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Satiation in Discounted Utility

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In this paper, we propose a model of intertemporal choice that explicitly incorporates satiation due to previous consumption in the evaluation of the utility of current consumption. In the discounted utility (DU) model, the utility of consumption is evaluated afresh in each time period. In our model, the utility of current consumption represents an incremental utility from the past level. When the time interval between consumption periods is large, and there are, therefore, no carryover effects, our model coincides with the DU model. For short time intervals between consumption periods, the satiation due to previous consumption lowers the utility of current consumption. Several implications of our model are examined, and comparisons with the DU model and the habituation model are made.

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1. Introduction

In this paper, we consider the problem in which the consequences of a decision accrue over time. We examine the case where consequences are consumption streams. This case is common in economics and in decision analysis, in which maximization of the utility of consumption subject to a budget constraint is assumed both in modeling and in theoretical analysis. Our main focus is on the discrete model wherein consumption occurs at discrete time periods.

The discounted utility (DU) model is the dominant model of intertemporal choice. In this model, consumption now is given a greater weight than consumption later. Further, the total utility of a consumption stream is additively separable across periods. Samuelson (1937) introduced the DU model and Koopmans (1960) provided an axiomatic justification for the additive separable form with positive discount rate. In those works, however, both Samuelson and Koopmans recognized the limitation of the logical appeal of the consumption independence assumed in the DU model. Simply stated, consumption independence requires that the utility of current consumption does not depend on past consumption. It is easy to see that the utility of current consumption (spicy food today) may depend on past consumption (spicy food yesterday), especially when the time interval between periods is small. For some consumption goods, such as a vacation or a particular movie, consumption independence may not hold even when time periods are separated by as much as a year. For example, it would not be uncommon to hear: "I don't want to go to Washington this year; I just went there last year." To account for the effects of past consumption appropriately, one must recognize that there is some satiation due to past consumption and that the utility of current consumption is an increment over the satiation level.

Our model is based on the following simple idea: Starting with a utility of zero, consumption in Period 0 (now) takes one to a higher level of utility. This utility begins to decay, but may be higher than zero at the beginning of Period 1 (retained utility). The utility of consumption in Period 1 is the incremental utility from the retained level. The total utility is the discounted sum of incremental utilities in each period. We will refer to the incremental utility in a period as the experienced utility.

Clearly, if in each period the utility level decays back to the neutral level of zero, then our model particularizes to the DU model. In this case, utility is computed afresh from the zero level in each period. At the other extreme, if there is no decay in the utility of consumption between periods, then the experienced utility becomes smaller and smaller in subsequent periods (assuming concave utility).

A key parameter in our model is the *satiation retention factor*, which captures the carryover effect of consumption from one period to the next. A satiation retention factor equal to zero indicates no carryover effect, whereas a satiation retention factor equal to one indicates that the entire consumption effect is carried over and, therefore, the experienced utility in the next period is measured from the utility level reached in the previous period.

Bell (1974) provided an early attempt to address this problem and proposed a model of utility discounts applied to cumulative income. Bell's model is not meant for consumption streams and does not particularize to the DU model. Our model encompasses both the DU model and Bell's model as special cases.

A related model that captures the effect of past consumption is the habituation (HA) model (Wathieu 1997, 2004; Ryder and Heal 1973). In that model, the reference point is updated each period and the utility of consumption is defined as the increment over the reference point. The HA model has a behavioral motivation, whereas our model is a normative extension of the DU model. We will compare our model with the HA model in §7.

In §2, we describe our satiation model (SA), which we view as a normative extension of the DU model that permits utility retention. To motivate the satiation model, we dedicate a subsection to highlight the problem caused by assuming separability over time, when in fact there is satiation due to previous consumption. Some typical analysis employed to illustrate the implications of the DU model will be used here to show the implications of the SA model. Another subsection describes the elicitation of the two parameters of the SA model. In §3, we compare the optimal consumption paths of the SA model and the DU model. Whereas in the DU model the consumption path is always decreasing, consumption over time in the SA model exhibits richer patterns. In the DU model, consumption levels and associated experienced utility and total utility always increase as income (budget) increases. In contrast, in the SA model per-period experienced utility may increase, remain the same, or decrease. In §4, we compare the optimal duration of constant consumption and show that it may depend on both the discount rate and the satiation retention factor.

Indirect utility of income is derived in §5, where we show that satiation may "concavify" the indirect utility for money. In §6, we examine the impact of satiation on the allocation of a budget to two or more consumption goods. These comparisons with the DU model provide insight into the behavior and properties of the SA model. In §7, we provide a comparison of the SA model with the HA model. Finally, in §8, conclusions are provided.

2. Satiation Across Time Periods

2.1. The Satiation Model

Consider a consumption stream $(x_0, x_1, \ldots, x_{T-1})$, where x_t is the consumption in period t. The DU model evaluates the total utility of the consumption stream as

$$\mathrm{DU}(x_0, \ldots, x_{T-1}) = \sum_{t=0}^{T-1} \delta^t u(x_t), \tag{1}$$

where $u(x_t)$ is the utility of consumption x_t in period t, and δ^t is the discount factor associated with period t. The discount factor $0 < \delta \leqslant 1$ captures impatience in the sense that the utility of present consumption is weighted more than the utility of future consumption.

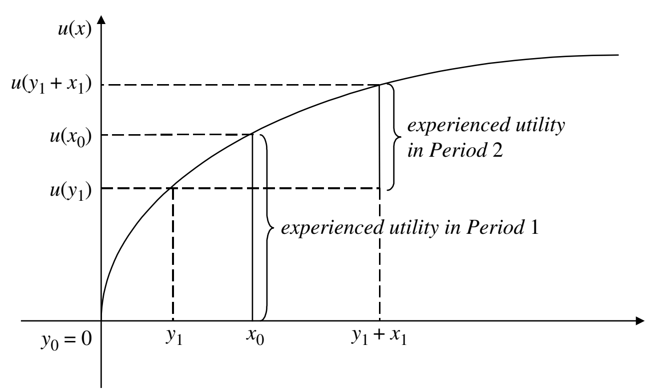
The DU model was axiomatized by Koopmans (1960) and Koopmans et al. (1964) for countable infinite streams. A key feature of the DU model is the separability over time. Thus, the utility derived from present consumption is not affected by previous consumption. The assumption of separability over time does not enjoy the same degree of normative appeal as the assumption of independence in expected utility theory. Koopmans (1960) recognizes this limitation and states, "one cannot claim a high degree of realism for such a postulate, because there is no clear reason why complementarity of goods could not extend over more than one time period" (p. 292). As an example, the utility derived from Chinese food for dinner may depend on what one had for lunch.

There are consumption goods such as movies and vacations where the enjoyment of the current consumption depends on the time elapsed since the previous consumption. The consumption creates a stock in memory that diminishes over time. Similarly, for consumption goods that satisfy biological needs such as food and exercise, each instance of consumption creates a stock and the later consumption provides utility starting from this stock. In our SA model, the contribution of the current consumption is over the satiation level achieved due to previous consumption. Thus, the carrier of utility is the increment from current satiation rather than current consumption. Suppose that the satiation level is y at the beginning of a period. A consumption x in this period yields a utility $u(y+x) - u(y)$ rather than $u(x)$. Clearly, when $y = 0$ and assuming $u(0) = 0$, the experienced utility is simply $u(x)$. In this case, our model coincides with the DU model, as the utility is computed afresh each period.

