A Supplier's Optimal Quantity Discount Policy Under Asymmetric Information

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In the supply-chain literature, an increasing body of work studies how suppliers can use incentive schemes such as quantity discounts to influence buyers' ordering behaviour, thus reducing the supplier's (and the total supply chain's) costs. Various functional forms for such incentive schemes have been proposed, but a critical assumption always made is that the supplier has full information about the buyer's cost structure. We derive the optimal quantity discount policy under asymmetric information and compare it to the situation where the supplier has full information.
(*Supply Contracts*; *Coordination*; *Lot Sizing*; *Quantity Discounts*; *Asymmetric Information*)

1. Introduction

The most well-established framework for studying coordination in supply chains is perhaps that of choosing lot sizes in a tightly coupled system with lot-for-lot production. Imagine a single supplier with high setup costs shipping to a single buyer with low setup costs, where the supplier's production lot size is equal to the lot size shipped to the buyer. Clearly, letting either party determine lot size independently would lead to inefficient outcomes. Starting with (among others) Goyal (1976), Monahan (1984), and Lee and Rosenblatt (1986), the joint economic lot-sizing literature has examined the case where the supplier wishes to induce the buyer to choose a higher lot size than she would of her own accord. Reviews by Goyal and Gupta (1989) and Weng (1995) show how coordination can be achieved in integrated lot-sizing models with deterministic demand. Their work and that of others provides valuable insights into how and when quantity discount schemes can be used to achieve jointly optimal outcomes. Weng (1995) shows that, as long as the quantity discount offered does not affect demand, a quantity discount scheme can indeed yield jointly optimal lot sizes. Several different types of quantity discount schemes have been proposed in the literature, including all-units and incremental discounts, and with a variety of functional forms. Appropriately designed, any of these schemes can lead the buyer to choose the jointly optimal lot size and make both parties better off than without any form of coordination.

A critical assumption made throughout this literature, though, is that the supplier has full information, and can design the quantity discount scheme accordingly; this rarely will be true in practice. In this paper we drop that full information assumption and derive the supplier's optimal quantity discount scheme when the buyer holds private information about her cost structure. Specifically, imagine that a supplier and buyer are about to start doing business together, and that the supplier's setup costs are (considerably) larger than the buyer's. We examine the lot-for-lot production system that is traditional in this context, so the supplier's production lot size equals the transfer lot size. The simplest mode of operating would be to agree on a price and let the buyer order as frequently as she wishes; clearly this will lead to a much higher order frequency than the supplier would like. The

0025-1909/00/4603/0444$05.00
1526-5501 electronic ISSN

supplier wants to induce the buyer to order less frequently, realizing she will be reluctant to do so, but, as he cannot observe the buyer's cost structure, he does not know how "reluctant" she truly is. The supplier needs to design an incentive mechanism to overcome this. The buyer's reluctance can stem from several causes and is often not easily quantifiable even to the buyer; from the supplier's standpoint, the buyer's "holding costs" are anything but known. (One could perform the analyses in this paper with information asymmetry about the buyer's setup costs, but, as these are generally assumed to be far less than the supplier's, this case is less interesting.)

In this paper we compare two contracts: the supplier's optimal contract under full information (case FI) and under asymmetric information (case AI). The asymmetric information case can also be interpreted as the optimal contract to offer to a group of heterogeneous buyers when the supplier cannot price-discriminate. Lal and Staelin (1984) were unable to obtain a closed-form solution to a closely related problem: Given N groups of buyers of different sizes, with holding costs, order costs, and demand rates varying between groups but not within groups, what is the optimal pricing policy? Our analysis is a partial solution to their problem: we only vary the buyer's holding costs, but derive the optimal quantity discount policy for an arbitrary continuum of buyer types. In §2, we develop the basic model under full information. Section 3 derives the optimal contract under asymmetric information. We compare the performance of both schemes in §4; conclusions and further research are discussed in §5.

