Fairness and Social Risk II: Aggregated Analyses

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This paper is the second of a two-paper study of fairness issues for decisions that affect the benefits received and the risks encountered by a population. The study examines fairness for individuals and for homogeneous groups within the population. It considers fairness both for population benefit-risk profiles and for probability distributions over profiles.

Our study bases fairness on notions of envy among individuals and groups. The first paper focused on fairness in profiles and profile distributions when benefits and risks are not aggregated. The present paper uses individual preferences to assess fairness when benefits and risks are aggregated within groups or over the population. It concentrates on intergroup envy measures and fairness indices that account for ways that aggregated benefits and risks might be allocated to members of groups or to groups within the population.
(*Fairness; Envy; Social Risk; Distributive Justice; Aggregated Benefits and Risks*)

1. Introduction

This paper is the second in a two-part study of envy and fairness in a population of N individuals partitioned into n homogeneous groups $G_1, G_2, \ldots, G_n$ whose members experience benefits and risks in an uncertain environment. As noted in Fishburn and Sarin (1994) (henceforth FS94), we regard the study as a foundation for approaches to fairness analysis that could contribute to policy decisions in which issues of fairness or distributive justice among individuals and groups play an important role. Examples include facility location decisions (parks, hospitals, airports, roads, schools, jails, waste treatment plants) and government budget decisions for taxes, public assistance, arts support, and other programs that may affect groups differently. FS94 outlined our general formulation and analyzed fairness from a perspective in which benefits and risks are not aggregated over groups or over the entire population. The present paper considers envy and fairness based in part on aggregates of benefits and risks within groups. As before, we analyze nonprobabilistic benefit-risk profiles as well as profile distributions that model uncertainty by probability distributions over profiles. Our primary aim is to develop tractable indices of fairness among groups based on simple counting measures that do not involve interpersonal utility comparisons but do account for individuals' and groups' perceptions of how well they are treated in comparison to other individuals and groups.

The following example illustrates a fundamental difference between the unaggregated perspective of FS94 and the present aggregated perspective. We consider a four-person population composed of two two-person groups, $G_1 = \{1, 2\}$ and $G_2 = \{3, 4\}$. Let $[b, r] = ((b_1, r_1), (b_2, r_2), (b_3, r_3), (b_4, r_4))$ denote a profile of benefit-risk pairs in which benefits b_i and risks r_i for individuals are scaled from 0 to 100 with more b_i preferred to less and less r_i preferred to more. We assume for illustrative purposes that benefits and risks aggregate additively and can be transferred among individuals. Suppose $[b, r]$ is given by the initial profile row of the following display:

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	G_1		G_2	
	$i=1$	$i=2$	$i=3$	$i=4$
Initial Profile	(60, 35)	(30, 15)	(50, 20)	(50, 20)
Aggregated Profile	(90, 50)		(100, 40)	
An Allocation	(65, 30)	(35, 10)		

an allocation of G_2 totals to individuals in G_1.

Suppose further that person 1 is happy to face substantial risks for larger benefits, person 2 is safety minded and will forego some benefits for lower risks, and persons 3 and 4 are attracted by good benefits with moderate risks. It is then plausible that each person likes his or her own benefit-risk pair as much as any other pair in $[b, r]$. In this case, $[b, r]$ is envy-free for both individuals and groups from the unaggregated perspective of FS94, and we say that $[b, r]$ is totally fair to every person and each group.

Because we assume that both benefits and risks aggregate additively, the aggregated benefit-risk pairs for the groups are

$$(\mathbf{B}_1, \mathbf{R}_1) = (90, 50) \quad \text{for } G_1,$$

$$(\mathbf{B}_2, \mathbf{R}_2) = (100, 40) \quad \text{for } G_2.$$

Because G_2 enjoys greater total benefits and less total risks than G_1, it seems reasonable that group 1 will envy group 2 in the aggregate. Indeed, if the totals for G_2 of (100, 40) were allocated to members of G_1 in the manner shown in the preceding display, so person 1 gets (65, 30) and person 2 gets (35, 10), then each member of group 1 would prefer his or her new benefit-risk pair to the original (b_i, r_i), thus providing a clear sense of group 1's envy based on its members' preferences. Group 2's aggregate dominance, revealed by $\mathbf{B}_2 > \mathbf{B}_1$ and $\mathbf{R}_2 < \mathbf{R}_1$, is not required for this envy conclusion, but it does make the conclusion more obvious.

We refer to the type of group envy illustrated by the example as allocational envy. It is one of several aggregation-based envy notions used later to define fairness indices. The next section precedes our discussion of envy and fairness with a summary of the formulation in FS94, supplemented by remarks on aggregation. Further introduction, including comments on interpersonal comparisons, alternative approaches, and our distinction between equity and fairness, is provided by FS94. See also the references in §4.

§3 begins our discussion of intergroup envy and fairness by considering whether the individuals in one group would be better off if their benefit-risk pairs were replaced by an allocation of another group's benefits and risks that accounts for different group sizes. The envy measure defined by this comparison is extended by expectation in two ways to define fairness indices for probability distributions on benefit-risk profiles. We also describe a related approach to intergroup envy and fairness based on marginal distributions for groups.

§4 shows how our perspective can be adapted for basic data on aggregated group benefits and risks that lack information about individuals' benefit-risk pairs. It considers situations in which groups need not have definite rules for allocations to members as well as situations with known allocation rules.

Section 5 describes a simplified approach that bases individuals' preferences on equal divisions of group aggregates, or on what we refer to as per capita benefit-risk pairs. It also notes variations of our usual Pareto envy measure (nobody worse off, somebody better off) that account for majority improvements and proportions of groups that would be better off with another group's per capita benefit-risk pair. Section 6 illustrates the approach of §5 by the decision of where to locate a plant to process hazardous wastes. Section 7 provides a concluding overview.

