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Individual Decision Making and Investor Welfare

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This article analyses and quantifies the costs of suboptimal decision making for an investor with a multi-period horizon. In light of the empirical evidence that investors are too conservative and hold portfolios that are insufficiently diversified, we evaluate the costs of suboptimal equity participation both analytically and using simulation, and also estimate the costs of suboptimal diversification using simulation. We find that suboptimal leverage imposes only modest costs on the investor for reasonable parameter values. While the costs of inadequate diversification can be very high, we find that, because of the higher returns on small firms, an equally weighted portfolio of as few as five randomly chosen firms can provide the same level of expected utility as the value weighted market portfolio.

(J.E.L.: G11, G18, G23).

Introduction

The growth in defined contribution (DC) corporate pension plans,[1] in which unsophisticated individuals are required to make the asset allocation decisions that will have a large effect on their retirement wealth, as well as recent proposals to privatize social security,[2] have given rise to concerns that individual plan participants will suffer large welfare costs as the result of their suboptimal decisions. Such concerns are reinforced both by recent claims of systematic biases in individual decision making (Kahneman and Tversky, 1979; Kahneman and Riepe, 1998; Olsen, 1998), and by evidence of widespread financial ignorance. For example, a survey of the financial knowledge of Americans by Merrill Lynch Inc. has prompted Bernheim (1996) to claim that 'This evidence depicts a crisis in financial planning ... most Americans

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[1] Brennan (1997) reports that while there were 45 million Americans in defined benefit (DB) plans and only 25 million in defined contribution (DC) plans in 1995, the total value of DC plan assets was $1,300 billion versus $1,500 billion for the DB plans.

[2] See, for example, Feldstein (1995).

are not making prudent financial decisions' and another authority to opine that 'Examining (the investment strategy of) their (defined contribution) plan is beyond the expertise of most Americans.'[3] Concern with the level of investment expertise of the individual investor has led to calls for additional financial education,[4] and even a national pension education programme. According to Berg (1995)[5] the US Department of Labor has launched a 'national pension education program aimed at drawing the attention of American workers to the importance of taking personal responsibility for their retirement security'.

DC pension plans, whether they are administered by employers or employer-chosen administrators as is typical in the USA, or by other intermediaries as in the privatized Chilean social security system, presuppose that individuals are able to choose security portfolios that are at least approximately efficient for them. It is therefore important to consider the extent to which lack of sophistication is likely to lead individual investors to incur significant welfare losses by choosing inefficient portfolios, and to quantify the likely magnitude of these losses. To the extent that markets are informationally efficient, 'one stock is as good as another',[6] and unsophisticated investors are protected against the risk of buying overpriced securities (or missing the opportunity to buy underpriced ones). However, even in an efficient market, investors may depart significantly from an optimal asset allocation strategy, and may invest in too few stocks to achieve efficient diversification.[7] In this article, we take a small step towards evaluating the likely magnitude of the welfare costs of inappropriate asset allocation and inadequate diversification. Our basic framework is that of an investor who is concerned with maximizing the expected value of a von Neumann Morgenstern utility function defined over wealth at the end of the investment horizon. In the interests of simplicity, we eschew more elaborate utility specifications such as recursive utility (Epstein and Zin, 1989), and other specifications which impose a myopic concern with intermediate rates of return.[8] We adopt a utility of final wealth

[3] J. Carter Beese, Commissioner of the Securites and Exchange Commission, *Defined Contribution Plan Investing*, April 13, 1993.

[4] One might question the value of much of this financial education to the extent that it increases investors' equity allocations by propagating the *canard* that stocks in the USA have outperformed bonds for every 30-year holding period since 1871 (Siegel, 1998). Such mechanistic extrapolations of history ignore the fact that the future returns on stocks must depend on their current level; they also ignore the recent history of Japan for example, where the Nikkei index languishes around 14,000, some 66 per cent below its high of around 42,000 reached in 1989.

[5] Cited in Bernheim (1996).

[6] The dartboard competition run by the *Wall Street Journal*, in which the stock selections of investment analysts are compared with a random selection, suggests that even investment professionals have great difficulty in outperforming a random selection.

[7] Dybvig (1988) describes how inefficient dynamic strategies may impose costs on investors in an efficient market.

[8] Benartzi and Thaler (1998) suggest that investors may be concerned with the distribution of the *short-term* returns on their investments, rather than the distribution of wealth at the horizon as assumed here.

assumption because it enables us to avoid difficult issues associated with labour income, and because our primary concern is with retirement planning, and we think that, as a first approximation, it is reasonable to separate the retirement portfolio management issue from more general issues concerned with savings behaviour.[9]

Within this framework of expected utility of final wealth maximization, we consider two aspects of the portfolio decision: how much to allocate between stocks and cash,[10] and how many stocks to include in the portfolio. Extensions to more complete settings – including stochastic interest rates, time varying risk premia, and more elaborate utility specifications – represent challenging and interesting extensions of our approach.

Three previous papers have considered related issues. First, Ackerlof and Yellen (1985) show that small deviations from optimal ('rational') decisions that have small welfare consequences for individuals may have major consequences for equilibrium; since our concern is not with the the equilibrium pricing of securities, this paper is not directly relevant. Second, Cochrane (1989) shows in the context of lifetime consumption allocation under certainty that first-order deviations from the optimal consumption plan have only second-order consequences for utility. Most recently, Canner *et al.* (1997), in a paper that is most closely related to this, contrast the portfolios recommended for investors by financial advisors with what their interpretation of financial theory implies is optimal, and show that, although the composition of the recommended portfolios is far from 'optimal', these portfolios plot close to the efficient frontier in mean–variance space.

In Section 1, we review some of the evidence on the biases and imperfections of individual investment decision making. In Section 2, we present a simple analytical framework for assessing the welfare costs of suboptimal asset allocation, and use this to calibrate the likely magnitude of welfare losses for reasonable parameter values. We confirm our analytical results, which rely on distributional assumptions, by Monte Carlo simulation on US stock market returns drawn from the CRSP tape for the period 1926 to 1997. Our general conclusion is that the costs of excessive conservatism in asset allocation are likely to be modest, at least as measured relative to typical levels of fees levied for portfolio management services, despite the fact that asset allocation is 'the major determinant of investment performance' (Brinson *et al.*, 1995). In Section 3 we turn our attention to diversification within the stock market and use Monte Carlo simulation on individual stock data to assess the welfare costs

[9] Cochrane (1989) argues that the welfare costs of suboptimal savings-consumption decisions are likely to be modest.

[10] We assume a constant interest rate which makes long-term bonds a redundant asset class. For models of optimal portfolio allocation between bonds, stocks and cash when interest rates are stochastic, see Brennan *et al.* (1997), Brennan and Xia (1998), Campbell and Viceira (1998), and Cantirelli (1998).

of imperfect diversification.[11] We find that, because of the higher returns realized by small firms, an equally weighted (EW) portfolio of as few as five stocks with monthly rebalancing can provide the same level of expected utility for a relatively risk-tolerant investor as an investment in the value weighted (VW) market index. However, when a more realistic buy and hold strategy is considered, it takes as many as 50 stocks to match the performance of the VW market index for the more risk-averse investor. Thus, there are signifiant gains from portfolio diversification. Section 4 concludes.