2. The Basic Model: Full Information in a Lot-for-Lot System

Let k_s and k_b denote the setup costs incurred by the supplier and buyer respectively and let h_b denote the buyer's unit holding cost per unit time. (All notation is summarized in Table 1.) We assume k_s and k_b are constant and common knowledge. Two decisions are considered: order lot size, Q, and the contract specifying the quantity discount $P(Q)$ per unit time from supplier to buyer. The buyer's operating cost is $C_b(h_b, Q) = (k_b d/Q) + (h_b/2)Q$; $Q_b(h_b) = \sqrt{2k_b d/h_b}$ is the lot size that minimizes $C_b(h_b, Q)$. We assume that supplier and buyer work under a lot-for-lot system, as is common in literature on integrated buyer-supplier lot sizing. (This assumption is relaxed in Corbett and de Groote 1997). Under lot-for-lot, the supplier's cost function is $C_s(Q) = k_s d/Q$ so that his individual optimum Q_s is the largest allowable lot size. The system (or joint) operating cost is $C_j(h_b, Q) = C_b(h_b, Q) + C_s(Q) = [(k_b + k_s)d/Q] + \frac{1}{2}h_b Q$ and the corresponding jointly optimal lot size is $Q_j(h_b) = \sqrt{2(k_b + k_s)d/h_b}$. The assumption that the supplier has the larger setup cost, i.e. $k_s \geq k_b$, is the starting point for the joint economic lot sizing literature.

Table 1 **Notation**

Notation	
$i = FI, AI$	cases considered
FI	full information
AI	asymmetric information with revelation
k_b, k_s	buyer's and supplier's setup costs
h_b	buyer's unit holding costs per period
$[\underline{h}_b, \bar{h}_b]$	range of buyer holding cost h_b
$F(h_b)$, $f(h_b)$	supplier's prior distribution and density over h_b
$E[X]$	expectation of a random variable X
d	demand per period
C_b, C_s, C_j	buyer, supplier, and joint cost function (excluding discount)
Q	lot size
Q_b, Q_s, Q_j	buyer's, supplier's, and jointly optimal lot size (without contracting)
$P_i(h_b)$	payment from supplier to buyer, as a function of h_b, under contract i
$\dot{P}(h_b) = D_{h_b}P(h_b)$	partial derivative of $P(h_b)$ with respect to h_b
TC_b^+, TC_s^+	maximum (net) cost level acceptable to buyer and supplier
$TC_{b,i}$, $TC_{s,i}$, $TC_{j,i}$	buyer, supplier, and joint net cost function in case i, after discount

Assuming that trade takes place, the supplier's and buyer's total or net costs are their operating costs net of the discount $P(Q)$, so that $TC_s(Q) := C_s(Q) + P(Q)$ and $TC_b(h_b, Q) := C_b(h_b, Q) - P(Q)$. If no trade takes place, their total costs are equal to their reservation net cost levels TC_s^+ and TC_b^+, respectively: They will choose not to trade with each other if the net costs (after discount) of doing so would exceed these reservation values. These could be the costs of the market alternatives or, if there are none, TC_b^+ could be the cost of the market opportunity the buyer forgoes by not trading with the supplier, and TC_s^+ could be the cost to the supplier of converting the process to supply a different market altogether. If their joint costs under Q_j exceed $TC_s^+ + TC_b^+$, they will definitely not trade; this gives the condition that $\sqrt{2d(k_b + k_s)h_b} \leq TC_s^+ + TC_b^+$. Only the buyer knows h_b. This condition can be rewritten to give an upper bound $\bar{h}_b$ where

$$h_b \leq \bar{h}_b \leq \frac{(TC_s^+ + TC_b^+)^2}{2d(k_b + k_s)}. \tag{1}$$

If (1) is met *under full information*, mutually beneficial trade can always take place. Under asymmetric information, however, (1) is necessary but not sufficient. Below, we let the supplier choose a cut-off point h_b^* such that he will choose not to trade with buyers with $h_b > h_b^*$, as the supplier's total costs would then exceed TC_s^+. Assume that the supplier holds a prior distribution $F(h_b)$ over h_b with support $[\underline{h}_b, \bar{h}_b]$. Demand per unit time d is known and constant. In particular it is not affected by the lot sizing or contracting decisions. The supplier has the initiative to propose contracts but the buyer may refuse them. This corresponds to a principal-agent framework with supplier as principal and buyer as agent with "adverse selection" (ex ante information asymmetry) but no "moral hazard" (unobservability of effort); see Laffont and Tirole (1993) for more on these concepts.

