

# Cournot Competition Under Yield Uncertainty: The Case of the U.S. Influenza Vaccine Market

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This paper is inspired by the recurring mismatch between demand and supply in the U.S. influenza vaccine market. Economic theory predicts that an oligopolistic market with unregulated but costly entry will experience excess entry and oversupply, not the undersupply observed in the market for influenza vaccine in recent years. In this paper, we examine the interaction between yield uncertainty, a key characteristic of many production processes, including that for influenza vaccine, and firms' strategic behavior. We find that yield uncertainty can contribute to a high degree of concentration in an industry and a reduction in the industry output and the expected consumer surplus in equilibrium. We use parameter values loosely based on the U.S. influenza vaccine market to numerically illustrate the impact of yield uncertainty.

*Key words:* Cournot competition; influenza vaccine; yield uncertainty; market structure

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## 1. Introduction

Many competitive industries are characterized by substantial yield uncertainty in their production process. Some examples include semiconductors, food processing, biopharmaceuticals, and resource-based industries such as mining and agriculture. Although extensive research has been done on competition and yield uncertainty independently, the intersection of the two areas is, surprisingly, almost empty. In this paper, we analyze the impact of production yield uncertainty on the equilibrium outcome and the social optimum for the classic Cournot model of oligopolistic competition with endogenous entry. To help illustrate the type of forces underlying firms' entry and production decisions and how they might be affected by yield uncertainty, we provide a brief background on the U.S. influenza vaccine market, the context which inspired this work. After that, we formulate the research questions that we set out to answer and which apply equally to the other examples mentioned above.

Every year, influenza results in nearly 36,000 deaths from related complications (Thomson et al. 2003)

and annual health care costs of \$11–\$18 billion (World Health Organization 2002). Recent seasons have seen vaccine shortages, which is surprising given that vaccination is the most cost-effective method of protection against influenza. It has been suggested that a high degree of market concentration<sup>1</sup> combined with production-related problems has resulted in these shortages (Scherer 2007, *The New York Times* 2004, *The Boston Globe* 2004).

Various reasons have been put forth for the high degree of market concentration such as low price, insufficient incentives, and uncertain demand (*Forbes* 2004, *Newsweek* 2004, *Time* 2004). However, a cursory inspection of the market characteristics does not conclusively support these claims. The price for influenza vaccine, unlike other vaccines, is not controlled by the

<sup>1</sup> The number of firms producing injectible influenza vaccine (which accounts for more than 98% of the influenza vaccine market) for the United States has declined steadily from more than a dozen in the 1970s to around five in the 1990s to a low of two in 2004 (Brown 2004). Currently, three manufacturers (Sanofi Pasteur, Novartis, and GlaxoSmithKline) are licensed to supply injectible influenza vaccine to the U.S. market.

government (Danzon et al. 2005) and has increased from \$2 to around \$8 per dose in the past five years (*Forbes* 2004). Also, the demand for influenza vaccine has been increasing steadily over the past decade as can be seen from the immunization rates (O'Mara 2003). Other possible reasons for exit of firms include mergers and acquisitions, plant closures resulting from inability to meet stringent regulatory standards, the lack of coverage under the National Vaccine Injury Compensation Program for vaccine producers (until July 1, 2005), and the market for vaccines being less profitable and much smaller compared with that for other pharmaceutical products. Danzon et al. (2005) argue that high country-specific regulatory cost is one of the key factors that would drive the long term equilibrium in the U.S. flu vaccine market to be characterized by one or two suppliers.

Production problems in the influenza vaccine market arise primarily from the combination of long production lead time, short immunization season, and frequent changes in the vaccine composition. The manufacturing process for influenza vaccine involves growing the virus in chicken eggs and later extracting, purifying, inactivating, and packaging the vaccine, which takes six to eight months (Gerdil 2002). Hence, the manufacturers have to decide on the production quantity long before complete information about demand is available. In addition, due to the continuous change in the constituent strains, unused vaccine from the previous season cannot be utilized in the current season. Williams (2005) and Yadav (2005) provide a detailed discussion of these distinguishing features of the influenza vaccine supply chain and suggest various improvement opportunities.

A key process characteristic that further exacerbates the production challenges mentioned above is the yield uncertainty: the inherent uncertainty regarding the quantity of vaccine that can be obtained per chicken egg due to the uncertain growth characteristics of the viral strains (National Influenza Vaccine Summit 2006, National Vaccine Advisory Committee 2003, PowerMed 2005, Government Accountability Office 2001). The magnitude of the challenge posed by the yield uncertainty in influenza vaccine production is illustrated by quotes such as “the yield of candidate strains sometimes is not as high as desired which results in fewer doses, or strains may take

additional time to obtain optimal yields, resulting in delays in the availability of vaccine” (National Vaccine Advisory Committee 2003) and “the first [major factor contributing to the delay in vaccine availability in 2001] was that two manufacturers had unanticipated problems growing one of the two new influenza strains introduced into the vaccine for 2000–01” (Government Accountability Office 2001).

The above discussion suggests that the effects of production yield uncertainty might be amplified if the market is concentrated. However, yield uncertainty could itself potentially affect firms' profitability and, consequently, the market structure. To better understand the interplay between yield uncertainty and market structure, we formulate a two-stage game of oligopolistic competition with endogenous entry. In the first stage, firms simultaneously decide whether to enter the market by incurring a fixed cost of entry. In the second stage, each entering firm selects the target production quantity. Then, each firm's production yield is realized, the actual quantity produced is brought to the market, and price emerges according to the traditional model of quantity (Cournot) competition. We use this model to answer the following questions:

- (i) What is the impact of yield uncertainty on the quantity produced by each firm, total output of the industry, and total number of firms in the market under competitive equilibrium?
- (ii) What is the impact of yield uncertainty on consumer surplus?
- (iii) What conditions result in less entry and lower production as compared with the social optimum?

The rest of the paper is structured as follows. Section 2 reviews related literature from operations management (OM), economics, and public health economics on yield uncertainty, competition, and vaccinations, respectively. In §3, we present the basic model. Section 4 outlines the main results concerning the competitive equilibrium and socially optimal solutions, whereas §5 discusses numerical experiments with parameter values loosely based on the U.S. influenza vaccine market. We provide some concluding remarks in §6.

## 2. Literature Review

This work draws on and contributes to two distinct streams of literature. First, we extend the literature

on the stochastically proportional yield model in OM to a competitive setting. Second, we show how yield uncertainty affects the existing results in the oligopoly literature that discusses various models of competition with endogenous entry.

### 2.1. Yield Uncertainty

The model of yield uncertainty employed here has been widely used in the OM literature and is referred to as the stochastically proportional yield model. However, most of this OM literature considers the impact of yield uncertainty on either the production planning decisions of a single firm or procurement decisions of a single firm buying from multiple non-competing suppliers with uncertain yields (Yano and Lee 1995). Henig and Gerchak (1990), in a single-firm model, show using an approximation that a higher yield variance results in lower optimal target production quantity. The first part of our analysis shows that this result extends to a competitive setting. Anupindi and Akella (1993) and Gerchak and Parlar (1990) discuss the value of diversification in the case of a given number of unreliable suppliers. More recently, Federgruen and Yang (2005) and Dada et al. (2007) have analyzed the problem of procurement from multiple suppliers with differing reliability and cost. In the second part of our analysis, the number of suppliers is endogenously determined through an entry game.

Carr et al. (2005) consider a competitive model of demand and capacity uncertainty. They show that a reduction in yield uncertainty can reduce the firm's profit, if process improvement leads to an effective overcapacity in the industry resulting in stiffer price competition. One of our results is consistent with this, but in our model the increase in quantity produced is a rational decision rather than a direct outcome as in Carr et al. (2005). Moreover, Carr et al. (2005) do not consider entry decisions, and their main focus is on studying the interaction between process improvement and competitive forces. In short, we contribute to the OM literature by studying the impact of yield uncertainty on strategic decisions of the firm such as entry and production quantity.

### 2.2. Oligopolistic Competition and Entry

Vives (1999) provides a detailed account of the vast literature related to the Cournot (1838) model of

oligopolistic competition. Part of our paper focuses on the question of firm entry in the context of oligopoly. Mankiw and Whinston (1986) compare the number of firms in the "free-entry equilibrium" with the number of firms that a social planner would choose. They show that, under a decreasing inverse demand function and convex cost structure, entry of an additional firm decreases the output of incumbents. Ignoring the integer constraint on the number of firms, this effect is sufficient to ensure that there will always be excess entry relative to the social optimum. Von Weizsäcker (1980) reaches similar conclusions using a linear inverse demand function and numerical examples. We show that, in contrast to Mankiw and Whinston (1986), adding yield uncertainty leads to less entry than optimal in a homogeneous goods market with business stealing effect.

