Known, Unknown, and Unknowable Uncertainties

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Abstract

In normative decision theory, the weight of an uncertain event in a decision is governed solely by the probability of the event. A large body of empirical research suggests that a single notion of probability does not accurately capture peoples' reactions to uncertainty. As early as the 1920's, Knight made the distinction between cases where probabilities are known and where probabilities are unknown. We distinguish another case – the unknowable uncertainty – where the missing information is unavailable to all. We propose that missing information influences the attractiveness of a bet contingent upon an uncertain event, especially when the information is available to someone else. We demonstrate that the unknowable uncertainty – the case where the missing information is inaccessible to everyone including the experimenters – falls in preference somewhere in between the known and the unknown uncertainty.

Keywords: *uncertainty*.

1 Introduction

In normative decision theory, uncertainty about the occurrence of an event is treated by the single dimension of probability. Further, choices whose rewards are contingent on uncertain events are governed solely by the probability and not by the nature of events. All forms of uncertainty are therefore treated alike. Thus, the two events "Heads on the toss of a fair coin" and "Strait Times Index goes up next week" carry the same weight in decision making if their subjective probabilities are identical.

The purpose of this paper is to examine whether people treat all forms of uncertainty in the same way. Our thesis is that the three forms of uncertainty that we call known, unknown, and unknowable are useful distinctions in the study of choice. By and large, people feel most comfortable with the uncertainty when probabilities are known (objective chance device, for example) and are generally agreed upon. They feel least comfortable with the uncertainty for which they do not know the probability, but for which the probability could be known or is known by someone else (Ellsberg Urn with unknown proportion, for example). The intermediate case is the uncertainty for which it is reasonable to assert that no one knows the probability (proportion of yellow versus red M&M candies in an unopened bag).

The distinction between known and unknown probabilities dates back at least to Knight (1921), with his risk versus uncertainty dichotomy. Keynes (1921) argued that both probability and the weight of evidence supporting the probability influence decisions. Savage (1954) and de Finetti (1937) argued that such distinctions have no role in normative decision theory. The famous Ellsberg paradox, however, demonstrates that the uncertainty about probabilities (ambiguity or vagueness) can affect peoples' decision-making behavior (Ellsberg (1961)).

Several researchers, including Becker and Brownson (1964), Slovic and Tversky (1974) and MacCrimmon and Larsson (1979) found strong support for ambiguity avoidance. Ambiguity about probabilities was manipulated both as a second order distribution (Curley and Yates (1985), Kahn and Sarin (1988), Boiney (1993)) and as a lack of familiarity with an event, such as Pierce Industries stock price will go up or down (MacCrimmon (1965)). The ambiguity avoidance was confirmed with sophisticated subjects (Hogarth and Kunreuther (1989)) as well as in experimental market settings (Sarin and Weber (1993)). Camerer and Weber (1992) provide a review of the literature on decisions under ambiguity.

In an important study, Heath and Tversky (1991) demonstrated that people prefer to bet on events about which they feel more knowledgeable or competent. They showed examples in which people preferred a bet on "football" or "politics" to matched chance events even though the former events have vague probabilities. Fox and Tversky (1995) extended these results by showing that the perception of knowledge can be manipulated by a suggested comparison to others who are more knowledgeable.

In normative decision theory, the attractiveness of a bet (overall value function) depends only on the probabilities of events and the utilities associated with payoffs. For a given state of information, the choices do not depend on whether one feels more or less sure about the probabilities. It appears, at least descriptively, that ambiguity or the missing information about probabilities can affect the attractiveness of a bet. Frisch and Baron (1988) conjectured that ambiguity avoidance is driven by the "salience of missing information." Generally speaking, people tend to prefer specificity of probability to vagueness in probability. Thus, a bet on the toss of a fair coin is often deemed more attractive that a bet on the toss of a thumbtack, presumably because in the latter case there is missing information about probabilities.

When information is missing (proportion of white or yellow balls in a bag is unknown), people may overweigh the worse possibilities and adjust their choice on the side of caution. In the Women's World Cup '99 soccer tournament, the U.S. drew North Korea, Nigeria, and Denmark in its group of first round matches. The U.S. team had never played North Korea or Nigeria prior to the World Cup. "You could call this the unknown group, because we really do not know North Korea or Nigeria very well," U.S. coach Tony DiCicco said after the draw. Tiffany Milbrett, a forward on the U.S. team, expressed her fear of the unknown: "Personally, I am a little scared of unknown teams. Often those types of teams are scrappy teams and scrappy teams' strengths tend to match up against our weaknesses pretty well." Subsequently, the U.S. defeated Denmark 3-0, North Korea 3-0, and Nigeria 7-1.