1. Empirical Evidence on Individual Investment Decision Making

Bernheim (1996) writes: 'The increasing popularity of 401(k)'s[12] leaves critical decisions concerning participation, contributions, and investments in the hands of employees. Many employees choose to contribute little, or nothing at all, while others invest heavily in safe, low return, fixed-income funds.' O'Neill (1990, 1993) provides evidence that investors both have insufficient diversification and exhibit 'excessive conservatism' in selecting investments. This is confirmed by the finding that the most popular investment in DC plans is the employer's own equity which accounts for around 30 per cent of investment portfolios, despite the fact that its return has significant diversifiable risk which is likely to be highly correlated with the return on the employee's human capital;[13] the second most popular investment is the guaranteed investment certificate (around 23 per cent) which is an insurance company liability with a fixed annual return (Brennan, 1997). Benartzi and Thaler (1998) report evidence that many participants in DC plans allocate their contributions according to the '$\frac{1}{n}$' rule; that is, they allocate their funds equally across all the bond and stock funds that are included in the plan, thus making their initial asset allocation the passive result of the mix of funds selected for inclusion by the plan managers. Samuelson and Zeckhauser (1988) report what they term a 'status quo bias' in investment decision making: they find that participants in the TIAA/CREF pension plan tend not to alter the ratio of their

[11] There is an older literature studying how many securities are required to achieve an adequate level of diversification; see Evans and Archer (1968) for example. However, these authors did not attempt to assess the economic costs of imperfect diversification as we do.

[12] These are employer adminstered, tax-advantaged, retirement accounts; 96 per cent of company plans allow participants to choose between different funds but not to invest in individual stock (with the exception of the employer's own stock) (Greenwich Associates, 1998).

[13] Benartzi and Thaler (1998) report the results of a survey by John Hancock Financial Services which found that a majority of respondents thought that money market funds (which invest only in high-grade short-term paper and under normal conditions maintain their net asset value at par) were riskier than government bonds, and felt that their own company stock was safer than a diversified portfolio.

contributions[14] to the fixed income and equity funds in response to the returns earned on these two asset classes – as a result, over time their asset allocations become heavily influenced by the history of realized returns on the different asset classes. Friend and Blume (1975) report that individuals who do own stocks typically own only one or two different stocks, while Mankiw and Zeldes (1991) found that only 47.7 per cent of consumers holding more than $100 000 in liquid assets in 1984 held any stock at all directly.[15] Collectively, this evidence is consistent with the notion that individual investors are too conservative in their asset allocation, allow their allocation to be determined passively by the number and types of funds offered by the plan sponsor and by the returns experienced on the different asset classes in their portfolio and, in situations where they have discretion, tend to diversify too little.

In other research, Elton *et al.* (1989) have shown that commodity funds are sold on the basis of their exceptional return performance prior to issuance, but that they fail to deliver similar results after they become public, for the good reason that their performance is random, and only those that have performed exceptionally well are brought to the market creating an *ex-post* selection bias in the pre-issue performance. Similarly, Weiss (1989), Peavey (1990), and Wang *et al.* (1992) have found that new issues of closed-end funds and REITS are overpriced, unlike other initial public offerings; and that, compared with other initial public offerings, they offer higher underwriting selling fees and have substantially greater individual investor participation. Finally, we observe that investors purchase mutual funds with load fees when they could purchase no-load funds. These suboptimal behaviours are undoubtedly costly for the investors who pursue them.

On the other hand, other examples of ‘irrational’ behaviour may simply represent what Merton Miller (1977) refers to as ‘neutral mutations’ in an informationally efficient market. For example, Sirri and Tufano (1993) and Patel *et al.* (1991) report that investor purchases of mutual funds are unduly influenced by recent good performance even though that shows no persistence. Patel *et al.* (1991) and Warther (1994) find that the fraction of their funds flow that individuals direct to the purchase of mutual funds is an increasing function of recent past returns on the market – they refer to this behaviour as ‘barn-door closing’.[16] Odean (1998) finds evidence that individual investors have a

[14] At the time of the survey, a participant could change his or her allocation of new contributions between the two funds (but not re-allocate previously invested funds).

[15] There is extensive anecdotal evidence of naivety among investors. For example, only 12 per cent of respondents to a poll of Americans were able to distinguish between a load and a no-load mutual fund, although the former imposes a sales charge of as much as 8 per cent while the latter does not (*New York Times*, March 2, 1997). A survey of 750 mutual fund investors revealed that they expected to earn an average annual return over the next ten years of no less than 22.2 per cent (*New York Times*, April 6, 1997).

[16] However, Brennan and Cao (1996) present a model in which the portfolio behaviour described by Patel *et al.* (1991) and Warther (1994) is rational for investors who are less well informed than the average.

strong tendency to sell profitable investments and to hold on to losers. While this would be a 'neutral mutation' (except for tax consequences) in an informationally efficient market, Odean argues that the winners that are sold on average outperform the market by 2.4 per cent over the next year, while the losers that are kept underperform the market by 1 per cent. Thus, if one accepts Odean's evidence of market inefficiency, selling a winner instead of a losing stock costs the investor a total of 3.4 per cent.

In summary, there is extensive evidence that investors behave in a suboptimal fashion in a variety of ways. Therefore, it is important to assess the welfare costs of suboptimal investment strategies. In the next section, we consider the costs of suboptimal leverage.

2. The Welfare Cost of Suboptimal Leverage

2.1. A Simple Analytical Model

To analyse the welfare costs of following a non-optimal asset allocation policy, consider an investor who is concerned with maximizing the expected value of an iso-elastic utility function defined over his wealth at the end of T periods, W_T:

$$U(W_T) = \frac{W_T^{\gamma}}{\gamma} \tag{1}$$

where $\gamma < 1$. The investor's coefficient of relative risk aversion (RRA) is given by $1 - \gamma$.

The investor may invest in a riskless asset whose instantaneous rate of return is a constant, r, or in a risky asset. The price of the risky asset, S, follows the stochastic differential equation

$$\frac{dS}{S} = \mu dt + \sigma dz \tag{2}$$

where μ and σ are constants, and dz is the increment to a Gauss Wiener process.

If x is the fraction of wealth allocated to the risky asset, the investor's wealth evolves according to

$$\frac{dW}{W} = [r + x(\mu - r)]dt + x\sigma dz \tag{3}$$

Under this portfolio strategy, wealth at the end of T periods is lognormally distributed

$$\ln W_T \sim \mathrm{N}\left(\ln W_0 + \left[r + x(\mu - r) - \frac{1}{2}x^2\sigma^2\right]T,\, x^2\sigma^2 T\right)$$

Then, since $\ln U(W) = -\ln\gamma + \gamma \ln W$, the investor's expected utility under the portfolio allocation x, given initial wealth W_0, $\mathrm{E}U(W_0, x)$, may be written as

$$\mathrm{E}U(W_0, x) = \frac{W_0^\gamma}{\gamma} \exp\left\{\gamma\left[r + x(\mu - r) - \frac{1}{2}x^2\sigma^2\right]T + \frac{1}{2}\gamma^2 x^2 \sigma^2 T\right\} \tag{4}$$

