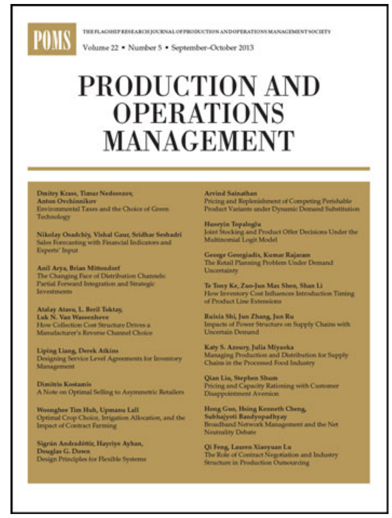
Accepted Manuscript

Title: Coping with Gray Markets: The Impact of Market Conditions and Product Characteristics

Authors: Reza Ahmadi, Foad Iravani, Hamed Mamani

DOI: http://dx.doi.org/doi:10.1111/poms.12319
Reference: POMS 12319
To appear in: Production and Operations Management



Please cite this article as: Ahmadi Reza., et al.,
Coping with Gray Markets: The Impact of Market Conditions and Product Characteristics.
Production and Operations Management (2014), http://dx.doi.org/doi:10.1111/poms.12319

Coping with Gray Markets: The Impact of Market Conditions and Product Characteristics

Reza Ahmadi* (rahmadi@anderson.ucla.edu)
Foad Iravani† (firavani@uw.edu)
Hamed Mamani† (hmamani@uw.edu)

September, 2014

Abstract

Gray markets, also known as parallel imports, have created fierce competition for manufacturers in many industries. We analyze the impact of parallel importation on a price-setting manufacturer that serves two markets with uncertain demand, and characterize her policy against parallel importation. We show that ignoring demand uncertainty can take a significant toll on the manufacturer's profit, highlighting the value of making price and quantity decisions jointly. We find that adjusting prices is more effective in controlling gray market activity than reducing product availability, and that parallel importation forces the manufacturer to reduce her price gap while demand uncertainty forces her to lower prices. Furthermore, we explore the impact of market conditions (such as market base, price sensitivity, and demand uncertainty) and product characteristics ("fashion" vs. "commodity") on the manufacturer's policy towards parallel importation. We also provide managerial insights about the value of strategic decision-making by comparing the optimal policy to the uniform pricing policy that has been adopted by some companies to eliminate gray markets entirely. The comparison indicates that the value of making price and quantity decisions strategically is highest for moderately different market conditions and non-commodity products.

Keywords: gray markets, parallel importation, parallel markets, strategic pricing, demand uncertainty, uniform pricing.
History: Received: February 2013, Revised: July 2013, January 2014, June 2014. Accepted: August 2014 by Amiya Chakravarty.

1 Introduction

Manufacturers around the world confront new pressures with the trade of their brand name products in unauthorized distribution channels known as gray markets. Gray markets primarily emerge when manufacturers offer their products in different markets at different prices. Price differentials may motivate enterprises or individuals to buy products from authorized distributors in markets with a lower price and sell them in markets with a higher price. Gray market channels may operate in the same market as the authorized distributors, or bring parallel imports from another market.

*Anderson School of Management, University of California, Los Angeles, 90095
†Foster School of Business, University of Washington, Seattle, 98195

This article has been accepted for publication and undergone full peer review but has not been through the copyediting, typesetting, pagination and proofreading process, which may lead to differences between this version and the Version of Record. Please cite this article as doi: 10.1111/poms.12319

Each year products worth billions of dollars are diverted to gray markets. In the IT industry alone, the approximate value of gray market products was $58 billion dollars and accounted for 5 to 30 percent of total IT sales, according to a 2008 survey conducted jointly by KPMG and The Alliance for Gray Market and Counterfeit Abatement (KPMG 2008). In the pharmaceutical industry, 20% of the products sold in the United Kingdom are parallel imports (Kanavos and Holmes, 2005). In communications, nearly 1 million iPhones were unlocked in 2007 and used on unauthorized carriers worldwide (*New York Times*, 2008). International versions of college textbooks, drinks, cigarettes, automobile parts, luxury watches, jewelry, electronics, chocolates, and perfumes are among the numerous products that are traded in gray markets (Schonfeld, 2010).

Unlike counterfeits, products traded in gray markets are genuine. Growing numbers of efficient global logistics networks help gray markets reach more customers faster. Advancing web technology and a rapidly growing online retail sector also boost gray markets. Amazon, eBay, Alibaba, Kmart, and Costco are among the retailers known to have sold gray goods (Bucklin, 1993; Schonfeld, 2010).

As to benefit and harm, opinions about gray markets are mixed. Manufacturers generally consider gray markets harmful because products diverted to gray markets end up competing with those sold by authorized distributors, and unauthorized channels get a free ride from expensive advertising and other manufacturer efforts to increase sales. Also, brand value may erode as products become available to segments that the manufacturer deliberately avoided. Gray markets, however, can benefit manufacturers by generating extra demand and deterring competitors.

The existing literature on gray markets largely focuses on pricing decisions in deterministic settings. While consumer demand can be accurately estimated for some products or markets, in many cases manufacturers are challenged with high uncertainty in demand. In this paper, we consider a manufacturer that operates in two markets with uncertain demand under the threat of competition from a parallel importer. If the manufacturer were to charge different prices across the markets, the parallel importer could buy the product in the low-price market and transfer it to sell in the high-price market. The manufacturer can control gray market activities through two operational levers: price and quantity. Consumers base their purchase decisions on prices offered by the manufacturer and the parallel importer and their perception of the gray market relative to the authorized channel.

Our paper makes three contributions to the literature. First, we extend the existing models on parallel importation (e.g., Ahmadi and Yang, 2000; Xiao et al., 2011) to incorporate demand uncertainty and quantity decisions in a Stackelberg game model. To the best of our knowledge, this paper is the first work that analyzes both price and quantity decisions of a manufacturer that faces parallel importation. The solution to the competition model determines the manufacturer's reaction to gray market activities, such as ignoring, allowing, or blocking parallel imports – referred to as the manufacturer's policy. Our analysis shows that the presence of parallel importation compels the manufacturer to reduce her[1] price gap, while uncertainty in demand compels her to reduce her price in both markets. Moreover, reducing the price gap is more effective in controlling gray market activity than creating scarcity in the low price market unless the manufacturer is better off exiting the low price market completely. We show that ignoring demand uncertainty and using prices that are optimal in a deterministic setting can severely hurt the manufacturer's profit and result in a suboptimal policy towards parallel importation; therefore, it is important to design a model that accounts for both demand uncertainty and gray markets.

Our second contribution is that we explore the impact of market conditions and product characteristics on the manufacturer's policy. The term market conditions refers to relative market bases, price sensitivities, and demand uncertainties, while the term product characteristics represents consumers' perception of parallel imports relative to the authorized channel. If consumers strongly prefer to buy the product from the manufacturer, we refer to the product as a *"fashion"* item, whereas we refer to it as a *"commodity"* when consumers are indifferent between buying from the manufacturer and buying from the gray market. We observe that when the product is a fashion item, the manufacturer mainly ignores the gray market. When the relative perception of parallel imports is moderately high, meaning that the product is in transition from a fashion item to a commodity, and market conditions are somewhat different, the manufacturer is better off allowing parallel importation. Finally, when the product is a commodity and the competition from the parallel importer intensifies, the manufacturer's policy is to block the gray market. These observations provide guidelines for determining a company's policy towards parallel importation.

Our third contribution is that we provide managerial insights about the value of making price and quantity decisions strategically as opposed to using myopic policies in the presence of parallel importation. In particular, we compare the manufacturer's profit under the strategic pricing policy to the profit of the myopic uniform pricing policy, which has been used by some companies, such as TAG Heuer and Christian Dior (Antia et al., 2004). These international companies charge the same price in all markets to eliminate price differentials and parallel importation entirely. Although implementing a uniform pricing policy is easier, companies that adopt this policy forgo the benefit

[1] Throughout the paper, we refer to the manufacturer as a female and to the parallel importer as a male.

of price discrimination in the strategic pricing policy. Our comparison shows that, on the one hand, when market conditions are moderately different and the product has not become a commodity, the additional profit of strategic pricing is significant and can be as high as 28%; therefore, it is important that companies follow strategic pricing in these situations. On the other hand, we observe that when market conditions are either too similar or too different and the product is a commodity, uniform pricing becomes a good alternative to strategic pricing.

This paper is organized as follows. We review the literature in Section 2. In Section 3, we describe the modeling framework and assumptions. Section 4 presents the main results of the competition model and Section 5 discusses the observations and managerial insights. We conclude the paper in Section 6 and offer directions for future work. Finally, the appendix explores some extensions to our model such as the effects of exchange rate, correlated demand, and relaxing some of the model assumptions.

2 Literature Review

Despite the ubiquity of gray markets, this topic occupies a relatively small niche in the interface of marketing and operations management literature. Existing marketing and economics research into gray markets can be divided into two groups of studies. The first group includes empirical studies and qualitative discussions about factors leading to the emergence of gray markets, e.g., Dutta et al. (1999), Maskus (2000), Ganslandt and Maskus (2004), and Antia et al. (2004).

The second group of studies includes analytical models for the price decision and whether or not gray markets should be deterred. Dutta et al. (1994) study the optimal policy towards retailers selling across their territories. Bucklin (1993) examines the claims made by trademark owners and gray market dealers and draws public policy implications. Li and Maskus (2006) find that parallel imports inhibit innovation and diminish welfare if the manufacturer deters parallel imports with a high wholesale price. Matsushima and Matsumura (2010) and Chen (2009) explore the ramifications of parallel imports for intellectual property holders and manufacturers. These studies suggest that manufacturers should tolerate some level of territorial restriction violation. Xiao et al. (2011) show that whether a manufacturer sells directly or through a retailer is critical to determining the increase or reduction in manufacturer profit due to parallel importation. Shulman (2013) shows that competing retailers may divert to gray markets even if it does not increase total sales. Autrey et al. (2013) consider two firms that engage in a Cournot competition in a domestic market and face gray market activities when they enter a foreign market. They find that when the products are close substitutes, it is better to decentralize the management structure in the foreign market. Ahmadi and Yang (2000) investigate the interaction between a manufacturer and a parallel importer in a deterministic setting with endogenous prices. They show that not only does parallel importation increase total sales, but it can also increase manufacturer profit.

