Optimal exploration investments under price and geological-technical uncertainty: a real options model

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This article develops a real options model for valuing natural resource exploration investments (e.g. oil or copper) when there is joint price and geological-technical uncertainty. After a successful several-stage exploration phase, there is a development investment and an extraction phase. All phases are optimized contingent on price and geological-technical uncertainty.

Several real options are considered. There are flexible investment schedules for all exploration stages and a timing option for the development investment. Once the mine is developed, there are closure, opening and abandonment options for the extraction phase. Our model maintains a relatively simple valuation structure by collapsing price and geological-technical uncertainty into a one-factor model.

We apply the model to a copper exploration prospect and find that a significant fraction of total project value is due to the operational, development and exploration options available to project managers.

1. Introduction

We present a real options model for valuing natural resource exploration investments (e.g. oil or copper) when there is joint price and geological-technical uncertainty. Price risk refers to output market value while geological-technical risk applies to reserves, development investments and cost structure. Continuous-time Brownian motions are used to model both uncertainty processes assuming a futures market for output prices,[1] and a declining geological-technical risk level as exploration investments are undertaken. In case of finding an economically feasible mine, there may be a development investment phase, to be followed by an extraction phase. All phases are optimized contingent on price and geological-technical uncertainty.

Several real options are considered. The exploration investment schedule is flexible and may be stopped and/or resumed at any moment depending on cash flow expectations, which in turn depend on current commodity price and geological-technical expectations. The model allows for several exploration phases, each one with its own investment schedule and probabilities of success. In the event of an exploration success there is a timing option for the development investment, and closure, opening and abandonment options for the extraction phase.

The model has the virtue of maintaining a relatively simple structure by collapsing price and geological-technical uncertainty into a one-factor model. The model can be applied to value oil or other natural resource investments and has been used by a major copper company to value real exploration prospects. We solve the model using implicit finite-difference numerical methods and present results for a specific case.

Our model follows the rapidly increasing literature on the real options approach for the valuation of investments under uncertainty. Among them, Majd and Pindyck (1989) include the effect of the learning curve by considering that accumulated production reduces unit costs, Trigeorgis (1993) combines real options and their interactions with financial flexibility, McDonald and Siegel (1986) and Majd and Pindyck (1987) optimize the investment rate, and He and Pindyck (1992) and Cortazar and Schwartz (1993) determine two optimal control variables. This approach has been used to analyse uncertainty on many underlying assets, including exchange rates (Dixit, 1989), costs (Pindyck, 1993) and commodities (Ekern, 1988). Finally, many asset types and problems have been modelled using this approach, including natural resource investments, environmental and new technology adoption, and strategic and competitive options (Trigeorgis, 1996, 2000; Brennan and Trigeorgis, 2000; Dixit and Pindyck, 1994).

Our model follows the Brennan and Schwartz (1985) model for valuing natural resource investments. Closely related papers on the modelling of undeveloped oil fields or the software implementation approach are Paddock, Siegel, and Smith (1988), Cortazar and Schwartz (1997), and Cortazar and Casassus (1998). Other models for valuing oil contingent claims include Smith and McCardle (1998, 1999), Lehman (1989), and Trigeorgis (1990).

2. The model

2.1. *Modelling the exploration investment decision*

Exploration of natural resources typically involve following several stages, each one with an investment schedule and with associated success and failure probabilities. A representation of n exploration stages is presented in Figure 1 with the following notation:

- X^j Value of the exploration project at the initial point in stage j
- I^j Present value of the investment during stage j
- T^j Time of exploration stage j
- p^j Probability of success stage j
- H Value of the project at the end of the exploration stages, conditional on success

We can view the exploration project X as an infinitely compounded option that may be continuously exercised as the exploration investment is undertaken. The model assumes that at any point in time investment can be stopped or resumed depending on the expected value of the project, which in turn depends on the geological and technical data, as will be explained later.

