FLOW MANAGEMENT TO OPTIMIZE RETAIL PROFITS AT THEME PARKS

KUMAR RAJARAM and REZA AHMADI

Decisions, Operations and Technology Management, The Anderson School at UCLA, 110 Westwood Plaza, Los Angeles, California 90095-1481
kumar.rajaram@anderson.ucla.edu • reza.ahmadi@anderson.ucla.edu

In many theme parks, stores are located within major attractions to sell related merchandise. Sales at such stores form a significant portion of theme park profits. Typically, store sales depend upon visitor flows through the attraction, customer satisfaction with the attraction, and the merchandise at the store. In addition, such stores constitute a unique retail environment, as visitor flows to attractions can be managed and stores are not competitors, but belong to the same parent company. This provides the opportunity to increase store sales by interfacing park operations, which manages visitor flows by setting schedules and capacity of attractions, with the store-level merchandising process, which determines which and how much product to order.

Motivated by a study at Universal Studios Hollywood (USH), we develop a flow management model to link park operations with store-level merchandising. This model sets the capacities and schedules of the major attractions to increase visitor flows to high-profit retail areas subject to visitor satisfaction, capacity, scheduling, and flow-balance constraints. In addition, this model serves as an important tool to generate and evaluate various strategies aimed at increasing theme park profitability at USH.

Received September 2000; revision received October 2001; accepted June 2002.
Subject classifications: Industries, recreation: theme parks. Schedules, applications: visitor flow management. Programming, integer: heuristic and relaxation.
Area of review: OR Practice.

1. INTRODUCTION

The service sector represents a major portion of the American economy. It is estimated that the entertainment industry, including segments such as movies, television, and theme parks represents about 7.2% of this sector (U.S. Census Bureau 1998). Theme parks are an important component within the entertainment industry. There are 450 theme parks around the country, which generated $9.1 billion in revenues in 1999 and represent around 4.6% of the entertainment industry (International Association of Amusement Parks and Attractions 1997).

Customers at theme parks typically pay a fixed fee for access to all attractions any number of times during a fixed time period (ranging from one day to one year). Thus, profit management in this environment has focused on increasing visitors to parks by marketing and improving customer satisfaction during visits through design, quality, and accessibility of attractions to ensure repeat visits (Wells 1989, Valenti 1997). As a direct consequence, a record number of more than 309 million people visited theme parks in 1999 (*USA Today*, April 28; 2000). Stores featuring merchandise related to an attraction (usually located within the vicinity of the attraction) form an important part of the profits generated for the theme park in addition to the entrance fees. Store profits have been steadily increasing and comprise over 40%of total theme park profits (estimated from *Amusement Industry Abstract* 1997).

Traditionally, park operations are responsible for choosing capacities and schedules of attractions to minimize disruptions, for managing visitor flows to achieve reasonable wait times, and for maximizing customer satisfaction. On the other hand, the merchandising function at each store is responsible for designing, selecting, and determining the order quantities of the merchandise associated with the attraction to maximize store profits. Typically, product demand at these stores is related to the visitor flow through the associated attraction, the popularity of the attraction, and the nature of the assortment at the store. This, coupled with a unique retail environment at a theme park in that visitor flows can be managed and individual stores belong to the same company and are not competitors, offers the potential for increasing store profits by interfacing park operations and the merchandising process at individual stores.

To better understand this interface, the second author conducted a field study at Universal Studios Hollywood (USH). In this study, he collected data by conducting a comprehensive survey and used regression analysis to explore the data. The main findings of the regression analysis were:

(1) Visitor flow patterns over time affect total store profits.

(2) Schedules and capacity of attractions can influence visitor flow patterns over time.

(3) In addition to the capacity, schedule, and visitor satisfaction with an attraction, profits at the associated store are strongly influenced by the merchandising process that determines which and how much product to order.

0030-364X/03/5102-0175 $05.00
1526-5463 electronic ISSN

Operations Research © 2003 INFORMS
Vol. 51, No. 2, March–April 2003, pp. 175–184

Details on the regression analysis that led to the above findings are described in the subsequent sections of the paper. Guided by these findings, we constructed a flow management model to explicitly link park operations and store-level merchandising. In this model, we set the capacities and the schedule of attractions to manage park flows to maximize overall store profits. This is subject to flow balance, capacity, scheduling, work-force resource availability, and customer satisfaction constraints measured by the minimum number of rides a customer could take per visit across all attractions at the park.

The problem of coordination between park operations and store-level merchandising faced at theme parks can be considered to be a part of a broader stream of research that focuses on understanding the benefits of coordination between the operations and the marketing functions of a firm. Karmarkar (1996) provides a comprehensive classification and summary of research in this area. While this link has been extensively explored in manufacturing (Eliashberg and Steinberg 1993), we have found no equivalent investigation into the service industry in general and particularly into the theme park industry with its unique characteristics.

The problems of theme park operations management and retail profit management have been separately studied in different contexts. Ahmadi (1997) considers the theme park flow management problem at the Six Flags Magic Mountain theme park in Southern California. In that paper, models are developed for the estimation of ride capacity and ideal customer transition patterns, and the results of these models are used as inputs to optimal capacity management and tour design models. However, in this paper, store profits are not considered and flows are managed to optimize a measure of customer satisfaction rather than overall theme park profits. There have been several streams of research that have demonstrated the potential for improving retail profits using a structured model-based approach (Rajaram 1998). However, none of this work considers this problem in the unique retail environment of the theme park.

