Competition and Structure in Serial Supply Chains with Deterministic Demand

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Supply chains often consist of several tiers, with different numbers of firms competing at each tier. A major determinant of the structure of supply chains is the cost structure associated with the underlying manufacturing process. In this paper, we examine the impact of fixed and variable costs on the structure and competitiveness of supply chains with a serial structure and price-sensitive linear deterministic demand. The entry stage is modeled as a simultaneous game, where the players take the outcomes of the subsequent post-entry (Cournot) competition into account in making their entry decisions. We derive expressions for prices and production quantities as functions of the number of entrants at each tier of a multitier chain. We characterize viability and stability of supply-chain structures and show, using lattice arguments, that there is always an equilibrium structure in pure strategies in the entry game. Finally, we examine the effects of vertical integration in the two-tier case. Altogether, the paper provides a framework for comparing a variety of supply-chain structures and for studying how they are affected by cost structures and by the number of entrants throughout the chain.
(*Supply Chains; Competition; Pricing; Production; Entry; Fixed Costs*)

1. Introduction

In many manufacturing industries, such as automotive, consumer appliances, electronic equipment, and apparel, multitier supply chains, with different companies at different tiers of the chain, are common. Furthermore, the structure of supply chains can vary substantially. There are many PC assemblers, but a few chip manufacturers dominate at their own tier. The automotive industry has few final assemblers, but many manufacturers for most parts. Traditional economic arguments tell us that the number of entrants at any tier depends on the barriers to entry. The number of entrants then determines the intensity of competition at each tier, and consequently each entrant's profitability. However, the exact interactions between structure, fixed costs, and entry and production decisions are not obvious. Moreover, interactions across tiers are even less clear. Here lies the focus of this paper.

Competition in supply chains has received some attention in management studies in addition to the economic literature. Porter's (1980) industry analysis distinguishes five forces: competition between incumbents, supplier power, buyer power, new entrants, and technological changes, substitutes, and complements. Our model provides a framework for analyzing the first four of these forces. We also study vertical integration. The model is thus fairly ambitious; we address its limitations later.

We develop an explicit model of entry and post-entry competition in serial multitier supply chains with deterministic demand. This model allows us

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to examine the impact of fixed and variable costs on entry and competition at each tier. We model entry as a simultaneous game, where players take the outcomes of the post-entry (Cournot) competition into account in making their entry decisions. For the post-entry stage, we obtain explicit expressions for prices, quantities, and profits. These results in turn allow us to examine the entry game fairly explicitly.

As expected, there are interaction effects between different tiers in the chain. The number of entrants, and prices and quantities produced at any tier of a chain, depend on the number of entrants at all other tiers; we are able to quantify these effects. As a result, we can address specific questions that go beyond the traditional qualitative strategic analyses:

- What is the impact of fixed and variable costs at any tier on prices, quantity, and profits at any other tier?
- What is the effect of "buyer power" or "supplier power" on prices, quantities, and profits for contiguous and for noncontiguous tiers in the supply chain? What is the marginal effect of an additional entry downstream or upstream?
- What is the interaction between concentration at any one tier and competitiveness in any other? How does upstream or downstream consolidation affect a firm's competitiveness?
- How many entrants can survive at each tier? Which of these combinations of entrants reflect equilibrium situations?
- How is end-market demand reflected in the demand seen by upstream tiers? If the demand curve or the costs of production or entry change, how could the number of entrants at each tier change?
- How do vertically integrated and unintegrated structures compare?
- Perhaps most important, is there a unique structure toward which a system with given fixed and variable costs will converge?

In §2, we review the relevant literature. Section 3 examines post-entry competition in a two-tier chain; §4 characterizes the entry game. Section 5 generalizes all results to multitier chains. Section 6 studies vertical integration. Section 7 contains our conclusions.

2. Literature Survey

This paper draws on and contributes to distinct strands of literature. An extensive body of work focuses primarily on optimization of a supply network owned by a single party. Examples of this include the distribution system optimization work reviewed in Geoffrion and Powers (1995). The literature on multiechelon inventory systems, starting with Clark and Scarf (1960), studies inventory policies for a given supply-chain structure.

We focus on the structure of (multitier) supply chains in a competitive environment, rather than on the operations within them. The post-entry competition model draws heavily on the successive oligopoly literature in economics, which is largely limited to two tiers. Machlup and Taber (1960) give an early overview of successive monopoly. Greenhut and Ohta (1979) show, using an approach similar to that in §3, that vertical integration of successive oligopolists leads to higher output and lower prices. Abiru (1988) extends this to a wider range of cases, and in §6 we confirm their findings in our context. Tyagi (1999) shows that downstream entry has ambiguous effects on consumers and on downstream incumbents. Ziss (1995) studies horizontal mergers in two-tier supply chains with two entrants at each tier, and shows that upstream merger leads to higher final prices, whereas downstream merger can reduce prices. Such findings are far from obvious a priori, hence the importance of formally modeling competition in multitier supply chains.

Measures of competitiveness include demand elasticity (Tirole 1988), the Lerner index of monopoly power, and the ratio of margins at successive tiers (Bresnahan and Reiss 1985). We extend all three measures to the multitier case to answer the questions posed earlier. Related work includes Waterson's (1980) analysis of suppliers selling to multiple markets, Vickers's (1995) study of regulation in multitier competition, and Choi's (1991) model of product differentiation with two manufacturers selling to a common retailer.

Our model goes further by analyzing multiple tiers, although with specific assumptions on demand and cost structures. We also examine the entry game itself with fixed costs of entry, rather than only post-entry

competition. Bresnahan and Reiss (1990) study entry in a market dominated by a monopolist. Spence (1977) shows how incumbents may invest in overcapacity to prevent entry. Karmarkar and Pitbladdo (1993) investigate pricing and entry in a single tier when capacity can only be added in finite increments.

Finally, we provide a rigorous analysis of some aspects of vertical integration. The transaction cost economics literature (Coase 1937, Williamson 1985) asks when firms will vertically integrate and when market-based transactions are preferable. Although we ignore many of the determinants of vertical integration studied by Grossman and Hart (1986), we find, intriguingly, that supply-chain profits increase after vertical integration of successive monopolists but decrease after integration of oligopolists.

3. Post-Entry Competition in a Two-Tier Supply Chain

We first examine competition in a supply chain with a given number of entrants in each tier; in §4 we study how this number of entrants is determined in the entry game. Consider a supply chain with two tiers, final product assembly (Tier 1) and parts fabrication (Tier 2). Assume that the demand for the (single) final product is characterized by a linear inverse demand function $p_1 = a_1 - b_1 Q_1$, where a_1 and b_1 are parameters, Q_1 is the quantity supplied to the market, and p_1 is the price. This may also be viewed as a linear approximation of the actual demand function. Suppose that n_1 firms have entered the downstream tier. We assume quantity competition, thus adopting the Cournot framework, as price competition with more than one firm will lead to marginal cost pricing. Suppose that each downstream firm (indexed by j) has infinite production capacity and chooses a production quantity $q_{1,j}$, taking the production plans of other firms as given. Each downstream firm has to buy a single "part" (or bundle or kit of parts) from suppliers. Without loss of generality, assume that one part is required to produce one unit of the final product. The price of a part (i.e., the price in Tier 2) is p_2. Gross profit of downstream firm j is

$$\begin{aligned}\Pi_{1,j} &= (p_1 - v_{1,j} - p_2)q_{1,j} \\ &= (a_1 - b_1 Q_{1,-j} - b_1 q_{1,j} - v_{1,j} - p_2)q_{1,j}, \qquad (1)\end{aligned}$$