Contingent on an exploration success the project may be developed by investing a present value of I^i_d during a development time of T^i_d. These values depend on the characteristics of the mine i found. The model considers a perpetual timing option by allowing the delay of this development investment. Once the decision to invest is made, there is no possibility of stopping the development investment.

When the development investment is concluded, the mine enters into the extraction phase. We use Brennan and Schwartz (1985) for modelling this phase considering opening, closing and abandonment options.

2.2. *Modelling price and geological-technical risk*

Exploration prospects of natural resources (e.g. oil, copper) are very risky investments because both output prices (which have been widely studied) and output quantities and development/production costs are uncertain.

Commodity price risk has long been modelled using no arbitrage finance models in a continuous-time

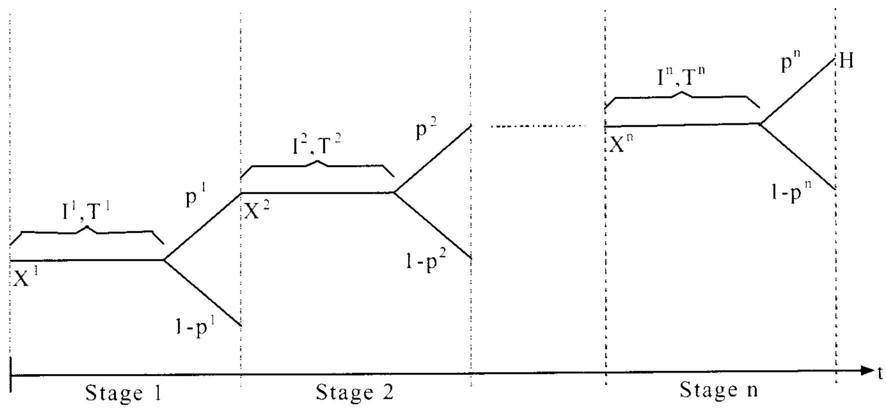

Figure 1. Illustration of the n exploration stages of the project.

setting. Even though recent multi-factor price models are very promising for explaining commodity price behaviour (Schwartz, 1997; Cortazar *et al.*, 1999), for simplicity in this paper we use a standard one-factor constant-convenience-yield model for risk-neutral prices:

$$\frac{dS}{S} = (r - c)\, dt + \sigma_S\, d\omega_S \qquad (1)$$

with the following notation:

- S Spot unit price of copper
- r Risk free real rate of interest, assumed constant
- c Convenience yield on holding one unit of Copper
- σ_S Instantaneous volatility of returns on holding one unit of copper
- $d\omega_S$ Increments to standard Gauss-Wiener process

While price risk is constant during all three phases of the project (exploration, development and extraction) the geological-technical risk is much higher during exploration phase. This geological-technical risk may be decomposed into two parts: one is the success or failure in finding an economically feasible mine, and the other is related to the particular characteristics of the (eventual) mine. While the first part has already been modelled using the discrete success-failure probabilities at each of the exploration stages, the second one, defined by mine reserve levels, development investments, production schedules and cost structures, is explained in what follows.

Before undertaking any exploration investment, uncertainty on the value of the expected final reserve, conditional on success, is at its greatest level. The characteristics of the eventual mine are highly unknown ranging from a modest to a highly profitable one. In a real options framework it is clear that exploration investments should optimally be undertaken considering both the expected mine value and its distribution.

As exploration investment is undertaken, uncertainty on the final characteristics of the mine decreases and price risk becomes comparatively more important. The Brennan and Schwartz (1985) model of a copper mine, for example, considers only price risk once development investment starts. But it is clear that an appropriate exploration investment model should add to its price risk the effect of a declining geological-technical risk.