Our paper is related to existing industrial organization models in which there is uncertainty related to the number of entrants. Janssen and Rasmusen (2002) analyze a Bertrand model in which each of the  $N$  firms do not know the number of active competitors in the market before deciding the price, but know only the probability that each of the  $N - 1$  competitors is active. They derive a mixed strategy equilibrium, in which industry profits are positive and decline with the number of entrants and compare the results with that of a Cournot model with similar uncertainty. However, they do not endogenize the firms' entry decision.

Quirnbach (1993) presents a model of research and development (R&D) competition in which firms undertake R&D based on a publicly available idea by incurring a fixed cost and succeed with some probability. Subsequently, the successful firms compete in the product market. He studies the impact of the nature of downstream conduct (Cournot, Bertrand, and perfect collusion) on R&D investment and finds several cases of underinvestment compared with the optimal solution depending on the R&D cost and probability of success. Creane (2006) reaches a similar conclusion for the special case of a Cournot model with endogenous entry. However in these models, the uncertainty is related to the success of entry and not the production quantity. We show later that these models can be considered as a special case of our model where the yield distribution is Bernoulli. Our formulation

is more general and also allows us to obtain closed-form expressions for the equilibrium number of firms and socially optimal number of firms, and analyze the impact of uncertainty on other market outcomes of interest, such as the total quantity brought to market and consumer surplus.

### 3. Model Formulation

#### 3.1. Modeling the Supply

We assume that the industry consists of  $n$  firms denoted by  $i \in \{1, 2, \dots, n\}$  possessing identical manufacturing processes. If  $\bar{q}_i$  is the production quantity targeted by firm  $i$  (for example, as reflected by the total number of chicken eggs chosen for vaccine production), then the actual quantity produced is given by  $q_i = \alpha_i \bar{q}_i$ , where  $\alpha_i$  is a random variable reflecting the random yield for firm  $i$ . Because the yield uncertainty results in a random proportion of the target quantity being produced, this multiplicative model is also known as the stochastically proportional yield model. Yano and Lee (1995) mention that this model is appropriate when relatively large batch sizes are used, when the variation of the batch size from production run to production run tends to be small, or when the yield losses might be relatively predictable for any particular set of conditions, but the conditions are not predictable. All of these criteria are met in the case of influenza vaccine production and also appear to hold for the other examples mentioned in §1. We assume that  $\alpha_i$  is identically and independently distributed for all firms, with  $\mu = E[\alpha_i]$  and  $\sigma^2 = \text{Var}[\alpha_i] \forall i$ . We include two marginal costs: (i)  $c_1$  per unit target quantity and (ii)  $c_2$  per unit actually produced. In the context of influenza vaccine production, the first cost is driven by the number of chicken eggs and the second cost corresponds to the cost of bottling and packaging the actual vaccine produced. Note that our formulation includes the model in Creane (2006) and Quirnbach (1993) as a special case, where each  $\alpha_i$  is distributed according to a Bernoulli distribution with  $\text{Prob}\{\alpha_i = 1\} = \rho$ , with a slight semantic difference. In their models,  $\rho$  is interpreted as a probability of success at the entry stage; in our model,  $\rho$  would be interpreted as the probability that the firm produces a nonzero quantity.

#### 3.2. Modeling the Market

We model competition among firms as a two-stage game. First, the firms simultaneously decide whether to enter the industry. Each entering firm incurs a fixed cost,  $f$ . We assume Cournot (1838) competition among the entering firms in the second stage. The manufacturers for influenza vaccine decide on production quantities six to eight months before the onset of the flu season, consistent with Cournot competition. Each firm sets its target production quantity,  $\bar{q}_i$ . After that, each firm's yield  $\alpha_i$  is realized, total production  $Q = \sum_i \alpha_i \bar{q}_i$  occurs, and price  $p$  is set according to a linear inverse demand function:  $p = a - bQ$ . The linear functional form substantially simplifies the analysis and allows us to obtain closed-form expressions for equilibrium and optimal outcomes. For the influenza vaccine market, the reservation price  $a$  could depend on the efficacy of the vaccine, which might not be known to the firm while setting the target production quantity,  $q_i$ . This can be included in our model provided the efficacy is independent of the production yield.

We solve this two-stage game using backward induction. We first solve the second-stage game for a given number of firms in the industry and derive the equilibrium target production quantities and profits as a function of this number. Then, we analyze the first-stage game to find the equilibrium number of firms in the market.

### 4. Equilibrium of the Two-Stage Game

#### 4.1. Postentry Competition Under Yield Uncertainty

In the second stage, given that there are  $n$  firms in the industry, each firm decides a target quantity  $\bar{q}_i$  at a cost of  $c_1 \bar{q}_i$ . The uncertainty is resolved during the production process, and  $q_i = \alpha_i \bar{q}_i$  is the actual quantity produced at a cost of  $c_2 q_i$ . The market price is given by the inverse demand function  $p = a - b(\sum_{j=1}^n q_j)$ , and the expected profit of the  $i$ th firm is given by  $\Pi_i(q_i) = E_{\alpha_i}[(a - b(\sum_{j=1}^n q_j))q_i - c_1 \bar{q}_i - c_2 q_i]$ . Substituting for  $q_i = \alpha_i \bar{q}_i$  in this expression and noting that  $e$  and  $\alpha_i$  are independent, we obtain

$$\Pi_i(\bar{q}_i) = E \left[ \left( a - b \left( \sum_{j=1}^n \alpha_j \bar{q}_j \right) \right) \alpha_i \bar{q}_i - (c_1 + \alpha_i c_2) \bar{q}_i \right]. \quad (1)$$

Let  $\Pi_i^*$  denote the maximum profit of the  $i$ th firm. Then, writing  $\bar{q}_{-i}$  to denote the decisions of all firms other than  $i$ , the decision problem of the  $i$ th firm is

$$\Pi_i^* = \max_{\bar{q}_i \geq 0} \Pi_i(\bar{q}_i, \bar{q}_{-i}^*). \quad (2)$$

The equilibrium is found by solving the following set of equations:

$$\left. \frac{\partial \Pi_i(\bar{q}_i)}{\partial \bar{q}_i} \right|_{\bar{q}_i = \bar{q}_i^*} = (a - c) E[\alpha_i] - 2b\bar{q}_i^* E[\alpha_i^2] - b E \left[ \alpha_i \sum_{i \neq j} \alpha_j \bar{q}_j^* \right] = 0 \quad \forall i, \quad (3)$$

where we have defined  $c = c_1/\mu + c_2$ . To ensure that all the entering firms produce a positive quantity, we assume that  $a > c$ . This is similar to the condition for a Cournot model with deterministic production except that, in our case, it also imposes a condition on the expected yield:  $\mu > c_1/(a - c_2)$ . Define the coefficient of variation  $\delta = \sigma/\mu$ . Because we are primarily interested in analyzing the impact of yield uncertainty on market and socially optimal solutions, we keep  $\mu$  constant and only analyze the effect of changes in  $\sigma$ . Hence, we can express all our results in terms of  $\delta$  rather than  $\sigma$ , which simplifies the exposition considerably. For  $E[\alpha_i] = \mu$  and  $\text{Var}[\alpha_i] = \sigma^2 \forall i$ , the system of Equations (3) has a unique solution, which is characterized below.

**LEMMA 1.** *The second-stage Cournot game with yield uncertainty has a unique equilibrium in which*

- (i) *The target quantity of each firm is given by  $\bar{q}_i^* = ((a - c)/\mu)/(b(n + 1 + 2\delta^2)) \forall i$ ;*
- (ii) *The expected quantity produced by each firm is given by  $E[q_i^*] = \mu\bar{q}_i^* = (a - c)/(b(n + 1 + 2\delta^2)) \forall i$ ;*
- (iii) *For given  $n$ , each firm's target quantity and expected quantity is decreasing in the yield uncertainty as measured by  $\delta$ ;*
- (iv) *The expected profit of each firm is given by  $\Pi_i^*(n) = ((a - c)^2(\delta^2 + 1))/(b(n + 1 + 2\delta^2)^2) \forall i$ ; and*
- (v) *For fixed  $n$ , each firm's expected profit is first increasing and then decreasing in  $\delta$  if  $n > 3$ , and monotone decreasing in  $\delta$  if  $n \leq 3$ .*

All proofs are provided in Appendix A1. Note that in the absence of any uncertainty, i.e.,  $\delta = 0$ , the expected quantity produced decreases to  $q_i^* = (a - c)/(b(n + 1))$ , whereas expected profit reduces to

$\Pi_i^* = ((a - c)^2)/(b(n + 1)^2)$ : both familiar from Cournot competition without yield uncertainty. To understand result (v), note that for given  $n$  and  $\mu$ , higher variance of the yield distribution ( $\sigma^2$ ) leads each firm in the equilibrium to target and, in expectation, to produce a smaller quantity. The expression for marginal profit in (3) shows that the yield uncertainty affects both the marginal revenue and the marginal cost, but that its variance affects only the marginal revenue. For a given  $n$ , the marginal revenue is decreasing in both  $\bar{q}_i$  and  $\sigma^2$ . As a result, the equilibrium target quantity is decreasing in  $\sigma^2$ . To understand the impact of yield uncertainty on the expected profit of the firm, note that  $\partial(MR)_i/\partial\delta^2 = (\partial(MR)_i/\partial\bar{q}_i)(\partial\bar{q}_i/\partial\delta^2)$ . We know from result (i) that  $\partial\bar{q}_i/\partial\delta^2$  is always negative.  $\partial(MR)_i/\partial\bar{q}_i$  is positive for smaller  $\bar{q}_i$  and then negative, because the marginal revenue is decreasing in  $\bar{q}_i$ . As a result, the marginal revenue is increasing in  $\delta^2$  for higher values of  $\bar{q}_i$  (which corresponds to smaller values of  $\delta^2$ ) and decreasing for smaller values of  $\bar{q}_i$  (which corresponds to higher values of  $\delta^2$ ). This drives result (v). Note that the impact of the yield uncertainty appears only through the first and the second moment due to the linearity of the demand model.

