# Rational Shopping Behavior and the Option Value of Variable Pricing

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When a product's price fluctuates at a store, how should rational, cost-minimizing shoppers shop for it? Specifically, how frequently should they visit the store, and how much of the product should they buy when they get there? Would this rational shopping behavior differ across Every Day Low Price (EDLP) and Promotional Pricing (HILO) stores? If shoppers are rational, which retail price format is more profitable, EDLP or HILO? To answer these questions, we develop a normative model that shows how rational customers should shop when the price of the product is random.

We derive a closed-form expression for the optimal purchasing policy and show that the optimal quantity to purchase under a given price scenario is linearly decreasing in the difference between the price under that scenario and the average price. This purchase flexibility due to price variability has a direct impact on shopping frequency. Indeed, the benefit of this purchase flexibility can be captured via an "option value" that implicitly reduces the fixed cost associated with each shopping trip. Consequently, rational shoppers should shop more often and buy fewer units per trip when they face higher price variability.

Our results suggest that if two stores charge the same average price for a product, rational shoppers incur a lower level of expenditure at the store with a higher price variability. Since stores with different price variabilities coexist in practice, we expect stores with higher price variability to charge a higher average price. Thus, given two stores, a higher relative mean price for a given item should be indicative of higher price variability, and vice versa.

These model implications are tested using multicategory scanner panel data from 513 households and pricing data for three stores (two EDLP stores and one HILO store) and 33 product categories over a two-year period. We find strong empirical support for the model implications. (*Rational Shopping; EDLP; HILO; Retail Pricing Format*)

## 1. Introduction

To increase store revenue, retailers must develop a price format to entice their shoppers both to shop more frequently and increase their purchase quantity during each visit. Shopping frequency is critical because it increases the chance of "spontaneous purchase."[1] However, shopping frequency and purchase quantity are interrelated and they both depend on the price format of the retailer. In order to develop an effective pricing format for increasing store revenue, it is important for the retailer to understand the impact of price format on both shopping frequency and purchase quantity simultaneously.

This paper addresses two related issues that deal with how consumer behavior is influenced by retail pricing format. First, we examine both analytically and

[1] A recent article in *Progressive Grocer* reports that up to 50% of a consumer's purchases result from unplanned, in-store decisions.

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empirically how retail price format influences shopping frequency *and* the purchase quantity of a cost-minimizing or rational consumer.[2] Second, we study the relative profitability of different retail price formats. Specifically, if shoppers are rational, which retail price format is more profitable, Every Day Low Pricing (EDLP) or Promotional Pricing (HILO)?[3] If both kinds of stores coexist in equilibrium (e.g., Lal and Rao 1997), how should the average prices of items in HILO stores be set relative to those in EDLP stores? This paper aims to provide answers to these important managerial questions.

Previous research (e.g., Blattberg et al. 1981, Krishna 1992, Assuncao and Meyer 1993, Krishna 1994) has examined the influence of price promotions on purchase quantity and consumption rate. The underlying assumptions of this stream of research are: (a) shoppers visit the store periodically (e.g., weekly); and (b) there are no fixed transaction costs associated with each store visit. The key findings of this stream of research are that, when consumers are more certain about the timing of deals, the average optimal purchase quantity on deal occasions is higher, and stockpiling in response to price promotions rationally leads to increased consumption for the product. In this paper, we develop a different model to capture the transaction cost of shopping and nonperiodic store visits. In addition, we examine the issue of shopping frequency, which has not been examined explicitly in prior research.

A second stream of research focuses on the influence of pricing format (i.e., EDLP and HILO) on purchase quantity. For instance, Mulhern and Leone (1990) conducted an event study of a discrete change in a store's price format. Their time-series analysis implied that sales increased when the store switched from EDLP to HILO. In a more recent paper, Hoch et al. (1994) investigated the impact of category-level price changes on sales response. They conducted an experiment at a retail chain which agreed to systematically alter prices at eighty stores for 26 product categories. Relatively inelastic response to price changes led the researchers to conclude that an EDLP format might be undesirable as a strategy for increasing purchase quantity. Both studies, however, did not have access to household-level data for examining the relationship between retail price format and the shopping behavior of an individual household. In this paper, we use multi-category panel data from Information Resources Inc. (IRI) to study how retail price format affects shopping behavior.

This paper makes three contributions to the literature on how price format affects shopping behavior. First, our model enables us to develop closed-form expressions for optimal purchase quantity and shopping frequency that are amenable to comparative static analysis.[4] Second, our model allows us to explain why, in practice, a HILO store tends to charge a higher average price. Third, we test our model implications on household-level purchase data for 33 product categories and store-level pricing data of about 3,000 stock keeping units (SKUs) at three different stores over a two-year period.

This paper is organized as follows. The next section presents a mathematical, single-product model to capture the shopping behavior of a rational household. It has the following implications. First, for a rational shopper (with a fixed consumption rate), it is optimal to: (1a) buy fewer units (on average) during each visit to the HILO store; and therefore, (1b) make more frequent visits to the HILO store, when compared to an EDLP store. Second, for stores with different price variabilities to be viewed as equally competitive by rational consumers, (2a) the store with a higher average price should have a higher price variability, and (2b) the store with a higher price variability should have a higher average price. In §3, we test these model implications using a scanner panel database provided by IRI. Our empirical

[2] We begin by modeling a rational consumer's shopping frequency and purchase quantity for a single product (or brand). Note that we implicitly model the behavior of a brand-loyal consumer, as the brand and product-category purchase decisions are identical in this context. The same assumption has been made in prior research (e.g., Helsen and Schmittlein 1992, Assuncao and Meyer 1993).

[3] In its purest version, EDLP is a pricing strategy in which the store adopts a constant price for each brand in each category. HILO pricing is a strategy in which the store adjusts the price from time to time.

[4] Kalymon (1971) was first to characterize the general structure of the optimal purchase policy under price uncertainty. We consider a more specific shopping context than that of Kalymon to derive stronger predictions for grocery shopping behavior. Specifically, our model formulation allows us to investigate the impact of price variability on purchase quantity and shopping frequency explicitly.

analyses strongly support all model implications (1a), (1b), (2a), and (2b). In §4, we extend the model to analyze the consumer's optimal consumption rate and end the paper with some concluding remarks and suggestions for future research.

## 2. Rational Shopping under Price Uncertainty

Consider a risk-neutral shopper who consumes a product with a random unit price at a known consumption rate. The shopper makes both *planned* and *unplanned* trips to the store.[5] A planned trip occurs when the product runs out, and hence, the shopper has to buy the product during the trip.[6] We call this purchase a *planned purchase*. An unplanned trip (from the perspective of the product under consideration) occurs when other products run out. Since each unplanned trip occurs before the product under consideration runs out, the shopper has the option not to buy the product during an unplanned trip. If, however, the shopper decides to buy the product during an unplanned trip, the purchase is called an *unplanned purchase*.

The sequence of events associated with each kind of shopping trip can be described as follows. In a planned trip, the shopper incurs a travel-related fixed cost, $K > 0$, that captures the imputed cost associated with the travel time (in order to enhance readability, we have included a list of notation at the end of the paper). While the shopper has knowledge of the price distribution (Alba et al. 1994), she does not observe the actual realized price prior to the store visit. Only upon arriving at the store does the shopper observe the realized price of the product and decide on the purchase quantity. The shopper pays a purchasing-related transaction cost, $k > 0$, that incorporates the imputed cost associated with the shopping time and time waiting in line, and pays the purchasing cost of buying the product.

In an unplanned trip, the shopper does not incur a travel-related fixed cost for the product under consideration, i.e., $K = 0$. When the shopper is at the store, she observes the realized price of the product and then decides on the purchase quantity. If the shopper decides not to buy, this will cost her nothing. If, however, the shopper decides to buy, i.e., makes an unplanned purchase—then she pays a purchasing-related transaction cost $k > 0$ and the purchasing cost of the product. Thus, the key difference between planned and unplanned trips is whether the trip is associated with a travel-related fixed cost $K > 0$ before the price realization is observed.

The decision problem of the shopper on a planned trip is to determine the optimal purchase quantity $Q(p)$, upon observing the price realization $p$, after incurring a travel-related fixed cost $K$. Unplanned shopping has been studied by Kalymon (1971), Golabi (1985), Helsen and Schmittlein (1992), Assuncao and Meyer (1993), Ozekici and Parlar (1993), and Krishna (1994), among others. These authors consider a *periodic* review model in which the shopper freely obtains information about the realized price at the beginning of each period. Thus, the model is equivalent to having the shopper make unplanned visits to the store periodically without incurring the travel-related fixed cost; i.e., $K = 0$. Given the inventory level $I$ and the realized price $p$ at the beginning of each time period, the buyer must decide the purchase quantity that minimizes the expected cost of satisfying all consumption needs. Given a purchasing-related transaction cost $k > 0$, Kalymon (1971) was the first to characterize the structure of the optimal purchasing policy as a $(w(p), W(p))$ policy that can be described as follows: if $I \geq w(p)$, then it is optimal to buy nothing. If, however, $I < w(p)$, then it is optimal to buy $W(p) - I$ units to bring the inventory level up to $W(p)$. While the structure of the optimal policy is known, the specific ways in which shopping frequency and purchase quantity vary with price variability are not.