Besides pessimism, a bet with missing information may seem less attractive because of justification or regret. Locke (1690) states: "He that judges without informing himself to the utmost that he is capable, can not acquit himself of judging amiss." Raiffa (1984) echoes the similar sentiment and observes that the kibitzer is often internalized and the divided self anticipates the ex post regret. Experiencing anxiety or nervousness when deciding with missing information is not uncommon.

In addition to the missing information, the knowledge or competence that one feels in evaluating the bet influences the attractiveness of a bet (Heath and Tversky (1991), Fox and Tversky (1995), Keppe and Weber (1995)). Heath and Tversky (1991) found that people prefer bets about which they feel especially competent or knowledgeable to the equivalent probability chance bets. Fox and Tversky (1995) found that a bet is deemed less attractive if more knowledgeable individuals are also evaluating the same bet. The suggested comparison with more knowledgeable individuals leads to recognition of inferior knowledge that one possesses, which diminishes the attractiveness of the gamble.

Our main hypothesis is that the attractiveness of a bet is influenced by several factors over and beyond probabilities and payoffs. One factor that we examine in our empirical studies is the role of missing information in its impact on attractiveness. Our conjecture is that people find the missing information more palatable when it is unavailable to all (unknowable case) compared to the case when the missing information is possessed by others or can be easily obtained (unknown case). The known bet is deemed more attractive than the unknowable bet because for the known bet probabilities are precisely specified. For the unknowable bet, the missing information, even if unavailable to all, has an effect of reducing its attractiveness. In the unknown case, one feels ignorant compared to others and therefore less confident in one's choice. In the unknowable case, the missing information is unavailable to all; therefore, one does not feel a particular information disadvantage. This distinction between the known, unknown and unknowable cases is consistent with the comparative ignorance hypothesis of Fox and Tversky (1995).

2 Experiments

A large body of empirical literature considers situations where uncertainty is known or unknown. The known uncertainty is simply the case where probability is precisely specified. In the case of unknown uncertainty, the subject does not know the probability, but believes that some other person may know it (experimenter, for example). We now distinguish the unknowable uncertainty where a subject believes that probabilities, to some reasonable degree, are unknown to everyone. In an experimental design, it is not possible to be completely confident that all subjects do indeed believe that the situation they are dealing with represents an unknowable uncertainty. Our experimental manipulations are therefore approximations of the condition of unknowable uncertainty.

Study 1

In this study, the hypothetical Ellsberg urn was used to manipulate the known, unknown, and unknowable uncertainty (Exhibit 1). The subjects were not shown the bags and were not paid for their participation. This experimental manipulation is similar to that used by Yates and Zukowski (1976).

EXHIBIT 1 HERE

There are seven conditions. In Condition 1, the subjects evaluate all three types of probabilities – they state their willingness to pay for bets contingent on each type of probability. In Condition 2, the subjects evaluate the unknown and unknowable probabilities. In Condition 3, the subjects evaluate the known and unknowable probabilities and in Condition 4 they evaluate the known and unknown probabilities.

Thus, Conditions 1 through 4 are comparative. In non-comparative Conditions 5, 6, and 7, the subjects evaluate only one of the three probabilities.

We recruited 158 undergraduate students from three separate classes. In Class 1, 23 students received Condition 1. In Class 2, we randomly assigned 88 students to three groups. Each of the three groups received either Condition 2, 3, or 4. Thus, each group received only the description of two types of probabilities. In Class 3, 47 students were randomly assigned to three groups where each group received either Condition 5, 6, or 7. Each group in this class received the description of only one type of probability. The results of this experiment are summarized in Exhibit 2.

EXHIBIT 2 HERE

In Condition 1, we observe that the mean price of the known bet is the highest and that of the unknown bet is the lowest with the unknowable bet in the middle. All pairwise price differences in Condition 1 (known versus unknown, known versus unknowable, and unknowable versus unknown) are significant ($p < 0.05$). In Condition 2, the mean price of the unknowable bet is significantly higher than that of the unknown bet ($p < 0.05$). In Condition 3, the mean price of the known bet is only marginally higher that that of the unknowable bet ($p < 0.15$). In Condition 4, the mean price of the known bet is significantly higher than for the unknown bet ($p < 0.05$).

Under comparative conditions, it is clear that the subjects find the missing information, when it is unavailable to all (unknowable case), more palatable than when the missing information is available to someone else (unknown case). Under non-comparative conditions, we were unable to detect significant differences among the mean prices of the known, unknown, and unknowable bets. Fox and Tversky (1995) also found that under non-comparative condition the mean prices of the known and unknown bets are similar.