To model geological-technical risk, different approaches may be followed. One alternative could be to define a vector of geological-technical variables that affect mine value and specify a stochastic process for each of them. In this case the value of the (already explored) mine would be a function of output price, S, as well as of a vector $\mathbf{G}$ of mine characteristics, $\{G_1, G_2, \ldots G_N\}$, which could include development investment, extraction rate, costs, etc. Thus, the value of the expected mine would be defined as $H(S, G_1, G_2, \ldots, G_N)$. In this setting, model complexity increases with the dimension of the mine-characteristics vector $\mathbf{G}$ due to both the amount of the information required to specify the multivariate process for the state variables, and to the added effort of solving a multi-factor model.

We take a simpler approach that provides a reasonable approximation for many cases and helps keep the model tractable. Instead of asking geologists/mining engineers to specify the level and multivariate process for all relevant mine characteristics, we ask them only to determine a representative set of possible mine-types, with their probabilities of occurrence, that could be found attending current prospect characteristics. Each mine-type is defined using the parameters required by the Brennan and Schwartz (1985) model, including total reserves, development investment amounts and schedule, and the production schedule (amounts and costs), with the associated opening and closing costs. Thus we are able to value each of the possible mines as a function of output price using the Brennan and Schwartz (1985) model. Using the conditional probabilities for each mine-type, we obtain the *expected* mine value (as a function of output price) as well as an initial empirical *distribution* of mine values that we define as the geological-technical risk.

To obtain the process for this geological-technical risk we start by defining a one-dimensional state variable G to represent this risk. We know the initial empirical *distribution* of mine values associated with this risk (obtained using the initial probability assessment) and assume that after the exploration investment concludes there will be no residual geological-technical uncertainty. In the absence of better information on the particular characteristics of the exploration process, we assume that initial geological-technical uncertainty is reduced continuously as exploration investment is undertaken.

Once we have defined mine value as a function of two state variables, output spot price, S, and geological-technical risk, G, we use the fact that both factors may be assumed to be independent and in many cases may be collapsed into one state variable, Z. This makes the model very simple to implement, while providing a reasonable approximation to the project value.

To formalize the model we define a geological-technical risk factor G (for example the amount of mineral in a mine) that follows a zero-drift constant volatility Brownian motion, as follows:[2]

$$\frac{dG}{G} = \sigma_G\, d\omega_G \qquad (2)$$

This geological-technical risk factor is assumed to be independent of output price S:

$$d\omega_S\, d\omega_G = 0 \qquad (3)$$

Mine value, $H(S, G)$, can be modelled as a function of output price, S, and the geological-technical variable, G.

We can now define a new state variable Z, a function of S and G, such that:

$$H(Z) \equiv H(S, G) \tag{4}$$

$$\text{and} \quad Z \equiv F(S, G) \tag{5}$$

Applying Ito's lemma and using equations (1) and (2), we obtain:

$$dZ = \left(F_S S(r - c) + \frac{1}{2} F_{SS} S^2 \sigma_S^2 + \frac{1}{2} F_{GG} G^2 \sigma_G^2 \right) dt + F_S S \sigma_S \, d\omega_S + F_G G \sigma_G \, d\omega_G \tag{6}$$

We can further simplify model implementation by assuming:

$$Z = SG \tag{7}$$

This is equivalent to assuming that an increase in any of the two factors (S or G) has a similar effect on mine value. The process for this new state variable becomes:

$$\frac{dZ}{Z} = (r - c) \, dt + \sigma_S \, d\omega_S + \sigma_G \, d\omega_G \tag{8}$$

The new state variable, Z, can be seen as a modified commodity price with the same drift as the original one, S, but with an increased volatility:

$$\sigma_Z = \sqrt{\sigma_S^2 + \sigma_G^2} \tag{9}$$

For many projects equation (7) represents a very convenient approximation; for others it is not only a good approximation, but holds perfectly. This is the case, for example, if G represents total mine reserves, A_1 is a constant that depends on a fixed extraction rate[3] and A_2 is a fixed cost. Then project value H is defined by:

$$H(S, G) = A_1 SG + A_2 \quad \text{or} \quad H(Z) = A_1 Z + A_2 \tag{10}$$

and equation (7) holds.

