Note: Optimal Ordering Decisions with Uncertain Cost and Demand Forecast Updating

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We determine the optimal ordering policy for a retailer who has two instants to order a seasonal product from a manufacturer prior to a single selling season. While the demand is uncertain, the retailer can improve the forecast by utilizing the market signals observed between the first and second instants. However, because of the nature of the manufacturing environment, the unit cost at the second instant is uncertain and could be higher (or lower) than the unit cost at the first instant. To determine the profit-maximizing ordering strategies at both instants, the retailer has to evaluate the trade-off between a more accurate forecast and a potentially higher unit cost at the second instant. We present a nested newsvendor model for determining the optimal order quantity at each instant and characterize the conditions under which it is optimal for the retailer to delay its order until the second instant.
(*Inventory; Uncertain Cost; Demand Forecast Updating*)

1. Problem Definition

Consider a situation in which a retailer has two instants to order a seasonal product so that the total quantity ordered at both instants will arrive before the start of the selling season.[1] While the demand is uncertain, the retailer can improve the demand forecast by utilizing the market signals (such as market trends, advance bookings, etc.) observed between the first and second instants. Hence, it is beneficial for the retailer to delay its order until the second instant. However, because of the nature of the manufacturing environment, the unit cost that the retailer has to pay at the second instant is uncertain and could be higher (or lower) than the unit cost at the first instant.[2] To

[1] We consider the case in which the retailer cannot place an order after the start of the selling season. This is especially reasonable when the replenishment lead time is longer than the selling season. However, if the replenishment lead time is shorter than the selling season, it is possible for the retailer to place an order during the early selling season. This would allow the retailer to obtain a more accurate demand forecast by using the actual sales that occurred during the early selling season to update its demand forecast. The reader is referred to the innovative work developed by Fisher and Raman (1996).

[2] There are many plausible causes for fluctuating unit cost. First, the fluctuation in currency exchange rate and hyper inflation could cause the unit cost to change (Austin 1990, and Carter and Vickery 1988). Next, shortage of certain key components such as DRAM (Willett 1993) and excessive inventory of certain key components such as memory chips (Kurawarwala and Matsuo 1996) could cause the unit cost to go up or down, respectively. Therefore, even though the manufacturer's production costs at the second instant are higher as a result of shorter lead-time available, overall costs could be higher or lower.

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determine the profit-maximizing order quantities at both instants, the retailer has to evaluate the trade-off between a more accurate demand forecast and a potentially higher unit cost at the second instant.

Donohue (1998) develops a model that aims to determine an efficient supply contract, in terms of wholesale price and return policy, which ensures coordination between the manufacturer and the distributor. While the focus of her model is different from ours, her model deals with a subproblem that is similar to the problem addressed here. However, our problem deals with a more general situation than her subproblem in two ways. First, we consider the case in which the cost at the second instant is uncertain and could be higher or lower than the cost at the first instant. In contrast, Donohue considers the case in which the cost at the second instant is known and is higher than the cost at the first instant. Second, we consider the case in which the value of the information observed between the first and second instants varies from worthless to perfect, while Donohue considers only the case of perfect information. Further, our paper utilizes the analytical results derived for the special cases of worthless information and perfect information to generate conjectures that characterizes the impact of the value of information on the total expected order quantity and the total expected profit. The resulting insights can be of value to the retailer as well as the manufacturer. Besides Donohue (1998), recent articles that use demand forecast updates to improve buying decisions include papers by Eppen and Iyer (1997a, b), Iyer and Bergen (1997), and others. However, these papers assume that there is no price uncertainty. Kouvelis and Gutierrez (1997) consider price uncertainty in a newsvendor problem in a global market but do not include the effect of forecast updating in the paper. To our knowledge, this paper is the first to examine the issue of demand forecast updating and uncertain unit costs.

2. The Model

We introduce the following notation. At Instant 1, let c be the unit cost and q be the order quantity. At Instant 2, the unit cost equals c_l with probability β and c_h with probability $(1-\beta)$, where $c_l \le c \le c_h$.[3] Let q_l and q_h be the order quantity to be placed when the cost at Instant 2 is equal to c_l and c_h, respectively. The retailer collects revenue p for each unit sold and collects the salvage value $v < c_l$ for each unsold unit. At Instant 1, the retailer estimates that the demand, denoted by D, has a distribution $F(\cdot)$ and density function $f(\cdot)$. The retailer utilizes the market information I observed between Instants 1 and 2 to update the probability distribution of D. Let:

$\varphi(I, D)$ = the joint density of I and D;

$g(I)$, $G(I)$ = the marginal density and distribution of I, respectively; and

$h(D|I)$, $H(D|I)$ = the conditional density and distribution of D given I, respectively.