This paper is organized as follows. In the next section, we briefly describe USH, the theme park where we conducted the field study. We then briefly describe the data collection process and present a preliminary data analysis. In §3, we provide details on the regression analysis conducted on this data and use the results to motivate the flow management linking park operations with store-level merchandising. Section 4 describes the model for park flow management. We develop an easily implementable heuristic to solve this problem and establish a tight upper bound on the optimal solution to this problem to evaluate the performance of the heuristic. We test the heuristic and the upper bound by performing computational experiments on real data gathered from the survey and also use the heuristic to conduct a numerical analysis to develop structural insights into this problem that will be useful for its practical implementation. In the concluding section, we describe the main results and their implications when the flow management model is applied at USH.

2. FIELD STUDY

USH is the largest film and television studio in the world and is located in a 400-acre facility in Southern California. Within this studio, USH operates a theme park, which opened in 1964. The unique feature of this theme park is that its attractions are associated with popular motion pictures such as *Jurassic Park, ET*, and *Back to the Future*. The basic concept behind these attractions is to enable visitors to personally experience a part of these movies. In addition, within or near these attractions are stores that sell associated merchandise and contribute significantly to park profits. There are a wide variety of stores ranging from Hollywood-style clothes to unique products. For example, *Universal Ranch* consists of western apparel and hard goods while *Take Two* sells signature Universal Studios souvenirs.

The park is divided into two main sections: the Entertainment Center (at the top of the hill) and the Studio Center (at the bottom of the hill). The Starway, the world's largest escalator, connects these two distinct sections. The Entertainment Center has mostly show-type attractions where the visitors are part of the audience, while the Studio Center consists mostly of ride-type or more interactive attractions. This layout of the park has implications for the visitor flow patterns within the park. For instance, it is usually recommended by studio guides that visitors finish everything they want to see at the bottom of the Starway before going up to the top, because traveling up the Starway takes about 10 to 15 minutes and time could be wasted going back and forth. In addition, visitors can take a backlot tram tour to understand how special effects in movies are created, although capacity and schedules at the other attractions significantly affect the wait time at this attraction.

While there are more than 20 attractions at the park, it has been observed that some attractions consistently have a larger number of visitors and greater occupancy levels. Such attractions are classified as major attractions. For example, *Jurassic Park*, *ET*, and *Back to the Future* are all considered major attractions. The capacity and schedules of major attractions affect visitor flows in the park. In addition, waiting lines for such attractions are much longer in the middle of the day, although they usually shorten dramatically before the park closes. It has also been noted that products related to the major attractions are very popular, thus influencing merchandise selection and sales at the associated stores.

In addition to the attractions, the USH theme park has about 15 restaurants and more than 20 merchandising establishments associated with these attractions. The park is usually open from 9 a.m. until 7 p.m., with peak hours usually occurring between 2 p.m. and 3 p.m. On average, around 16,000 visitors visit USH each day, and the number of visitors in the park during peak hours can be as high as 14,000. During holidays such as Thanksgiving and Christmas there are usually over 40,000 visitors to this park.

The total annual profits at USH are over $50 million, and the proportion of profits from stores has been increasing over the years at a greater rate than profits from ticket receipts. Realizing the growing importance of this component in total profits, USH wanted to explore strategies to further increase these profits by better managing visitor flows to the stores, which led to our involvement.

2.1. Data Collection and Analysis

The first step of our study was to collect data to analyze how visitor flow patterns over time affect total store profits, how schedules and capacity of attractions can influence visitor flow patterns over time, and finally, how the merchandising process affects store profits. To collect the required data, we chose seven different weeks during the period from March 1, 1999 through September 6, 1999. This period was chosen because it covered the spring break holiday (during March) and the peak holiday season between Memorial Day and Labor Day (from May through September) in the United States. We chose one week from each month and collected data on each day of this week to account for changes in intraday flow patterns due to the volume of visitors arriving in the park.

On each survey day, we distributed a survey termed as the "diary survey," which records the movements of several visitors throughout the day. Details on how we developed, tested, refined, and implemented this survey can be found in Ahmadi and Rajaram (2000). The objectives of the diary surveys were to collect data to perform regression analysis and to estimate parameters for the flow management model. To meet these objectives, we requested the following information in the diary: name of attraction, time in (this is the time they entered the queue for the attraction), time out (time they exited the attraction), customer satisfaction rating for each attraction (on a scale of one to seven, with seven being the highest), and dollars they spent at each store associated with the attraction. We conducted this survey on each day of seven different weeks over a seven-month period from March through September 1999. Finally, on the survey days, we collected data on store profits over time across all the stores associated with the major attractions and also recorded the schedule and capacities at each of these attractions.

We first wanted to verify that the data from the 49 survey days chosen for our analysis were representative of typical traffic arrival patterns at different times at this park. To do so, we calculated the average arrivals per hour for each day of the week, using the survey data over the seven-week period. We also calculated the average arrival per hour for each day of the week, using two years of historical data recorded by USH. We then calculated the correlation between the average arrival per hour from the survey data and the historical data, and found this to be at least 90% for each day of the week. While the total average number of visitors between the days in each comparison varied, their distribution in arrival to the park seemed stable. Because much of the analysis in this section uses visitor arrival patterns per hour, it seems reasonable to generalize the results to the other days.