Golabi (1985), Helsen and Schmittlein (1992), and Krishna (1994) consider a shopping scenario in which both the travel-related fixed cost $K = 0$ and the purchasing-related transaction cost $k = 0$. When

[5] A recent survey conducted by Block and Morwitz (1998) confirmed that both planned and unplanned shopping trips exist with roughly equal frequency.

[6] We conducted a conservative test for evidence of purchases taking place when product inventory is zero. We estimated inventory for each household, category, and shopping trip (assuming fixed consumption and linear draw-down, cf. Bucklin and Lattin (1991)) and counted the number of times purchases occurred when estimated inventory was equal to zero. On average, such purchases occurred 15% of the time. The minimum and maximum were 9% and 21% for dryer softeners and flavored soda, respectively.

demand is known, Golabi (1985) is first to show that the optimal purchasing policy in each period can be prescribed by a sequence of critical price levels, and that the optimal purchase quantity depends on where the realized price (at each period) falls in the critical price levels. In grocery shopping, it is plausible to have the demand (in a period) depend on the consumption rate, which could be a decision variable itself. This motivates Assuncao and Meyer (1993) to extend Golabi's unplanned shopping model to the case where the consumption rate is a decision variable that depends on the inventory level at the beginning of the period and the price observed during that period. They show that that stockpiling in response to price promotions rationally leads to increased rates of consumption for the product. Again, the exact functional relationships between purchase quantity as well as shopping frequency and price variability are not derived.

When applied to grocery shopping, Kalymon's and Golabi's unplanned shopping models are reasonable if the shopper visits the store periodically (e.g., weekly) and the modeling focus is *only* on purchase quantity at the store.[7] The underlying assumptions of the model ($K = 0$ and periodicity of store visits) are restrictive because some shoppers visit stores irregularly and make planned trips to the store.[8] Furthermore, the assumption of periodicity in store visits must be relaxed if shopping frequency is to be modelled explicitly.

Most prior research in marketing has focused on periodic and unplanned shopping. This observation has motivated us to develop a model that allows the shopper to make both planned and unplanned trips to the store in a nonperiodic fashion. Our shopper's objective is to determine the optimal purchasing policy so as to minimize the long run average relevant cost per unit time. When the shopper makes both planned and unplanned trips, the characterization of the optimal policy is still an open research question. We assume the shopper adopts a two-part purchase policy: (a) If the trip is planned, the shopper buys $Q(p)$ units if the realized price is $p$; and (b) if trip is unplanned, the shopper buys according to Kalymon's $(w(p), W(p))$ purchase policy. This purchasing policy structure allows us to better position our work vis-à-vis the existing work.

The fluctuation in the retail price of the product is specified by a stationary probability distribution (or zero order price distribution) that consists of $S$ price scenarios, where each price scenario $s$ corresponds to the case in which the unit price is $p_s$.[9] Prior to the store visit, the shopper does *not* know the price scenario of the product. Based on the observed empirical frequencies of prices, however, the shopper knows the likelihood (or the probability) $\pi_s$ for each price scenario $s$, where $\Sigma_{s=1}^{S} \pi_s = 1$.[10] Let $\mu_P$ be the average price and let $\sigma_P^2$ be the variance of the price, where:

$$\mu_P = \sum_{s=1}^{S} \pi_s \cdot p_s, \tag{2.1}$$

$$\sigma_P^2 = \sum_{s=1}^{S} \pi_s \cdot (p_s - \mu_P)^2. \tag{2.2}$$

We present our model formulation below. We first consider a basic model in which the shopper makes only planned trips. We then show that the key model implications of this basic model remain unchanged when it is extended to include unplanned trips. Throughout the

[7] These models were originally developed to describe purchasing behavior of a manufacturing division in which pricing information of raw materials can be obtained easily without making a trip (via phone or computer terminal). In addition, the purchase-related fixed cost $k$ is often large (e.g., a truck delivering the ordered raw material) compared to the travel-related cost $K$.

[8] We test the null hypothesis that shopping is periodic by computing the following statistic for each household. First, let $T_{n-1}$ (and $T_n$) denote the elapsed time between trip $n-1$ and trip $n$ (and between trip $n$ and trip $n+1$). Furthermore, let $X_n = T_n / T_{n-1}$. If a household visits the store periodically, then $X_n \sim N(1, \sigma^2)$. We find that only 36 (out of 513) households show evidence of periodic shopping (i.e., only 36 households have $X_n$ that are not significantly different from 1).

[9] The zero-order price distribution is assumed for analytical tractability. In addition, we analyzed the average times between consecutive purchases in our 33 product categories and found them to be rather long (from 12 to 108 days with an average of 50 days). Thus, shoppers are unlikely to use current observed prices to form future price expectations.

[10] This type of consumer behavior assumes that, in the absence of explicit advertising information, prior to the store visit, the consumer has knowledge of the price distribution, but not the realization. This assumption is common to many types of pricing models (e.g., Lal and Rao 1997). There is also evidence that consumers have reasonable knowledge of the range of prices for frequently-bought grocery products (e.g., Krishna et al. 1991; Dickson and Sawyer 1990).

model analysis, we assume that the consumption rate of the shopper is fixed at $r$. A constant consumption rate is reasonable for nonfood products such as bathroom tissue and detergents, etc. Therefore, our model implications are less likely to hold true for some food products such as frozen yogurt or cookies, in which the consumption rate is potentially a function of price variability. In §4, we extend the model to allow the consumer to choose an optimal level of consumption rate to maximize her utility.

### 2.1. The Planned Shopping Model

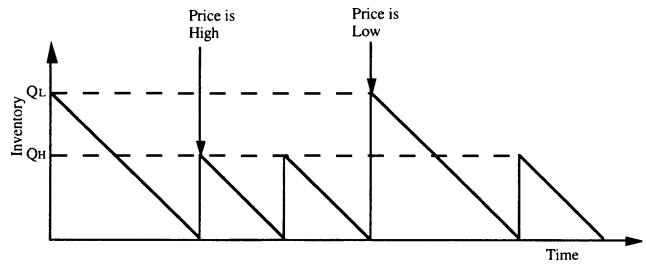
The basic model considers a shopper who makes *only* planned trips to a store. Since the shopper makes only planned trips, each purchase occurs when the inventory drops to zero and the shopper must incur a travel-related cost $K$ to observe the realized price. In addition, the shopper must pay the purchasing-related fixed cost $k$ in order to purchase $Q_s$ units under price scenario $s$. By noting that the effective fixed cost for each purchase is equal to $K + k$ and that the shopper buys when the inventory level $I = 0$, one can re-cast this problem as a special case of the model developed by Kalymon (1971).[11] Our simpler model allows us to obtain a closed-form expression for the optimal purchase quantity and examine the impact of price variability on purchase quantity and shopping frequency.

Suppose the shopper purchases $Q_s$ units under price scenario $s$. Then the elapsed time until the next purchase is given by $Q_s / r$. Figure 1 depicts the inventory pattern and purchasing quantities under a two-price (high / low price) scenario.

The objective of the shopper is to choose the purchase quantity $Q_s$ for each price scenario $s$ so that the total long-run average relevant cost per unit time $\bar{C}(Q_1, \ldots, Q_s, \ldots, Q_S)$ is minimized. The relevant cost includes the travel-related fixed cost, the purchasing-related transaction cost, the inventory cost, and the cost of purchase. The travel-related fixed cost per planned shopping trip, $K$, incorporates the imputed cost associated with the travel time incurred during each store visit. The

[11] To elaborate, suppose we treat $K + k$ as the fixed cost $K$, specified in Kalymon's model, and we set $w(s) = 0$ and $W(s) = Q_s$ for each price scenario $s$. Then it is easy to see that Kalymon's model reduces to our planned shopping model.

**Figure 1** **Inventory Pattern and Purchase Quantities Under Planned Shopping**




purchasing-related transaction cost, $k$, captures the imputed cost associated with the shopping time and time waiting in line. Inventory cost is charged at $h$ per unit per unit time.[12] As shown in Figure 1, the inventory cost incurred until the next purchase is given by $h \cdot Q_s / 2 \cdot Q_s / r$. The purchasing cost under price scenario $s$ is $p_s \cdot Q_s$. Thus, the relevant cost per unit time (until the next purchase) under scenario $s$ is given by:

$$\frac{K + k + p_s \cdot Q_s + h \cdot Q_s^2 / (2 \cdot r)}{Q_s / r}.$$

The expression for the function $\bar{C}(Q_1, \cdots Q_S)$ is given in the following lemma.