Let $E_X(\cdot)$ = expectation taken over random variable X, $x^+ = \text{Max}(0, x)$, $x \wedge y = \text{Min}(x, y)$, and $x \vee y = \text{Max}(x, y)$. The retailer's problem can be defined as:

$$\text{(P)} \quad EP^* = \underset{q \ge 0}{\text{Maximize}}\{-qc + E_I\{\underset{q_l, q_h \ge 0}{\text{Maximize}} \cdot \Pi(q, I, q_l, q_h)\}\}, \quad (1)$$

where

$$\begin{aligned}\Pi(q, I, q_l, q_h) &= E_{D|I}[\beta\{p(D \wedge (q + q_l)) + v(q + q_l - D)^+ - q_l c_l\} \\ &\quad + (1-\beta)\{p(D \wedge (q + q_h)) \\ &\quad + v(q + q_h - D)^+ - q_h c_h\}]. \end{aligned} \quad (2)$$

The first term $-qc$ corresponds to the ordering cost incurred at Instant 1, while the second term $E_I\{\text{Maximize}_{q_l, q_h \ge 0}\Pi(q, I, q_l, q_h)\}$ corresponds to the optimal expected profit (excluding the ordering cost incurred at Instant 1). Therefore, the retailer's problem (P) is to determine the optimal ordering deci-

[3] For simplicity, we focus only on the case in which the unit cost at the second instant is either high or low. We assume that the retailer knows the probability and magnitude of price change; that is, the values of β, c_l, and c_h are known. Often, the retailer can check the previous pricing patterns and estimate the parameters for future consideration. In reality, the manufacturer could offer several different pricing levels (with a certain probability associated with each level) at Instant 2. The scope of the model can be similarly generalized to this case as the underlying trade-offs are the same.

sions, denoted by (q^*, q_l^*, q_h^*), to maximize the total expected profit. Our approach for solving problem (P) is to first determine the optimal $q_l^*(q)$ and $q_h^*(q)$ for any given q, and then determine the optimal q^*. First, for any given q, problem (P) reduces to:

$$\text{(Ps)} \quad \underset{q_l, q_h \geq 0}{\text{Maximize}}\ \Pi(q, I, q_l, q_h).$$

Let $t_l = q + q_l$ and let $t_h = q + q_h$, where t_l (t_h) corresponds to the total amount to be ordered at both instants when the cost at Instant 2 is c_l (c_h). By substituting $q_l = t_l - q$ and $q_h = t_h - q$ into (2), subproblem (Ps) can be rewritten as:

$$\begin{aligned}\text{(Ps}'\text{)} \quad \underset{t_l, t_h \geq q}{\text{Maximize}}[&\beta\{(p - c_l)t_l \\ &- (p - v)\int_{D=-\infty}^{t_l} (t_l - D)h(D|I)dD \\ &+ qc_l\} + (1 - \beta)\{(p - c_h)t_h \\ &- (p - v)\int_{D=-\infty}^{t_h} (t_h - D)h(D|I)dD \\ &+ qc_h\}]. \end{aligned} \tag{3}$$

Since the objective function of subproblem (Ps′) is a separable concave function of t_l and t_h, one can show that:

PROPOSITION 2.1. *The optimal solution to subproblem (Ps′) can be expressed as follows*:

$$\begin{aligned}(t_l^*, t_h^*) &= [(t_l(I) \vee q), (t_h(I) \vee q)], \quad \textit{where} \\ t_l(I) &= H^{-1}[(p - c_l)/(p - v)|I], \\ t_h(I) &= H^{-1}[(p - c_h)/(p - v)|I]. \end{aligned} \tag{4}$$

In addition, if the random variable $D|I$ is stochastically increasing in I, then there exists threshold values of $i_l(q)$, $i_h(q)$ with $i_h(q) \geq i_l(q)$, $t_l(i_l(q)) = q$ and $t_h(i_h(q)) = q$ such that:

$$t_l^* = \begin{cases} q, & \textit{if } I \leq i_l(q), \\ t_l(I), & \textit{if } I > i_l(q), \end{cases} \tag{5}$$

$$t_h^* = \begin{cases} q, & \textit{if } I \leq i_h(q), \\ t_h(I), & \textit{if } I > i_h(q). \end{cases} \tag{6}$$

Furthermore, the optimal order quantity at Instant 2 is given by: $q_l^* = t_l^* - q$ *and* $q_h^* = t_h^* - q$.