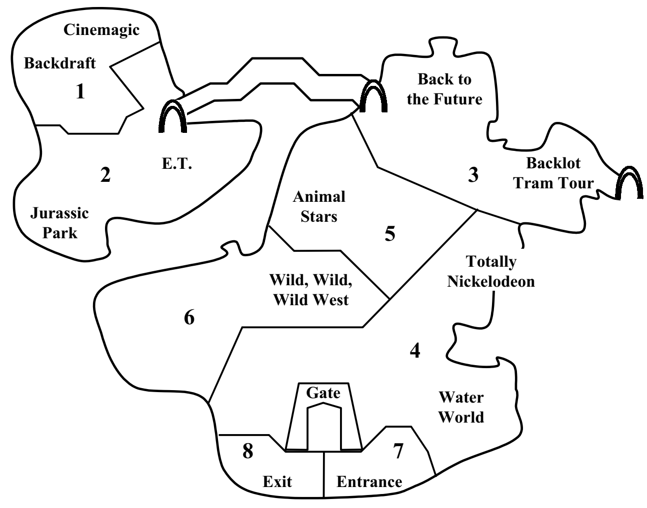
To assess whether the spending patterns of customers in the survey were representative of the larger set of customers who visit the theme park, we computed the fraction of sales at each store for all the customers who participated in the survey. We then compared this to the fraction of sales at each store using total aggregate sales and found it to be very similar. We also computed the average sales per customer in the survey and found that it was almost identical to the average sales per customer calculated using historical data. These results suggest that the spending habits of customers in the survey were representative of the larger sample of customers who visit USH.

Next, to analyze visitor flow patterns, we segmented the park into eight sections consisting of six areas, the entrance, and the exit, as shown in Figure 1. The six areas are largely characterized by their major attractions. The criteria we used in segmenting the areas were to segregate the major thoroughfares in separate areas and to balance the number of major attractions among the areas. Based on the diary survey, we computed the proportion of visitors over time in a given area and also computed the proportion of visitors over the six areas in a given time. Thus, given the total number of visitors in the park, we can use these percentage estimates to calculate visitors in an area at a given time. These values are important in several of the regression analyses performed in the next section. Using the survey, we also computed the fraction of visitors who move from the entrance to a particular area at a given time and the fraction of visitors who move from one area to another at a given time. These fractions serve as important inputs to the flow management model.

3. FINDINGS

In this section, we provide details on the regression analysis of the data and synthesize our findings. These findings provide insights that led to the optimization model described in this paper. We start by analyzing the relationship between store profits and visitor flows.

Figure 1. Layout of Universal Studios Hollywood.



3.1. Profits and Visitor Flows

To analyze the relationship between total store profits and visitor flows in the park during that period, we developed a multiple linear regression. In this regression, the dependent variable was defined as total store profits at a given time, while the independent variables were defined as the estimated number of visitors at each area at different times. The adjusted R^2 for this regression was over 96%, and significant at an α risk level of 0.03, where the α risk level is the probability that we conclude the regression is significant when it is not. In addition, we have found that the t-statistic associated with each variable in this regression was significant with an individual α level of 0.10 or less. This analysis suggests that total store profits are strongly affected by visitor flows in the park across time. From this regression, we also concluded that total store profits are negatively correlated with the estimated number of visitors in areas 2 and 3 (i.e., at the bottom section and backlot tour), and that they are positively correlated with the number of visitors in areas 1, 4, 5, and 6 (all other areas). This is because stores in areas 2 and 3 contribute a smaller share to total store profits than stores in all the other areas.

3.2. Visitor Flows, Capacity, and Scheduling of Attractions

To analyze how the capacity and schedules at the associated main attraction affect visitor flows at a particular area, we constructed a multiple linear regression at the main attractions in each area. In this regression, the dependent variable is defined as the number of visitors at each attraction at a given time, and the independent variables are defined as the capacity of the attraction at that time and the schedule on a given day (with 1 representing a scheduled start during that hour and 0 otherwise).

It is important to note that in this regression, visitor flows themselves could influence capacity. The problem in which an independent variable could be influenced by a dependent variable in regression analysis is commonly referred to as the identification problem. To address the identification problem in our regression, we used two standard methods, namely, the indirect least squares and the two-stage least squares method (Rao and Miller 1971). In the indirect least squares method, we express capacity as a function of the schedules and the workforce availability and substitute this function into the original regression with visitor flows. In the two-stage least squares method, we first develop a regression for capacity as a function of visitor flows and workforce levels, and then use the estimated value for capacity from this regression in the original regression relating visitor flows with capacity and schedules. Table 1 lists the adjusted R^2 along with the α level for the regression and the independent variables representing the capacity and schedule for the major attractions in each area. Here, the adjusted R^2 is based on the lower value from these methods. This analysis implies that visitor flows at a particular area are indeed strongly affected by the capacity and schedules of the major attractions in that area.

To test the goodness of fit of these regressions, in each area we used half the data to fit the regression and calculated the difference between predicted and actual visitor flows for the other half of the data. This calculation showed that the average absolute error between predicted and actual visitor flows across all areas was 9.6% of the actual visitor flows. The results in this section, coupled with the results of §3.1, provide a logical basis to develop the flow management model using the capacity and schedules of the major attractions as decision variables in maximizing store profits in the theme park.

3.3. Store Profits, Visitor Satisfaction, Capacity, and Scheduling of Attractions

To assess how individual store profits are affected by visitors' satisfaction with the associated attraction and its capacity and schedule (or, implicitly, visitor flows using the results of §3.2), we considered the store associated with the leading attraction for a given area and developed a multiple linear regression. In this regression, the dependent variable is defined as the total profit at the store during a given time, and the independent variables include the visitor satisfaction ratings and the capacity level at that time for the associated attraction, as well as an indicator variable for schedules at the attraction (1 if it starts during that

Table 1. Regression results for analysis relating visitor flows to schedules and capacity at major attractions.