In the operations management literature, there exists a rich body of research on optimal pricing and quantity decisions with stochastic demand (Petruzzi and Dada, 1999; Chan et al., 2004); however, these studies ignore gray market activities. Recently, a few papers have analyzed quantity decisions and coordination in supply chains that face gray markets. In these papers, however, either price is exogenous or demand is deterministic. Dasu et al. (2012) consider a decentralized supply chain with exogenous pricing in which a retailer could salvage leftover inventory or sell it to the gray market. Altug and van Ryzin (2013) consider a manufacturer selling a product through a large number of retailers that sell their excess inventory to a domestic gray market. They assume a market-clearing price for the gray market, but an exogenous retail price. Hu et al. (2013) study a reseller who takes advantage of a supplier quantity discount offer and diverts a portion of orders to a gray market. They show that when the reseller's batch inventory holding cost is high, the gray market improves channel performance. Su and Mukhopadhyay (2012) consider a deterministic setting in which a manufacturer offers a quantity discount to one dominant retailer and multiple fringe retailers. Krishnan et al. (2013) study the impact of gray markets on a decentralized supply chain with one manufacturer and two retailers that may divert the product to the gray market, when demand is assumed to be deterministic.

Our work differs from the foregoing in that we analyze the impact of parallel importation on a vertically integrated manufacturer who must set both prices and quantities before demand uncertainty is resolved. We analyze the effect of market conditions and product characteristics on the manufacturer's policy and explore the value of a strategic reaction to parallel importation.

3 Modeling Assumptions and Framework

Consider a manufacturer who produces a single product at per-unit cost c and sells it in two separate markets. The manufacturer chooses price p_1 and quantity q_1 in market 1, and chooses price p_2 and quantity q_2 in market 2. We assume that there are no capacity constraints, and that unsatisfied demand in both markets are lost. For ease of exposition, we assume holding costs, lost-sales costs,

Table 1: Notations

Parameters	
N_i, b_i, ϵ_i	base, price sensitivity, and demand uncertainty of market $i = 1, 2$
(L_i, U_i), μ_i, σ_i	domain, expected value, and standard deviation of ϵ_i
$f_i(x), F_i(x), h_i(x)$	probability density, cumulative distribution, and hazard rate functions of ϵ_i
c	manufacturer's unit production cost
c_G	parallel importer's unit transfer cost
δ	consumer's relative perception of parallel imports
Manufacturer's Variables	
p_i, q_i	price and quantity in markets $i = 1, 2$
$\overline{\pi}$	expected total profit when there are no parallel imports
π	expected total profit in the presence of the parallel importer
π^d	expected total profit in the presence of the parallel importer for deterministic demand
Manufacturer's Optimal Variables	
$\widetilde{p}_i$, $\widetilde{q}_i$	when there are no parallel imports
$\widetilde{p}_i^{\,d}$, $\widetilde{q}_i^{\,d}$	when there are no parallel imports and demand is deterministic
p_i^*, q_i^*	in the presence of the parallel importer
p_i^{*d}, q_i^{*d}	in the presence of the parallel importer for deterministic demand
Parallel Importer's Variables	
q_G, p_G, π_G	quantity, price, and profit

and salvage values are zero, though these parameters can be added to our model.

The demand in both markets is stochastic and depends on price. In particular, we assume that demand in market $i = 1, 2$ is defined as $\mathcal{D}_i(p_i, \epsilon_i) = d_i(p_i) + \epsilon_i = N_i - b_i p_i + \epsilon_i$ in which $d_i(p_i)$ denotes the deterministic component of demand, N_i denotes the market base, b_i denotes the consumer sensitivity to price change (following Petruzzi and Dada, 1999; Ahmadi and Yang, 2000; Xiao et al., 2011; Su and Mukhopadhyay, 2012; Krishnan et al., 2013), and ϵ_i represents the stochastic component of demand. We assume that ϵ_i takes its value in the interval (L_i, U_i), and denote the probability density and cumulative distribution functions of ϵ_i with $f_i(x)$ and $F_i(x)$, respectively. The expected value and the standard deviation of ϵ_i are denoted with μ_i and σ_i. We assume that ϵ_1 and ϵ_2 are statistically independent and discuss the effect of correlated demands in Appendix. We also assume that ϵ_1 and ϵ_2 satisfy the Increasing Failure Rate (IFR) property and their hazard rate functions, denoted by $h_i(x) = \frac{f_i(x)}{1-F_i(x)}$, is increasing; i.e., $h_i'(x) > 0$. This property holds for many common distributions such as normal and uniform (Lariviere and Porteus, 2001). Table 1 summarizes the notations.

We analyze the competition between the manufacturer and a parallel importer using a Stackelberg game model and the following sequence of events: (1) the manufacturer acts as the leader and chooses her price and quantity for both markets before demand uncertainties are resolved; (2) having observed manufacturer's prices, the parallel importer may decide to buy the product from the manufacturer in the low-price market and transfer to the high-price market for resale if the price gap makes the venture sufficiently profitable. The parallel importer must choose the quantity to buy from the manufacturer and set his selling price in the high-price market; (3) demand uncertainty for the authorized channel is resolved in both markets and the total profits of the manufacturer and the parallel importer (if he transfers the product) are realized.

We make two assumptions about the parallel importer for tractability. First, we assume that the parallel importer places his order before other customers and the manufacturer cannot distinguish the parallel importer's order. Due to their economic incentives, most gray marketers hire agents to swiftly purchase products. Also, in most situations it is very difficult for a manufacturer to distinguish between orders received from customers who intend to buy the product for personal use and orders placed by gray market agents, especially when orders are placed through the Internet. KPMG's survey of IT companies reports that 42% of the respondents do not have a process to identify or monitor gray market activities, and that 62% of the respondents have no formal training programs to educate their staff or customers about gray market issues. Antia et al. (2004) note that methods for uncovering gray markets can be very costly and in many cases gray markets cannot be identified and stopped completely as they counteract manufacturers' efforts to track the diversion of products to gray markets. In the appendix we discuss the effects of relaxing this assumption and show that the outcomes of our model are fairly robust.

Our second assumption about the parallel importer is that he makes his decisions based on an estimate of average demand and does not have the capability to estimate the uncertainty he would face. We believe this assumption is reasonable because, unlike major manufacturers, most gray marketers typically have low capital and cannot invest in market research to estimate the parameters of demand distribution. Our numerical experiments in the appendix show that relaxing this assumption has little impact on the main insights of our model.

Finally, we assume, without loss of generality, that the direction of parallel importation is from market 1 to market 2. That is, if the manufacturer were able to achieve an ideal price discrimination in the two markets without worrying about a gray market, then market 1 would be the low price market and market 2 would be the high price market. To formalize this argument, let $\overline{\pi}$ represent the manufacturer's expected total profit if there were no parallel imports. Then the manufacturer solves

$$\max_{p_1,p_2,q_1,q_2} \overline{\pi} = E_{\epsilon_1,\epsilon_2}\left[p_1 min\Big\{q_1, \mathcal{D}_1\left(p_1,\epsilon_1\right)\Big\} + p_2 min\Big\{q_2, \mathcal{D}_2\left(p_2,\epsilon_2\right)\Big\} - c(q_1+q_2)\right]. \tag{1}$$

This is the classic price–setting newsvendor problem (Petruzzi and Dada, 1999) with two independent markets. The optimal prices, $\widetilde{p_1}$ and $\widetilde{p_2}$, satisfy

$$N_i - 2b_i\widetilde{p_i} + z_i\left(\widetilde{p_i}\right) - \int_{L_i}^{z_i(\widetilde{p_i})} F_i(x)dx + cb_i = 0 \quad (i=1,2), \tag{2}$$

in which $z_i\left(\widetilde{p_i}\right) = F_i^{-1}\left(1 - \frac{c}{\widetilde{p_i}}\right)$ with $F_i^{-1}(x)$ denoting the inverse of $F_i(x)$. The optimal quantities are equal to $\widetilde{q_i} = d_i(\widetilde{p_i}) + z_i\left(\widetilde{p_i}\right)$ (Xu et al., 2010). Throughout this paper, we assume that $\widetilde{p_2} > \widetilde{p_1}$.

4 Analysis of Competition

We now assume that a parallel importer is present. By entering the high-price market (market 2), the parallel importer engages in a Stackelberg game with the manufacturer. In what follows, we use backward induction to characterize the equilibrium of this two-stage game.

4.1 Parallel Importer's Problem

For given price and quantity decisions of the manufacturer, define q_G to be the quantity that the parallel importer buys from the manufacturer in market 1 and let p_G be the gray market price in market 2. We assume that the parallel importer incurs a cost of c_G to transfer one unit of the product to market 2. This cost represents the shipping cost and all other costs associated with distributing the product in market 2 (e.g., translating the user manual, repackaging, tariffs).

When there are no parallel imports in market 2, some customers buy the product from the manufacturer and some do not. Once the parallel importer enters market 2 and offers the product at price p_G, the market divides into three segments as depicted in Figure 1. The first segment is customers who continue to buy the product from the manufacturer. The second segment contains customers who buy the product from the parallel importer. Some of these customers initially bought from the manufacturer, but now switch to the parallel importer (the distance between the dashed lines) and some had not considered buying the product before due to the higher price charged by the authorized channel. The third segment contains those who had not bought the product before and continue to refrain from doing so even after the parallel importer enters the market.