Differentiating with respect to x, the investor's optimal allocation to the risky asset, x^*, is

$$x^* = \frac{\mu - r}{(1-\gamma)\sigma^2} = \frac{S}{\mathrm{RRA} \times \sigma} \tag{5}$$

where

$$S \equiv \frac{\mu - r}{\sigma}$$

is the Sharpe ratio for the risky asset and $\mathrm{RRA} \equiv (1-\gamma)$ is the investor's coefficient of relative risk aversion. Define $\overline{W}_0(\mathrm{E}U, x)$ as the level of initial wealth required to achieve a given level of expected utility, $\mathrm{E}U$, under the (possibly non-optimal) investment policy, x. Then the quantity $\overline{W}_0[\mathrm{E}U(W_0, x^*), x]$ is the initial wealth required to achieve, under policy x, the same level of expected utility as is achieved under the optimal policy, x^*, starting with initial wealth W_0. Since x^* is, by definition, optimal, it follows that

$$\overline{W}_0[\mathrm{E}U(W_0, x^*), x] > W_0$$

Define

$$\rho(x) \equiv \frac{\overline{W}_0}{W_0}$$

as the ratio of the wealth required under policy x, to the wealth required under policy x^*, to achieve a given level of expected utility. Then from equation (4) we have

$$\rho(x) = \exp\left\{(x^* - x)\sigma T\left[S - \frac{1}{2}(x^* + x)\sigma \mathrm{RRA}\right]\right\} \tag{6}$$

where $x^*$is given by equation (5). It follows that $\rho(x)$ is a measure of the inefficiency of strategy x. For example, a value of 1.15 means that it requires 15 per cent more wealth to achieve a given level of expected utility following strategy x than it would require under the optimal policy; in other words, an investor who is following strategy x is effectively throwing away $1 - \frac{1}{1.15} =$ 13.04 per cent of his wealth.

To evaluate expression (6) for different investment strategies, x, it is necessary to specify the Sharpe ratio, S, and the standard deviation of the risky asset return, σ, as well as the investment horizon, T, and the coefficient of relative risk aversion, RRA. MacKinlay (1995) reports (annual) Sharpe ratios of 0.27 for the CRSP value weighted (VW) index for the period July 1963 to December 1991, and of 0.32 for the S&P500 index for the period January 1981 to June 1992. We therefore choose a value of 0.30 for the (annual) Sharpe ratio. Campbell *et al.* (1997) report an annualized standard deviation for the return on the VW market index for the period 1962–1994 of 15.0 per cent, and this is the value we use.

Table 1 reports the values of $\rho(x)$ when $S = 0.3$, $\sigma = 0.15$ and $T = 20$ years for different values of the coefficient of relative risk aversion, RRA, and different values of x. For the parameters chosen, the first line of the table shows that the optimal leverage ratio is unity for RRA = 2, while $\rho(0.5) = 1.12$. That is, a portfolio strategy that holds 50 per cent of its investment in stocks when it is optimal to hold 100 per cent, requires only 12 per cent more wealth to achieve the level of expected utility yielded by the optimal strategy. Similarly for RRA = 3, the optimal leverage ratio is 0.67; $\rho(0.3) = 1.09$, and $\rho(1.0) = 1.08$ so the cost of deviating 50 per cent up or down from the optimal allocation for 20 years is less than 10 per cent of initial wealth. As risk aversion increases, the cost of holding half the optimal allocation in stocks decreases.[17] In general, the costs of taking too small a position in stocks are surprisingly small,[18] and the cost of deviations from the optimum are approximately symmetric for positive and negative deviations. The table shows that, in general, the cost of investing too little in stocks is likely to be modest, while the cost of investing more than is optimal may be significant for the more risk-averse investors since, for them, an allocation of 100 per cent to stocks may represent several times the optimal allocation. The important finding is that the cost of excessive conservatism, about which concern has been expressed, seems small. As shown in Table 2, the costs of suboptimal behaviour are reduced by about 50 per cent when the horizon is reduced from 20 years to 10 years. For example, for an investor with RRA = 2, the cost of investing only 50 per cent of his wealth in stocks when it is optimal to invest 100 per cent falls from 12 per cent when the horizon is 20 years to only 6 per cent when the horizon is 10 years.

Further insight into the costs of suboptimal policies can be obtained by considering the indifference curves in (μ_p, σ_p) space of an investor with a given value of γ, where μ_p and σ_p represent the mean and standard deviation of the return on his portfolio. Rewriting equation (4) in terms of $\mu_p(x)$ and $\sigma_p(x)$ gives

$$\text{(7)} \qquad \mathrm{E}U(W_0, x) = \frac{W_0^\gamma}{\gamma} \exp\left\{\gamma\left[\mu_p(x) - \frac{1}{2}\sigma_p^2(x)\right]T + \frac{1}{2}\gamma^2\sigma_p^2(x)T\right\}$$

[17] In Tables 1 and 2, the italicized cells correspond to values of x that are closest to half the optimal value, x^*.

[18] At least, they are surprising to us. Some colleagues with whom we have spoken regard these costs as large. We compare them with other costs below.

Table 1: Analytic Estimates of the Welfare Costs of Non-optimal Equity Allocation for a 20-year Horizon

RRA	x^*	$x = 0$	$x = 0.1$	$x = 0.2$	$x = 0.3$	$x = 0.4$	$x = 0.5$	$x = 0.6$	$x = 0.7$	$x = 0.8$	$x = 0.9$	$x = 1.0$	$x = 1.1$
2	1.0	1.57	1.44	1.33	1.25	1.18	*1.12*	1.07	1.04	1.02	1.00	1.00	1.00
3	0.67	1.35	1.24	1.16	*1.09*	1.05	1.02	1.00	1.00	1.01	1.04	1.08	1.14
4	0.50	1.25	1.15	1.08	*1.04*	1.01	1.00	1.01	1.04	1.08	1.15	1.25	1.38
5	0.40	1.20	1.11	*1.05*	1.01	1.00	1.01	1.05	1.11	1.20	1.32	1.50	1.74
6	0.33	1.16	1.08	*1.02*	1.00	1.01	1.04	1.10	1.20	1.34	1.54	1.82	2.21
7	0.29	1.14	*1.06*	1.01	1.00	1.02	1.08	1.17	1.31	1.52	1.81	2.23	2.84

Note: The table gives values of $\rho(x)$, the wealth level required with equity allocation x, to reach the level of expected utility achievable with a $1 initial investment under the optimal equity allocation, x^*, for different levels of relative risk aversion, RRA. The table assumes that the annual Sharpe ratio is 0.30, the annual standard deviation of the return on the equity portfolio, $\sigma = 0.15$, and that the investor's time horizon (T) is 20 years. The italicized cells correspond to values of x that are closest to 50 per cent of x^*.

Table 2: Analytic Estimates of the Welfare Costs of Non-optimal Equity Allocation for a 10-year Horizon

RRA	x^*	$x = 0$	$x = 0.1$	$x = 0.2$	$x = 0.3$	$x = 0.4$	$x = 0.5$	$x = 0.6$	$x = 0.7$	$x = 0.8$	$x = 0.9$	$x = 1.0$	$x = 1.1$
2	1.0	1.25	1.20	1.16	1.12	1.08	*1.06*	1.04	1.02	1.01	1.00	1.00	1.00
3	0.67	1.16	1.11	1.08	*1.05*	1.02	1.01	1.00	1.00	1.01	1.02	1.04	1.07
4	0.50	1.12	1.08	*1.04*	1.02	1.01	1.00	1.01	1.02	1.04	1.08	1.12	1.18
5	0.40	1.09	1.05	*1.02*	1.01	1.00	1.01	1.02	1.05	1.09	1.15	1.22	1.32
6	0.33	1.08	1.04	*1.01*	1.00	1.00	1.02	1.05	1.10	1.16	1.24	1.35	1.49
7	0.29	1.07	*1.03*	1.01	1.00	1.01	1.04	1.08	1.15	1.23	1.35	1.49	1.69

Note: The table gives values of $\rho(x)$, the wealth level required with equity allocation x, to reach the level of expected utility achievable with a $1 initial investment under the optimal equity allocation, x^*, for different levels of relative risk aversion, RRA. The table assumes that the annual Sharpe ratio is 0.30, the annual standard deviation of the return on the equity portfolio, $\sigma = 0.15$, and that the investor's time horizon (T) is 10 years. The italicized cells correspond to values of x that are closest to 50% of x^*.