The primary aim of the paper is to describe conceptually-viable approaches to intergroup fairness based in part on aggregated data. Because the information demands of some of our proposals may be unrealistic or excessive, we mention simplifications that could increase applicability. We have not tried to exhaust the possibilities that arise from our formulation, but we hope that enough has been said to indicate some of those possibilities. One deliberate omission is intergroup envy comparisons that use group preferences instead of individuals' preferences. If group preference relations are accessible, they can be integrated into methods described in FS94 or the present paper to assess fairness according to methods developed in our study.

2. Formulation and Aggregation

We assume that a population of N individuals is divided into n homogeneous groups $G_1, G_2, \ldots, G_n$ with N_j the number of people in G_j and $N_1 + N_2 + \cdots + N_n = N$. Each person will experience some benefit-risk pair $(b, r) \in B \times R$ during a fixed time period. Elements of the benefits set B and the risks set R could be multidimensional and encompass public as well as private goods and risks.

The set of G_j *profiles* is $(B \times R)^{N_j}$. A generic G_j profile is

$$(b, r)_j = ((b_{1j}, r_{1j}), (b_{2j}, r_{2j}), \ldots, (b_{N_j j}, r_{N_j j})),$$

where (b_{ij}, r_{ij}) is the benefit-risk pair of person i in group j. The set of *population profiles* is $(B \times R)^N$. A generic population profile is

$$\begin{aligned}[b, r] &= ((b_1, r_1), (b_2, r_2), \ldots, (b_N, r_N)) \\ &\quad \text{without group designations} \\ &= ((b, r)_1, (b, r)_2, \ldots, (b, r)_n) \\ &\quad \text{with group designations.}\end{aligned}$$

We use subscript i for individuals and subscript j for groups. Boldface is reserved for aggregates.

We associate with each group profile $(b, r)_j$ an aggregated benefit-risk pair $(\mathbf{B}_j, \mathbf{R}_j)$ and a per capita benefit-risk pair $(\mathbf{b}_j, \mathbf{r}_j)$. Aspects of $\mathbf{B}_j$ and $\mathbf{R}_j$ may be obtained by summing over individuals in G_j when benefit and risk dimensions support additive aggregation. Division by N_j would then give corresponding values for the per capita aspects for $\mathbf{b}_j$ and $\mathbf{r}_j$. Other dimensions, including many public features, may have binary yes-no variables that are the same in $(\mathbf{B}_j, \mathbf{R}_j)$ and $(\mathbf{b}_j, \mathbf{r}_j)$. Examples include public goods such as a park or street lights, as well as undesirable factors such as pollution, excessive noise, and plant odors. However aggregates might be defined, it is important for comparison purposes to have a clear understanding of their structure.

Aggregated profiles and *per capita aggregated profiles* are defined from group aggregates in the natural ways:

$$[\mathbf{B}, \mathbf{R}] = ((\mathbf{B}_1, \mathbf{R}_1), (\mathbf{B}_2, \mathbf{R}_2), \ldots, (\mathbf{B}_n, \mathbf{R}_n));$$

$$[\mathbf{b}, \mathbf{r}] = ((\mathbf{b}_1, \mathbf{r}_1), (\mathbf{b}_2, \mathbf{r}_2), \ldots, (\mathbf{b}_n, \mathbf{r}_n)).$$

Two special notations for group aggregates will be needed later. The first accounts for the fact that a number of G_j profiles may have the same aggregated benefit-risk pair. Let F_j denote the function that maps G_j profiles into aggregated benefit-risk pairs for the group:

$$F_j((b, r)_j) = (\mathbf{B}_j, \mathbf{R}_j).$$

Then $F_j^{-1}(\mathbf{B}_j, \mathbf{R}_j)$ is the set of all G_j profiles that have aggregate $(\mathbf{B}_j, \mathbf{R}_j)$. If $[b, r] = ((b, r)_1, \ldots, (b, r)_n)$, its associated aggregated profile is

$$[\mathbf{B}, \mathbf{R}] = (F_1((b, r)_1), \ldots, F_n((b, r)_n)).$$

And the set of all population profiles (with group designations) that have $((\mathbf{B}_1, \mathbf{R}_1), \ldots, (\mathbf{B}_n, \mathbf{R}_n))$ as their aggregated profile is

$$F_1^{-1}(\mathbf{B}_1, \mathbf{R}_1) \times \cdots \times F_n^{-1}(\mathbf{B}_n, \mathbf{R}_n).$$

Our second special notation identifies aggregated benefit-risk pairs for different groups that are equivalent on a per capita (pc) basis. For distinct $j, k \in \{1, \ldots, n\}$, let

$$(\mathbf{B}_j, \mathbf{R}_j) \approx_{\text{pc}} (\mathbf{B}_k, \mathbf{R}_k) \quad \text{mean that} \quad (\mathbf{b}_j, \mathbf{r}_j) = (\mathbf{b}_k, \mathbf{r}_k),$$

where it is understood that $(\mathbf{b}_j, \mathbf{r}_j)$ is the aggregated per capita benefit-risk pair that corresponds to $(\mathbf{B}_j, \mathbf{R}_j)$. We use $\approx_{\text{pc}}$ and F_j^{-1} when we consider how one group would fare if it could allocate the aggregated benefit-risk pair of another group to its own members in place of their original benefit-risk pairs after proper account is taken of different group sizes. If benefits and risks are additive on all dimensions, then

$$(\mathbf{B}_j, \mathbf{R}_j) \approx_{\text{pc}} (\mathbf{B}_k, \mathbf{R}_k) \Leftrightarrow \frac{1}{N_j}(\mathbf{B}_j, \mathbf{R}_j) = \frac{1}{N_k}(\mathbf{B}_k, \mathbf{R}_k).$$

The preceding definitions apply only to profiles without consideration of probabilities. Our notations for the latter are similar to those in FS94. P_0 is the set of all finite-support probability distributions on $B \times R$, and P denotes the set of profile distributions. That is, each $p \in P$ is a finite-support probability distribution on the population profile set $(B \times R)^N$.

