The Impact of Alternative Performance Measures on Single-Period Inventory Policy

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Abstract

While many firms and researchers have developed various supply chain solutions, there are many underlying reasons why these solutions have not been adopted in practice. Some key reasons, as articulated by Lee and Billington (1992), include organizational barriers, coordination challenges among marketing, manufacturing, and logistics, technical challenges in the area of information systems, as well as conflicting supply chain performance metrics. Other key reasons are due to alternative performance measures besides total expected relevant cost, which include sales target, product substitution, product clearance, sales per square foot, etc. In order to understand how these alternative performance measures affect the supply chain solution, we make an initial attempt to analyze how alternative measures would affect the simplest form of inventory policy, namely, the newsvendor solution. To identify various alternative measures and to explore how such order decisions are made, we conducted a simple experiment by giving a single-period inventory problem to 250 MBA students and 6 professional buyers who order fashion items. We observed that both groups select their order quantities less than the newsvendor solution and made their ordering decisions based on various specific performance metrics besides total expected cost. These observations have motivated us to analyze how these performance metrics would affect the ordering decision. Our analysis indicates that, under these performance metrics, it is rational for the decision maker to order less than the newsvendor solution.

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1 Introduction

Most supply chain models and analyses tend to focus on developing solutions that minimize total expected relevant cost. However, there are many reasons why these supply chain solutions are rarely implemented in practice. Firstly, when dealing with products with short life cycles or manufacturing processes with uncertain and limited supply capabilities, these solutions tend to breakdown. Fisher (1997) provided some insightful ideas regarding ways to develop the right supply chain solutions for the right products. Secondly, as articulated by Lee and Billington (1992), there are many obstacles for implementing these supply chain solutions, which include organizational barriers, coordination challenges among marketing, manufacturing, and logistics, as well as technical challenges in the area of information systems, demand planning process, as well as conflicting supply chain performance metrics. Thirdly, even though expected cost is an important performance measure, there are other equally important performance measures, which include sales target, product substitution, product clearance, sales per square foot, etc. When considering alternative performance measures, traditional models and their solutions may not be robust.

In order to understand how these alternative performance measures affect the supply chain solution, we make an initial attempt to analyze how alternative measures would affect the simplest form of inventory policy, namely, the newsvendor solution. We select the newsvendor problem as our initial attempt because it is the simplest single-period inventory problem with a simple optimal ordering decision. Despite its simple and intuitive form, the newsvendor solution has not been widely used in practice for single-period inventory problems. Of course, it is clear that the newsvendor solution will not be used when the buyers are not fully aware of the information necessarily to determine the newsvendor solution, e.g., the tradeoff between the overstock cost and the understock cost. However, it is unclear how one selects an order quantity if all the necessary information is known. To enhance our understanding of this issue, we designed an experiment based on a single-period ordering scenario in which full information, the cost tradeoff and the demand distribution parameter values, is provided. The cost tradeoff in the scenario was chosen so that the newsvendor solution exceeded the mean demand forecast.

We conducted our experiment on 250 MBA students at the Anderson School at UCLA
and the Owen School at Vanderbilt, and we conducted informal interviews with 6 professional buyers.\footnote{Note that all MBA students have 3.5 years of working experience. However, to avoid any pre-conceived bias, we select MBA students who have not been exposed to the newsvendor solution. In addition, note that these 6 buyers are employees of Sears, J.C. Penny, Relax the Back stores, Bugle Boy, and Arizona, and they are professional buyers who order fashion items on a regular basis. The intention of this experiment was not meant to be an empirical survey. Instead, it was meant to gain some basic insights into how decision makers (MBA students and professional buyers) select an appropriate order quantity when confronted with a single-period inventory problem.} We observed that both groups generally select order quantities much less than the newsvendor solution and that both groups made their ordering decisions based on various specific performance metrics besides total expected cost. In addition, most MBA students order according to the mean (demand forecast), while most professional buyers order below the mean\footnote{One may argue that this happens because the MBA students and the buyers may not know the exact newsvendor solution. However, we observed that the respondents are aware of the fact that the understock cost is higher than the overstock cost. As such, if they determine their order decision purely based on the notion of expected cost, then they should order at least above the mean.} These observations have motivated us to analyze how these performance metrics would affect their ordering decision. Our analysis indicates that, under these performance metrics, it is rational for the decision maker to order less than the newsvendor solution.

This paper is divided into two halves. In this first half, we consider the ordering behavior of the MBA students – the tendency to order according to the mean. Since these MBA students are not professional buyers, we cannot identify specific underlying reasons for their ordering behavior. Instead, we attempt to justify the ordering behavior by also considering short-term and risk-related concerns, not just the expected profit concern as with the newsvendor problem. We introduce new performance measures that reflect these concerns and show that when evaluated on the basis of these concerns, ordering according to the mean demand forecast, may, in fact, not be a poor decision.

In the second half, we consider the ordering behavior of professional buyers, using information provided during the informal interviews of 6 buyers. Besides selecting an appropriate order quantity for the scenario, we asked each buyer to articulate their performance metrics (incentives) and additional concerns that might impact their order decision. The performance metrics identified include: meeting a profit target, meeting targets on both sales and gross margin, and minimizing excess inventory. Additional concerns identified include in-
ventory holding cost during the season, cannibalization of next season’s demand by leftover product, availability of a substitutable product, and a limited production budget. Although the buyers worry about most of these performance metrics and additional concerns, examining their combined effect on order quantity is difficult. Instead, we consider each of these performance metrics/additional concerns separately, and we show that the consideration of each causes the buyer to lower the order quantity from the newsvendor solution. Thus, the buyer will order much less than the newsvendor solution when he takes most of the metrics/concerns into consideration. (In our case, the buyer’s average order quantity was even less than the mean.)

We believe this paper makes three contributions. First, we show that decision makers order significantly less than the newsvendor problem in a single-period setting in which full cost and demand information is provided. Due to the universality of the newsvendor solution in academic literature and the degree of “under-ordering” that occurs, this observation is significant. Second, we examine the single-period inventory problem with alternative performance measures and analyze the impact of these measures on the ordering decisions, and illustrate how these measures would cause a buyer to order a smaller quantity. Third, we believe our analysis can serve as a building block for developing more complicated supply chain models with alternative performance measures and we hope our results may motivate other researchers to consider alternative performance measures besides expected cost or profit when developing inventory models or supply chain models.

The paper is organized as follows. In section 2, we first describe the traditional newsvendor problem and its solution. We describe the scenario of a single-period inventory problem that was given to the MBA students and the professional buyers, and report the results of our experiment. In section 3, we examine four performance gaps measures that reflect different concerns of the decision maker. By comparing the performance when ordering the mean to that when ordering the newsvendor solution, we show that, with these new concerns, ordering according to the mean demand forecast may, in fact, not be a poor decision. In section 4, we present the performance metrics and additional concerns raised by the buyers during our informal interviews. By considering each of these issues separately, we show that the consideration of these performance metrics and additional concerns will cause the buyer to lower the order quantity from the newsvendor solution. Thus, the inclusion of each of these
additional concerns helps to justify the buyers’ ordering of quantities below the newsvendor solution, often below the mean. We conclude this paper in section 5.

2 The Newsvendor Problem

Consider a single-period inventory problem in which a buyer has to decide on the order quantity of a product that has a single selling season. The objective is to maximize the expected net profit. This single-period inventory problem is known as the newsvendor problem, which was first addressed by Arrow, Harris and Marschak in 1951. To introduce the classical newsvendor problem, let us define the following notation:

\[ D = \text{demand over the selling season}, \]
\[ F(.) = \text{probability distribution of } D, \text{ where } E(D) = \mu \text{ and } V(D) = \sigma^2, \]
\[ Q = \text{order quantity}, \]
\[ p = \text{selling price}, \]
\[ c = \text{unit cost}, \]
\[ s = \text{salvage value}, \]
\[ C_u = \text{understock cost} = p - c, \]
\[ C_o = \text{overstock cost} = c - s, \]
\[ r = \text{critical ratio} = \frac{C_u}{C_u + C_o}, \]
\[ \Phi(.) = \text{standard normal distribution function, } \phi(.) = \text{standard normal density function, and} \]
\[ k = \text{optimal critical fractile} = \Phi^{-1}(r) = \Phi^{-1}\left(\frac{p-c}{p-s}\right). \]

For any realization of demand \( D \), the net profit to be generated from ordering \( Q \) units is denoted as \( \pi(Q, D) \), where

\[ \pi(Q, D) = p \min(Q, D) + s(Q - D)^+ - cQ = (p - s)\min(Q, D) - (c - s)Q. \quad (2.1) \]

Notice that this profit function can be rewritten as

\[ \pi(Q, D) = C_u D - C_u(D - Q)^+ - C_o(Q - D)^+. \quad (2.2) \]
If the goal is to determine the order quantity $Q^*$ that maximizes the expected net profit $E(\pi(Q, D))$, then this problem is known as the newsvendor problem. In addition, the optimal solution is known as the newsvendor solution, where $Q^* = F^{-1}(r)$. To simplify the exposition, we assume $C_u > C_o$ so that $Q^* > \mu$ throughout this paper.