LEMMA 1. *The long run average relevant cost per unit time, $\bar{C}(Q_1, \ldots, Q_S)$, is pseudo-convex in $Q_1, \ldots, Q_S$, where $\bar{C}(Q_1, \ldots, Q_S)$ is given by:*

$$\bar{C}(Q_1, \ldots, Q_S) = \frac{K + k + \Sigma_{s=1}^{S} [\pi_s \cdot p_s \cdot Q_s + \pi_s \cdot h \cdot Q_s^2 / (2 \cdot r)]}{\Sigma_{s=1}^{S} [\pi_s \cdot Q_s / r]}. \quad (2.3)$$

PROOF. See Appendix. □

Since $\bar{C}(Q_1, \ldots, Q_S)$ is pseudo-convex, the optimal purchasing policy $(Q_1^*, \ldots, Q_S^*)$ satisfies the first-order conditions. By examining the first-order conditions, we can determine the optimal purchase quantity $Q_s^*$. In preparation, let

[12] To simplify the exposition of the model, we present the case in which the inventory holding cost is independent of the price scenario. However, the model has been extended to the case where the inventory holding cost is dependent on the price scenario. For this more general case, it can be shown that the analysis and the implications of the model remain the same.

$$\hat{K} = K + k - \frac{r}{2 \cdot h} \cdot \sigma_P^2. \tag{2.4}$$

PROPOSITION 1. *The optimal purchasing policy* $(Q_1^*, \ldots, Q_S^*)$ *can be expressed as*

$$Q_s^* = \sqrt{\frac{2 \cdot \hat{K} \cdot r}{h}} - \frac{r}{h} \cdot (p_s - \mu_P). \tag{2.5}$$

*The expected optimal purchase quantity during any store visit, denoted by* $\mu_{Q^*}$, *is*

$$\mu_{Q^*} = \sqrt{\frac{2 \cdot \hat{K} \cdot r}{h}}. \tag{2.6}$$

*The expected optimal time until the next purchase, denoted by* $\mu_{T^*}$, *is*

$$\mu_{T^*} = \sqrt{\frac{2 \cdot \hat{K}}{r \cdot h}}. \tag{2.7}$$

*The optimal long run average expenditure per unit time (i.e., the cost of purchases per unit time excluding the holding and fixed costs), denoted by* $\overline{E^*}$, *is*

$$\overline{E^*} = r \cdot \mu_P - r \cdot \sigma_P^2 \cdot \sqrt{\frac{r}{2 \cdot \hat{K} \cdot h}}. \tag{2.8}$$

*Finally, the optimal long run average relevant cost per unit time, denoted by* $\overline{C^*}$, *where* $\overline{C^*} = \bar{C}(Q_1^*, \ldots, Q_S^*)$, *is*

$$\overline{C^*} = \mu_P \cdot r + \sqrt{2 \cdot \hat{K} \cdot r \cdot h}. \tag{2.9}$$

PROOF. See Appendix. □

Observe from (2.4) that it is quite possible to have $\hat{K} < 0$. To simplify the analysis presented in this paper, we shall assume that $K + k$ is sufficiently large so that $\hat{K} > 0$, and that

$$\sqrt{\frac{2 \cdot \hat{K} \cdot r}{h}} - \frac{r}{h} \cdot (p_s - \mu_P) > 0 \quad \text{for all } s.$$

This assumption enables us to guarantee that $Q_s^*$ as stated in (2.5) is nonnegative.[13]

We now interpret the results stated in Proposition 1. First, observe from (2.5) that a rational shopper will adjust the purchase quantity (linearly) according to the observed price. Specifically, the shopper will buy more (less) than the expected optimal purchase quantity

$$\mu_{Q^*} = \sqrt{\frac{2 \cdot \hat{K} \cdot r}{h}}$$

when the observed price $p_s$ is lower (higher) than the average price $\mu_P$. This adjustment is likely to be higher for products (such as nonperishable products) that have lower inventory holding costs and that have a high consumption rate.

Second, observe from (2.4) that $\hat{K} \leq K + k$, and $\hat{K}$ can be interpreted as the "adjusted" fixed cost per visit under random price shopping. To see this, consider the case in which the store increases its price variance from $\sigma_P^2 = 0$ to $\sigma_P^2 > 0$ while keeping the average price $\mu_P$ fixed. Applying (2.9), one can compare the shopper's optimal relevant cost per unit time for $\sigma_P^2 = 0$ and for $\sigma_P^2 > 0$. It is easy to check that the shopper's optimal relevant cost per unit time reduces by $\sqrt{2 \cdot r \cdot h} \cdot (\sqrt{K + k} - \sqrt{\hat{K}})$ as the store increases the price variance. Observe that the savings generated from price fluctuation are captured in fixed cost reduction from $K + k$ to the "adjusted" fixed cost per visit $\hat{K}$. Price variability thus provides the shopper the option of buying less at higher prices and more at lower prices and this "option value" is shown to be equivalent to a reduction in the fixed cost per visit.[14]

[13] In the event where

$$\sqrt{\frac{2 \cdot \hat{K} \cdot r}{h}} - \frac{r}{h} \cdot (p_s - \mu_P) < 0,$$

$Q_s^*$ could be negative. However, since the long run average relevant cost $\bar{C}(Q_1, \ldots, Q_S)$ is pseudo-convex, it is optimal to truncate those negative $Q_s^*$ to a minimal positive level, say, $Q_s^* = 1$ (in order to satisfy consumption needs).

[14] While it is well known that price variability gives consumers flexibility in product purchasing, our result with respect to store visit behavior is new to the marketing literature.

The fact that the adjusted fixed cost $\hat{K}$ decreases as the variance of the price $\sigma_P^2$ increases enables us to explain Proposition 1 as follows.

• **Expected Optimal Purchase Quantity**. Since $\hat{K}$ decreases as the variance of the price $\sigma_P^2$ increases, (2.6) implies that the expected optimal purchase quantity $\mu_{Q^*}$ is decreasing in $\sigma_P^2$. This implies that the shopper will purchase fewer units per trip, on average, as the price fluctuation increases.

• **Expected Optimal Elapsed Time**. Since $\hat{K}$ decreases as the variance of the price $\sigma_P^2$ increases, (2.7) implies that the expected elapsed time until the next purchase, denoted by $\mu_{T^*}$, is decreasing in $\sigma_P^2$. Thus, a rational shopper should make more frequent trips to the store as the variance of the price $\sigma_P^2$ increases.[15]

• **Optimal Long-Run Average Expenditure**. Since $\hat{K}$ decreases as the variance of the price $\sigma_P^2$ increases, Equation (2.8) implies that the optimal long-run average expenditure per unit time $\overline{E^*}$ decreases when the variance of the price increases. Since price variability provides the flexibility to shop economically by buying more at lower prices and buying less at higher prices, the shopper will spend less on average in the long run when the price fluctuation increases (assuming other things, such as $\mu_P$, remain the same).

• **Optimal Long-Run Average Relevant Cost**. Since $\hat{K}$ decreases as the variance of the price $\sigma_P^2$ increases, (2.9) implies that the optimal long-run average relevant cost per unit time is decreasing in $\sigma_P^2$. Thus, the shopper will find that it is "cheaper" to shop at a store that has higher price fluctuation (assuming other things remain the same).

## 2.2. The Unplanned Shopping Model

The basic model can be extended to include unplanned trips. As indicated above, we assume a purchasing policy that 'combines' the structure of the purchasing policy for planned trips (presented in Proposition 1) and for unplanned trips (developed by Kalymon (1971)). Specifically, our *combined purchasing policy* can be described as follows. First, when $I = 0$, the shopper makes a planned trip: pays a fixed cost $K$, observes the price scenario $s$, makes a planned purchase by paying a transaction cost $k$, and buys $Q_s$ units. (Note that the purchase quantity $Q_s$ could be different from that stated in (2.5) because the shopper now must take the purchases during unplanned trips into consideration.) The inventory level after planned shopping is $Q_s$. Next, when $I > 0$, the shopper makes an unplanned trip and observes the price without incurring the travel-related fixed cost ($K = 0$). If the price scenario is $s$, the shopper follows the $(w_s, W_s)$ policy to determine the purchase quantity; if $I \geq w_s$, then the shopper buys nothing. If, however, $I < w_s$, then the shopper makes an unplanned purchase by paying a transaction cost $k$, and buys $W_s - I$ units so as to bring the inventory level up to $W_s$.

As it turns out, it is very complex to determine the optimal values for $Q_s$, $w_s$ and $W_s$. In order to obtain a closed-form expression for the optimal purchasing policy so that we can perform comparative statics and formulate hypotheses for empirical testing, we consider a special case of the combined purchasing policy by imposing three simplifying assumptions. These assumptions are: (a) $Q_s = W_s$ for all $s$; (b) $w_s = w$ for all $s$, where $w \leq W_s$ for all $s$; and (c) the time between any consecutive unplanned trips is exponentially distributed with a rate $u$, where $u$ is exogenously fixed.[16]

By imposing these simplifying assumptions, we were able to develop closed-form expressions for the optimal purchase quantity, optimal shopping frequency, etc. for a given threshold value $w$. These closed-form expressions are similar to those in Proposition 1 and yield

[15] In other words, price variability increases the rational consumer's incentives for increasing shopping frequency because of the increased option value (i.e., there is a higher probability that the shopper will observe a favorable price).

[16] Assumption (a) requires that the order-up-to levels depend on the observed price scenario $s$ only, regardless of whether the trip is planned or unplanned. Assumption (b) imposes a condition that the lower threshold $w$ is the same for all price scenarios. By imposing that $w \leq W_s$ for all $s$, the shopper buys nothing when $I \geq w$ during an unplanned trip, and buys $W_s - I$ units when $I < w$ (so as to bring the inventory up to $W_s$). The assumption is reasonable if the shopper is reluctant to buy during an unplanned trip when the inventory is sufficiently high. Assumption (c) has been shown to be a reasonable assumption for modeling consumer purchases (see, for example, Morrison and Schmittlein (1988) and Gupta and Morrison (1991)). Under assumption (c), the unplanned trips follow a stationary Poisson process (see, for example, Proposition 3.1 on page 176 in Ross (1980)).

implications identical to those in Proposition 1. In addition, we show that shoppers who shop at HILO stores tend to make more unplanned purchases than those shoppers who shop at EDLP stores (detailed analysis is given in Ho et al. (1997).)