Reducing model complexity at the exploration phase by collapsing all geological-technical risk into one factor, with an appropriate (increased) volatility, is convenient because it allows for representing many geological-technical factors by their joint effect on mine value. The fact that all these factors are orthogonal to the market factor, (represented in this model by commodity price), allows for this reduction in the state space with little accuracy-loss while retaining the possibility of being individually considered during the extraction phase. Should there be any additional knowledge on the way exploration investment modifies geological-technical risk for any particular case, adjustments to the above model should be undertaken.

2.3. *Estimation of geological-technical volatility*

As we have stated above, all the geological-technical risk is represented by the initial distribution of the expected mine value at the beginning of the development stage. Before initiating the exploration stage we have an expected mine value and the variance for this distribution as a function of output price. The estimation of the geological-technical volatility σ_G needs to be consistent with the variance of the expected mine value. By applying Ito's lemma to the expected value of the mine for any given output price S (such that all the volatility is due to the geological-technical risk), we obtain the relation to pin down σ_G.

2.4. *Valuing the exploration investment project*

In order to value the exploration project we start by valuing the alternative mines that could be obtained if exploration is successful. Then, we value the development investment decision, to conclude valuing the exploration phase.

Before exploration investments begin, there is a set of alternative mines that could eventually be obtained, should the exploration phases be successful. Each one of the M possible mines is assumed to have an associated probability of occurrence, conditional on exploration success, α^i, such that:

$$\sum_{i=1}^{M} \alpha^i = 1 \tag{11}$$

2.4.1. Value of a developed mine. Each of the M possible mines is valued using the Brennan and Schwartz (1985) model:[4]

$$\max_{q_p^i} \left[\frac{1}{2} V_{SS}^i S^2 \sigma_S^2 + (r - c) S V_S^i - q_p^i V_Q^i + q_p^i (S - a^i) - (r + \lambda) V^i \right] = 0 \tag{12}$$

$$\frac{1}{2} W_{SS}^i S^2 \sigma_S^2 + (r - c) S W_S^i - m^i - (r + \lambda) W^i = 0 \tag{13}$$

Subject to:

$$W^i(S_0^{*i}, Q) = 0 \tag{14}$$

$$V^i(S_1^{*i}, Q) = \max[W^i(S_1^{*i}, Q) - K_1^i, 0] \tag{15}$$

$$W^i(S_2^{*i}, Q) = V^i(S_2^{*i}, Q) - K_2^i \tag{16}$$

$$W_S^i(S_0^{*i}, Q) = 0 \tag{17}$$

$$V^i_S(S^{*i}_1, Q) = \begin{cases} W^i_S(S^{*i}_1, Q) & \text{if } W^i(S^{*i}_1, Q) - K^i_1 \geq 0 \\ 0 & \text{if } W^i(S^{*i}_1, Q) - K^i_1 \leq 0 \end{cases} \quad (18)$$

$$W^i_S(S^{*i}_2, Q) = V^i_S(S^{*i}_2, Q) \quad (19)$$

$$V^i(S, 0) = 0 \quad (20)$$

$$W^i(S, 0) = 0 \quad (21)$$

$$V^i(0, Q) = 0 \quad (22)$$

$$W^i(0, Q) = 0 \quad (23)$$

$$\lim_{S \to \infty} V_{SS}(S, Q) = 0 \quad (24)$$

$$\lim_{S \to \infty} W^i_{SS}(S, Q) = 0 \quad (25)$$

We use the following notation:

V^i Value of mine i when it is optimal to be open
W^i Value of mine i when it is optimal to be closed
Q Reserves
q^i_p Production rate of mine i
S^{*i}_0 Critical Price below which it is optimal to abandon
S^{*i}_1 Critical Price below which it is optimal to close
S^{*i}_2 Critical price above which it is optimal to open
K^i_1 Cost of closing the mine i
K^i_2 Cost of opening the mine i
m^i Annual maintenance cost of keeping a mine closed
λ Country risk (or probability of expropriation)

2.4.2. Value of an undeveloped mine. We first assume the development investment decision has been made, but is still under way. The value of the mine must now satisfy the following equation:

$$\frac{1}{2} U^i_{SS} S^2 \sigma^2_S + (r - c) S U^i_S - U^i_T - (r + \lambda) U^i = 0 \quad (26)$$

Subject to:

$$U^i(0, T) = 0 \quad (27)$$

$$\lim_{S \to \infty} U^i_{SS}(S, T) = 0 \quad (28)$$

$$U^i(S, 0) = V^i(S, Q) \quad (29)$$

Then, we can define H as the value of the mine before the development investment is made, which must satisfy:

$$\frac{1}{2} H^i_{SS} S^2 \sigma^2_S + (r - c) S H^i_S - (r + \lambda) H^i = 0 \quad (30)$$

Subject to:

$$H^i(0) = 0 \quad (31)$$

$$H^i(S) = \begin{cases} H^i(S) & \text{if } S \leq S^{*i}_d \\ U^i(S, T^i_d) & \text{if } S \geq S^{*i}_d \end{cases} \quad (32)$$

We use the following notation:

I^i_d Development investment
T^i_d Time of development stage (after the investment has been done)
S^{*i}_d Critical Price above which it is optimal to invest

Once each of the mines are valued, we can obtain an expected mine value by multiplying each of the values by its probability α^i.

$$H(S) = \sum_{i=1}^{M} \alpha^i H^i(S) \quad (33)$$

2.4.3. Value of an exploration project. Finally, we solve for the value of the exploration project. We consider that while the project is under way and exploration investment is undertaken, the value of the project is X, and while the project is optimally stopped, the value is Y.

Notice that while exploration investment is under way (X), relevant volatility is higher than when it is temporally stopped (Y),[5] because geological-technical information is only obtained with investment.

It is possible to solve for each stage j, with $j = 1, n$. To do this we start by solving for stage $j = n$ and work our way backwards until $j = 1$. We now present the equations for an intermediate stage j.

$$\max_{q^j_i} \left[\frac{1}{2} X^j_{ZZ} Z^2 \sigma^2_Z + (r - c) Z X^j_Z + q^j_i X^j_I - q^j_i - (r + \lambda + \gamma^j) X^j \right] = 0 \quad (34)$$

$$\frac{1}{2} Y^j_{ZZ} Z^2 \sigma^2_S + (r - c) Z Y^j_Z - (r + \lambda) Y^j = 0 \quad (35)$$

Subject to:

$$X^j(0, I) = 0 \quad (36)$$

$$Y^j(0, I) = 0 \quad (37)$$

$$\lim_{Z \to \infty} X^j_{ZZ}(Z, I) = 0 \quad (38)$$

$$X^j(Z, I^j) = X^{j+1}(Z, 0) \quad \text{if } Z \geq Z^{*j} \quad (39)$$

$$Y^j(Z, I^j) = Y^{j+1}(Z, 0) \quad \text{if } Z \leq Z^{*j} \quad (40)$$

For the last stage these last two boundary conditions are replaced by

$$X^n(Z, I^n) = H(Z) \quad \text{if } Z \geq Z^{*n} \quad (41)$$

$$Y^n(Z, I^n) = H(Z) \quad \text{if } Z \leq Z^{*n} \quad (42)$$

We use the following notation:

I Accumulated exploration investment at that stage
q^j_i Investment rate stage j

I^j Present value of the investment during stage j
T^j Time of exploration stage j
p^j Probability of success stage j
Z^{*j} Critical Price for investing at stage j
γ^j Poisson probability of success stage j[6]

3. Results

The model can be applied to oil and other commodity exploration prospects. We have applied it to several exploration prospects available to a copper company. To our knowledge there is no analytical solution to this compound option system of equations even if we consider infinite resource profiles during the extraction stage. In what follows we present a specific case and report the results of solving the above exploration model using implicit finite-difference numerical methods.