The size of these segments is determined by the prices set by the manufacturer and the parallel importer. Size is also affected by the consumers' perception (valuation) of gray–market products

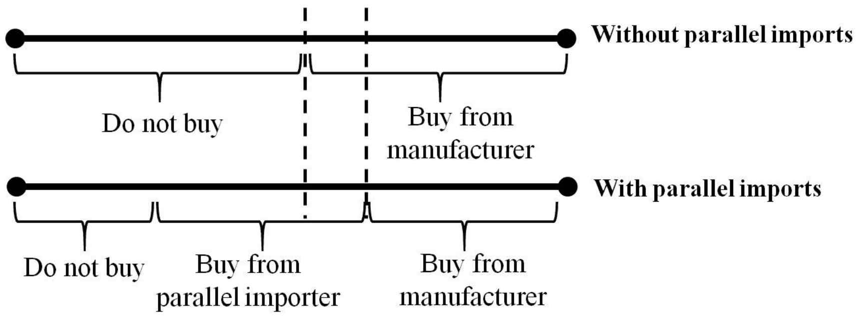


Figure 1: Segmentation of market 2 before and after parallel importation

relative to their valuation of products distributed by the manufacturer, which we denote with $0 < \delta < 1$. A low value of δ implies that consumers strongly prefer to buy the product from the authorized channel (unless the gray market price is significantly lower), whereas a high value of δ implies that consumers are somewhat indifferent between buying from the authorized channel and buying from the gray market. When consumers buy gray market products, they usually lose warranty and after–sales services from manufacturers or have to pay higher fees for such services. Also, even though gray markets carry genuine products, some consumers may not fully trust gray markets and suspect that gray market products are counterfeits. Therefore, consumers perceive gray markets to be inferior to authorized channels, valuing instead the peace of mind they get when they buy a product from authorized channels (Huang et al. 2004, Thompson 2009). How inferior consumers perceive parallel imports can be attributed to characteristics of the product. For example, warranty and other post–sales services are more important for some products than others, making gray markets more inferior for such products.

To determine the size of market segments, we note that the linear demand function $N_2 - b_2 p_2$ can be interpreted as the expected demand for the product when the consumers' net surplus of buying the product from the manufacturer is equal to $\theta - \frac{b_2 p_2}{N_2}$, in which θ is uniformly distributed between 0 and 1 and represents the heterogeneity in consumer taste. Given that consumers have a lower valuation of parallel imports compared to products distributed in the authorized channel, we assume that the net surplus of consuming parallel imports is equal to $\delta\theta - \frac{b_2 p_G}{N_2}$. Now, if θ_1 is the taste of the consumer who is indifferent between buying from the manufacturer and buying from the parallel importer, then $\theta_1 - \frac{b_2 p_2}{N_2} = \delta\theta_1 - \frac{b_2 p_G}{N_2}$ which gives $\theta_1 = \frac{p_2 - p_G}{N_2(1-\delta)} b_2$. Similarly, if θ_2 is the boundary between the segment that buys from the parallel importer and the segment that does not buy the product at all, we can write $\delta\theta_2 - \frac{b_2 p_G}{N_2} = 0$ which means $\theta_2 = \frac{b_2 p_G}{\delta N_2}$. Therefore, the net deterministic demand for the manufacturer is $N_2(1-\theta_1) = N_2 - \frac{p_2 - p_G}{1-\delta} b_2$ and the net demand for the parallel importer is $N_2(\theta_1-\theta_2)=\frac{\delta p_2-p_G}{\delta(1-\delta)}b_2$. However, because the parallel importer buys the product from the manufacturer in market 1, his order quantity is limited by q_1. Therefore, the parallel importer's profit maximization problem is

$$\max_{p_G} \quad \pi_G=(p_G-p_1-c_G)\,q_G$$

in which $q_G=min\left(\frac{\delta p_2-p_G}{\delta(1-\delta)}b_2,q_1\right)$.

Proposition 1 *For given prices p_1 and p_2, let $\psi(p_1,p_2)=\frac{\delta p_2-p_1-c_G}{2\delta(1-\delta)}b_2$ be the parallel importer's desired quantity. If the manufacturer's quantity in market 1 is large enough to fulfill the parallel importer's desired quantity, i.e., $q_1>\psi(p_1,p_2)$, then the parallel importer's optimal price and quantity will be*

$$p_G=\frac{\delta p_2+p_1+c_G}{2}, \qquad q_G=max\left(0,\psi(p_1,p_2)\right). \tag{3}$$

Otherwise, $p_G=\delta p_2-\dfrac{\delta(1-\delta)q_1}{b_2}$ and $q_G=q_1$.

All proofs are provided in the supplementary document. Equation (3) shows the parallel importer's optimal decisions when he is not constrained by manufacturer's quantity. The parallel importer incurs a cost of p_1+c_G to purchase the product in market 1 and transfer it to market 2. Clearly, if this cost is above the manufacturer's authorized price in market 2, p_2, transferring the product will not be profitable. However, because consumers in market 2 have a lower perception of gray market products, the importer's total purchase and transfer cost should be even lower (below δp_2) to justify his entry to the competition. As a result, the gray market is profitable if and only if $\delta p_2>p_1+c_G$. When product availability is low, the parallel importer charges $p_G=\delta p_2-\frac{\delta(1-\delta)q_1}{b_2}$ to sell q_1.

4.2 Strategic Manufacturer's Problem

Having obtained the parallel importer's optimal decisions, we now find the manufacturer's optimal price and quantity decisions. In doing so, we also characterize the manufacturer's optimal response to gray market activities. Specifically, hereafter we make the following distinction between the manufacturer's *decisions* and her *policy*. We use decision to refer to the manufacturer's price and quantity values, and use policy to describe the manufacturer's high-level reaction to parallel importation. The manufacturer's decisions can result in one of the following three policies:

Ignore the parallel importer. Under this policy, the manufacturer continues to use her decisions that are optimal in the absence of gray markets; i.e., $\widetilde{p_i}$ and $\widetilde{q_i}$. We will later show that the manufacturer uses this policy only if the gray market can be automatically eliminated by using prices $\widetilde{p_1}$ and $\widetilde{p_2}$. Intuitively this would be the case when consumers' relative perception of parallel imports is very low ($\delta \ll 1$) or the parallel importer's transfer cost, c_G, is so high that it would be too costly for him to transfer the product. This policy also arises when $\widetilde{p_1}$ and $\widetilde{p_2}$ are fairly close to each other. In this situation, these prices would render the gray market unprofitable unless δ is extremely high or c_G is extremely small.

Block parallel imports. If the difference between $\widetilde{p_1}$ and $\widetilde{p_2}$ is large enough for the gray market to operate, then the manufacturer may decide to block the parallel importer. The manufacturer has two levers to block the gray market.

Block using prices. In this policy, the manufacturer blocks the gray market by altering her prices such that $\delta p_2 = p_1 + c_G$. This could be an effective policy when the price difference in the absence of gray markets $(\widetilde{p_2} - \widetilde{p_1})$, consumers' perception of parallel imports (δ), and the importer's transfer cost (c_G) are such that the gray market could (barely) exist; however, a small reduction in the price gap between the two markets could make parallel importation no longer profitable. The manufacturer may also opt for this policy when the parallel importer could emerge as a strong competitor. This can happen when the consumers' perception of parallel imports is very high. The gray market, if allowed, could undercut the manufacturer and gain a significant portion of market 2.

Block using quantity. In this policy, the manufacturer blocks the gray market by making the product unavailable to the parallel importer. Since there is usually no way of identifying the orders placed by parallel importers, this policy is equivalent to exiting (or not entering) market 1. The manufacturer may choose this policy when the consumers' perception of parallel imports is very high ($\delta \approx 1$) and the manufacturer's optimal prices in the absence of the gray market are significantly different across the markets ($\widetilde{p_1} \ll \widetilde{p_2}$). In this situation, the manufacturer would lose a significant portion of her profit if she stays in both markets and blocks the gray market with prices. Therefore, she foregoes the relatively small profit in market 1 to eliminate parallel importation and only operates in market 2 using price $\widetilde{p_2}$. The impact of parallel imports on market entry/exit decisions has been witnessed by the pharmaceutical industry (*2020health*, 2011).

Allow parallel imports. Under this policy, the manufacturer allows the parallel importer to resell the product in market 2; i.e., she sets p_1 and p_2 such that $\delta p_2 - p_1 - c_G > 0$, and sets $q_1 > 0$. The manufacturer would opt for this policy when $\widetilde{p_1}$ and $\widetilde{p_2}$ are moderately different and the consumers' perception of parallel imports is neither too high nor too low. Blocking the parallel importer in such a setting requires a relatively significant deviation from otherwise optimal decisions. Furthermore, because consumers still highly value the authorized channel, the parallel importer is not a grave threat to the manufacturer. In this situation, the manufacturer would allow the gray market to emerge simply because blocking the parallel importer is more costly than allowing him.

The next proposition describes the impact of parallel imports on the manufacturer's demand, which has also been observed in deterministic settings (e.g., Ahmadi and Yang, 2000).

Proposition 2 *When the parallel importer transfers the product, the manufacturer's demand in market 1 increases by q_G, and her demand in market 2 decreases by δq_G.*

From Figure 1, we see that the manufacturer's demand in market 2 reduces because some consumers switch to the parallel importer. However, the segment of market 2 that buys the product from the parallel importer increases the manufacturer's demand in market 1. Overall, the manufacturer's demand goes up because parallel importation provides the product at a lower price and induces the consumers that have a lower willingness-to-pay to buy the product. The manufacturer could directly offer the product at a discounted price, but doing so through the authorized channel would lead to consumer confusion and severe demand cannibalization. Although the parallel importer also cannibalizes the demand of the authorized channel, this effect is alleviated because the importer is not affiliated with the manufacturer and has a lower reputation in the market.

To characterize the manufacturer's optimal decisions and policy, we first present an interesting result of our model. Proposition 1 states that the parallel importer ideally wants to transfer $\psi(p_1, p_2) = \frac{\delta p_2 - p_1 - c_G}{2\delta(1-\delta)} b_2$ units to market 2, but the actual quantity that he will obtain may be lower than $\psi(p_1, p_2)$ if q_1 is small. However, the next proposition shows that the manufacturer will not limit the parallel importer with quantity when she chooses prices that allow parallel importation.

Proposition 3 *If the manufacturer's prices (p_1, p_2) are such that the gray market could exist, i.e., $\delta p_2 - p_1 - c_G > 0$, then the optimal policy of the manufacturer is to either (a) block the parallel importer using quantity; i.e., $q_1^* = 0$, or (b) allow the parallel importer and provide a large enough quantity in market 1 to fulfill his entire order; i.e., $q_1^* \geq \psi(p_1, p_2)$.*

This proposition suggests that a mixed policy of allowing gray market activities through prices, but limiting the size of the gray market through quantity (i.e., allowing partial importation) is not optimal. Put differently, once she decides to allow gray market activities, the manufacturer would not reduce product availability to limit parallel importation. We note that this result depends on our assumption that the parallel importer is the first customer who receives the product. However, our experiments in the appendix indicate that even if this assumption is relaxed, the manufacturer will not limit the gray market by reducing her quantity. That is, prices are more effective in controlling gray market activities than reducing product availability.