### 2.3. Price Format: EDLP Versus HILO Stores

We now utilize Proposition 1 to investigate two empirical phenomena, the impact of price format on a rational consumer's shopping behavior and the relationship between average price and price variability if the stores are to be viewed equally competitive by the rational shopper. We compare two stores under EDLP and HILO pricing formats. For each store $m$, the travel-related fixed cost is $K(m)$, the purchasing-related fixed cost is $k(m)$, the average price is $\mu_P(m)$, and the variance of the unit price is $\sigma_P^2(m)$, where $m$ = EDLP, HILO.

**2.3.1. Shopping Behavior: Expected Purchase Quantity and Expected Time Until Next Purchase.** Recall that the "adjusted" fixed costs for the case of planned and unplanned shopping (i.e., $\hat{K}$ given in (2.4)) decrease as the price fluctuation increases (i.e., as $\sigma_P^2$ increases). This observation has two implications. First, (2.6) implies that the optimal expected purchase quantity $\mu_{Q^*}$ decreases as $\sigma_P^2$ increases. Thus, a rational shopper will purchase a smaller quantity on average when the price fluctuation increases. Second, (2.7) implies that the optimal expected time until the next purchase, $\mu_{T^*}$ decreases as the price fluctuation increases. Thus, we have proven the following corollary:

COROLLARY 1. *Suppose that both stores have the same fixed costs* ($K$(EDLP) = $K$(HILO) *and* $k$(EDLP) = $k$(HILO)) *and that the variance of the price at store* HILO *is higher* (*i.e.*, $\sigma_P^2$(HILO) > $\sigma_P^2$(EDLP)). *Then the expected optimal purchase quantity is lower for store* HILO *than for store* EDLP, *and the expected optimal elapsed time until the next purchase at any store after purchasing at store* HILO *is shorter than for store* EDLP.

Corollary 1 enables us to formulate the following hypotheses:

HYPOTHESIS 1a. *The average purchase quantity is lower for the* HILO *store than for the* EDLP *store; i.e.*, $\mu_{Q^*}$(HILO) < $\mu_{Q^*}$(EDLP).

HYPOTHESIS 1b. *The average elapsed time until the next purchase is shorter for the* HILO *store than for the* EDLP *store; i.e.*, $\mu_{T^*}$(HILO) < $\mu_{T^*}$(EDLP).

**2.3.2. Retail Price Format: Mean Price and Price Variance.**

**Identical Mean Price.** Consider the case in which both stores have the same average price (i.e., $\mu_P$(HILO) = $\mu_P$(EDLP)) and fixed costs (i.e., $K$(HILO) = $K$(EDLP) and $k$(HILO) = $k$(EDLP)). Since $\sigma_P^2$(HILO) > $\sigma_P^2$(EDLP) and $\mu_P$(HILO) = $\mu_P$(EDLP), (2.9) implies that the minimum long-run average relevant cost associated with the HILO store is lower than that of the EDLP store; i.e., $\overline{C^*}$(HILO) ≤ $\overline{C^*}$(EDLP). Thus, under these conditions, the rational shopper will prefer the HILO format to the EDLP format.

**Identical Optimal Long-Run Average Expenditure.** Let $\overline{E^*}$(EDLP) and $\overline{E^*}$(HILO) be the optimal long run average expenditure per unit time for a shopper shopping at stores EDLP and HILO, respectively. From equation (2.8), $\overline{E^*}$(EDLP) = $\overline{E^*}$(HILO) if and only if:

$$[\mu_P(\text{HILO}) - \mu_P(\text{EDLP})] = \sigma_P^2(\text{HILO})\cdot\sqrt{\frac{r}{2\cdot\hat{K}(\text{HILO})\cdot h}} - \sigma_P^2(\text{EDLP})\cdot\sqrt{\frac{r}{2\cdot\hat{K}(\text{EDLP})\cdot h}} \tag{2.10}$$

Observe that the function

$$y^2\cdot\sqrt{\frac{r}{2(K+k-r/(2h)\cdot y^2)\cdot h}}$$

is increasing in $y$. Combining this observation and the fact that $\sigma_P^2$(HILO) > $\sigma_P^2$(EDLP), we can conclude that $\mu_P$(HILO) > $\mu_P$(EDLP) if and only if $\sigma_P^2$(HILO) > $\sigma_P^2$(EDLP). Thus, we expect the HILO store to charge a higher average price in order to receive revenue from the product category that is identical to that received by EDLP store. (This implication is also valid for the case when the shopper makes both planned and unplanned trips. We omit the details.)

**Identical Optimal Long-Run Average Relevant Cost.** Let $\overline{C^*}$(EDLP) and $\overline{C^*}$(HILO) be the optimal

long-run average relevant cost per unit time for a loyal shopper shopping at stores EDLP and HILO, respectively. Consider the case in which both stores would like to be viewed as equally competitive, in the sense that $\overline{C^*}(\text{EDLP}) = \overline{C^*}(\text{HILO})$. By examining (2.9), it is easy to verify that, for the stores to be viewed as equally competitive, $\mu_P(\text{HILO}) > \mu_P(\text{EDLP})$ if and only if $\sigma_P^2(\text{HILO}) > \sigma_P^2(\text{EDLP})$. This leads us to the following corollary.

COROLLARY 2. *Suppose that both stores have the same fixed costs* ($K(\text{EDLP}) = K(\text{HILO})$ *and* $k(\text{EDLP}) = k(\text{HILO})$), *and would like to be viewed as equally competitive* (*either* $\overline{E^*}(\text{EDLP}) = \overline{E^*}(\text{HILO})$ *or* $\overline{C^*}(\text{EDLP}) = \overline{C^*}(\text{HILO})$). *Then both stores would select a pricing format that satisfies the following property*: $\mu_P(\text{HILO}) > \mu_P(\text{EDLP})$ *if and only if* $\sigma_P^2(\text{HILO}) > \sigma_P^2(\text{EDLP})$.

Corollary 2 enables us to state the following hypotheses:

HYPOTHESIS 2a. *Given any pair of stores and any single SKU, the probability of observing a higher average price in one store is higher if that store also has a higher price variability.*

HYPOTHESIS 2b. *Given any pair of stores and any single SKU, the probability of observing higher price variability in one store is higher if that store also has a higher average price.*

### 2.4. Managerial Implications

Our model results have implications for a cost-sensitive shopper. First, we have shown that the option value due to HILO pricing is equivalent to the reduction of fixed cost per visit (i.e., travel and purchasing related fixed costs) for the shopper. Hence, all things being equal (i.e., both stores impose the same fixed costs and have the same average prices), it is optimal for the shopper to buy at the store that has the highest price variance (because of lower long-run average relevant cost and lower expenditure per unit time). Second, as the store increases the price variance while keeping the average price fixed, it is optimal for the shopper to shop more frequently, buy fewer units (on average) per trip, and spend less per unit time.

Our model results have implications for retailers as well. We have shown that the HILO pricing format is more effective in enticing shoppers to make more frequent trips to the store. Since, however, the HILO pricing format provides more flexibility for shoppers to buy more when the price is low and buy less when the price is high, the revenues from the product category under consideration will be lower to the store per unit time. Thus, there is no dominant pricing format. Specifically, the HILO pricing format (EDLP pricing format) increases (decreases) shopping frequency but it generates lower (higher) revenue from the product category under consideration. This may explain why both pricing formats coexist in practice. Later, in §4, we shall show an additional benefit of HILO pricing is that it may increase the consumption rate of the product category, which leads to a corresponding increase in store revenue.

## 3. Empirical Analysis and Hypothesis Testing

### 3.1. Data

This section presents various empirical tests for the hypotheses presented in the last section. The scanner panel data are drawn from a single IRI Market in a large metropolitan area in the United States. The database contains purchasing information for 33 product categories (9 non-food and 24 food products) over a two-year period (June 1991 to June 1993),[17] covering a total of 66,694 shopping visits taken by 513 households. There are three stores located within the same neighborhood (i.e., within a 3-mile radius) and all the trips are to one of these three stores, which allows us to control somewhat for the travel-related fixed cost associated with a store visit. Hereafter we refer to these stores as $\text{EDLP}_1$, $\text{EDLP}_2$ and HILO. Stores $\text{EDLP}_1$ and $\text{EDLP}_2$ explicitly advertise as operating an EDLP format and store HILO is a HILO store; all three stores are from different chains.[18]

### 3.2. Hypothesis Testing: Purchase Quantity and Time Until Next Purchase

Since our model addresses rational shopping under price uncertainty for a *single* product, we test

[17] The consumption rate for nonfood products is likely to be fairly constant, while the consumption rate for food products may be price-dependent. For completeness, we shall test our hypotheses for both food and nonfood product categories.

[18] We also examine the *actual* pricing behavior of the stores to confirm that their pricing practice is consistent with the advertised price formats (see Ho et al. 1997).