Setting the left-hand side of (7) equal to a constant and simplifying, the investor's indifference curves in (μ_p, σ_p) space are defined by

$$\mu_p - \frac{1}{2}(1-\gamma)\sigma_p^2 = \text{ constant}$$

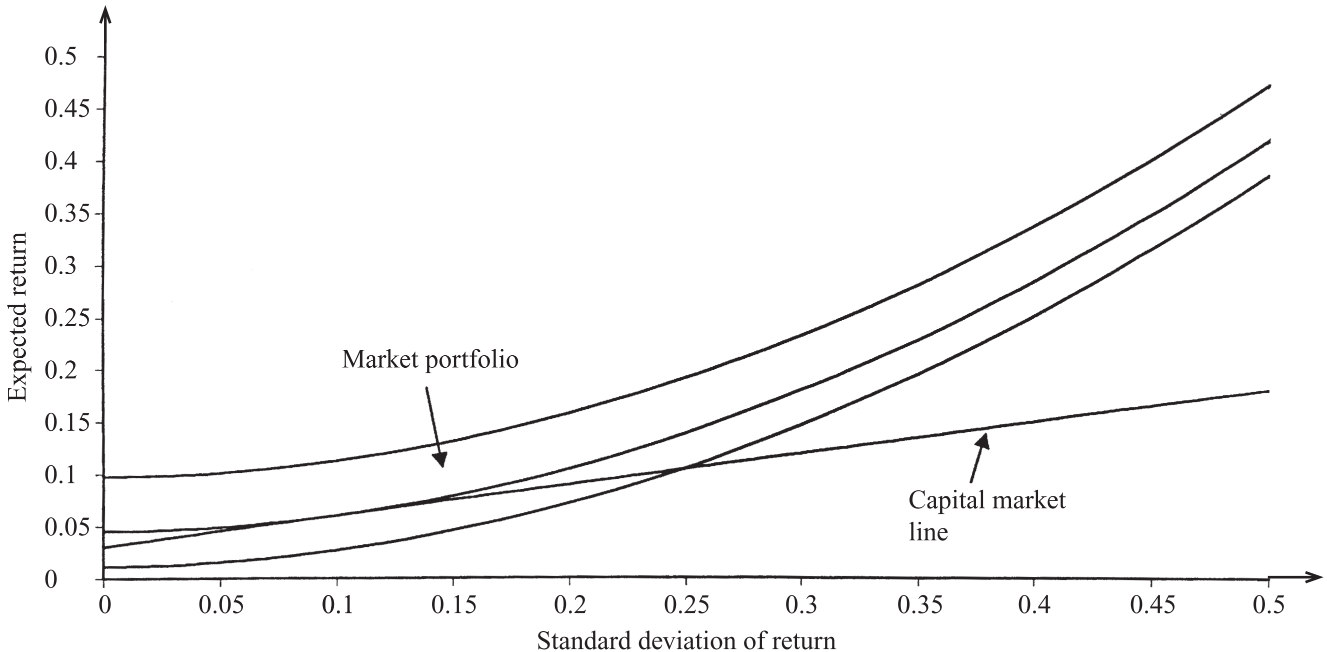
Figure 1 plots the indifference curves of an investor for whom RRA = 3 along with the 'capital market line' which is drawn on the assumption that the riskless interest rate is 3 per cent and the (annual) Sharpe ratio is 0.30, implying a market risk premium of 4.5 per cent if the standard deviation of the market return is 15 per cent. The capital market line is the locus of (μ_p, σ_p) combinations that are attainable by varying *x*, the portfolio allocation to equities. It may be seen from the figure (as well as from Tables 1 and 2) that the optimal portfolio for this investor has a standard deviation of 10 per cent and a mean return of 6 per cent. The certainty equivalent (CE) rate of return for the optimal portfolio is represented by the intersection of the indifference curve through the optimal portfolio with the vertical axis; for this investor, the CE rate of return for the optimal portfolio is 4.5 per cent. Notice, however, that the capital market line is virtually coincident with the indifference curve for standard deviations ranging from about 8 per cent to about 12 per cent. Thus, while the investor's optimal portfolio has a standard deviation of 10 per cent, he is virtually indifferent between the optimal portfolio and other portfolios along the capital market line that are attainable by varying the portfolio allocation parameter *x*, and therefore the portfolio standard deviation, by 20 per cent around the optimum. Even larger departures from optimality have relatively small welfare implications; for example, a portfolio allocation to equities that is only 40 per cent of the optimal allocation ($\sigma = 0.04$) implies a CE rate of return of 4 per cent, or only $\frac{1}{2}$ per cent less than that of the optimal allocation. As the figure makes clear, it is the curvature or second derivative of the indifference curve

$$\frac{\partial^2 \mu}{\partial \sigma^2} = 1 - \gamma$$

that determines the welfare loss associated with departures from the optimal point on the capital market line, and, for moderate degrees of risk aversion,[19] the curvature is low.

Our general conclusion that the cost associated with a given proportional departure from the optimal allocation to equities is likely to be modest is reinforced by two further considerations. First, the value for the Sharpe ratio that we have assumed is based on historical data for the USA and, as Brown *et*

[19] There is considerable controversy among economists about what constitutes a 'reasonable' level of risk aversion, but most authorities would accept that a value of RRA exceeding 10 would constitute very high risk aversion. With a Sharpe ratio of 0.3 and a market standard deviation of 15 per cent an investor will invest all his wealth in the equities if RRA = 3.



Note: Indifference curves of an investor with RRA = 3 and the straight line originating at 3 per cent on the vertical axis with a slope (Sharpe ratio) of 0.3.

Figure 1: Optimal portfolio choice in mean-standard deviation space.

al. (1995) have argued, this historical estimate may be subject to substantial positive selection bias. To the extent that the true Sharpe ratio is lower than 0.3, the welfare loss due to investing a given proportion of the optimal amount in equities will be less than we have calculated. For example, when RRA $= 3$, if the true Sharpe ratio is only 0.15 rather than 0.30, the optimal allocation to equities drops to 0.33 from 0.67, and the cost of investing half the optimal amount in equities ($x = 0.165$ instead of 0.33) drops to 2 per cent from 8 per cent, when the horizon is 20 years. A second, and related, consideration is that we have treated the market risk premium as *known*, whereas it is at best an estimate and, as Brennan (1998) has shown, estimation risk significantly reduces the optimal allocation to equities for investors who are more risk averse than the logarithmic utility function (RRA $= 1$).