Given $p \in P$, p_{ij} denotes the marginal probability distribution in P_0 that is induced by p for individual i in G_j, and p^j denotes the marginal distribution of p on G_j profiles $(b, r)_j$. In addition, $\mathbf{p}$ is the joint probability distribution induced by p on aggregated profiles $[\mathbf{B}, \mathbf{R}]$,

and $\mathbf{p}^j$ is the marginal of p on aggregated benefit-risk pairs $(\mathbf{B}_j, \mathbf{R}_j)$ for G_j.

Individual preferences continue to play a key role in our aggregated perspective. We let $\succsim_{ij}$ denote the preference-or-indifference relation on P_0 of individual i in G_j: $p_{ij} \succsim_{ij} q_{ij}$ signifies that the individual prefers p_{ij} to q_{ij} or is indifferent between the two. The asymmetric (strict preference) and symmetric (indifference) parts of $\succsim_{ij}$ are $\succ_{ij}$ and $\sim_{ij}$ respectively. When a and a' are individual benefit-risk pairs in $B \times R$, $a \succsim_{ij} a'$ means that $p_{ij} \succsim_{ij} q_{ij}$ when $p_{ij}(a) = q_{ij}(a') = 1$. As in FS94, it is assumed that $\succsim_{ij}$ is a weak order that is independent of others' positions in the sense that, for all $p, q \in P$ for which $p_{lk} = q_{lk}$ for all $(l, k) \neq (i, j)$, individual i in G_j likes p as much as q if and only if $p_{ij} \succsim_{ij} q_{ij}$.

3. Allocational Envy and Fairness

Our primary aim in the next few sections is to develop notions of intergroup envy and associated fairness indices that are based on within-group aggregates and allocations of aggregates to group members. We begin with comparisons between profiles $(b, r)_j$ and $(b, r)_k$ of different groups.

Roughly speaking, group G_j allocationally envies the profile $(b, r)_k$ of group G_k to its own profile $(b, r)_j$ if there is a way to allocate the per capita equivalent for G_j of G_k's aggregated benefit-risk pair to the members of G_j so that each member likes his or her allocated position as much as that specified in $(b, r)_j$ and at least one member strictly prefers the allocated position. We denote this type of allocational envy by $e_{jk}[(b, r)_j, (b, r)_k] = 1$, with $e_{jk}[(b, r)_j, (b, r)_k] = 0$ (no envy) otherwise. Formally,

$$
\begin{aligned}
&e_{jk}[(b, r)_j, (b, r)_k] = 1 \\
&\quad \text{if } (\mathbf{B}'_j, \mathbf{R}'_j) \approx_{\text{pc}} (\mathbf{B}_k, \mathbf{R}_k) \text{ and there is} \\
&\quad \text{a } (b', r')_j \in F_j^{-1}(\mathbf{B}'_j, \mathbf{R}'_j) \quad \text{such that} \\
&\quad (b'_{ij}, r'_{ij}) \succsim_{ij} (b_{ij}, r_{ij}) \quad \text{for } i = 1, \ldots, N_j, \text{ and} \\
&\quad (b'_{ij}, r'_{ij}) \succ_{ij} (b_{ij}, r_{ij}) \quad \text{for some } i \in \{1, \ldots, N_j\}; \\
&e_{jk}[(b, r)_j, (b, r)_k] = 0 \quad \text{otherwise.}
\end{aligned}
$$

Two aspects of this definition deserve emphasis. First, it applies only to the two groups in question without direct reference to what may occur in other groups. Second, the definition uses a Pareto criterion, and no consideration is given to strengths of preference that individuals may feel in comparing alternative allocations. For example, an allocation of $(b, r)_k$ to the members of G_j is assumed to be envy-free if $N_j - 1$ people in G_j strongly prefer the allocated position to their original position but one dissenter mildly prefers the original position.

Group j's total envy in population profile $[b, r] = ((b, r)_1, \ldots, (b, r)_n)$ is the number of other groups it envies:

$$e_j[b, r] = \sum_{k \neq j} e_{jk}[(b, r)_j, (b, r)_k].$$

Thus $e_j[b, r] \in \{0, 1, \ldots, n - 1\}$ for each j. A group that is relatively well off may have $e_j = 0$, whereas another group all of whose members could be better off with an allocation of any other group's equivalent benefit-risk pair will have $e_j = n - 1$. It could also happen that $e_j = 0$ for all j. An example with three groups and unidimensional benefits and risks is illustrated by the aggregated per capita benefit-risk pairs

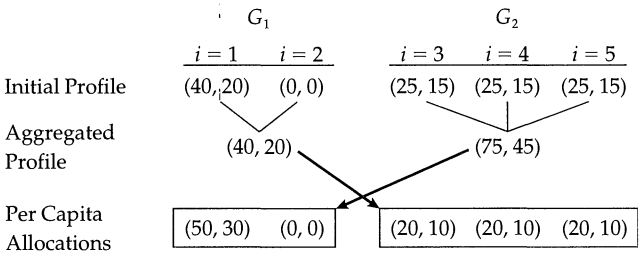
$$(\mathbf{b}_1, \mathbf{r}_1) = (60, 30),$$

$$(\mathbf{b}_2, \mathbf{r}_2) = (50, 25),$$

$$(\mathbf{b}_3, \mathbf{r}_3) = (40, 20).$$

If the members of G_1 are relatively risk prone, the members of G_3 are relatively safety minded, and the members of G_2 prefer a more even balance between benefits and risks, then we could have $e_1 = e_2 = e_3 = 0$.