where $v_{1,j}$ is that firm's variable unit production cost and $Q_{1,-j} := Q_1 - q_{1,j}$ is the total production by all others in Tier 1, which firm j takes as given. $\Pi_{1,j}$ is concave in $q_{1,j}$ whenever $b_1 > 0$, so (1) is maximized when first-order conditions hold, i.e., when $q_{1,j} = (a_1 - b_1(Q_1 - q_{1,j}) - v_{1,j} - p_2)/2b_1$. If Tier 1 has n_1 firms, all with the same variable cost $v_{1,j} = v_1$, then the system of equations defining $q_{1,j}$ has a symmetric solution $q_{1,j} = q_1 = Q_1/n_1$. Solving for q_1, we have $q_1 = (a_1 - v_1 - p_2)/((n_1+1)b_1)$, or

$$Q_1 = n_1 q_1 = \frac{n_1}{n_1+1}\frac{a_1 - v_1 - p_2}{b_1}. \qquad (2)$$

Substituting into the inverse demand function, we have

$$p_1 = \frac{1}{n_1+1}a_1 + \frac{n_1}{n_1+1}(v_1 + p_2) \qquad (3)$$

so that, as the number of entrants increases, the price approaches variable cost. Because p_2 depends on n_2, the number of entrants in Tier 2, Π_1 depends on both n_1 and n_2; let $\mathbf{n}$ denote the vector (n_1, n_2). Operating profit per Tier 1 firm, $\Pi_1(\mathbf{n})$, is

$$\Pi_1(\mathbf{n}) = (p_1 - v_1 - p_2)q_1 = \frac{1}{b_1}\left(\frac{a_1 - v_1 - p_2}{n_1+1}\right)^2. \qquad (4)$$

Now consider the upstream tier of suppliers. The number of parts demanded by final assemblers at a price p_2 is equal to the quantity Q_1 of the final product. Hence, the demand for parts is given by $Q_2 = Q_1$ as in (2). This is the derived demand curve for the upstream tier and can be written as

$$p_2 = (a_1 - v_1) - b_1\left(\frac{n_1+1}{n_1}\right)Q_2 = a_2 - b_2 Q_2, \qquad (5)$$

where $a_2 = (a_1 - v_1)$ and $b_2 = b_1((n_1+1)/n_1)$. The linearity of p_2 allows us to apply the same model there. With n_2 firms upstream, we have the analogous results:

$$Q_2 = n_2 q_2 = \left(\frac{n_2}{n_2+1}\right)\frac{a_2 - v_2}{b_2}, \qquad (6)$$

$$p_2 = \left(\frac{1}{n_2+1}\right)a_2 + \left(\frac{n_2}{n_2+1}\right)v_2. \qquad (7)$$

Substituting back the expressions for a_2, b_2, and p_2 and simplifying, we get

$$Q_1 = \left(\frac{n_1 n_2}{(n_1+1)(n_2+1)}\right)\left(\frac{a_1 - v_1 - v_2}{b_1}\right), \tag{8}$$

$$p_1 = \frac{(n_1+n_2+1)a_1 + n_1 n_2 (v_1+v_2)}{(n_1+1)(n_2+1)} = a_1 - \frac{n_1 n_2 (a_1 - v_1 - v_2)}{(n_1+1)(n_2+1)}, \tag{9}$$

$$\Pi_1(\mathbf{n}) = \frac{1}{b_1}\left(\frac{n_2}{n_2+1}\right)^2\left(\frac{a_1 - v_1 - v_2}{n_1+1}\right)^2, \tag{10}$$

$$Q_2 = Q_1 = \left(\frac{n_1 n_2}{(n_1+1)(n_2+1)}\right)\left(\frac{a_1 - v_1 - v_2}{b_1}\right), \tag{11}$$

$$p_2 = \left(\frac{1}{n_2+1}\right)(a_1 - v_1) + \left(\frac{n_2}{n_2+1}\right)v_2, \tag{12}$$

$$\Pi_2(\mathbf{n}) = \frac{1}{b_1}\left(\frac{n_1}{n_1+1}\right)\left(\frac{a_1 - v_1 - v_2}{n_2+1}\right)^2. \tag{13}$$

The operating profits can also be expressed as $\Pi_1(\mathbf{n}) = b_1 q_1^2$ and $\Pi_2(\mathbf{n}) = b_1((n_1+1)/n_1)q_2^2$. Note that all the expressions are affected by parameters for both tiers. Profits in both tiers are equally affected by variable costs at either tier. Note that p_2 does not depend on n_1, the number of customers; Corollary 3.1 in Tyagi (1999, p. 69) lists exactly for which types of demand functions (beyond the linear form we use here) this result holds. Expressions (8) and (9) immediately give:

Proposition 1. *For the two-tier supply chain, the quantity Q_1 produced for the final market increases with the number of entrants at either level, and the price decreases.*

Let the total operating profit Π_t accruing to the supply chain be defined as $\Pi_t(n_1, n_2) = n_1\Pi_1(n_1) + n_2\Pi_2(n_2)$.

Proposition 2. *For any a_1, b_1, v_1, and v_2, $\Pi_t(n_1, n_2)$ has the following properties*:

1. *If there is a single entrant at either stage ($n_1 = 1$ or $n_2 = 1$), then $\Pi_t(n_1, n_2)$ increases with the number of entrants at the other stage (n_1 or n_2).*
2. $\Pi_t(n_1, n_2) = \Pi_t(n_2, n_1)$.
3. *For $n_1 > (n_2+1)/(n_2-1)$ or $n_2 > (n_1+1)/(n_1-1)$, $\Pi_t(n_1, n_2)$ decreases for increasing n_1 or n_2.*
4. $\lim_{n_2\to\infty}\Pi_t(1, n_2) = \lim_{n_1\to\infty}\Pi_t(n_1, 1) = \Pi_t^*$.
5. *$\Pi_t(n_1, n_2)$ achieves its maximum $\Pi_t^* = (a_1 - v_1 - v_2)^2/4b_1$, at (n_1, n_2) equal to (2, 3) or (3, 2).*

Proof. Parts 1, 3, and 4 are easy to verify. Part 2 follows from

$$b_1\frac{\Pi_t(n_1, n_2)}{(a_1 - v_1 - v_2)^2} = \frac{n_1}{(n_1+1)^2}\left(\frac{n_2}{n_2+1}\right)^2 + \frac{n_1}{n_1+1}\frac{n_2}{(n_2+1)^2} = \frac{n_1 n_2^2 + n_1^2 n_2 + n_1 n_2}{(n_1+1)^2(n_2+1)^2}.$$

For part 5, relaxing integrality of n_1 and n_2, a necessary condition for an extreme point of $\Pi_t(n_1, n_2)$ is that $n_2 = (n_1+1)/(n_1-1)$, so that (2, 3) and (3, 2) are the only (integer) solutions, which together with the preceding parts completes the proof. □

Part 1 shows that if there is a monopolist at one tier, the total chain benefits from increased competition at the other tier. Part 2 shows that total profits are symmetric in the number of entrants at each tier (regardless of variable costs). Part 3 shows that for n_1 and n_2 large enough, total profits decrease as competition increases. Parts 4 and 5 show that total profits are maximized by either two entrants in one tier and three in the other, or a monopolist combined with an infinitely competitive stage.

