac{q_j}{2}) - \frac{q_i^2}{\rho}.$$

Since this function is strictly concave in q_i , the best response for agent i is defined by the following first order condition that is linear in the choices of each agent:

$$(1 - \frac{q_j}{2}) - \frac{2q_i}{\rho} = 0.$$

Solving yields $q = \frac{2\rho}{\rho+4}$.

Now, we can compare this level of investment to those in the base model. First, consider the case of full monitoring. The agents always invest more when the conflict of interest is mild:

$$\frac{2\rho}{\rho+4} > \frac{\rho}{4} \iff \rho < 4.$$

This is not surprising because in both scenarios, the agents tell the truth. However, when a low-type project does not generate a payoff for the agent, there is an additional incentive to avoid bad realizations¹⁰.

We can now contrast this when exaggeration is allowed, which introduces a tradeoff for the principal. On one hand, as we showed before, financial exaggeration increases the incentives to

¹⁰The cost parameter ρ is bounded by 4, assuring that the probability q_i is maximized at one.

invest in quality because prior beliefs play a role in evaluation. At the same time, in the base case, the agent may benefit even when his project is low value, which lowers the incentive to invest in quality. It turns out that for small values of ρ , investment is higher in the base case with exaggeration. To see this,

$$\frac{2\rho}{\rho+4} < \frac{\sqrt{2\rho}}{4} \iff \rho < 12 - 8\sqrt{2} \approx 0.69.$$

This increase in investment can also generate a higher payoff to the principal. To make the comparison, note that exaggeration in the base case can only be better if investment is higher, which happens when $\rho < 0.69 < 8/9$. Hence, we need only compare the payoffs for low values of ρ . Comparing these payoffs yields that exaggeration with a more severe conflict of interest is preferred whenever:

$$1 - \left(1 - \frac{2\rho}{\rho+4}\right)^2 < \frac{19}{12} \frac{\sqrt{2\rho}}{4} \iff \rho < \hat{\rho} \approx 0.504.$$

6 Asymmetric Agents

So far, we have considered that the two agents are ex ante identical. In many settings, this may not be the case. In particular, when a principal manages two divisions within a firm, the human capital, outside options, labor quality, and initiative could vary considerably across agents. This heterogeneity might in turn affect the incentives between the agents and the outcomes of competition. For example, the agent in a weaker division may decrease effort because he has a lower probability of securing capital. In what follows, we analyze the effect of exaggeration on these incentives.

Let us suppose that the cost of quality for the two agents is such that

$$R_a < \rho_b < \rho_a < 2, \tag{7}$$

where R_a as defined analytically in the Appendix.¹¹ Thus, while one agent has a higher investment cost, the weaker agent's cost is still above a certain threshold, so that the asymmetry is not too large. The following proposition characterizes the equilibrium of the game.

¹¹Setting this lower bound on ρ_b implies an upper bound on the cost of quality for agent b and assures that a pure strategy equilibrium exists.

Proposition 4. (*Asymmetric Agents*) Suppose that (7) holds. Then, there exists an equilibrium with exaggeration and

$$q_a^* = \frac{\sqrt{2\rho_a(\rho_a + \rho_b)\sqrt{\rho_a\rho_b} - 4\rho_a^2\rho_b}}{2(\rho_a - \rho_b)} \quad q_b^* = \sqrt{\frac{\rho_b}{\rho_a}}q_a. \quad (8)$$

Further,

$$\frac{q_b^*}{\rho_b} > \frac{q_a^*}{\rho_a}. \quad (9)$$

According to Proposition 4, $q_b^* < q_a^*$, which is not surprising given that $\rho_b^* < \rho_a^*$. Based on our characterization in Proposition 1, agent a will be more inclined to exaggerate his performance at $t = 1$ than the weaker agent b . However, this has an interesting effect on both agents at $t = 0$. Higher expected exaggeration leads agent a to invest more in quality at $t = 0$, which is consistent with our previous results. However, agent b has an even higher relative incentive to increase quality due to this heightened competition when exaggeration is present.

To see this, let's compare the ratio of each agent's quality choice with exaggeration q_i^* to their choice under full monitoring $\frac{\rho_i}{4}$.

$$\frac{q_a^*}{\frac{\rho_a}{4}} < \frac{q_b^*}{\frac{\rho_b}{4}} \Rightarrow \frac{q_a^*}{\rho_a} < \frac{\sqrt{\frac{\rho_b}{\rho_a}}q_a^*}{\rho_b} \Rightarrow \frac{\rho_b}{\rho_a} < \sqrt{\frac{\rho_b}{\rho_a}}, \quad (10)$$

which is always the case. Given this, there is a spill-over effect of exaggeration. When the stronger agent will be expected to invest more in quality and exaggerate, this causes the weaker one to work harder. This result may have normative implications. Monitoring may not only be suboptimal because it lowers effort provision, but it may also remove a complementarity between asymmetric agents. Allowing for exaggeration abets a complementarity that encourages weaker divisions to invest in higher quality and effort provision.