The same point can be made in more detail to illustrate specific aspects of our approach here and in FS94. Suppose benefits and risks are unidimensional and additive, and there are two different-sized groups: $G_1 = \{1, 2\}$ and $G_2 = \{3, 4, 5\}$. The following array shows an initial profile along with allocations of per capita equivalent benefit-risk pairs:



Assume that each individual has an additive utility function over individual benefit-risk pairs of the form

$$u_i(b, r) = b - k_i r, \quad \text{for } (b, r) \in B \times R,$$

and that the tradeoff-parameter values for the five people are

$$k_1 = 1.5, \quad k_2 = 2.0, \quad k_3 = k_4 = k_5 = 0.5.$$

The aggregated G_2 benefit-risk pair, $(\mathbf{B}_2, \mathbf{R}_2) = (75, 45)$, has per capita equivalent (50, 30) for G_1. The $G_2 \to G_1$ allocation to individuals 1 and 2 in the preceding array reflects the fact that no allocation of (50, 30) makes each individual as well off as initially. That is, $u_1(b, r) \geq u_1(40, 20)$ and $u_2(50 - b, 30 - r) \geq u_2(0, 0)$ cannot hold simultaneously. We make three further observations for this example.

(i) There is no envy *within* either group from the unaggregated perspective for the initial profile. We have $u_1(40, 20) \geq u_1(0, 0)$ and $u_2(0, 0) \geq u_2(40, 20)$ for G_1; there is no envy in G_2 because of equal division.

(ii) There is envy *between* groups from the unaggregated perspective. In particular, $u_i(40, 20) > u_i(25, 15)$ for $i = 3, 4, 5$, so everyone in G_2 envies individual 1.

(iii) There is no allocational envy because it is impossible to make allocations within either group of the per capita equivalent pair from the other group that makes everyone in the allocating group as well off as initially. Hence $e_j[b, r] = 0$ for $j = 1, 2$.

The group envy measure $e_j[b, r]$ can be combined over groups in various ways to define profile fairness indices. As in FS94, we structure indices so that fairness decreases as index value increases. Three indices that are similar to indices used in FS94 for individuals in the unaggregated setting are

$$M_1[b, r] = \sum_j e_j[b, r]/n,$$

$$M_2[b, r] = |\{j : e_j[b, r] > \lambda_n\}|,$$

$$M_3[b, r] = \max_j e_j[b, r].$$

M_1 is the average number of other groups that a group envies, M_2 is the number of groups whose envy count exceeds a threshold value λ_n, and M_3 focuses on the group that envies the most other groups. Depending on circumstances, each index could apply to situations in which an organization wants to maximize intergroup allocational fairness, or minimize unfairness, subject to various feasibility constraints. For example, M_3 may be most relevant when one or two groups are traditionally disadvantaged relative to others, and M_2 or M_1 could apply when the aim is to achieve low envy for as many groups as possible.

The preceding envy measure and fairness indices extend in the natural way by expectation to profile distributions. We have

$$\mu_{jk}(p) = \sum_{(B\times R)^N} p[b, r]e_{jk}[b, r]$$

as the probability that G_j allocationally envies G_k in $p \in P$, and

$$\mu_j(p) = \sum_{k \neq j} \mu_{jk}(p) = \sum_{(B\times R)^N} p[b, r]e_j[b, r]$$

as the expected number of other groups that G_j envies. Expected values of fairness indices are given by $M(p) = \Sigma\, p[b, r]M[b, r]$. Order-of-summation reversal gives

$$M_1(p) = \sum_j \mu_j(p)/n,$$

but reversal does not apply for M_2 and M_3 because their $M[b, r]$ counts precede expectation. Alternatives that take expectations first are

$$M_2'(p) = |\{j : \mu_j(p) > \lambda_n\}| \quad \text{and} \quad M_3'(p) = \max_j \mu_j(p).$$

M_2' is the number of groups whose expected number of other groups envied exceeds the threshold, and M_3' focuses on the group with maximum expectation envy.

The preceding indices are holistic in the sense that they are designed for population profiles and profile distributions without separation into group marginals. We conclude this section by describing a marginal approach in which the basic comparison is between a probability distribution p^j on G_j profiles and a probability distribution q^k on G_k profiles.

We consider first the set $P^j(q^k)$ of marginal distributions on G_j profiles induced by q^k through per capita equivalence and allocation. Given q^k, let $\mathbf{q}^k$ be the associated probability distribution on aggregated benefit-risk pairs $(\mathbf{B}_k, \mathbf{R}_k)$ for G_k. For each such pair that has positive probability under $\mathbf{q}^k$, choose a G_j profile $(b, r)_j$ in $F^{-1}(\mathbf{B}_j, \mathbf{R}_j)$ for $(\mathbf{B}_j, \mathbf{R}_j) \approx_{\text{pc}} (\mathbf{B}_k, \mathbf{R}_k)$, and denote by p^*

the probability distribution on G_j profiles induced by $\mathbf{q}^k$ and these choices. $P^j(q^k)$ is the set of all p^* formed in this manner. The distributions in P_0 for individual i in G_j induced by p^j and p^* are p^j_{ij} and p^*_{ij} respectively. Allocational envy based on marginal distributions can then be defined as follows:

$$e_{jk}(p^j, q^k) = 1$$

if there is a $p^* \in P^j(q^k)$ such that

$p^*_{ij} \gtrsim_{ij} p^j_{ij}$ for $i = 1, \ldots, N_j$, and $p^*_{ij} >_{ij} p^j_{ij}$

for some $i \in \{1, \ldots, N_j\}$;

$$= 0 \quad \text{otherwise.}$$

Group j's total marginal allocational envy for profile distribution p with group marginals $p^1, \ldots, p^n$ is

$$e_j(p) = \sum_{k \neq j} e_{jk}(p^j, p^k).$$

This measure can be used as before to form fairness indices based on averages or other aspects of the $e_j(p)$. An advantage over prior indices is that one requires only the marginal distributions and does not need to know how they fit together to form p because if profile distributions p and q have the same group marginals then they have the same index value.