2.2. Simulation Results

The results reported in Tables 1 and 2 rest on the assumption that the stochastic process for stock returns is adequately described by the geometric Brownian Motion, equation (1), and that the portfolio is rebalanced continuously to maintain the equity proportion, x. To test the robustness of the results to the relaxation of these assumptions, the expected utility of non-optimal portfolio policies was calculated from actual returns, selected by Monte Carlo simulation. The basic data are the monthly excess returns (in excess of the 30-day Treasury Bill rate) on the CRSP VW market index for the period January 1926 to December 1997. The mean (annualized) excess return was 8.22 per cent, and the (annualized) standard deviation of the excess return was 19.1 per cent, yielding an annual Sharpe ratio of 0.432. This Sharpe ratio and standard deviation were then used to calculate the optimal equity allocation according to equation (5) for each value of RRA as shown in the third line of Table 3. Then, for each value of the equity portfolio allocation, x, shown in the left-hand column of Table 3, the expected utility was estimated by simulating portfolio returns for 10 and 20 years. For each simulation, the riskless interest rate was taken as 3 per cent and equity returns were obtained by drawing at random (with replacement) from the 864 element vector of market excess returns and adding the result to the riskless interest rate. Two portfolio strategies were evaluated for each value of x. The first one assumes that the portfolio is rebalanced monthly to the assumed equity proportion. The second strategy assumes that the initial equity allocation is set equal to x, but that no rebalancing takes place; this 'buy-and-hold' strategy is intended to represent the type of inefficiency in portfolio allocations induced by the *status quo* bias described by Samuelson and Zeckhauser (1988). The simulations were repeated 100 000 times for each horizon and value of RRA, and the values of $\rho(x)$ for the two strategies are reported in Table 3. In this table, the upper entry

Table 3: Monte Carlo Estimates of the Welfare Costs of Non-optimal Equity Allocations for 10- and 20-year Horizons with and without Rebalancing

	RRA = 2 $T = 10$ $x^* = 1.13$	RRA = 2 $T = 20$ $x^* = 1.13$	RRA = 3 $T = 10$ $x^* = 0.76$	RRA = 3 $T = 20$ $x^* = 0.76$	RRA = 4 $T = 10$ $x^* = 0.57$	RRA = 4 $T = 20$ $x^* = 0.57$	RRA = 5 $T = 10$ $x^* = 0.45$	RRA = 5 $T = 20$ $x^* = 0.45$	RRA = 6 $T = 10$ $x^* = 0.38$	RRA = 6 $T = 20$ $x^* = 0.38$	RRA = 7 $T = 10$ $x^* = 0.32$	RRA = 7 $T = 20$ $x^* = 0.32$
$x = 0.0$	1.58	2.50	1.36	1.85	1.26	1.59	1.21	1.45	1.17	1.37	1.14	1.31
	1.58	2.50	1.36	1.85	1.26	1.59	1.21	1.45	1.17	1.37	1.14	1.31
$x = 0.1$	1.46	2.14	1.26	1.59	1.17	1.37	1.12	1.26	1.09	1.18	1.07	1.14
	1.42	1.92	1.23	1.46	1.15	1.28	1.10	1.19	1.07	1.14	1.06	1.10
$x = 0.2$	1.36	1.86	1.18	1.39	1.10	1.22	*0.06*	*1.12*	*1.04*	*1.07*	*1.02*	*1.04*
	1.32	1.64	1.15	1.29	1.09	1.16	*1.05*	*1.10*	*1.03*	*1.06*	*1.02*	*1.04*
$x = 0.3$	1.28	1.63	1.12	1.25	*1.05*	*1.11*	1.02	1.04	1.01	1.01	1.00	1.00
	1.24	1.46	1.10	1.18	*1.05*	*1.09*	1.02	1.05	1.01	1.03	1.01	1.03
$x = 0.4$	1.21	1.46	*1.07*	*1.14*	1.02	1.04	1.00	1.01	1.00	1.00	1.01	1.02
	1.18	1.33	*1.06*	*1.11*	1.02	1.05	1.01	1.03	1.01	1.03	1.02	1.04
$x = 0.5$	1.15	1.32	1.03	1.07	1.00	1.01	1.00	1.01	1.02	1.04	1.05	1.09
	1.13	1.24	1.03	1.06	1.00	1.03	1.01	1.03	1.02	1.05	1.04	1.07
$x = 0.6$	*1.10*	*1.22*	1.01	1.02	1.00	1.00	1.02	1.04	1.06	1.12	1.12	1.22
	1.09	*1.17*	1.01	1.03	1.01	1.03	1.02	1.05	1.05	1.09	1.08	1.13
$x = 0.7$	1.06	1.13	1.00	1.00	1.02	1.03	1.07	1.13	1.14	1.27	1.23	1.45
	1.06	1.11	1.00	1.02	1.02	1.04	1.05	1.10	1.10	1.17	1.15	1.24
$x = 0.8$	1.04	1.07	1.00	1.00	1.05	1.09	1.14	1.27	1.26	1.51	1.40	1.79
	1.03	1.06	1.00	1.01	1.04	1.08	1.10	1.18	1.18	1.29	1.25	1.41
$x = 0.9$	1.02	1.03	1.01	1.03	1.10	1.20	1.24	1.48	1.42	1.85	1.65	2.28
	1.02	1.03	1.01	1.03	1.09	1.16	1.19	1.34	1.32	1.54	1.45	1.75

Note: The table gives estimates of the values of $\rho(x)$, the wealth level required with equity allocation x, to reach the level of expected utility achievable with a $1 initial investment under the optimal equity allocation, x^*, for different levels of relative risk aversion, RRA, and time horizon, T. Each row of the table contains two entries for $\rho(x)$. The upper entry assumes that the portfolio is rebalanced monthly to maintain the proportionate equity allocation x. The lower entry assumes that a fraction x of the initial investment is allocated to equities, *and that there is no subsequent rebalancing*. The table is constructed by Monte Carlo simulation. The riskless interest rate is assumed to be 3 per cent and the excess return on the equity portfolio is calculated by drawing uniformly with replacement from the excess returns (over the 1-month Treasury bill rate) of the CRSP VW market portfolio for the period January 1926 to December 1997. The italicized cells correspond to values of x that are closest to 50 per cent of x^*.

in each cell corresponds to the rebalancing strategy, while the lower entry corresponds to the buy-and-hold strategy, and the italicized cells correspond to values of x that are approximately 50 per cent of the optimal values.

In comparing the results reported in Table 3 with those reported in Tables 1 and 2, allowance must be made for the fact that Table 3 is constructed using the empirical Sharpe ratio of 0.432 for the period 1926–1997 to compute x^*, instead of the value of 0.30 used in the previous tables; in addition, the standard deviation of the market index return is 19.1 per cent instead of 15 per cent as assumed previously. To determine whether the differences between the tables are due to differences in the parameter values or to the relaxation of the lognormal assumption, Table 4 reports *analytical* estimates of $\rho(x)$ for a rebalancing strategy, using the same values of the Sharpe ratio and standard deviation of the equity return as underlie the data used in the simulations in Table 3. The values for the rebalancing strategy in the two tables are almost identical, confirming the validity of the analytical expression (6) for $\rho(x)$ when the portfolio is rebalanced monthly. Therefore, the differences between the values reported in Table 3 and those in Tables 1 and 2 are due almost entirely to the different values of the Sharpe ratio and the standard deviation of the equity return. The higher Sharpe ratio underlying Table 3 implies that the opportunity costs of investing less than the optimal amount in equities are higher than reported in Tables 1 and 2. For example, in Table 1 when RRA = 2, the optimal allocation to equities is 1.00, and the opportunity cost of taking half the optimal position is 12 per cent. In Table 3 when the horizon is 20 years, and RRA = 2, the optimal allocation to equities is 1.13 and the opportunity cost of allocating only 0.56 to equities is about 26 per cent. When RRA = 4, the opportunity cost of half the optimal allocation to equities is about 6 per cent; the corresponding figure for 20 years is about 11 per cent. With the exception of RRA = 2 where the figure is higher, the opportunity cost of investing only half the optimal amount in equities for 20 years as reported in Table 3 is around 10–15 per cent of the initial investment.