Major Attraction (Associated Area)	Adjusted R^2	α Risk Level for Regression	α Risk Level for Schedule	α Risk Level for Capacity
Backdraft (1)	0.9	0.001	0.003	0.004
Cinemagic (1)	0.8	0.005	0.007	0.008
Jurassic Park (2)	0.95	0.0008	0.001	0.0009
ET (2)	0.94	0.0008	0.0009	0.001
Back to the Future (3)	0.97	0.0003	0.0004	0.003
Backlot Tram Tour (3)	0.9	0.0009	0.001	0.0011
Water World (4)	0.85	0.007	0.009	0.008
Nickelodeon (4)	0.8	0.0048	0.006	0.0055
Animal Stars (5)	0.75	0.01	0.011	0.012
Wild, Wild, Wild West (6)	0.88	0.001	0.0014	0.0013

Table 2. Regression results for analysis relating store profits to visitor satisfaction ratings, capacity and schedules of major attractions.

Store Names	Associated Major Attraction and (Area)	Adjusted R^2 for Regression	α Risk Level for Regression	α Risk Level for Visitor Satisfaction	α Risk Level for Capacity	α Risk Level for Schedules
Backdraft Souvenirs	Backdraft (1)	0.55	0.15	0.16	0.17	0.18
ET's Toy Closet	ET (2)	0.6	0.1	0.11	0.15	0.13
Jurassic Outfitters	Jurassic Park (2)	0.65	0.07	0.08	0.07	0.08
Backlot Tram Tour	Backlot Tram Tour (3)	0.7	0.06	0.08	0.06	0.07
Tram Central	Backlot Tram Tour (3)	0.58	0.12	0.12	0.13	0.12
Tram Kiosk	Backlot Tram Tour (3)	0.6	0.012	0.015	0.013	0.014
Time Travelers	Back to the Future (3)	0.62	0.09	0.01	0.01	0.009
Nickelodeon Too	Nickelodeon (4)	0.7	0.05	0.06	0.05	0.07
Animal Stars Store	Animal Stars (5)	0.67	0.06	0.08	0.08	0.07
Universal Ranch Store	Wild Wild Wild West (6)	0.65	0.06	0.07	0.08	0.06
Silver Screen	Wild Wild Wild West (6)	0.7	0.04	0.05	0.06	0.07
Take Two	Wild Wild Wild West (6)	0.72	0.03	0.06	0.05	0.04

hour and 0 otherwise). We developed this regression for 12 stores, each associated with a major attraction, covering all six areas. These results are summarized in Table 2. Here it is important to recognize that some but not all of the expected profits at a store are due to visitor satisfaction with an attraction and to visitor flows to the store. The residuals of the regression are the "merchandising effect," which indicates the degree to which the profits of the stores are attributable to the choice of assortment on the shelf.

To better understand the merchandising effect, we define the conversion efficiency ratio for a store as the ratio of the number of transactions to the number of visitors entering the store in a given time period. This provides a measure of the effectiveness of merchandise planning at a given store. We computed the conversion efficiency ratios at the store associated with a main attraction across all areas. We found that stores with low conversion efficiency ratios corresponded with stores with low R^2 in Table 2. This suggests that the merchandising effect is important in these areas and that lowered conversion ratios could be attributable to poor merchandise planning. On the other hand, stores with high ratios have a high R^2, suggesting that once the merchandise is well positioned, profits are determined largely by visitor flows.

3.4. Synthesis of Key Findings

The main findings of our analysis are as follows. In §3.1, we find that at any given time, total profits are affected by visitor flows in different areas. For instance, total profits are negatively correlated with the number of people in the backlot tour and bottom section. This is intuitive because there are not many stores in these areas. Second, profits are positively correlated with visitors in all other areas, because this is where the main stores are located. Given this, it may be desirable to manage visitor flows to favor areas with potentially high retail profit. To achieve this, natural variables would be to manipulate the schedules and capacity of the main attraction. The results in §3.2 imply that these parameters affect visitor flows. This provides a direct motivation to develop a flow management model to maximize total profits at the theme park by optimally setting capacity and scheduling across all of the major attractions.

In §3.3, when we perform a multiple regression relating individual store profits to the visitor satisfaction rating and to the flow management variables (i.e., capacity and schedules of the attraction), we find that there is a notable residual in all cases. This residual is representative of the effect of the merchandise and its stock levels at these stores. To measure the effectiveness of merchandise planning, we developed the conversion efficiency ratio for a store, which is defined as the ratio of the number of transactions to the number of visitors entering the store during a given time period. This measure is low for stores with high residuals, suggesting the potential to improve merchandising at these stores. In addition, this measure can be used to identify stores with merchandise selection and stocking problems. Conversely, when this measure is high, we find that residuals in the regression are low, suggesting that once merchandise planning is well executed, manipulating flows could further increase profits. This provides additional motivation for the flow management model developed in the next section.

4. FLOW MANAGEMENT MODEL

Motivated by the findings of the field study, which suggest that total store profits are affected by visitor flow patterns and that visitor flow patterns to an attraction can be managed by setting its capacity and schedule, we developed a flow management model for linking park operations with store profits. Given the store merchandise and associated average profits per visitor from the store associated with the attraction, the flow management model aims at setting the capacities and schedules of all attractions in the park. The objective is to manage visitor flows to maximize total store profits across the park, subject to flow balance, capacity, scheduling, workforce resource availability, and customer satisfaction constraints. Depending on the type of attraction, we assume that resources can be moved from one period to the next. For example, operators are often cross-trained and, depending on queue lengths, are assigned to one attraction during one period and another attraction during the next period. In addition, we also assume that the capacity at any particular attraction can be altered, for instance, by changing the number of operators or by bringing on more cars for a ride, or both. These assumptions are consistent with our observations at USH and at several other theme parks.