4. Intergroup Envy for Aggregates

This section and the next section consider intergroup envy based on aggregated profiles $[\mathbf{B}, \mathbf{R}] = ((\mathbf{B}_1, \mathbf{R}_1), \ldots, (\mathbf{B}_n, \mathbf{R}_n))$ and probability distributions on such profiles. There are two main reasons for approaching envy and fairness from this basis rather than the more detailed basis of population profiles and profile distributions. First, it can greatly simplify the data requirements for analysis. Second, it accounts for situations in which group data are available only in aggregated form. This encompasses policy decisions that are concerned primarily with the treatment of different groups. Examples include allocation of funds for public safety to each community in an area, and location of a facility that creates benefits for community members but also imposes occupational or environmental risks. In such cases, it may be up to the groups themselves to decide how aggregated benefit-risk pairs are to be allocated to group members. Our aggregated basis therefore covers two-stage processes in which policy-level decisions for groups are followed by intragroup decisions for individual allocations: see Eliashberg and Winkler (1981). The latter can be analyzed separately for within-group envy and fairness by methods discussed in FS94 and elsewhere, including Kolm (1971), Feldman and Kirman (1974), Varian (1974, 1975), Raiffa (1982), Tadenuma and Thomson (1991) and Brams and Taylor (1995). Depending on circumstances and procedures, groups may be able to devise envy-free allocations for their members, but they may have to settle for considerably less.

The present section describes a modification of the approach in the preceding section for the aggregated profile basis. It also considers situations in which each group has a fixed rule f_j for mapping aggregated benefit-risk pairs into group profiles. Such rules may be based on income levels, health status, or age. Taxation to reduce environmental risks or improve sanitation that depends on income or property values is an example of a fixed allocation rule. The next section looks at simplifications that use only per capita allocations as the basis for envy comparisons.

Our first definition of intergroup envy for aggregated benefit-risk pairs considers all possible ways that G_j can allocate $(\mathbf{B}_j, \mathbf{R}_j)$ to its members and asks, for each way, whether the per capita equivalent of $(\mathbf{B}_k, \mathbf{R}_k)$ could be allocated to G_j in a Pareto superior manner. This definition of intergroup envy is practical only when the number of possible allocations to G_j is small, as might be the case if there is a fixed allocation rule among members of G_j or if the number of possible allocations is constrained. We take

$$e_{jk}[(\mathbf{B}_j, \mathbf{R}_j), (\mathbf{B}_k, \mathbf{R}_k)] = 1$$

if for every $(b, r)_j \in F^{-1}(\mathbf{B}_j, \mathbf{R}_j)$

there is a $(b', r')_j \in F^{-1}(\mathbf{B}'_j, \mathbf{R}'_j)$

with $(\mathbf{B}'_j, \mathbf{R}'_j) \approx_{\text{pc}} (\mathbf{B}_k, \mathbf{R}_k)$ such that

$(b'_{ij}, r'_{ij}) \gtrsim_{ij} (b_{ij}, r_{ij})$ for $i = 1, \ldots, N_j$, and

$(b'_{ij}, r'_{ij}) >_{ij} (b_{ij}, r_{ij})$ for some $i \in \{1, \ldots, N_j\}$;

$$= 0 \quad \text{otherwise.}$$

Group j's total envy in the aggregated profile $[\mathbf{B}, \mathbf{R}]$ is

$$e_j[\mathbf{B}, \mathbf{R}] = \sum_{k \neq j} e_{jk}[(\mathbf{B}_j, \mathbf{R}_j), (\mathbf{B}_k, \mathbf{R}_k)]$$

which, in the manner of the preceding section, can be combined over groups to define fairness indices such as

$$M_1[\mathbf{B}, \mathbf{R}] = \sum_j e_j[\mathbf{B}, \mathbf{R}]/n,$$

$$M_2[\mathbf{B}, \mathbf{R}] = |\{j : e_j[\mathbf{B}, \mathbf{R}] > \lambda_u\}|, \quad \text{and}$$

$$M_3[\mathbf{B}, \mathbf{R}] = \max_j e_j[\mathbf{B}, \mathbf{R}].$$

Expectations for a probability distribution p on aggregated profiles give

$$\mu_{jk}(\mathbf{p}) = \sum_{[B,R]} \mathbf{p}[\mathbf{B}, \mathbf{R}] e_{jk}[(\mathbf{B}_j, \mathbf{R}_j), (\mathbf{B}_k, \mathbf{R}_k)]$$

as the probability that G_j allocationally envies G_k on an aggregated basis, and

$$\mu_j(\mathbf{p}) = \sum_{k \neq j} \mu_{jk}(\mathbf{p}) = \sum_{[\mathbf{B},\mathbf{R}]} \mathbf{p}[\mathbf{B}, \mathbf{R}] e_j[\mathbf{B}, \mathbf{R}]$$

as the expected number of other groups that G_j envies. The latter measure can be used as in the preceding section to define fairness indices, as in M_2' or M_3', or expectations of aggregated profile fairness indices might be used in the manner of $M(\mathbf{p}) = \Sigma\, \mathbf{p}[\mathbf{B}, \mathbf{R}] M[\mathbf{B}, \mathbf{R}]$.

Calculation of envy and fairness in the aggregated setting is conceptually more straightforward when every group has a fixed allocation rule f_j that maps members of $\{(\mathbf{B}_j, \mathbf{R}_j)\}$ into group profiles $f_j(\mathbf{B}_j, \mathbf{R}_j)$ in $F^{-1}(\mathbf{B}_j, \mathbf{R}_j)$. We could then take

$$\begin{aligned} e'_{jk}[(\mathbf{B}_j, \mathbf{R}_j), (\mathbf{B}_k, \mathbf{R}_k)] &= 1 \\ &\text{if, when } (b, r)_j = f_j(\mathbf{B}_j, \mathbf{R}_j), \\ &(\mathbf{B}'_j, \mathbf{R}'_j) \approx_{\text{pc}} (\mathbf{B}_k, \mathbf{R}_k), \text{ and} \\ &(b', r')_j = f_j(\mathbf{B}'_j, \mathbf{R}'_j), (b'_{ij}, r'_{ij}) \gtrsim_{ij} \\ &(b_{ij}, r_{ij}) \text{ for } i = 1, \ldots, N_j, \text{ and} \\ &(b'_{ij}, r'_{ij}) >_{ij} (b_{ij}, r_{ij}) \text{ for some } i \in \{1, \ldots, N_j\}; \\ &= 0 \quad \text{otherwise.} \end{aligned}$$

Expectation and definitions of fairness indices would follow the pattern described above.