The need to coordinate stems from the fact that the buyer, in optimizing her own costs, does not consider the cost impact for the supplier, leading to $Q_b < Q_j < Q_s$. The supplier can influence the buyer's choice of Q by specification of the contract $P(Q)$, leading the buyer to minimize her net costs $TC_b(h_b, Q) = C_b(h_b, Q) - P(Q)$. By doing so, he can induce the buyer to choose a Q that reduces his operating cost $C_s(Q)$ and the total system cost $TC_j(h_b, Q)$. However, this is done at the expense of the discount $P(Q)$. In designing a contract, the supplier must therefore trade off the *efficiency* of the outcome against the *sharing* of the resulting efficiency gains between the two parties. It is easy for the supplier to induce efficiency by passing on his setup cost, charging the buyer k_s for each delivery, thus making the buyer internalize all variable costs (the "individually rational and responsible decision" (IRRD) approach suggested by Joglekar and Tharthare 1990). The buyer will then choose the jointly optimal lot size Q_j. To make this contract palatable to the buyer (regardless of h_b) the supplier must offer a high discount; this approach is therefore efficient but the resulting cost sharing is unattractive for the supplier.

In the principal-agent framework, the supplier first proposes a "menu of contracts" or discount scheme, specifying the discount $P(q)$ offered for any lot size q. The buyer decides whether or not to accept the contract and, if she accepts, chooses some order lot size Q. The supplier gives the buyer a discount of $P(Q)$. The supplier's problem can be formalized as follows:

$$\mathcal{S} \quad \min_{Q,P(Q)} E[TC_s(Q)] = E[P(Q) + C_s(Q)] \tag{2}$$

subject to

$$\textbf{IC: } Q = \arg\min_q \{TC_b(h_b, q)\} = \arg\min_q \{C_b(h_b, q) - P(q)\} \quad \forall\ h_b \in [\underline{h}_b, \bar{h}_b] \tag{3}$$

$$\textbf{IRb: } TC_b(h_b, Q) = C_b(h_b, Q) - P(Q) \leq TC_b^+ \quad \forall\ h_b \in [\underline{h}_b, \bar{h}_b] \tag{4}$$

$$\textbf{IRs: } TC_s(Q) = C_s(Q) + P(Q) \leq TC_s^+. \tag{5}$$

The expectation in (2) will be defined more precisely below. The supplier minimizes his expected total cost taking into account the buyer's reaction to the contract as expressed in the two constraints. Equation (3) is the *incentive-compatibility* (IC) constraint, and accounts for the buyer's selection of a lot size that minimizes her net costs including the discount. Equation (4) is the buyer's *individual-rationality* (IRb) constraint, and ensures the buyer's participation: The net costs incurred by the buyer (again after discount) must be at most equal to the reservation costs TC_b^+. If we were to omit IRb, the supplier could set $P(Q)$ arbitrarily negative. No buyer would ever wish to contract with such a supplier, hence the need to restrict the supplier's behavior by including IRb. Equation (5) is the equivalent condition for the supplier, and ensures that he will not end up worse off from the contracting process than from the outside alternative. The range over which IRs must hold is defined below.