### 2.1 A Simple Experiment

To understand how one would select an order quantity when confronted with a single-period problem with full information regarding the cost tradeoff and demand information, we have designed a simple experiment that is based on the following scenario of a single-period inventory problem. Five independent demand forecasts for a fashion product are 1000, 3000, 5000, 7000, and 9000 units, respectively. Selling price $p$ is $100 per unit, cost $c$ is $60 per unit, and markdown price (or salvage value) $s$ is $45 per unit. Each interviewee is requested to select an appropriate order quantity. Based on this scenario, the mean demand forecast is 5000 units, the standard deviation of the demand forecast is 3200, $C_u = \$40$, $C_o = \$15$, and the newsvendor solution is 6800, 6950, or 7000, depending on whether the distribution was taken to be uniform, normal, or discrete with equal probabilities for each of the five values. Throughout this paper, we shall use 6950 as the newsvendor solution.

We presented this scenario to 250 first year MBA students in 6 different class sections at The Anderson School at UCLA and The Owen School at Vanderbilt, and 6 professional buyers whose jobs are to order fashion items for stores such as Sears, J.C. Penny, and Relax the Back or for apparel manufacturers such as Bugle Boy and Arizona. The first year MBA students, at the time when the scenario was presented, had no exposure to any newsvendor type models, while some of the professional buyers are trained in the area of purchasing management. The average order quantity statistics generated from our experiments are provided in Table 1.

Observe from Table 1 that the MBA students ordered 5284 on average, well below the newsvendor solution of 6950. Although the under-ordering from the newsvendor solution was expected, the magnitude of under-ordering was larger than we expected. The average

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3The details of this experiment is provided in the Appendix.
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*A one-sided T-test measured relative to the mean forecast (5000 units).

Table 1: Order quantity results.

Order quantity is very close to the mean forecast of 5000, not the newsvendor solution. Using a null hypothesis that the mean order quantity is less than the mean forecast of 5000 (using a one-sided t-test), we could only reject the hypothesis in two of the six class sections (at a 0.05 level of significance) but could reject it overall. Thus, the mean order quantity is larger than mean forecast; however, the difference is small (for example, the null hypothesis that the order quantity is at least 150 units higher than the mean cannot be accepted at 0.05 significance). More interestingly, 111 out of the 250 MBA students (44%) chose to order exactly the mean (5000), by far the most common decision (the next most common was 7000 with 13%).

Since most our of MBA students selected the mean demand forecast as the order quantity, it raises the following question that we attempt to address in Section 3: When compared to the newsvendor solution, is the most common decision (the mean demand forecast) a poor order decision from the perspective of the decision-maker? Ordering according to the mean is sub-optimal if we assume a particular demand distribution and assume that the objective is to maximize expected profit. However, we believe decision-makers have multiple performance concerns, not just expected profit. For example, with the “one-shot” nature of the single-

4While Eeckhoudt et al. (1995) argue that the buyer is concerned about expected utility instead of expected cost, we believe that the buyer has multiple concerns. Our belief is supported by comments made by the MBA students to justify their ordering decision. Some concerns for those selecting the mean or lower include: potential profit loss (‘chance of huge profit loss if product doesn’t sell’, ‘minimize potential loss if the item has to be marked down’), likelihood of making a profit (‘Even if you’re one standard deviation off, you still make a profit [with 5000]’), and excess inventory (‘would not want to risk over-buying for the summer’, ‘I don’t want to be stuck with excess inventory’).
period problem, decision-makers could be more risk averse and feel more comfortable with a safer decision. This short-term perspective is different from the traditional perspective that focuses on the expected profit as the performance measure.

The order quantities selected by the professional buyers were quite striking. Although one buyer ordered 6500, two ordered 5000 and three ordered only 3000. The resulting average order quantity was 4250, a significantly lower value than 5284, the average order quantity selected by 250 MBA students.\(^5\) The implication of this significant difference in order quantities is that the professional buyers have some unique issues or concerns that cause them to order below the newsvendor solution and even below the mean demand forecast! To help determine these underlying reasons, we conducted an informal study in which we asked the buyers to describe their performance metrics and other concerns when placing an order. In Section 4, we discuss our findings and analyze the impact on the order quantity of each of the issues identified as important by the buyers.

3 Order According to the Mean

Motivated by the fact that most of our MBA students selected the mean demand forecast as the order quantity, we introduce a number of measures that account for the one-shot nature of the single-period problem. These measures allow us to compare the performance of having the mean demand forecast \(\mu\) and the newsvendor solution \(Q^*\) as order decisions.

3.1 Max-Min Performance Criterion

Consider the case in which the decision maker either does not know the underlying demand distribution or does not impose a specific demand distribution. In this case, the decision maker has to select an order quantity based on the given demand scenarios. When the

\(^5\)Although it was certainly not the intent of this study to rigorously prove the difference in buyers’ and our MBA students’ ordering behavior, we did perform a two-sample t-test to determine if the difference between the two average order quantities was statistically significant. Despite the small sample size, the level of significant was found to be 0.053.
underlying demand distribution is not known, it is quite common for the decision maker to make the ordering decision based on the max-min criterion; i.e., select an order quantity that will result in the highest minimum profit. In other words, if the worse case happens under each decision, which order decision would result in the least worst profit outcome?

Consider the case in which the demand $D$ takes on one of the following five possible values: $d_1, d_2, d_3, d_4, d_5$ where $d_1 < d_2 < d_3 < d_4 < d_5$. To simplify our exposition, let us consider the case in which these 5 possible values correspond to the demand forecasts specified in our experiment. Specifically, $d_1 = 1000, d_2 = 3000, d_3 = 5000, d_4 = 7000, d_5 = 9000$. If the demand distribution is discrete with equal probabilities, then the mean (or median) demand forecast $\mu = d_3 = 5000$ and the newsvendor solution $Q^* = d_4 = 7000$.

Since $c > s$, we can see from (2.1) that $\pi(Q, D)$ is decreasing in $D$. Thus, for an ordering decision, the minimum payoff occurs when the smallest demand is realized; i.e., when $D = d_1$. Comparing the profits under the mean demand forecast $\mu = d_3$ and the newsvendor solution $Q^* = d_4$, we have:

$$\pi(d_4, d_1) - \pi(d_3, d_1) = (p - s)d_1 - (c - s)d_4 - (p - s)d_1 + (c - s)d_3 = (c - s)(d_3 - d_4) < 0.$$  (3.1)

Thus, by the above analysis, the mean demand forecast $\mu = d_3$ is actually a better decision than the newsvendor solution $Q^* = d_4$ when the decision maker evaluates his decision based on the max-min criterion. As such, we can conclude that the mean demand forecast may not be a poor choice from the decision-maker’s perspective.