Finally, we observe from the simulated results in Table 3 that the inefficiency costs for the buy-and-hold strategy are generally smaller than, but close to, those for the rebalancing strategy.[20] We have ignored transaction costs. Taking these into account would tip the balance further in favour of the buy-and-hold strategy for non-taxable accounts. Hence, there is no evidence that the *status quo* bias imposes any significant costs on investors.

In summary, the cost of excessive conservatism as measured by the opportunity cost of investing half the optimal amount in equities for 20 years is of the order of 5–12 per cent of the initial investment using the figures in Table 1, and around 10–15 per cent of the initial investment using the data in Table 3.

[20] This is consistent with the popular wisdom that even annual rebalancing is ‘not worth the effort’ (Waggoner, 1998).

Table 4: Analytical Estimates of the Welfare Costs of Non-optimal Equity Allocations for 10- and 20-year Horizons with Continuous Rebalancing for Parameter Estimates Corresponding to the Data used for the Monte Carlo Simulations Reported in Table 3

	RRA = 2 $T = 10$ $x^* = 1.13$	RRA = 2 $T = 20$ $x^* = 1.13$	RRA = 3 $T = 10$ $x^* = 0.76$	RRA = 3 $T = 20$ $x^* = 0.76$	RRA = 4 $T = 10$ $x^* = 0.57$	RRA = 4 $T = 20$ $x^* = 0.57$	RRA = 5 $T = 10$ $x^* = 0.45$	RRA = 5 $T = 20$ $x^* = 0.45$	RRA = 6 $T = 10$ $x^* = 0.38$	RRA = 6 $T = 20$ $x^* = 0.38$	RRA = 7 $T = 10$ $x^* = 0.32$	RRA = 7 $T = 20$ $x^* = 0.32$
$x = 0.0$	1.59	2.54	1.36	1.86	1.26	1.59	1.21	1.45	1.17	1.36	1.14	1.31
$x = 0.1$	1.47	2.17	1.26	1.69	1.17	1.37	1.12	1.25	1.09	1.18	1.07	1.14
$x = 0.2$	1.37	1.88	1.18	1.40	1.10	1.22	*1.06*	*1.12*	*1.03*	*1.07*	*1.02*	*1.04*
$x = 0.3$	1.29	1.65	1.12	1.25	*1.05*	*1.11*	1.02	1.04	1.01	1.01	1.00	1.00
$x = 0.4$	1.22	1.48	*1.07*	*1.15*	1.02	1.04	1.00	1.01	1.00	1.00	1.01	1.02
$x = 0.5$	1.16	1.34	1.04	1.07	1.00	1.01	1.00	1.01	1.02	1.03	1.04	1.08
$x = 0.6$	*1.11*	*1.23*	1.01	1.03	1.00	1.00	1.02	1.04	1.06	1.12	1.10	1.22
$x = 0.7$	1.07	1.15	1.00	1.00	1.01	1.03	1.06	1.12	1.12	1.26	1.20	1.44
$x = 0.8$	1.04	1.08	1.00	1.00	1.04	1.08	1.12	1.25	1.22	1.48	1.34	1.79
$x = 0.9$	1.02	1.04	1.01	1.02	1.09	1.18	1.20	1.44	1.35	1.82	1.53	2.34

Note: The table gives estimates of the values of $\rho(x)$, the wealth level required with equity allocation x, to reach the level of expected utility achievable with a $1 initial investment under the optimal equity allocation, x^*, for different levels of relative risk aversion, RRA, and time horizon, T. The riskless interest rate is assumed to be 3 per cent, the annual Sharpe ratio is 0.432, and the annual standard deviation of the return on the equity portfolio is 0.19. These values correspond to the CRSP VW market portfolio for the period January 1926 to December 1997, whose returns are used to construct the Monte Carlo estimates in Table 3. The italicized cells correspond to values of x that are closest to 50 per cent of x^*.

2.3. Management Fees

To place the costs of suboptimal investment strategies in perspective, it is useful to compare them with reasonable levels of management fees. Consider a management fee which is levied each year at the rate c on the market value of the portfolio assets at the beginning of the year. Then, following Ross (1978), it may be shown that the present value of the fees to be levied over the next T years on a portfolio with initial value S_0, is

$$[1 - (1 - c)^T]S_0$$

This expression may be understood by noting that the fraction of the portfolio taken by the first year fee is c, leaving a fraction $(1 - c)$ of the original portfolio to the investor; on this fraction a further fractional fee of c is levied in the second year; thus the fraction of the value of the initial portfolio allocated to the second year fee is $c(1 - c)$, and the fraction of the portfolio not allocated to either the first or second year fees is

$$1 - c - c(1 - c) = (1 - c)^2$$

so that the present value of the fees to be paid over the first two years is

$$[1 - (1 - c)^2]S_0$$

If the fee is charged continuously at the rate c on the current market value of the portfolio, the corresponding expression for the present value of the fees is

$$[1 - \mathrm{e}^{-cT}]S_0$$

It follows that $\pi(c, T)$, the amount that must be invested in a portfolio with a continuous fee rate c, to achieve the same final wealth after T years as \$1 invested in a portfolio without fees is given by

$$\pi(c, T) = \mathrm{e}^{cT}$$

Values of $\pi(c, T)$ are shown in Table 5 for annual fee rates ranging from $\frac{1}{2}$ per cent to 2 per cent.[21] Taking the $1\frac{1}{2}$ per cent fee as representative, we see

Table 5: The Cost of Portfolio Management Fees for Different Investment Horizons

T	$c = 0.005$	$c = 0.010$	$c = 0.015$	$c = 0.020$
5 years	1.025	1.051	1.078	1.105
10 years	1.051	1.105	1.162	1.221
15 years	1.078	1.162	1.252	1.350
20 years	1.105	1.221	1.350	1.492

Note: The table gives values of $\pi(c, T)$, the wealth level required with a continuous management fee at the rate c to achieve the same payoff as \$1 invested in a portfolio on which no management fee is levied when the investment horizon is T years.

[21] For comparison, the median expense ratio on US equity mutual funds was 1.57 per cent in 1991. (*Business Week*, March 23, 1991).

that the cost of this fee is 35 per cent for a 20-year horizon and 16.2 per cent for a 10-year horizon. These costs are roughly comparable to those shown in Table 3 for RRA = 2 when the allocation to equities is 0.5, or less than 50 per cent of the optimal allocation of 1.13. For higher values of RRA, the costs of the $1\frac{1}{2}$ per cent management fee considerably exceed the opportunity costs associated with investing only 50 per cent of the optimal amount in equities. They are comparable to the opportunity costs of investing *double* the optimal amount in equities.

In summary, it appears that the opportunity costs of an error of a factor of 2 in the allocation to equities is comparable to having a portfolio management fee of $1\frac{1}{2}$ per cent per year. Since such fee levels are commonplace, we conclude that the problem of portfolio management fees is at least as important as (and probably more important than) the purported problem of excessive conservatism in individual investor portfolio choice. Moreover, there is no evidence that the tendency of investors to allow their asset allocations to be determined passively by the realized returns on the different asset classes by their failure to rebalance, imposes any significant welfare cost.