Thus, the yield uncertainty decreases the target quantity and total expected quantity brought to market. As a result, the market price increases. The relative magnitude of these two effects determines the overall impact of the yield uncertainty on the expected revenue and, hence, the expected profit.

## 4.2. Entry Game

Next, we focus on the first stage of the game. We assume that there is a large population of identical potential entrants. Each of these potential entrants has a reservation profit level of zero. All firms simultaneously decide whether to enter the market or not. We are not interested in which specific firms out of the potential population enter, but only in the equilibrium number of entrants. For  $n^* \in \mathbb{N}$  to be the equilibrium number of firms in the industry, we must have  $\Pi_i^*(n^*) \geq f$  and  $\Pi_i^*(n^* + 1) \leq f$ , because otherwise entering firms are losing money or earning sufficient profits to attract additional entrants. Temporarily relaxing the integer constraint, the equilibrium number of entrants,  $x^* \in \mathbb{R}_+$  satisfies  $\Pi_i^*(x^*) = f$ . Let  $n_u^* \in \mathbb{N}$  denote the equilibrium number of firms under yield

uncertainty and  $n_d^* \in \mathbb{N}$  be the corresponding equilibrium number of firms for the deterministic case. Similarly, let  $x_u^*, x_d^* \in \mathbb{R}_+$  be the respective equilibrium numbers after relaxing the integer constraints. Let  $\lfloor n \rfloor$  denote the largest integer less than or equal to  $n$ .

**LEMMA 2.** *The number of firms in the industry at equilibrium with and without yield uncertainty is given by  $n_u^* = \lfloor (((a-c)/\sqrt{bf})\sqrt{1+\delta^2}) - (1+2\delta^2) \rfloor$  and  $n_d^* = \lfloor ((a-c)/\sqrt{bf}) - 1 \rfloor$ , respectively.*

We now use these results to determine the impact of yield uncertainty on the equilibrium number of firms,  $n_u^*$ , using the deterministic equilibrium number,  $n_d^*$ , as a benchmark. One might expect that uncertainty always (weakly) decreases the number of entrants in equilibrium, but the following proposition shows that that is not necessarily true.

**PROPOSITION 1.** *The equilibrium number of firms under uncertainty ( $n_u^*$ ) and in the deterministic case ( $n_d^*$ ) satisfy (i)  $n_u^* \leq n_d^*$  if  $\{(a-c)/\sqrt{bf} > 4 \text{ and } \delta \geq \delta_e^*\}$  or  $(a-c)/\sqrt{bf} \leq 4$  and (ii)  $n_u^* \geq n_d^*$  if  $\{(a-c)/\sqrt{bf} > 4 \text{ and } \delta \leq \delta_e^*\}$ , where  $\delta_e^* \triangleq \sqrt{((a-c)/(2\sqrt{bf}) - 1)^2 - 1}$ .*

Recall that  $\lfloor ((a-c)/\sqrt{bf}) - 1 \rfloor = n_d^*$ . Using the result from Lemma 1 for  $n_d^* > 3$  and fixing  $n$  at  $n_d^*$ , we see that profit would first increase for small values of  $\delta^2$ , attracting new entrants, and then decrease for large values of  $\delta^2$ , lowering the equilibrium number of firms. Hence, beyond a threshold level of yield uncertainty  $\delta_e^*$ , the equilibrium number of firms  $n_u^*$  would drop below  $n_d^*$ . For  $n_d^* \leq 3$ , the profit is always decreasing in uncertainty for fixed  $n$ . Hence, the number of entrants would always be smaller under uncertainty.

Note that  $(a-c)/\sqrt{bf}$  can also be interpreted as a measure of market attractiveness: a market with larger  $(a-c)/\sqrt{bf}$  can support more firms in equilibrium. Thus, if the market is not attractive enough, then any amount of yield uncertainty (weakly) decreases the number of firms at equilibrium. However, if the market is attractive enough, then limited yield uncertainty can actually result in entry of more firms than the deterministic case.

### 4.3. Industry Output

Clearly related to the number of entrants, we are also interested in the impact of yield uncertainty on the expected total quantity produced in equilibrium,

because that is directly linked to the consumption, for example, the number of vaccinations in the case of influenza vaccine market. Let  $Q_d^*$  be the total quantity produced in equilibrium in the absence of uncertainty and  $E[Q_u^*]$  the expected total quantity produced in equilibrium under uncertainty. In Proposition 1, we have seen that limited levels of uncertainty can lead to increased entry. However, the next proposition shows that uncertainty cannot lead to higher total supply.

**PROPOSITION 2.**  *$E[Q_u^*] \leq Q_d^* \forall \delta \geq 0$ ; i.e., the expected quantity produced by the market under yield uncertainty is less than or equal to that in the deterministic case.*

The above result is true even for those levels of uncertainty where the number of firms at equilibrium is higher than in the base case, i.e.,  $\delta_e^* > \delta > 0$ . Two effects are at play here. By Proposition 1, a small amount of uncertainty can lead to more firms entering than in the deterministic case. However, by Lemma 1, each of these firms sets a smaller target production quantity. Proposition 2 shows that the second effect dominates the first effect. For the influenza vaccine case, the results show that yield uncertainty can lead to higher market concentration and will always reduce the total vaccine supply.

### 4.4. Consumer Surplus

It might seem that yield uncertainty will reduce the expected consumer surplus because, on average, fewer units are brought to the market (Proposition 2) and sold at a higher expected price (decreasing inverse demand function) as a result of yield uncertainty. We investigate this phenomenon here. In equilibrium, let  $E[CS_u(Q_u^*)]$  denote the expected consumer surplus under uncertainty, and let  $CS_d(Q_d^*)$  denote the consumer surplus in the absence of uncertainty. Formally, when the total quantity produced is  $Q$ , the total expected consumer utility is  $E[\int_0^Q (a - bu) du]$ , and the expected payment by the consumers is  $E[(a - bQ)Q]$ . Hence, the expected consumer surplus for the two cases is given by  $CS_d(Q_d^*) = (b/2)(Q_d^*)^2$  and  $E[CS_u(Q_u^*)] = E[\int_0^{Q_u^*} (a - bu) du - (a - bQ_u^*)Q_u^*] = (b/2)E[(Q_u^*)^2] = (b/2)((E[Q_u^*])^2 + \text{var}(Q_u^*))$ . We know from Proposition 2 that the expected total quantity is smaller under uncertainty. However, by definition,  $\text{var}(Q_u^*)$  is greater under uncertainty than in the deterministic case, so the combined effect is not

immediately obvious. Proposition 3 shows that the combined effect of yield uncertainty is a decrease in consumer surplus.

**PROPOSITION 3.** *The expected consumer surplus in equilibrium is always smaller under yield uncertainty than in the deterministic case; i.e.,  $E[CS_u(Q_u^*)] \leq CS_d(Q_d^*) \forall \delta > 0$ .*

Thus, we find that the increase in the variance of the total quantity produced is not enough to compensate for the reduction in the expected total production. As a result, consumers are always worse off in the presence of yield uncertainty. Note that the effect of uncertainty here is different than in the risk-pooling model in the inventory literature. In that context, price is fixed and firms (or society) incur a fixed penalty per unit of overproduction or underproduction. Adopting a newsvendor-type cost function would imply that the value of a dose of vaccine (or the cost of not receiving a dose) is equal for all consumers, whereas in the influenza vaccine context, the value of a dose of vaccine is far higher to high-risk populations than to others. We shall further investigate the impact of yield uncertainty on consumer surplus using numerical analysis in §5.

#### 4.5. First-Best Solution

As a benchmark against which to compare the preceding results, we formulate and solve the decision problem of a social planner who wants to maximize the total social welfare.