To illustrate our model in a two-period setting, consider a consumption stream (x_0, x_1), where x_0 is the current-period consumption and x_1 is the consumption in the following period. For simplicity, assume an initial satiation $y_0 = 0$. In the current period, one experiences a utility level equal to $u(y_0 + x_0) - u(y_0)$, $y_0 = 0$, $u(0) = 0$. In the next period, one starts out with a satiation level of y_1. The experienced utility of consumption x_1 is therefore $u(y_1 + x_1) - u(y_1)$. Figure 1 illustrates the two components of total utility of the consumption stream (x_0, x_1). The total utility of (x_0, x_1) in the SA model is

$$\mathrm{SA}(x_0, x_1) = [u(y_0 + x_0) - u(y_0)] + \delta[u(y_1 + x_1) - u(y_1)], \tag{2}$$

Figure 1. Total utility in the satiation model.



where y_1 is the satiation level produced by previous consumption and y_0 is given and assumed to be zero in Figure 1. In contrast, the DU model evaluates the experienced utility of each consumption afresh:

$$\text{DU}(x_0, x_1) = u(x_0) + \delta u(x_1).$$

It is easy to see that $\text{SA}(x_0, x_1) = \text{DU}(x_0, x_1)$ if $y_0 = y_1 = 0$. We further specify that the satiation level from one period to the next diminishes by a factor γ (the satiation retention factor), thus $y_1 = \gamma(y_0 + x_0)$. While the DU model requires only one parameter, δ, our SA model requires two parameters, δ and γ. In §2.3, we will discuss how both δ and γ can be derived from choices.

In the multiperiod case, the SA model is written as

$$\text{SA}(x_0, \ldots, x_{T-1}) = \sum_{t=0}^{T-1} \delta^t [u(y_t + x_t) - u(y_t)], \tag{3}$$

$$y_{t+1} = \gamma(y_t + x_t), \quad t = 0, \ldots, T-1, \text{ with } y_0 \text{ given}, \tag{4}$$

where $\delta, \gamma \in [0, 1]$. Clearly, if $\gamma = 0$, then the SA model reduces to the DU model (1). On the other end, if $\gamma = 1$, then our model is identical to Bell's (1974) model. Bell's model does not include utility decay and is more appropriate for income streams, as originally intended, rather than for consumption.

It is easily seen by recursive substitution in (4) that the satiation level in a period is simply the cumulative discounted consumption in previous periods. That is,

$$y_{t+1} = \gamma^{t+1} y_0 + \gamma^{t+1} x_0 + \gamma^t x_1 + \cdots + \gamma^2 x_{t-1} + \gamma x_t. \tag{5}$$

If consumption ceases at period t, then the satiation level decays geometrically at a rate of γ, i.e., $y_{t+k} = \gamma^k y_t$. Hence, after $k^* = \ln 0.5/\ln\gamma$ periods, the satiation level would be reduced by half. The satiation half-life k^* may vary a great deal based on the nature of consumption. For most consumption goods (food, exercise, and entertainment activities), the satiation half-life may be a week or less, but for other consumption goods (visiting a theme park, taking a vaccine) the effects of past consumption may last a long time. Thus, the satiation retention factor γ depends on the length of the interval between time periods and the type of good being consumed. For large time intervals between two time periods (relative to k^*), γ can be assumed to be approximately zero and the SA model particularizes to the DU model. Conversely, when the length of interval between two time periods is small (relative to k^*), then the satiation level may be quite high and, at the extreme, may equal the level of accumulated past consumption. The experienced utility of the current consumption is therefore significantly smaller than the utility of consumption computed afresh without any previous satiation (the value used in the DU model). The total utility given by the SA model will therefore be smaller than the total utility given by the DU model.

We clarify that the utility in our model is assumed to be cardinal (von Neumann and Morgenstern 1947, Krantz et al. 1971, Dyer and Sarin 1979) and all parameters are elicited using preferences. Further, the use of the term "satiation" in our paper refers to a decrease in the marginal utility induced by high levels of past consumption. The same meaning appears in Loewenstein and Angner (2003). This is not to be confused with the assumption of "local nonsatiation," often used in economics (Mas-Colell et al. 1995), which rules out the local maximum in the utility curve, i.e., points where the utility is flat in all directions. In our study, utility is increasing in consumption, but at a decreasing rate.

Throughout the paper, we model the consumption of nondurable goods, i.e., goods that yield experienced utility only in the period when they are consumed. A durable good (e.g., a house) can be accommodated in both the DU and the SA models by representing it as a consumption stream over several periods. Similarly, if current consumption adds future capabilities—for example, a tennis lesson enhances future competence—then outcomes should be appropriately modified to include such effects. Satiation is still valid and accounts for fatigue and the preference for spreading out the lessons over time.

The idea that experienced utility depends on past consumption is not new. Ryder and Heal (1973), Constantinides (1990), Becker (1996), and Sundaresan (1989), among others, explore habit formation and addiction using DU models with $u(x - y)$ in the per-period evaluation. In these models, the marginal utility increases with y so that current consumption increases the desirability of future consumption. While these models capture adjacent complementarity, our SA model accounts for adjacent substitutability. Chakravarty and Manne (1968) propose models where instant utility depends on the rate of change of consumption.

To obtain the continuous-time version of our SA model, we let $\hat{x}(t)$ be a flow, measured in the units of a consumption good per unit of time. In the SA model, the evaluation of such a flow would be given by

$$\text{SA}(\hat{x}(t)) = \int_0^T \delta^t u'(y(t))\hat{x}(t)\, dt, \tag{6}$$

$$y(t) = \int_0^t \gamma^{t-s} \hat{x}(s)\, ds,$$

where u' is the derivative of the per-period utility. To see this, we consider a small time interval between t and $t + \Delta$, so that $x_t = \hat{x}(t)\Delta$. Setting $\varepsilon = x_t$, we have

$$\begin{aligned} \delta^t u'(y(t))\hat{x}(t)dt &\approx \delta^t \frac{u(y_t + \varepsilon) - u(y_t)}{\varepsilon} \hat{x}(t)\Delta \\ &= \delta^t [u(y_t + x_t) - u(y_t)]. \end{aligned}$$

Equation (6) particularizes into Bell's continuous model for $\gamma = 1$ (to see this, note that for $\gamma = 1$, $dy(t)/dt = \hat{x}(t)$).

Note that u' in (6) is a decreasing function: the higher the satiation level, the lower the experienced utility. More importantly, Hindy et al. (1992) and Hindy and Huang (1992) have provided conditions for continuous models to ensure that consumption on nearby dates are perfect substitutes. Essentially, the continuous-time experienced utility has to be of the form $u(\hat{x}, y) = \hat{x}a(y) + b(y)$ for some arbitrary continuous functions a and b. The continuous form of the DU models, $u(\hat{x}, y) = u(\hat{x})$, does not satisfy substitutability. This has prompted the adoption of so-called duration models having $u(\hat{x}, y) = u(y)$ (Heaton 1995, Hindy et al. 1997). Our paper invites the use of $\hat{x}u'(y)$ as the continuous-time experienced utility, which, in contrast to the duration model, has the property that the experienced utility in the intervals where $\hat{x}(t) = 0$ is always zero, independent of the level of $y(t)$.

In the next section, we discuss an example that highlights the problem of assuming consumption independence when the time interval between two consumptions is small. We will use this example to compare the SA model with the DU model.