The prospect we present considers that before exploration starts the geologists have an exploration investment plan with four exploration stages, each one with its own investment schedule and probabilities of success (see Table 1). In case of failure at one stage the exploration is abandoned. In case of success the project could be stopped or resumed, depending on the expected value of the mine. This expected value varies due to both changes in prices and/or on the expected geological-technical characteristics of the mine.

If the exploration stage is successful, the project enters into the development stage. In the specific case we are evaluating, we consider 11 different development/extraction plans, each one represented by a mine profile. Each of these possible mines has particular development and production schedules. For example, Table 2 and Table 3 describe one of the mine profiles ('Mine 1' in Figure 2) that has a probability of 15% of occurrence.

Figure 2 shows the value of all mines contingent on output price using the Brennan and Schwartz (1985) model. The Expected Deposit is the expected mine value obtained by multiplying each of the values by its occurrence probability.

Figure 3 presents the optimal exploration investment schedule. If the expected mine value exceeds the critical expected mine value, then exploration investment should proceed; if it does not, then it should be stopped.

Table 1. Description of the exploration stages of the project.

Exploration stage	Investment (MUS$)	Time of exploration (years)	Success probability
1	3	1	0.2
2	15	3	0.3
3	12	2	0.3
4	12	2	0.8

Table 2. Investments of the development stage for mine 1.

Year	Investment (MUS$)
1	123.2
2	246.4
3	123.2

Table 3. Costs and production schedule of the extraction stage for mine 1.

Maintenance costs (MUS$/Yr.)		0.45
Closure costs (MUS$)		70.4
Opening costs (MUS$)		48.3
Year	Production (M Lbs)	Operating Costs (MUS$)
1	248	81.1
2	248	81.9
3	248	97.5
4	248	83.6
5	248	84.4
6	248	100
7	248	86.1
8	248	87
9	248	102.6
10	248	88.7
11	248	89.6
12	166.4	60.7

Finally, Table 4 shows the value of the exploration project at the beginning of the exploration stage and how can it be decomposed in terms of option values associated with each stage. The optimal value of the exploration project (total value) is decomposed into: (a) the value of the project if there is no flexibility (or there is no volatility) at any stage (*Value without options*); (b) the value added for optimally opening, closing or abandoning the mine during the extraction stage (*Operational options*); (c) the extra value for optimally deferring the development investment (*Development options*) and; (d) the additional value for optimally investing during the exploration stage (*Exploration option*). The results are presented contingent on the value of the expected mine before development investment is made.

For example, suppose that at the beginning of the exploration stage our best estimation for the value of the mine that we can potentially develop is 500. In this value we consider the output price and estimations regarding possible reserves, investments, costs and productions during development and extraction phases. If we don't consider any flexibility during the life of the project, the value of the exploration project is −11.44. To calculate this value, we evaluate the project considering that the investment rate during exploration stage j is always q_i^j even when it is optimal to wait, that the development investment is done immediately once the exploration stage is finished (in all mine profiles),