7 Uninformative Reporting

One assumption that may appear critical to the results in this paper is that the agents both choose θ before observing their project's quality. In what follows, we relax this assumption. This leads to a cheap talk scenario in which both agents have an incentive to lie when they get a low value. In fact,

it is a weakly dominant strategy to do so. To see this, suppose that an agent realizes his project is $\tau_i = L$. If he tells the truth, he realizes a positive payoff only if his competitor also reports $s_j = L$. However, in this case he is better off reporting $s_i = H$ because he will get allocated the resource for sure. Because the principal has a posterior belief of q_i that the outcome is a high-type, it is rational to take a chance rather than allocating the resource to someone who admits to having a low-type project.

However, as we show shortly, this leads to an even higher investment in quality. Indeed, the results in the paper imply that the principal typically prefers less information, not more. In the limit, if the principal were to ignore reports altogether, she is strictly better off.¹² Given this, the results in the paper are robust to different timing and modeling assumptions.

Suppose that the reports given by the agents are completely uninformative. Then, the game between the agents is essentially a full-information, all-pay auction with a symmetric, convex cost of bidding, and a bid cap. Each agent simultaneously chooses $q_i \in [0, 1]$. The agent with the higher value of q receives a payoff of one, but both agents lose their investments, $C_i(q_i) = \frac{q_i^2}{\rho_i}$.

Proposition 5. (*Uninformative Reporting*)

i. Suppose that $\rho \geq 2$. Then $q_i = q_j = 1$ is the unique equilibrium.

ii. If $1 < \rho < 2$, then there exists a unique mixed strategy Nash equilibrium in which

$$q_i = \begin{cases} Q \sim F(x) = \frac{x^2}{2-\rho} & \text{with probability } \frac{2}{\rho} - 1 \\ 1 & \text{with probability } 2(1 - \frac{1}{\rho}) \end{cases}$$

iii. If $\rho \leq 1$, there exists a unique mixed strategy Nash equilibrium in which

$$q_i \sim F(x) = \frac{x^2}{\rho}.$$

For all ρ , the payoff to the principal is higher with uninformative reporting than with strategic exaggeration or with full monitoring.

¹²We show below that uninformative reporting generates a higher payoff for the principal than the equilibrium with exaggerated (but informative) reports when the agents are symmetric. This ranking may reverse when the agents are asymmetric. Analytical results that support this latter finding are available from the authors.

This result arises because q is publicly observable and there is no other information to judge the situation. However, it does alleviate the concern that the main findings of this paper are simply special to the specifics of the posed model.

8 Conclusion

For decades, financial economists have focused on ways to mitigate the agency problems that arise between shareholders and managers, investors and entrepreneurs, and lenders and borrowers. Whether due to adverse selection or moral hazard, agency conflicts are ubiquitous in corporate finance as an important determinant of firm size, capital structure, corporate governance, and firm value. Generally, the field has viewed perfect monitoring as a panacea, as long as it is not too costly.

In this paper, we show that this is not the complete story. If agents forecast that they will have the opportunity to hide information or act in their own self-interest, it is possible that they will provide higher effort provision anticipating this. In our model, agents compete for resources, which in some cases leads them to exaggerate the outcome of their projects. Knowing this, they work harder to win, which benefits the principal who allocates the capital. In such case, having less information may benefit the principal.

Our findings, then, can be viewed as a more general contribution. Indeed, one can think of many settings to apply this analysis besides financial markets: marriage markets, education, litigation, and elections, to name a few¹³. Investigating these settings is the subject of current research.

¹³For example, see Boleslavsky and Cotton (2014).

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Appendix A

Proof of Proposition 1.

The proof follows from Lemmas A1-A3. At the reporting stage, investments (q_a, q_b) with $q_a \geq q_b$ are taken as given, because these were chosen in the previous stage. Agents simultaneously choose (θ_a, θ_b) , their level of exaggeration. The posterior belief that a project with a bad report is high quality is 0, since no high quality projects receive bad reports. The posterior belief, g_i , that project i is high quality given a good report ranges from $g_i = q_i$ when $\theta_i = 1$ to $g_i = 1$ when $\theta_i = 0$. For all levels of report inflation $\theta_i \in [0, 1]$, Bayes' rule provides a one-to-one mapping between θ_i and g_i , with $g_i = q_i / (q_i + (1 - q_i)\theta_i)$, and $\partial g_i / \partial \theta_i < 0$. When characterizing the equilibrium of the reporting stage, we work with the choice of g_i directly for convenience. If an agent chooses g_i , the probability with which the posterior is equal to g_i is equal to q_i / g_i . Therefore the expected payoff of agent i is given by:

$$u_i(g_i, g_j) = \begin{cases} \frac{q_i}{g_i}(1 - \frac{q_j}{g_j}) + \frac{1}{2}(1 - \frac{q_i}{g_i})(1 - \frac{q_j}{g_j}) & \text{if } g_i < g_j \\ \frac{q_i}{g_i} + \frac{1}{2}(1 - \frac{q_i}{g_i})(1 - \frac{q_j}{g_j}) & \text{if } g_i > g_j \\ \frac{q_i}{g_i}(1 - \frac{1}{2}\frac{q_j}{g_j}) + \frac{1}{2}(1 - \frac{q_i}{g_i})(1 - \frac{q_j}{g_j}) & \text{if } g_i = g_j \end{cases}$$

Lemma A1. *If $q_b \geq 1 - q_a$ then the unique Nash equilibrium of the second stage game is $g_a = g_b = 1$.*

Proof. If agent j chooses $g_j=1$, then the best possible deviation from $g_i = 1$ is $g_i = q_i$. By choosing this deviation, agent i assures that if the other agent's project receives an L and is thus revealed to be low-type, agent i payoff is 1. Thus, $g_a = g_b = 1$ is a Nash equilibrium if and only if for each agent i

$$q_i(1 - \frac{q_j}{2}) + \frac{1}{2}(1 - q_i)(1 - q_j) \geq 1 - q_j \leftrightarrow q_i + q_j \geq 1$$

Thus, the equilibrium of the second stage is fully revealing if and only if $q_b \geq 1 - q_j$ ■

Lemma A2. *If $\frac{1}{2}(1 - q_a) < q_b < 1 - q_a$ then the mixed strategy Nash equilibrium is as follows:*

$$g_a = \begin{cases} q_a & \text{with prob } 1 - \phi_1 - \phi_2 = 1 - \frac{2q_b}{q_b + q_a} \\ G \sim F(x) = \frac{x^2 - q_a^2}{4(1 - q_b)(1 - q_a - q_b)} & \text{with prob } \phi_1 = (\frac{2q_b}{q_b + q_a})(1 - \frac{q_a + 2q_b - 1}{q_b(q_a + q_b)}) \\ 1 & \text{with prob } \phi_2 = (\frac{2q_b}{q_b + q_a})\frac{q_a + 2q_b - 1}{q_b(q_a + q_b)} \end{cases}$$

$$g_b = \begin{cases} G \sim F(x) = \frac{x^2 - q_a^2}{4(1-q_b)(1-q_a-q_b)} & \text{with prob } \lambda = 1 - \frac{q_a + 2q_b - 1}{q_b(q_a + q_b)} \\ 1 & \text{with prob } 1 - \lambda = \frac{q_a + 2q_b - 1}{q_b(q_a + q_b)} \end{cases}$$

Proof. Note, all probabilities are positive and sum to one, and the density of G is given by $f(x) = x/(2(1-q_b)(1-q_a-q_b))$. The support of G is $[q_a, 2 - q_a - 2q_b]$, and for the parameters of the proposition, the top of the support is in $[0, 1]$. To show that the proposed strategies constitute a mixed strategy Nash equilibrium, we verify that each agent is indifferent among all pure strategies inside the support and that no pure strategy outside the support delivers a better expected payoff against the mixed strategy of the other player.¹⁴

Agent b 's expected payoff from pure strategy p in the support of its mixed strategy:

$$u_b = \begin{cases} \frac{q_b}{p}(1 - \phi_1 \int_p^{2-q_a-2q_b} f(s) \frac{q_a}{s} ds - \phi_2 q_a) + \frac{1}{2}(1 - E[\frac{q_a}{g_a}])(1 - \frac{q_b}{p}) & \text{if } p \in [q_a, 2 - q_a - 2q_b] \\ q_b(1 - \frac{\phi_2 q_a}{2}) + \frac{1}{2}(1 - E[\frac{q_a}{g_a}])(1 - q_b) & \text{if } p = 1 \end{cases}$$

Substitution and simplification gives:

$$u_b = \begin{cases} \frac{q_b}{p} \frac{q_b^2 + pq_a}{(q_a + q_b)^2} + \frac{1}{2} \left(\frac{2q_b^2}{(q_a + q_b)^2} \right) (1 - \frac{q_b}{p}) & \text{if } p \in [q_a, 2 - q_a - 2q_b] \\ q_b \left(1 - \frac{q_a}{2} \left(\frac{2q_b}{q_b + q_a} \right) \frac{q_a + 2q_b - 1}{q_b(q_a + q_b)} \right) + \frac{1}{2} \left(\frac{2q_b^2}{(q_a + q_b)^2} \right) (1 - q_b) & \text{if } p = 1 \end{cases}$$