4. Entry Decisions in the Two-Tier Supply Chain

4.1. The Entry Game

So far, we analyzed competitive behavior in a two-tier supply chain with n_i firms at tier i. Here we examine viability and stability of supply chain structures. In a viable structure, all firms earn nonnegative net profits (after incurring the fixed costs of entry). A structure is stable in the short term if no firm has an incentive to unilaterally reverse its decision whether or not to enter. A structure is stable in the long term if even simultaneous entry in multiple tiers is not profitable. Let there be K_i potential entrants in tier i, where K_i is sufficiently large not to be exhausted; all potential entrants at all tiers are independent firms. Firms look ahead to the post-entry stage, and will not enter if their operating profits will not cover the cost of entry. Any firm that does not enter earns zero net profits;

firms that do enter engage in the post-entry competition studied in the previous section.

We assume all firms make their entry decision simultaneously; although this clearly raises some questions about the exact nature of the entry process (which we discuss later), the identification of viable supply-chain structures and characterization of Nash equilibria that it allows is useful. We restrict ourselves to pure-strategy equilibria; in the Appendix we analyze the mixed extension of this game (Vorob'ev 1977, p. 91) and show why pure strategies are of most interest. The strategy space for firm j at tier i is given by $\mathcal{S}_{ij} := \{0, 1\}$, where the strategy $s_{ij} = 1$ if and only if the firm enters. We do not care exactly which firms enter at any tier i, only how many do so, as they are symmetric. Therefore, we can focus on the number of entrants $n_i := \sum_{j=1}^{K_i} s_{ij}$ at tier i instead of on individual firm decisions s_{ij}. Let $\mathbf{s}_i \in \{0, 1\}^{K_i}$ be the vector of entry decisions s_{ij} of firms in tier i, so any structure (n_1, n_2) corresponds to a set of strategies $\mathcal{S}(n_1, n_2) := \{(\mathbf{s}_1, \mathbf{s}_2) | \sum_{j=1}^{K_i} s_{ij} = n_i\}$. Any pair (n_1, n_2) that corresponds to a Nash equilibrium $(\mathbf{s}_1, \mathbf{s}_2)$ is an *equilibrium structure*. The conditions for (n_1, n_2) to be an equilibrium structure in the entry game are

$$\Pi_1(n_1+1, n_2) \le F_1 \le \Pi_1(n_1, n_2), \tag{14}$$

$$\Pi_2(n_1, n_2+1) \le F_2 \le \Pi_2(n_1, n_2). \tag{15}$$

No firm at either tier could be strictly better off by unilaterally reversing its decision whether or not to enter. In general, for any given n_2, there is a unique integer n_1 that satisfies (14). (If an integer n_1 satisfies the first part of (14) with equality, then both n_1 and $n_1' := n_1 + 1$ satisfy (14)). For $\mathbf{n} = (n_1, n_2)$ to be an equilibrium structure in the entry game, it must satisfy both (14) and (15). To examine the nature of equilibrium structures, let $\mathbf{e_i}$ be the ith unit vector and define the set of viable structures $\mathcal{W} = \{\mathbf{n} : \Pi_i(\mathbf{n}) \ge F_i$ for $i = 1, 2\}$ and the set of equilibrium structures $\mathcal{N} = \{\mathbf{n} \in \mathcal{W} : \Pi_i(\mathbf{n} + \mathbf{e_i}) \le F_i$ for $i = 1, 2\}$. The set $\mathcal{W}$ contains all $\mathbf{n}$ in which firms at both tiers earn nonnegative net profits after fixed costs of entry. Equilibrium structures preclude further unilateral profitable entry (after fixed costs) at either tier. The conditions defining $\mathcal{W}$ can be rewritten as

$$n_2 \ge \frac{k_1(n_1+1)}{1-k_1(n_1+1)}, \tag{16}$$

$$n_2 \le \frac{1}{k_2}\left(\frac{n_1}{n_1+1}\right)^{\frac{1}{2}} - 1, \tag{17}$$

where k_1 and k_2 are given by

$$k_1 = \frac{\sqrt{b_1 F_1}}{a_1 - v_1 - v_2} \text{ and } k_2 = \frac{\sqrt{b_1 F_2}}{a_1 - v_1 - v_2}.$$

The sets $\mathcal{W}$ and $\mathcal{N}$ have some useful properties. We will sometimes use two regularity conditions: A monopolist always earns positive net profits, regardless of entry at other tiers (18), and too many firms at any tier all lose money (19). Let $\mathbf{n}_{-i}$ be the vector $\mathbf{n}$ without its ith component.

$$\Pi_i(1, \mathbf{n}_{-i}) > F_i \text{ for all } i, \mathbf{n}_{-i} \text{ with } n_k \ge 1 \text{ for all } k. \tag{18}$$

For all i, there exists $\tilde{n}_i < K_i$ such that

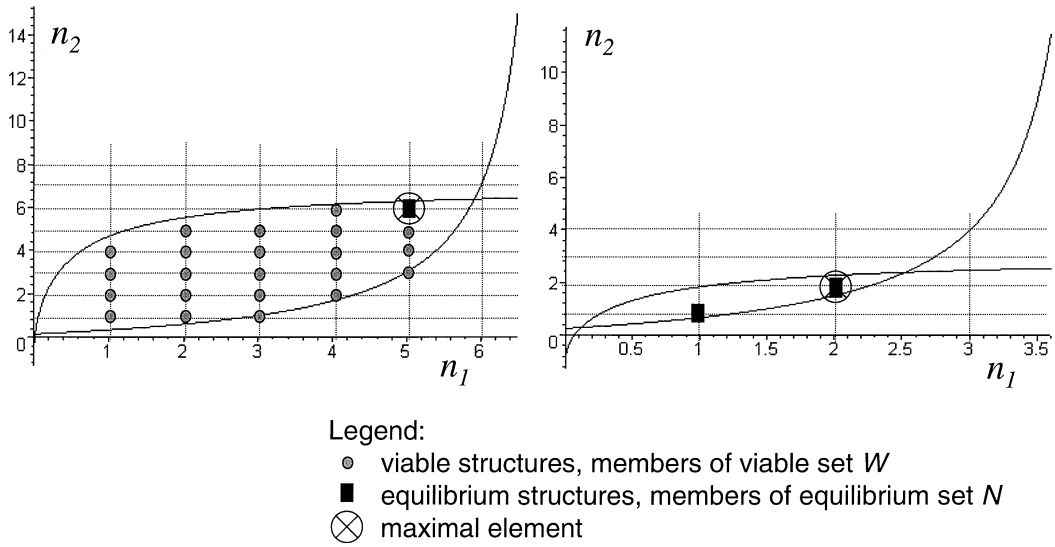
$$\Pi_i(\tilde{n}_i, \mathbf{n}_{-i}) < F_i \text{ for all } \mathbf{n}_{-i}. \tag{19}$$

Condition (19) is the formal version of our earlier assumption that K_i be "sufficiently large" not to restrict $\mathcal{W}$ or $\mathcal{N}$; for $K_i = \infty$ it is trivially satisfied, since $\lim_{n_i \to \infty} \Pi_i(\mathbf{n}) = 0$. If (18) holds, $\mathcal{W}$ is nonempty (as it contains (1,1)), and (19) implies $k_1 > 0$ and $k_2 > 0$. Figure 1 shows two examples of $\mathcal{W}$.