To develop the flow management model, we consider a theme park with m attractions and let $i \varepsilon (1, \ldots, m)$ index the set of attractions. Each attraction can operate at n capacity levels and let $j \in (1, \ldots, n)$ index the set of capacity levels. We consider T time periods and let $t \in (1, \ldots, T)$ index the set of time periods. To choose the capacity level and schedule of the attractions, define the variables:

$$y_{ijt} = \begin{cases} 1 & \text{if attraction } i \text{ is operated at capacity level} \\ & j \text{ at time period } t, \\ 0 & \text{otherwise.} \end{cases}$$

$F_{sit} =$ number of visitors moving from attraction s to attraction i at the beginning of time period t. Here, we also consider the case $s = i$, to include the possibility that a customer immediately revisits attraction i.

$Q_{it} =$ number of visitors in line at attraction i in the beginning of period t.

$S_{it} =$ number of visitors served by attraction i during period t.

We are given:

$W_i =$ average profit per visitor from the store associated with attraction i.

$A_t =$ number of visitors who arrive at the park at the beginning of time period t.

$P_{it} =$ fraction of arriving visitors at time period t who first go to attraction i.

$P_{sit} =$ fraction of visitors who move from attraction s to attraction i at the beginning of period t, including the case $s = i$.

$C_{ij} =$ capacity of attraction i at level j per time period.

$b_{ij} =$ workforce resources required if attraction i is run at capacity level j.

$B_t =$ total workforce resources available at the park during time period t.

$M =$ the minimum number of trips a visitor likes to make across all attractions at the park per visit. This includes multiple trips to the same attractions. M is defined as a specified percentile of visitor preferences and can also be regarded as a customer service parameter. For example, based on previously collected data by USH, setting $M = 10$ would set the minimum ride requirements for 90% of the visitors.

The flow management model can be represented by the following linear mixed-integer program, which we call the Theme Park Profit Maximization Problem (TPPMP):

$$\text{(TPPMP)} \quad Z = \text{Max} \sum_{s=1}^{m} \sum_{i=1}^{m} \sum_{t=1}^{T} W_i F_{sit} \tag{1}$$

subject to

$$\sum_{s=1}^{m} F_{sit} = \sum_{s=1}^{m} P_{sit} S_{s(t-1)} \quad \forall i, t, \tag{2}$$

$$Q_{it} = Q_{i(t-1)} + \sum_{s=1}^{m} F_{sit} + A_t P_{it} - S_{it} \quad \forall i, t, \tag{3}$$

$$S_{it} \leqslant Q_{it} \quad \forall i, t, \tag{4}$$

$$S_{it} \leqslant \sum_{j=1}^{n} C_{ij} y_{ijt} \quad \forall i, t, \tag{5}$$

$$\sum_{j=1}^{n} y_{ijt} = 1 \quad \forall i, t, \tag{6}$$

$$\sum_{i=1}^{m} \sum_{j=1}^{n} b_{ij} y_{ijt} \leqslant B_t \quad \forall t, \tag{7}$$

$$\sum_{t=1}^{T} \sum_{i=1}^{m} S_{it} \geqslant M \left(\sum_{t=1}^{T} A_t \right), \tag{8}$$

$$Q_{it}, F_{sit}, S_{it} \geqslant 0 \quad \forall i, t, \tag{9}$$

$$y_{ijt} \in \{0, 1\} \quad \forall i, t. \tag{10}$$

Objective function (1) is chosen to maximize total profits across the entire park. Constraints (2) and (3) represent visitor flow conservation and queue length for each attraction at each period respectively. Constraints (4) and (5) together ensure that the number of visitors served at an attraction is no more than the size of the queue at the attraction before it's started or the capacity of that attraction during that period. The condition that only one capacity level per period is chosen at a given attraction is enforced by (6), while (7) ensures that the choice of capacity levels across all rides meets the available workforce resources in the park during a given time period. Constraint (8) enforces the condition that the total visitors served at all attractions in the park during periods 1 to T is required to be over a certain value. This value depends on the total number of visitor arrivals and the minimum number of rides each visitor likes to take, defined by M. (9) and (10) impose nonnegativity and integrality conditions, respectively.

Note that the TPPMP is NP-Complete. To see this result, observe that the TPPMP contains a knapsack problem defined by (7), which is known to be NP-Complete. Therefore, by restriction, the TPPMP is NP-Complete. In light of this result, it is unlikely that we could solve large, real problems to optimality. We confirmed this in our application. Consequently, we elected to develop the following heuristic to solve this problem.

4.1. Heuristic

We describe the two-phase heuristic that we developed to solve the TPPMP. In the first phase of this heuristic, we determine a variety of allocations of workforce resources to the attractions. In the second phase, we use these allocations to determine flow patterns to maximize overall park profits.

Phase 1: Allocation Phase. In the first phase of this heuristic, we solve the following subproblem that we call the Workforce Allocation Problem (WAP) to determine the allocation of workforce resources to the attractions:

$$\text{(WAP)} \quad W = \text{Max} \sum_{t=1}^{T} \sum_{i=1}^{m} \sum_{j=1}^{n} W_i C_{ij} y_{ijt} \tag{11}$$

Subject to

(6), (7) and (10).

Objective function (11) of the WAP is structured to allocate the workforce resources of the park across periods to maximize the capacity of the rides in the highest retail profit generating areas, subject to constraints (6), (7), and (10). As this problem is decomposable by time, we omit the subscript t without loss of generality, but solve this problem for each of the T time periods. The k best solutions for this problem during a given time period are computed using the following algorithm.

Step 1. Construct a layered network $G = (N, E)$ with the following characteristics:

Layers. The number of layers in the network is equal to $m+2$. Layers 0 and $m+1$ include only one node corresponding to the source node S and the sink node S', respectively. Here, S represents all the attractions before the allocation of workforce resources and S' represents all of the attractions after the allocation of workforce resources. Layers 1 through m correspond to attractions 1 through m.