Further simplification gives $u_b = q_b/(q_a + q_b)$ in both cases. Thus all pure strategies in the support of b 's mixed strategy give the same expected payoff. Choosing any pure strategy $\hat{g} \in (2 - q_a - 2q_b, 1)$ is dominated by choosing $g = 2 - q_a - 2q_b$, because the probability of winning is the same for both pure strategies, but g is more likely to generate a good realization. It is also straightforward to verify that choosing q_b is dominated by the equilibrium mixed strategy. Thus all pure strategies in the support of b 's mixed strategy give the same expected payoff against a 's mixed strategy, and no strategy outside the support gives b a higher payoff. Thus, b 's mixed strategy is a best response to a 's mixed strategy.

A symmetric analysis applies to agent a . Simplifying the utility function gives $u_a = q_a/(q_a + q_b)$, and by a symmetric argument as above, one can show that a mixed strategy is a best response to b 's mixed strategy. ■

¹⁴We follow a similar approach in the proof to the next lemma as well. Alternative derivation of the equilibrium from the indifference conditions, which also shows uniqueness, is available for both equilibrium cases upon request.

Lemma A3. If $q_b \leq \frac{1}{2}(1 - q_a)$ then the mixed strategy Nash equilibrium is as follows:

$$g_a = \begin{cases} q_a & \text{with probability } 1 - \phi = 1 - \frac{2q_b}{q_b + q_a} \\ G \sim F(x) = \frac{x^2 - q_a^2}{4q_b(q_a + q_b)} & \text{with probability } \phi = \frac{2q_b}{q_b + q_a} \end{cases}$$

$$g_b = G \sim F(x) = \frac{x^2 - q_a^2}{4q_b(q_a + q_b)}$$

Proof. Note, all probabilities are positive and sum to one, and the density of G is given by $f(x) = x/(2q_b(q_a + q_b))$. The support of G is $[q_a, q_a + 2q_b]$, and for the parameters of the proposition, the top of the support is in $[0, 1]$. As before, we establish indifference between all pure strategies played with positive probability, and show that no pure strategy outside of the mixing distribution results in higher expected payoffs.

Consider agent b 's expected payoff from a pure strategy $p \in [q_a, q_a + 2q_b]$ in the support of its mixed strategy:

$$u_b = \frac{q_b}{p} (1 - \phi \int_p^{q_a + 2q_b} f(s) \frac{q_a}{s} ds) + \frac{1}{2} (1 - E[\frac{q_a}{g_a}]) (1 - \frac{q_b}{p})$$

Substitution and simplification gives:

$$u_b = \frac{q_b}{p} (1 - \frac{2q_b}{q_a + q_b} q_a \frac{q_a + 2q_b - p}{2q_b(q_a + q_b)}) + \frac{1}{2} (1 - \frac{2q_b}{q_a + q_b} \frac{q_a}{(q_a + q_b)} - (1 - \frac{2q_a}{q_a + q_b})) (1 - \frac{q_b}{p})$$

Further simplification gives $u_b = q_b/(q_a + q_b)$. Following the same argument as in the proof to the previous lemma, one can show that b mixed strategy is a best response to a 's mixed strategy.

Consider agent a 's expected payoff from a pure strategy p in the support of its mixed strategy:

$$u_a = \frac{q_a}{p} (1 - \int_p^{q_a + 2q_b} f(s) \frac{q_b}{s} ds) + \frac{1}{2} (1 - E[\frac{q_b}{g_a}]) (1 - \frac{q_a}{p}) \quad \text{if } p \in [q_a, q_a + 2q_b]$$

Substitution and simplification gives $u_a = q_b/(q_a + q_b)$. Following the same argument as before, one can show that a mixed strategy is a best response to b 's mixed strategy. ■

Proof of Proposition 2.

Proof. Consider a symmetric pure strategy Nash equilibrium $q_i = q_j = q \leq 1/2$. Because $q \leq 1/2$, $q \leq 1 - q$. Therefore, over the region $q_i \leq 1 - q$, selecting $q_i = q$ must be optimal. Hence,

a necessary condition for (q, q) to constitute a Nash equilibrium with $q \leq 1/2$ is the following stationarity condition:

$$\left. \frac{d}{dq_i} \left(\frac{q_i}{q_i + q} - \frac{q_i^2}{\rho} \right) \right]_{q_i=q} = 0 \iff 2q_i^3 + 4qq_i^2 + 2q^2q_i - q\rho \Big|_{q_i=q} = 0 \iff 8q^3 - q\rho = 0 \quad (\text{A1})$$