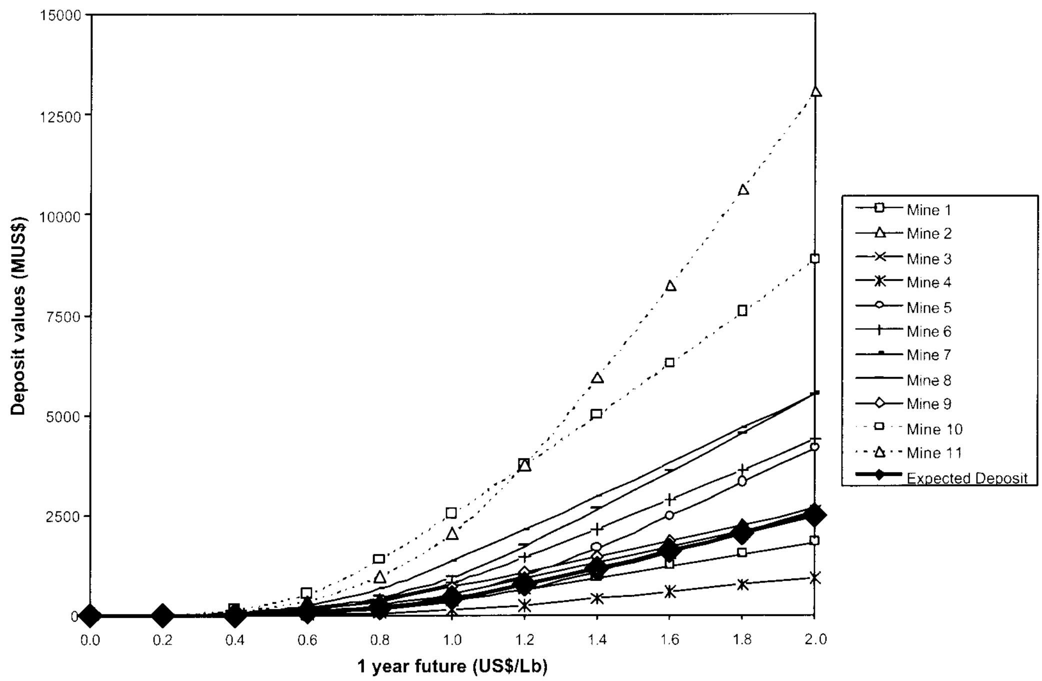

Figure 2. Values before development investment for the different mine-profiles and the expected deposit contingent on output price using the Brennan and Schwartz (1985) model.