Nodes (N). Nodes other than the source and the sink node in the network are identified by a couplet $(i, j(i))$, where $i(1 \leqslant i \leqslant m)$ denotes the layer number or, equivalently, the attraction and $j(i)$ represents a potential value for its workforce requirement, where $\text{Min}(\sum_{p=1}^{i} b_p^L, B) \leqslant j(i) \leqslant \text{Min}(\sum_{p=1}^{i} b_p^U, B)$, $b_p^L = \min_j\{b_{pj}\}$, and $b_p^U = \max_j\{b_{pj}\}\ \forall p$. Here, we assume that $j(i)$ increases incrementally by one unit in its range so that there are $\text{Min}(\sum_{p=1}^{i} b_p^U, B) - \text{Min}(\sum_{p=1}^{i} b_p^L, B)$ nodes in each layer.

Directed Arcs (E). Excluding the source and the sink node, each directed arc $(i, j(i)) \rightarrow (i+1, j(i+1))$ connects a pair of nodes from adjacent layers in the network. Node S is connected to all nodes in layer 1 with $(S) \rightarrow (1, j(1))$ representing a directed arc from the source to a node in the first layer. Finally, all nodes in the final layer m are connected to the sink node S', with $(m, j(m)) \rightarrow (S')$ representing a directed arc from a node in the last layer to the sink.

Costs. Let $C(x)$ denote the cost of arc x in E. Then, the cost of each arc can be written as

$$C((S) \rightarrow (1, j(1))) = 0 \quad \forall j(1), \tag{12}$$

$$C((m, j(m)) \rightarrow (S')) = 0 \quad \forall j(m), \tag{13}$$

$$C((i, j(i)) \rightarrow (i+1, j(i+1))) = \begin{cases} W_i C_{ij} & \text{if } j(i+1) - j(i) = b \in \{b_{ij} / j = 1 \text{ to } n\} \\ & \quad \forall i, j(i), \\ \infty & \text{Otherwise.} \end{cases} \tag{14}$$

Equations (12) and (13) ensure that the costs of traversing a directed arc from the source node to nodes in layer 1 or from nodes in layer m to the sink node is 0. The costs of traveling from a node in the set $(i, j(i))$ to a node in the set $(i+1, j(i+1))$ is represented by (14). Here, these costs are set to the capacity required at the attraction weighted by the profit generated at its retail area of location if the resource allocation at node $j(i+1)$ represents a feasible workforce requirement level at attraction $i+1$. Otherwise, these costs are set to a large number for an infeasible resource allocation level.

Step 2. The k longest paths between the source and sink node determine the k best solutions to the WAP. We find the k longest paths using the well-known double-sweep method (Phillips and Garcia-Diaz 1981, pp. 72–77). The complexity of this method can be shown to be of order $O(kB^3)$ (Phillips and Garcia-Diaz 1981, p. 90). Each longest path gives the associated capacity allocation at each attraction at a given period.

Steps 1 and 2 are repeated for each time period to find the k best capacity allocations during every period.

Phase 2: Flow Phase. Let $Z(y^r)$ represent the linear programming solution to the TPPMP when the workforce and the corresponding capacity allocations y^r at each period are fixed according to the rth longest path. We set the heuristic solution to $Z^H = \text{Max}_{r=1 \text{ to } k}\{Z(y^r)\}$. Next, we establish an a priori worst-case bound on our heuristic.

PROPOSITION 1.

$$Z/Z^H \leqslant \alpha = \max_i \left(\frac{\max_j (C_{ij})}{\min_j (C_{ij})} \right).$$

PROOF. Given the capacity allocation levels at each attraction and time period, the TPPMP is a pure linear program defined by (2)–(5) and (8)–(9). To provide a generalized representation of the optimal solution to this program, we represent this in the standard general matrix form, where $Z = \text{Max}\{P^T x / Ax = g,\ x \geqslant 0\}$. Let B represent the optimal basis of this linear program and P_B^T correspond to the coefficients of the objective function corresponding to this basis. Then, it is well known that any optimal solution to this linear program will be of the form $Z = P_B^T B^{-1} g$.

Let g correspond to $g_{\max}$ when we set all the right-hand-side coefficients C_{ij} corresponding to (5) to the maximum values at each attraction and time period. For this case, let $B_{\max}^{-1}$ represent the optimal basis and $P_{B,\max}^T$ represent the optimal objective coefficients corresponding to this basis. The optimal value of the associated linear program is $Z_{\max} = P_{B,\max}^T B_{\max}^{-1} g_{\max}$. Similarly, let $Z_{\min} = P_{B,\min}^T B_{\min}^{-1} g_{\min}$ when all C_{ij} corresponding to (5) are set to the minimum value at each attraction and time period. It is important to note that $Z_H \geqslant Z_{\min}$ and that $Z \leqslant Z_{\max}$. Combining these inequalities, we get $Z/Z_H \leqslant Z_{\max}/Z_{\min}$.

Next, let

$$\alpha = \max_i \left(\frac{\max_j (C_{ij})}{\min_j (C_{ij})} \right).$$

Observe that $Z_{\max} = P_{B,\max}^T B_{\max}^{-1} g_{\max} \leqslant P_{B,\min}^T B_{\min}^{-1} \alpha g_{\min} = \alpha P_{B,\min}^T B_{\min}^{-1} g_{\min} = \alpha Z_{\min}$, because α scales the right-hand side of (5) from the lowest possible capacity level for *each* attraction and time period to the highest possible capacity level across *all* attractions and time periods. This, by definition, is larger than the highest possible value of C_{ij} at *each* individual attraction and time period used to calculate $Z_{\max}$. Thus, $Z_{\max}/Z_{\min} \leqslant \alpha$.