3.2 Performance Gaps

We now consider the case in which the decision maker imposes a specific demand distribution. Instead of comparing the expected profits, we consider the following performance gaps that enable us to compare the performance of having the mean demand forecast $\mu$ and the newsvendor solution $Q^*$ as order decisions. Let:

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6While we are considering 5 possible demand realizations here, we notice that the generalization to an arbitrary number of odd demand realizations is trivial, the only caveat being that the newsvendor solution must be greater than the median (mean) demand realization.
1. $\Delta(Q^*, \mu, D)$ be the **Profit Gap** that measures the difference in net profits between ordering $Q^*$ and $\mu$, where

$$\Delta(Q^*, \mu, D) = \pi(Q^*, D) - \pi(\mu, D),$$

(3.2)

2. $P\{Gain\}$ be the **Gain Probability** that measures the probability that ordering $Q^*$ would yield a higher profit than ordering $\mu$, where

$$P\{Gain\} = P\{\Delta(Q^*, \mu, D) > 0\},$$

(3.3)

3. $E(I(Q^*, \mu, D))$ be the **Expected Inventory Gap** that measures the difference in expected leftover inventory between ordering $Q^*$ and $\mu$, where

$$E(I(Q^*, \mu, D)) = E((Q^* - D)^+) - E((\mu - D)^+),$$

and

(3.4)

4. $E(\Delta(Q^*, \mu, D))$ be the **Expected Profit Gap** that measures the expected difference in net profits between ordering $Q^*$ and $\mu$, where

$$E(\Delta(Q^*, \mu, D)) = E(\pi(Q^*, D)) - E(\pi(\mu, D)).$$

(3.5)

In general, the first two performance gaps evaluate performance based on short-term profit concerns. The profit gap compares the profits associated with the newsvendor solution $Q^*$ and the mean demand forecast $\mu$ for a given demand realization. Instead of comparing the expected profits, the gain probability measures the probability that the newsvendor solution $Q^*$ yields a higher profit than the mean demand forecast $\mu$ does. The third performance gap compares the expected leftover inventories associated with the newsvendor solution and the mean demand forecast. This performance gap reflects the concern about excess inventory. The fourth performance gap measure compares the expected profits associated with the newsvendor solution and the mean demand forecast. Note that the fourth performance gap is always positive because the newsvendor solution $Q^*$ maximizes the expected profit.

To analyze these four performance gaps analytically, let us assume that the demand is taken to be normally distributed with a mean $\mu$ and a standard deviation $\sigma$. In this case, the mean demand forecast is equal to $\mu$ and the newsvendor solution, $Q^* = F^{-1}(r)$, can be simplified as:

$$Q^* = \mu + k\sigma.$$  

(3.6)
To simplify our analysis, we shall restrict our attention to the case in which the overstock cost $C_o$ satisfies: $C_u \geq C_o \geq 0$. When $C_u \geq C_o \geq 0$, $1 \geq \frac{C_o}{C_u+C_o} \geq 0.5$, $k = \Phi^{-1}(r) \geq 0$, and $Q^* \geq \mu$. The following Proposition specifies the performance gaps in closed form expressions:

**Proposition 1** Suppose that the demand is normally distributed with a mean $\mu$ and a standard deviation $\sigma$ and that $C_u \geq C_o \geq 0$. Then:

$$\Delta(Q^*, \mu, D) = \begin{cases} -C_o k \sigma & \text{if } D \leq \mu \\ -C_o k \sigma + (C_u + C_o)(D - \mu) & \text{if } \mu < D \leq \mu + k \sigma \\ C_u k \sigma & \text{if } D > \mu + k \sigma \end{cases}$$ (3.7)

$$P\{\text{Gain}\} = P\{\Delta(Q^*, \mu, D) > 0\} = 1 - \Phi((1 - r)k) \leq 0.5$$ (3.8)

$$E(I(Q^*, \mu, D)) = \sigma(\phi(k) + k\Phi(k) - \phi(0)) \geq 0$$ (3.9)

$$E(\Delta(Q^*, \mu, D)) = (C_u + C_o)[\phi(0) - \phi(k)]\sigma \geq 0$$ (3.10)

**Proof.** All proofs are given in the Appendix.

Consider each gap in proposition 1 independently. It follows from the definition of Profit Gap $\Delta(Q^*, \mu, D)$ and (3.7) that ordering $Q^*$ (instead of $\mu$) will reduce profit by $C_o k \sigma$ when $D \leq \mu$, and will increase profit by $C_u k \sigma$ when $D > \mu + k \sigma$. Thus, ordering the newsvendor solution $Q^*$ (instead of the demand forecast $\mu$) could yield a maximum profit loss of $L(Q^*, \mu)$ and a maximum profit gain of $G(Q^*, \mu)$, where:

$$L(Q^*, \mu, D) = -C_o k \sigma$$ (3.11)

$$G(Q^*, \mu, D) = C_u k \sigma$$ (3.12)

Thus, for a buyer who is more concerned with the profit under the low demand scenario, i.e., more concerned with maximum loss than with the maximum profit gain, the profit gap would suggest that the demand forecast is not a poor decision, and in fact, may be a better decision than the newsvendor solution.
It follows from the definition of the Gain Probability and (3.8) that the probability that ordering $Q^*$ would yield a higher profit than ordering $\mu$ is less than 0.5. Thus, by the gain probability evaluation, the demand forecast is then always a superior decision. Using expression (3.9), we can see the obvious result that ordering the demand forecast will result in lower expected excess inventory. Once again this suggests that the demand forecast may be a superior decision when a different evaluation method is used. Finally, (3.10) shows the increase in expected profit that results from ordering the newsvendor solution instead of the demand forecast.

Certainly the actual magnitude, not just the sign, of the performance gaps are critical in using the performance gaps to assess whether or not the demand forecast is a poor decision. We seek to understand when (under what values of the problem parameters) the performance gaps are large or small. Using (3.11), (3.12), and the expressions of the performance gaps presented in Proposition 1, it can be shown that:

**Proposition 2**

1. The maximum loss $L(Q^*, \mu)$ is decreasing in $C_u$, and it is decreasing in $C_o$ first and then increasing in $C_o$.

2. The maximum gain $G(Q^*, \mu)$ is increasing in $C_u$, and it is decreasing in $C_o$.

3. The gain probability $P\{\text{Gain}\}$ is decreasing in $C_u$ first and then increasing in $C_u$. In addition, it is decreasing in $C_o$ first and then increasing in $C_o$.

4. The expected inventory gap $E(I(Q^*, \mu, D))$ is increasing in $C_u$ and is decreasing in $C_o$.

5. The expected profit gap $E(\Delta(Q^*, \mu, D))$ is increasing in $C_u$ and is decreasing in $C_o$.

While most of the results in proposition 2 are intuitive, we note the non-intuitive, quasi-convex behavior of $L(Q^*, \mu)$ with respect to $C_o$ and of $P\{\text{Gain}\}$ with respect to $C_o$ and $C_u$. To further examine this behavior, we analyzed a numerical example with $\mu = 100$ and $\sigma = 10$. Figures 1 and 2 show $L(Q^*, \mu)$, $G(Q^*, \mu)$, and $P\{\text{Gain}\}$ as a function of $C_u$ and $C_o$, respectively. In Figure 1, $C_o$ was held at 5, and in Figure 2, $C_u$ was held at 10.
Observe from Figures 1 and 2 that the short term performance gaps justify ordering the demand forecast when $C_u$ and $C_o$ are very close but do not justify ordering the demand forecast when $C_u$ and $C_o$ are very far apart (in relative sense). This observation is intuitive. When $C_u$ and $C_o$ are close in value, the short term performance gaps are small since the value of $Q^*$ is very close to the value of $\mu$. When $C_o$ and $C_u$ are very far apart in value, the performance gaps are large — the gain probability is close to its maximum value of 0.5 and the maximum profit gain is many times greater than the maximum profit loss. Thus, for such disparately different overage and underage cost values, short term performance measures do not justify ordering according to the demand forecast. However, for “intermediate” values of $C_o$ relative to $C_u$, the short term performance measures can offer strong justification for ordering according to the demand forecast. Since the probability gain is quasi-convex in $C_o$ and $C_u$ according to Proposition 2, the performance gap often achieves its minimum (strongest bias for ordering the demand forecast) at intermediate values of $C_o$ and $C_u$. For instance, observe from Figure 2 that the probability gain achieves its minimum when $C_o = 3$. In addition, the maximum profit loss has this same quasi-convex property (although only in $C_o$), achieving its smallest (most negative value) at $C_o = 4$. Both these values of $C_o$ are “intermediate” relative to the value of $C_u = 10$. For such intermediate values of $C_o$ and $C_u$, the maximum profit gain is often not overwhelmingly larger than the maximum profit loss and the gain probability is small for such intermediate values of $C_o$ and $C_u$. Thus, while the expected profit gap would show the demand forecast order to be a poor decision, these short-term measures would show the opposite.