In this case h_b is observed by the supplier. The optimal contract is equivalent to the selection of the joint economic lot size $Q_j(h_b)$, as in Goyal (1976), Banerjee (1986), and others, leaving all efficiency gains to the supplier. This outcome can be implemented with the contract

$$P_{FI}(h_b, Q) = \begin{cases} C_b(h_b, Q_j(h_b)) - TC_b^+ & \text{if } Q \geq Q_j(h_b) = \sqrt{\dfrac{2(k_s + k_b)d}{h_b}} \\ -TC_b^+ & \text{otherwise} \end{cases} \quad (6)$$

The buyer will choose $Q_j(h_b)$, resulting in net costs

$$TC_{b,FI}(h_b) = TC_b^+ \quad (7)$$

$$TC_{s,FI}(h_b) = C_j(h_b, Q_j(h_b)) - TC_b^+ = \sqrt{2(k_s + k_b)dh_b} - TC_b^+. \quad (8)$$

This is known as the first-best solution. Condition (1) guarantees that $TC_{s,FI}(h_b) \leq TC_s^+$ for all $h_b \in [\underline{h}_b, \bar{h}_b]$. Below we evaluate contracts when the supplier has no information about h_b other than the range $[\underline{h}_b, \bar{h}_b]$ and a prior distribution $F(h_b)$.

3. Optimal Contract Under Asymmetric Information (Case AI)

The approach we take here relies on the revelation principle (explained below) and closely follows Laffont and Tirole (1993) and Corbett (1997). We reformulate the problem as a direct revelation game and use optimal control theory to derive the supplier's optimal contract. A common assumption needed here is the following:

Assumption 1. *Decreasing reverse hazard rate*: $D_{h_b}[f(h_b)/F(h_b)] \leq 0$.

Many common distributions satisfy Assumption 1, including uniform, normal, logistic, chi-squared, and exponential; see Shaked and Shanthikumar (1994, p. 24) for more on the reverse hazard rate. If $f(x)$ defined on any (possibly infinite) interval $[l, u]$ satisfies Assumption 1, its truncation $f_T(x)$ to the interval $[\underline{h}_b, \bar{h}_b]$, defined by $f_T(x) := f(x)/(F(\bar{h}_b) - F(\underline{h}_b))$ does too. See Bagnoli and Bergstrom (1989) for an extensive discussion of the related condition of log-concavity and distributions which meet these conditions. Distributions with thin tails cause problems; to rule these out, we use an additional assumption to guarantee incentive-compatibility:

Assumption 2. $[F(h_b)/h_b f(h_b)] \leq (k_s/k_b)$ *for all* h_b.

Neither of these assumptions (1 and 2) on $F(h_b)$ are in fact necessary conditions.

Instead of proposing a single contract $P(Q)$ as in the full-information case, the supplier now offers a *menu of contracts* $\{Q, P(Q)\}$, letting the buyer choose a specific $(Q, P(Q))$-pair from the menu. We parameterize Q and P on h_b; offering a $\{Q(h_b), P(h_b)\}$ menu is equivalent to a $\{Q, P(Q)\}$ menu, though that equivalence relation need not exist in closed form. Whether or not contracting is explicitly based on h_b, the supplier can always infer h_b after the fact from the buyer's selection of $(Q, P(Q))$. The contracting procedure is then as follows. At the outset, the buyer knows h_b, unobserved by the supplier. The supplier offers a menu $\{Q(\hat{h}_b), P(\hat{h}_b)\}$, linking lot size $Q(\hat{h}_b)$ to the discount $P(\hat{h}_b)$ for any $\hat{h}_b \in [\underline{h}_b, \bar{h}_b]$ the buyer announces. The buyer chooses lot size $Q(\hat{h}_b)$ and discount $P(\hat{h}_b)$, effectively announcing $\hat{h}_b$. After contracting, lot size is fixed (forever) at $Q(\hat{h}_b)$, and the buyer receives a per-period discount of $P(\hat{h}_b)$. Buyer and supplier incur net costs per period of $C_b(h_b, Q(\hat{h}_b)) - P(\hat{h}_b)$ and $C_s(Q(\hat{h}_b)) + P(\hat{h}_b)$, respectively. Later we see that, under the supplier's optimal menu of contracts, if constraint IRb is satisfied for some h_b^*, it will also be satisfied for all $h_b \leq h_b^*$. For large values of h_b, though, constraint IRs may not hold. To ensure that both players' individual-rationality constraints are satisfied, the supplier can set h_b^* such that IRb and IRs are met for all $h_b \leq h_b^*$ and such that both parties will revert to their outside alternatives whenever $h_b > h_b^*$. This is explained more formally below.