PROOF. All proofs are given in Gurnani and Tang (1998).

By substituting the optimal values of q_l^* and q_h^* (stated in Proposition 2.1) into (2), one can use the same argument given in Brown and Lee (1997) (using the convexity argument developed by Heyman and Sobel 1984, p. 525) to show that the optimal solution to Problem (P), q^*, must satisfy the first-order condition:

$$\begin{aligned}&\beta[(p - c_l)G(i_l(q)) - (p - v)\int_{-\infty}^{i_l(q)} H(q|I)g(I)dI + c_l] \\ &+ (1 - \beta)[(p - c_h)G(i_h(q)) - (p - v) \\ &\cdot \int_{-\infty}^{i_h(q)} H(q|I)g(I)dI + c_h] - c + \lambda = 0, \quad \lambda q = 0, \end{aligned} \tag{7}$$

where $\lambda \geq 0$ corresponds to the Lagrangian multiplier associated with the constraint $q \geq 0$. The first-order condition given in (7) cannot be solved analytically for general probability distributions $G(I)$ and $H(D|I)$. To obtain some basic insights, we consider the case in which D and I has a joint distribution that is bivariate normal.

3. Optimal Order Quantities

Suppose that the joint distribution of information I and demand D, $\varphi(I, D)$, is a bivariate normal distribution with means m and μ, standard deviations s and σ, and the correlation coefficient ρ.[4] Then it is well known (Bickel and Doksum 1977) that $(D|I)$ is also

[4] Our selection of the bivariate normal distribution is motivated by the innovative work of Fisher and Raman (1996) and by Brown and Lee (1997). By assuming that the joint distribution of D and I is bivariate normal, we apply the novel solution developed by Brown and Lee (1997) to determine the optimal ordering policy. (Even though Brown and Lee's approach was developed for analyzing supply contracts, we found their approach can be applied to determine the optimal ordering policy in this section.)

normally distributed with mean μ' and standard deviation σ', where:[5]

$$\mu' = \mu + \rho(I - m)\sigma/s \tag{8}$$

and

$$\sigma' = \sigma\sqrt{1 - \rho^2}. \tag{9}$$

Notice from (9) that $\sigma' \leq \sigma$. Hence, the bivariate normal distribution does allow us to capture how information I enables the retailer to obtain a more accurate demand forecast. We consider the case in which $1 \geq \rho \geq 0$ so that $D|I$ is stochastically increasing in I. In this case, Proposition 2.1 implies that the value of $i_l(q)$ and $i_h(q)$ satisfy $t_l(i_l(q)) = q$ and $t_h(i_h(q)) = q$, respectively. By using (4) and the mean and standard deviation of $D|I$ given in (8) and (9), we have:

$$i_l(q) = m + \frac{s}{\rho\sigma}\left[q - \mu - \sigma\sqrt{1 - \rho^2}\Phi^{-1}\left(\frac{p - c_l}{p - v}\right)\right], \tag{10}$$

$$i_h(q) = m + \frac{s}{\rho\sigma}\left[q - \mu - \sigma\sqrt{1 - \rho^2}\Phi^{-1}\left(\frac{p - c_h}{p - v}\right)\right], \tag{11}$$

where $\Phi(\cdot)$ is the standard normal cumulative distribution function. By substituting the above expressions for $i_l(q)$ and $i_h(q)$ into (7), we can simplify (7) but we cannot obtain closed form expressions for the optimal order quantities for the case when $1 > \rho > 0$. In the remainder of this section, we first analyze two special cases $\rho = 0$ and $\rho = 1$, and obtain closed form expressions for the optimal order quantities, the optimal expected profit, and the optimal expected total order quantity. Then we compare the optimal solution associated with the case of $\rho = 0$ to that of $\rho = 1$. The comparison enables us to develop various conjectures regarding the impact of ρ (where $1 > \rho > 0$) on the optimal expected total order quantity and the optimal expected profit. For the general case of ρ (that is, $1 > \rho > 0$), we analytically prove some properties and the rest are examined numerically.