We use the inequality $Z_{\max}/Z_{\min} \leqslant \alpha$ along with the result $Z/Z_H \leqslant Z_{\max}/Z_{\min}$ to get

$$Z/Z^H \leqslant \alpha = \max_i \left(\frac{\max_j (C_{ij})}{\min_j (C_{ij})} \right). \quad \square$$

In practice, we have found that $\alpha = 1.42$ for the USH theme park and is equal to around 1.5 for the Six Flags Magic Mountain theme park also located in Southern California (Ahmadi 1997). Thus, even under the a priori worst-case performance criteria, this heuristic provides an efficient basis to address this problem.

4.2. Upper Bounds

To evaluate the quality of this heuristic, we develop an upper bound on the TPPMP. To compute this bound, we introduce $\mu \geqslant 0$, a vector of Lagrange multipliers associated with constraint (5), which links the continuous and binary variables in this problem. We relax this constraint to decompose the problem into the following subproblems:

$$\text{(WMP)} \quad Z_1(\mu) = \text{Max} \sum_{i=1}^{m} \sum_{t=1}^{T} \sum_{j=1}^{n} \mu_{it} C_{ij} y_{ijt}$$

subject to

(6), (7), and (10),

$$\text{(FMP)} \quad Z_2(\mu) = \text{Max} \sum_{s=1}^{m} \sum_{i=1}^{m} \sum_{t=1}^{T} (W_{it} F_{sit} - \mu_{it} S_{it})$$

subject to

(2) to (4), (8) and (9),

$$\text{(RTPPMP)} \quad Z(\mu) = Z_1(\mu) + Z_2(\mu).$$

The Lagrangean dual corresponding to (RTPPMP) is given by

$$\text{(DTPPMP)} \quad \overline{Z} = \min_{\mu \geqslant 0} \{Z_1(\mu) + Z_2(\mu)\}.$$

The Relaxed Theme Park Profit Management Problem (RTPPMP) consists of a Workforce Management Problem (WMP) and a Flow Management Problem (FMP). The WMP determines the best workforce resource allocation to maximize the capacity of the park. The FMP determines the ideal visitor flow patterns to optimize total store profits. Of course, the actual solution values for the decision variables may be of little significance. The dual variables μ may be interpreted as the value of operating the ride in the WMP, while they represent the cost of ride operation in the FMP. The solution to the DTPPMP provides a tight upper bound $\overline{Z}$ on the TPPMP.

It is important to observe that the WMP is identical in structure to the WAP in the allocation phase of the two-phase heuristic, and therefore, it is solved in a similar manner changing the arc costs appropriately. The FMP is a standard linear program, which is solved using the MINOS5 solver in GAMS (Brooke et al. 1992). We solve the Lagrangean dual problem (DTPPMP) using a subgradient method (Bertsekas 1995) to choose the multipliers used to tighten the bound as much as possible. This procedure was incorporated using a specialized *C* program and linked to GAMS.

4.3. Computational Experiments

In this section, we conduct computational experiments to evaluate the actual performance of the heuristic using several "realistic" subproblems extracted from the data set from USH. Here, each subproblem is defined by the time periods, the number of attractions, and the possible capacity levels at each attraction. We used two levels for time periods and four levels each for attraction and capacity, generating 32 (i.e., $2 \times 4 \times 4$) subproblems. Based upon a prior survey by USH and the 90th percentile of visitors in this survey, we set $M = 10$. We tried to solve the TPPMP corresponding to these problems using leading commercial software tools such as the OSL solver in GAMS and CPLEX (1995). However, we found that these tools were unable to generate feasible solutions to the larger subproblems. This provides validation for developing and using the two-phase heuristic we have developed to address this problem.

Next, we solved each subproblem using the two-phase heuristic. A specialized *C* program was written for the allocation phase of this heuristic and we set $k = 10$ in computing the k longest paths in the second step of this phase. The second phase was solved using the MINOS5 solver in GAMS, which is designed to solve linear programs effectively. To evaluate the performance of the heuristic, we also generated a Lagrangean dual-based upper bound using the procedure outlined in §4.2 for each corresponding problem. Define the percentage gap of the heuristic as $100(\overline{Z} - Z^H)/Z^H$. Across the 32 subproblems, the percentage gap varied from 0.8% to 7%, with an average of approximately 3%. We also computed the a priori worst-case performance for any heuristic for these subproblems using Proposition 1. This varied from 120% to 150%, averaging around 140%. These results provide a strong justification for using the two-phase heuristic to solve larger-sized practical problems. In the concluding section, we use the heuristic to solve the specific problem at USH and to discuss the main results and their implications.

4.4. Numerical Analysis

In this section, we analyze the structure of the solutions of the 32 subproblems considered in the computational study. The purpose of this analysis is to use the solution structure to develop generalizable insights that can then be used to develop robust operating policies in the park. In the solutions to each of the 32 subproblems, we found that attractions associated with areas with high retail profits were assigned more frequent shows with smaller operating capacities, while attractions associated with areas with low retail profits were assigned less frequent shows with higher operating capacities. In addition, we also found that the solution to each subproblem suggests a more staggered closing of the park, in which attractions in lower-profit retail areas close a few hours before attractions at the high-profit retail areas. In the final section, we discuss how these aspects of the solution structure affect visitor flows and store profit.

To test the sensitivity of the solution structure to changes in retail profits per area, we changed the weights representing average retail profits per area in the objective function, so that the previous high retail profit areas were lowest, and vice versa. Even in this case, we observed the same solution structure, suggesting that these results are robust to changes in retail profits per area.

Finally, to understand the consequences of deviating from the solution structure, we performed the following analysis for each of the 32 subproblems. In this analysis, attractions associated with high-profit retail areas were now scheduled less frequently with higher capacities, while attractions associated with low-profit retail areas were scheduled more frequently with lower capacities. In addition, we closed the attractions at the low-profit retail areas later than attractions at the high-profit retail areas. We now found that across the 32 subproblems, total retail profits dropped by over 50% on average, ranging from 20% to 70%. These results indicate the importance of interfacing park operations with store merchandising decisions.