Note that over the region $q_i < 1 - q$ the payoff function is strictly concave in q_i , and thus this first order condition defines a maximum over this region. Because $q = 0$ cannot be an equilibrium, the only positive value of q satisfying this necessary condition that could arise in equilibrium is

$$q^* = \frac{\sqrt{2\rho}}{4}$$

To be consistent with the initial assumption that $q \leq 1/2$, it must be that $\rho \leq 2$. Thus, for $\rho \leq 2$ selecting q^* as a response to q^* dominates any other possible value of $q_i < 1 - q^*$. To show that investment level q^* constitutes the unique symmetric equilibrium, profitable deviations above $1 - q^*$ must be ruled out. Note that for $q_i > 1 - q^*$, the derivative of agent i 's payoff function is

$$\frac{d}{dq_i} \left(\frac{1}{2}(1 + q_i - q^*) - \frac{q_i^2}{\rho} \right) = \frac{1}{2} - \frac{2q_i}{\rho}$$

For $q_i > 1 - q^*$

$$\frac{1}{2} - \frac{2q_i}{\rho} < \frac{1}{2} - \frac{2(1 - q^*)}{\rho} = \frac{\rho + \sqrt{2\rho} - 4}{2\rho} \leq 0 \text{ for } \rho \leq 2$$

Hence, when $\rho \leq 2$, no deviations above $1 - q^*$ are profitable.

Next, consider a symmetric pure strategy equilibrium $q_i = q_j = q > 1/2$. Because $q > 1/2$, $q > 1 - q$. Therefore, over the region $q_i > 1 - q$, selecting $q_i = q$ must be optimal. Hence, a necessary condition for (q, q) to constitute a Nash equilibrium with $q > 1/2$ is the following stationarity condition:

$$\left. \frac{d}{dq_i} \left(\frac{1}{2}(1 + q_i - q_j) - \frac{q_i^2}{\rho} \right) \right]_{q_i=q} = 0 \iff \frac{1}{2} = \frac{2q}{\rho} \quad (\text{A2})$$

Note that over the region $q_i > 1 - q$ the payoff function is strictly concave in q_i , and thus this first order condition defines a maximum over this region. Thus, the only positive value of q satisfying this necessary condition that could arise in equilibrium is

$$q^* = \frac{\rho}{4}$$

To be consistent with the initial assumption that $q > 1/2$, it must be that $\rho > 2$. Thus, for $\rho > 2$ selecting q^* as a response to q^* dominates any other possible value of $q_i > 1 - q^*$. To show that investment level q^* constitutes the unique symmetric equilibrium, profitable deviations below $1 - q^*$ must be ruled out. Note that for $q_i < 1 - q^*$, the agent's payoff function is strictly concave, and thus has no more than one peak. The derivative of agent i 's payoff function is

$$\frac{d}{dq_i} \left(\frac{q_i}{q_i + q^*} - \frac{q_i^2}{\rho} \right) = \frac{q^*}{(q_i + q^*)^2} - \frac{2q_i}{\rho}$$

For $q_i < 1 - q^*$ the derivative is larger than at $q_i = 1 - q^*$. Hence, for $q_i < 1 - q^*$,

$$\frac{q^*}{(q_i + q^*)^2} - \frac{2q_i}{\rho} > q^* - \frac{2(1 - q^*)}{\rho} > 0 \text{ for } \rho > 2$$

Hence, when $\rho > 2$, no deviations below $1 - q^*$ are profitable. ■

Proof of Corollary 1.

Proof. In the fully-revealing benchmark, $q^* = \frac{\rho}{4}$. When exaggeration is allowed, $q^* = \frac{\sqrt{\rho}}{2\sqrt{2}}$.

$$q^* = \frac{\sqrt{\rho}}{2\sqrt{2}} > q^* = \frac{\rho}{4},$$

iff

$$\rho < 2.$$

■

Proof of Proposition 3.

Proof. First, we begin with the principal's payoff for symmetric investments $q_a = q_b = q$. Because we consider symmetric agents, the equilibrium of the investment stage will also be symmetric. This calculation will therefore facilitate the comparison of principal payoff under exaggeration to the principal payoff in the no exaggeration benchmark. Given the posterior belief realizations of

(g_a, g_b) generated by the equilibrium mixed strategies, define $g_m = \max[g_a, g_b]$ and $g_n = \min[g_a, g_b]$.