Proposition 3. *If $\mathcal{W}$ is nonempty, it is a lattice. If $k_1 > 0$ and $k_2 > 0$, $\mathcal{W}$ is bounded and contains a maximal element $(\bar{n}_1, \bar{n}_2)$ and a minimal element $(\underline{n}_1, \underline{n}_2)$: For any (n_1, n_2) in $\mathcal{W}$, $\bar{n}_1 \ge n_1 \ge \underline{n}_1$ and $\bar{n}_2 \ge n_2 \ge \underline{n}_2$. $\bar{n}_1$ and $\bar{n}_2$ increase with a_1 and decrease with b_1, v_1, v_2, F_1, and F_2; $\underline{n}_1$ and $\underline{n}_2$ decrease with a_1, and increase with b_1, v_1, v_2, F_1, and F_2.*

Proof. Suppose $\mathbf{n} = (n_1, n_2)$ and $\mathbf{m} = (m_1, m_2)$ are both in $\mathcal{W}$. Define $\mathbf{u} := \mathbf{n} \wedge \mathbf{m}$, i.e., $u_1 := \max\{n_1, m_1\}$ and $u_2 := \max\{n_2, m_2\}$, and similarly $\mathbf{l} := \mathbf{n} \vee \mathbf{m}$, i.e., $l_1 := \min\{n_1, m_1\}$ and $l_2 := \min\{n_2, m_2\}$. Then, by definition, $\mathcal{W}$ is a lattice if and only if both $\mathbf{l}$ and $\mathbf{u}$ are also in $\mathcal{W}$. This is easily verified, as the right-hand sides of (16) and (17) are both increasing in n_1. We have $n_1, n_2 \ge 0$; for any (n_1, n_2) in $\mathcal{W}$, (16) and (17) yield $n_1 \le (1-k_1)/k_1$ and $n_2 \le (1/k_2) - 1$, which

Figure 1 Two Examples of Sets $\mathscr{W}$ of Viable Supply Chain Structures with Equilibrium Structures and Maximal Element



Note. The two figures show examples of viable sets $\mathscr{W}$, using Expressions (16) and (17). The left-hand graph uses $k_1 = k_2 = 1/8$, which gives a relatively well-behaved viable set in that all points can be reached from another point in $\mathscr{W}$ by a marginal entry decision. The right-hand graph uses $k_1 = 1/5$ and $k_2 = 1/4$, in which case there is no way to reach the maximal element (2,2) from the only other element (1,1) by a single firm entering.

ensures $\mathscr{W}$ is bounded. Lattice properties and boundedness ensure that $\mathscr{W}$ has a minimal and a maximal element. Comparative statics of $(\bar{n}_1, \bar{n}_2)$ and $(\underline{n}_1, \underline{n}_2)$ are easily verified. □

Proposition 4. *The maximal viable structure is also an equilibrium. It is the structure that produces the largest quantity with the lowest end market price.*

Proof. If $(\bar{n}_1, \bar{n}_2)$ were not contained in $\mathscr{N}$, entry would still be possible, a contradiction. The second part follows directly from Proposition 1. □

$\mathscr{W}$ "expands" with market size a_1; new viable structures may have more firms than the previously "most competitive" structure or fewer firms than the previously "least competitive" structure. When $k_1 > 0$ and $k_2 > 0$, a Nash equilibrium (in pure strategies) in the entry game exists, and hence also an equilibrium structure. The lower bound in (16) increases in F_1, reducing the size of $\mathscr{W}$; a similar argument holds for F_2. The maximal element of $\mathscr{W}$ must then decrease as fixed costs increase. Less obvious at the outset is that a reduction in fixed costs in one tier can lead to more entrants in equilibrium in the other tier. A drastic drop in fixed costs upstream will increase entry there, but may also lead to increased entry of customers. Later, in §5, we generalize these findings to multitier supply chains.

For computational purposes, one may prefer to relax the integrality of n_1 and n_2 in the definition of $\mathscr{W}$ to get $\widehat{\mathscr{W}} := \{(x_1, x_2) \in \mathbb{R}^2_+ : \Pi_1(x_1) \geq F_1 \text{ and } \Pi_2(x_2) \geq F_2\}$.

Proposition 5. *$\widehat{\mathscr{W}}$ is a convex lattice, and all statements in Proposition 3 apply to $\widehat{\mathscr{W}}$ too.*

Proof. The condition $\Pi_1(x_1) \geq F_1$ leads to an inequality of the form $x_2 \geq g_1(x_1)$ for some convex g_1, and similarly $\Pi_2(x_2) \geq F_2$ leads to $x_2 \leq g_2(x_1)$ for some concave g_2. The set $\widehat{\mathscr{W}}$ is defined by the region between g_1 and g_2 and must therefore be convex. □

4.2. The Entry Process

We assumed simultaneous entry, which ignores exactly *which* firms from the set of potential entrants will actually enter. In the Appendix, we extend the analysis to allow mixed strategies where all potential entrants determine their probability of entry, and characterize other equilibria in addition to the pure-strategy equilibria above. An alternative model is that firms enter one at a time, at either tier, and that entry stops when the marginal firm cannot cover its entry

cost. At first sight, this conforms to the notion of Nash equilibrium employed above. However, below we show an equilibrium structure that cannot result from one firm entering at a time. Alternatively, one could assume aggressive entry behavior with perfect foresight; potential entrants line up at each tier and continue entering as long as there is some further entry pattern that gives them profits, even though they might lose money in the current structure. If all firms follow this decision rule, all entrants will (eventually) earn nonnegative profits, and entry will stop at the maximal element. The equilibrium structures in $\mathcal{N}$ are only stable under myopic entry behavior; allowing perfect foresight, only the maximal structure is stable.

Consider the right-hand side of Figure 1, where $k_1 = 1/5$ and $k_2 = 1/4$ in (16) and (17). Then $\mathcal{W}$ contains (1,1) and (2,2) but neither (1,2) nor (2,1). This means that although (2,2) is viable, it cannot be reached by marginal entry from (1,1). One firm each in Tier 1 and Tier 2 would have to coordinate their entry decision. Or, with perfect foresight, a firm entering Tier 1 to change the structure from (1,1) to (2,1) knows that some firm will then profitably enter Tier 2, thus justifying the first firm's entry decision. Moreover, (1,1) itself need not be viable, but (2,1) could be; this happens when, for instance, $k_1 = 0.1$ and $k_2 = 0.4$. (Note that $(1,2) \notin \mathcal{W}$, as then $\mathcal{W}$ would no longer be a lattice.) In this case, Regularity Condition (18) does not hold, and to "start" the entry process, two firms in Tier 1 and one firm in Tier 2 would have to coordinate.

More formally, define a 1-path as a sequence $\{\mathbf{n}\}_k$ such that exactly one n_i increases by 1 at each step. We have seen that there may be no 1-path from the minimal to the maximal element. Even if we also allow (1,1)-paths, i.e., a sequence $\{\mathbf{n}\}_k$ such that both n_1 and n_2 may increase by 1 at each step, there is not always a path to a viable structure, since (1,1) itself need not be in $\mathcal{W}$. It is unclear whether any reasonable class of paths connects (0,0) to any viable structure, or the minimal to the maximal element of $\mathcal{W}$. Such analysis would help us understand under what conditions, for instance, a new entrant in one tier might unleash a rash of entry elsewhere in the supply chain.

5. The M-tier Serial Supply Chain

5.1. Prices, Quantities, and Profits in the M-tier Supply Chain

The extension of the post-entry competition results to M tiers is straightforward. Number the tiers from 1 (downstream) to M (upstream). Set $v_0 := 0, \mathbf{n} = (n_1, n_2, \ldots, n_M)$, and define $V_j = \sum_{i=1}^{j} v_i$, $N_i = \frac{n_i}{n_i+1}$ and $P_{jk} = \Pi_{i=j}^{k} N_i = \Pi_{i=j}^{k}\left(\frac{n_i}{n_i+1}\right)$.