5. APPLICATION

We tested the TPPMP with data from the field study. The objective of this analysis was to determine the schedule and capacity at each major attraction to optimize park profits at USH. Major attractions are defined as those attractions that consistently have a larger number of visitors and greater occupancy levels than the typical attraction. We used the TPPMP to address this problem. The test problem consisted of 10 major attractions over 10 time periods (between 9 a.m. and 6 p.m.), and each attraction had 20 potential capacity levels. We were not able to find feasible solutions for this model using GAMS. In contrast, the specialized *C* program of the two-phase heuristic produced a solution within a few minutes on a Dell desktop computer. To assess the quality of this solution, we also generated a Lagrangean dual-based upper bound for the TPPMP. The resulting upper bound was, on average, around 2% higher than the heuristic solution.

Next, we computed profits using existing capacity and schedules and compared it to the profits from the two-phase heuristic. This analysis suggested that had the schedules and capacity of the two-phase heuristic been implemented, the average profits during the test period would have increased by over 9%. This translates to a potential increase in annual profits of over \$6 million. To better understand the underlying reasons for this improved profitability, we compared the existing schedule to the solution provided by the heuristic. We found that the heuristic solution suggested more frequent shows with smaller operating capacities at attractions located in high-profit retail areas and less frequent shows with higher capacities in attractions located in low-profit retail areas. In effect, this allocation of capacity and schedules ensures that visitors make more frequent visits to the high-profit areas and less frequent trips to low-profit areas, which, in turn, improves the potential to generate larger total store profits. In addition, a lowered operating capacity results in longer waiting lines, offering opportunities for some members of families waiting in line to explore the high-profit stores associated with the attraction. We also found that the solution provided by the heuristic suggests a more staggered closing of the park, in which attractions at lower-profit retail areas close a few hours before attractions at the higher-profit retail areas. Increasing the capacity of people in the attractions in lower-profit areas permits such staggered schedules, which ultimately draws visitors to these high-profit areas.

We also used this heuristic to assess the impact of the changes in schedules and capacities of rides in visitor flow patterns and store profits. It is also being used to understand the benefits of capacity expansion and of changes to schedules at major attractions, and to evaluate the impact of new attractions and stores in the park on visitor flows and store profits. In addition, the solution to this model identifies corridors with a consistently high volume of visitor flows. This analysis provides guidance to locate carts, kiosks, and walking salespeople to further increase retail profits.

As noted by the third finding of our field study, the merchandising process at a store influences store profits and the conversion efficiency ratio could be used as a basis to identify stores with merchandise selection and stocking problems. Our current research focuses on developing and implementing models that choose merchandise and its stock levels to maximize profits at these stores. The results of this research will be reported in a follow-up paper.

In conclusion, we believe the model presented in this paper addresses an important problem in the theme park industry. In addition, it can be used to design and evaluate a range of strategies based on interfacing park operations with store-level merchandising. Such strategies could play an important role in increasing profitability across different types of theme parks.

ACKNOWLEDGMENTS

The authors gratefully acknowledge the support provided by Universal Studios Hollywood and the field study office at the Anderson School at UCLA. They are also grateful to Professors Arthur Geoffrion, Uday Karmarkar, Don Morrison, Fred Murphy, Garrett van Ryzin, and Rakesh Sarin for several helpful comments on this paper. Finally, this paper has benefited from the suggestions made by the reviewers.

REFERENCES

Ahmadi, R. H. 1997. Managing capacity and flows at theme parks. *Oper. Res.* **45**(1) 1–13.

——, K. Rajaram. 2000. Flow and merchandise management at theme parks. Working paper, Anderson School at UCLA, Los Angeles, CA.

Bertsekas, D. P. 1995. *Non Linear Programming*. Athena Scientific, Belmont, MA.

Brooke, A., D. Kendrick, A. Meeraus. 1992. *GAMS: A User's Guide*. The Scientific Press, San Francisco, CA.

CPLEX Optimization Inc. 1995. *CPLEX Version 4.0 Manual*. Incline Village, NV.

Eliashberg, J., R. Steinberg. 1993. Marketing-production joint decision-making. J. Eliashberg, G. L. Lilien, eds. *Handbooks in OR-MS*, Vol 3, Chapter 18. Elsevier Science Publishers, B.V., Amsterdam, The Netherlands.

International Association of Amusement Parks and Attractions. 1997. *Amusement Industry Abstract*. Retrieved April 1998 ⟨http://www.iaapa.org⟩.

Karmarkar, U. S. 1996. Integrative research in marketing and operations management. *J. Marketing Res.* **33** 125–133.

Phillips, D. T., A. Garcia-Diaz. 1981. *Fundamentals of Network Analysis*. Prentice Hall, Englewood Cliffs, NJ.

Rajaram, K. 1998. Merchandise planning in fashion retailing: Models, applications and analysis. Unpublished doctoral dissertation, The Wharton School, University of Pennsylvania, Philadelphia, PA.

Rao P., R. L. Miller. 1971. *Applied Econometrics*. Wadsworth Publishing Company, Inc., Belmont, CA.

United States Census Bureau. 1998. *Service Annual Survey Publication* ⟨http://www.census.gov⟩.

USA Today. 2000. Gannett Co. Inc., McLean, VA, April 28.

Valenti, M. 1997. The fantastical meets the practical. *Mech. Engrg.* (December) 66–71.

Wells, F. 1989. Marketing factors and customer satisfaction. *Quality Progress* **22**(February) 49–51.