Fixed allocation rules for groups also raises the possibility of defining envy proportionally as the fraction of group members who would be better off when another group's per capita equivalent benefit-risk pair is allocated to the initial group. Proportional envy can also be considered within other settings, as in FS94 and other sections of the present paper. We say more about it in the next section for the equal-division approach.

5. Equal-Division Approaches

The definitions of intergroup envy in preceding sections place demands on information and computation that are unrealistic for many situations. We therefore consider simplifications that retain the spirit of our allocational approach but alleviate informational demands. The simplest allocation rule for this purpose is the rule that divides a group's aggregated benefit-risk pair equally among its members. Thus, for envy comparisons, we consider the aggregated per capita benefit-risk pair $(\mathbf{b}_j, \mathbf{r}_j)$ that corresponds to $(\mathbf{B}_j, \mathbf{R}_j)$ as the benefit-risk pair for each person in G_j. We do not propose equal division as the actual rule or procedure that a group ought to use, but suggest it only for fairness assessment. Our approach in this section therefore separates envy and fairness calculations from processes that determine actual group profiles.

We mention three ways to define intergroup envy based on equal division. Given per capita benefit-risk pairs $(\mathbf{b}_j, \mathbf{r}_j)$ and $(\mathbf{b}_k, \mathbf{r}_k)$, let α_{jk} and β_{jk} denote the number of people in G_j that prefer $(\mathbf{b}_k, \mathbf{r}_k)$ to $(\mathbf{b}_j, \mathbf{r}_j)$ and that prefer $(\mathbf{b}_j, \mathbf{r}_j)$ to $(\mathbf{b}_k, \mathbf{r}_k)$ respectively, so $\alpha_{jk} + \beta_{jk} \leq N_j$:

$$\begin{aligned} &\alpha_{jk}[(\mathbf{b}_j, \mathbf{r}_j), (\mathbf{b}_k, \mathbf{r}_k)] \\ &\qquad = |\{i \in \{1, \ldots, N_j\} : (\mathbf{b}_k, \mathbf{r}_k) >_{ij} (\mathbf{b}_j, \mathbf{r}_j)\}|, \\ &\beta_{jk}[(\mathbf{b}_j, \mathbf{r}_j), (\mathbf{b}_k, \mathbf{r}_k)] \\ &\qquad = |\{i \in \{1, \ldots, N_j\} : (\mathbf{b}_j, \mathbf{r}_j) >_{ij} (\mathbf{b}_k, \mathbf{r}_k)\}|. \end{aligned}$$

The three envy definitions are

$$\begin{aligned} e^{(1)}_{jk}[(\mathbf{b}_j, \mathbf{r}_j), (\mathbf{b}_k, \mathbf{r}_k)] &= 1 \quad \text{if } \alpha_{jk} > 0 \text{ and } \beta_{jk} = 0, \\ &= 0 \quad \text{otherwise;} \\ e^{(2)}_{jk}[(\mathbf{b}_j, \mathbf{r}_j), (\mathbf{b}_k, \mathbf{r}_k)] &= 1 \quad \text{if } \alpha_{jk} > N_j/2, \\ &= 0 \quad \text{otherwise.} \\ e^{(3)}_{jk}[(\mathbf{b}_j, \mathbf{r}_j), (\mathbf{b}_k, \mathbf{r}_k)] &= \alpha_{jk}/N_j. \end{aligned}$$

Table 1 Computation of Pairwise and Total Envy for Each Community If the Plant is Located at Arcadia

		Envy of the Community when Plant is Located at Arcadia					
		Arcadia	Bellflower	Cerritos	Downey	Encino	Total Envy
Envy by the Community	Arcadia	n.a.	(0, 1, 0.7)	(0, 1, 0.7)	(0, 1, 0.7)	(0, 1, 0.7)	(0, 4, 0.7)
	Bellflower	(0, 1, 0.6)	n.a.	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0, 1, 0.15)
	Cerritos	(0, 0, 0.18)	(0, 0, 0)	n.a.	(0, 0, 0)	(0, 0, 0)	(0, 0, 0.045)
	Downey	(0, 1, 0.55)	(0, 0, 0)	(0, 0, 0)	n.a.	(0, 0, 0)	(0, 1, 0.1375)
	Encino	(0, 0, 0.12)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	n.a.	(0, 0, 0.03)

Measure $e_{jk}^{(1)}$ is similar to preceding measures. Its demands for envy are strongly relaxed by $e_{jk}^{(2)}$, which says that G_j envies G_k if a majority of G_j's members prefer the other group's per capita pair to its own pair. This may be more suitable than $e^{(1)}$ because per capita allocations can differ significantly from actual group profiles. Measure $e_{jk}^{(3)}$ assesses G_j's envy of G_k as the proportion of its members who believe they would be better off with the other group's per capita benefit-risk pair. It is clearly more refined than the others and could be more suitable when the extent of envy in a proportional sense is an issue.

Our three measures may differ in their ease of assessment. If one group's per capita pair dominates another's by giving greater benefits at lower risks, the picture is clear for all three. When dominance does not obtain, $e^{(2)}$ and $e^{(3)}$ could be estimated by sampling, especially for large groups. In general, smaller sample sizes would be needed for majority evaluation than for estimating α_{jk}/N_j accurately.