**Table 1 Average Quantities and Interpurchase Times**

| Product Category | Purchase Quantity | | | | Interpurchase Time | | | |
|---|---|---|---|---|---|---|---|---|
| | *Q1* | *Q2* | *Q3* | *Q4* | *Q1* | *Q2* | *Q3* | *Q4* |
| Nonfood Products | | | | | | | | |
| Analgesics | I | S | S | S | I | I | S[a] | S[b] |
| Bar Soap | I | S[a] | S | S | I | I | N | N |
| Bathroom Tissue | S | S | S | S | S | S | S | S |
| Canned Catfood | I | S | S | S | I | I | I | S |
| Dryer Softeners | I | N | N | I | I | I | I | N |
| Liquid Detergents | S | S | S | S | I | I | I | S |
| Paper Towels | I | S | S | S | N[a] | I | I | S |
| Toothpaste | I | S | S | S | I | I | I | N |
| Washer Softeners | I | I | S | S | I | I | I | N |
| Food Products | | | | | | | | |
| Bacon | I | I | I | S | I | I | I | S[a] |
| Barbecue Sauce | I | I | I | S[a] | I | I | I | I |
| Butter | I | S | S | S | I | I | I | I |
| Cereal (Regular) | I | I | I | S | I | S | S | S |
| Cookies (Chip-filled) | I | N | I | S | S | I | I | S[b] |
| Cookies (Sandwich Cream) | I | I | I | S | I | I | I | I |
| Crackers (Flavored) | I | S[a] | S[a] | I | S[b] | I | I | S |
| Crackers (Soda) | I | I | S[a] | I | N | I | I | S |
| Eggs | S | I | I | S | I | I | I | S |
| Frankfurter Sausage | I | S | I | I | I | I | I | I |
| Frozen Pizza | I | S | S | S | I | S | S[a] | S |
| Frozen Yogurt | I | I | I | I | I | I | I | S |
| Ice Cream | S | S[a] | S | S | I | I | I | S |
| Margarine | S[b] | S | S | S | I | I | S | S |
| Meat Sauce | I | I | I | N | I | I | I | I |
| Peanut Butter | I | S | I | S | I | I | I | S |
| Potato Chips | I | I | S | S | I | I | S | I |
| Pretzels | I | I | I | S | I | I | I | S[b] |
| Soda (Cola) | S | S | S | S | I | S | S | S |
| Soda (Flavored) | S | S | S | S | S[b] | S | S | S |
| Spaghetti Sauce | I | I | S | S | I | I | I | N |
| Sugar | I | S | I | S | I | I | I | I |
| Tortilla Chips | S | S[b] | S[a] | S | I | I | I | S |
| Yogurt | I | I | I | N | I | I | S[a] | S |

I = inconclusive (i.e., nonsignificant *F*-statistic).
S = supported; N = not supported. In all cases $p < 0.01$.
[a] $p < 0.05$.
[b] $p < 0.10$.

Hypotheses 1a and 1b using information from each of the thirty-three product categories individually and proceed as follows. First, for every product category, we use the information in the IRI Stub Files to define a "standard unit" of product.[19] We then record, for each household, the amount of product purchased on each store visit. For each category we estimate a one-way analysis of variance (ANOVA) model that specifies its null hypothesis as $\mu_{Q^*}(\text{EDLP}_1) = \mu_{Q^*}(\text{EDLP}_2) = \mu_{Q^*}(\text{HILO})$. Similarly, to test Hypothesis 1b we identify the date on which a purchase takes place, for each category and household. We then compute the elapsed time until the next purchase (at *any* store) in this category by the household. We perform a one-way ANOVA with the null hypothesis $\mu_{T^*}(\text{EDLP}_1) = \mu_{T^*}(\text{EDLP}_2) = \mu_{T^*}(\text{HILO})$.

Note, however, that in order for these tests to be valid, we require the aggregate consumption within a category to be approximately constant across households and stores. It can be easily shown from (2.6) and (2.7) that $\mu_{Q^*}$ is increasing in the consumption rate $r$ while $\mu_{T^*}$ is decreasing in the consumption rate $r$. This implies that, if (say) higher consumption households systematically shop at EDLP stores we could find support for H1a and no support for H1b. That is, we would observe households that buy larger quantities on average *and* shop more frequently at EDLP stores. Thus, in order to control for this sort of heterogeneity in the scanner panel data, we compute an estimate of each household's product-category specific consumption rate and then sort households into four consumption quartiles on a category-by-category basis. (This allows for the possibility that a household that appears in the lowest consumption group (Q1) for bacon may appear in the highest group (Q4) for ice cream.) The household's category-specific consumption rate is estimated by the total purchase quantity in all stores over the fixed two year time horizon (Bucklin and Lattin 1991).

The ANOVAs are then computed separately *within* each consumption quartile. Table 1 presents a 33 by 8 matrix of ANOVA results for each product category and the four consumption quartiles for both average purchase quantities and average inter-purchase times. In order to facilitate exposition and interpretation of the information in the table, we report a single letter indicating support (or absence of support) for the hypotheses.[20] We use three letter designations: *S* and *N* indicate

[19] For example, a standard unit of bacon is 16 oz; buying a 32 oz pack therefore constitutes a purchase of two standard units.

[20] Full supporting results are available from the authors upon request.

that the hypothesis was supported or not supported, given that the ANOVA model was significant ($p < 0.01$); $I$ indicates that the ANOVA model was not significant.[21] When the $F$-statistic is significant, Hypothesis 1a is considered supported in a product category if $\mu_{Q^*}(\text{EDLP}_1) \geq \mu_{Q^*}(\text{HILO})$ and $\mu_{Q^*}(\text{EDLP}_2) \geq \mu_{Q^*}(\text{HILO})$ (and at least one of the inequalities is strictly greater). It is not supported if $\mu_{Q^*}(\text{EDLP}_1) \leq \mu_{Q^*}(\text{HILO})$ and $\mu_{Q^*}(\text{EDLP}_2) \leq \mu_{Q^*}(\text{HILO})$ (with at least one of the inequalities strictly less), and inconclusive otherwise. We use an analogous procedure to classify the results for average interpurchase times, which we expect to be shorter for the HILO store.

The following observations assist in interpretating Table 1. First, note that by definition, each quartile has the same number of households; however, the higher consumption quartiles naturally contain more store visits and purchase observations for a given category under consideration. Second, the average interpurchase time is naturally more variable than the average quantity (the natural range of quantities purchased is much smaller than the range of possible interpurchase times). Together, these two observations imply that we are likely to obtain more instances of statistically significant results in the domain of average quantities, and in the higher consumption quartiles. Table 1 confirms this with 26 of 33 categories supporting H1a in Q4 and 21 of 33 categories supporting H1b in Q4. In addition, we obtain a greater number of instances of support for H1a (71) than for H1b (39). In percentage terms, we find that 54% of the quantity cells indicate support ($S$) for H1a, 42% are inconclusive ($I$) and only 4% run counter to H1a ($N$). For average interpurchase time we have 30% ($S$), 64% ($I$), and 6% ($N$), respectively. Considering H1a and H1b together, we find that 23 categories (70%) support both hypotheses.[22] In addition, the highly penetrated, more frequently purchased products (e.g., bathroom tissue, liquid detergents, margarine, and both types of soda) consistently provide the strongest levels of support for H1a and H1b.

[21] Only one of the significant ANOVA models cannot be classified as either $S$ or $N$. We classify this special case as $I$. In this case, $\text{EDLP}_1$ has the highest purchase quantity but $\text{EDLP}_2$ has the lowest purchase quantity.

[22] Further analysis of the 13 instances of nonsupport (5 for average quantities, 8 for average interpurchase times) shows that in all cases where H1a is not supported $\mu_{Q^*}(\text{HILO}) > \mu_{Q^*}(\text{EDLP}_1) \geq \mu_{Q^*}(\text{EDLP}_2)$; for H1b $\mu_{T^*}(\text{HILO}) = \mu_{T^*}(\text{EDLP}_i) \geq \mu_{T^*}(\text{EDLP}_j)$ in 6 of 8 cases and $\mu_{T^*}(\text{HILO}) > \mu_{T^*}(\text{EDLP}_1) \geq \mu_{T^*}(\text{EDLP}_2)$ in the remaining two instances.

Thus, overall we find very strong empirical support for both H1a and H1b. Further confirmation of this claim can be obtained by considering the probability of support given a significant model. The following table shows this.

| | Quantity | | | | Timing | | | |
|---|---|---|---|---|---|---|---|---|
| | Q1 | Q2 | Q3 | Q4 | Q1 | Q2 | Q3 | Q4 |
| Percent ($S$\|significant) | 100% | 90% | 95% | 93% | 67% | 100% | 90% | 81% |

### 3.3. Hypothesis Testing: Mean Price and Price Variance

We test Hypotheses 2a–b by comparing the relative mean prices and price variability across the three stores. Specifically, we are interested in examining the following questions: If for a given SKU the HILO store has a higher price variability (relative to an EDLP store), does it also charge a higher average price? Conversely, do stores that charge a higher average price have higher price fluctuation? We use the prices of all SKUs that are common to any two stores to test Hypotheses 2a–b. On average, there are about 3,000 common SKUs between any pair of stores (we have three stores and three paired comparisons). For each SKU $i$ in store $m$, we determine the mean and standard deviation of the weekly prices over a 104-week period. For any pair of stores $(m_1, m_2)$, let $\Delta^i_{\mu_P}(m_1, m_2) = \mu^i_P(m_1) - \mu^i_P(m_2)$ be the difference in the average price of SKU $i$ between stores $m_1$ and $m_2$, and let $\Delta^i_{\sigma_P}(m_1, m_2) = \sigma^i_P(m_1) - \sigma^i_P(m_2)$ be the difference in the standard deviation. To formally test whether the relationship between average prices provides any information about relative price variability and vice versa, we reformulate Hypotheses 2a and 2b using conditional probabilities:

$$\text{Prob}(\Delta^i_\mu(m_1, m_2) > 0 \mid \Delta^i_\sigma(m_1, m_2) > 0) = \text{Prob}(\Delta^i_\mu(m_1, m_2) > 0), \tag{3.1}$$

$$\text{Prob}(\Delta^i_\mu(m_1, m_2) > 0 \mid \Delta^i_\sigma(m_1, m_2) < 0) = \text{Prob}(\Delta^i_\mu(m_1, m_2) > 0), \tag{3.2}$$

$$\text{Prob}(\Delta^i_\sigma(m_1, m_2) > 0 \mid \Delta^i_\mu(m_1, m_2) > 0) = \text{Prob}(\Delta^i_\sigma(m_1, m_2) > 0), \tag{3.3}$$