2.2. Motivation of the Model

In the DU model, it is implicitly assumed that there is no satiation due to consumption that is carried over across time periods. Thus, the utility of consumption in a period remains unaffected by the consumption in the previous period. The assumption that the utility in each period is computed afresh may be reasonable if the time interval between two periods is relatively large. Thus, the total utility is simply the sum of the discounted utilities as in (1). A serious problem, however, arises when the time interval between two periods is relatively small and there is a lingering effect of previous consumption on the experienced utility of the current consumption. In this case, the DU model will overstate the total utility. To see this problem, we consider the example below. We will assume that the utility function is strictly concave, representing diminishing marginal utility of consumption. Note that in models (1) and (2) we assume a unit time interval (say a year), and δ and γ reflect the discount and satiation retention factors, respectively, for the unit time. If we set the time interval between periods to Δ time units, then the appropriate discount and satiation factors are δ^Δ and γ^Δ, respectively. In doing so, we fix the number of periods; hence, the time span of the model is $T\Delta$.

Example 1. Consider two consumption streams $A \equiv (2c, 0)$ and $B \equiv (c, c)$, where B is obtained by postponing consumption c in A from the first period (now) to the second period (later). The time interval between the two periods is Δ. Then, $\text{DU}(A) = u(2c) + \delta^\Delta u(0)$ and $\text{DU}(B) = u(c) + \delta^\Delta u(c)$.

If the time interval between now and later is short, then $\Delta \to 0$, $\delta^\Delta \to 1$, and $\text{DU}(A) \to u(2c) + u(0)$, whereas $\text{DU}(B) \to 2u(c)$. The strict concavity of u implies that $2u(c) > u(2c) + u(0)$, so that the decision maker experiences a positive jump in total utility evaluation by instantly delaying the reception of a part of $2c$. Exploiting this opportunity, the decision maker could consume c/n over n consecutive instants and obtain a total utility of $nu(c/n)$.

The previous example indicates that a desirable convergence property for a multiperiod utility function $U(x_0, x_1)$ is that as $\Delta \to 0$, $U(x_0, x_1) \to u(x_0 + x_1)$, i.e., x_0 and x_1 should become perfect substitutes. A simple example makes our argument vivid. Suppose you consume a pizza now and another pizza a month later. Then, it seems appropriate to compute total utility by adding the utilities derived from consuming one pizza now and one pizza a month from now with some suitable discounting. If, however, you consume one pizza now and another soon after consuming the first one, then you do not receive a total utility that is twice the utility of one pizza, as the model in (1) implies. Instead, you get the utility of consuming two pizzas, which is likely to be less than twice the utility derived from consuming one pizza because of diminishing marginal utility. By continuity at $\Delta = 0$, the desirable property implies $u(x_0) + u(x_1) = u(x_0 + x_1)$. It is well known (Aczél 1966) that such a functional equation is satisfied only if $u(x) = kx$ for some positive k. Thus, the paradox in the example arises because the additive separability in (1) and the strict concavity of u are incompatible with the linear behavior of U when $\Delta \to 0$.

In contrast with the additive model (1), which assumes that the utility in each period is computed afresh, the SA model possesses the right convergence properties. The difficulty represented in Example 1 for short time intervals ($\Delta \to 0$) therefore does not arise. Note that the satiation factor in (2) is now γ^Δ. Assume that the initial satiation level y_0 is zero.

$$\text{SA}(A) = \text{SA}(2c, 0) = u(2c),$$

$$\text{SA}(B) = \text{SA}(c, c) = u(c) + \delta^\Delta[u(c\gamma^\Delta + c) - u(c\gamma^\Delta)].$$

As $\Delta \to 0$, both δ^Δ and $\gamma^\Delta \to 1$ and $\text{SA}(B) \to u(2c)$, satisfying the desirable local substitution property that instant delay in consumption should not create a discontinuity in the utility evaluation and produce a jump in total utility as in the DU model. Basically, in the SA model, one receives a utility of $u(c)$ in the first period. Because the second period is only an instant away, there is no depreciation in the satiation level. In the second period, therefore, the experienced utility is $u(2c) - u(c)$ and the total utility in the two periods is simply $u(c) + u(2c) - u(c) = u(2c)$.

Although the above example is extreme ($\Delta = 0$, $\gamma = 1$), it is generally true that for any $\gamma > 0$, the DU model will overstate the total utility. While the convergence property of the SA model is a desirable feature of the SA model, the accounting of satiation is justified whenever the time interval of the model is of the same order of magnitude as the satiation half-life. Using time-series aggregate consumption in the United States with monthly data, Heaton (1995) finds that consumption is relatively substitutable over a period of about four months. More generally, for a given time interval, satiation has to be taken into account for those consumption goods for which γ^{Δ} is nonnegligible. The smaller the Δ, the larger the list of goods for which satiation is significant.

2.3. Elicitation of the Discount Factor δ and the Satiation Retention Factor γ

The SA model requires two parameters, δ and γ, and the initial value of satiation, y_0. A variety of models such as the HA model, the exponential forecasting model, and the learning model require an initial value. To elicit δ and γ, assume that the per-period utility $u(x)$ has been assessed with standard methods and set $u(0)=0$. There are several ways to elicit δ and γ. A simple way is to seek d so that

$$(x, 0) \sim (0, x+d), \qquad (7)$$

where x is some appropriate level of consumption. Now,

$$u(y_0+x)-u(y_0)=\delta[u(\gamma y_0+x+d)-u(\gamma y_0)]. \qquad (8)$$

Next, elicit d' so that

$$(x, 0) \sim (x/2-d', x/2-d'). \qquad (9)$$

We now obtain the second equation,

$$u(y_0+x)-u(y_0)=u\left(y_0+\frac{x}{2}-d'\right)-u(y_0) + \delta\left[u\left(y_1+\frac{x}{2}-d'\right)-u(y_1)\right], \qquad (10)$$

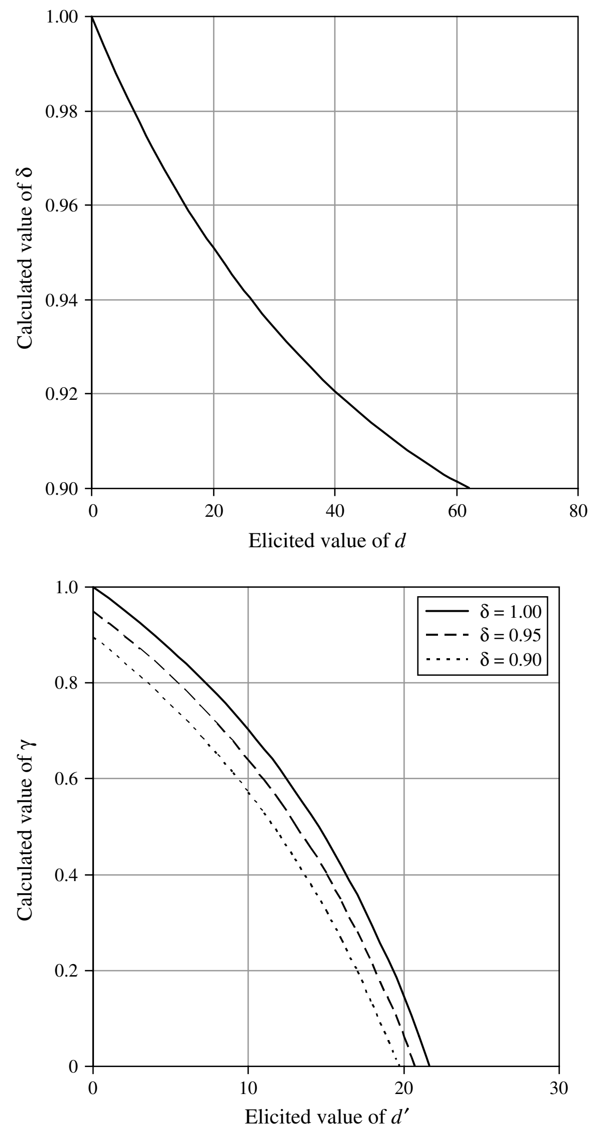
where $y_1=\gamma(y_0+x_0)$. For any given y_0, Equations (8) and (10) can be solved to yield δ and γ.