Each intergroup envy measure can be used in ways indicated earlier to define fairness indices. Because the techniques involved are straightforward, we omit details.

6. Example

We illustrate the approach of the preceding section by considering the location of a plant for processing low-level hazardous waste. We begin by assuming a fixed compensation benefit, and then discuss the determination of fair compensation. A modification near the end of the section brings benefit uncertainty into play.

Suppose the waste processing plant is to be built in one of five communities: Arcadia, Bellflower, Cerritos, Downey, or Encino. Each community wants the plant somewhere else when no compensation is involved, so this decision is an instance of the classic NIMBY (not in my backyard) phenomenon that is prevalent in environmental problems.

We suppose that the community that ends up with the plant receives a per capita benefit b (such as a subsidy or tax relief) for the risks involved. We make two simplifying assumptions: (i) the community in which the plant is built receives all benefits and bears all risks; (ii) benefits and risks are distributed equally among community members. A community's outcome of all benefits and risks will be denoted by ALL, and of no benefits and risks by NONE.

Our approach depends crucially on individuals' benefit-risk tradeoffs in the communities. Given b, let α_j denote the proportion of people in community j who prefer ALL to NONE. Sample values for these proportions, expressed as percents, are as follows:

Arcadia	Bellflower	Cerritos	Downey	Encino
30%	60%	18%	55%	12%

For example, only 18% of the people in Cerritos prefer ALL to NONE, whereas a majority in Bellflower would rather see the plant built in their community than elsewhere. We assume for simplicity that the other 82% of Cerritosians prefer NONE to ALL, but indifference could be incorporated in the analysis. A forced-choice voting or sampling procedure with yes/no responses could be used to assess α_j for each community.

The three envy definitions in §5 yield the ($e^{(1)}$, $e^{(2)}$, $e^{(3)}$) envy triples in Table 1 when the preceding α_j apply

Table 2 Computation of Average Envy for Each Community

		Plant Located at				
		Arcadia	Bellflower	Cerritos	Downey	Encino
Total Envy by the Community	A	(0, 4, 0.7)	(0, 0, 0.075)	(0, 0, 0.075)	(0, 0, 0.075)	(0, 0, 0.075)
	B	(0, 1, 0.15)	(0, 0, 0.4)	(0, 1, 0.15)	(0, 1, 0.15)	(0, 1, 0.15)
	C	(0, 0, 0.045)	(0, 0, 0.045)	(0, 4, 0.82)	(0, 0, 0.045)	(0, 0, 0.045)
	D	(0, 1, 0.1375)	(0, 1, 0.1375)	(0, 1, 0.1375)	(0, 0, 0.45)	(0, 1, 0.1375)
	E	(0, 0, 0.03)	(0, 0, 0.03)	(0, 0, 0.03)	(0, 0, 0.03)	(0, 4, 0.88)
Average Envy		(0, 6/5, 0.2125)	(0, 1/5, 0.1375)	(0, 6/5, 0.2425)	(0, 1/5, 0.15)	(0, 6/5, 0.2575)

and the plant is to be built in Arcadia. For example, (0, 1, .7) in the first row and second column of Table 1 indicates that if the plant is located in Arcadia then:

someone in Arcadia does not envy Bellflower [$e^{(1)} = 0$],
a majority in Arcadia envies Bellflower [$e^{(2)} = 1$],
70% of the Arcadians envy Bellflower [$e^{(3)} = .7$].

Total envy is noted in the last column of the table. The third entry in the Total Envy column is computed as an average and interpreted as the average proportion envied.

Table 2 shows the total envy of each community for the three envy definitions and for each of the five possible plant locations. The first column in Table 2 is simply the last column of Table 1, since that table calculated total envy when the plant is located at Arcadia. The other four columns are constructed similarly. Average envy is noted in the last row of Table 2. Using $e^{(1)}$, no discrimination can be made among the sites on the fairness criterion. This is because $e^{(1)}$ is 0 unless *everyone* in a group prefers the other group's allocation. Although inappropriate in the present case, this measure may be relevant when everyone along a route must agree to build a highway or a canal. Definition $e^{(2)}$ requires only that a majority prefer ALL to NONE. In our example, Bellflower and Downey are tied as the preferred sites on this fairness criterion. Finally, $e^{(3)}$ yields the proportion of people in a community who believe they would be better off by accepting the plant in their community. Bellflower invokes the least envy using this criterion. We note that the relative ranking of communities by our fairness indices does not change when sites are added or deleted.

The preceding scenario assumes that benefits provided to a community for accepting the hazardous waste plant have been determined exogenously. A simple yes/no voting scheme then provides sufficient information to compute measures of fairness. We now consider a somewhat more complex case in which a public agency wishes to *determine* the appropriate level of benefits that may induce one or more communities to accept the plant while the process is viewed as fair by all communities involved.

To illustrate this, consider three communities where the hazardous waste plant could be located: Arcadia, Bellflower, and Downey. We now need to know how much compensation individuals in each community would accept (WTA) to assume the risks associated with the plant. A contingent valuation (CV) technique may be used for elicting WTA values from individuals through the use of carefully designed and administered sample surveys: see, e.g., Mitchell and Carson (1989). The CV method has been used for about twenty years for environmental risk valuation even though it has been criticized for several reasons. In our illustration, we assume that either the CV method or some other elicitation/survey procedure is used to provide WTA values as shown in Table 3.