Proposition 6. *For an M-tier serial supply chain with n_i entrants at tier i, the prices, quantities, and resulting operating profits for the firms at tier i are given by*

$$p_i = (1 - P_{iM})(a_1 - V_{i-1}) + P_{iM}(V_M - V_{i-1}), \tag{20}$$

$$Q_i = n_i q_i = Q = P_{1M}\frac{a_1 - V_M}{b_1}, \tag{21}$$

$$\begin{aligned}\Pi_i(\mathbf{n}) &= \left(\frac{b_1}{P_{1,\,i-1}}\right) q_i^2 = \frac{1}{n_i^2} P_{iM} P_{1M} \frac{(a_1 - V_M)^2}{b_1} \\ &= \frac{1}{(n_i+1)^2} P_{i+1,\,M}^2 P_{1,\,i-1} \frac{(a_1 - V_M)^2}{b_1}.\end{aligned} \tag{22}$$

Proof. By recursion from the two-tier case, follow the derivation preceding Proposition 1. Adding a third tier and substituting for a_2 and b_2, we find the derived demand function for Tier 3:

$$p_3 = (a_1 - v_1 - v_2) - \frac{b_1}{N_1 N_2} Q_3.$$

Analogously, the derived demand function for tier i is

$$p_i = \left(a_1 - \sum_{k=1}^{i-1} v_k\right) - \frac{b_1}{\Pi_{k=1}^{i-1} N_k} Q_i.$$

Solving this gives

$$p_i = (1 - N_i)\left(a_1 - \sum_{k=1}^{i-1} v_k\right) + N_i(v_i + p_{i+1})$$

and

$$Q_i = \frac{a_1 - \sum_{k=1}^{i} v_k - p_{i+1}}{b_1/(\Pi_{k=1}^{i} N_k)},$$

or

$$p_i = (1 - N_i)(a_1 - V_i) + v_i + N_i p_{i+1}$$

and

$$Q_i = \frac{a_1 - V_i - p_{i+1}}{b_1/P_{1i}}.$$

There are M tiers, so $p_{M+1} = 0$ and $Q_i = Q_M = (a_1 - V_M)/(b_1/P_{1M})$ for any i, which confirms (21). The demand function at tier i is

$$\begin{aligned} p_i = a_i - b_i Q_M &= (a_1 - V_{i-1}) - (b_1/P_{1,i-1})\frac{a_1 - V_M}{b_1/P_{1,M}} \\ &= (a_1 - V_{i-1}) - P_{iM}(a_1 - V_M) \\ &= (1 - P_{iM})(a_1 - V_{i-1}) + P_{iM}(V_M - V_{i-1}), \end{aligned}$$

which proves (20). For (22), write $\Pi_i(\mathbf{n}) = (p_i - v_i - p_{i+1})Q_i/n_i$; substituting for p_i and rearranging terms gives

$$\Pi_i(\mathbf{n}) = n_i(1 - N_i)(b_1/P_{1i})q_i^2 = N_i(b_1/P_{1i})q_i^2. \qquad \square$$

From (22), variable costs at tier i have the same effect on total profits in that tier as do variable costs at any other tier. Prices at each tier and total quantity are based on total variable costs; the drivers of profitability specifically for tier i are concentration n_i and fixed costs F_i. Using (20) and (21), we can again analyse how prices and quantities change with the number of entrants at each tier.

Proposition 7. *Prices and quantities in the M-tier supply chain*:

1. *For an M-tier serial supply chain, the quantity Q produced at any tier $i = 1, 2, \ldots, M$ increases with the number of entrants at any tier in the chain.*

2. *The price per unit p_i at tier $i = 1, 2, \ldots, M$ decreases with the number of entrants in any upstream tier (from i to M) and is unchanged with the number of entrants in a downstream tier (from 1 to $i-1$).*

Those same expressions also show that if we are interested in one specific part of a supply chain, we can aggregate the other tiers to facilitate the analysis.

Proposition 8. *Aggregation of tiers*: *Each tier i can be characterized by the variable cost v_i and the ratio N_i. Any set of contiguous tiers $\{i_1, \ldots, i_2\}$, where $i_2 \geq i_1$, may be aggregated into one tier i', characterized by $N_{i'} = \Pi_{k=i_1}^{i_2} N_k$ and $V_{i'} = \sum_{k=i_1}^{i_2} v_k$. Replacing the tiers $\{i_1, \ldots, i_2\}$ by this aggregate tier leaves the results for the rest of the chain unchanged.*

The total operating profit accruing to an M-tier chain is given by $\Pi_t(\mathbf{n}) = \sum_{i=1}^{M} n_i \Pi_i(\mathbf{n})$. Then we have the following generalization of Proposition 2.

Proposition 9. *Properties of total profits in an M-tier chain*:

1. $\Pi_t(\mathbf{n}) = P_{1M}(1 - P_{1M})\left(\frac{(a_1 - V_M)^2}{b_1}\right)$.

2. *For $\mathbf{n} = (n_1, n_2, \ldots, n_M)$, the number of entrants at each tier of the chain, let $\mathbf{n}' = (n_{[1]}, n_{[2]}, \ldots, n_{[M]})$ be any reordering of the elements of $\mathbf{n}$. Then $\Pi_t(\mathbf{n}) = \Pi_t(\mathbf{n}')$.*

3. *For any serial supply chain $\Pi_t(\mathbf{n}) \leq 1/4((a_1 - V_M)^2/b) = \Pi_t^*$.*

4. *For an M-tier chain, let $\mathbf{n}^*$ be any reordering of $(M, M+1, \ldots, 2M-1)$. Then $\Pi_t(\mathbf{n}^*) = \Pi_t(\mathbf{n})$, i.e., there is always an $\mathbf{n}$ at which Π_t^* is attained.*

5. *Suppose that Π_t^* is attained at $\mathbf{n}^*$ for M-tier supply chains. Then for $(M+1)$-tier chains, $\lim_{k\to\infty} \Pi_t^*(\mathbf{n}^*, k) = \Pi_t^*$. This also holds for any rearrangement of $(\mathbf{n}^*, k)$.*

Proof. Part 1 follows by substitution into $\Pi_t(\mathbf{n}) = (p_1 - V_M)Q$ and 2 follows from 1. Since $P_{1M} \leq 1/2$ implies $P_{1M}(1 - P_{1M}) \leq 1/4$, 3 holds. Parts 4 and 5 are easily verified, using

$$P_{1M} := \Pi_{i=1}^{M}\left(\frac{n_i}{n_i + 1}\right) = \left(\frac{M}{M+1}\right)\left(\frac{M+1}{M+2}\right)\cdots \left(\frac{2M-2}{2M-1}\right)\left(\frac{2M-1}{2M}\right). \qquad \square$$

Part 2 shows that total profits do not depend on competitiveness at specific tiers, regardless of how variable costs differ among tiers. Part 4 gives a structure that always maximizes total profits in an M-tier chain, in which the average number of suppliers per tier increases with M. After adding a tier and reallocating variable costs to keep total supply-chain value-added constant ($V'_{M+1} = V_M$), more firms at each tier generate the same total profits. Conversely, removing a level of intermediaries that adds no value could cause consolidation upstream and downstream.

5.2. The Entry Game in the M-tier Supply Chain

For the entry stage, we obtain similar results as before. Define the set of viable structures $\mathscr{W}_M = \{\mathbf{n} : \Pi_i(\mathbf{n}) \geq F_i$ for $i = 1, 2, \ldots, M\}$; under Regularity Condition (18), $\mathscr{W}_M$ is nonempty. Also define the equilibrium set $\mathscr{N}_M = \{\mathbf{n} \in \mathscr{W}_M : \Pi_i(\mathbf{n} + \mathbf{e_i}) \leq F_i$ for $i = 1, 2, \ldots, M\}$.