**Table 2 The Proportions**

| Event | Proportions | | |
|---|---|---|---|
| | EDL$P_1$ − EDL$P_2$ | EDL$P_1$ − HILO | EDL$P_2$ − HILO |
| $\text{Prob}(\Delta^i_\mu(m_1, m_2) > 0)$ | 0.6475 | 0.2868 | 0.2450 |
| $\text{Prob}(\Delta^i_\mu(m_1, m_2) > 0 \mid \Delta^i_\sigma(m_1, m_2) > 0)$ | 0.6357 | 0.5667 | 0.5568 |
| $\text{Prob}(\Delta^i_\mu(m_1, m_2) > 0 \mid \Delta^i_\sigma(m_1, m_2) < 0)$ | 0.6638 | 0.0804 | 0.0589 |
| $\text{Prob}(\Delta^i_\sigma(m_1, m_2) > 0)$ | 0.5783 | 0.4244 | 0.3738 |
| $\text{Prob}(\Delta^i_\sigma(m_1, m_2) > 0 \mid \Delta^i_\mu(m_1, m_2) > 0)$ | 0.5677 | 0.8385 | 0.8494 |
| $\text{Prob}(\Delta^i_\sigma(m_1, m_2) > 0 \mid \Delta^i_\mu(m_1, m_2) < 0)$ | 0.5978 | 0.2578 | 0.2195 |

$$\text{Prob}(\Delta^i_\sigma(m_1, m_2) > 0 \mid \Delta^i_\mu(m_1, m_2) < 0) = \text{Prob}(\Delta^i_\sigma(m_1, m_2) > 0). \tag{3.4}$$

Under Hypotheses 2a and 2b we asserted that for a pair of stores, there is an "information" relationship between the mean and the standard deviation of the price. In the context of Equations (3.3)–(3.6), we aim to test whether the conditional and the unconditional probabilities are equal. We conduct proportion tests to investigate our hypotheses.[23]

Table 2 presents, for each pair of stores, the unconditional and conditional probabilities; Table 3 shows the accompanying test statistics. Note that for comparisons between EDLP stores (EDLP$_1$ − EDLP$_2$), there were no significant differences between the conditional and unconditional probabilities. However, in the case of the EDLP-HILO comparisons (EDLP$_1$ − HILO, EDLP$_2$ − HILO), all were highly significant. Hence, stores that charge higher average prices tend to have higher price variance and vice versa. Thus, we have strong support for Hypotheses 2a and 2b.

### 3.4. Summary

In summary, we find strong support for all four hypotheses. Thus we conclude that shoppers who visit stores with higher price variability tend to (1a) purchase smaller quantities of product per visit, (1b) shop more frequently, and that (2a, b) stores that charge higher average prices tend to have higher price variability and vice versa (there is information content in relative average prices and price variance).

[23] The proportions test requires the event $\Delta^i_x(m_1, m_2) > 0$ where $x = \mu, \sigma$ to be statistically independent among different SKUs. With a sample of 3,000 SKUs, there are about 4.5 million pairs of random variables. We randomly sample 150 pairs from the 4.5 million pairs and determine the correlation between each pair of random variables. Overall, the analysis indicates that the correlation between the random variables is small. Details of the analysis are available from the authors upon request.

## 4. Discussion

In this section, we relax the assumption that the consumption rate is fixed and allow the shopper to choose her consumption rate to maximize her utility. As we shall see, our result complements that of Assuncao and Meyer (1993). Essentially, Assuncao and Meyer (1993) examined the first order effect of price on the optimal consumption rate in a period by showing that it increases as the observed price decreases. We analyze the second order effect of price on the optimal average consumption rate by showing that it increases as the price variance $\sigma_P^2$ increases.

### 4.1. Optimal Consumption Rate

Let $V(r)$ be the value of consuming $r$ units of the product per unit time, subtracting all costs associated with that consumption rate. Following Assuncao and Meyer (1993), a separable utility function $V(\cdot)$ can be written as:

$$V(r) = U(r) - \overline{C^*}(r), \tag{4.1}$$

where $U(r)$ is the utility derived from consuming $r$ units of product per unit time and $\overline{C^*}(r)$ is the optimal long run relevant cost per unit time. By substituting $\overline{C^*}(r)$ from (2.9) into the equation above, the utility maximization problem can be rewritten as:

**Table 3 The Proportion Tests**

| Hypothesis | $t$-statistics | | |
|---|---|---|---|
| | EDL$P_1$ − EDL$P_2$ | EDL$P_1$ − HILO | EDL$P_2$ − HILO |
| $\text{Prob}(\Delta^i_\mu(m_1, m_2) > 0 \mid \Delta^i_\sigma(m_1, m_2) > 0) = \text{Prob}(\Delta^i_\mu(m_1, m_2) > 0)$ | −0.8247 | 17.0773 | 18.5325 |
| $\text{Prob}(\Delta^i_\mu(m_1, m_2) > 0 \mid \Delta^i_\sigma(m_1, m_2) < 0) = \text{Prob}(\Delta^i_\mu(m_1, m_2) > 0)$ | 1.0235 | −16.4874 | −16.5325 |
| $\text{Prob}(\Delta^i_\sigma(m_1, m_2) > 0 \mid \Delta^i_\mu(m_1, m_2) > 0) = \text{Prob}(\Delta^i_\sigma(m_1, m_2) > 0)$ | −0.7390 | 21.1140 | 22.6534 |
| $\text{Prob}(\Delta^i_\sigma(m_1, m_2) > 0 \mid \Delta^i_\mu(m_1, m_2) < 0) = \text{Prob}(\Delta^i_\sigma(m_1, m_2) > 0)$ | 1.1087 | −12.1213 | −11.785 |

$$\text{Max}_r V(r) = \text{Max}_r\{U(r) - \mu_P \cdot r - \sqrt{2 \cdot \hat{K} \cdot r \cdot h}\}. \quad (4.2)$$

To reflect diminishing marginal return per unit time from consuming at rate $r$, $U(.)$ is often specified as a concave function. The commonly used semilog function, i.e., $U(r) = \beta_0 + \beta_1 \cdot \log(r)$, is assumed here to illustrate how one might go about determining the optimal consumption rate. Since $U(r) > 0$, we have $r > r_0 = e^{-\beta_0/\beta_1}$. Hence, it suffices to consider $r$ such that $r > r_0$. Differentiating $V(r)$ with respect to $r$, we have:

$$V'(r) = \frac{\beta_1}{r} - \mu_P - \frac{2 \cdot (K + k) \cdot h - 2 \cdot \sigma_p^2 \cdot r}{2 \cdot \sqrt{2 \cdot (K + k) \cdot h \cdot r - \sigma_p^2 \cdot r^2}}. \quad (4.3)$$

Notice that the sum of the first two terms on the right-hand side can be interpreted as the marginal return per unit time from consuming $r$ net the average price $\mu_P$; i.e., $U'(r) - \mu_P$. It is reasonable to assume that the consumption rate is such that $U'(r) - \mu_P > 0$. Therefore, we shall focus on the case when $\beta_1 / r - \mu_P > 0$. This condition holds when $r > r_1$, where $r_1 = \beta_1 / \mu_P$. In this case, the optimal consumption rate $r^*$ must be located between $r_0$ and $r_1$. Thus, it is sufficient to restrict our attention to $r \in (r_0, r_1)$.

PROPOSITION 2. *If*

$$V'(r_0) > 0 \quad \textit{and} \quad \sigma_p^2 < \frac{(K + k) \cdot h \cdot \mu_P}{\beta_1},$$

*then the optimal consumption rate $r^*$ increases with price fluctuation $\sigma_P^2$.*

PROOF. See Appendix. □

The first condition $V'(r_0) > 0$ implies that the net marginal return of the utility is positive for $r = r_0$. This condition is reasonable and guarantees that the optimal consumption rate $r^* > r_0$. The second condition

$$\sigma_P^2 < \frac{(K + k) \cdot h \cdot \mu_P}{\beta_1}$$

is reasonable because it is equivalent to the condition that requires the long-run average relevant cost $\overline{C^*}(r)$ to be increasing in $r \in (r_0, r_1)$ for any average price $\mu_P$. Under these conditions, Proposition 2 implies that a utility maximizing shopper will increase her average consumption in the face of price fluctuation. This result shows that even if the consumption rate is independent of stockpiling or inventory level, it is rational for the shopper to increase the consumption rate under price fluctuation. Thus, a higher consumption rate is potentially an additional benefit of the HILO pricing format.