“First best” denotes the solution to the social planner’s problem of maximizing the total social welfare or the total surplus of society by choosing the number of firms ( $n$ ) and the target production quantity of each firm ( $\bar{q}_i$ ) (Vives 1999). This presupposes the existence of an omnipotent and omniscient benevolent agency, possibly government, that can costlessly and perfectly control both the structure of the industry and the conduct of the firms in the industry. Then, the social planner’s problem can be formulated as

$$\max_{\bar{q}_i, n} E[W(Q, n)] = \max_{\bar{q}_i, n} E \left[ \int_0^Q (a - bQ) dq \right] - E[cQ] - nf,$$

where  $Q = \sum_{i=1}^n q_i = \sum_{i=1}^n \alpha_i \bar{q}_i$  denotes the total quantity produced by  $n$  firms. The first term is the total

expected consumer utility, i.e., the area under the demand curve for consumers who do purchase, and the second term is the expected variable cost of production if  $Q(n)$  is the total quantity produced. The third term is the total cost of entry incurred by society if  $n$  firms enter the industry. Simplifying, we obtain the following social planner’s problem:

$$\max_{\bar{q}_i, n} E[W(Q(n), n)] = \max_{\bar{q}_i, n} (a - c) E(Q) - \frac{b}{2} E(Q^2) - nf. \quad (4)$$

This problem can be solved optimally by first fixing  $n$  and optimizing over  $\bar{q}_i$ , which we call the quantity problem. In the second step, we substitute the optimal  $\bar{q}_i$  in the original problem and optimize over  $n$ . We call this the structural problem.

**LEMMA 3.** *Let  $\bar{q}_i^{fb}$  denote the first-best planned production quantity of the  $i$ th firm and let  $q_i^{fb}$  denote the corresponding actual quantity produced. Then, (i)  $\bar{q}_i^{fb} = (2(a - c)\mu) / (b[(n + 1)\mu^2 + 2\sigma^2]) = 2\bar{q}_i^*$  and (ii)  $E[q_i^{fb}] = (2(a - c)) / (b(n + 1 + 2\delta^2))$ , where  $\delta = \sigma/\mu$  is as defined earlier.*

For a given number of firms, the socially optimal target quantity and expected production quantity for each firm is twice that under competition. The next step is to characterize the socially optimal number of firms. Substituting the expression for  $\bar{q}_i^{fb}$  in (4) and simplifying, the structural problem is

$$\max_n E[W(n)] = \max_n \frac{2(a - c)^2 n(1 + \delta^2)}{b(n + 1 + 2\delta^2)^2} - nf, \quad (5)$$

and the result is characterized in the following proposition.

**PROPOSITION 4.** *Let  $n^{fb}$  denote the number of firms in the first best solution. Then,  $1 \leq n^{fb} < 1 + 2\delta^2$ . Also,  $n^{fb} = 1$  if  $(2(1 + \delta^2))/\delta \geq (a - c)/\sqrt{bf}$ .*

This is in accordance with the existing intuition that the first-best solution in a deterministic setting involves having a benevolent monopoly that produces the socially optimal quantity because society then incurs the fixed cost of entry only once. However, under yield uncertainty ( $\delta > 0$ ), the society might or might not want supplier diversification; i.e.,  $n^{fb} > 1$ .

This again depends on the market attractiveness captured by  $(a-c)/\sqrt{bf}$ . Proposition 4 shows that if the market is not very attractive ( $(a-c)/\sqrt{bf} < 4$ ), it is still socially optimal to have a monopoly; i.e.,  $n^{fb} = 1$  regardless of the level of uncertainty, as  $(2(1+\delta^2))/\delta \geq 4$ . However, if the market is sufficiently attractive, there exist levels of uncertainty for which society prefers multiple suppliers because the value of supply diversification is greater than the additional fixed cost of entry. In particular,  $1+2\delta^2 \geq 2$ , or  $\delta \geq \sqrt{1/2}$  is a necessary condition for  $n^{fb} > 1$ .

#### 4.6. Second-Best Solution

Due to the restrictive assumptions required for the first-best solution, we shall focus in greater detail on the case where the social planner can regulate the number of firms in the industry but cannot regulate their conduct, so that the entering firms engage in Cournot competition in the postentry game. This solution is referred to as the second-best structural regulation or simply “second-best” (Vives 1999). The social planner’s problem in this case is given by

$$\max_n E[W(Q(n), n)] = \max_n (a-c)E(Q) - \frac{b}{2}E(Q^2) - nf. \quad (6)$$

Substituting  $E(Q) = (n(a-c))/(b(n+1+2\delta^2))$  from Lemma 1 and

$$E(Q^2) = ((a-c)^2 n(n+\delta^2))/(b^2(n+1+2\delta^2)^2)$$

in (6), we obtain

$$\max_n E[W(Q(n), n)] = \max_n \frac{(a-c)^2}{2b} \left[ 1 - \frac{(1+2\delta^2)^2 + n\delta^2}{(n+1+2\delta^2)^2} \right] - nf. \quad (7)$$

Relaxing  $n$  to  $x \in \mathbb{R}_+$ , it can be verified that  $E[W(Q(x), x)]$  is strictly concave. Hence, by restricting  $n \in \mathbb{N}_+$ ,  $E[W(Q(n), n)]$  can have at most two maximizers. Let  $n_u^{sb}$  denote the element of this set of two maximizers and let  $n_d^{sb}$  denote the deterministic optimum. We begin by analyzing the deterministic case and then extend the analysis to the case with uncertainty.

**PROPOSITION 5.** *In the absence of yield uncertainty, the second-best number of firms is given by  $n_d^{sb} \in \{\lfloor ((a-c)/\sqrt{bf})^{2/3} - 1 \rfloor, \lfloor ((a-c)/\sqrt{bf})^{2/3} - 1 \rfloor + 1\}$ . Also,  $n_d^{sb} - 1 \leq n_d^*$ .*

This result, in the absence of yield uncertainty, is identical to Proposition 2 in Mankiw and Whinston (1986). If one were to relax the integrality condition on the number of firms, the result would reduce to  $x_d^{sb} \leq x_d^*$ . Thus, in the absence of yield uncertainty, the equilibrium number of firms can be less than the second-best number of firms, but not by more than one. This is because in the second-best outcome, the firms are making positive profits, causing more firms to enter. However, we show that including yield uncertainty can change the relationship between the second-best number of firms,  $n_u^{sb}$ , and the equilibrium number of firms under uncertainty,  $n_u^*$ . The result is summarized in the following proposition.

**PROPOSITION 6.** *The number of firms in equilibrium,  $n_u^*$ , and in the second-best solution,  $n_u^{sb}$ , satisfy (i)  $n_u^{sb} \geq n_u^*$  if  $\delta \geq \delta_{sb}^*$  and (ii)  $n_u^{sb} - 1 \leq n_u^*$  if  $\delta < \delta_{sb}^*$ , where  $\delta_{sb}^* > 0$  solves  $(a-c)/\sqrt{bf} = (2(1+2\delta^2)\sqrt{1+\delta^2})/(2+\delta^2)$ .*

This shows that if the yield uncertainty is larger than a certain threshold, then the number of firms in equilibrium will be less than in the second-best case, in direct contrast to Mankiw and Whinston (1986). Because the total expected quantity produced,  $(n(a-c))/(b(n+1+2\delta^2))$ , is increasing in  $n$ , the industry undersupplies at equilibrium whenever uncertainty is higher than that threshold. In contrast, for a low level of uncertainty, the outcome is the same as that in the deterministic case. From Propositions 6 and 1, we see that yield uncertainty decreases the equilibrium number of entrants for  $\delta \geq \delta_e^*$  and results in less entry than the social optimum for  $\delta \geq \delta_{sb}^*$ . The next result compares the relative magnitudes of the two.

**PROPOSITION 7.**  *$\exists K > 4$  such that  $\delta_e^* > \delta_{sb}^*$  iff  $(a-c)/\sqrt{bf} > K$ .*

Proposition 7 shows that the relationship between  $\delta_e^*$  and  $\delta_{sb}^*$  depends on the value of  $(a-c)/\sqrt{bf}$ . Thus, if the market is relatively more attractive (higher values of  $(a-c)/\sqrt{bf}$ ), yield uncertainty can result in less entry than the socially optimal solution, despite an increase in the number of entrants compared with the deterministic case. Similarly, if the market is relatively less attractive (smaller values of  $(a-c)/\sqrt{bf}$ ), yield uncertainty can result in more entry than is socially optimal, despite a reduction in the number of entrants compared with the deterministic case. This suggests that the yield uncertainty also impacts the



social optimum; society values supply diversification as the level of uncertainty increases, and whether entry in unregulated equilibrium is more or less than the socially optimal solution depends on the relative magnitude of these two effects. We further illustrate these effects using numerical examples in the next section.