If $e^{(1)}$ is deemed appropriate, then the public agency will minimize envy by setting per capita benefits at $24 and locating the plant at Downey. We assume for convenience that at $24 per capita benefits, everyone in Downey prefers the plant. This solution will be totally fair and invoke no envy from any of the three communities involved. By $e^{(2)}$, however, per capita benefits will

Table 3 Per Capita Compensation Needed to Accept the Plant

	(Willingness to Accept, WTA)		
Cumulative Proportion	Arcadia	Bellflower	Downey
<1%	$4	$5	$4
25%	$11	$9	$8
50%	$19	$14	$17
75%	$27	$23	$19
>99%	$35	$30	$24

be set at $14 and a totally fair solution obtained by locating the plant at Bellflower, since 50% of Bellflowerites will accept $14, but fewer than 50% of the other communities' members would accept $14. Average envy for each community by $e^{(3)}$ is shown in Table 4. The first row of Table 4 gives the proportion of people in a community who are willing to accept the plant if per capita benefits are set at $20. These numbers can be obtained through interpolation from Table 3 or by direct elicitation from individuals. The second row of Table 4 shows average envy levels for each community at the $20 per capita benefit level. Notice that Downey is the preferred choice based on the proportional envy criterion if benefit levels are set at $20 per capita. The third and fourth rows show similar computations for a $15 benefit level. At this reduced level, Bellflower becomes the preferred choice based on $e^{(3)}$. Similar computations can be made for other levels of benefits. It is interesting to note that the envy level of a community, say Arcadia, will be high if benefits are too low and the plant is located in Arcadia. The envy level for Arcadia will also be high if benefits are high and the plant is located elsewhere. Such a conclusion is consistent with our intuition that a community will complain of getting a bad deal if benefits are low and the plant is located there, or of being robbed of a good deal if benefits are high and the plant is located elsewhere.

Finally, we consider the case in which benefits are uncertain, as might be true for location of a chemical plant, a power plant, or an airport. Assume for simplicity that per capita benefits are $20 or $15 with a 50-50 chance of each and that the decision is to locate the facility in one of the three communities used in Table 4.

Envy measure $e^{(1)}$ requires consensus, and therefore all three locations are equally fair or unfair: the expected envy index is 0 for all three communities. Under measure $e^{(2)}$, locating the plant at Bellflower will result in the lowest expected envy. This is because when per capita benefits are $20, it can be inferred from the WTA values of Table 3 that both Arcadia and Downey will envy Bellflower. However, when benefits equal $15, neither will envy Bellflower. The expected envy values for locating the plant at Arcadia, Bellflower, and Downey are $\frac{5}{6}$, $\frac{1}{3}$, and $\frac{5}{6}$ respectively: see Table 5. Measure $e^{(3)}$ yields expected envy values of 0.755 for Arcadia, 0.63 for Bellflower, and 0.605 for Downey. Thus Downey would be the preferred location under the proportional envy measure.

The above illustrations, though simplified, show that the measures of fairness proposed in this paper can be obtained using well-established procedures such as voting or the CV method. An important area for further applied research is to design decision processes that result in fair solutions for classes of decision problems that involve risks to human health and environment. The foundational principles that we suggest in this paper are robust with respect to a variety of decision situations. These principles, however, do not point to only one approach for application to a specific decision situation. Alternative schemes, such as negotiation, arbitration, auction, siting jury, a quota system, or a market mechanism, may be required to achieve fair benefit-risk allocations in certain situations.

7. Discussion

The study of fairness presented in this paper and FS94 grew out of earlier work on equity and public risk reported in Keeney (1980), Keeney and Winkler (1985), Keller and Sarin (1988), Fishburn and Straffin (1989), and Fishburn and Sarin (1991) among others. Equity notions in those papers focused on balanced treatment of individuals or groups based primarily on ethical criteria. By contrast, fairness is based primarily on preferences of individuals (or

Table 4 Average Envy Computations Using Proportional Criterion

	Arcadia	Bellflower	Downey
Benefits = \$20	0.55	0.65	0.80
Average Envy	$\frac{2(0.45) + 0.65 + 0.8}{3}$ (0.78)	$\frac{2(0.35) + 0.55 + 0.8}{3}$ (0.68)	$\frac{2(0.2) + 0.55 + 0.65}{3}$ (0.53)
Benefits = \$15	0.40	0.55	0.45
Average Envy	$\frac{2(0.6) + 0.55 + 0.45}{3}$ (0.73)	$\frac{2(0.45) + 0.40 + 0.45}{3}$ (0.58)	$\frac{2(0.55) + 0.40 + 0.55}{3}$ (0.68)

groups) and how they perceive themselves in relation to others. Profiles or distributions of risks, or of benefits and risks, that have little envy and are therefore fairest may also be relatively equitable, but there is no necessary correlation between the two concepts.

The first paper of our fairness study, FS94, looked at notions of envy and fairness among individuals or groups when benefits and risks are not aggregated within groups or across the population. The present paper has focused on comparisons between groups that explicitly account for aggregated benefits and risks, thus providing a picture of the treatments of different groups along various dimensions of benefits and risks. Its key to measures of envy is ways that aggregates for one group can be allocated to another group after adjustments are made for different group sizes. An array of fairness indices was developed for profiles and profile distributions on the basis of envy measures. Our indices emphasize different aspects of intergroup envy that may be appropriate for different types of situations and policy decisions.

A policy that is totally fair with respect to each group and each individual in the population may not be attainable in practical settings. An enlightened decision maker may nevertheless be interested in preserving as much fairness as possible in the design and evaluation of alternative policies. Our work provides some foundational principles and measures of fairness that can be used to shed light on the degree of fairness implied by a policy.

Table 5 Expected Envy Index for $e^{(2)}$ and $e^{(3)}$

		Plant Located at		
		Arcadia	Bellflower	Downey
$e^{(2)}$	Benefits = \$20	2/3	2/3	2/3
	Benefits = \$15	3/3	0/3	3/3
	Expected Envy	5/6	1/3	5/6
$e^{(3)}$	Benefits = \$20	0.78	0.68	0.53
	Benefits = \$15	0.73	0.58	0.68
	Expected Envy	0.755	0.63	0.605

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Accepted by Robert T. Clemen; received August 3, 1994. *This paper has been with the authors* $4\frac{1}{2}$ *months for* 2 *revisions.*