### 4.2. Summary and Future Research

In this paper, we have attempted to analyze the impact of price format on the shopping frequency and purchasing behavior of a rational shopper. The closed-form expression for the optimal shopping policy enables us to elegantly characterize the optimal shopping policy of a rational shopper and to examine the strengths and weaknesses of retail price formats. Our model results show that there is no dominant pricing format and provide a rational underpinning for why HILO and EDLP price formats coexist in practice (c.f., Lal and Rao 1997). Finally, our multi-category scanner panel database allows us to test specific model implications involving rational shopping frequency and purchasing behavior. Overall, we find support for our model implications. Thus, we conclude that shoppers who visit stores with higher price variability tend to (1a) purchase smaller quantities of product per visit, (1b) shop more frequently, and that (2a, b) stores charging higher average prices tend to have higher price variability and vice versa (there is information content in relative average prices and price variance).

There are research questions related to multi-category pricing that we think deserve attention. For instance, what is the impact of multi-category pricing format on the shopping frequency and purchasing behavior of a rational shopper? How should retailers coordinate their price formats for individual product categories (HILO for certain products while EDLP for others) so as to maximize store revenue? We intend to pursue these questions in future research.[24]

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## Notation: In the Order of Appearance

Let:

$T_n$ = elapsed time between trip $n$ and trip $n + 1$

$s$ = index for price scenario, where $s = 1, \ldots, S$

$p_s$ = the realized price for price scenario $s$

$\pi_s$ = probability of a particular price scenario $s$

$\mu_X$ = expected value of any random variable $X$; for example, $\mu_P$ = average price

$\sigma_X^2$ = variance of any random variable $X$; for example, $\sigma_P^2$ = variance of price

$Q_s$ = purchase quantity under price scenario $s$ during a planned trip

$r$ = consumption rate in units per unit time

$\bar{C}(Q_1, \ldots, Q_S)$ = long run average relevant cost per unit time for the planned shopping model

$K$ = travel-related fixed cost associated with a planned shopping trip

$k$ = purchasing-related fixed cost associated with a purchase

$h$ = inventory holding cost per unit per unit time

$\hat{K}$ = 'adjusted' fixed cost associated with a planned purchase

$Q_s^*$ = optimal purchase quantity during a planned trip

$\mu_{Q^*}$ = expected optimal purchase quantity per trip

$\mu_{T^*}$ = expected optimal time until the next purchase

$\overline{E^*}$ = optimal long run average expenditure per unit time

$\overline{C^*}$ = optimal long run average relevant cost per unit time

$m$ = index for store

$i$ = index for SKU

$\mu_P^i(m)$ = average price of SKU $i$ at store $m$

$\Delta_{\mu_P}^i(m_1, m_2)$ = difference in average price of SKU $i$ between store $m_1$ and $m_2$

$\Delta_{\sigma_P}^i(m_1, m_2)$ = difference in standard deviation of price of SKU $i$ between store $m_1$ and $m_2$

$I$ = inventory level before an unplanned purchase (a random variable)

$w_s$ = reorder point associated with price scenario $s$ during an unplanned trip

$W_s$ = order-up-to level associated with price scenario $s$ during an unplanned trip

$w$ = common reorder point as a result of assumption (b)

$u$ = the rate of a shopper making unplanned trips

$V(r)$ = value of consuming $r$ units per unit time net all costs associated with that consumption

$U(r)$ = utility derived from consuming $r$ units

### Appendix

PROOF OF LEMMA 1. Since each planned trip occurs when the product runs out, the shopper makes a planned purchase during each store visit. Let $T_n$, $n \geq 1$ be the random variable that corresponds to the elapsed time until the next planned purchase after the $n$th visit (i.e., planned purchase). Clearly, $T_n$ depends on the price observed during the $n$th visit. Since the price scenarios (i.e., $p_s$) observed during different store visits are assumed to be independent and identically distributed (*i.i.d*), the purchase quantity during different store visits are *i.i.d*. In this case, we can conclude that the times until next purchase, $T_n$, $n \geq 1$, are *i.i.d*. Let $N(t)$ be the counting process that specifies the number of store visits from time 0 to time $t$. Clearly, $\{N(t), t \geq 0\}$ is a renewal process, and the time until the next planned purchase is a renewal cycle.

Next, let $C_n$ be the total cost (i.e., the fixed cost of a shopping trip (travel and purchasing related fixed costs), the purchasing cost, and the inventory holding cost until the next purchase) incurred during the $n$th visit. For simplicity, we assume that $C_n$ is incurred immediately after the $n$th visit. Let:

$$TC(t) = \sum_{n=1}^{N(t)} C_n, \tag{5.1}$$

where $TC(t)$ represents the total cost incurred by time $t$. Let $E(C_n) = \mu_C$ and $E(T_n) = \mu_T$. Notice that $\mu_C$ corresponds to the expected total cost incurred during a visit and $\mu_T$ corresponds to the expected time elapsed until the next purchase. In this case, it is easy to show that,

$$\mu_C = K + k + \sum_{s=1}^{S}\left[\pi_s \cdot p_s \cdot Q_s + \pi_s \cdot h \cdot \frac{Q_s^2}{2 \cdot r}\right] \text{ and that}$$

$$E(T) = \sum_{s=1}^{S}\left\{\pi_s \cdot \frac{Q_s}{r}\right\}.$$

In this case, we can apply Proposition 4.1 in Ross (1980), which states that the long run average relevant cost is equal to the expected cost incurred during a renewal cycle (i.e., expected cost incurred until the next purchase) divided by the expected length of the renewal cycle (i.e., expected time until the next purchase), to show the following is true: If $\mu_C < \infty$ and $\mu_T < \infty$, then with probability 1:

$$\bar{C}(Q_1, \ldots, Q_N) = \frac{E(TC(t))}{t} \to \frac{\mu_C}{\mu_T} \text{ as } t \to \infty.$$

By substituting $\mu_C$ and $\mu_T$ into the above expression, we have $\bar{C}(Q_1, \ldots, Q_S)$ given by (2.3).

Next, we shall prove that $\bar{C}(Q_1, \ldots, Q_S)$ is pseudo-convex. First, let $y_s = \pi_s \cdot (Q_s/r)$. Then we can rewrite the function $\bar{C}(Q_1, \ldots, Q_S)$ in terms of $y_s$. Specifically, we have:

$$\bar{C}(y_1, \ldots, y_S) = \frac{g_1(y_1, \ldots, y_S)}{g_2(y_1, \ldots, y_S)}. \tag{5.2}$$

where the function $g_1(y_1, \ldots, y_S) = K + k + \Sigma_{s=1}^{S}\ [r \cdot p_s \cdot y_s + (r \cdot h)/(2 \cdot \pi_s) \cdot y_s^2]$, and $g_2(y_1, \ldots, y_S) = \Sigma_{s=1}^{S}\ [y_s]$. Since $g_1(y_1, \ldots, y_S)$ is quadratic and $g_2(y_1, \ldots, y_S)$ is linear, the function $\bar{C}(y_1, \ldots, y_S)$ takes on the form of a quadratic function divided by a linear function. In this case, we can apply Avriel's (1976, p. 156) result that a function is pseudo convex when the function takes on the form of dividing a quadratic function by a linear function, to show that the function $\bar{C}(y_1, \ldots, y_S)$ is pseudo-convex.

PROOF OF PROPOSITION 1. Recall from Lemma 1 that the function $\bar{C}(.)$ is pseudo-convex in $Q_1, \ldots, Q_s$. We can apply Theorem 6.7 in Avriel (1976), which states that the minimum of any pseudo-convex function satisfies the first order conditions, to show that the optimal purchasing policy $(Q_1^*, \ldots, Q_S^*)$ must satisfy the first-order conditions; i.e., $(\partial\bar{C}/\partial Q_s) = 0$, $s = 1, \ldots, S$. By taking the partial derivatives of $\bar{C}(Q_1, \ldots, Q_S)$ in (2.3) with respect to $Q_s$, $s = 1, \ldots, S$ and by setting $(\partial\bar{C}/\partial Q_s) = 0$, it is easy to show that the optimal purchasing policy satisfies:

$$Q_j^* = Q_i^* + \frac{r}{h} \cdot (p_i - p_j),\ j, i \in \{1, \ldots, S\}. \tag{5.3}$$

Since

$$\text{Min}_{Q_1, \ldots, Q_S}\bar{C}(Q_1, \ldots, Q_S) = \text{Min}_{Q_1}\{\text{Min}_{Q_2, \ldots, Q_S}\bar{C}(Q_1, \ldots, Q_S)\},$$

we can substitute $Q_s$, $s = 2, \ldots, S$ in (5.3), as a function of $Q_1$, into the function $\bar{C}(Q_1, \ldots, Q_S)$, and obtain $\bar{C}(Q_1, \ldots, Q_S) = \bar{C}(Q_1, Q_1 + (p_1 - p_2) \cdot r/h, \ldots, Q_1 + (p_1 - p_S) \cdot r/h)$. Let $\overline{C^*} = \bar{C}(Q_1^*, \ldots, Q_S^*)$. In this case, we have:

$$\overline{C^*} = \text{Min}_{Q_1}\left(\frac{K + k + \Sigma_{s=1}^{S}\left[\pi_s \cdot p_s \cdot \left(Q_1 + (p_1 - p_s) \cdot \frac{r}{h}\right) + \pi_s \cdot \frac{h \cdot (Q_1 + (p_1 - p_s) \cdot \frac{r}{h})^2}{2r}\right]}{\left(\Sigma_{s=1}^{S}\left[\pi_s \cdot \left(Q_1 + (p_1 - p_s) \cdot \frac{r}{h}\right)\right]\right)/r}\right). \tag{5.4}$$