The *revelation principle* (Laffont and Tirole 1993, p.

120) states that if there is an optimal contract for the supplier, then there exists an optimal contract under which the buyer will truthfully reveal her holding cost. This allows us to restrict our attention to such revelation mechanisms. Intuitively, this is easy to see: the supplier can predict exactly how a buyer with holding cost h_b would behave, and therefore what $\hat{h}_b$ she would announce when faced with any given quantity discount scheme. Therefore, he can construct a mapping $\hat{h}_b(h_b)$ and use this in designing the discount scheme $\{Q(\hat{h}_b), P(\hat{h}_b)\}$. The revelation principle allows us to formulate an incentive-compatibility constraint on $P_{AI}(h_b)$, which requires that it is optimal for a buyer with holding cost h_b to indeed reveal $\hat{h}_b = h_b$. Write the derivative $D_{h_b}P(h_b)$ of $P(h_b)$ with respect to h_b as $\dot{P}(h_b)$. Presented with a contract $\{Q_{AI}(h_b), P_{AI}(h_b)\}$, the buyer chooses which $\hat{h}_b$ to reveal by solving

$$\mathcal{B}_{AI} \min_{\hat{h}_b} \left\{ \frac{k_b d}{Q_{AI}(\hat{h}_b)} - \frac{h_b}{2} Q_{AI}(\hat{h}_b) - P_{AI}(\hat{h}_b) \right\}. \tag{9}$$

Taking the first-order condition with respect to $\hat{h}_b$ and requiring it be satisfied at $\hat{h}_b = h_b$ yields the incentive-compatibility constraint:

$$\dot{P}_{AI}(h_b) = \left(\frac{h_b}{2} - \frac{k_b d}{Q_{AI}(h_b)^2} \right) \dot{Q}_{AI}(h_b). \tag{10}$$

The common formulation of the buyer's individual-rationality constraint IRb requires that the contract is acceptable to any buyer, regardless of h_b:

$$\begin{aligned} TC_b^+ &\geq TC_b(h_b, Q(h_b)) \\ &= C_b(Q(h_b)) - P(h_b) \quad \forall \;\; h_b \in [\underline{h}_b, \bar{h}_b]. \end{aligned} \tag{11}$$

We relax this requirement, and give the supplier the option to refuse to trade with buyers with $h_b > h_b^*$. By default, IRb is then met for any $h_b > h_b^*$, and for $h_b \leq h_b^*$ IRb becomes

$$\begin{aligned} TC_b^+ &\geq TC_b(h_b, Q(h_b)) \\ &= C_b(Q(h_b)) - P(h_b) \quad \forall \;\; h_b \in [\underline{h}_b, h_b^*]. \end{aligned} \tag{12}$$

Of course, if $h_b^* \geq \bar{h}$, the two are equivalent. To find the optimal menu of contracts, the supplier solves the optimal control problem:

$$\begin{aligned} \mathcal{S}_{AI} \quad & \min_{Q(h_b), P(h_b), h_b^* \in [\underline{h}_b, \bar{h}_b]} E_{h_b}[TC_s(h_b)] \\ &= E_{h_b \leq h_b^*}\left[\frac{k_s d}{Q(h_b)} + P(h_b) \right] + E_{h_b > h_b^*}[TC_s^*] \end{aligned} \tag{13}$$

subject to the incentive-compatibility and individual-rationality constraints. More details on how to solve this problem are provided in Corbett and de Groote (1997).