4 Order Below the Newsvendor Solution

Recall from Table 1 that the professional buyers tend to order below the newsvendor solution and 3 out of 6 buyers even order below the mean demand forecast! In addition to asking each buyer to select an appropriate order quantity when conducting our experiment, we asked each buyer independently about various factors (performance metrics and additional concerns)
that would affect his ordering decision. The important factors identified by the buyers can be divided into two general categories: performance metrics/incentives and additional concerns. In this section, we present analytical models to show how each of these factors would cause the buyer to lower his order quantity.

4.1 Performance Metrics and Incentives

Based on our informal interview with the 6 buyers from 5 different companies, we explicitly inquired about four potential performance metrics: total product profit, gross margin, excess inventory, and stockouts. Product profit and gross margin were consistently ranked as the two most important metrics. The only exception came from a buyer who indicated that sales volume and gross margin were the two most important metrics. More importantly, the buyers indicated that they were generally measured on their ability to attain a ‘target’ for profit, gross margin, or sales. With a target, a buyer’s objective is really to maximize the probability of achieving the target, not to maximize or minimize a certain performance measure as with the newsvendor problem. This change in objective can change the optimal order quantity considerably.

4.1.1 Attaining a Target Profit

Consider a buyer whose objective is to maximize the likelihood of achieving a profit target.

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7 This standard set of questions are given in the Appendix.
8 Most of the buyers did indicate that excess inventory and stockouts were used as metrics, although they did not have the same importance as profit, gross margin, and sales. The buyers were also asked if lost sales due to a stock out (not just the stockout occurrences) are measured. Only two buyers indicated that their firms made attempts to measure lost sales by counting customers that request an out-of-stock product. Since lost sales is either not measured or under-estimated while excess inventory is explicitly measured, there is an obvious tendency to order less than the newsvendor solution.
9 In later questions, two other buyers, who had previously indicated that gross margin targets were important, indicated that sales volume was an important metric.
10 Four indicated that targets were used, one indicated that targets were not used, and two left the target question blank.
Proposition 3 Suppose a buyer has a profit target of $\tau$. Then, the order quantity $\hat{Q}$ that maximizes the likelihood of achieving this profit target; i.e., $\hat{Q} = \text{Argmax}\{P[\pi(Q, D) \geq \tau] : Q \geq 0\}$, is given by $\hat{Q} = \tau/(p - c)$.

The implication of proposition 3 is that the optimal order decision $\hat{Q}$ will be less than the newsvendor solution $Q^*$ unless the profit target $\tau$ is sufficiently high. Specifically, $\hat{Q} < Q^*$ if $\tau < (p - c)Q^*$. This condition is met for all but unreasonably high profit targets. To illustrate this, we consider two reasonable profit targets: (1) the maximum expected profit $E[\pi(Q^*, D)]$ generated from the newsvendor problem, and (2) the profit generated from the expected demand $\mu(p - c)$; i.e., the product of the expected demand and the profit margin on each product sold at regular price (not at markdown).

First, let us consider the case when the profit goal $\tau = E[\pi(Q^*, D)]$. It is easy to show that:

$$\tau = E[\pi(Q^*, D)] = (p - c)Q^* - (p - s)E(Q^* - D)^+ \leq (p - c)Q^*. \quad (4.1)$$

As $\tau \leq (p - c)Q^*$, we can apply Proposition 3 to show that $\hat{Q} = \tau/(p - c) \leq Q^*$. Therefore, when the profit target is $\tau = E[\pi(Q^*, D)]$, it is optimal for the buyer to order less than the newsvendor solution. For the scenario given to the MBA students and the professional buyers, the optimal order quantity using this profit target would be 3540, well below the mean!

Next, consider the case when the profit goal $\tau = \mu(p - c)$. This would be a target we might expect management to set since it is simply the mean demand forecast times the profit margin. For this case, we can show an interesting result:

**Corollary 1** Under $\tau = \mu(p - c)$, the optimal order quantity is given by $\hat{Q} = \mu$.

Thus, under the profit target $\tau = \mu(p - c)$, the best decision for the buyer to order is the demand forecast.
4.1.2 Attaining a Sales Target and a Gross Margin Target

Consider the case in which a buyer’s success is measured on his ability to meet targets on sales and gross margin. Sales before markdown is \( \rho(Q, D) = p \min(D, Q) \), and the target is denoted \( \tau_s \mu p \), \( \tau_s > 0 \). Note that \( \mu p \) is the expected sales volume so \( \tau_s \) represents the percent of the expected sales volume. Gross margin is \( \pi(Q, D)/cQ \), and the target is denoted \( \tau_m \), \( 0 < \tau_m < 1 \).

On one hand, if the only objective is to maximize the sales target, the buyer will simply order as many units as possible. On the other hand, if the only objective is to maximize the probability of achieving the gross margin target, the buyer will order as few units as possible. Thus, a more meaningful objective is to maximize the weighted probability of achieving both targets. This objective will balance the desire for high gross margin (with low order quantities) with the desire for large sales volume (with high order quantities). Let \( w \) and \( 1 - w \) represent the relative importance to the buyer of achieving the targets on sales before markdown and gross margin, respectively. The buyer’s problem is then:

\[
\max_{Q \geq 0} \quad wP[\rho(Q, D) \geq \tau_s \mu p] + (1 - w)P[\pi(Q, D)/cQ \geq \tau_m],
\]

where the optimal solution is denoted by \( \tilde{Q} \).

**Proposition 4** Suppose a buyer has a sales target of \( \tau_s \mu p \) and a gross margin target of \( \tau_m \). Let \( \theta = [c(1 + \tau_m) - s]/(p - s) \leq 1 \). If

\[
\frac{w}{1 - w} \geq \frac{F(\tau_s \mu \theta)}{1 - F(\tau_s \mu)},
\]

then \( \tilde{Q} = \tau_s \mu \). Else \( \tilde{Q} \) is set as small as possible.

Proposition 4 characterizes the optimal order quantity \( \tilde{Q} \). Since \( \frac{w}{1 - w} \) is increasing in \( w \), condition (4.3) holds when the weight \( w \) is sufficiently large; i.e., when the probability of meeting the sales target is relatively more important. In this case, the buyer will order \( \tau_s \mu \), just enough to ensure that the sales target will be met if enough demand occurs. Ordering more does nothing to increase the probability of meeting the sales target but does lower the probability of achieving the gross margin target. Condition (4.3) does not hold when meeting
the sales target is less important (i.e., when \( w \) is low). In this case, the buyer should ignore the sales target and order as little as possible to maximize the probability of achieving the gross margin target.

It is easy to check from Proposition 4 that \( \bar{Q} \leq \tau_s \mu \). Thus, the condition \( \tau_s < Q^*/\mu \) is sufficient to guarantee that \( \bar{Q} \leq Q^* \). This sufficient condition is met as long as the sales target is not too over-inflated. Although sales target \( \tau_s \mu p \) is usually over-inflated so that \( \tau_s > 1 \), it is unlikely it is so over-inflated that this sufficient condition \( \tau_s \mu p < Q^* p \) does not hold. For example, in our experiment given to the buyers and the MBA students, \( \tau_s \) would have to be greater than 140%; i.e., the sales target is 40% above the sales based on the average demand, to violate this sufficient condition. We should also note that if the sales target over-inflation is very large (large \( \tau_s \)), the condition (4.3) will no longer hold, and the optimal decision is to order as little as possible. Thus, in this case, we no longer need to meet the sufficient condition \( \tau_s < Q^*/\mu \) to guarantee that \( \bar{Q} < Q^* \). The sales target is so large that the chance of getting enough demand to achieve the target is very small. Thus, it is optimal to give up on the sales target and simply maximize the probability of achieving the gross margin target.