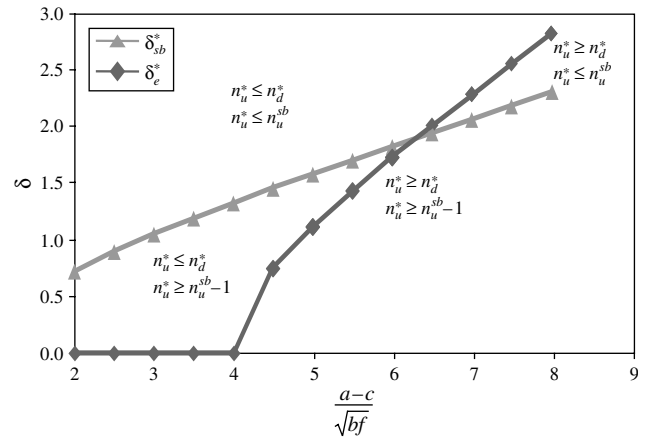
### 5. Numerical Illustrations

In this section, we use numerical examples to illustrate the effects of yield uncertainty on market outcome and on the socially optimal solution derived in the previous section. For the demand and cost parameters, we use values that are loosely based on the context of the U.S. influenza vaccine market (as explained in Appendix A2). However, given the unavailability of verifiable data, this analysis should be viewed as illustrative of the interaction between yield uncertainty and competition rather than as an actual representation of the U.S. vaccine market. Specifically, we aim to identify the relative levels of yield uncertainty for which various theoretical results hold as well as numerically explore the regions for which we could not derive analytical results. We also illustrate the impact of yield uncertainty on various equilibrium outcomes: the number of entrants, total quantity produced, and consumer surplus.

Propositions 1, 3, and 6 show that the impact of yield uncertainty on the equilibrium outcomes and social optimum depends on the magnitude of the coefficient of variation  $\delta$  compared with the thresholds  $\delta_e^* \triangleq \sqrt{((a-c)/(2\sqrt{bf})-1)^2-1}$  and  $\delta_{sb}^* \triangleq \{\delta > 0: (a-c)/\sqrt{bf} = (2(1+2\delta^2)\sqrt{1+\delta^2})/(2+\delta^2)\}$ . Note that these thresholds depend on the problem parameters only through the composite measure of market attractiveness,  $(a-c)/\sqrt{bf}$ . For the parameter values considered,  $(a-c)/\sqrt{bf}$  ranges from 1.84 to 7.12, as seen in Table 1. However, to focus on cases where at least one firm enters the market in equilibrium, we only consider values greater than 2, because  $(a-c)/\sqrt{bf} = n_u^* + 1$ .

Figure 1 shows the plot of  $\delta_e^*$  and  $\delta_{sb}^*$  as functions of  $(a-c)/\sqrt{bf}$ , verifying Proposition 6 that  $\delta_e^* > \delta_{sb}^*$  for large values of  $(a-c)/\sqrt{bf}$ . The figure is divided into four regions depending on the impact of yield uncertainty and market attractiveness on the equilibrium number of firms (Proposition 1), and the difference between equilibrium number of firms and

Figure 1 Relative Threshold Value of  $\delta_e^*$  and  $\delta_{sb}^*$  for Different Levels of Market Attractiveness



socially optimal number of firms (Proposition 6). Very low levels of yield uncertainty ( $\delta < \min\{\delta_e^*, \delta_{sb}^*\}$ ) attracts more firms to the market than the deterministic case and results in more entry than is socially optimal. However, very high levels of yield uncertainty ( $\delta > \max\{\delta_e^*, \delta_{sb}^*\}$ ) attracts fewer firms to the market than the deterministic case and results in less entry than is socially optimal. However, for medium levels of uncertainty ( $\min\{\delta_e^*, \delta_{sb}^*\} < \delta < \max\{\delta_e^*, \delta_{sb}^*\}$ ), the impact depends on the level of market attractiveness.

The above findings are confirmed in Table 2, which displays the equilibrium number and socially optimal number of firms for different levels of market attractiveness and yield uncertainty. We also see that there are several combinations of parameters where the Mankiw and Whinston (1986) result of excess entry is reversed. Table 2 also shows that for sufficiently attractive markets, the socially optimal number of firms ( $n_u^{sb}$ ) first increases and then decreases in the level of yield uncertainty, indicating that it is socially optimal in some cases to reduce entry if the level of yield uncertainty is very high. This is counter to the intuition that it would be socially optimal to have more firms under high levels of yield uncertainty

Table 1 Parameter Values for Numerical Examples

Parameter	$b$ (\$/million)	$a$ (\$)	$c$ (\$)	$f$ (\$ million)	$M$ (million)	$(a-c)/\sqrt{bf}$
Value	0.026	8	3	40	300	3.87
Range	0.026–0.040	8–12	3–5	40–100	300	1.84–7.12

**Table 2** Impact of Yield Uncertainty on the Equilibrium Number and Socially Optimal Number of Firms

$(a-c)/\sqrt{bf}$	$\delta=0$	$\delta=0.5$	$\delta=1.0$	$\delta=1.5$	$\delta=2.0$	$\delta=2.5$	$\delta=3.0$	$\delta=3.5$
2.0	1,1	0,1	0,1	0,0	0,0	0,0	0,0	0,0
2.5	1,1	1,1	0,1	0,0	0,0	0,0	0,0	0,0
3.0	2,2	1,2	1,2	0,1	0,0	0,0	0,0	0,0
3.5	2,2	2,2	1,2	<b>0,2</b>	0,1	0,0	0,0	0,0
4.0	3,2	2,2	2,3	1,2	<b>0,2</b>	0,0	0,0	0,0
4.5	3,2	3,3	3,3	2,3	<b>1,3</b>	0,1	0,0	0,0
5.0	4,2	4,3	4,3	3,4	2,3	<b>0,2</b>	0,1	0,0
5.5	4,2	4,3	4,4	4,5	3,4	<b>1,4</b>	<b>0,2</b>	0,0
6.0	5,2	5,3	5,4	5,5	4,5	<b>2,4</b>	<b>0,3</b>	0,1
6.5	5,2	5,3	6,4	6,5	5,5	4,5	<b>1,4</b>	<b>0,2</b>
7.0	6,2	6,3	6,4	7,5	6,6	5,6	<b>3,5</b>	<b>0,3</b>
7.5	6,2	6,3	7,5	8,6	7,6	6,7	<b>4,6</b>	<b>1,5</b>

Notes. Each cell is a tuple  $n_u^*, n_u^{sb}$ .  $\delta=0$  denotes the deterministic case. The numbers in bold indicate the cases where the Mankiw and Whinston (1986) result is reversed.

because the social planner has to trade off the fixed cost of entry with the increased supply of goods.

Next, we examine how the level of yield uncertainty and the market attractiveness jointly affect the expected industry output and expected consumer surplus in equilibrium. Table 3 displays the ratio of expected industry output under uncertainty to that in the deterministic case. It verifies the result in Proposition 2 that expected industry output under uncertainty is always smaller than that in the deterministic

**Table 3** Impact of Yield Uncertainty on the Expected Industry Output in Equilibrium

$(a-c)/\sqrt{bf}$	$\delta=0$	$\delta=0.5$	$\delta=1.0$	$\delta=1.5$	$\delta=2.0$	$\delta=2.5$	$\delta=3.0$	$\delta=3.5$
2.0	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2.5	1.00	0.80	0.00	0.00	0.00	0.00	0.00	0.00
3.0	1.00	0.60	0.38	0.00	0.00	0.00	0.00	0.00
3.5	1.00	0.86	0.38	0.00	0.00	0.00	0.00	0.00
4.0	1.00	0.76	0.53	0.21	0.00	0.00	0.00	0.00
4.5	1.00	0.89	0.67	0.36	0.13	0.00	0.00	0.00
5.0	1.00	0.91	0.71	0.44	0.23	0.00	0.00	0.00
5.5	1.00	0.91	0.71	0.53	0.31	0.09	0.00	0.00
6.0	1.00	0.92	0.75	0.57	0.37	0.15	0.00	0.00
6.5	1.00	0.92	0.80	0.63	0.43	0.27	0.06	0.00
7.0	1.00	0.93	0.78	0.65	0.47	0.32	0.16	0.00
7.5	1.00	0.93	0.82	0.69	0.51	0.36	0.20	0.04

Notes. Each cell contains the ratio of expected industry output under uncertainty to that in the absence of uncertainty  $E[Q_u]/Q_d$ . The expected industry output is decreasing in yield uncertainty.