A simpler elicitation is possible when $y_0=0$. If $y_0=0$, then (8) gives

$$\delta = u(x)/u(x+d). \qquad (11)$$

One can now solve for γ using (10). For $y_0=0$, Figure 2 shows the relationship between d and δ, as well as between d' and γ. The calculations assume an exponential utility with a risk tolerance of 50. Clearly, when d is larger, the future is weighted less and δ is smaller. For a given δ, a smaller d' implies a higher level of satiation. For example, when $\delta=1$, $(x/2, x/2)$ is preferred to $(x, 0)$ because of concavity of the utility function. If $\gamma=0$, then one would require a large d' so that $(x/2-d', x/2-d')$ becomes indifferent to $(x, 0)$. The role of d' is to lower the utility of the equal consumption profile. Another way to lower the utility is to have a high γ as satiation reduces the utility in the second period. For a fixed δ, as γ increases one would need a smaller d' to obtain the required indifference.

Figure 2. Given the values of d and d', and assuming $y_0=0$, we first determine $\delta=u(100)/u(100+d)$ and then use (10) to calculate γ.



Note. Here, $u(x)=1-e^{-x/50}$.

3. Optimal Consumption Path for the Satiation Model

Suppose that a decision maker (agent or consumer) wishes to consume a total of I units in T periods. I can also be thought of as a budget constraint, together with the simplifying assumption that the prices are constant over time and are normalized to one. What is the consumption amount x_t in period t, $t=0$ to $T-1$, that will maximize the total utility?

It is well known that the optimal consumption path for the DU model is decreasing ($x_t < x_{t-1}$) if the discount factor is less than one. The optimal consumption path is obtained by solving

$$\max \sum_{t=0}^{T-1} \delta^t u(x_t)$$
$$\text{s.t.} \sum_{t=0}^{T-1} x_t = I.$$

The first-order conditions for the above problem require that $\delta^t u'(x_t)$ be equal to a constant, and thus $u'(x_{t-1}) = \delta u'(x_t)$, which yields $x_{t-1} < x_t$ for a concave utility function and $0 < \delta < 1$. Essentially, the consumption is set so that the ratio of marginal utilities in two consecutive periods is equal to the discount factor. For the special case of $u(x) = \ln(x)$, $x_t = \delta x_{t-1}$, $t = 1$ to $T-1$, and $x_0 = I/\sum_{t=0}^{T-1} \delta^t$. When $\delta = 1$, $x_0 = \cdots = x_{T-1} = I/T$, and the constant consumption path is the optimal path. Note that the per-period consumption x_t and the experienced utility $u(x_t)$ both increase as I increases.

To derive the optimal consumption path for the SA model, we solve

$$\max \sum_{t=0}^{T-1} \delta^t [u(y_t + x_t) - u(y_t)]$$
$$\text{s.t.} \quad y_{t+1} = \gamma(y_t + x_t), \quad t = 0, \ldots, T-1,$$
$$\sum_{t=0}^{T-1} x_t = I,$$

where y_0 is the initial satiation level that is given or is assumed to be zero. The first set of T constraints can be substituted using $x_t + y_t = y_{t+1}/\gamma$. If λ is the Lagrange multiplier of the last constraint, then we can write the first-order conditions in terms of y_t, $t = 1, \ldots, T$, as

$$u'(y_t/\gamma) - \delta\gamma u'(y_t) = \lambda(1-\gamma)/\delta^{t-1}, \quad t = 1, \ldots, T-1, \quad (12)$$

$$u'(y_T/\gamma) = \lambda/\delta^{T-1}. \quad (13)$$

If we define $f(y) \equiv u'(y/\gamma) - \delta\gamma u'(y)$, then we can rewrite (12) as

$$f(y_t) = \lambda(1-\gamma)/\delta^{t-1}, \quad t = 1, \ldots, T-1. \quad (14)$$

For a given utility function and the values of δ and γ, we can solve for $y_1, \ldots, y_T$ and thus derive the optimal consumption levels $x_0, \ldots, x_{T-1}$ because $x_t = (y_{t+1}/\gamma) - y_t$.

The case where the discount factor $\delta = 1$ is especially instructive in seeing how the optimal consumption paths of the DU model and the SA model differ. In the DU model, the optimal consumption is simply $x_t = I/T$, $t = 0$ to $T-1$. In the SA model, when $\delta = 1$, Equation (14) yields that y_t is constant, $t = 1, \ldots, T-2$. Let $\hat{y}$ be this constant value. Now, $x_t = (y_{t+1}/\gamma) - y_t$, so that $x_t = \hat{x} = \hat{y}(1-\gamma)/\gamma$, $t = 1, \ldots, T-2$, $x_0 = (\hat{y}/\gamma) - y_0$, and $x_{T-1} = (y_T/\gamma) - \hat{y}$. Thus, when discounting is not relevant, the optimal consumption path for any concave utility function is a constant consumption except for the first and the last periods (i.e., x_0 and x_{T-1}).

Compared to the DU model, the optimal consumption path is a bit more complex in the SA model. The optimal consumption path is decreasing as in the DU model, except for the beginning and end effects. Essentially, assuming the initial satiation level to be zero, the SA model yields a U-shaped consumption pattern with high consumption both at the beginning and at the end periods. In the intervening periods, the consumption path shows a decreasing pattern. The higher initial consumption is due to the assumption of a zero initial satiation level, which increases experienced utility in period $t = 0$. The higher final consumption is a consequence of not having an after-effect of satiation beyond T.

Although the reasons for such a consumption pattern have to do with the satiation factor, this high-low-high optimal pattern of consumption can be identified in several real-life situations. Consider, for example, the optimal design of a vacation plan, an MBA course, a speech, or a concert. In many such instances, it is recommended to pay special attention to a good beginning and a great ending, and to only maintain sufficiently satisfactory levels in the middle.

3.1. The Power Utility Function and the Optimal Consumption Path

We now consider the optimal consumption path for the power utility function. The power form for the per-period utility has been widely used because of its mathematical tractability. The power form we use is $u(x) = x^{1-\beta}/(1-\beta)$ if $\beta \neq 1$, and $u(x) = \ln x$ if $\beta = 1$. For this form, the proportional risk aversion is β. For tractability, we assume that the initial satiation level $y_0 = \alpha_0 I$ is a fraction of the total consumption budget. In this case, $f(y) = \gamma(\gamma^{\beta-1} - \delta)/y^\beta$ is a hyperbola, and (13) and (14) yield

$$y_t = \left(\frac{\delta^t \kappa}{\lambda}\right)^{1/\beta} \quad \text{for } t = 1, \ldots, T-1,$$
$$\text{where } \kappa = \frac{\gamma(\gamma^{\beta-1} - \delta)}{\delta(1-\gamma)}, \quad \text{and} \quad (15)$$