4.2 Additional Concerns

The buyers were provided with a list of concerns and asked to rank them according to the impact (if any) they have on the order quantity decisions.\textsuperscript{11} In addition, the buyers were asked to suggest other concerns that affected their decision. The ranking were highly dependent on the individual buyer. However, each of the concerns, except for limited storage space issue, was viewed as important (achieving a 1, 2, or 3 ranking) for at least one buyer. We consider each of the concerns, showing that the inclusion of each in the newsvendor problem would result in an optimal order quantity that is less than the newsvendor solution. As in the previous subsection, we restrict our attention to the case in which the overstock cost \( C_o \) satisfies: \( C_u \geq C_o \geq 0 \).

\textsuperscript{11}The concerns listed were generated from informal discussions with a number of different purchasing managers and from suggestions by colleagues. In addition, the list of questions is provided in the Appendix.
4.2.1 Inventory Holding Cost

Consider the impact of inventory holding costs. In the newsvendor problem, it is implicitly assumed that the selling season is rather short, and hence, the cost of holding inventory throughout the selling season is negligible. However, in retailing, there are inventory holding costs that include the financial burden of carrying the inventory throughout the selling season, as well as storage and handling costs. Additionally, implicit costs are incurred because inventory could deprive the shelf space of other products, lowering the sales per square foot as well as the selling opportunity for other products.

To model the implication of this implicit inventory holding cost, let \( h \) be the inventory holding cost per unit over the selling season. Since the inventory at the beginning of the selling season is \( Q \) and the inventory at the end of the selling season is \( (Q - D)^+ \), the inventory holding cost incurred during the selling season is equal to \( \frac{Q+(Q-D)^+}{2}h \). Subtract the inventory holding cost from the net profit given in (2.1) and use the fact that \((Q-D)^+ = Q - \min(Q, D)\), the “modified” net profit \( \pi(Q, D) \) becomes:

\[
\pi(Q, D) = (p + h/2 - s)\min(Q, D) - (c + h - s)Q
\]

Define \( p' = p + h/2 \) and \( c' = c + h \). Suppose we view \( p' \) and \( c' \) as the “modified” selling price and unit cost, respectively. Then (4.4) is analogous to (2.1) when \( p \) and \( c \) are replaced by \( p' \) and \( c' \). In this case, the corresponding “modified” understock cost \( C'_u = p' - c' = p - h/2 < C_u \) and the “modified” overstock cost \( C'_o = c' - s > C_o \). Thus, imposing the inventory holding cost would reduce the understock cost \( C_u \) and increase the overstock cost \( C_o \). Since \( k = \Phi^{-1}(r) \), \( k \) is increasing in \( C_u \) and decreasing in \( C_o \). This observation and (3.6) imply that the newsvendor solution \( Q^* \) decreases when we impose the inventory holding cost. Thus, when one is concerned with the inventory holding cost, it is rational for the buyers to order less than the newsvendor solution.

4.2.2 Product Substitution

In the traditional newsvendor problem, it is assumed that the retailer would suffer from lost sales when the product is out of stock. However, in retailing, the shopper may simply
opt to buy a similar or related product. Thus, the retailer may not lose the 100% of the profit margin because product substitution is prevalent in retailing.\textsuperscript{12} In practice, assessing the magnitude of the impact of product substitution on the optimal order quantity for an individual product is very difficult, and it is currently an open research problem in Marketing research. The reader is referred to Ho and Tang (1998) for details. Measuring product substitutability requires an understanding of which other products can act as substitutes and of the fraction of demand for which a particular substitute product will be accepted. It also requires a deeper understanding of the demand structures. The buyer must be able to estimate the marginal distributions of the substitutable products as well as the demand correlations between each of the products. Even if the degree of substitutability and the demand information could be accurately estimated, modeling of the problem is complex and does not offer a simple solution like the critical fractile solution.

Despite the fact that it is difficult to quantify and to analyze the substitution effect on the order quantity, buyers do indicate that it is importance in terms of its impact on how much they should order. To gain insight into the impact of product substitution on order quantity, we consider a simple model of two substitutable products. The demand for products 1 and 2 are given by the random variables $D_1$ and $D_2$, respectively. The buyer’s order quantities for the products are $Q_1$ and $Q_2$. Let $\alpha_{ij}$ represent the proportion of demand for product $i$ that can be satisfied by substituting product $j$. Let the unit price, cost, and salvage value for product $i$ be by $p_i$, $c_i$, and $s_i$, respectively. The total profit for both products is then given by:

$$
\pi(Q_1, Q_2, D_1, D_2) = p_1 \min(Q_1, D_1) + p_2 \min(Q_2, D_2) - c_1 Q_1 - c_2 Q_2
$$

$$
+ p_1 \min[\alpha_{21}(D_2 - Q_2)^+, (Q_1 - D_1)^+] + p_2 \min[\alpha_{12}(D_1 - Q_1)^+, (Q_2 - D_2)^+]
$$

$$
+ s_1[(Q_1 - D_1)^+ - \alpha_{21}(D_2 - Q_2)^+]^+ + s_2[(Q_2 - D_2)^+ - \alpha_{12}(D_1 - Q_1)^+]^+.
$$

An explanation of (4.5) is as follows. The buyer receives revenue $p_i$ for each unit of demand

\textsuperscript{12}According to a recent study conducted by the Coca-cola Retailing Research Council and Andersen Consulting in 1997, the retailer suffer from 46% loss in revenue on average instead of 100% when there the product is out of stock.
or product \( i \) that occurs up to \( Q_i \) and pays \( c_i \) for each unit produced. The fraction of product \( i \) unfilled demand that can be filled by product \( j \) is given by \( \alpha_{ij}(D_i - Q_i)^+ \). The buyer receives revenue on the minimum of this substitutable excess demand and the excess available substitute product \( (Q_j - D_j)^+ \). Any excess inventory that is not used by the other product’s demand is salvaged.

By examining the terms in (4.5), one can rewrite the expression for \( \pi(Q_1, Q_2, D_1, D_2) \) as follows:

\[
\pi(Q_1, Q_2, D_1, D_2) = (p_1 - c_1)Q_1 - (p_2 - c_2)Q_2 \\
-(p_1 - s_1)[(Q_1 - D_1)^+ - \alpha_{21}(D_2 - Q_2)^+]^+ \\
-(p_2 - s_2)[(Q_2 - D_2)^+ - \alpha_{12}(D_1 - Q_1)^+]^+. \tag{4.6}
\]

Let \( Q_1^* \) and \( Q_2^* \) be the newsvendor solutions, i.e., the expected profit maximizing order quantities if product substitution is ignored. Let \( Q_1' \) and \( Q_2' \) be the expected profit maximizing order quantities if product substitution is considered. We first demonstrate the effects on the optimal order quantities of product substitution numerically. We explore the two product profit function given by (4.6) with \( \alpha_{12} = \alpha_{21} = \alpha \). We assume the demands \( D_1 \) and \( D_2 \) are governed by a bivariate normal with parameters \( \mu_1, \mu_2, \sigma_1, \sigma_2, \) and \( \rho \). For the numerical example, we set \( p_1 = p_2 = \$100, c_1 = c_2 = \$50, s_1 = s_2 = \$35, \mu_1 = \mu_2 = 100, \sigma_1 = \sigma_2 = 20 \). Figure 3 shows the resulting value of \( Q' = Q_1' = Q_2' \) as a function of the correlation \( \rho \) for four values of substitutability \( \alpha \), namely, \( \alpha = 0, 0.25, 0.5, 1 \). As one would expect, high levels of substitutability and low values of correlation biases the optimal order quantity away from the newsvendor solution \( Q^* \) and towards the demand forecast \( \mu \). We should also note that increasing the number of substitutable products should further bias the optimal order quantity towards \( \mu \).

