The "Price of Information": Inventory Management with Limited Information about Demand

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1. Introduction

Traditional stochastic inventory models assume full knowledge of the demand distribution. However, in practice, it is often difficult to completely characterize the demand distribution, especially with little or outdated historical data. In these uncertain environments, good inventory management has to be robust, i.e., perform well under most demand scenarios.

In this paper, we study the newsvendor problem with limited information about the demand distribution. In particular, we derive robust order quantities that minimize the maximum regret, defined as the opportunity cost from not acting optimally, had we had full information about the demand distribution. In addition, we quantify the value of additional information and highlight that knowing that the demand distribution has a regular shape is in general more valuable than knowing its mean and variance.

In the newsvendor model, a firm has to decide how much to order, denoted by Q, before a selling season, without knowing the demand, in order to maximize its expected profit $\Pi_F(Q)$, that is

$$\max_{Q\geq 0} \Pi_F(Q) \doteq pE_F[\min\{D, Q\}] - cQ, \tag{1}$$

where c is the unit order cost and p is the unit selling price. The expectation is taken with respect to the demand D with distribution F. The problem is a concave optimization problem and, if F is continuous, the optimal order quantity Q^* is well known to satisfy the following optimality condition: $F(Q^*) = 1 - c/p$.

Despite its wide range of applications and extensions, the newsvendor model is not always appropriate in practice, as it might be difficult to estimate the demand distribution. Very often, the demand is subject to several factors that are beyond the firm's control and

is therefore hard to predict. Moreover, because of the shortening of product lifecycles, companies introduce new products with very little historical data. In these uncertain business environments, it is desirable for the inventory decision to be independent of any assumption about the demand distribution.

The maximin criterion. The traditional paradigm for robust optimization is the maximin approach: the firm chooses an order quantity that maximizes its worst-case profit. Scarf (1958) and Gallego and Moon (1993) applied the maximin approach to the newsvendor problem when only the mean and the variance of the distribution are known. Formally, with the maximin objective, the firm solves the following problem:

$$\max_{Q \geq 0} \min_{F \in \mathcal{F}} \Pi_F(Q),$$

where $\mathcal{F}$ is the class of all nonnegative demand distributions specified by the available information (e.g., mean and variance). The maximin approach is conservative since it focuses on the worst-case scenario. In some cases, the maximin optimal decision might be not to order at all (for instance, when the only available information is the mean demand).

The minimax regret criterion. To avoid the conservative decisions associated with the maximin objective, Savage (1951) suggested the minimax regret criterion: the firm minimizes the regret about not acting optimally.

In our setting, the order decision Q is compared to the optimal solution of the classic newsvendor problem (1), had we had full information about the demand distribution. By contrast, Vairaktarakis (2000) analyzed the regret in the newsvendor problem by comparing Q to the demand realization D, and Schweitzer and Cachon (2000) showed experimental evidence of such a regret affecting practical decision making. However, a decision criterion based on that type of regret cannot exploit partial demand information (moments or shape of the distribution).

Suppose that the demand distribution is F. If the newsvendor had known it, she would have ordered $F^{-1}(1 - c/p)$ units (by solving problem (1)) and would have had an expected profit of $\Pi_F(F^{-1}(1 - c/p))$. Instead, without knowledge of the demand distribution, her expected profit is $\Pi_F(Q) = pE_F[\min\{Q, D\}] - cQ$. The opportunity loss, or "regret", from ordering Q instead of $F^{-1}(1 - c/p)$ units is then equal to $\{\Pi_F(F^{-1}(1 - c/p)) - \Pi_F(Q)\}$. In general, the newsvendor hedges her decision against all possible demand distributions and

solves the following optimization problem:

$$\min_{Q\geq 0}\max_{F\in\mathcal{F}}\left\{\Pi_F(F^{-1}(1-\frac{c}{p}))-\Pi_F(Q)\right\}. \tag{2}$$

The objective balances the regret about ordering too much with the regret about ordering too little. Therefore, the minimax regret objective is less conservative than the maximin criterion.