We can now formulate the Stochastic Stackelberg Game (SSG) to characterize the manufacturer's optimal decisions and policy

$$\max_{p_1,p_2,q_1,q_2} \pi = E_{\epsilon_1,\epsilon_2}\Big[p_1 min\left\{q_1, \mathcal{D}_1\left(p_1,\epsilon_1\right) + q_G(p_1,p_2)\right\} \\ + p_2 min\left\{q_2, \mathcal{D}_2\left(p_2,\epsilon_2\right) - \delta q_G(p_1,p_2)\right\} - c(q_1+q_2)\Big], \quad (4)$$

where $q_G(p_1,p_2) = max(0,\psi(p_1,p_2))$ in light of Proposition 3. The next proposition characterizes the solution to the SSG.

Proposition 4 *Suppose the manufacturer does not leave market 1. Let $\widehat{p}_1$ be the solution to the following equation*

$$\delta\left(N_1 - 2b_1\widehat{p}_1 + cb_1 + z_1\left(\widehat{p}_1\right) - \int_{L_1}^{z_1(\widehat{p}_1)} F_1(x)dx\right) \\ + N_2 - 2b_2\left(\frac{\widehat{p}_1 + c_G}{\delta}\right) + cb_2 + z_2\left(\frac{\widehat{p}_1 + c_G}{\delta}\right) - \int_{L_2}^{z_2\left(\frac{\widehat{p}_1+c_G}{\delta}\right)} F_2(x)dx = 0. \quad (5)$$

Then, $\widehat{p}_1$ is unique and

(a) If $\delta\widetilde{p}_2 - \widetilde{p}_1 - c_G \leq 0$, then the manufacturer's optimal policy is to ignore the parallel importer. Thus, $p_1^* = \widetilde{p}_1$ and $p_2^* = \widetilde{p}_2$.

(b) If $\delta\widetilde{p}_2 - \widetilde{p}_1 - c_G > 0$ and $\eta > 0$, where

$$\eta = N_1 - 2b_1\widehat{p}_1 + \frac{c_G}{2\delta\left(1-\delta\right)}b_2 + z_1\left(\widehat{p}_1\right) + c\left(b_1 + \frac{b_2}{2\delta}\right) - \int_{L_1}^{z_1(\widehat{p}_1)} F_1\left(x\right)dx, \quad (6)$$

then the optimal policy is to block the parallel importer by setting $p_1^* = \widehat{p}_1$ and $p_2^* = \frac{\widehat{p}_1+c_G}{\delta}$.

(c) If $\delta\widetilde{p}_2 - \widetilde{p}_1 - c_G > 0$ and $\eta \le 0$, then it is optimal to allow parallel importation. In this case, p_1^* and p_2^* satisfy the following equations:

$$N_1 - 2b_1p_1^* + \frac{2\left(\delta p_2^* - p_1^*\right) - c_G}{2\delta\left(1-\delta\right)}b_2 + c\left(b_1 + \frac{b_2}{2\delta}\right) + z_1\left(p_1^*\right) - \int_{L_1}^{z_1\left(p_1^*\right)} F_1\left(x\right)dx = 0,$$

$$N_2 - 2b_2p_2^* - \frac{2\left(\delta p_2^* - p_1^*\right) - c_G}{2\left(1-\delta\right)}b_2 + c\frac{b_2}{2} + z_2\left(p_2^*\right) - \int_{L_2}^{z_2\left(p_2^*\right)} F_2\left(x\right)dx = 0.$$

(d) The manufacturer's optimal quantities are

$$q_1^* = d_1\left(p_1^*\right) + q_G\left(p_1^*, p_2^*\right) + z_1\left(p_1^*\right), \qquad q_2^* = d_2\left(p_2^*\right) - \delta q_G\left(p_1^*, p_2^*\right) + z_2\left(p_2^*\right).$$

The manufacturer controls the parallel importer's order quantity through her prices. However, changing the prices also affects the demand of the authorized channels. Therefore, she should choose prices that balance these effects. For given prices p_1 and p_2, if the parallel importer is allowed to transfer the product, then the change in the manufacturer's expected total profit will be $(p_1 - \delta p_2 - c(1-\delta))q_G(p_1, p_2)$, which is negative because the importer would enter only if $\delta p_2 - p_1 > c_G$. This means that, while total demand increases according to Proposition 2, the manufacturer's profit would always be less in the presence of the parallel importer. Thus, if $\widetilde{p}_1$ and $\widetilde{p}_2$ happen to eliminate the importer (i.e., $\delta\widetilde{p}_2 - \widetilde{p}_1 - c_G \le 0$), then the manufacturer simply ignores the parallel importer.

When $\delta\widetilde{p}_2 - \widetilde{p}_1 - c_G > 0$, the manufacturer has to change her prices and deviate from $\widetilde{p}_1$ and $\widetilde{p}_2$. In this scenario, the optimal policy would be to either block the parallel importer (by setting $\delta p_2 - p_1 - c_G = 0$), or to allow him (by setting $\delta p_2 - p_1 - c_G > 0$). Thus, one can solve the SSG by imposing the constraint $\delta p_2 - p_1 - c_G \ge 0$. The parameter η defined in (6) is simply the shadow price of this constraint for $(\widehat{p}_1, \frac{\widehat{p}_1 + c_G}{\delta})$. Proposition 4 shows that the optimal blocking price, $\widehat{p}_1$, and its corresponding shadow price, η, are the factors that determine whether the optimal policy is to allow or block the parallel importer. If $\widehat{p}_1$ makes the corresponding shadow price positive, then the constraint will be tight and the manufacturer will block the importer via $\widehat{p}_1$ and $\frac{\widehat{p}_1 + c_G}{\delta}$. However, if the shadow price is non-positive, then allowing parallel importation is the optimal policy.

We note that when δ approaches 1, (6) is positive and the manufacturer will always block the parallel importer. In this situation, products in the gray market become perfect substitutes for products in the authorized channel and the competition is highly intense. Therefore, the parallel importer could gain a significant size of market 2 if allowed.

Corollary 1 *If consumers' relative perception of parallel imports is very high, i.e., if $\delta \approx 1$, then the manufacturer blocks parallel importation.*

Proposition 4 assumes that the manufacturer is better off staying in both markets. The following result provides a necessary condition for when it is better for the manufacturer to exit (not enter) market 1 and only operate in market 2.

Proposition 5 *If the optimal policy for the manufacturer is to block the parallel importer with quantity (leave market 1) for given model parameters, then* $N_1 - b_1(\delta\widetilde{p_2} - c_G) + z_1(\delta\widetilde{p_2} - c_G) \leq 0$.

The interpretation of this necessary condition is that $\delta\widetilde{p_2} - c_G$ must be too high to generate a positive demand (hence a positive quantity) in market 1. Otherwise, the manufacturer can increase her profit by selling the product at price $\delta\widetilde{p_2} - c_G$ in market 1 and still block parallel importation. Next we look at how the presence of the parallel importer changes the manufacturer's prices.

Proposition 6 *If the presence of the parallel importer forces the manufacturer to alter her otherwise optimal prices (i.e., $\delta\widetilde{p_2} - \widetilde{p_1} - c_G > 0$), then the manufacturer always increases her price in market 1 and reduces her price in market 2. That is, $p_1^* > \widetilde{p_1}$ and $p_2^* < \widetilde{p_2}$.*

The presence of the parallel importer forces the manufacturer to reduce her price gap, whether she allows or blocks parallel imports. When the manufacturer allows the importer to transfer the product, she increases her price in market 1 because the importer generates extra demand in that market. The manufacturer also reduces her price in market 2 because of the competition from the parallel importer in that market. If the manufacturer's policy is to block the importer, she has to choose prices so that $\delta p_2 - p_1 - c_G = 0$. Doing so by increasing p_1 or reducing p_2 alone severely hurts the manufacturer's profit in the authorized channels. Therefore, she reduces the price gap by adjusting p_1 upward and p_2 downward.

Next, we analyze the effect of demand uncertainty on the manufacturer's prices when she faces parallel imports. For this purpose, we define the DSG as the deterministic version of the SSG in which ϵ_1 and ϵ_2 are replaced with their expected values, μ_1 and μ_2, as follows

$$\text{(DSG)} \quad \max_{p_1,p_2} \ \pi^d = \left(p_1^d - c\right)\left(\mathcal{D}_1\left(p_1^d, \mu_1\right) + q_G(p_1,p_2)\right) + \left(p_2^d - c\right)\left(\mathcal{D}_2\left(p_2^d, \mu_2\right) - \delta q_G(p_1,p_2)\right).$$

Let p_1^{*d} and p_2^{*d} denote the solution to the DSG and define $q_1^{*d} = N_1 + \mu_1 - b_1 p_1^{*d}$ and $q_2^{*d} = N_2 + \mu_2 - b_2 p_2^{*d}$. The next proposition compares the prices of the SSG to those of the DSG.

Proposition 7 *Given the same average demand, the optimal prices in the stochastic demand case (SSG) are always less than the optimal prices when demand is assumed to be deterministic (DSG); i.e.,* $p_1^* < p_1^{*d}$ *and* $p_2^* < p_2^{*d}$.

Prior work has shown a similar result for a single market with additive uncertainty in the absence of parallel importation. For example, Petruzzi and Dada (1999) show that the optimal price of a price-setting newsvendor who serves a single market is always below the optimal price when demand is equal to the average of the stochastic demand. Proposition 7 extends this result to the setting in which two markets are connected by parallel importation. We note that this result holds regardless of the policies adopted by the manufacturer under the SSG and DSG scenarios.

Proposition 6 shows that the existence of a gray market forces the manufacturer to increase p_1 and reduce p_2. In order to make a similar comparison between the quantities released in each market, we note that parallel importation influences the manufacturer's quantities in two ways. First, it increases the demand in market 1 by $q_G(p_1, p_2)$ and decreases the demand in market 2 by $\delta q_G(p_1, p_2)$. Thus, the manufacturer would like to increase q_1 and decrease q_2 accordingly. Second, given the directions of price change in Proposition 6, the demand of the authorized channel in market 1 decreases while the demand of the authorized channel in market 2 increases. While we were not able to prove the magnitude of these effects analytically, our extensive numerical experiments indicated that the second effect proves to be stronger. Consequently, in the presence of a gray market, the manufacturer keeps a lower quantity in market 1 and a higher quantity in market 2. We can prove this result analytically when demand is deterministic in both markets.