$$y_T = \gamma\left(\frac{\delta^{T-1}}{\lambda}\right)^{1/\beta}. \quad (16)$$

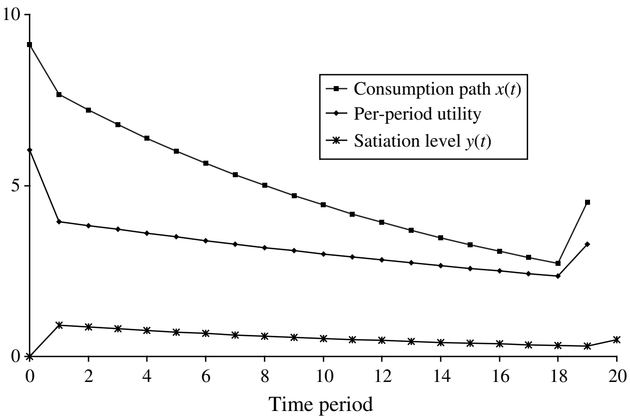
From these equations, we use $x_t = (y_{t+1}/\gamma) - y_t$ and $\sum_{t=0}^{T-1} x_t = I$ to derive

$$x_0 = \frac{1}{\gamma}\left(\frac{\delta\kappa}{\lambda}\right)^{1/\beta} - \alpha_0 I, \quad \alpha_0 \text{ given}, \quad (17)$$

$$x_t = \left(\frac{\delta^t \kappa}{\lambda}\right)^{1/\beta}\left(\frac{\delta^{1/\beta}}{\gamma} - 1\right), \quad t = 1, \ldots, T-2, \quad (18)$$

$$x_{T-1} = \left(\frac{\delta^{T-1}}{\lambda}\right)^{1/\beta}(1 - \kappa^{1/\beta}), \quad (19)$$

Figure 3. Optimal consumption path x_t, with its associated satiation level y_t, and per-period utility $u(y_t + x_t) - u(y_t)$.



Note. Parameter values are $\delta = 0.97$, $\gamma = 0.1$, $\beta = 0.5$, and $\lambda = 0.225$.

$$\left(\frac{\kappa}{\lambda}\right)^{1/\beta} = \frac{I(1+\alpha_0)}{\left(\frac{1-\gamma}{\gamma}\right)^{1/\beta} \frac{\delta^{T/\beta}}{(\gamma^{\beta-1}-\delta)^{\beta}} + \frac{1-\gamma}{\gamma}\frac{\delta^{1/\beta}-\delta^{T/\beta}}{1-\delta^{1/\beta}}}. \quad (20)$$

There are three possible solutions for $x_0, \ldots, x_{T-1}$. The "interior" solution depicted in Figure 3 is obtained when $\gamma^{\beta} < \delta < \gamma^{\beta-1}$. In this case, the consumption is strictly decreasing except for the last period. A jump in consumption in the last period occurs because there is no penalty for an increased satiation level. In the intermediate periods, the consumption is held down to keep satiation levels low. The two extreme solutions are $x_0 = I$ and $x_t = 0$, $t = 1, \ldots, T-1$; $x_t = 0$, $t = 0, \ldots, T-2$, and $x_{T-1} = I$. In the first extreme solution, all consumption occurs in the very first period, and this case is obtained when $\delta \leqslant \gamma^{\beta}$. In the second extreme solution, all consumption occurs in the last period, and this case is obtained when $\delta \geqslant \gamma^{\beta-1}$. In the example in Figure 3, if we assume $\delta = 1$, then the optimal consumption is constant except for the first and last periods.

3.2. Experienced Utility

For the power utility function, we now examine what happens to the experienced utility when income or the total consumption level I increases. In the discussion of the results of this subsection, we adopt an extramathematical interpretation of experienced utility $u(y_t + x_t) - u(y_t)$ as a measure of "happiness." Under this interpretation, higher levels of utility lead to greater happiness. Still, utility in our model is preference based, i.e., a mathematical representation of preferences. This case is of interest when an interior solution holds ($\gamma^{\beta} < \delta < \gamma^{\beta-1}$) because in the extreme cases, all consumption occurs either in the initial period or in the last period.

The results depend on whether $\beta < 1$, $\beta = 1$ (logarithmic case), or $\beta > 1$. Using (15) and (18), the expression for the experienced utility in period t is given by

$$u(y_t + x_t) - u(x_t) = \left(\frac{\delta^t \kappa}{\lambda}\right)^{(1-\beta)/\beta} \frac{(\delta^{1/\beta}/\gamma)^{1-\beta} - 1}{1-\beta}. \quad (21)$$

In the above expression, the sign of $(\delta^{1/\beta}/\gamma)^{1-\beta} - 1$ is crucial. If $\beta < 1$, then this sign is always positive. Plugging in the expression for κ/λ from (20) shows that the experienced utility is proportional to $I^{(1-\beta)}/(1-\beta)$. Thus, for $\beta < 1$, as I increases, experienced utility also increases. We should clarify that the optimal consumption path assumes that the decision maker rationally anticipates satiation and therefore does not immediately increase consumption as income rises (at high levels of income). A myopic decision maker may increase consumption in the initial periods if income rises and thus reach a high level of satiation, thereby actually reducing the experienced utility levels in subsequent periods.

Experienced utility as a function of I is shown in Figure 4. If $\beta < 1$, the same increasing pattern that one would obtain from the DU model appears.

In the logarithmic case, $\beta = 1$ and the experienced utility $u(y_t + x_t) - u(y_t)$ is constant and, hence, independent of I. This counterintuitive result is clear if we recall from (15), (18), and (20) that x_t and y_t are proportional to I, so that

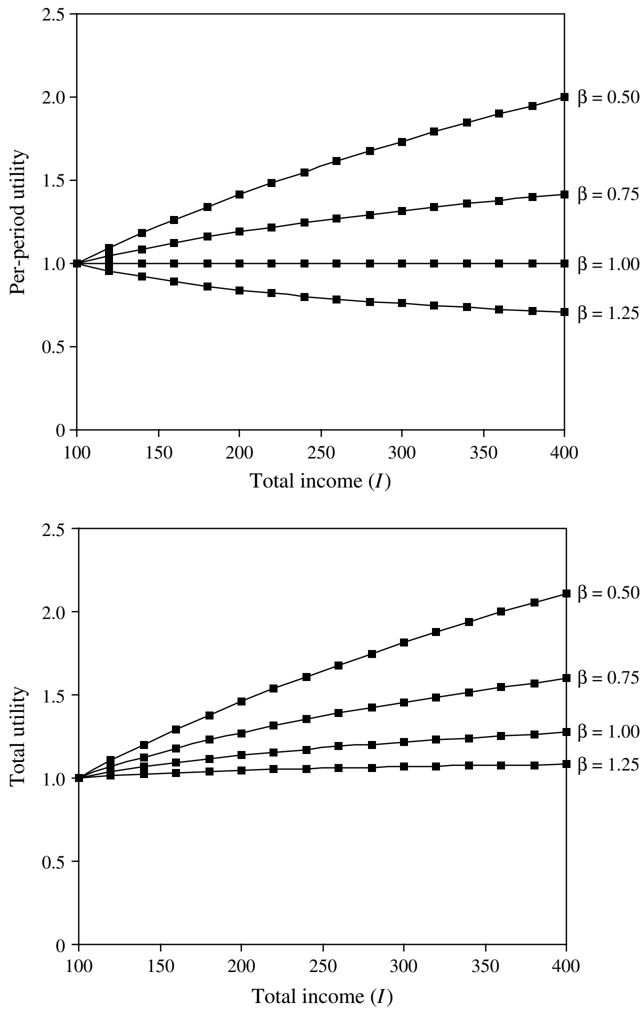
$$u(y_t + x_t) - u(y_t) = \ln(\theta_t I + \xi_t I) - \ln(\theta_t I) = \ln\left(\frac{\theta_t + \xi_t}{\theta_t}\right). \quad (22)$$

With the logarithmic utility function, as income increases, both consumption levels and satiation levels increase. The net result is that the experienced utility in any given period t remains the same, so more money and higher consumption do not buy greater happiness. Figure 4 depicts this result.