Let $\bar{c}$ be the expected unit cost at the second instant, where $\bar{c} = \beta c_l + (1 - \beta)c_h$. Let $L(k)$ be the right linear loss function of a standard normal distribution at k, where:

$$L(k) = \int_k^\infty (z - k)\phi(z)dz. \tag{12}$$

Special Case 1: Worthless Information I ($\rho = 0$)
Since the joint distribution of (I, D) is bivariate normal, I and D are independent when $\rho = 0$ (Bickel and Doksum 1977). Thus, $\mu' = \mu$ and $\sigma' = \sigma$. Let:[6]

$$r = \Phi^{-1}\left(\frac{p - \dfrac{(c - \beta c_l)}{(1 - \beta)}}{p - v}\right),$$

$$r_l = \Phi^{-1}\left(\frac{p - c_l}{p - v}\right), \quad \text{and} \quad r_h = \Phi^{-1}\left(\frac{p - c_h}{p - v}\right). \tag{13}$$

PROPOSITION 3.1. (A) *If* $c \geq \bar{c}$, *then*:

1. *The optimal order quantity* (q^*, q_l^*, q_h^*) *can be expressed as*:

$$q^* = 0, \quad q_l^* = \mu + r_l\sigma, \quad \textit{and} \quad q_h^* = \mu + r_h\sigma. \tag{14}$$

2. *The optimal expected total order quantity,* $TQ^* = q^* + \beta q_l^* + (1 - \beta)q_h^*$, *is given as*:

$$TQ^* = \mu + [\beta r_l + (1 - \beta)r_h]\sigma. \tag{15}$$

3. *The optimal expected profit for the retailer,* EP^* *given in* (1) *can be expressed as*:

$$EP^* = (p - \bar{c})\mu - \sigma[\beta(c_l - v)r_l + (1 - \beta)(c_h - v)r_h] - (p - v)\sigma[\beta L(r_l) + (1 - \beta)L(r_h)]. \tag{16}$$

[5] The bivariate normal distribution allows us to obtain simple expressions for μ' and σ' and a tractable solution to problem (P). To elaborate, if one uses the conjugate prior distributions to determine the posterior distribution of the updated demand, then the mean and the standard deviation of the posterior distribution is quite complex and would complicate the analysis significantly.

[6] Notice that the terms r, r_l, and r_h correspond to the "classic newsvendor" critical fractiles when the selling price is p, the salvage value is v, and the unit costs are $(c - \beta c_l)/(1 - \beta)$, c_l, and c_h, respectively. As we shall see, the terms r, r_l, and r_h have direct association with q^*, q_l^*, and q_h^*, respectively. For instance, under certain conditions, q^* takes on the form of the newsvendor solution; i.e., $q^* = \mu + r\sigma$.

(B) *If* $c < \bar{c}$, *then*:

1. *The optimal order quantity* (q^*, q_l^*, q_h^*) *can be expressed as*:

$$q^* = \mu + r\sigma, \quad q_l^* = [r_l - r]\sigma, \quad \text{and} \quad q_h^* = 0. \tag{17}$$

2. *The optimal expected total order quantity,* TQ^*, *can be expressed as*:

$$TQ^* = \mu + [\beta r_l + (1 - \beta)r]\sigma. \tag{18}$$

3. *The optimal expected profit for the retailer,* EP^*, *can be expressed as*:

$$EP^* = (p - c)\mu - (c - v)\sigma r - \beta(c_l - v)\sigma(r_l - r) - (p - v)[\beta L(r_l) + (1 - \beta)L(r)]\sigma. \tag{19}$$

In Case (A), as $c \geq \bar{c}$, (14) implies that it is optimal to delay the order and to order according to the classic newsvendor solution at the second instant. This result is intuitive because when unit cost at the first instant is higher, there is no point to ordering early. Next, in Case (B), as $c < \bar{c}$, it can be checked from (13) that $r_l > r$. In this case, (17) suggests that it is optimal to order q^* at the first instant, and place an additional order $q_l^* = (r_l - r)\sigma$ only when there is a price promotion at the second instant. Observe that q_l^* can be viewed as the difference between two newsvendor solutions because $q_l^* = (\mu + r_l\sigma) - (\mu + r\sigma) = (r_l - r)\sigma$. Since the unit cost at the first instant is lower than the expected second instant cost, and information I has no value, most of the ordering is done at the first instant. An order may still be placed at the second instant but only if the price offered is lower than the first instant price.