Proposition 8 *In the DSG, the optimal quantity in market 1 (market 2) in the presence of parallel imports is less (greater) than the optimal quantity when there are no parallel imports, i.e.,* $q_1^{*d} < \widetilde{q}_1{}^d$ *and* $q_2^{*d} > \widetilde{q}_2{}^d$.

Ahmadi and Yang (2000) made the same observation when the manufacturer selects the block policy. We show that the same direction of changes is valid regardless of the policy.

5 Discussion and Managerial Insights

This section presents numerical experiments that respond to the motivating questions raised in the introduction. We first highlight the value of developing a model that jointly incorporates the competition from parallel importers and demand uncertainty. We then provide managerial insights that can help address some policy questions of interest to manufacturers, such as the transition of the optimal policy as model parameters vary, and the value of strategic decision-making in the presence of a gray market.

Table 2 lists the parameter values of the experiments. N_1 and N_2 values were chosen between $1,000$ and $10,000$ to capture markets with approximately same size ($N_1/N_2 = 1$) as well as markets with significantly different sizes ($N_2/N_1 = 1/10$ and $N_2/N_1 = 10$). b_1 and b_2 varied from 1 to 4 to represent markets with very similar sensitivity to price ($b_1/b_2 = 1$) and markets with very different sensitivity to price ($b_2/b_1 = 1/4$ and $b_2/b_1 = 4$). We chose equidistant values from each range to keep the total number of experiments manageable. We set the manufacturer's production cost to 250 and 550, a low and high production cost relative to market bases. The sample data in Li and Maskus (2006) suggests that transport costs to and import duties of the United States range from 3% to 10% of FOB value[2] for most products. In our model, we are only concerned with the parallel importer's transfer cost and not his shipping mode. Therefore, we set c_G to 3% and 10% of c. We assumed ϵ_1 and ϵ_2 to have the same distribution, but not necessarily the same parameters. We focused on normal and uniform distributions as they are widely used in the literature (e.g., Schweitzer and Cachon, 2000; Yao et al., 2006). Because the behaviors we observed for the uniform distribution were not significantly different from those for the normal distribution, we only present the results for the normal distribution. We set μ_i to $0.1N_i$ and changed σ_i such that the coefficient of variation of ϵ_i was 0.5, 1, and 2. When running the experiments, we considered all parameter combinations except those that resulted in market 1 price being greater than or equal to market 2 price, violating the direction of import. Therefore, we considered 4490 cases for our experiments.

5.1 Value of a Joint Model

We have developed a unified framework to analyze the decisions of a manufacturer that faces gray market activities and uncertain demand. As pointed out in the literature review, prior work in marketing and operations management largely dealt with price and quantity decisions in the presence of either a parallel importer or uncertain demand. Our model studies the simultaneous effects of both parallel importation and demand uncertainty.

[2] FOB is an acronym for Free on Board. It is a type of sea transport contract in which the seller pays for transportation of the goods to the port of shipment, plus loading costs. The buyer pays cost of marine freight transport, insurance, unloading, and transportation from the arrival port to the final destination. The passing of risks occurs when the goods are loaded on board at the port of shipment (http://www.iccwbo.org).

Table 2: Parameter values for numerical experiments

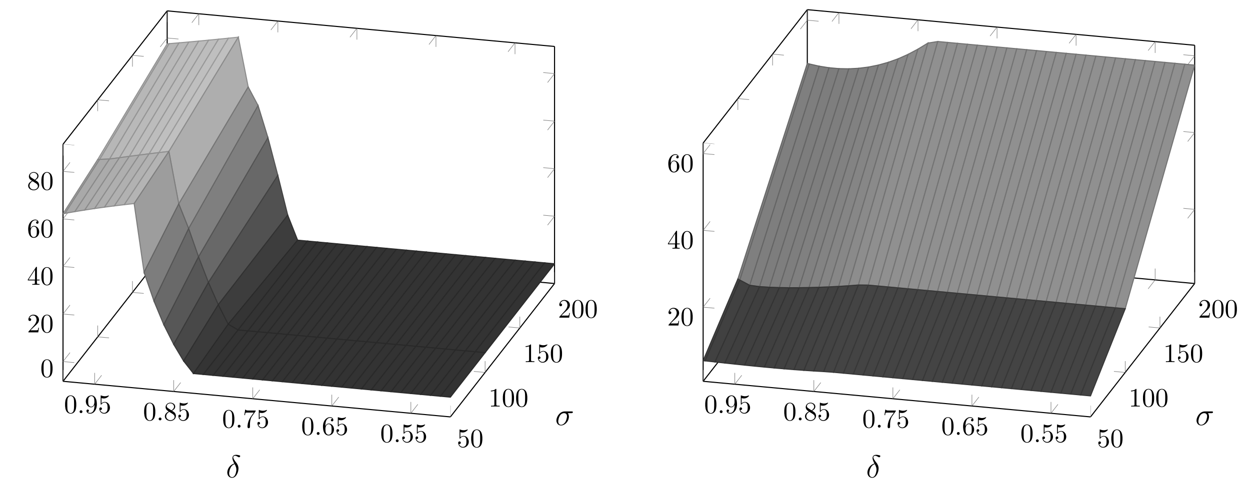
N_i $(i = 1, 2)$	$\{1,000,\ 4,000,\ 7,000,\ 10,000\}$
b_i $(i = 1, 2)$	$\{1,\ 2,\ 3,\ 4\}$
μ_i $(i = 1, 2)$	$0.1N_i$
σ_i $(i = 1, 2)$	$\{0.5\mu_i,\ \mu_i,\ 2\mu_i\}$
c	$\{250,\ 550\}$
c_G	$\{0.03\,c,\ 0.1\,c\}$

5.1.1 Cost of Ignoring Parallel Importation

In the first set of experiments, we looked at how much profit the manufacturer would forfeit if she does not take into consideration the possibility of parallel importation and treats each market separately. In our experiments, we observed that the magnitude of profit losses can be as high as 70%, depending on the consumers' perception of parallel imports. Figure 2(a) is an example of our experiments and shows the percentage of the manufacturer's profit loss when she continues to use $\widetilde{p_1}$ and $\widetilde{p_2}$ for different values of δ and $\sigma_1 = \sigma_2 = \sigma$, when $N_1 = N_2 = 1,000, b_1 = 3, b_2 = 2, c = 250,$ and $c_G = 25$. When δ is very low, parallel importation is not a threat and there is no profit loss. As δ increases, however, the profit loss increases. Even for relatively moderate values of δ, the manufacturer can lose between 10% to 20% of her profit if she does not take the parallel importer into account. For larger values of δ, profit losses reach 60%.

5.1.2 Cost of Ignoring Demand Uncertainty

We also evaluated the profit loss to the manufacturer when she is aware of the presence of the parallel importer, but does not take demand uncertainty into consideration. For this purpose, we first solved the DSG and obtained its optimal solution, $(p_1^{*d}, p_2^{*d}, q_1^{*d}, q_2^{*d})$. We then evaluated the profit of the SSG for the optimal deterministic decisions. In our test set, we observed that the reduction in the manufacturer's profit can be as high as 57% depending on the magnitude of demand uncertainty. When consumers' relative perception of parallel imports is very high, the percentage of profit loss decreases in δ because the price gap becomes narrower both in the deterministic model and in the stochastic model. One exception to this behavior for very high values of δ, however, is when the market parameters dictate that the manufacturer exit market 1 in the stochastic model,



(a) Error percentage of using $\widetilde{p_1}$ and $\widetilde{p_2}$ as a heuristic solution for the SSG.

(b) Error percentage of using the solution to the DSG as a heuristic solution for the SSG.

Figure 2: Error percentage of ignoring parallel imports or demand uncertainty.

while the optimal policy in the deterministic model is to stay in both markets. In this situation, the percentage of profit loss increases in δ. Figure 2(b) illustrates one example of our experiments when $N_1 = N_2 = 1,000, b_1 = 3, b_2 = 2, c = 250$, and $c_G = 25$ in which ignoring demand uncertainty is detrimental to the manufacturer and she could lose as much as 57% of her profit. Therefore, it is crucial that manufacturers account for both uncertainty in demand and parallel importation in making price and quantity decisions.

5.2 How Do Market Conditions and Product Characteristics Determine Policy?

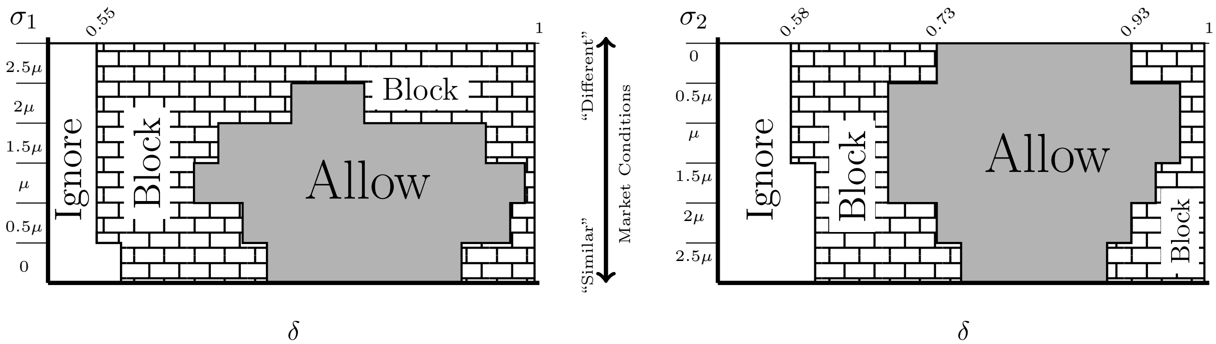
Though determining the optimal price and quantity decisions of the manufacturer has been the main objective of our model, an important high-level question for any manufacturer is: What is the best policy to cope with the gray market? In this section, we examine the manufacturer's optimal policy through the lens of two important dimensions of our model: product-specific parameters and market-specific parameters, of which an important one is demand uncertainty in each market.