Proposition 10. *If $\mathscr{W}_M$ is nonempty, it is a semilattice, bounded (and therefore finite), and contains a maximal element that is also contained in $\mathscr{N}_M$. Similarly, $\widehat{\mathscr{W}}_M$,*

defined as the relaxation of $\mathscr{W}_M$ to $\mathbb{R}_+^M$, is a bounded semilattice and has a maximal element.

Proof. Decompose $\Pi_i(\mathbf{n})$ by writing $\Pi_i(\mathbf{n}) = T_1(\mathbf{n}_i)T_2(\mathbf{n}_{-i})$, where

$$T_1(n_i) = \frac{1}{(n_i+1)^2} \text{ and } T_2(\mathbf{n}_{-i}) = P_{i+1,M}^2 P_{1,i-1}\frac{(a-V_M)^2}{b_1}.$$

Then $T_1(n_i)$ is decreasing in n_i and $T_2(\mathbf{n}_{-i})$ is increasing in all n_j for $j \neq i$. Take $\mathbf{n}$ and $\mathbf{m}$ such that $\Pi_i(\mathbf{n}) \geq F_i$ and $\Pi_i(\mathbf{m}) \geq F_i$ for all i. Let $\mathbf{u} := \mathbf{n} \wedge \mathbf{m}$, i.e., $u_i := \max\{n_i, m_i\}$ for all i. Then $T_2(\mathbf{u}_{-i}) \geq T_2(\mathbf{n}_{-i})$ and $T_2(\mathbf{u}_{-i}) \geq T_2(\mathbf{m}_{-i})$. Without loss of generality, assume $u_i = m_i$. Then $\Pi_i(\mathbf{u}) = T_1(m_i)T_2(\mathbf{u}_{-i}) \geq T_1(m_i)T_2(\mathbf{m}_{-i}) \geq F_i$, so that $\mathbf{u} \in \mathscr{W}_M$ as required. The proof for $\widehat{\mathscr{W}}_M$ is analogous. □

5.3. Measures of Competitiveness in the M-tier Supply Chain

One question in the introduction concerned the impact of concentration of one tier on competitiveness in any other tier. To look at this, we focus on demand elasticity, the Lerner index of monopoly power, and relative margins. Elasticity of (imputed) demand faced by tier i, ϵ_i, measures the marginal change in demand caused by a marginal change in price. It is defined (see, e.g., Tirole 1988, pp. 218–219) as

$$\epsilon_i := -\frac{\partial Q_i}{\partial p_i}\bigg/\frac{Q_i}{p_i} = \frac{p_i}{a_1 - V_{i-1} - p_i},$$

so a higher (positive) elasticity reflects that a price increase will cause a larger drop in demand. Substituting the equilibrium prices from (20) gives the local demand elasticity in equilibrium.

Proposition 11. *Demand elasticity in equilibrium at tier i of an M-tier supply chain is given by*

$$\epsilon_i = \frac{a_1 - V_{i-1}}{P_{iM}(a_1 - V_M)} - 1.$$

ϵ_i is decreasing as one moves upstream and in the number of entrants n_j for all tiers $j \geq i$. If one adds a new tier $M+1$ and allocates variable costs at tiers i through $M+1$ in any way such that $V'_{M+1} \geq V_M$, ϵ_i increases.

Proof. Straightforward, noting that V_i is increasing in i and $P_{iM} \leq 1$ for all i. □

The longer the supply chain, the more price-sensitive each tier is. A small price change upstream has less impact on total quantity than downstream. This means that models in which price is assumed constant may be more applicable to upstream parts of supply chains than to downstream parts. The overview in Ganeshan et al. (1999) suggests that articles in which price is constant do indeed focus more often on manufacturers and upstream tiers.

The Lerner index L_i of competitiveness of an industry at tier i is (Tirole 1988, pp. 218–219) the ability of firms to charge more than marginal costs. L_i is defined as $L_i := (p_i - c'_i)/p_i$, where c'_i is the marginal cost at tier i. Under perfect competition, price equals marginal costs, so $L_i = 0$; higher L_i signals more monopoly power. With linear costs $L_i := (p_i - v_i - p_{i+1})/p_i$ now substitute equilibrium prices from (20).

Proposition 12. *The Lerner index in equilibrium at tier i of an M-tier supply chain is given by*

$$L_i = \left(\frac{1}{n_i+1}\right)\left(\frac{P_{i+1,M}(a_1 - V_M)}{(a_1 - V_{i-1}) - P_{iM}(a_1 - V_M)}\right).$$

L_i is decreasing in the number of entrants n_i at that tier, and increasing in the number of entrants n_j at all upstream tiers $j > i$. If $n_{i+1} \leq n_i$ for all i, L_i increases as one moves upstream. If one adds a new tier $M+1$ and allocates variable costs at tiers i through $M+1$ in any way such that $V'_{M+1} \geq V_M$, L_i decreases.

Proof. Again straightforward, noting that P_{iM} is increasing in n_j for all $j \geq i$. □

Firms in longer supply chains have less monopoly power, consistent with the higher demand elasticity above. Even when $n_i = n$ for all i, the Lerner index increases as one moves upstream: The closer one is to the final market, the more competitive the industry is. More entrants in tier i mean less monopoly power for firms in that tier and more monopoly power for firms at downstream tiers. The first-order equilibrium conditions under Cournot competition can be written as $L_i\epsilon_i = 1/n_i$ for all i, analogous to the single-tier case in Tirole (1988, pp. 218–219).

Finally, consider the ratio of margins at tier $i-1$ and tier i, defined as $R_i := (p_{i-1} - v_{i-1} - p_i)/(p_i - v_i - p_{i+1})$, following the two-tier successive monopoly analysis in Bresnahan and Reiss (1985).

PROPOSITION 13. *The ratio of margins at tier $i-1$ and tier i is given by $R_i = n_i/(n_{i-1}+1)$. R_i is increasing in the number of entrants n_i at tier i and decreasing in the number of entrants n_{i-1} at the downstream tier $i-1$.*

This ratio does not depend on the number of entrants at any other tier than the two being compared. This suggests that the effects of entry at tiers further upstream or downstream are the same for tiers $i-1$ and i. In the successive monopoly case with $n_{i-1} = n_i = 1$, we get $R_i = 1/2$, the classic double marginalization result under linear demand.

6. Vertically Integrated and Unintegrated Firms

We now let firms simultaneously enter both tiers of a two-tier chain. Use primes to denote the integrated firm case; if all n' firms are vertically integrated, prices and quantities are determined as in the single-tier case, with variable costs (v_1+v_2) and fixed costs of entry (F_1+F_2).

$$Q' = n'q' = \frac{n'}{n'+1}\frac{a_1-v_1-v_2}{b_1}, \tag{23}$$

$$p' = \frac{1}{n'+1}a_1 + \frac{n'}{n'+1}(v_1+v_2), \tag{24}$$

$$\Pi'(n') = (p'-v_1-v_2)q' = \frac{1}{b_1}\left(\frac{a_1-v_1-v_2}{n'+1}\right)^2, \tag{25}$$

where, in equilibrium, n' satisfies

$$\Pi'(n'+1) \le F_1+F_2 \le \Pi'(n'). \tag{26}$$

Compare this with the unintegrated case with $n' = n_1 = n_2 = n$ entrants at each tier, so that the total entry costs are the same in both cases. (We compare structures in the integrated and unintegrated case for any given n; such structures are not necessarily equilibria in either case.) The unintegrated case delivers lower quantities to the market at higher prices, for any n. The difference is greatest for small n, and disappears as n becomes large (which, by (26), implies $F_1 = F_2 = 0$). Profits in the unintegrated case are

$$\Pi_1(n) = \frac{1}{b_1}\left(\frac{n}{n+1}\right)^2\left(\frac{a_1-v_1-v_2}{n+1}\right)^2 \text{ and}$$

$$\Pi_2(n) = \frac{1}{b_1}\left(\frac{n}{n+1}\right)\left(\frac{a_1-v_1-v_2}{n+1}\right)^2,$$