Special Case 2: Perfect Information I $(\rho = 1)$

When $\rho = 1$, $\mu' = \mu + (I - m)\sigma/s$ and $\sigma' = 0$. Also, final demand is known after observing I, where $D|I = \mu + (I - m)\sigma/s$. Let:[7]

$$\tilde{r} = \Phi^{-1}\left(\frac{\bar{c} - c}{\bar{c} - v}\right). \tag{20}$$

[7] Note that $\tilde{r}$ is analogous to r given in (13) (i.e., for the case when $\rho = 0$). It corresponds to the critical fractile for the order at the first instant and is defined for $c \leq \bar{c}$.

PROPOSITION 3.2. (A) *If* $c \geq \bar{c}$, *then*:

1. *The optimal order quantity,* $(q^*, q_l^*(I), q_h^*(I))$, *can be expressed as*:

$$q^* = 0, \quad \text{and} \quad q_l^*(I) = q_h^*(I) = \mu + (I - m)\sigma/s. \tag{21}$$

2. *The optimal expected total order quantity,* TQ^*, *can be expressed as*:

$$TQ^* = \mu. \tag{22}$$

3. *The optimal expected profit for the retailer,* EP^*, *can be expressed as*:

$$EP^* = (p - \bar{c})\mu. \tag{23}$$

(B) *If* $c < \bar{c}$, *then*:

1. *The optimal order quantity,* $(q^*, q_l^*(I), q_h^*(I))$, *can be expressed as*:

$$\begin{aligned} q^* &= \mu + \tilde{r}\sigma, \quad \text{and} \\ q_l^*(I) &= q_h^*(I) \\ &= [\mu + (I - m)\sigma/s - q^*]^+ \\ &= \sigma[(I - m)/s - \tilde{r}]^+. \end{aligned} \tag{24}$$

2. *The optimal expected total order quantity,* TQ^*, *can be expressed as*:

$$TQ^* = \mu + \sigma[\tilde{r} + L(\tilde{r})]. \tag{25}$$

3. *The optimal expected profit for the retailer,* EP^*, *can be expressed as*:

$$EP^* = (p - c)\mu - (c - v)\sigma\tilde{r} - (\bar{c} - v)L(\tilde{r})\sigma. \tag{26}$$

Proposition 3.2 has a similar interpretation as Proposition 3.1. In Case (A), as $c \geq \bar{c}$, it is optimal to delay the ordering decision and order according to the known demand $D|I = \mu + (I - m)\sigma/s$ at the second instant. Next, in Case (B), as $c < \bar{c}$, it is optimal to order according to a newsvendor type solution at the first instant. Then, as final demand $D|I$ becomes known after observing I, the retailer should place additional order at the second instant so as to meet the known demand. This implies that, if the information I reveals that the final demand $D|I = \mu + (I - m)\sigma/s$ is less than q^*, then no further order would be necessary; i.e., $q_l^*(I) = q_h^*(I) = 0$.

Table 1 Impact of ρ on the Optimal Order Quantities and Optimal Total Expected Profit

	Impact of ρ (as ρ increases)				
	Case (A) when $c \geq \bar{c}$		Case (B) when $c < \bar{c}$		
	Low Service Level	High Service Level	Low Service Level	High Service Level	*Comment*
q^*	0	0	decrease	decrease	*corroborates Proposition* 3.4
TQ^*	increase	decrease	increase	decrease	*corroborates Conjecture* 1
EP^*	increase	increase	increase	increase	*corroborates Conjecture* 2

Worthless Information Versus Perfect Information
Consider Case (A) where $c \geq \bar{c}$. By comparing the optimal expected total order quantity TQ^* given in (15) and (22) and the optimal expected total profit stated in (16) and (23), one can prove the following corollary for Case (A):[8]

COROLLARY 3.3. *If $c \geq \bar{c}$, the following comparisons can be made.*

1. *When r_l, $r_h < 0$ (i.e., when the corresponding service level is low—less than* 0.5*), the expected total order quantity TQ^* is higher when $\rho = 1$ as compared to the case when $\rho = 0$. Similarly, when r_l, $r_h > 0$ (i.e., when the corresponding service level is high—greater than* 0.5*), the optimal expected total order quantity TQ^* is lower when $\rho = 1$ as compared to when $\rho = 0$.*
2. *The optimal expected profit EP^* is higher when $\rho = 1$ as compared to when $\rho = 0$.*

Corollary 3.3 enables us to make the following *conjectures* for the case when $1 > \rho > 0$:

1. When r, r_l, r_h, $\tilde{r} < 0$, the optimal expected total order quantity TQ^* is increasing in ρ. When r, r_l, r_h, $\tilde{r} > 0$, the expected total order quantity TQ^* is decreasing in ρ.
2. The optimal expected profit EP^* is increasing in ρ for a fixed β.