More specifically, we define *market conditions* as the aggregate effect of market-based parameters, such as standard deviation of demand uncertainties, market bases, and price sensitivities. We say market conditions are *"similar"* (*"different"*) if the parameters of the markets are such that the price gap would naturally be small (large) even if there were no parallel imports; i.e., small (large) $\widetilde{p_2} - \widetilde{p_1}$.

The parameter that represents product characteristics in our model is δ. A low value of δ means that parallel imports and authorized-channel products are quite distinct in the eyes of consumers, whereas a very high value of δ means that consumers' valuations of the gray market and the authorized channel are very close. The higher the δ, the more intense the competition between the manufacturer and the parallel importer. We define *"commodity"* items as products for which consumers have a relatively high perception of parallel imports and are almost indifferent between the authorized channel and the gray market (i.e., $\delta \approx 1$). At the other end of the spectrum, we define *"fashion"* items as products for which consumers have a relatively low perception of parallel imports ($\delta \ll 1$) and strongly prefer to buy the product from the authorized channel. We emphasize that the term *"fashion"* does not necessarily refer to the brand of the product in our context. Rather, we use this term (and *"commodity"*) to concisely convey the degree of consumers' inclination towards the authorized channel. The factors that can influence the perception of parallel imports include the maturity of the product, consumers' knowledge of and familiarity with the product, and the risks associated with buying from the gray market.

Figure 3 depicts the simultaneous effects of product characteristics (δ) and market conditions on the regions characterizing the optimal policy when $N_1 = N_2 = 4,000, b_1 = 2, b_2 = 1, c = 550$, and $c_G = 55$. To further highlight the effect of demand uncertainty, Figure 3a illustrates the optimal policies when demand in market 2 is known ($\sigma_2 = 0$), whereas Figure 3b demonstrates the same when demand in market 1 is deterministic ($\sigma_1 = 0$). In both graphs, the horizontal axis represents the product characteristic, ranging from fashion to commodity from left to right; the vertical axis represents the standard deviation of demand in the market with uncertain demand (as a measure of market conditions ranging from "similar" to "different"). Note that in Figure 3a, the bottom row corresponds to $\sigma_1 = 0$; given that $\sigma_2 = 0$ as well, this row represents prior work in the literature that assumes deterministic demand in both markets. If demand in market 1 becomes variable ($\sigma_1 > 0$), a special case of Proposition 7 indicates that $\widetilde{p}_1 < \widetilde{p}_1^d$. Given that $\sigma_2 = 0$ (and $\widetilde{p}_2 = \widetilde{p}_2^d$), inclusion of demand variability in market 1 means more "different" market conditions between markets 1 and 2 (i.e., $\widetilde{p}_2 - \widetilde{p}_1 > \widetilde{p}_2^d - \widetilde{p}_1^d$). Note that the values of σ_2 in Figure 3b are sorted in the reverse order of σ_1 values in Figure 3a. This is because if demand in market 2 becomes variable, then $\widetilde{p}_2 < \widetilde{p}_2^d$. Given that $\sigma_1 = 0$, inclusion of demand variability in market 2 means more "similar" market conditions between markets 1 and 2 ($\widetilde{p}_2 - \widetilde{p}_1 < \widetilde{p}_2^d - \widetilde{p}_1^d$). Note that the top row in Figure 3b is same as the bottom row in Figure 3a.

First, we observe the same pattern in both graphs. When the product is a fashion item and



(a) $\sigma_2 = 0$ (b) $\sigma_1 = 0$

Figure 3: Effects of demand uncertainty and product characteristics on manufacturer's policy

market conditions are somewhat similar, the manufacturer can ignore the parallel importer. As parallel imports gain acceptance from consumers and/or market conditions somewhat differ, the manufacturer's policy is to block the parallel importer by slightly deviating from her otherwise optimal decisions. When there is a higher perception of parallel imports or when market conditions are moderately different, it is no longer beneficial for the manufacturer to block the gray market, as it requires large deviations from her otherwise optimal decisions in each market. In this region, parallel imports are allowed into market 2. Finally, when the product is a commodity or when market conditions are significantly different, the manufacturer goes back to blocking the parallel importer. We note that the gray market can be blocked with price or quantity. In our experiments, we observed that blocking with quantity may arise only when the market conditions are vastly different and the product is a commodity.

Second, Figure 3 readily shows the impact of demand uncertainty on the manufacturer's optimal policy towards parallel importation. As demand uncertainty increases in either market, the manufacturer may change her policy from block to allow and/or from allow to block. This observation shows that ignoring demand uncertainty not only may have a significant impact on the manufacturer's profit, as illustrated in Section 5.1.2, but it may also affect her optimal policy towards the gray market; the manufacturer may block (or allow) parallel importation when it is not optimal to do so. Finally, we note that the effect of demand uncertainty in the two markets are not the same. Increasing demand variability in market 1 makes the two markets more different, while increasing demand variability in market 2 renders the two markets more similar.

While Figure 3 plots the optimal policy regions for when demand uncertainty is chosen as a proxy for market conditions, we observe a similar behavior when market bases or price sensitivities are used to represent market conditions.

5.3 Strategic Versus Uniform Pricing

Implementing the optimal price and quantity decisions prescribed by the SSG requires estimating the value of parameters, such as the relative perception of parallel imports, δ, and the parallel importer's transfer cost, c_G, among others specific to the gray market. Antia et al. (2004) report that some manufacturers have adopted a uniform pricing policy and charge the same price for their products across all markets to eliminate gray markets entirely. The uniform pricing policy requires less information and facilitates price coordination. Nevertheless, this policy bears the risk of fluctuations in exchange rates and the manufacturer parts with the added profit from using the market-specific prices that the SSG provides.

We conducted experiments to provide insights into the impact of market conditions and product characteristics on the extra profit the manufacturer would earn by using the SSG recommendations. The optimal uniform price, p^u, can be obtained by solving (1), while enforcing $p_1 = p_2$. Figure 4 represents a common behavior we observed in our experiments where $N_1 = N_2 = 10,000$, $b_2 = 1$, $c = 250$, $c_G = 25$, and $\sigma_1 = \sigma_2 = 2,000$. It illustrates the ratio of the manufacturer's profit under the SSG to her profit under the uniform pricing policy as a function of δ for various market conditions captured with relative price sensitivities, b_1/b_2. We observed the same behavior when market conditions were represented by other market parameters.

We observe two important behaviors in Figure 4. First, despite the benefits of the uniform pricing policy such as easier implementation, there is a large range of δ and market conditions in which the manufacturer's profit will be significantly higher, as high as 28%, if she uses strategic prices rather than using a uniform price. Therefore, in many situations it is indeed crucial that companies put effort into market research to obtain the necessary information for strategic pricing.

The second observation is that the benefit of strategic pricing is greatest when market conditions are not too similar or too different, namely they are *moderately* different, and the product has not turned to a commodity. The ratio of profits decreases with δ and when $b_1/b_2 = 4$ and δ is high, the ratio equals one. Also, when δ is sufficiently below one, the additional profit from strategic pricing is higher for $b_1/b_2 = 3$ (moderately different market conditions) than for $b_1/b_2 = 2$ (similar market conditions) and $b_1/b_2 = 4$ (different market conditions). The reason for this pattern is that when market conditions are very similar, the price gap in the absence of parallel imports, $\widetilde{p_2} - \widetilde{p_1}$, is

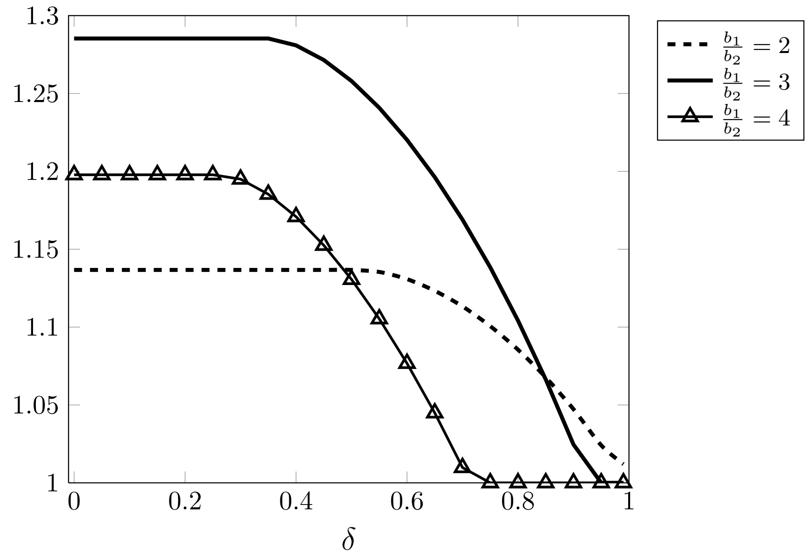


Figure 4: Ratio of optimal profit in the SSG to the uniform pricing profit

naturally small and the manufacturer blocks the parallel importer with prices for most values of δ. Since $\widetilde{p_1} \leq p^u \leq \widetilde{p_2}$ and $\widetilde{p_2} - \widetilde{p_1}$ is small, the manufacturer will not lose too much if she charges p^u in both markets. When market conditions are very different, the manufacturer would not be able to charge a single price that attracts significant portions of both markets simultaneously. Thus, $p^u = \widetilde{p_2}$ and the manufacturer leaves market 1 for the sake of the higher profit in market 2. For strategic pricing, although the manufacturer has the opportunity to boost her profit by charging markets differently, if she stays in both markets, she will have to charge very different prices. The parallel importer will exploit the high price gap and transfer a large amount of the product, reducing the manufacturer's profit in market 2. As a result, the small profit from market 1 no longer pays off the loss of profit to the parallel importer in market 2, especially when δ is very high and the product is a commodity. Therefore, the manufacturer eventually decides to only operate in market 2, which means that the strategic and uniform pricing policies become identical.

In summary, we find that strategic pricing leads to a significant increase in profits for non-commodity products when the two markets are moderately different. In extreme cases, however, when markets are either too similar or too different and the product is a commodity, the uniform pricing policy can be considered a viable alternative to strategic pricing.