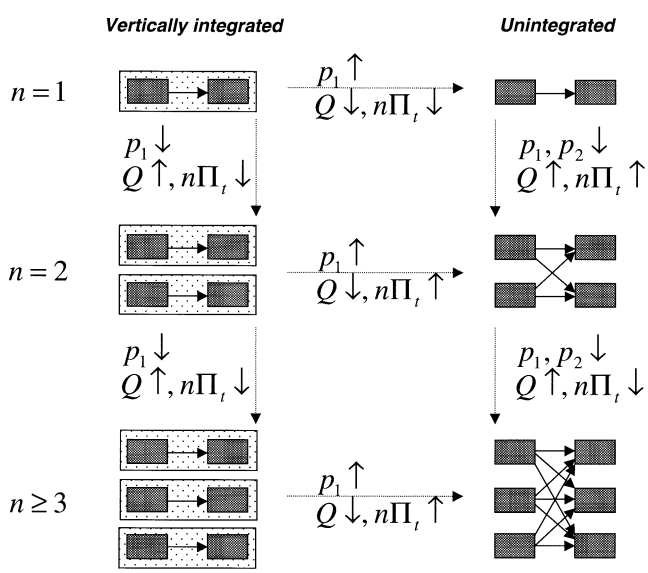
from which it follows that

$$n(\Pi_1(n)+\Pi_2(n)) = n\left[\left(\frac{n}{n+1}\right)^2 + \left(\frac{n}{n+1}\right)\right]\cdot\Pi'(n) > n\Pi'(n) \quad \forall n \in \mathbb{N}, n > 1. \tag{27}$$

For $n = 1$, this reflects the double marginalization principle (Tirole 1988): A vertically integrated monopolist earns more profits than two successive unintegrated monopolists combined. For $n > 1$, the opposite holds: Operating profits for the chain are always larger for the unintegrated case. The benefits of vertical integration (from avoiding double marginalization) are less than the benefits of allowing competition at each tier (from lower input prices p_2). Figure 2 summarizes these comparisons.

If $n > 1$ unintegrated firms could survive at each tier, it does not follow that n integrated firms could survive too. For small F_1 and F_2, the unintegrated case could support almost twice as many firms, but

Figure 2 **Comparing Vertically Integrated and Unintegrated Supply Chains**



Note. The chart summarizes the effects of vertical integration and of increased entry in a two-tier chain. Moving down the left-hand column, we add vertically integrated firms; moving down the right-hand column, we add equal numbers of unintegrated firms at each tier. Moving from right to left, we vertically integrate the firms. From $n \ge 3$ tiers onwards, the effects remain the same.

customers prefer integration. Allow n_1 and n_2 to be different in the unintegrated case, and let n_2 go to ∞. Then p_2 in the unintegrated chain converges to marginal costs v_2, and p_1 and Q converge to the integrated p' and Q'. $\Pi_2(n_1, n_2)$ vanishes, and $\Pi_1(n_1, n_2)$ converges to the integrated profits $\Pi'(n_1)$.

Next, let n' integrated and (n_1, n_2) unintegrated firms exist simultaneously. Then the inverse demand function is $p_1 = a_1 - b_1(Q+Q')$, where Q and Q' denote the total production from the unintegrated and integrated sectors, respectively. We can show that $Q = N_1N_2(A-Q')$ and $Q' = N'(A-Q)$, where $A := (a_1 - v_1 - v_2)$, $N_i := n_i/(n_i+1)$, and $N' := n'/(n'+1)$, Solving this gives

$$Q = Q_1 = Q_2 = \frac{N_1N_2(1-N')A}{1-N_1N_2N'}, \tag{28}$$

$$Q' = \frac{N'(1-N_1N_2)A}{1-N_1N_2N'}. \tag{29}$$

Given $\mathbf{n} = (n_1, n_2, n')$, operating profits per firm reduce to $\Pi_1(\mathbf{n}) = b_1q_1^2$, $\Pi_2(\mathbf{n}) = (b_1/N_1)q_2^2$, and $\Pi'(\mathbf{n}) = b_1q'^2$. We again compare the special case with n entrants at each tier of the unintegrated sector and n integrated entrants (where, again, these structures need not represent equilibria). The ratio of total profits in the two sectors is given by

$$\frac{\Pi_1(n)+\Pi_2(n)}{\Pi'(n)} = \frac{n}{2n+1},$$

so that integrated firms are more profitable by a factor of over 2 than competing unintegrated firms. In this special case, integrated firms have a higher motivation to enter for any n. However, it is quite possible for unintegrated firms in one tier to earn higher net profits than the integrated firms.

7. Conclusions

We have developed a model that considers entry decisions and post-entry competition in multitier serial supply chains. The model is a step toward a rigorous foundation for the analysis of competitive strategy in supply chains. It encompasses four of Porter's (1980) five forces in a simplified but multitier setting. We characterize and compute equilibrium structures and provide comparative statics.

The analysis yields several insights. While it is conceptually clear that prices and quantities are affected by the number of entrants at other tiers, the explicit expressions we derive permit a much sharper understanding of these interactions. The number of entrants n_i at tier i affects other tiers through the factors $n_i/(n_i+1)$ and $1/(n_i+1)$, implying that further entry beyond, say, 5 has little impact. We also show that one can aggregate multiple tiers into one.

The downstream quantity and price depend on the number of entrants at all tiers, but not on their exact location by tier. For example, a two-tier chain with (n_1, n_2) entrants looks the same to the end customer as a chain with (n_2, n_1) entrants. Also, the relative profitability of firms in different tiers only depends on the respective concentration of firms and fixed costs, and not on their respective variable costs.

We characterize viable supply chain structures and find that more entrants at any stage permit more entrants to survive at all other stages. The set of viable structures is a semilattice. If a viable structure exists, then there is a maximal equilibrium structure, the supply-chain structure that will emerge if all potential entrants have perfect foresight. This result permits us to define a sequential entry game with a unique solution. It is easy to compute prices, quantities, and the set of maximal equilibrium structures for a given example. One can speculate about how the rapid changes in technology, including the advent of e-commerce, will affect the structure of entire supply chains, not only the tier directly affected.

Finally, we examine vertical integration. In our setting, integration always leads to a more competitive supply chain in the sense that prices decrease and quantities increase. Vertical integration of successive monopolists increases their joint profits, but integration of successive oligopolists always reduces total profits. This suggests that the benefit of avoiding double marginalization does not outweigh the benefit of upstream competition in the successive oligopoly case. When both integrated and unintegrated firms coexist, the integrated chains do perform better.

Future work includes extensions to supply networks with more complex forms. Examples include distribution networks, assembly networks, systems with recyling and remanufacturing, and substitutes

and complementary products. This will allow us to study competition between supply chains with different technologies. Other possible extensions include multiple products, alternative production technologies, capacity constraints, heterogeneous production costs and entry costs, and multiattributed products and product positioning decisions. Finally, one could allow nonlinear demand and production cost functions.