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**Figure 3 about here**

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To demonstrate analytically that the addition of product substitution in this model lowers
the optimal order quantity\textsuperscript{13}, consider (4.6) with $p_1 = p_2 = p$, $c_1 = c_2 = c$, $s_1 = s_2 = s$, and $\alpha_{12} = \alpha_{21} = 1$ (full substitution). The total profit function becomes

\[
\pi(Q_1, Q_2, D_1, D_2) = (p - c)(Q_1 + Q_2) - (p - s)[(Q_1 - D_1)^+ - (D_2 - Q_2)^+]^+ - (p - s)[(Q_2 - D_2)^+ - (D_1 - Q_1)^+]^+, \tag{4.7}
\]

which reduces to

\[
\pi(Q_1, Q_2, D_1, D_2) = (p - c)(Q_1 + Q_2) - (p - s)(Q_1 + Q_2 - D_1 - D_2)^+. \tag{4.8}
\]

Then, applying standard newsvendor solution methodology, we find that

\[
Q'_1 + Q'_2 = H^{-1}\left(\frac{p - c}{p - s}\right) \leq F^{-1}\left(\frac{p - c}{p - s}\right) + G^{-1}\left(\frac{p - c}{p - s}\right) = Q'_1 + Q'_2 \tag{4.9}
\]

where $H(\cdot)$ is the distribution of $D_1 + D_2$, $F(\cdot)$ is the marginal distribution of $D_1$, and $G(\cdot)$ is the marginal distribution of $D_2$. Thus, the total quantity ordered under product substitution is lower than the newsvendor solution.\textsuperscript{14}

\subsection{Leftover Inventory Cannibalizes Future Sales}

Consider the possibility that left-over product from one season will cannibalize the sales of the next season’s products. Let $D'$ denotes the demand of the product in the next selling season. Let $p$ and $c$ be the selling price and the unit cost of the product to be sold during the next selling season, respectively. For simplicity, we assume that the order quantity $Q$ for the current season is less than the total demand of two selling seasons; i.e., $Q < D + D'$ with

\textsuperscript{13}Under the assumption that a single buyer (or cooperative set of buyers) make the ordering decisions on all the substitutable products.

\textsuperscript{14}For the simple case in which $D_1$ and $D_2$ are independent and identically distributed normal random variables with mean $\mu$ and standard deviation $\sigma$, we note that

\[
Q'_1 = Q'_2 = \mu + \sqrt{2}/2\sigma\Phi^{-1}\left(\frac{p - c}{p - s}\right) < \mu + \sigma\Phi^{-1}\left(\frac{p - c}{p - s}\right) = Q'_1 = Q'_2. \tag{4.10}
\]
probability 1. Also, since our focus is on the order quantity \( Q \) for the current selling season, we assume that the buyer can order the exact quantity to meet the effective demand for the next selling season. The total profit generated during both selling seasons can be expressed as:

\[
\pi(Q, D, D') = p \min(Q, D) + s(Q - D)^+ - cQ + (p - c)(D' - (Q - D)^+) \tag{4.11}
\]

The first three terms correspond to the profit generated from the current selling season (including the markdown sales at the end of the current selling season). Suppose that \((Q - D)^+\) units are sold as markdowns that cannibalize the sales of the product in the next selling season. Then the effective demand for the next selling season is equal to \((D' - (Q - D)^+)\).

Since we assume that the buyer can order the exact quantity to meet the effective demand for the next selling season, the fourth term represents the profit incurred during the next selling season. By rearranging the terms in (4.11), we can rewrite \(\pi(Q, D, D')\) as follows:

\[
\pi(Q, D, D') = C_u(D + D') - C_u(D - Q)^+ - C_o'(Q - D)^+,
\]

where \(C_u = (p - c)\) as before and \(C_o' = (c - s) + (p - c) = C_o + (p - c)\). Let \(\bar{Q}\) denote the order quantity that maximizes the expected total profit generated from both seasons. To characterize \(\bar{Q}\), let us compare the expressions between \(\pi(Q, D, D')\) in (4.12) and \(\pi(Q, D)\) in (2.2). It is easy to see that, when the buyer is concerned that the sales of the overstock at the end of the selling season will cannibalize the sales of the product in the next selling season, the understock cost \(C_u\) remains the same while the imputed overstock cost increases; i.e., \(C_o' > C_o\). As the overstock cost increases, it is easy to show that \(\bar{Q} < Q^*\). Thus, when the buyer is concerned about the sales of the markdowns in the current season would cannibalize the sales of the product in the next selling season, the buyer should order less than the newsvendor solution \(Q^*\).

### 4.2.4 Budget Constraint

Finally, consider the effect of limited production budget. Consider \(n\) different seasonal items (with cost and demand parameters denoted by the subscript \(i\)) constrained by an overall budget \(B\). Then, the problem becomes

\[
\max \sum_{i=1}^{n} E(\pi_i(Q_i, D_i)) \quad \text{subject to}
\]
\[ \sum_{i=1}^{n} c_i Q_i \leq B, \text{where} \]

\[ \pi_i(Q_i, D_i) = (p_i - s_i) \min(Q_i, D_i) - (c_i - s_i) Q_i \]  (4.13)

Let \( \lambda \geq 0 \) represent(s) the Lagrange multiplier on the constraint. Then, the optimal order quantity can be represented as (see Nahmias and Schmidt, 1984) \( \hat{Q}_i^* = F^{-1}(\hat{r}_i) \) where \( \hat{r}_i = \frac{p_i - c_i - \lambda c_i}{p_i - s_i} \). Since \( \lambda \geq 0, \hat{r}_i \leq \frac{p_i - c_i}{p_i - s_i} = r_i \). Therefore, \( \hat{Q}_i^* \leq Q_i^* \). Thus, when one is concerned about the budget constraint, it is natural for the buyer to order less than the newsvendor solution. The results are analogous for limited storage space.

\section{Conclusion}

In this work, we explored how alternative performance measures affect the ordering behavior for seasonal fashion goods. We noticed that most decision makers often order less than the newsvendor solution. We used a number of methods to illustrate and to justify or explain this phenomenon: a controlled experiment in the form of an ordering scenario, an informal study of buyers using a standard set of questions, and analytical models. The controlled experiment illustrates that thenon-professional buyers (MBA students) tend to order below the newsvendor solution and most of them order according to the mean demand forecast. The professional buyers tend to order below the newsvendor solution and most of them order below the mean demand forecast. We show that, from a different perspective than the standard newsvendor model, ordering according to the mean demand forecast may not be a poor decision. An informal study of buyers was conducted to identify issues that might impact their ordering decision. Incorporating these issues into analytical models, we were able to show that it is rational for the buyers to order below the newsvendor solution.

Although this work examined one type of ordering decision for a single industry, we think it has implications for how we should be studying other ordering problems. The major implication is that we should consider specific performance metrics in objective functions, not just the standard approach of maximizing expected profit or minimizing expected cost. Companies are imposing multiple performance metrics and tying them explicitly or implicitly to
employees’ pay and promotion. Currently, it is unclear exactly how the performance metrics impact ordering decisions. Inclusion of these metrics in modeling the decision problems gives more realistic ordering decisions and allows us to close the gap between theory and practice. We hope our analysis can serve as a building block for developing more complicated supply chain models with alternative performance measures and we hope our results may motivate other researchers to consider alternative performance measures besides expected cost or profit when developing inventory models or supply chain models.

References


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15We note that companies, not just individual decision-makers, are subject to a variety of different metrics. Financial analysts judge the current and future success or a company based on a variety of metrics. Each metric has different importance depending on the company, the state of the industry, etc. These metrics have great impacts on ordering decisions. Consider the programmable logic segment of the semiconductor industry between 1994 and 1996. Although ordering decisions in the semiconductor industry are not one-time, the product lifetimes are sufficiently short, obsolescence risk is very high, and inventory value decreases rapidly. Thus, the problem has similarities to the fashion good problem. However, unlike the fashion good problem, no under-ordering occurred. Despite the extremely high overstock cost, these firms tended to carry 80 - 100 days of supply (a high number even considering the 2-3 month lead times). One likely reason for this behavior was that the industry was still relatively young, and financial analysts placed stronger emphasis on revenue than profit. Within these companies, delinquent orders, which represented missed revenue, received a much higher profile than inventory. Thus, metrics are very different in different industries and these differences greatly affect ordering behavior, something that inventory models should capture.


6 Appendix

Before we present the proof for Propositions 1 and 2, let us introduce the following lemmas that would prove useful.

**Lemma 1** For any $x \geq 0$, we have:

\[
\frac{d\Phi^{-1}(x)}{dx} = \frac{1}{\phi(\Phi^{-1}(x))} \tag{6.1}
\]

\[
\frac{d\phi(x)}{dx} = -x\phi(x) \tag{6.2}
\]

\[
\frac{d\phi(\Phi^{-1}(x))}{dx} = -\Phi^{-1}(x) \tag{6.3}
\]

\[
\frac{d\Phi^{-1}(x)\phi(\Phi^{-1}(x))}{dx} = 1 - [\Phi^{-1}(x)]^2 \tag{6.4}
\]

**Proof.** It is easy to prove the above statements by using calculus and by considering the standard normal density function $\phi(.)$. We omit the details.