The principal's expected payoff for a particular combination of (g_m, g_n) is given by

$$u_p(g_m, g_n) = g_m \frac{q}{g_m} + (1 - \frac{q}{g_m}) g_n \frac{q}{g_n} = q(2 - \frac{q}{g_m}) \quad (\text{A3})$$

If the agent with the less-inflated disclosure policy, and higher posterior g_m , generates a good report, then the principal will accept that project, giving the principal a payoff of g_m . This event occurs with probability $\frac{q}{g_m}$ (the first term in expression (A3)). If the project with higher g_i generates a bad report, then the principal knows for sure that the project is low quality. In this instance, if the more-inflated project generates a good report, (probability $\frac{q}{g_n}$) the principal accepts it, giving the principal an expected payoff of g_n . Thus, the principal's ex ante expected payoff in this equilibrium is equal to

$$u_p^* = E[q(2 - \frac{q}{g_m})] = q(2 - qE[\frac{1}{g_m}]) \quad (\text{A4})$$

In order to calculate the principal's ex ante expected payoff in this equilibrium, we need to determine the expected value of the inverse of the maximum order statistic from the equilibrium mixed strategies, $E[\frac{1}{g_m}]$. We consider three cases in turn:

Case I: $q \leq \frac{1}{3}$. When $q_a = q_b = q \leq \frac{1}{3}$, the equilibrium mixed strategy of each agent is to randomize over support $[q, 3q]$ using distribution function $F(x) = \frac{x^2 - q^2}{8q^2}$ with corresponding density $f(x) = \frac{x}{4q^2}$. The expectation in question, $E[\frac{1}{g_m}]$, is therefore¹⁵

$$\int_q^{3q} 2(\frac{1}{x})(\frac{x^2 - q^2}{8q^2}) \frac{x}{4q^2} dx = \frac{5}{12q}$$

Thus, when $q \leq 1/3$ the principal's ex ante equilibrium expected payoff is

$$q(2 - q\frac{5}{12q}) = (\frac{19}{12})q$$

Case II: $\frac{1}{3} \leq q \leq \frac{1}{2}$. Here, the equilibrium mixed strategy of each agent is as follows. With probability $\phi = \frac{3q-1}{2q^2}$ choose $g = 1$. With probability $1 - \phi$ randomize over support $[q, 2 - 3q]$

¹⁵The density of the maximum order statistic g_m is $2f(x)F(x)$

using distribution function $F(x) = (x^2 - q^2)/(4(1 - q)(1 - 2q))$ with corresponding density $f(x) = x/(2(1 - q)(1 - 2q))$. Thus, $E[\frac{1}{g_m}]$ equals

$$1 - (1 - \phi)^2 + (1 - \phi)^2 \int_q^{2-3q} 2\left(\frac{1}{x}\right)\left(\frac{x^2 - q^2}{4(1 - q)(1 - 2q)}\right)\left(\frac{x}{2(1 - q)(1 - 2q)}\right)dx = \frac{1}{12q^4}(32q^3 - 27q^2 + 9q - 1).$$

Thus for $\frac{1}{3} \leq q \leq \frac{1}{2}$ the principal expected payoff simplifies to

$$(1 - 9q + 27q^2 - 8q^3)/(12q^2)$$

Case III: $q \geq \frac{1}{2}$. In this case, agents use fully revealing grading strategies in the second stage and the principal's ex ante equilibrium expected payoff is

$$2q - q^2$$

Summarizing, in a subgame in which $q_a = q_b = q$, the principal's ex ante expected payoff when financial exaggeration is possible is equal to:

$$u_p^* = \begin{cases} \frac{19}{12}q & \text{if } q \leq \frac{1}{3} \\ (1 - 9q + 27q^2 - 8q^3)/(12q^2) & \text{if } \frac{1}{3} \leq q \leq \frac{1}{2} \\ 2q - q^2 & \text{if } q > \frac{1}{2} \end{cases} \quad (\text{A5})$$

Now, we can consider the payoff to the principal for different values of $\rho < 2$. Substituting into (A5) yields (6). Taking the difference between (5) and (6) yields

$$\begin{aligned} & 1 - (1 - \frac{\rho}{4})^2 - \frac{19}{12} \frac{\sqrt{2\rho}}{4} & \text{if } \rho \leq \frac{8}{9} \\ & 1 - (1 - \frac{\rho}{4})^2 - \frac{1}{12(\frac{\sqrt{2\rho}}{4})^2} (1 - 9\frac{\sqrt{2\rho}}{4} + 27(\frac{\sqrt{2\rho}}{4})^2 - 8(\frac{\sqrt{2\rho}}{4})^3) & \text{if } \frac{8}{9} \leq \rho \leq 2 \end{aligned}$$

Plotting these payoff functions clearly shows that when investment is endogenous, the principal's evaluator payoff is higher when exaggeration is allowed. ■

Proof of Proposition 4.