Appendix. Mixed Strategies in the Entry Game

In this Appendix, we formulate the mixed extension to the entry game (see Definition 3.1.3 in Vorob'ev 1977, p. 91), and derive some interesting results. First, any firm choosing an entry probability $p \in]0, 1[$ in equilibrium earns zero expected net profits. Second, there is always a mixed strategy equilibrium in which all firms in tier i choose the same probability of entry. Third, any structure $\mathbf{n}$ is an equilibrium structure in the pure strategy game if and only if it is an equilibrium in the mixed extension. Firm j in tier i now chooses entry probability p_{ij}. Let $\mathbf{p}_i$ denote the K_i-dimensional vector of entry probabilities for tier i, and $\mathbf{P}$ the set of the $\mathbf{p}_i$ for all M tiers. After a supply chain configuration $\mathbf{n}$ is realized, all firms engage in Cournot competition, as before. Let the probability of configuration $\mathbf{n}$ emerging be given by $h(\mathbf{n})$. Given $\mathbf{n}$, the operating profit of each entrant in tier i is $\Pi_i(\mathbf{n})$; if $n_j = 0$ for any j, set $\Pi_i(\mathbf{n}) = 0$ for all i. Let $E_{ij}(\mathbf{P})$ denote expected operating profits conditional upon firm j entering tier i, so that

$$E_{ij}(\mathbf{P}) = \sum_{n_i=1}^{K_i} \sum_{\substack{k\neq i\\ k=1}}^{M} \sum_{n_k=0}^{N_k} h(\mathbf{n} \mid \text{firm } j \text{ in tier } i \text{ enters })\Pi_i(n_i, \mathbf{n}_{-i}). \quad (30)$$

The first result shows that a firm will only choose a strictly mixed strategy ($p_{ij} \notin \{0, 1\}$) if its expected profits upon entry are zero.

Proposition 14. *Any firm choosing $p_{ij} \in]0, 1[$ in equilibrium in the mixed extension will earn zero expected net profits.*

Proof. Firm ij's expected net profits are $p_{ij}[E_{ij}(\mathbf{P}) - F_i]$. If the term between brackets, which does not depend on p_{ij}, is nonzero, the optimal p_{ij} must be either zero or one. □

Write $\mathbf{1}_i$ for the K_i-dimensional vector of ones, and $\mathbf{P}_{-i}$ for the set of $\mathbf{p}_k$ for all $k \neq i$.

Lemma 1. *Regularity Conditions* (18) *and* (19) *imply that for all i, j and $\mathbf{P}$, there is $\epsilon > 0$ such that $E_{ij}(\epsilon\mathbf{1}_i, \mathbf{P}_{-i}) > F_i$ and $E_{ij}(\mathbf{1}_i, \mathbf{P}_{-i}) < F_i$.*

Proof. Ignoring terms $n_i > 1$ in (30), we know that

$$E_{ij}(\epsilon\mathbf{1}_i, \mathbf{P}_{-i}) > \sum_{\substack{k\neq i\\ k=1}}^{M} \sum_{n_k=0}^{K_k} (1-\epsilon)^{K_i-1} h(\mathbf{n}_{-i})\Pi_i(1, \mathbf{n}_{-i}).$$

Setting

$$\epsilon = 1 - \left(\frac{F_i}{\sum_{\substack{k\neq i\\ k=1}}^{M} \sum_{n_k=0}^{K_k} h(\mathbf{n}_{-i})\Pi_i(1, \mathbf{n}_{-i})} \right)^{\frac{1}{K_i-1}} > 0$$

satisfies the first inequality. For the second, note that $E_{ij}(\mathbf{1}_i, \mathbf{P}_{-i}) = \sum_{\substack{k\neq i\\ k=1}}^{M} \sum_{n_k=0}^{K_k} h(\mathbf{n}_{-i})\Pi_i(K_i, \mathbf{n}_{-i}) < F_i$. □

Lemma 2. *For any i and any set $\mathbf{P}_{-i}$, there is a unique entry probability vector $\mathbf{p}_i$ with elements $p_{ij} = p_i$ and such that $p_i E_{ij}(\mathbf{p}_i, \mathbf{P}_{-i}) = p_i F_i$ for all j.*

Proof. Let $\epsilon_1 \geq \epsilon_2$ be common entry probabilities of all firms in tier i, and $H_{i1}(n_i)$ and $H_{i2}(n_i)$ the corresponding cumulative distribution functions for the resulting number of entrants in tier i. Then $1 - H_{i1}(n_i) \geq 1 - H_{i2}(n_i)$ for all n_i. Write

$$E_{ij}(\epsilon\mathbf{1}_i, \mathbf{P}_{-i}) = \sum_{\substack{k\neq i\\ k=1}}^{M} \sum_{n_k=0}^{K_k} \left[\sum_{n_i=1}^{K_i} \binom{K_i - 1}{n_i - 1} \epsilon^{n_i-1}(1-\epsilon)^{K_i-n_i} \Pi_i(n_i, \mathbf{n}_{-i}) \right] h(\mathbf{n}_{-i}),$$

where the term between square brackets is decreasing in ϵ by Proposition 9.1.2 in Ross (1996, p. 405), because of the inequality comparing $H_{i1}(n_i)$ and $H_{i2}(n_i)$, and because $\Pi_i(n_i, \mathbf{n}_{-i})$ is decreasing in n_i. $E_{ij}(\epsilon\mathbf{1}_i, \mathbf{P}_{-i})$ is continuous in ϵ, so the result follows from the previous lemma. □

In other words, for all $\mathbf{P}_{-i}$, there is a $p_i(\mathbf{P}_{-i})$ that makes all firms in tier i exactly indifferent between entering and not entering. We then have the main result for the mixed extension:

Proposition 15. *Under Regularity Conditions* (18) *and* (19), *there is always a mixed strategy equilibrium in which all firms in tier i use the same entry probability p_i.*

Proof. Restrict $\mathbf{P}$ to have $p_{ij} = p_i \forall i, j$, and adapt notation to write $\mathbf{P} \in [0, 1]^M$. Define the mapping $g(\mathbf{P})$ on $[0, 1]^M \to [0, 1]^M$ as $g(\mathbf{P}) := \{p_i(\mathbf{P}_{-i})\}_{i=1}^M$. Then g is defined on a nonempty compact convex subset of $\mathbb{R}^M$. By Lemma 2, $g(\mathbf{P})$ is nonempty and unique (and hence convex); because $E_{ij}(\mathbf{P})$ is continuous in all p_{ij}, g is too, so by Kakutani's fixed point theorem (Takayama 1985, p. 259), there exists $\mathbf{P} \in [0, 1]^M$, such that $g(\mathbf{P}) = \mathbf{P}$. □

The existence of a mixed strategy equilibrium is in itself not surprising (and follows directly from the finiteness of the entry game; Theorem 3.1 in Myerson 1991, p. 95). However, in this case there always exists a mixed strategy equilibrium *which is symmetric within tiers*. This does not rule out other more complex equilibria, as the second result confirms:

Proposition 16. *Any structure $\mathbf{n}$ that corresponds to an equilibrium in the pure strategy game also corresponds to a pure-strategy equilibrium in the mixed extension.*

Proof. Let $\mathbf{n}$ be an equilibrium structure in the pure strategy game; any firm j in tier i that decided not to enter can now choose an entry probability p_{ij}, so its expected net profits are $p_{ij}[\Pi_i(n_i + 1, \mathbf{n}_{-i}) - F_i]$. Because $\mathbf{n} \in \mathcal{N}_M$, the term between square brackets is either negative, in which case the firm optimally continues not to enter ($p_{ij} = 0$), or it is zero, in which case, any $p_{ij} \in [0, 1]$ is an optimal response. Analogous reasoning applies to any firm that did originally decide to enter. Conversely, any pure strategy equilibrium in the mixed extension must clearly be an equilibrium in the original pure-strategy game. □

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