---

**Proof of Proposition 1** Substitute $Q^*$ (given in (3.6)) and $\mu$ into (2.1), (3.2) can be rewritten as:

\[
\Delta(Q^*, \mu, D) = (p - s)[\min(Q^*, D) - \min(\mu, D)] - (c - s)k\sigma
\]

By considering the fact that $(c - s) = C_o$ and $(p - s) = (C_u + C_o)$, the above equation can be simplified to (3.7). By examining (3.7), $\Delta(Q^*, \mu, D) > 0$ when $D > \mu + (1-r)k\sigma$. In this case, $P\{Gain\} = 1 - \Phi((1-r)k) \leq 0.5$, since $C_u \geq C_o$, $r \geq 0.5$, $k \geq 0$, and $\Phi((1-r)k) \geq 0.5$. This proves (3.8).

Next, take the expectation of $\pi(Q^*, D)$ with respect to $D$, and transform the probability distribution of $D$ to the standard normal distribution $\Phi(.)$, one can use the property of standard normal distribution ($\int x\phi(x)dx = -\phi(x)$), to show that:

\[
E(\pi(Q^*, D)) = (p - c)\mu - (p - s)\sigma\phi(k) + k\sigma[(p - c) - (p - s)\Phi(k)]
\]
Since \( k = \Phi^{-1}(\frac{C_u}{C_u + C_o}) \) and since \( C_u = (p - c) \) and \( C_o = (c - s) \), the above equation can be simplified as:

\[
E(\pi(Q^*, D)) = (p - c)\mu - (p - s)\sigma\phi(k)
\]

By using the same approach, one can show that:

\[
E(\pi(\mu, D)) = (p - c)\mu - (p - s)\sigma\phi(0)
\]

Since \( (p - s) = C_u + C_o \) and since \( \phi(0) \geq \phi(k) \) for any \( k \geq 0 \), (3.10) follows immediately from the above equations.

Finally, take the expectation of \((Q^* - D)^+\) with respect to \( D \), and transform the probability distribution of \( D \) to the standard normal distribution \( \Phi(,\) \), one can use the property of standard normal distribution (\( \int x\phi(x)dx = -\phi(x) \)), to show that:

\[
E((Q^* - D)^+) = \sigma[\phi(k) + k\Phi(k)]
\]

By using the same approach, we can show that:

\[
E((\mu - D)^+) = \sigma[\phi(0)]
\]

It follows from the above equations, we can show that

\[
E(I(Q^*, \mu, D)) = \sigma(\phi(k) + k\Phi(k) - \phi(0))
\]

Notice that \( E(I(Q^*, \mu, D)) = 0 \) when \( k = 0 \). In addition, one can apply (6.2) to show that \( \frac{dE(I(Q^*, \mu, D))}{dk} \geq 0 \). Hence, we can conclude that \( E(I(Q^*, \mu, D)) \geq 0 \). This completes the proof.

Proof of Proposition 2 Note that \( k = \Phi^{-1}(r) \) and that \( r = \frac{C_u}{C_u + C_o} \). Since \( \frac{dL}{dC_u} = \frac{dL}{dr} \cdot \frac{dr}{dC_u} \), we can apply (6.1) to (3.11) and show that \( \frac{dL}{dC_u} < 0 \). Next, using the same approach, we have:

\[
\frac{dL}{dC_o} = -\Phi^{-1}(r)\sigma + r(1 - r) \cdot \frac{1}{\phi(\Phi^{-1}(r))} \cdot \sigma
\]

Thus, \( L \) is increasing in \( C_o \) if \( r(1 - r) > \Phi^{-1}(r)\phi(\Phi^{-1}(r)) \) and is decreasing in \( C_o \), otherwise. Observe the following. First, since \( 1 \geq r \geq 0.5 \), \( r(1 - r) \) is decreasing in \( r \). Second, as \( r \) increases from 0.5 to 1, (6.4) implies that \( \Phi^{-1}(r)\phi(\Phi^{-1}(r)) \) is first increasing in \( r \) and
then decreasing in $r$. Third, $r(1 - r) > \Phi^{-1}(r)\phi(\Phi^{-1}(r))$ when $r = 0.5 + \epsilon$ and $r(1 - r) < \Phi^{-1}(r)\phi(\Phi^{-1}(r))$ when $r = 1 - \epsilon$, where $\epsilon$ is a sufficiently small number. Combine these three observations and the fact that $C_o$ decreases from $C_u$ to 0 as $r$ increases from 0.5 to 1, one can show that $L$ is first decreasing in $C_o$ and then increasing in $C_o$.

Next, since $k = \Phi^{-1}(r)$, and $\frac{dG}{dc_u} = \frac{dG}{dr} \cdot \frac{dr}{dc_u}$, we can apply (6.1) to (3.12) and show that $\frac{dG}{dc_u} > 0$. By using the exact same approach, we can show that $\frac{dG}{dc_u} < 0$.

Since $k = \Phi^{-1}(r)$, (3.8) implies that $P\{Gain\}$ is decreasing in the term $\Phi((1 - r)\Phi^{-1}(r))$. By applying the chain rule, we have:

$$\frac{d\Phi((1 - r)\Phi^{-1}(r))}{dC_u} = \phi((1 - r)\Phi^{-1}(r)) \cdot \frac{d(1 - r)\Phi^{-1}(r)}{dr} \cdot \frac{dr}{dC_u} \tag{6.5}$$

By applying (6.1), one can show that:

$$\frac{d(1 - r)\Phi^{-1}(r)}{dr} = -\Phi^{-1}(r) + (1 - r) \cdot \frac{1}{\phi(\Phi^{-1}(r))}$$

Substitute the above equation into (6.5) and observe the fact that, $\frac{dr}{dc_u} > 0$, it can be easily shown that $P\{Gain\}$ is decreasing in $C_u$ if $(1 - r) > \Phi^{-1}(r)\phi(\Phi^{-1}(r))$ and is increasing in $C_u$, otherwise. Let us observe the following. First, since $1 \geq r \geq 0.5$, the term $(1-r)$ is decreasing in $r$. Second, (6.4) implies that $\Phi^{-1}(r)\phi(\Phi^{-1}(r))$ is first increasing in $r$ and then decreasing in $r$. Third, $(1 - r) > \Phi^{-1}(r)\phi(\Phi^{-1}(r))$ when $r = 0.5 + \epsilon$ and $(1 - r) < \Phi^{-1}(r)\phi(\Phi^{-1}(r))$ when $r = 1 - \epsilon$, where $\epsilon$ is a sufficiently small number. Combine these three observations and the fact that $C_u$ increases as $r$ increases from 0.5 to 1, one can show that $P\{Gain\}$ is first decreasing in $C_u$ and then increasing in $C_u$. By using the exact same approach, one can show that $P\{Gain\}$ is first decreasing in $C_o$ and then increasing in $C_o$.

Let us differentiate (3.10) with respect to $C_u$. Since $k = \Phi^{-1}(r)$, we can apply the chain rule and (6.3) to show that:

$$\sigma\left[\frac{dE(\Delta(Q^*, \mu, D))}{dc_u}\right] = \phi(0) - \phi(k) + (1 - r)\Phi^{-1}(r)$$

Since $\phi(0) \geq \phi(k)$ for any $k \geq 0$, we can conclude that $E(\Delta(Q^*, \mu, D))$ is increasing in $C_u$. By using the exact same approach, one can show that $E(\Delta(Q^*, \mu, D))$ is decreasing in $C_o$.