A problem with using randomization to achieve the unknowable condition is that the approach induces a second order distribution over the probability. The manipulation therefore may not represent an ambiguity that is characterized by missing information or lack of knowledge. We now employ a non-urn context to study the known, unknown, and unknowable uncertainties.

Study 2

In this study, instead of drawing a ball from an urn, the subjects draw a candy from a bag of M&M's candies (labeled as Game 1). The unknowable condition is approximated by asking the subjects to draw a candy from a bag of M&M's candies that has not been opened. In the unknown condition, the bag has been opened and counted, but the subjects do not know the relative proportion of different colors. In another manipulation, the subjects guess which of the two apples that have not yet been cut has more seeds (labeled as Game 2) which represents the unknowable condition. In the unknown condition, apples have been cut and seeds counted, but the subjects do not know the results.

A bag of M&M's candies contains candies of six different colors. Therefore, we defined an event A ≡ {red, blue, or orange} and the complement of A as the event $\overline{A}$ ≡ {green, brown, or yellow}. The subjects provided prices for the bet on A (if A occurs win $100; otherwise nothing) as well as for the bet on $\overline{A}$. Our pre-tests revealed that the mean probability of event A is approximately 0.5. Because of randomness in the manufacturing and bagging process, the proportion of colors in A or $\overline{A}$ differs from one bag to the other. M&M/MARS Company verified that the true distribution in the peanut M&M's candies contains 50% green, brown, or yellow and 50% red, blue, or orange M&M's with the actual proportion varying from one bag to the next.

We employed the more stringent non-comparative design for these experiments. We did not use the comparative design because in Study 1 we were able to establish a difference in the known, unknown, and unknowable cases in the comparative setting, but could not detect the difference in the non-comparative setting.

We have three independent groups of 235 undergraduate students who participated in this study. The experiments were conducted in three separate classrooms. Group 1 (n = 93) was given the unknowable case. The unknowable case consisted of two games: Game 1 is drawing a candy from a bag of M&M's that is not opened yet and Game 2 is guessing which of the two apples has more seeds when the apples are not cut yet. Subjects physically saw the M&M's bag as well as the apples, but did not actually play the bet subsequent to their evaluations. For Game 1, the questions are depicted below:

1. *If the candy you draw is <u>a red, a blue, or an orange candy</u> you win $100; otherwise, you win nothing. Suppose that you are offered a ticket to play this game. What is the <u>most</u> that you would be willing to pay for a ticket to play this game? $______*

2. *If the candy you draw is <u>a green, a brown, or a yellow candy</u> you win $100; otherwise, you win nothing. Suppose that you are offered a ticket to play this game. What is the <u>most</u> that you would be willing to pay for a ticket to play this game? $______*

Group 2 (n = 79) was assigned the known case. The known case consisted of only Game 1 (M&M's candies) and not Game 2 (Apples). To implement the known case, the proportion of red, blue, and orange candies (A) and of green, brown, and yellow candies ($\overline{A}$) was kept to be 0.5 each. The content of the bag was revealed to the subjects so that everybody knew that the probability of winning is precisely 0.5.

Group 3 (n = 63) was assigned the unknown case. The unknown case consisted of two games. In Game 1, they were asked to draw a candy from a bag of M&M's that had been opened and counted by the experimenter, but they did not know the relative proportion. In Game 2, they were asked to guess which of the two apples had more seeds where the apples had been cut and the seeds counted by the experimenter, but they did not know the results. We physically showed a bag of M&M and apples to all groups, except for the subjects in Group 2 who only saw the M&M bag. Four subjects from Group 1, three subjects from Group 2, and one subject from Group 3 were discarded because of dominance violation (price = $0 or $100) or non-response.

In Game 1, the subjects provided their willingness to pay to bet on event A as well as on event $\overline{A}$. Since there was virtually no difference in the mean prices for A and $\overline{A}$, we report the average price ((WTP (A) + WTP ($\overline{A}$))/2).

EXHIBIT 3 HERE

The results are presented in Exhibit 3. For Game 1, the mean price of the known bet is clearly higher than that of the unknown bet ($p < 0.05$). The mean price of the known bet is only marginally higher than that of the unknowable bet ($p < 0.1$). The mean price of the unknowable bet is higher than that of the unknown bet, but the difference is statistically insignificant. For Game 2, the mean price of the unknowable bet is significantly higher than that of the unknown bet ($p < 0.05$).