To simplify the right hand side of the above expression, the following transformation is useful: $x = Q_1 + r/h(p_1 - \mu_P)$. By substituting $Q_1 = x - (p_1 - \mu_p) \cdot r/h$ into (5.4) and by rearranging the terms, it can be shown that $\text{Min}_{Q_1}\bar{C}(Q_1, Q_1 + (p_1 - p_2) \cdot r/h, \ldots, Q_1 + (p_1 - p_S) \cdot r/h)$. $= \text{Min}_x\ \bar{C}(x)$, where $\bar{C}(x)$ is given by:

$$\bar{C}(x) = \frac{\hat{K} + \mu_p \cdot x + \frac{h}{2r} \cdot x^2}{\frac{x}{r}}. \tag{5.5}$$

It is easy to check that $\bar{C}(x)$ is convex in $x$. By setting $\partial\bar{C}(x)/\partial x = 0$, it can be shown that the optimal value, $x^*$, satisfies $x^* = \sqrt{2 \cdot \hat{K} \cdot r/h}$. By substituting $x^*$ into $x = Q_1 + r/h \cdot (p_1 - \mu_P)$, we obtain $Q_1^*$ as stated in the proposition. Then by substituting $Q_1^*$ into (5.3), we obtain $Q_s^*$. Since $K + k$ is sufficiently large, all $Q_s^*$ stated in (2.5) are nonnegative. In this case, we can conclude that $Q_s^*$ is an interior point that satisfies the first-order conditions. Thus, $Q_s^*$ is a global minimum.

Next, given the expressions for $Q_s^*$, one can show that the expected purchase quantity, $\mu_{Q^*}$, the expected time elapsed until the next purchase, $\mu_{T^*}$, and the long run average expenditure per unit time, $\overline{E^*}$, are as given in the proposition. Finally, by substituting the expression for $x^*$ into $\bar{C}(x)$, it is easy to show that $\overline{C^*} = \bar{C}(Q_1^*, \ldots, Q_S^*) = \sqrt{2 \cdot \hat{K} \cdot r \cdot h} + \mu_p \cdot r$. This completes the proof.

PROOF OF COROLLARY 1. First note that $\mu_{Q^*}$ is increasing in $\hat{K}$. However, $\hat{K} = K + k - r/2h \cdot \sigma_P^2$ is decreasing in the price variability, $\sigma_P^2$. Since $\sigma_P^2(\text{HILO}) > \sigma_P^2(\text{EDLP})$ then $\mu_{Q^*}(\text{HILO}) < \mu_{Q^*}(\text{EDLP})$. Hence, the purchase quantity at the EDLP store is higher than at the HILO store. Next, when the consumption rates are the same for both stores, the expected time until the next purchase is given by $\mu_{Q^*}(\text{HILO})/r < \mu_{Q^*}(\text{EDLP})/r$. Therefore, we have shown that the expected time until the next purchase after purchasing the product at a HILO store is shorter. This completes the proof.

PROOF OF COROLLARY 2. When both stores want to be viewed as equally competitive ($\overline{C^*}(\text{EDLP}) = \overline{C^*}(\text{HILO})$), it is easy to check from (2.9) or (2.4) that the condition $\overline{C^*}(\text{EDLP}) = \overline{C^*}(\text{HILO})$ can be expressed as:

$$\mu_P(\text{EDLP})r + \sqrt{2\left(K(\text{EDLP}) + k(\text{EDLP}) - \sigma_P^2(\text{EDLP}) \cdot \frac{r}{2h}\right) \cdot r \cdot h}$$

$$= \mu_P(\text{HILO})r + \sqrt{2\left(K(\text{HILO}) + k(\text{HILO}) - \sigma_P^2(\text{HILO}) \cdot \frac{r}{2h}\right) \cdot r \cdot h}.$$

It follows from the supposition that $K(\text{EDLP}) = K(\text{HILO}) = K$ and $k(\text{EDLP}) = k(\text{HILO}) = k$, so we can simplify:

$$(\mu_P(\text{HILO}) - \mu_p(\text{EDLP}))\cdot r = \sqrt{2\left(K + k - \sigma_P^2(\text{EDLP})\cdot\frac{r}{2h}\right)\cdot r\cdot h} - \sqrt{2\left(K + k - \sigma_P^2(\text{HILO})\cdot\frac{r}{2h}\right)\cdot r\cdot h}.$$

In order for the above equation to hold, we must have $\sigma_P^2(\text{HILO}) > \sigma_P^2(\text{EDLP})$ if and only if $\mu_P(\text{HILO}) > \mu_P(\text{EDLP})$. This completes the proof.

PROOF OF PROPOSITION 2. Let us recall from (4.3) that the first order condition $V'(r) = 0$ can be rewritten as:

$$\frac{(2\cdot\mu_p\cdot r - 2\cdot\beta_1)^2}{r} = \frac{(2\cdot(K+k)\cdot h - 2\cdot\sigma_p^2\cdot r)^2}{2\cdot(K+k)\cdot h - \sigma_p^2\cdot r}. \tag{5.6}$$

In preparation, let the left-hand side be $F(r)$ and the right-hand side $G(r, \sigma_p^2)$. Note that $F(.)$ is not a function of $\sigma_P^2$, that both $F(r)$ and $G(r, \sigma_p^2)$ are decreasing and convex in $r$ for any fixed value of $\sigma_P^2$. In addition, $G(r, \sigma_p^2)$ is decreasing in $\sigma_P^2$ for any given value of $r$. We shall utilize these characteristics to prove our result.

Our proof is structured as follows. We first show that there is a unique solution $r^* \in (r_0, r_1)$ that satisfies $V'(r) = 0$. Then we show that this particular $r^*$ is increasing in $\sigma_P^2$.

First, since $V'(r_0) > 0$, we have $F(r_0) > G(r_0, \sigma_P^2)$. Also, since $\sigma_P^2 < (K + k)\cdot h\cdot\mu_P/\beta_1$ and since $r_1 = \beta_1/\mu_P$, it is easy to show that $G(r_1, \sigma_P^2) > F(r_1)$. In this case, we have $V'(r_0) > 0$ and $V'(r_1) < 0$. It follows from the mean-value theorem, there exists an $r \in (r_0, r_1)$ such that $V'(r) = 0$; i.e., the two functions $F(r)$ and $G(r, \sigma_P^2)$ cross at least once within the region $(r_0, r_1)$.

To argue for the uniqueness of $r \in (r_0, r_1)$ that has $V'(r) = 0$, we shall show that $F(r)$ and $G(r, \sigma_P^2)$ cross exactly once within the region $(r_0, r_1)$. First, since $F(r_0) > G(r_0, \sigma_P^2)$ and $F(r_1) < G(r_1, \sigma_P^2)$, the number of cross points within the region $(r_0, r_1)$ must be odd. Second, since $F(r) = G(r, \sigma_P^2)$ is a cubic equation, the two functions cross, at most, three times for any real $r$. Third, the two functions cross at least once outside the region $(r_0, r_1)$. To see that, consider the range $((K + k)\cdot h/\sigma_P^2, 2\cdot(K + k)\cdot h/\sigma_P^2)$. Since $\sigma_P^2 < (K + k)\cdot h\cdot\mu_p/\beta_1$, this range lies outside the region $(r_0, r_1)$. In this case, it is easy to show that $F((K + k)\cdot h/\sigma_P^2) > G((K + k)\cdot h/\sigma_p^2, \sigma_P^2)$ and $F(2\cdot(K + k)\cdot h/\sigma_p^2) < G(2\cdot(K + k)\cdot h/\sigma_p^2, \sigma_p^2)$. By using the same argument presented earlier in the proof, it is easy to see that the two functions cross at least once within the range $((K + k)\cdot h/\sigma_P^2, 2\cdot(K + k)\cdot h/\sigma_P^2)$. Combining these three observations, we can conclude that the two functions cross exactly once at $r^* \in (r_0, r_1)$. Notice that $r^*$ is a maximum because $V'(r_0) > 0$.

We now show the unique cross point $r^*$ increases in $\sigma_P^2$. We prove this by contradiction. Suppose that $\sigma_{P1}^2 < \sigma_{P2}^2$ and that the corresponding cross points are $r^*(\sigma_{P1}^2) > r^*(\sigma_{P2}^2)$. It is easy to check that $G(r^*(\sigma_{P2}^2), \sigma_{P1}^2)) < F(r^*(\sigma_{P2}^2))$. Since $r^*(\sigma_{P2}^2)$ is a cross point, $F(r^*(\sigma_{P2}^2)) = G(r^*(\sigma_{P2}^2), \sigma_{P2}^2)$. Combining these two observations, we have $G(r^*(\sigma_{P2}^2), \sigma_{P1}^2)) < G(r^*(\sigma_{P2}^2), \sigma_{P2}^2)$, which contradicts the fact that the function $G$ is decreasing in $\sigma_P^2$. This completes the proof.

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