Proposition 1. *In the optimal discount scheme under asymmetric information, the supplier will only trade with buyers with $h_b \leq h_b^*$ where h_b^* is the solution to*

$$TC_s^+ = \frac{k_s d}{Q_{AI}(h_b^*)} + P_{AI}(h_b^*), \tag{14}$$

and h_b^* is increasing in TC_s^+. For $h_b \in [\underline{h}_b, h_b^*]$, the lot size and discount scheme are given by

$$Q_{AI}(h_b) = \sqrt{\frac{2(k_s + k_b)d}{h_b + \dfrac{F(h_b)}{f(h_b)}}} \tag{15}$$

$$\dot{P}_{AI}(h_b) = \left(\frac{h_b}{2} - \frac{k_b d}{Q_{AI}(h_b)^2} \right) \dot{Q}_{AI}(h_b). \tag{16}$$

For $h_b \in]h_b^*, \bar{h}_b]$, no trade takes place and both players revert to their outside alternatives, incurring costs TC_s^+ and TC_b^+, respectively. When the prior $F(h_b)$ is uniform, the optimal policy and corresponding cost levels for $h_b \in [\underline{h}_b, h_b^*]$ are:

$$Q_{AI}(h_b) = \sqrt{\frac{2(k_s + k_b)d}{2h_b - \underline{h}_b}} \tag{17}$$

$$\begin{aligned} P_{AI}(Q) = \frac{h_b Q}{4} - \frac{(k_s - k_b)d}{2Q} + \tfrac{1}{2}\sqrt{2(k_s + k_b)(2h_b^* - \underline{h}_b)d} \\ - TC_b^+ \end{aligned} \tag{18}$$

$$\begin{aligned} TC_{b,AI}(h_b) = TC_b^+ - \tfrac{1}{2}\sqrt{2(k_s + k_b)d}\,\big(\sqrt{2h_b^* - \underline{h}_b} \\ - \sqrt{2h_b - \underline{h}_b}\big) \end{aligned} \tag{19}$$

$$\begin{aligned} TC_{s,AI}(h_b) = \tfrac{1}{2}\sqrt{(k_s + k_b)d}\,\big(\sqrt{2h_b^* - \underline{h}_b} + \sqrt{2h_b - \underline{h}_b}\big) \\ - \tfrac{1}{2}(h_b - \underline{h}_b)Q_{AI}(h_b) - TC_b^+ \end{aligned} \tag{20}$$

$$TC_{j,AI}(h_b) = \sqrt{2(k_s + k_b)d(2h_b - \underline{h}_b)} - \tfrac{1}{2}(h_b - \underline{h}_b)Q_{AI}(h_b) \qquad (21)$$

There is a probability $1 - F(h_b^*)$ of no trade taking place. For $h_b \le h_b^*$, $Q_{AI}(h_b)$ is decreasing in h_b. In general, h_b^* cannot be found explicitly as it is impossible to write $P_{AI}(h_b)$ explicitly; more detailed analysis of h_b^* is left for further research. For TC_s^+ sufficiently large, $h_b^* = \bar{h}_b$, which is the case (implicitly) traditionally assumed in the economics literature.

Whether it can be written in closed form or not, $P_{AI}(h_b)$ can always be interpreted as a quantity discount. Both $Q_{AI}(h_b)$ and $P_{AI}(h_b)$ are strictly decreasing in h_b, so there is a one-to-one mapping $P_{AI}(Q)$ which itself is increasing: A larger order quantity Q will lead to a larger lump-sum discount $P_{AI}(Q)$ (and hence a larger per-unit discount $P_{AI}(Q)/d$). For the uniform case, the quantity discount can be written explicitly as in (18). Here, the discount scheme $P_{AI}(Q)$ is increasing and concave in Q. It is interesting to note that the variable part of the discount depends on the "best practice" value $\underline{h}_b$, while the constant part also depends on the "worst-case" value h_b^*. Furthermore, supplier's and buyer's costs are decreasing in h_b; the buyer's net costs are (by construction) equal to TC_b^+ at h_b^*. At $\underline{h}_b$, the resulting lot size $Q_{AI}(\underline{h}_b)$ is equal to the jointly optimal lot size; as h_b increases, the discrepancy between $Q_{FI}(h_b)$ and $Q_{AI}(h_b)$ increases, their ratio being $Q_{AI}(h_b)/Q_{FI}(h_b) = \sqrt{h_b/\{h_b + [F(h_b)/f(h_b)]\}}$. The expressions derived in Proposition 1 may superficially resemble those in Min (1992); that paper however addresses the very different pricing problem where total demand depends on the price charged by the supplier, but where there is no lot-size-related coordination issue of any kind.