3. Welfare Costs of Suboptimal Diversification

To assess the costs of holding an inadequately diversified portfolio, Monte Carlo simulation was used to choose a starting year, and the securities to be held, in an equally weighted (EW) portfolio over a given holding period; the security returns were then drawn from the CRSP file. For example, for the EW portfolio with a T-year holding period, for each simulation run, i, a portfolio formation year between 1926 and $1997 - T$ was chosen with equal probability; then N securities were chosen at random from those listed in January of that year, where each security that was listed had an equal probability of being chosen; returns on these securities were weighted equally to form monthly portfolio returns over the next T years,[22] and the cumulative return per \$1 initially invested, W_{TNi}, was calculated. This process of selecting a year and then the securities to be included in the portfolio for the next T years was repeated 10 000 times to yield an estimate of the probability distribution of wealth outcomes from a T year, N security equally weighted (EW) portfolio strategy. The certainty equivalent (CE) wealth outcome of this strategy for an investor with a risk-aversion parameter, γ, $\mathrm{CE}(T, N; \gamma)$ was then estimated by

$$\mathrm{CE}(T, N; \gamma) = \left[\frac{1}{10\,000} \sum_{i=1}^{10\,000} W_{TNi}^{\gamma} \right]^{\frac{1}{\gamma}} \tag{8}$$

22 Where a security return was missing it was replaced by the return on another randomly chosen security for the same month.

The procedure was repeated for a value-weighted (VW) probability portfolio; whereas for the EW portfolio each security listed in January of the portfolio formation year has an equal chance of being included in the portfolio, for the VW probability portfolio, the probability of being included in the portfolio is proportional to the equity market value in January of the portfolio formation year; however, both strategies weight equally the returns on the securities that are included in the portfolio, and correspond to strategies in which the portfolio weights are rebalanced monthly to be equal.

The CE wealth is the sure amount to be received at the investment horizon per dollar invested that would make the investor as well off as following the specified policy. The CE wealth outcomes are shown in Table 6 for different investment horizons (T), numbers of securities in the portfolio (N) and coefficients of relative risk aversion (RRA) for both the EW portfolio (which is shown in Roman) and the VW probability portfolio (which is shown in italic). The two rows at the foot of the table show the CE wealths for strategies of investing in the EW CRSP and the VW CRSP portfolios where the starting date is chosen randomly from the Januaries between 1926 and $1997 - T$. Each column in Table 6 shows the effect of increasing the number of securities held in the portfolio, holding constant the investment horizon (T) and risk aversion (RRA). As expected, the CE wealth from investing in a risky portfolio is greater, the less risk averse is the investor, the longer the investment horizon, and the more diversified is the investment portfolio. It is immediately apparent that the welfare costs of inadequate diversification can be very large indeed, and that they potentially dwarf the costs of suboptimal leverage. For example, for an investor with a RRA coefficient of 2, the CE wealth from investing in a 1-security portfolio for 10 years is \$0.36. This is equivalent to losing 64 per cent of one's wealth *for sure* over 10 years! For more risk-averse investors, the situation is even worse: for a RRA coefficient of 7, the 1-security policy is equivalent to a CE loss of as much as 99 per cent over 10 years. A 5-security portfolio policy does lead to a CE wealth increase in all cases, but there are still significant welfare gains from going from an EW 20-security portfolio to a 50-security portfolio; these are in the range of 7–25 per cent of the CE wealth of the 20-security strategy. To place the CE wealth gains in perspective, note that the \$16.15 20-year CE for an investor with RRA = 2 who purchases a 50-security portfolio corresponds to a certain annual rate of return of 14.9 per cent over 20 years; the corresponding CE rate of return for an investor with RRA = 7 is 12.5 per cent. These CE rates of return fall to 14.1 per cent and 10.1 per cent when the number of securities is reduced from 50 to 10. This means that the less risk-averse investor could afford to pay an annual 'management' fee of 0.8 per cent to achieve the additional diversification, while the more risk-averse investor could afford to pay as much as 2.4 per cent.

The CE wealth yielded by the EW portfolio (in italic) exceeds the certainty equivalent wealth of the corresponding VW portfolio (in Roman) whenever the number of securities in the portfolio exceeds one; this reflects

Table 6: Certainty Equivalent (CE) Payoffs for *Equal Probability* and *Value-Weighted Probability* Random Portfolios formed with Equal Weights under Portfolio Strategies Rebalanced Monthly

	RRA = 2 $T = 10$	RRA = 2 $T = 20$	RRA = 3 $T = 10$	RRA = 3 $T = 20$	RRA = 4 $T = 10$	RRA = 4 $T = 20$	RRA = 5 $T = 10$	RRA = 5 $T = 20$	RRA = 6 $T = 10$	RRA = 6 $T = 20$	RRA = 7 $T = 10$	RRA = 7 $T = 20$
$N = 1$	0.36	1.19	0.05	0.13	0.02	0.05	0.01	0.03	0.01	0.02	0.01	0.02
	1.00	*1.15*	*0.21*	*0.10*	*0.08*	*0.03*	*0.05*	*0.02*	*0.03*	*0.01*	*0.03*	*0.01*
$N = 5$	3.01	12.02	2.38	9.60	1.87	7.86	1.51	6.60	1.26	5.69	1.09	5.01
	2.50	*7.47*	*2.13*	*6.02*	*1.76*	*4.77*	*1.44*	*3.86*	*1.19*	*3.23*	*1.01*	*2.82*
$N = 10$	3.50	13.91	3.02	11.84	2.55	10.14	2.15	8.76	1.93	7.68	1.61	6.87
	2.71	*8.26*	*2.41*	*7.18*	*2.12*	*6.27*	*1.87*	*5.51*	*1.64*	*4.89*	*1.45*	*4.38*
$N = 20$	3.73	15.11	3.30	13.35	2.85	11.91	2.41	10.67	2.04	9.50	1.75	8.53
	2.79	*8.64*	*2.53*	*7.71*	*2.28*	*6.87*	*2.06*	*6.17*	*1.87*	*5.60*	*1.71*	*5.13*
$N = 50$	3.89	16.15	3.52	14.63	3.15	13.38	2.79	12.30	2.49	11.39	2.23	10.63
	2.83	*8.83*	*2.58*	*7.96*	*2.35*	*7.19*	*2.16*	*6.53*	*1.99*	*6.02*	*1.85*	*5.57*
EW portfolio	3.34	12.80	2.97	11.54	2.68	10.16	2.44	9.39	2.21	8.68	2.03	8.08
VW portfolio	*2.38*	*6.29*	*2.03*	*5.23*	*1.75*	*4.29*	*1.54*	*3.66*	*1.37*	*3.28*	*1.25*	*2.98*

Note: The table gives Monte Carlo estimates of the certainty equivalent wealth per dollar invested for an investor with a given level of relative risk aversion, RRA, who invests in an N security portfolio for T years. The estimates in roman type are for equal probability portfolios that are derived by choosing an initial portfolio formation year at random between 1926 and $1997 - T$, and choosing N securities at random from those listed on the CRSP file at the beginning of that year. The final wealth is calculated by compounding the monthly returns on an equally weighted portfolio of the selected securities over the following T years. 10 000 simulations were used. The estimates in italics correspond to the returns on equally weighted portfolios whose constituent securities are selected with a probability that is proportional to the value of the issuing firm's equity in January of the randomly chosen formation year (value-weighted probability portfolios).