Proof. Here we consider the model in which the agents' development costs are not identical, where $\rho_a > \rho_b$. If $\rho_a < 2$ and ρ_b is not too small, then a pure strategy equilibrium exists in the investment

stage, which can be explicitly characterized. Consider the possibility of an equilibrium in which $q_a + q_b < 1$. Ruling out profitable deviations for agent i that are inside $[0, 1 - q_j]$ requires that the following system of first order conditions holds:

$$\frac{q_b}{(q_a + q_b^2)} - \frac{2q_a}{\rho_a} = 0 \quad \frac{q_a}{(q_a + q_b^2)} - \frac{2q_b}{\rho_b} = 0$$

The only solution of this system in which both investments are positive is

$$q_a = \frac{\sqrt{2\rho_a(\rho_a + \rho_b)}\sqrt{\rho_a\rho_b} - 4\rho_a^2\rho_b}{2(\rho_a - \rho_b)} \quad q_b = \sqrt{\frac{\rho_b}{\rho_a}}q_a$$

and these are local maxima. The term inside the radical is always positive:

$$2\rho_a(\rho_a + \rho_b)\sqrt{\rho_a\rho_b} - 4\rho_a^2\rho_b > 0 \iff (\rho_a + \rho_b)^2\rho_a^3\rho_b - 4\rho_a^4\rho_b^2 > 0 \iff \rho_1^3\rho_2(\rho_1 - \rho_2)^2 > 0$$

Next note that if $\rho_a < 2$ then $q_a < 1/2$ and therefore $q_a + q_b < 1$. Indeed,

$$\begin{aligned} \frac{\sqrt{2\rho_a(\rho_a + \rho_b)}\sqrt{\rho_a\rho_b} - 4\rho_a^2\rho_b}{2(\rho_a - \rho_b)} &< \frac{1}{2} \iff \\ \sqrt{2\rho_a(\rho_a + \rho_b)}\sqrt{\rho_a\rho_b} - 4\rho_a^2\rho_b &< (\rho_a - \rho_b) \iff \\ 2\rho_a(\rho_a + \rho_b)\sqrt{\rho_a\rho_b} - 4\rho_a^2\rho_b &< (\rho_a - \rho_b)^2 \iff \\ 4(\rho_a + \rho_b)^2\rho_a^3\rho_b &< (4\rho_a^2\rho_b + (\rho_a - \rho_b)^2)^2 \iff \\ 4(\rho_a + \rho_b)^2\rho_a^3\rho_b - (4\rho_a^2\rho_b + (\rho_a - \rho_b)^2)^2 &< 0 \iff \\ -(\rho_a - \rho_b)^2((\rho_a - \rho_b)^2 + 4\rho_a^2\rho_b(2 - \rho_a)) &< 0 \iff \end{aligned}$$

To complete the characterization, we show that when q_b is sufficiently close to q_a , no deviations in the range $[1 - q_j, 1]$ are optimal for player i . In this region, each player's payoff function is

$$\frac{1}{2}(1 + q_i - q_j) - \frac{q_i^2}{\rho_i}$$

with derivative

$$\frac{1}{2} - \frac{2q_i}{\rho_i} = \frac{4(\frac{\rho_i}{4} - q_i)}{2\rho_i}$$

Hence if $q_i > \rho_i/4$ for both $i = a, b$, then each player's payoff function is decreasing in this region (when the other player plays his part of the strategy) and therefore no deviation in this interval could be beneficial. Observe first that if $q_a > \rho_a/4$ then $q_b > \rho_b/4$. Indeed,

$$q_a > \rho_a/4 \Rightarrow q_a \sqrt{\frac{\rho_b}{\rho_a}} > \sqrt{\frac{\rho_b}{\rho_a}} \rho_a/4 \Rightarrow q_b > \frac{\sqrt{\rho_a \rho_b}}{4} > \frac{\rho_b}{4}$$

Hence, $q_a > \rho_a/4$ is sufficient to rule out global deviations. For $\rho_a < 2$, this condition is satisfied whenever

$$\rho_b > R_a \equiv \frac{\rho_a^2 - 16\rho_a + 32 + (4\sqrt{2}\rho_a - 16\sqrt{2})\sqrt{2 - \rho_a}}{\rho_a}$$

$R_a < \rho_a$ for $\rho_a < 2$. Hence for $R_a < \rho_b < \rho_a < 2$, the investment levels above constitute an equilibrium.

■

Proof of Proposition 5.

Proof. First consider that $\rho \geq 2$. Suppose that agent j chooses $q_j = 1$. All pure strategies $q_i \in (0, 1)$ lead to payoff zero. Because investment is costly, and given agent j 's strategy, the best deviation from $q_i = 1$ is $q_i = 0$. Thus, provided

$$\frac{1}{2} - \frac{1}{\rho} \geq 0 \leftrightarrow \rho \geq 2$$

$q_i = 1$ is a best response to $q_j = 1$, and full investment is the unique equilibrium.