**Table 4** Impact of Yield Uncertainty on Consumer Surplus in Equilibrium

$(a-c)/\sqrt{bf}$	$\delta=0$	$\delta=0.5$	$\delta=1.0$	$\delta=1.5$	$\delta=2.0$	$\delta=2.5$	$\delta=3.0$	$\delta=3.5$
2.0	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2.5	1.00	0.80	0.00	0.00	0.00	0.00	0.00	0.00
3.0	1.00	0.45	0.28	0.00	0.00	0.00	0.00	0.00
3.5	1.00	0.83	0.28	0.00	0.00	0.00	0.00	0.00
4.0	1.00	0.65	0.43	0.14	0.00	0.00	0.00	0.00
4.5	1.00	0.86	0.59	0.27	0.09	0.00	0.00	0.00
5.0	1.00	0.88	0.64	0.34	0.15	0.00	0.00	0.00
5.5	1.00	0.88	0.64	0.43	0.23	0.05	0.00	0.00
6.0	1.00	0.89	0.68	0.47	0.27	0.10	0.00	0.00
6.5	1.00	0.89	0.75	0.54	0.33	0.19	0.04	0.00
7.0	1.00	0.91	0.71	0.56	0.36	0.22	0.10	0.00
7.5	1.00	0.91	0.76	0.61	0.41	0.26	0.13	0.03

Note. Each cell contains the ratio of expected consumer surplus under uncertainty to that in the absence of uncertainty  $E[CS_u]/CS_d$ .

case. We also note that the expected industry output decreases with increasing uncertainty. Table 4 shows the ratio of expected consumer surplus under yield uncertainty to that in the deterministic case. It verifies Proposition 3 and further illustrates that the expected consumer surplus is decreasing in the level of yield uncertainty. However, the ratio does not display any monotonicity with respect to the measure for market attractiveness  $((a-c)/\sqrt{bf})$ , indicating that in some cases the consumers might be better off in markets that are less attractive to firms.

### 6. Concluding Remarks

In this paper, we formulate a two-stage model of Cournot competition with endogenous entry to analyze the effect of yield uncertainty on firms' entry and production decisions. The model is inspired by the context of the market for influenza vaccines, but applies to other settings with yield uncertainty, fixed cost of entry, and Cournot competition. Our analysis leads to some interesting observations.

First, we find that a limited degree of yield uncertainty can make an industry more attractive to potential entrants than it would be with no yield uncertainty. This is because even if one firm has high yield, other firms may still experience low yield, restricting industry output and, hence, raising prices, so the first firm can benefit from high quantity and high prices. In contrast, for yield uncertainty higher

than a certain threshold, the market becomes less attractive for potential entrants even as the social planner wants more firms in the market to mitigate the effect of supply uncertainty. This results in cases where the number of firms in equilibrium is less than the number of firms in the socially optimal solution, a result that is contrary to the conventional result of excess entry in the oligopoly models without uncertainty.

Second, we find that consumer surplus is always lower under yield uncertainty than in the deterministic case. Although this result may appear intuitively obvious, it is worth reiterating that the underlying mechanism is not exactly the same as that in the risk-pooling literature. In the context of the influenza vaccine market, this implies that a regulator should consider subsidizing research on manufacturing processes with lower uncertainty as an alternative approach to increasing the social benefit of the influenza vaccine, rather than focus only on programs aimed at increasing vaccination rates.

Third, we find numerically that as yield uncertainty becomes very high, the equilibrium, and perhaps more surprisingly, the second-best solutions call for fewer firms to enter. We also find numerically that the interaction between market attractiveness and the effect of yield uncertainty is strong. In a market that is not very attractive, even with deterministic yield, a small amount of yield uncertainty can lead to drastic drops in industry output and consumer surplus. In more attractive markets, the effect of small degrees of yield uncertainty is much less pronounced.

There are several cases that we have not explored here. For instance, firms might have heterogenous and/or correlated yield distributions. Firms might have different production lead times, allowing the firms with smaller lead times to respond to yield realizations of the firms with longer lead times. The linear demand function we use definitely shapes some of our results; generalization of some of our results to nonlinear demand, although challenging, would be worthwhile.

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### Appendix A1. Proofs

**PROOF OF LEMMA 1.** The proof is analogous to that for the deterministic case. The  $i$ th firm solves the concave maximization problem in  $\bar{q}_i^*$  given by (2). Hence, the first-order condition in (3) is necessary and sufficient to obtain the equilibrium target quantities. Using  $E[\alpha_i \alpha_j] = E[\alpha_i] E[\alpha_j]$  (random variables  $\alpha_i$  and  $\alpha_j$  are independent),  $\text{Var}(\alpha_i) = E[\alpha_i^2] - (E[\alpha_i])^2 = \sigma^2$ ,  $E[\alpha_i] = \mu \quad \forall i$  (each firm has the same yield distribution), and simplifying (3), we obtain a unique solution to the above set of equations given by

$$\bar{q}_i^* = \frac{(a-c)\mu}{b[(n+1)\mu^2 + 2\sigma^2]} \quad \forall i. \quad (\text{A1})$$

The expected production quantity of the  $i$ th firm is given by

$$E[q_i^*] = E[\alpha_i] \bar{q}_i^* = \frac{(a-c)\mu^2}{b[(n+1)\mu^2 + 2\sigma^2]} = \frac{(a-c)}{b(n+1+2\delta^2)} \quad \forall i, \quad (\text{A2})$$

which is decreasing in  $\delta$ . This completes the proof for parts (i), (ii), and (iii). Substituting (A1) in (1) and simplifying yields

$$\Pi_i^*(n) = \frac{(a-c)^2(\delta^2 + 1)}{b(n+1+2\delta^2)^2}, \quad (\text{A3})$$

which proves part (iv). Differentiating (A3) with respect to (w.r.t.)  $\delta$ , we obtain  $\partial \Pi_i(\bar{q}_i^*) / \partial \delta = ((a-c)^2 2\delta(n-3-2\delta^2)) / (b(n+1+2\delta^2)^3)$ . Hence,  $\partial \Pi_i(\bar{q}_i^*) / \partial \delta < 0$  for  $\delta > \sqrt{(n-3)}/2$  if  $n > 3$  and  $\partial \Pi_i(\bar{q}_i^*) / \partial \delta < 0$  for  $\forall \delta > 0$  if  $n \leq 3$ , which proves part (v).

**PROOF OF LEMMA 2.** We first ignore the integer constraint on the number of firms and solve for  $x_u^*$  by using the condition  $\Pi_i^*(x_u^*) = f$ . Using (A3) and rearranging the terms we get

$$x_u^* = \frac{(a-c)}{\sqrt{bf}} \sqrt{1 + \delta^2} - (1 + 2\delta^2). \quad (\text{A4})$$

Because  $\Pi_i^*(x)$  is decreasing in  $x$ ,  $n_u^* = \lfloor x_u^* \rfloor = \lfloor ((a-c)/\sqrt{bf}) \cdot \sqrt{1 + \delta^2} - (1 + 2\delta^2) \rfloor$ , using (A4).

**PROOF OF PROPOSITION 1.** Because the derivatives w.r.t.  $\delta \geq 0$  and  $\delta^2$  have the same signs, we focus on the latter due to ease of analysis. Differentiating (A4) w.r.t.  $\delta^2$ , we get  $(dx_u^*/d\delta^2)|_{\delta^2=0} = ((a-c)/(2\sqrt{bf}))(1/\sqrt{1+\delta^2}) - 2|_{\delta^2=0} = (a-c)/(2\sqrt{bf}) - 2$ . For  $(a-c)/\sqrt{bf} \leq 4$ ,

$$\frac{dx_u^*}{d\delta^2} \Big|_{\delta^2 > 0} \leq \frac{dx_u^*}{d\delta^2} \Big|_{\delta^2 = 0} \leq 0 \implies x_u^* \leq x_d^* \implies n_u^* \leq n_d^* \quad \forall \delta > 0.$$

Next, consider the case  $(a - c)/\sqrt{bf} > 4$ . Substituting  $\delta = 0$  in (A4), we get  $x_d^* = (a - c)/\sqrt{bf} - 1$ . Comparing this with (A4) and simplifying, we obtain  $x_u^* \leq x_d^* \Leftrightarrow \delta \geq \delta_e^*$ , where  $\delta_e^* = \sqrt{((a - c)/(2\sqrt{bf}) - 1)^2 - 1}$ . Because  $\lfloor x_u^* \rfloor = n_u^*$  and  $\lfloor x_d^* \rfloor = n_d^*$ , we obtain  $\delta \geq \delta_e^* \Rightarrow n_u^* \leq n_d^*$  and  $\delta \leq \delta_e^* \Rightarrow n_u^* \geq n_d^*$ .