3. Discussions

In this paper, we have distinguished between three forms of uncertainty: known, unknown, and unknowable. Two experiments were conducted to examine subjects' reactions to the known, unknown, and unknowable uncertainties. The key result that emerges from these experiments is that the unknowable uncertainty (the subject does not know the probability, but believes that others also do not know the probability) is intermediate to the known and the unknown forms of uncertainty. The specificity of the probability (known case) is preferred to the vagueness in probability (unknown and unknowable cases). However the vagueness in probability is made more tolerable when others also lack the information about probability and are therefore perceived to be in the same boat (unknowable case) than when one is missing information that others may possess (the unknown case). Our findings are consistent with the prediction of the Comparative Ignorance Hypothesis of Fox and Tversky (1995). Essentially, comparative ignorance effects are weaker in situations for which counterfactual states of knowledge are less available; that is, probabilities are not readily knowable. If the experimenter knows the probability, the comparative ignorance effect is more pronounced and the subjects are reluctant to bet.

The distinction between the unknown and unknowable uncertainty depends on the assumption that a subject makes about the availability of information. In an experimental setting, it is easier to manipulate a subject's likely assumptions about the availability of information. For example in Study 1, we manipulated the usually unknown uncertainty of Ellsberg's Urn to be closer to unknowable uncertainty through randomization. Similarly in Study 2 by not opening the bag or by not cutting the apple in advance, the unknowable case was approximated. In a real-world situation, the context would determine whether the uncertainty is closer to known, unknown, or unknowable. Actuaries, for example, have access to common data sets that can be used to provide frequencies of accidents or mortalities. The uncertainty that the actuaries face is closer to the known case. A person who does not follow the stock market is likely to consider an investment in stocks as an unknown uncertainty. Buying a used car creates an unknown uncertainty for most people. Predictions of some types of natural hazards (Will a major hurricane pass through my town next year?) and acts of God (Will lightning damage my property?) may be treated closer to unknowable uncertainty by most people. Similarly, it may be reasonably conjectured that the outcome of a presidential race in the U.S. (Democrat versus Republican) in the year 2020 or the prediction of interest rates ten years from now are closer to the unknowable cases.

Subjects in our experiments exhibit the behavior that is likely to be observed in many real situations. In medical treatments, investment choices, and personal decisions such as the selection of a vacation destination, people are likely to prefer, *ceteris paribus*, the known uncertainty to the unknown uncertainty with the unknowable uncertainty somewhere in-between.

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Exhibit 1

Ellsberg Urn Manipulation of Known, Unknown, and Unknowable

Type of Probability	***Cases***
Known Probability	The experimenter filled a bag with 5 red poker chips and 5 black poker chips. You are allowed to examine the bag.
Unknown Probability	The experimenter filled a bag with 10 poker chips that are red and black, but you do not know the relative proportion. You are not allowed to examine the bag.
Unknowable Probability	The experimenter filled a box with 11 bags. The experimenter filled each bag with 10 poker chips that are red and black. Bag 1 has 0 red and 10 black. Bag 2 has 1 red and 9 black and so on. You are allowed to examine the bags. Next, you are asked to draw a bag from the box. The bag you draw is labeled as Bag C. You and the experimenter are not allowed to examine Bag C.

Exhibit 2

The Mean Willingness to Pay (WTP) in Comparative and Non-comparative Conditions for Known, Unknown, and Unknowable Probabilities

Condition	Known	Unknown	Unknowable
1. Comparative: Known, Unknown, Unknowable	$24.04, n = 23, SE = 4.50	$8.67, n = 23, SE = 2.69	$13.02, n = 23, SE = 2.46
2. Comparative: Unknown, Unknowable	– – –	$6.93, n = 28, SE = 2.72	$11.41, n = 28, SE = 2.0
3. Comparative: Known, Unknowable	$9.32, n = 33, SE = 2.13	– – –	$7.73, n = 33, SE = 2.45
4. Comparative: Known, Unknown	$9.63, n = 27, SE = 2.58	$4.93, n = 27, SE = 1.97	– – –
5. Non-comparative: Known	$7.60, n = 15, SE = 1.87	– – –	– – –
6. Non-comparative: Unknown	– – –	$7.38, n = 13, SE = 3.7	– – –
7. Non-comparative: Unknowable	– – –	– – –	$9.30, n = 19, SE = 2.73

Exhibit 3

The Mean Willingness to Pay (WTP) for Known, Unknown, and Unknowable Uncertainty

	Game 1 M&M's Candies	**Game 2 Apples**
Known	$11.95, n = 76, SE = 1.88	---
Unknown	$6.38, n = 62, SE = 1.51	$5.22, n = 62, SE = 1.16
Unknowable	$8.78, n = 89, SE = 1.70	$9.61, n = 89, SE = 1.73