Finally, when $\beta > 1$, then the experienced utility (21) actually decreases with income level I. Figure 4 depicts this case as well. It has been observed that lottery winners report no more happiness than nonwinners and, in one experiment, reported significantly less pleasure from mundane daily activities (Brickman et al. 1978).

A summary of the above results is that for a wide variety of utility functions, the SA model will indeed predict that experienced utility will increase with an increase in income or total consumption level. For some utility functions, the experienced utility remains the same (logarithmic) or even decreases (power form with $\beta > 1$) as income increases. In these cases, if the initial satiation level is low, then when income goes up, an increase in experienced utility occurs only in the initial period. In subsequent periods, or in steady state, the satiation levels go up, and therefore there is no increase in the experienced utility. More money therefore does not necessarily buy more happiness.

Figure 4. Normalizing so that the utility associated with $I = 100$ is one, we observe that total utility (bottom) always increases with I, but the experienced utility (top) increases with I if $\beta < 1$, is constant if $\beta = 1$, and decreases if $\beta > 1$.



Note. Here, $\delta = 0.5$ so that $\gamma^{\beta} < \delta < \gamma^{\beta-1}$ and $y_0 = 1$.

4. Optimal Duration of Constant Consumption

Consider the stationary consumption of a total of I units over T periods. In this case, $x_t = x = I/T$, $t = 0, \ldots, T-1$. Because consumption x is constant from period to period, the stationary satiation level is obtained by solving (4), which yields

$$y = \gamma(y + x). \tag{23}$$

Thus, $y = \gamma x/(1-\gamma)$. For simplicity, we assume that $y_0 = \gamma x/(1-\gamma)$ as well, so that the stationary satiation level y will hold for all periods. The experienced utility, $u(y+x) - u(y)$, is then given by $u(x/(1-\gamma)) - u(\gamma x/(1-\gamma))$. The total utility in (3) is written as

$$\sum_{t=0}^{T-1} \delta^t \left[u\left(\frac{I}{T(1-\gamma)}\right) - u\left(\frac{I}{T}\frac{\gamma}{1-\gamma}\right)\right]. \tag{24}$$

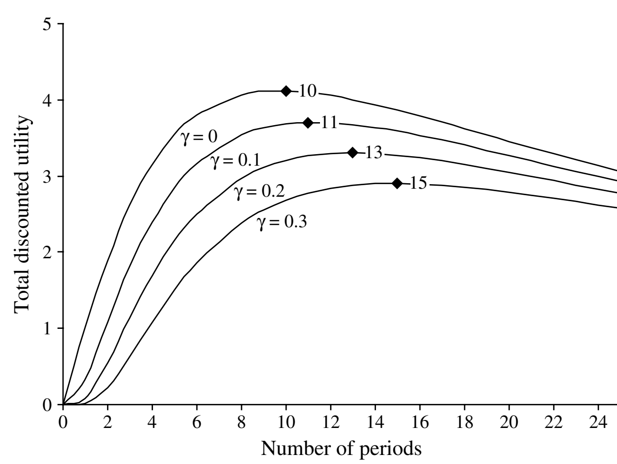
For a given utility function, one could ask: What is the optimal duration T^* over which to spread out the consumption of I units? Here, it is worth underlining the originality of this question. To the best of our knowledge, we do not know of any other paper that attempts to find such a T^* (probably because traditional models cannot make sense of this question, however intuitive). Usually, the question one considers in time preference is: How do you spread consumption over a given period? (e.g., Wathieu 1997).

The choice of the optimal T^* depends on two considerations. The discounting penalty is higher for a larger T; that is, the present value of utility for a distant consumption is low. The tendency, then, is to consume I in a smaller number of periods. On the other hand, the concavity of u and the presence of satiation display a preference for spreading out consumption over a longer duration. Thus, the optimal T^* is an interior value.

For an exponential utility function with a risk tolerance, ρ, equal to 10 and δ equal to 0.9, Figure 5 shows the relationship between the number of periods T during which consumption I is spread out and the total utility. The optimal T^* for the DU model ($\gamma = 0$) is 10 in this example. Thus, a consumption of $I/10$ in each of the 10 periods maximizes total utility. In the SA model with $\gamma > 0$, the optimal T^* is greater than 10. Further, a larger value of the satiation retention factor γ leads to an increase in optimal T^*. Thus, satiation strengthens the desire to spread out consumption over a longer number of periods. We also note that for a given T upon which consumption is spread out, the total utility is lower in the SA model than it is in the DU model.

For a power utility function, the optimal number of periods to spread out consumption depends only on the risk-aversion parameter β, and not on the satiation retention

Figure 5. Per-period utility is exponential, with $u(x) = 1 - e^{-x/10}$ and $\delta = 0.9$.



factor γ. This is a unique characteristic of the power family for which the marginal effect of T on both the discounting penalty and the desire to spread out consumption are proportional to a function of γ. Hence, γ disappears from the first-order condition and the optimal T does not depend on γ.

5. Indirect Utility for Income

It is common in decision analysis and economic applications to consider a utility function that is defined over income. Such a utility function is derived from an optimal consumption plan that an income can afford. Thus, the indirect utility of an income I is derived by solving

$$U(I) = \max \sum_{t=0}^{T-1} \delta^t u(x_t)$$
$$\text{s.t. } \sum_{t=0}^{T-1} x_t = I.$$

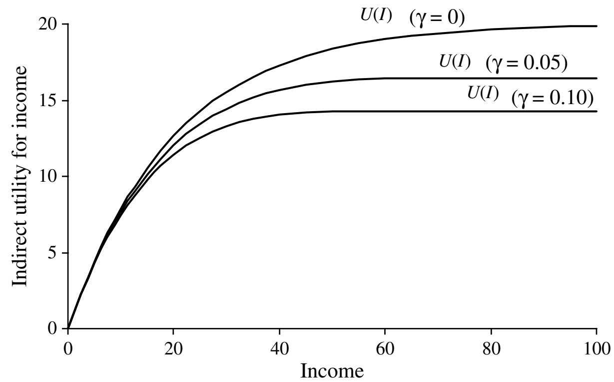
The indirect utility $U(I)$ will depend on the form of the per-period utility, $u(x)$. In the DU context, assuming an exponential utility function for the per-period utility, Smith (1998) derives the indirect utility of income and shows that the relationship between the risk tolerance of the per-period utility, ρ_c, and the derived risk tolerance of the indirect utility for income, ρ_I, may depend on the time resolution of uncertainty. Assuming a zero risk-free rate for lending and borrowing, Smith (1998) shows that if all uncertainties resolve at the beginning, then $\rho_I = T\rho_c$. This is consistent with the intuition that the risk tolerance for income, given a long planning horizon, should be much larger than the per-period risk tolerance for consumption. However, if the uncertainty about income level resolves just before the last period of consumption rather than in the initial period, then $\rho_I = \rho_c$. The intuition for this striking reduction in risk tolerance comes from the highly inconvenient information timing. Because the total budget becomes known only in the last period, additional income may only be spent in this last period.