2. Minimax regret order quantities

In this section, we derive the order quantities that minimize the newsvendor's maximum regret, with partial information about the demand distribution. In fact, the problem of identifying the worst-case demand distribution that is associated with the maximum regret in (2) can be formulated as a moment bound problem. Accordingly, our method of derivation relates to the work by Bertsimas and Popescu (2002) and Popescu (2005).

We derive the robust order quantities when the following information about the demand distribution is available: range; range and mean; mean; mean equal to the median; mean and symmetry; mode; mean, symmetry and unimodality. Table 1 summarizes some of the robust order quantities when the profit margin p/c is less than 4.

Table 1: Robust order quantities when $p/c \leq 4$

Information	$p \leq 2c$	$p \geq 2c$
Range $[A, B]$	$\frac{c}{p}A + \frac{p-c}{p}B$	
Mean μ	$\mu\frac{p-c}{p}$	$\frac{\mu}{4}\frac{p}{c}$
Mean/Median μ	$2\mu\frac{p-c}{p}$	
Mean/Median/Mode μ	$2\mu\frac{\sqrt{c(p-c)}}{p}$	$2\mu(1-\frac{\sqrt{c(p-c)}}{p})$

Interestingly, when the demand distribution has a known support or when the distribution is known to be symmetric, the robust order quantity is the same as the optimal solution of (1) with a uniform demand distribution.

In general, as the demand distribution is restricted to a "regular" shape, the robust order quantity is re-centered about the mean demand. In particular, when the demand distribution is known to be symmetric and unimodal, the order quantity is almost equal to the mean demand and almost independent of the profit margin.

Similarly, we characterize the robust order policy when the newsvendor has knowledge of the mean and the variance of the demand distribution. In contrast to the previous cases, the minimax regret order quantity has no explicit form, but it can be efficiently computed numerically. When the coefficient of variation is small, the minimax regret order quantity is similar to the maximin order quantity and also to the solution of the newsvendor problem (1) with a normal demand distribution. This suggests that in practice, when only the mean and the variance are known, and that the coefficient of variation is small, one can assume that the demand is normally distributed in solving (1) without great loss of optimality. Traditionally, the normal distribution is justified by the Central Limit Theorem. Our analysis shows that the normal distribution is also robust, both in terms of the maximin and the minimax regret objectives, as long as the coefficient of variation is small.

3. Expected Value of Additional Information

The optimal value of problem (2) quantifies the maximum opportunity loss from not having full information about the demand distribution. In other words, it measures the expected value of additional information (EVAI). Similarly to the order quantities, all EVAI's have an explicit form, except when only the mean and variance are known (in which case it can be computed numerically). Obviously, as one restricts the shape of the demand distribution, the EVAI decreases. Similarly, if the variance is known, the EVAI is a decreasing function of the variance.

Estimating the variance of the demand might be challenging with limited or outdated historical data. On the other hand, the general shape of the demand distribution (symmetry, unimodality) can be specified by the newsvendor's overall knowledge of the market. It turns out that the EVAI associated with a unimodal and symmetric distribution is lower than that of a distribution with a certain mean and variance, whose coefficient of variation is greater than .3. Therefore, in practice, joint efforts should be spent to characterize the demand distribution both qualitatively (shape) and quantitatively (moments).

4. Conclusions

In this paper, we propose a robust approach to inventory management with partial demand information. In particular, we derive order quantities that minimize the newsvendor's maximum regret. The minimax regret objective balances the opportunity costs from ordering

too little with those from ordering too much, and is consequently less conservative than the maximin approach. Most of the derived order quantities are simple functions, which makes them attractive for practical application. Alternatively, one can still use the classic newsvendor model in some situations with partial information, by carefully selecting the demand distribution (e.g., uniform if distribution is known to be symmetric; normal if only the mean and variance are known, and the coefficient of variation is small). We also show that the information that the demand distribution is "regular" is, in general, more valuable than knowing its mean and variance. Our analysis highlights the value of qualitatively characterizing the demand distribution, in addition to estimating its moments.

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