In addition, we can apply the lemma presented in Brown and Lee (1997) to show that:

PROPOSITION 3.4. *The optimal order quantity at the first instant, q^* is nonincreasing in ρ, where $1 \geq \rho \geq 0$.*

Proposition 3.4 implies that, as information I becomes more informative (i.e., as ρ increases), it is optimal for the retailer to reduce the order quantity at the first instant so that the retailer can place a better order at the second instant after observing I. This implication is consistent with the underlying concept of "accurate response" developed by Fisher and Raman (1996).

3.1. Numerical Analysis and Concluding Remarks
We now present numerical examples to corroborate the two aforementioned conjectures and Proposition 3.4.[9] We set $c = 40$, $p = 60$, $\mu = 2000$, $\sigma = 600$, $m = 1000$, $s = 300$. For the low service level case that yields r, r_l, r_h, $\tilde{r} < 0$, we set $c_h = 45$, $c_l = 35$, and $v = 10$. For the high service level case that yields r, r_l, r_h, $\tilde{r} > 0$, we set $c_h = 45$, $c_l = 39$, and $v = 37$. Table 1 summarizes the impact of ρ on the optimal order quantities (i.e., the order quantity at the first instant q^*, the expected total quantity TQ^*), and the optimal expected profit EP^*.

It is easy to check from Table 1 that the impact of ρ corroborates with Conjectures 1 and 2 as well as Proposition 3.4. Observe that the retailer's expected profit EP^* increases as ρ increases (i.e., as the value of the information increases). However, ρ has a significant impact on the manufacturer's profit as well. To

[8] For Case (B) where $c < \bar{c}$. If r, r_l, r_h, $\tilde{r} < 0$, we can show that TQ^* is higher when $\rho = 1$ as compared to the case when $\rho = 0$. However, in general, the comparisons of TQ^* and EP^* for the case $\rho = 0$ and $\rho = 1$ are quite complex and we are unable to prove the statements that are analogous to Corollary 3.3.

[9] Specifically, to evaluate the multidimensional integrals, we use the composite Newton-Coles formulae known as Simpson's rules (see Gerard and Wheatley 1994) and use standard concave programming techniques on the objective function to find the solution to the first-order condition (7). It is, however, important to note that the numerical integration is not very accurate for small values of ρ.

elaborate, let us consider the case in which the service level is high. Notice from Table 1 that the total expected order quantity TQ^* may decrease in ρ. This implies that the retailer may order less from the manufacturer, and hence, the manufacturer's profit may decrease as ρ increases. Therefore, when the service level is high, our result suggests that as the value of information about the demand increases (i.e., as ρ increases), the retailer's profit would increase while the manufacturer's profit would decrease. This observation should be of interest to the manufacturer and the retailer when evaluating different mechanisms for improving demand forecast.

The contribution of this paper is to determine the optimal ordering decisions for a retailer when facing uncertain cost and random demand for the product in the selling season, and to characterize the conditions under which the retailer would defer the ordering decision. While many researchers have addressed the issue of supply contracts and coordination (Lariviere 1998, Tsay et al. 1998, and the references therein), we plan to consider the impact of a supply contract on the order decision for the retailer in the future. Specifically, under certain supply contracts, the retailer has certain restrictions on the order quantity at each instant and may be forced to make a partial order at the first instant even when the expected cost at the second instant is lower. In addition, under other types of supply contracts, the retailer may face certain restrictions on the timing of the ordering decisions. As such, the retailer has to make good use of the market information to determine the optimal time for placing an order. Such development has been addressed recently by Li and Kouvelis (1997). Further, when there is a cost associated with obtaining market information, the retailer has to trade-off the cost (effort) versus the benefit of gathering market information. Overall, we believe that this paper enhances our understanding about the impact of uncertain cost and forecast updating in a supply chain.[10]

[10] This paper is based on an unpublished manuscript in 1996 entitled "Coordinating Ordering and Production Decisions with Decreasing Material Costs and Uncertain Demand," by Haresh Gurnani and Christopher S. Tang. The authors appreciate the valuable suggestions provided by the editor (Professor Hau Lee), the associate editor and three anonymous reviewers. Part of this research was conducted while Christopher S. Tang was visiting the Hong Kong University of Science and Technology in 1996. This research is supported in part by UCLA James Peters Research Fellowship.

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