6 Conclusion

In this paper, we analyzed the impact of parallel importation on a manufacturer's price and quantity decisions in an uncertain environment, and showed that reducing the price gap is more effective in controlling the gray market than reducing product availability. We found that the manufacturer's policy depends heavily on market conditions and product characteristics. When the product is a fashion item, the manufacturer mainly ignores the parallel importer. The manufacturer blocks the parallel importer when the product is a commodity. In that case, she may be forced to leave (or not enter) the less profitable market and only serve the more profitable market. Finally, if the product is in transition from a fashion item to a commodity, the manufacturer allows the parallel importer to operate if the market conditions are moderately different.

We also showed that strategic pricing is indeed significantly more valuable than a uniform pricing policy when the product is not a commodity and market conditions are moderately different. Thus, in these situations it is worth investing in market studies to have a better understanding of market parameters and consumer's perception of gray goods, and to set prices strategically. However, if market conditions are too similar or too different and the product is a commodity, uniform pricing is a good alternative to strategic pricing.

Some of the assumptions made throughout this paper can be relaxed to extend the model in several directions; these extensions are summarized here and discussed further in the appendix. First, our framework enables us to include the uncertainty in exchange rate between the two markets, which can be an important factor in international gray market activities. Second, our model can be extended to allow for demand correlation between the two markets. Specifically, we show that when demands follow a bivariate normal distribution, and the manufacturer can use the demand signal in market 1 to set her quantity decision for market 2, she earns higher profit and is more tolerant of parallel importation when markets are highly correlated. Third, our model assumes that the parallel importer places and receives his order before other customers; this assumption directly affects Proposition 3 and the results thereafter. In reality, it is likely that the authorized channel and parallel importer's demand are satisfied concurrently. When demand from both channels are fulfilled proportionally, our numerical results indicate that the statement of Proposition 3 and the qualitative nature of our results still hold true. Fourth, we assumed that the parallel importer's demand is perceived to be deterministic. In general, our numerical results illustrate that variability in parallel importer's demand reduces the manufacturer's profit because the parallel importer acts conservatively and sets a lower price. Furthermore, we find the effect of relaxing the deterministic demand assumption on our model and the insights to be minimal.

Our work has limitations that can be addressed in future research. It would be interesting to analyze the impact of parallel importation on the manufacturer in a multi-period setting. We assume that the manufacturer has unlimited capacity. Limited capacity can impact the manufacturer's allocation of quantities to each market, which then changes her prices. Also, because the importer can act as an agent who transfers the product between markets, he can influence the manufacturer's capacity investment decisions, especially when capacity costs are different across the markets. We leave these extensions for future research.

Acknowledgments

The authors are grateful for suggestions made by the anonymous reviewers, associate editor, and department editor throughout the review process.

References

2020health. 2011. Parallel importing and exporting of pharmaceuticals severely limits the options in designing an effective UK drug pricing scheme. (November 15), http://2020health.wordpress.com

Ahmadi, R., B. R. Yang. 2000. Parallel imports: Challenges from unauthorized distribution channels. *Marketing Science.* **19**(3) 279–294.

Altug, M., G. van Ryzin. 2013. Supply chain efficiency and contracting in the presence of gray market. *Working paper.*

Antia, K. D., S. Dutta, M. Bergen. 2004. Competing with gray markets. *Sloan Management Review.* **46**(1) 63-69.

Autrey, R. L., F. Bova., D. Soberman. 2013. Organizational structure and gray markets. *Working paper.*

Bucklin, L. P. 1993. Modeling the international gray market for public policy decisions. *International Journal of Research in Marketing.* **10** 387-405.

Cachon, G. P., M. A. Lariviere. 1999a. Capacity choice and allocation: Strategic behavior and supply chain performance. *Management Science.* **45**(8) 1091–1108.

Cachon, G. P., M. A. Lariviere. 1999b. An equilibrium analysis of linear, proportional and uniform allocation of scarce capacity. *IIE Transactions.* **31** 835–849.

Chan, L. M. A., Z. J. M. Shen, D. Simchi-Levi, J. Swann. 2004. Coordination of pricing and inventory decisions: A survey and classification. *Handbook of Quantitative Supply Chain Analysis: Modeling in the E-Business Era.* D. Simchi-Levi, S. D. Wu, Z. J. M. Shen. Kluwer Academic Publishers. 335-392.

Chen, H. 2009. Gray marketing: Does it hurt the manufacturers? *Atlantic Economic Journal.* **37**(1) 23–35.

Dasu, S., R. Ahmadi, S. Carr. 2012. Gray markets: A product of demand uncertainty and excess inventory. *Production Operations Management.* **21**(6) 1102–1113.

Dutta, S., M. Bergen, G. John. 1994. The governance of exclusive territories when dealers can bootleg. *Marketing Science.* **13**(1) 83–99.

Dutta, S., J. B. Heide, M. Bergen. 1999. Vertical territorial restrictions and public policy: Theories and industry evidence. *Journal of Marketing.* **63** 121–134.

Ganslandt, G., K. E. Maskus. 2004. Parallel imports and the pricing of pharmaceutical products: Evidence from the European Union. *Journal of Health Economics.* **23**(5) 1035-1057.

Hines, W. W., D. C. Montgomery. 1990. *Probability and Statistics in Engineering and Management Science.* J. Wiley, New York.

Hu, M., M. Pavlin., M. Shi. 2013. When gray markets have silver linings: All-unit discounts, gray markets and channel management. *Manufacturing Service Oper. Management.* **15**(2) 250-262.

Huang, J., B. C.Y. Lee., S. H. Ho. 2004. Consumer attitude toward gray market goods. *International Marketing Review.* **21**(6) 598–614.

Kanavos, P., P. Holmes. 2005. *Pharmaceutical Parallel Trade in the U.K.* The Institute for the Study of Civil Society. London, U.K.

KPMG. 2008. Effective channel management is critical in combating the gray-market and increasing technology companies bottom line. `http://www.agmaglobal.org`.

Krishnan, H., J. Shao., T. McCormick. 2013. Impact of gray markets on a decentralized supply chain. *Working paper.*

Lariviere, M. A., E. L. Porteus. 2001. Selling to a newsvendor: An analysis of price-only contracts. *Manufacturing Service Oper. Management.* **3**(4) 293-305.

Li, C., K. E. Maskus. 2006. The impact of parallel imports on investments in cost-reducing research and development. *Journal of International Economics.* **68** 443-455.

Maskus, K. E. 2000. Parallel imports. *World Economy.* **23**(9) 1269-1284.

Matsushima, N., T. Matsumura. 2010. Profit-enhancing parallel imports. *Open Economics Review.* **21**(3) 433–447.

Petruzzi N. C., M. Dada. 1999. Pricing and the Newsvendor Problem: A review with extensions. *Operations Research.* **47**(2) 183–194.

Schonfeld, M. 2010. Fighting grey goods with trademark law. *Managing Intellectual Property.* `http://www.pancommunications.com/_docs/burnslevinson20100601.pdf`

Schweitzer, M. E., G. P. Cachon. 2000. Decision bias in the newsvendor problem with a known demand distribution: Experimental evidence. *Management Sci.* **46**(3) 404–420.

Shulman, J. D. 2013. Product diversion to a direct competitor. *Working paper.*

Su X., S. K. Mukhopadhyay. 2012. Controlling power retailer's gray activities through contract design. *Production Operations Management.* **21**(1) 145–160.

Thompson, S. 2009. Gray power: An empirical investigation of the impact of parallel imports on market prices. *J. Ind. Compet. Trade* **9**(3) 219–232.

Xiao, Y., U. Palekar., Y. Liu. 2011. Shades of gray – The impact of gray markets on authorzied distribution channels. *Quantitative Marketing and Economics.* **9** 155–178.

Xu, M., Y. Chen., X. Xu. 2010. The effect of demand uncertainty in a price-setting newsvendor model. *European Journal of Operational Research.* **207**(2) 946–957.

Yao, L., Y. Chen, H. Yan. 2006. The newsvendor problem with pricing: Extensions. *International Journal of Management Science and Engineering Management.* **1**(1) 3–16.

Appendix. Model Extensions

Exchange Rate Uncertainty and Market–Specific Production Costs

We have assumed that the manufacturer produces the product at cost c and releases it in the markets. A manufacturer may produce her product at local facilities. Also, the exchange rate between the markets may be uncertain. We are able to incorporate both market–specific production costs and exchange rate uncertainty into our model. Suppose c_1 and c_2 denote the production cost in markets 1 and 2, respectively, and the exchange rate in market 2 is a random variable r with expected value μ_r, which is resolved after both the manufacturer and the parallel importer set their decisions. Then the parallel importer's problem changes to

$$\max_{p_G} \quad \pi_G = E_r\left[(rp_G - p_1 - c_G)\, q_G\right] = (\mu_r\, p_G - p_1 - c_G)\, q_G. \tag{7}$$

Propositions 2–3 continue to hold and $q_G(p_1,p_2) = max(0, \psi(p_1,p_2))$, but $\psi(p_1,p_2) = \frac{\mu_r \delta p_2 - p_1 - c_G}{2\delta(1-\delta)\mu_r} b_2$ and the entry condition for the parallel importer changes to $\mu_r\delta p_2 - p_1 - c_G > 0$. The manufacturer's profit–maximization problem in the SSG changes to

$$\max_{p_1,p_2,q_1,q_2} \pi = E_{\epsilon_1,\epsilon_2}\Big[p_1 min\left\{q_1, \mathcal{D}_1\left(p_1,\epsilon_1\right) + q_G(p_1,p_2)\right\} - c_1 q_1 \\ + \mu_r p_2 min\left\{q_2, \mathcal{D}_2\left(p_2,\epsilon_2\right) - \delta q_G(p_1,p_2)\right\} - \mu_r c_2 q_2\Big]. \quad (8)$$

We can characterize the optimal decisions and policy of the manufacturer by making appropriate adjustments for c_1, c_2, and μ_r in Proposition 4. We omit the details due to limited space.