the higher average returns on small stocks that play a proportionately more important role in the EW portfolios. It is a striking fact that, while the 1-security portfolios yield very low CEs, the 5-security EW portfolios have CEs that exceed those of the VW CRSP portfolios for all values of RRA less than 5 for the 10-year strategy and for all values of RRA for the 20-year strategy. That is, on the basis of the historical evidence, it is a superior strategy for an investor to invest in an EW monthly rebalanced portfolio of 5 stocks for 20 years than to invest in the VW CRSP index portfolio for the same period; the 20-security EW portfolios outperform the VW CRSP index portfolio for all values of RRA considered. Indeed the 50-security EW portfolios have CE that exceed those of the EW CRSP index portfolio. This is because the average geometric return for the randomly selected portfolios exceeds that of the EW CRSP index portfolio by about 1 per cent per annum. The main difference in average portfolio composition between the randomly selected portfolios and the EW CRSP index portfolio is that the securities included in the former are all listed at the beginning of the 10- or 20-year holding period, whereas the constituents of the CRSP portfolio are continually updated to reflect new listings. Therefore, the superior performance of the randomly selected portfolios is consistent with the abnormally low returns to new listings that have been documented by Loughran and Ritter (1995).[23]

The CE wealths of the VW portfolios (in italics) typically – i.e. except for the 1-security portfolios – fall between those of the EW and VW CRSP indices and are around 60 per cent of those of the corresponding EW portfolios. This is not too surprising since these portfolios, while composed of securities selected on the basis of their market capitalization, are actually equally weighted – their formation scheme is thus a hybrid of equal and value weighting.

The essential message of Table 6 is that there are significant diversification gains to be garnered by increasing the number of securities in the portfolio and that these gains, which are more pronounced for higher levels of risk aversion, are still increasing when the number of securities in the portfolio is as high as 20.

Table 7 repeats the analysis of Table 6 but for buy and hold strategies instead of monthly rebalancing strategies. For both the VW and the EW portfolios, the buy and hold CEs are always less than the corresponding monthly rebalancing strategy CEs shown in Table 6. Monthly portfolio rebalancing – which entails selling winners and buying losers – seems to pay, at least on a pre-transaction cost basis. This is in interesting contrast to our finding that for the asset allocation decision the buy and hold strategy performs just about as well as the monthly revision strategy as shown in Table 3. It is

[23] These authors report that Initial Public Offerings of common stock underperform benchmark securities by an average of 6.7 per cent per year over the five years following the offering during the period 1970–1990.

Table 7: Certainty Equivalent (CE) Payoffs for *Equal Probability* and *Value-Weighted Probability* Random Portfolios formed with Equal Weights under Buy and Hold Portfolio Strategies

	RRA = 2 $T = 10$	RRA = 2 $T = 20$	RRA = 3 $T = 10$	RRA = 3 $T = 20$	RRA = 4 $T = 10$	RRA = 4 $T = 20$	RRA = 5 $T = 10$	RRA = 5 $T = 20$	RRA = 6 $T = 10$	RRA = 6 $T = 20$	RRA = 7 $T = 10$	RRA = 7 $T = 20$
$N = 1$	0.42	1.03	0.07	0.13	0.03	0.05	0.02	0.03	0.01	0.02	0.01	0.02
	1.12	*1.50*	*0.25*	*0.17*	*0.08*	*0.06*	*0.05*	*0.04*	*0.03*	*0.03*	*0.03*	*0.02*
$N = 5$	2.42	6.92	1.77	4.43	1.23	2.98	0.94	2.14	0.77	1.69	0.67	1.43
	2.37	*6.16*	*1.97*	*4.57*	*1.57*	*3.26*	*1.26*	*2.40*	*1.03*	*1.79*	*0.89*	*1.45*
$N = 10$	2.69	8.35	2.10	5.92	1.59	4.07	1.24	2.89	1.01	2.26	0.87	1.79
	2.55	*6.77*	*2.21*	*5.42*	*1.91*	*4.27*	*1.65*	*3.34*	*1.44*	*2.67*	*1.29*	*2.26*
$N = 20$	2.87	9.01	2.32	6.81	1.81	5.16	1.43	4.04	1.18	3.30	1.00	2.89
	2.62	*7.13*	*2.32*	*5.90*	*2.04*	*4.88*	*1.81*	*4.11*	*1.62*	*3.49*	*1.47*	*3.00*
$N = 50$	3.03	9.68	2.55	7.56	2.10	5.75	1.74	4.48	1.49	3.63	1.31	3.08
	2.63	*7.20*	*2.34*	*6.04*	*2.10*	*5.07*	*1.88*	*4.34*	*1.71*	*5.47*	*1.57*	*3.40*
EW portfolio	3.34	12.80	2.97	11.54	2.68	10.16	2.44	9.39	2.21	8.68	2.03	8.08
VW portfolio	*2.38*	*6.29*	*2.03*	*5.23*	*1.75*	*4.29*	*1.54*	*3.66*	*1.37*	*3.28*	*1.25*	*2.98*

Note: The table gives Monte Carlo estimates of the certainty equivalent wealth per dollar invested for an investor with a given level of relative risk aversion, RRA, who invests in an N security portfolio for T years. The estimates in roman type are for equal probability portfolios that are derived by choosing an initial portfolio formation year at random between 1926 and $1997 - T$, and choosing N securities at random from those listed on the CRSP file at the beginning of that year. The final wealth is calculated for an initially equally weighted portfolio of the selected securities when a buy and hold strategy is followed over the following T years. 10 000 simulations were used. The estimates in italics correspond to the returns on equally weighted portfolios whose constituent securities are selected with a probability that is proportional to the value of the issuing firm's equity in January of the randomly chosen formation year (value-weighted probability portfolios).

possible that the superior returns of the monthly revision strategy are due, at least in part to biases caused by the bid–ask spread (Roll, 1983; Blume and Stambaugh, 1983; Conrad and Kaul, 1993). Since the buy and hold strategy is easier to implement than the monthly rebalancing strategy and is free of bias, we tend to place more weight on the results shown in Table 7. These show the same qualitative tendencies as Table 6. For example, for an investor with a 20-year horizon and RRA equal to 2 the CE of a 5-security EW portfolio strategy is 6.92 which is equivalent to an annual rate of return of 10.1 per cent; as the number of securities is increased to 50 the CE rises to 9.68 which is equivalent to an annual rate of return of 12.0 per cent. For more risk-averse investors, the CE gains are even larger. Thus, we conclude that the potential welfare costs of inadequate diversification may be high. Equally striking, however, is the relatively poor performance of the VW market index. This reflects the superior returns on small firms over our sample period.

4. Conclusion

In this article, we have used the expected utility paradigm to show that the welfare costs of suboptimal leverage decisions made by individual investors are likely to be relatively modest, particularly when they are compared to the typical levels of fees charged for the management of investment portfolios. On the other hand, we have also shown that the costs associated with inadequate levels of diversification are potentially very large, and that significant welfare gains accrue to increasing the number of securities in the portfolio. This suggests that public policy should be directed at ensuring that individuals who manage their own retirement portfolios hold sufficiently diversified portfolios, and that the management costs of these portfolios be not too high. There is much less cause to be concerned that individuals will choose portfolios that are insufficiently leveraged.

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