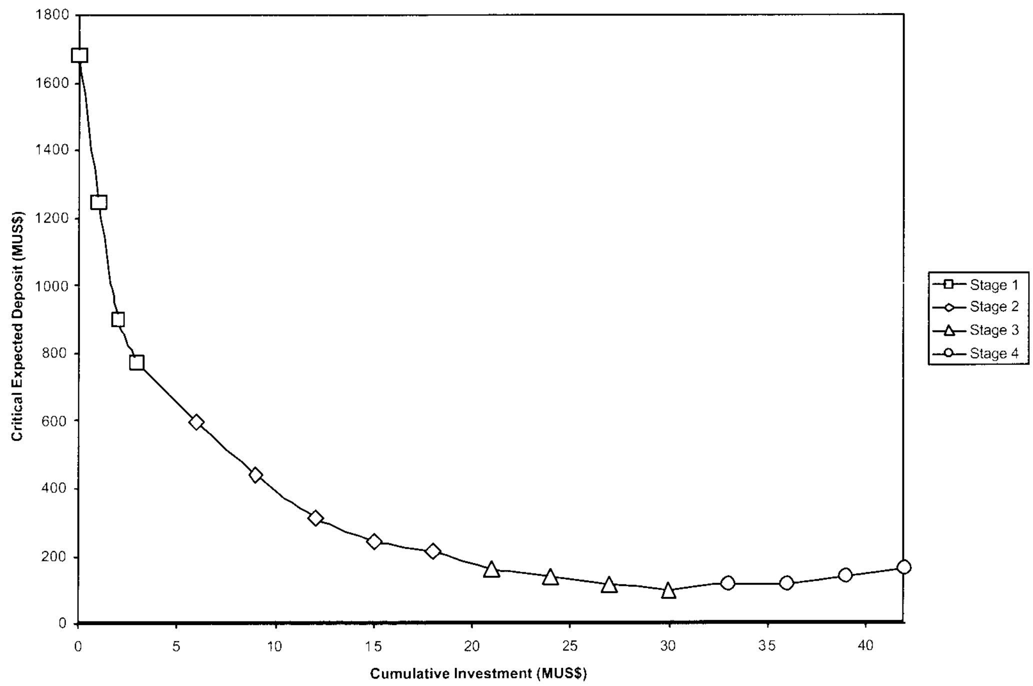

Figure 3. Optimal investing policy for the 4 exploration stages contingent on cumulative investment. If the expected deposit is above the critical value, it is optimal to invest.

Table 4. Sources of value of the exploration project in MUS$, when $r = 2.8\%$, $c = 6\%$, $\sigma_S = 40\%$ and $\lambda = 2\%$.

Expected deposit (MUS$)	Value without options	Operational options	Development option	Exploration option	Total value
500	−11.44	6.68	2.94	3.19	1.37
1000	−6.29	5.46	2.25	1.58	3.01
1500	−1.95	4.71	1.82	0.60	5.17
2000	2.26	4.15	1.49	0.20	8.09

and that the production rate during the extraction stage is always q_p^i (in all mine profiles), even if it is optimal to stop or abandon the mine. If we only consider operative options during the extraction phase, the value of the exploration project is -4.76 ($=-11.44+6.68$). To calculate this value we allow for an optimal operation of the mine (see equations (12) to (25)). If in addition we consider the development option the value increases to -1.82 ($=-11.44+6.68+2.94$). To get this value we optimally defer the development investment (see equations (30) to (32)). Finally to get the total value of the exploration project (and the exploration option value) we consider that we can optimally invest during exploration stage (see equations (34) to (42)). This will give a value for the exploration project of 1.37 and an exploration option value of 3.19.

4. Conclusions

We have presented a real options model for valuing natural resource exploration investments when there is joint price and geological-technical uncertainty. By collapsing both sources of uncertainty, price and geological-technical uncertainty, into a one-factor model for expected value we are able to maintain model simplicity, while retaining operational flexibility.

The model considers that the exploration investment schedule may be stopped and/or resumed at any moment depending on cash flow expectations, which depend on current commodity price and geological-technical expectations. Once all exploration phases are concluded the project is modelled as having the flexibility of postponing development investments and, once developed, as having the option to close or reopen production.

Results for a copper exploration prospect shows that a significant fraction of total project value is due to the operative, the development and the exploration options available to project managers.

Acknowledgments

We acknowledge financial support from FONDECYT and FONDEF. We thank Dean Paxson, an anonymous referee, participants of the 2000 European Financial Management Association Meeting, Athens, of the 3rd Annual Conference Real Options: Theory Meets Practice International, NIAS/Royal Academy of Arts and Sciences, Wassenaar/Leiden, 1999, and of the Real R&D Options Symposium, Manchester, 2000, for helpful comments, and Andrés Löwener for research assistance.

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Notes

1. Actually the model only requires that price risk may be hedged. This is easily satisfied when there is a futures market, but also if there is a portfolio of traded assets perfectly correlated to commodity spot prices.
2. To understand why the process has zero drift, suppose for simplicity that G represents only the risk associated with the total reserves. If we think that the reserves will increase at a certain rate during the exploration stage, then our estimation of the initial G should be updated such that it reflects the reserves that we are expecting by the end of the exploration stage, keeping G as a martingale.
3. For example if all reserves can instantaneously be extracted, then $A_1 = 1$.
4. Although in this specification it is straightforward to allow for extra cash flows like taxes, we have not included them for simplicity.
5. Recall that $\sigma_Z > \sigma_S$
6. Since the probability that no Poisson event occurs in interval $(0, T^j)$ (i.e. success of exploration stage j) is $e^{-\gamma^j T^j}$, it is easy to see that γ^j should be $-\ln(p^j)/T^j$.