**PROOF OF PROPOSITION 2.** With yield uncertainty, the total quantity produced at equilibrium is  $E[Q_u^*] = \sum_{i=1}^{n_u^*} E[q_i^*] = (n_u^*(a - c))/(b(n_u^* + 1 + 2\delta^2))$ , using (A2). Without yield uncertainty ( $\delta = 0$  and  $n_u^* = n_d^*$ ), we get  $Q_d^* = (n_d^*(a - c))/(b(n_d^* + 1))$ . Comparing the two and using  $n_d^* = \lfloor (a - c)/\sqrt{bf} - 1 \rfloor$  and  $n_u^* = \lfloor ((a - c)/\sqrt{bf})\sqrt{1 + \delta^2} - (1 + 2\delta^2) \rfloor$ ,

$$Q_d^* \geq E[Q_u^*] \iff (1 + 2\delta^2) \left[ \frac{a - c}{\sqrt{bf}} - 1 \right] \geq \left[ \frac{(a - c)}{\sqrt{bf}} \sqrt{1 + \delta^2} - (1 + 2\delta^2) \right]. \quad (A5)$$

Consider the following inequality:

$$(1 + 2\delta^2) \left[ \frac{a - c}{\sqrt{bf}} - 1 \right] \geq \frac{(a - c)}{\sqrt{bf}} \sqrt{1 + \delta^2} - (1 + 2\delta^2). \quad (A6)$$

Write LHS and RHS for left- and right-hand sides of (A6), respectively. RHS = LHS at  $\delta^2 = 0$ . Again, because we are only interested in the sign of the derivative, we can differentiate w.r.t.  $\delta^2$ . Note that  $\partial \text{LHS}/\partial \delta^2 = 2 \lfloor (a - c)/\sqrt{bf} - 1 \rfloor$  and  $\partial \text{RHS}/\partial \delta^2 = ((a - c)/\sqrt{bf})(1/(2\sqrt{1 + \delta^2})) - 2$ . So,

$$\begin{aligned} (\partial \text{RHS}/\partial \delta^2)|_{\delta^2 > 0} &< (\partial \text{RHS}/\partial \delta^2)|_{\delta^2 = 0} \\ &< (\partial \text{LHS}/\partial \delta^2)|_{\delta^2 = 0} = (\partial \text{LHS}/\partial \delta^2)|_{\delta^2 > 0}. \end{aligned}$$

Hence, (A6) and, consequently, (A5) hold  $\forall \delta > 0$ . Hence, we have  $Q_d^* \geq E[Q_u^*] \forall \delta \geq 0$ .

**PROOF OF PROPOSITION 3.** We know that  $E[CS_u(Q_u^*)] = (b/2)((E[Q_u^*])^2 + \text{Var}(Q_u^*))$ . Substituting  $E[Q_u^*] = n_u^* E[q_i^*]$ ,  $\text{Var}[Q_u^*] = (E[q_i^*])^2 n_u^* \sigma^2$ ,  $E[q_i^*] = (a - c)/(b(n_u^* + 1 + 2\delta^2))$  and simplifying, we obtain

$$E[CS_u(Q_u^*)] = \frac{(a - c)^2 n_u^* (n_u^* + \delta^2)}{2b(n_u^* + 1 + 2\delta^2)^2}. \quad (A7)$$

Hence, for the deterministic case ( $\delta = 0$  and  $n_u^* = n_d^*$ ),

$$CS_d(Q_d^*) = \frac{(a - c)^2 (n_d^*)^2}{2b(n_d^* + 1)^2}. \quad (A8)$$

Using the continuous relaxation of the above expressions and simplifying, we get

$$E[CS_u(Q_u^*)] = \frac{x_u^*(x_u^* + \delta^2)}{2f(1 + \delta^2)} \quad (A9)$$

and

$$CS_d(Q_d^*) = \frac{(x_d^*)^2}{2f}. \quad (A10)$$

Thus,  $E[CS_u(Q_u^*)] < CS_d(Q_d^*)$  if and only if  $x_u^*(x_u^* + \delta^2) < (x_d^*)^2(1 + \delta^2)$ . Using expressions for  $x_u^*$  and  $x_d^*$  we obtain

$$E[CS_u(Q_u^*)] < CS_d(Q_d^*) \iff \left( \frac{a - c}{\sqrt{bf}} - \frac{1 + 2\delta^2}{\sqrt{1 + \delta^2}} \right) \left( \frac{a - c}{\sqrt{bf}} - \sqrt{1 + \delta^2} \right) < \left( \frac{a - c}{\sqrt{bf}} - 1 \right)^2.$$

Note that, for  $x_u^*, x_d^* > 0$ , this condition is satisfied  $\forall \delta > 0$  because  $(1 + 2\delta^2)/\sqrt{1 + \delta^2} < \sqrt{1 + \delta^2} < 1 < (a - c)/\sqrt{bf}$ . Hence,  $E[CS_u(Q_u^*)] < CS_d(Q_d^*) \forall \delta > 0$ .

**PROOF OF LEMMA 3.** Note that  $E(Q) = E[\sum_{i=1}^n q_i] = E[\sum_{i=1}^n \alpha_i \bar{q}_i] = \mu \sum_{i=1}^n \bar{q}_i$ . Similarly,

$$\begin{aligned} E(Q^2) &= E \left[ \left( \sum_{i=1}^n q_i \right)^2 \right] = E \left[ \sum_{i=1}^n q_i^2 \right] + E \left[ \sum_{i \neq j} q_i q_j \right] \\ &= E[\alpha_i^2] \sum_{i=1}^n \bar{q}_i^2 + E[\alpha_i] E[\alpha_j] \sum_{i \neq j} \bar{q}_i \bar{q}_j \\ &= (\sigma^2 + \mu^2) \sum_{i=1}^n \bar{q}_i^2 + \mu^2 \sum_{i \neq j} \bar{q}_i \bar{q}_j. \end{aligned}$$

Substituting the expressions for  $E(Q)$  and  $E(Q^2)$  in (4) and simplifying, we obtain

$$\begin{aligned} \max_{\bar{q}_i, n} \left\{ E[W(Q(n), n)] \right. \\ \left. = (a - c) \mu \sum_{i=1}^n \bar{q}_i - \frac{b}{2} \left[ (\sigma^2 + \mu^2) \sum_{i=1}^n \bar{q}_i^2 + \mu^2 \sum_{i \neq j} \bar{q}_i \bar{q}_j \right] - nf \right\}. \end{aligned}$$

We first maximize over  $\bar{q}_i$ , keeping  $n$  fixed. The resulting objective function is jointly concave in  $\bar{q}_i$ , with first-order condition  $(a - c)\mu = (b/2)[(2\sigma^2 + \mu^2)\bar{q}_i + \mu^2 \sum_{k=1, k \neq i}^n \bar{q}_k] \forall i$ . This condition and, consequently, the optimal solution  $\bar{q}_i^*$  are symmetric in  $i$ . Summing over  $i$  and utilizing symmetry, we obtain  $\bar{q}_i^{*b} = (2(a - c)\mu)/(b[(n + 1)\mu^2 + 2\sigma^2])$ . Similarly, the expected quantity produced by each firm is given by  $E[\bar{q}_i^*] = (2(a - c))/(b(n + 1 + 2\delta^2))$ .

**PROOF OF PROPOSITION 4.** First, consider the continuous relaxation of (5) to  $x \in \mathbb{R}_+$ . The first-order condition w.r.t.  $x$  gives  $(2(a - c)^2(1 + \delta^2)(1 + 2\delta^2 - x))/(bf) = (x + 1 + 2\delta^2)^3$ . Note that the left-hand side is decreasing in  $x$  and positive only for  $x < 1 + 2\delta^2$ . The right-hand side is increasing in  $x$  and always positive. Because we require that the expected quantity produced is non-negative for any  $x$ , it is required that  $x \geq 1$ . Thus, if a solution exists to this equation, it is unique and lies in the range  $1 \leq x < 1 + 2\delta^2$ . Also, a necessary and sufficient condition for a solution to exist is given by  $((2(a - c)^2(1 + \delta^2)(1 + 2\delta^2 - x))/(bf))|_{x=1} > (x + 1 + 2\delta^2)^3|_{x=1}$  or, alternately,  $(2(1 + \delta^2))/\delta < (a - c)/\sqrt{bf}$ . If this condition is not satisfied, then  $n = x = 1$ , because otherwise it would imply that  $(dE[W(x)]/dx)|_{x=1} \leq 0$ .

**PROOF OF PROPOSITION 5.** Consider the continuous relaxation of (7) in  $x \in \mathbb{R}_+$  instead of  $n \in \mathbb{N}$ . Clearly,  $E[W(q(x), x)]$

is also a concave function of  $x$  and has a unique optimum given by the following first-order condition:

$$\begin{aligned} & \frac{\partial E[W(q(x), x)]}{\partial x} \\ &= \frac{(a-c)^2}{2b} \left[ \frac{2(1+2\delta^2)^2 + x\delta^2 - (1+2\delta^2)\delta^2}{(x+1+2\delta^2)^3} \right] - f = 0. \quad (A11) \end{aligned}$$

For  $\delta = 0$ , this yields  $x_d^{sb} = ((a-c)/\sqrt{bf})^{2/3} - 1$  and, hence,  $n_d^{sb} \in \{\lfloor x_d^{sb} \rfloor, \lfloor x_d^{sb} \rfloor + 1\}$ . Recall that  $n_d^* = \lfloor ((a-c)/\sqrt{bf}) - 1 \rfloor$  and  $x_d^* = ((a-c)/\sqrt{bf}) - 1$ . First check that  $x_d^{sb} \leq x_d^*$ . Now  $n_d^{sb} - 1 \leq \lfloor x_d^{sb} \rfloor \leq x_d^{sb} \leq x_d^*$ . Clearly,  $n_d^{sb} - 1 \leq x_d^* \Rightarrow n_d^{sb} - 1 \leq n_d^*$ .