Now, consider that $1 < \rho < 2$. Under the parameter range in the proposition, all values we claim are probabilities are in $[0, 1]$ and sum to one. Also, $F(x)$ is increasing on the support of Q which is $[0, \sqrt{2 - \rho}]$. To show that the proposed strategies constitute a mixed strategy Nash equilibrium, we verify that each agent is indifferent among all pure strategies inside the support and that no pure strategy outside the support delivers a better expected payoff against the mixed strategy of the other player. A derivation of the equilibrium from the indifference conditions, which also shows uniqueness, is available upon request.

Consider agent i 's expected payoff from a pure strategy p in the support of its mixed strategy:

$$u = \begin{cases} 0 & \text{if } p = 0 \\ (\frac{2}{\rho} - 1)F(p) - \frac{p^2}{\rho} & \text{if } p \in [0, \sqrt{2 - \rho}] \\ 1 - \frac{2(1 - \frac{1}{\rho})}{2} - \frac{1}{\rho} & \text{if } p = 1 \end{cases}$$

Substituting and simplifying gives:

$$u_{\beta} = \begin{cases} 0 & \text{if } p = 0 \\ (\frac{2}{\rho} - 1)\frac{p^2}{2 - \rho} - \frac{p^2}{\rho} = 0 & \text{if } p \in [0, \sqrt{2 - \rho}] \\ 1 - \frac{2(1 - \frac{1}{\rho})}{2} - \frac{1}{\rho} = 0 & \text{if } p = 1 \end{cases}$$

Thus all pure strategies in the support of agent i 's mixed strategy give the agent expected payoff zero. Choosing any pure strategy $\hat{q} \in (\sqrt{2 - \rho}, 1)$ is dominated by choosing $q = \sqrt{2 - \rho}$, because the probability of winning is the same for both pure strategies, but q is less costly. Thus all pure strategies in the support of i 's mixed strategy give the same expected payoff against j 's mixed strategy, and no strategy outside the support gives i a higher payoff.

Now, consider that $\rho \leq 1$. To show that the proposed strategies constitute a mixed strategy Nash equilibrium, we verify that each agent is indifferent among all pure strategies inside the support and that no pure strategy outside the support delivers a better expected payoff against the mixed strategy of the other player. A derivation of the equilibrium from the indifference conditions, which also shows uniqueness, is available upon request.

Consider agent i 's expected payoff from a pure strategy p in the support of its mixed strategy:

$$u = \begin{cases} 0 & \text{if } p = 0 \\ F(p) - \frac{p^2}{\rho} & \text{if } p \in [0, \sqrt{\rho}] \end{cases}$$

Substituting and simplifying gives that in both cases $u = 0$. Thus all pure strategies in the support of agent i mixed strategy give the agent expected payoff zero. Choosing any pure strategy $\hat{q} \in (\sqrt{\rho}, 1)$ is dominated by choosing $q = \rho$, because the probability of winning is the same for both pure strategies, but q is less costly. Thus all pure strategies in the support of i mixed strategy give the same expected payoff against j mixed strategy, and no strategy outside the support gives i a higher payoff.

Finally, we can compare payoff comparisons for all three cases. If $\rho > 2$, uninformative reporting is clearly optimal since the unique equilibrium involves $q_i = q_j = 1$. We consider the other two cases in turn and show that uninformative reporting is preferred to either strategic reporting or fully-revealing reporting. In order to make this calculation, we need to know the evaluator payoff in the absence of reporting.

Case: $1 \leq \rho \leq 2$ Observe that

$$E[Q^{(2)}] = 2 \int_0^{\sqrt{2-\rho}} \left(\frac{2x}{2-\rho}\right)\left(\frac{x^2}{2-\rho}\right)(x)dx = \frac{4}{5}\sqrt{2-\rho}$$

Where $Q^{(2)}$ represents the maximum of two draws of Q . The principal's expected payoff is therefore

$$1 - \left(\frac{2}{\rho} - 1\right)^2 + \left(\frac{2}{\rho} - 1\right)^2 E[Q^{(2)}] = \frac{4}{5\rho^2}((2-\rho)^{\frac{5}{2}} - 5(1-\rho))$$

Case: $\rho \leq 1$ Observe that the principal's expected payoff is given by:

$$E[Q^{(2)}] = 2 \int_0^{\sqrt{\rho}} \left(\frac{2x}{\rho}\right)\left(\frac{x^2}{\rho}\right)(x)dx = \frac{4}{5}\sqrt{\rho}$$

A simple plot reveals that both of these payoffs dominate both fully-revealing and strategic reporting derived earlier in the paper. ■