Although our analysis is motivated by Smith (1998), we assume no uncertainty; instead, we investigate the impact that satiation has on the indirect utility of income. In the SA model, the indirect utility of income, $U(I)$, is derived by solving

$$U(I) = \max \sum_{t=0}^{T-1} \delta^t [u(y_t + x_t) - u(y_t)]$$
$$\text{s.t. } y_{t+1} = \gamma(y_t + x_t), \quad t = 0, \ldots, T-1, \tag{25}$$
$$\sum_{t=0}^{T-1} x_t = I.$$

Satiation lowers the total utility and, therefore, the indirect utility for income decreases as γ increases. Assuming

Figure 6. Indirect utility for income.



Note. The model has $T = 20$ periods, no discounting ($\delta = 1$), and the per-period utility is $u(x) = 1 - e^{-x}$, or $\rho_c = 1$.

an exponential utility function, Figure 6 shows the indirect utility of income for $\gamma = 0$ (the DU model), $\gamma = 0.05$, and $\gamma = 0.1$. The discount factor, δ, is assumed to be one. Indirect utility decreases as the satiation retention factor increases. This is because at higher levels of satiation, experienced utility $[u(y + x) - u(y)]$ is lower. Interestingly, in the exponential case, the curvature of $U(I)$, measured by $-U''(I)/U'(I)$, is bounded by $1/T\rho_c$ and increases with I, approaching $1/\rho_c$ as I increases.

If the per-period utility is assumed to be of the power form, then the total utility in the SA model is proportional to $I^{1-\beta}/(1-\beta)$ (see §3.2). In this case, the coefficient of proportional risk aversion β of the indirect utility of money is precisely the same as the coefficient of proportional risk aversion of the per-period utility of consumption.

For any utility function, (25) can be solved to derive the indirect utility for income, $U(I)$. As the satiation retention factor increases, $U(I)$ decreases. The implied $-U''/U'$ could, however, be more complex for a general utility function. For the exponential case, $-U''/U'$ increases to $1/\rho_c$ as income increases. For the power case, $-U''/U'$ is β/I, which decreases as income increases.

6. Two Consumption Goods

We now consider the allocation of a budget among two or more consumption goods. By means of an example, we show that the SA model contradicts the intuitive notion that a higher marginal utility induces more consumption. Consider two consumption goods having the same price that is constant over time. We assume the following per-period utility:

$$u(x1, x2) = (x1)^{0.4} + 0.5(x2)^{0.4}, \tag{26}$$

where $x1$ and $x2$ are quantities of the two consumption goods. At $x1 = x2 = I/2$, an increment of consumption in Good 1 yields twice as much marginal utility as an equal increase of Good 2. Thus, in an optimal allocation, one expects a higher quantity of Good 1. Assume a discount factor $\delta = 1$, number of periods $T = 20$, and that the budget constraint requires that the total consumption not exceed 100 units. Because there is no discounting, the optimal consumption is constant in the DU model and is constant except for the first and last period in the SA model (see Figure 3).

In the DU model, the optimal consumption per period for the two goods is equal to $x1^* = 3.8$ and $x2^* = 1.2$, the quantity where the marginal utility of Good 1 equals the marginal utility of Good 2.

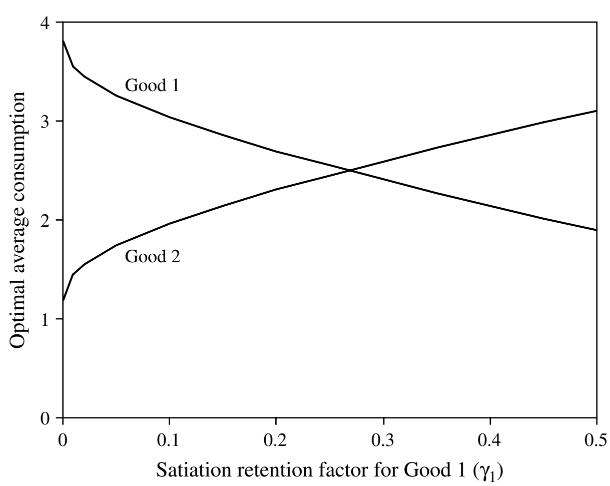
In the SA model, the average optimal consumption of Goods 1 and 2 depends on the satiation retention factors of the two goods, γ_1 and γ_2. Let us set $\gamma_2 = 0$. Clearly, when $\gamma_1 = 0$, we get exactly the same result as in the DU model. As we increase γ_1, we predict that the average consumption of Good 1 will decrease and that of Good 2 will increase. Assuming $y1_0, y2_0 = 0$, the total utility is given by

$$\sum_{t=0}^{T-1} \delta^t [u(y1_t + x1_t, y2_t + x2_t) - u(y1_t, y2_t)],$$

$$y1_t = \sum_{s=0}^{t} \gamma_1^{(t-s)} x1_t, \quad \text{and} \quad y2_t = \sum_{s=0}^{t} \gamma_2^{(t-s)} x2_t.$$

The relationship between the satiation retention factor γ_1 for Good 1 and the average optimal consumption is shown in Figure 7. Note that the total consumption per period of the two goods is always equal to five ($I = 100$; $T = 20$). Figure 7 shows that the average consumption of Good 1 equals that of Good 2 at $\gamma_1 = 0.25$. For $\gamma_1 > 0.25$, the average consumption of Good 2 exceeds that of Good 1. In summary, the optimal allocation of a budget among two or more goods will depend not only on their marginal utilities (DU model), but also on their relative satiation retention factors (SA model). A higher satiation (lower utility decay) would lead to lesser consumption.

Figure 7. Average optimal consumption of Goods 1 and 2 over 20 periods.



Note. $\delta = 1$, $u(x1, x2) = (x1)^{0.4} + 0.5(x2)^{0.4}$. Satiation retention factor for Good 2 is zero. Budget constraint takes the form of average total consumption of less than or equal to 5.

7. Comparison with the Habituation Model

The HA model proposed by Wathieu (1997), although quite different in structure than our model, has remarkably similar predictions to ours under some conditions. In the HA model, the experienced utility is given by $u(x - r)$, where r is the reference level that tracks past consumption using an exponential smoothing equation. Formally,

$$\text{HA}(x_0, \ldots, x_{T-1}) = \sum_{t=0}^{T-1} \delta^t u(x_t - r_t), \tag{27}$$

$$r_{t+1} = \alpha x_t + (1 - \alpha) r_t,$$
$$t = 0, \ldots, T-1, \text{ with } r_0 \text{ given.} \tag{28}$$

In both the SA model and the HA model, where present consumption has a lasting effect, the final period T plays a special role. One natural interpretation of this last period is the end of life. A second possibility is the case where the consumption of a certain good is constrained to some time window—for example, recreational activities undertaken during vacation—in which case the model would account for the consumption during this time period.

The SA model and the HA model both relax the noncomplementarity assumption of the DU model. Thus, in both models, the utility derived from current consumption depends on past consumption. While the HA model nicely accounts for the reported preference for increasing sequences, the SA model expands the DU model to account for utility decay. Accounting for utility decay is normative because satisfaction from consumption lingers over time. Consumption independence assumed in the DU model is akin to preference independence in multiattribute utility models and does not enjoy the same normative status as the independence axiom in the expected utility theory. The SA model is, therefore, consistent with rational preferences and reduces to the DU model for sufficiently large time intervals (utility decays to neutral zero level).

Both the satiation level y in the SA model and the reference point r in the HA model are discounted sums of past consumption. While higher values of y and r reduce the utility of current consumption, the roles of y and r in influencing experienced utility are, however, quite different.