4. Comparing the Contracts

Having derived the optimal contracts under full and under asymmetric information, the comparisons between them are now immediate. In the full information case, when $\underline{h}_b = \bar{h}_b$, we have $Q_{AI} = Q_{FI} = Q_j$ and $TC_{s,AI} = TC_{s,FI}$ and $TC_{b,AI} = TC_{b,FI} = TC_b^+$. Let $TC_{j,b}$ and $TC_{j,j}$ represent total joint costs without coordination and under the joint optimum respectively.

Proposition 2. *Under asymmetric information, the lot sizes satisfy*:

$$Q_b(h_b) \le Q_{AI}(h_b) \le Q_{FI}(h_b) = Q_j(h_b) \quad \forall\ h_b \in [\underline{h}_b, \bar{h}_b]. \qquad (22)$$

Global efficiency is reduced by information asymmetry but contracting is more efficient than no form of coordination:

$$TC_{j,b} \ge TC_{j,AI} \ge TC_{j,FI} = TC_{j,j} \quad \forall\ h_b \in [\underline{h}_b, \bar{h}_b]. \qquad (23)$$

The supplier's expected net costs increase under information asymmetry:

$$TC_s^+ \ge E_{h_b}[TC_{s,AI}] \ge E_{h_b}[TC_{s,FI}]. \qquad (24)$$

The buyer's net costs decrease when she has private information:

$$TC_b^+ = TC_{b,FI}(h_b, Q_{FI}(h_b)) \ge TC_{b,AI}(h_b, Q_{AI}(h_b)) \quad \forall\ h_b \in [\underline{h}_b, \bar{h}_b]. \qquad (25)$$

The first inequality in (22) follows from Assumption 2; the second follows directly from the definition of Q_{AI} and Q_{FI}. One cannot meaningfully compare $TC_{b,b}$ with $TC_{b,FI}$ and $TC_{b,AI}$ or $TC_{s,b}$ with $TC_{s,FI}$ and $TC_{s,AI}$ as this would require additional assumptions about payment flows before contracting; these assumptions would then drive the resulting comparisons.

5. Conclusions and Future Research

This paper has derived the optimal quantity discount scheme for the joint economic lot-sizing problem under asymmetric information. Strictly speaking, to implement a quantity discount scheme as a reduction of unit price, one would have to look into the impact on the holding cost structure. One could also combine this with letting total demand vary with the discount offered, as in Weng (1995). Such extensions should not materially alter the key qualitative insights of the current work. One can easily redo this analysis with uncertainty about the buyer's setup cost; this leads to similar qualitative results. Allowing uncertainty on several dimensions independently, e.g. following Lal and Staelin's (1984) model of heterogeneous buyer groups, does not seem manageable with current techniques. Finally, we need to further explore how else one might model the presence of outside alternatives rather than through the individual-rationality constraint commonly used in the economics literature, either using the relaxation proposed here or in some other way.[1]

[1] This paper is partly based on discussions with Xavier de Groote before his death in 1996. Financial support from INSEAD and from the Owen Graduate School of Management at Vanderbilt University is gratefully acknowledged, as are comments from Marty Larivière, the referees and the associate editor.

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Accepted by Christopher S. Tang; received September 15, 1997. This paper was with the authors 9 months for 2 revisions.