Since $k = \Phi^{-1}(r)$, we have $\frac{dE(1(Q^*, \mu, D))}{dc_u} = \frac{dE(1(Q^*, \mu, D))}{dk} \cdot \frac{dk}{dr} \cdot \frac{dr}{dc_u}$. In this case, we can apply (6.2) and (6.1) to show that $\frac{dE(1(Q^*, \mu, D))}{dc_u} \geq 0$. By using the exact same approach, one can show that $\frac{dE(1(Q^*, \mu, D))}{dc_u} \leq 0$. This completes the proof.
Proof of Proposition 3 Consider a general constant $\alpha$. If $\alpha \geq 0$,

$$P[(Q - D)^+ \leq \alpha] = E[1\{(Q - D)^+ \leq \alpha\}] = \int_{\infty}^{Q} 1 \, dF + \int_{Q}^{Q+\alpha} 1 \, dF + \int_{0}^{Q-\alpha} 0 \, dF = 1 - F(Q - \alpha) \quad (6.6)$$

Since $P[(Q - D)^+ \leq \alpha] = 0$ if $\alpha < 0$, we have that

$$P[(Q - D)^+ \leq \alpha] = \begin{cases} 1 - F(Q - \alpha) & : \alpha \geq 0 \\ 0 & : \alpha < 0 \end{cases} \quad (6.7)$$

The objective function becomes

$$P[\pi(Q, D) \geq \tau] = P[Q(p - c) - (p - s)(Q - D)^+ \geq \tau] = P[(Q - D)^+ \leq (p - c)/(p - s) Q - \tau/(p - s)] = \begin{cases} 1 - F[Q(c - s)/(p - s) + \tau/(p - s)] & : Q(p - c) \geq \tau \\ 0 & : Q(p - c) < \tau \end{cases} \quad (6.8)$$

using (6.7).

Thus, to maximize the objective function, $Q$ should be lowered as much as possible keeping $Q \geq \tau/(p - c)$. Thus, $\hat{Q} = \tau/(p - c)$. This completes the proof.

Proof of Proposition 4 Consider each term in the objective function separately. First, the probability of achieving the sales target is given by

$$P[\rho(Q, D) \geq \tau_s \mu p] = P[p_{\min}(Q, D) \geq \tau_s \mu p] = P[(Q - D)^+ \leq Q - \tau_s \mu] \quad (6.9)$$

using (2.1). Applying (6.7), we find that

$$P[\rho(Q, D) \geq \tau_s \mu p] = 1 - F(\tau_s \mu) \quad (6.11)$$

for $Q \geq \tau_s \mu$ and zero otherwise. Thus, the buyer should order at least $\tau_s \mu$. However, it is apparent from (6.10) that ordering more than $\tau_s \mu$ does nothing to further increase the probability.
Next, notice that the probability of achieving the gross margin target is given by:

\[
P[\pi(Q, D)/(cQ) \geq \tau_m] = P[(p - s)\min(Q, D) - (c - s)Q \geq \tau_mcQ] \tag{6.12}
\]
\[
= P[(Q - D)^+ \leq Q(p - c(1 + \tau_m))/(p - s)] \tag{6.13}
\]

using (2.1). Applying (6.7), we find that

\[
P[\pi(Q, D)/(cQ) \geq \tau_m] = 1 - F[Q(c(1 + \tau_m) - s)/(p - s)] \tag{6.14}
\]

for \(\tau_m < (p - c)/c\) (the gross margin can never be higher than the unit margin divided by the unit cost). Using (6.14), we can see that the gross margin target probability is maximized if \(Q\) is minimized.

Let \(\theta = (c(1 + \tau_m) - s)/(p - s)\). Then, the complete objective function is

\[
w[1 - F(\tau_s\mu)]1(Q \geq \tau_s\mu) + (1 - w)[1 - F(Q\theta)]. \tag{6.15}
\]

If

\[
w(1 - F(\tau_s\mu)) + (1 - w)(1 - F(\tau_s\mu\theta)) \geq (1 - w), \tag{6.16}
\]

the objective function is maximized by \(\bar{Q} = \tau_s\mu\). Otherwise, the decision is to order as little as possible.
Appendix: Survey Form

Scenario: Single Period Inventory Problem: The following scenario was given to MBA students and to professional buyers. In addition, the professional buyers were given the following additional instruction: “Please make a decision in the same manner as you would in your company, considering additional factors (performance metrics, additional operational concerns) are relevant.”

Consider the following scenario. *Rapido*, a designer swimwear manufacturing company, introduces new lines of swimsuits every summer. Every spring, the buyers of *Rapido* have to place an order with their contract manufacturer in Asia so that the order will arrive prior to the summer. You are responsible to determine the one-time order for Capri, one of the newly designed swimsuits. To do so, you have collected the predictions of the sales of Capri from other buyers, and the pricing information from the Marketing department. The information is given as follows:

- Sales predictions provided by 5 buyers: 1000, 3000, 5000, 7000, 9000 (These predictions yield: average prediction = 5000 units, and the standard deviation of the predictions = 3200 units)
- Selling price: $100 per unit
- Cost: $60 per unit
- Markdown price (so as to get rid of the leftovers): $45 per unit

1. How many units would you order? (There is no right or wrong answer. Just pick a number that you think is appropriate.)

2. What are the underlying reasons for choosing the order quantity specified in question 8?

Interview Questions given to Professional Buyers:

1. Consider a seasonal product that has been selected by a buyer (or assigned to him/her) in your company. The buyer is now responsible for determining the order quantity. Please rank (1 being highest) the following performance metrics in terms of its importance to the buyer’s overall performance success. Please indicate if there is a pre-set target for each of these metrics, as opposed to the buyer simply trying to achieve as positive a value as possible.

<table>
<thead>
<tr>
<th>RANK</th>
<th>Preset Target? (Y/N)</th>
<th>Metric</th>
</tr>
</thead>
<tbody>
<tr>
<td>_____</td>
<td>_____</td>
<td>Total Product Profit</td>
</tr>
<tr>
<td>_____</td>
<td>_____</td>
<td>Return on Investment or Gross Margin for product</td>
</tr>
<tr>
<td>_____</td>
<td>_____</td>
<td>Excess Inventory (demand is below the order quantity)</td>
</tr>
<tr>
<td>_____</td>
<td>_____</td>
<td>Out of Stock (demand exceeds the order quantity)</td>
</tr>
<tr>
<td>_____</td>
<td>_____</td>
<td>Other</td>
</tr>
<tr>
<td>_____</td>
<td>_____</td>
<td>Other</td>
</tr>
</tbody>
</table>
2. Name three implicit metrics for buyers in your company related to the accuracy of the order quantity decisions, i.e., unwritten metrics to evaluate a buyer’s performance.

   a. ____________________________________________________________

   b. ____________________________________________________________

   c. ____________________________________________________________

3. From the buyer’s viewpoint, rank the impact (1 being highest) of each of the following concerns on the buyers’ order quantity decisions.

   ____ The loss resulting from having to sell an overabundance of inventory at markdown (below cost) prices.
   ____ The potential loss of sales of the product from lack of available supply.
   ____ Financial budget (on production costs).
   ____ Limited storage space.
   ____ The possibility that the left-over products cannibalize the sales of the next season’s products.
   ____ The possibility that customers will be willing to buy a substitutable product (ex: a green shirt if blue is sold out) if their first choice is not available.
   ____ Cost of capital tied up in inventory.
   ____ Meeting a profit target.
   ____ Other ______________________________________________________
   ____ Other ______________________________________________________

4. Does your company keep track of amount of unsatisfied demand (due to out of stock) for each product?

   ____ YES       ____ NO

If YES, what is the measure of the unsatisfied demand?

___________________________________________________________________________

Is unsatisfied demand (due to out of stock) an explicit performance metric for individual buyers?

   ____ YES       ____ NO
**Figure 1**: The gain probability, maximum gain, and maximum loss performance gaps as a function of $C_u$ for $C_o = 5$, $\mu = 100$, and $\sigma = 10$.

**Figure 2**: The gain probability, maximum gain, and maximum loss performance gaps as a function of $C_o$ for $C_u = 10$, $\mu = 100$, and $\sigma = 10$. 
Figure 3: The effects of product substitution on the optimal order quantity, showing the effects of the degree of substitutability $\alpha$ and the correlation $\rho$ between product demands. The data shown is based on the data set $p_1 = p_2 = $100, $c_1 = c_2 = $50, $s_1 = s_2 = $35, $\mu_1 = \mu_2 = 100$, and $\sigma_1 = \sigma_2 = 20$. 