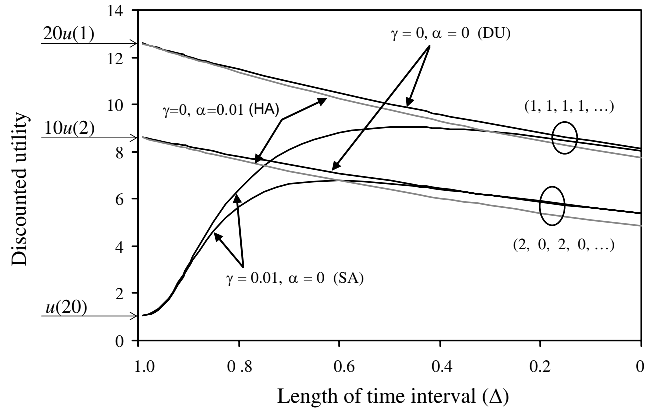
- In the SA model, if $x = y$, then the experienced utility $u(y + x) - u(y) = u(2x) - u(x) > 0$. In contrast, if $x = r$ in the HA model, then $u(x - r) = 0$. Therefore, the experienced utility in the SA model depends on the curvature of the utility function in the positive range, whereas in the HA model it depends on the shape of the utility function around zero, with both the positive and negative ranges playing a role.
- Normalizing $u(0) = 0$, and assuming $y, r > 0$, the experienced utility associated with no consumption is zero in the SA model, whereas it is negative in the HA model. More striking, and assuming concave utility, the HA model predicts that the incremental utility of x, $u(x - r) - u(-r)$,

decreases as we wait to consume x because r decays by a factor $(1-\alpha)$. The reason for this is that the craving for x diminishes with time. In contrast, the SA model predicts that the incremental utility of x, $[u(y+x)-u(y)]$, increases as we wait to consume x because y decays by a factor γ. Thus, the SA model predicts quite generally a preference for delaying consumption to allow the decay of the recent past, thus increasing utility. The HA model can also account for this preference, but requires u to be S-shaped and to have a sufficiently high value of r (Wathieu 2004).

- In the HA model, the shape of the consumption path can be decreasing, U-shaped, or increasing, depending on the discount factor. The SA model predicts a strong start and strong end (U-shaped) to the consumption sequence. Hence, the descriptive finding that people prefer U-shaped sequences is consistent with both models.
- Both the SA model and the HA model particularize to the DU model if $\gamma=0$ or $\alpha=0$, respectively. The implications of the two models for extremes of the other parameter are very different. If $\gamma=1$, then the SA model leads to an accumulation model where the total utility is simply the utility of total consumption. If $\alpha=1$, then $r_t=x_{t-1}$ and, in the case of constant consumption, the utility in each period, except the first, would be zero.
- The HA model was not designed to account for time intervals that are arbitrarily small and does not possess the convergence property discussed in Example 1. In fact, as Δ decreases, the α in the HA model goes to zero, and the HA model resembles the DU model. Specifically, consider two consumption streams $A=(1,1,\ldots,1)$ and $B=(2,0,2,\ldots,0,2,0)$, both lasting for $T=20$ periods. We will vary the time separation, Δ, between periods. For $\Delta=1$, we set $\delta=0.95$, $1-\alpha=0.99$, and $\gamma=0.01$. As shown in Figure 8 at $\Delta=1$, the three models do not differ much in evaluating the same consumption sequence. As Δ decreases from 1.0 to 0.5, the same consumption pattern is accelerated and takes place in half the time, the penalty for discounting decreases, and the utility evaluation increases for the three models. As Δ decreases further and approaches zero, DU(A) converges to $20u(1)$ and DU(B) converges to $10u(2)$. Because u is strictly concave, $20u(1)>10u(2)$. As shown in Figure 8, the HA evaluation of A and B also converges to separate distinct values. The SA model has the desirable property that as $\Delta\to 0$, the evaluation of both consumption streams converges to $u(20)$. Essentially, when $\Delta\to 0$, 20 units are consumed in a short time interval in both streams A and B, thereby producing a total utility of $u(20)$ for both.

In summary, the SA and the HA models are two different departures of the DU model. While the latter captures the reduction in experienced utility due to the effect of a changing reference point, the former captures the reduction in experienced utility due to satiation. This shows that habituation is different from satiation, and that it is possible to construct hybrid models that combine both aspects. In fact, using consumption data, Heaton (1995) finds "evidence for a preference structure in which consumption at nearby dates is substitutable and where habit over consumption develops slowly" (p. 681).

Figure 8. In all cases, $\delta=0.95$ and $u(x)=1-\exp(-x)$.



Note. The HA model uses $\alpha=0.01$ and the SA model uses $\gamma=0.01$.

8. Conclusions

We have presented a model of time preference that incorporates satiation into discounted utility. In the SA model, utility is affected by a retention factor, γ. If $\gamma=0$, then the utility decays back to the neutral level of zero (i.e., satiation level is equal to 0) and the utility of consumption is evaluated afresh in each period. In this case, the SA model particularizes to the DU model. For $\gamma>0$, there is some satiation level y that decays at rate $1-\gamma$, and experienced utility of consumption x is an increment over the utility at the satiation level $(u(y+x)-u(y))$. In the other extreme case, when $\gamma=1$, there is no decay in the utility. Total utility in this case is simply the utility of total consumption I over T periods, if there is no discounting ($\delta=1$). Such an extreme case occurs, for example, when the time interval between periods is small, and thus the entire consumption takes place in a short time. In the DU model, the optimal consumption when $\delta=1$ will provide a total utility of $(I/T)u(I/T)$, which may be substantially larger than $u(I)$ because of the concavity of the utility function.

The SA model is an extension of the DU model. Both models compute the total utility of a consumption stream as the sum of discounted utilities. Here is a list of the distinct features and predictions of the SA model:

- The utility derived from consumption in a period (experienced utility) is an increment over the satiation level achieved due to past consumption. Thus, if the satiation level is high, then the utility of additional consumption will be evaluated at a lower marginal rate (assuming concave utility).
- The optimal consumption path for the SA model is U-shaped, with high initial consumption, followed by a

lower and smoother consumption path and a higher final consumption. In contrast, the DU model predicts decreasing consumption paths, and the HA model may exhibit decreasing, U-shaped, or increasing consumption paths.

• Satiation influences the allocation of a budget among two or more goods. Basically, a good with higher satiation will be consumed in lower quantities, ceteris paribus.

• In the SA model, a greater consumption earlier stimulates lesser consumption later. Suppose that one prefers meat over chicken and chicken over fish. In a weekly menu plan, it may be optimal to relish meat three times interspersed with fish and chicken meals on two days each (Ratner et al. 1999). The SA model will permit such a preference, but a naïve implementation of the DU or the HA model will prescribe meat every day of the week.

• A somewhat surprising implication of the SA model is that with some utility functions, the experienced utility in intermediate periods (all periods except for the first and the last) does not increase as income or total consumption increases. With an increase in consumption, satiation also increases. Because the experienced utility is an increment over the utility at the satiation level, there is no net increase in the experienced utility. The decision maker will still prefer more income to less as the total utility of the former is higher, but in intermediate periods will not experience a greater experienced utility. This issue requires further empirical and behavioral exploration.

The principle of diminishing marginal utility has been a cornerstone of economic theory and psychology. Our satiation model provides a straightforward approach to incorporating this principle into a model of time preference. As one moves up the utility curve (due to past consumption), the utility of consumption is evaluated at a lower marginal rate. The total utility of a consumption stream is the sum of discounted incremental utilities.

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