**PROOF OF PROPOSITION 6.** We define  $x_u^*$  and  $x_u^{sb}$  corresponding to  $n_u^*$  and  $n_u^{sb}$ , respectively. Thus,  $x_u^{sb} = \{x: (\partial EW(x)/\partial x)|_{x=x_u^{sb}} = 0\}$ . First, to compare  $x_u^*$  and  $x_u^{sb}$ , we calculate  $(\partial EW(x)/\partial x)|_{x=x_u^*}$ . Using (A11) we get

$$\begin{aligned} & \left. \frac{\partial E[W(q(x), x)]}{\partial x} \right|_{x=x_u^*} \\ &= \frac{(a-c)^2}{2bf} \left[ \frac{2(1+2\delta^2)^2 + x_u^*\delta^2 - (1+2\delta^2)\delta^2}{(x_u^*+1+2\delta^2)^3} \right] - f. \end{aligned}$$

Also, because  $f = \Pi_i^*(x_u^*) = ((a-c)^2(1+\delta^2))/(b(x_u^*+1+2\delta^2)^2)$  at equilibrium,

$$\begin{aligned} & \left. \frac{\partial E[W(q(x), x)]}{\partial x} \right|_{x=x_u^*} \\ &= \frac{(a-c)^2}{2bf} \left[ \frac{2(1+2\delta^2)^2 + x_u^*\delta^2 - (1+2\delta^2)\delta^2}{(x_u^*+1+2\delta^2)^3} \right] - \frac{(a-c)^2(1+\delta^2)}{b(x_u^*+1+2\delta^2)^2} \\ &= \frac{(a-c)^2}{2bf(x_u^*+1+2\delta^2)^3} [2(1+2\delta^2)^2 - 3(1+2\delta^2)\delta^2 \\ & \quad - x_u^*\delta^2 - 2(x_u^*+1+2\delta^2)]. \end{aligned}$$

A sufficient condition for  $x_u^* \leq x_u^{sb}$  is  $(\partial E[W(q(x), x)]/\partial x)|_{x=x_u^*} \geq 0$ . Substituting  $x_u^*$  and simplifying gives  $x_u^* \leq x_u^{sb}$  if  $(2(1+2\delta^2)\sqrt{1+\delta^2})/(2+\delta^2) \geq (a-c)/\sqrt{bf}$ . In addition,  $n_u^* = \lfloor x_u^* \rfloor$  and  $n_u^{sb} \in \{\lfloor x_u^{sb} \rfloor, \lfloor x_u^{sb} \rfloor + 1\}$ . Thus,  $x_u^* \leq x_u^{sb} \Rightarrow n_u^* \leq n_u^{sb}$ . Combining these two conditions, we obtain  $(2(1+2\delta^2)\sqrt{1+\delta^2})/(2+\delta^2) > (a-c)/\sqrt{bf} \Rightarrow n_u^* \leq n_u^{sb}$ . It is easy to check that  $(2(1+2\delta^2)\sqrt{1+\delta^2})/(2+\delta^2)$  is an increasing function of  $\delta$ . Defining

$$\delta_{sb}^* = \left\{ \delta > 0: (2(1+2\delta^2)\sqrt{1+\delta^2})/(2+\delta^2) = (a-c)/\sqrt{bf} \right\},$$

the above condition is equivalent to  $n_u^* \leq n_u^{sb}$  if  $\delta \geq \delta_{sb}^*$ . This proves the first part of the proposition. For the second part of the proposition, combining the fact that  $(2(1+2\delta^2)\sqrt{1+\delta^2})/(2+\delta^2) \leq (a-c)/\sqrt{bf} \Rightarrow x_u^* \geq x_u^{sb}$  and  $x_u^* \geq x_u^s \Rightarrow n_u^* \geq n_u^{sb}$ , we obtain  $(2(1+2\delta^2)\sqrt{1+\delta^2})/(2+\delta^2) \leq (a-c)/\sqrt{bf} \Rightarrow n_u^* \geq n_u^{sb}$ . Note that the left-hand side of this expression is increasing in  $\delta$ , and using the same argument as above we conclude that  $n_u^* \geq n_u^{sb}$  if  $\delta \leq \delta_{sb}^*$ .

**PROOF OF PROPOSITION 7.** From Proposition 1,  $\delta_c^*$  is the solution to the equation  $(a-c)/(2\sqrt{bf}) = 1 + \sqrt{1+\delta^2}$  for

$(a-c)/\sqrt{bf} \geq 4$ , and  $\delta_{sb}^*$  is the solution to the equation  $(a-c)/(2\sqrt{bf}) = ((1+2\delta^2)\sqrt{1+\delta^2})/(2+\delta^2)$ . Note that the LHS of both the equations is the same. Hence, equating the RHS of the two equations gives us the value of  $\delta^2$  for which  $\delta_c^* = \delta_{sb}^*$ . Substituting this back in the original equations and simplification yields the value of  $(a-c)/\sqrt{bf}$  at that point; call it  $K$ . Clearly  $K \geq 4$ . Now, the RHS of both the equations is increasing in  $\delta^2$  and for  $(a-c)/\sqrt{bf} = 4$ ,  $\delta_c^* = 0 < \delta_{sb}^*$ . Hence,  $\delta_c^* \leq \delta_{sb}^*$  iff  $(a-c)/\sqrt{bf} \leq K$ .

## Appendix A2. Choice of Parameter Values for Numerical Analysis

This paper was inspired by the U.S. influenza vaccine market. Hence, we let our numerical analysis be guided, as much as possible, by the parameter values that correspond to that particular context rather than being based on completely arbitrary numbers. However, given the difficulty of obtaining the necessary data, these parameters should not be considered as an accurate representation of the U.S. situation; they serve only as inputs to our numerical examples.

For demand parameters, we assume uniformly distributed consumer valuations, which lead to a linear inverse demand function. Consider  $M$  individuals, who each demand zero or one unit of vaccine in a single-period context. The individuals differ only in the expected cost incurred if they do not get vaccinated, denoted by  $v$ . This cost reflects the likelihood of getting infected and the resulting costs of health care, lost income, etc. We assume perfect vaccination, i.e., after vaccination consumers stay perfectly healthy and do not incur any health care or other costs; relaxing this would not change our results.

Following Brito et al. (1991), we assume that  $v$  follows a uniform distribution  $F(v)$  on the range  $[\underline{v}, \bar{v}]$ . Let  $p$  be the price of one dose of vaccine, and let  $v^*$  be the valuation of the threshold consumer who is indifferent between getting vaccinated and not getting vaccinated, given price  $p$ . Then, for a rational consumer, who does not account for the positive externality of vaccination mentioned earlier,  $p = v^*$  and  $q(p) = (1 - F(v^*))M$  is the total demand at that price. Substituting  $F(v^*) = (v^* - \underline{v})/(\bar{v} - \underline{v})$  and  $p = v^*$ , we get  $q(p) = M((\bar{v} - p)/(\bar{v} - \underline{v}))$ . Defining  $a \triangleq \bar{v}$  and  $b \triangleq (\bar{v} - \underline{v})/M$ , the following inverse demand function is obtained:

$$p = a - bq. \quad (A12)$$

The population ( $M$ ) was chosen to be the U.S. population, which is approximately 300 million (U.S. Census Bureau 2004). The lower limit of the customer valuation ( $\underline{v}$ ) can be normalized to zero. The upper limit of the customer valuation ( $\bar{v}$ ) of \$8 was chosen based on anecdotal evidence (Nichol 2001) of the direct cost of vaccination and the fact that vaccination is a covered benefit under insurance for many customers. However, given the low reliability of this value, we let it vary between 8 and 12 for our analysis.

The variable cost ( $c$ ) of \$3 per dose was chosen based on the costs of procurement from the manufacturers

(O'Mara et al. 2003) and assuming an approximately 50% gross margin. The value of  $f$  is also not directly available. Gottlieb (2004) reports that an investment of approximately \$300 million is required to build a new influenza vaccine plant. A 10%–20% cost of capital on this investment translates into an annual fixed cost of \$30–\$60 million dollars. During the 2000–2001 season, Parkedale announced its departure from the influenza vaccine market, writing off \$45 million (Danzon et al. 2005). Based on these data, we chose a nominal value of \$40 million, but let it vary from \$20 million to \$100 million for our analysis. The nominal values of the parameters and their ranges are summarized in Table 1.

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