One can consider an alternative sequence of events where the manufacturer defers her pricing decisions until after the exchange rate uncertainty is resolved. Under this scenario, the model evolves to a three-stage game: The manufacturer makes her quantity decisions in stage 1; Having observed exchange rate r, the manufacturer sets her pricing decisions in stage 2; The gray market imports the product from market 1 to market 2 in stage 3. We can modify our framework to consider this sequence of events as follows. In stage 3, the gray market solves a problem similar to (7) where μ_r is replaced by r. In stage 2, the manufacturer solves a problem similar to (8) where μ_r is replaced by r and optimization is taken over p_1 and p_2 only. Finally, in stage 1 the manufacturer maximizes her expected total profit similar to (8) where p_1 and p_2 are replaced by their solutions from stage 2, μ_r is replaced by r, the expectation is taken over ϵ_1 and ϵ_2 as well as r, and optimization is taken over q_1 and q_2. We leave further analysis of gray market activities under exchange rate uncertainty with alternative sequence of events for future research.

Correlated Demand

Suppose that demand uncertainties, ϵ_1 and ϵ_2, are correlated and let $\zeta(\epsilon_1,\epsilon_2)$ be the joint probability density function and ρ represent the correlation of ϵ_1 and ϵ_2. Then our analysis continues to work if $f_i(x)$ and $F_i(x)$ represent the *marginal* probability density and distribution functions of ϵ_i, i.e., $f_1(x) = \int_{\epsilon_2=L_2}^{U_2} \zeta(x,\epsilon_2) d\,\epsilon_2$ and $f_2(x) = \int_{\epsilon_1=L_1}^{U_1} \zeta(\epsilon_1,x) d\,\epsilon_1$. This is true because the total expected profit of the manufacturer is the sum of the expected profit in each market. In particular, when $\zeta(\epsilon_1,\epsilon_2)$ is a bivariate normal distribution with means μ_1,μ_2, standard deviations σ_1,σ_2, and correlation ρ, we have $\epsilon_1 \sim N\left(\mu_1,\sigma_1^2\right)$ and $\epsilon_2 \sim N\left(\mu_2,\sigma_2^2\right)$ (Hines and Montgomery, 1990, p. 217); therefore, demand correlation does not alter the optimal decisions of the manufacturer.

To make demand correlation effective in the case of bivariate normal distribution, we consider an alternative sequence of events which is similar to the current sequence except that the manufacturer determines q_2 after ϵ_1 is resolved. Under this sequence, the manufacturer observes ϵ_1 and uses the correlation between ϵ_1 and ϵ_2 to decide what quantity of the product she wants to release in market

2. We still assume that the manufacturer determines both p_1 and p_2 before uncertainties are resolved because the parallel importer can make his decisions only when both prices are announced. For this alternative sequence of events, the manufacturer solves

$$\max_{p_1,p_2,q_1,q_2} \pi = E_{\epsilon_1}\left[p_1 \min\left\{q_1, \mathcal{D}_1\left(p_1, \epsilon_1\right) + q_G(p_1,p_2)\right\} - cq_1 + \pi_2(\epsilon_1)\right],$$

The term $\pi_2(\epsilon_1) = E_{\epsilon_2}\left[p_2 \min\left\{q_2, N_2 - b_2p_2 - \delta q_G(p_1,p_2) + \epsilon_2|\epsilon_1\right\} - cq_2\right]$ is the manufacturer's expected profit in market 2 given the realized uncertainty in market 1.

Using the properties of the bivariate normal distribution, $\epsilon_2|_{\epsilon_1=\varepsilon_1}$ has a normal distribution with mean $\mu_2 + \rho\frac{\sigma_2}{\sigma_1}(\varepsilon_1 - \mu_1)$ and standard deviation $\sigma_2\sqrt{1-\rho^2}$ (Hines and Montgomery, 1990, p. 218). Let $\phi(x)$ and $\Phi(x)$ denote the density and cumulative distribution functions of the standard normal distribution. Propositions 2–3 are independent of correlation and remain intact. Therefore, given prices (p_1, p_2) and ε_1, the optimal quantities will be $q_1^* = d_1\left(p_1\right) + q_G\left(p_1,p_2\right) + \mu_1 + z_1\left(p_1\right)\sigma_1$ and $q_2^* = d_2\left(p_2\right) - \delta q_G\left(p_1,p_2\right) + \mu_2 + \rho\frac{\sigma_2}{\sigma_1}(\varepsilon_1 - \mu_1) + z_2\left(p_2\right)\sigma_2\sqrt{1-\rho^2}$. Substituting the optimal quantities and using the property of the normal distribution, the manufacturer's total expected profit is $\pi = (p_1-c)(d_1(p_1)+q_G\left(p_1,p_2\right)+\mu_1) - p_1\phi\left(z_1\left(p_1\right)\right)\sigma_1 + (p_2-c_2)\left(d_2(p_2) - \delta q_G\left(p_1,p_2\right) + \mu_2\right) - p_2\phi\left(z_2\left(p_2\right)\right)\sigma_2\sqrt{1-\rho^2}$.

Therefore, we can find the optimal prices by using Proposition 4 with $F_1(x)$ being a normal distribution function with parameters μ_1 and σ_1 and $F_2(x)$ being a normal distribution function with parameters μ_2 and $\sigma_2\sqrt{1-\rho^2}$. We repeated our experiments to explore the effect of correlation. We found the price in market 2, and therefore the price gap, is higher for larger values of $|\rho|$.

Note that this observation is in line with Proposition 7, which states that prices get lower with uncertainty. When the correlation between two markets is high, the demand in market 2 is more predictable and closer to the deterministic demand case; therefore, the price of market 2 (and the price gap as a result) is higher when demand is highly correlated across the two markets. As a result the quantity and profit of the parallel importer are nondecreasing in $|\rho|$. Nevertheless, the manufacturer's profit is increasing in $|\rho|$ since the manufacturer sets q_2 after observing ϵ_1 and the higher the absolute value of the correlation, the lower the demand variability in market 2.

Demand Fulfillment

We now investigate the effect of relaxing our assumption that the parallel importer obtains the product before customers in the authorized channel. Suppose $\delta p_2 - p_1 - c_G > 0$ and the parallel importer wishes to transfer the product. If the manufacturer's quantity in market 1 is greater than the sum of the realized authorized channel demand and the parallel importer's *ideal* order quantity, i.e., $q_1 \geq d_1(p_1) + \varepsilon_1 + \psi(p_1, p_2)$, where ε_1 is a realization of ϵ_1, then the parallel importer can transfer $\psi(p_1, p_2)$ to market 2. However, if $q_1 < d_1(p_1) + \varepsilon_1 + \psi(p_1, p_2)$ and supply is less than total demand, then we assume that the quantities obtained by the authorized channel and the parallel importer are proportional to the size of their orders. Specifically, the authorized channel will obtain $\left(\frac{d_1(p_1)+\varepsilon_1}{d_1(p_1)+\varepsilon_1+\psi(p_1,p_2)}\right) q_1$ units and the parallel importer will obtain $\left(\frac{\psi(p_1,p_2)}{d_1(p_1)+\varepsilon_1+\psi(p_1,p_2)}\right) q_1$ units. A similar proportional allocation policy has been used in the supplier capacity assignment literature such as Cachon and Lariviere (1999a and 1999b).

Given ε_1, the parallel importer's optimal price and manufacturer's best response q_1 can be found. However, finding p_1^*, p_2^*, and q_2^* in closed form is not possible because the change in manufacturer's demand in market 2 depends on optimal p_G which in turn depends on the *realization* of ϵ_1. So in order to analyze the effect of this assumption we resort to numerical experiments. We find p_1^*, p_2^*, q_2^* using Monte Carlo simulation–optimization, sampling 5,000 values of ϵ_1 and ϵ_2 in each iteration. More specifically, we use Matlab optimization toolbox to find the values of (p_1, p_2, q_2) that maximize the average of the total profit for 5,000 values of demand uncertainty.

Table 3 illustrates a typical observation in our experiments. Mixed demand fulfillment refers to the proportional splitting scenario. The value of p_1^* (p_2^*) when the parallel importer obtains the product first provides an upper (lower) bound for p_1^* (p_2^*) in the mixed fulfillment setting. This is because competition from the gray market becomes weaker when the parallel importer only obtains a proportion of short supply. Therefore, the manufacturer can afford to increase the price gap across the two markets by lowering p_1 and increasing p_2. The case of $\delta = 0.7$ shows an interesting situation in which the parallel importer is blocked when he obtains the product first; however, the manufacturer allows some gray market activity when demand fulfillment is mixed. Furthermore, q_1^* is higher when demand fulfillment is mixed and, more importantly, it is still higher than the parallel importer's *ideal* order quantity, which is $\psi(p_1^*, p_2^*)$. Therefore, our numerical experiments indicate that the statement in Proposition 3 holds true even when the parallel importer may not be able to obtain the product before customers in the authorized channel.

Finally, we note that the manufacturer's profit under the two cases are fairly close. Overall, we observed a profit loss of less than 2% in most of our experiments. Therefore, while significantly simplifying model analysis, the assumption that the parallel importer obtains the product first does not appear to significantly affect the outcomes.

Table 3: Effect of mixed demand fulfillment

	Importer's demand is fulfilled first					Mixed demand fulfillment				
δ	p_1^*	p_2^*	q_1^*	$q_G(p_1^*, p_2^*)$	Mfg. profit	p_1^*	p_2^*	q_1^*	$q_G(p_1^*, p_2^*)$	Mfg. Profit
0.6	518	904	571	0	530015	518	904	571	0	530015
0.7	563	840	495	0	519771	561	845	512	13	520832
0.8	591	785	483	37	498601	588	794	524	71	501444
0.9	616	725	464	61	470549	615	735	523	118	473202

Uncertainty in Parallel Importer's Demand

So far, we have assumed that the parallel importer's demand is deterministic (or more precisely he perceives his demand to be deterministic). Suppose that the parallel importer's demand is defined as $D_G(p_2, p_G, \epsilon_G) = d_G(p_2, p_G) + \epsilon_G$, where $d_G(p_2, p_G) = \frac{\delta p_2 - p_G}{\delta(1-\delta)} b_2$ and ϵ_G is a random variable with distribution function $F_G(x)$ defined on (L_G, U_G), expected value μ_G, and standard deviation σ_G. Then, the parallel importer solves the following problem

$$\max_{p_G, q_G} \;\; \pi_G = E_{\epsilon_G}\left[p_G min\left\{q_G, D_G(p_2, p_G, \epsilon_G)\right\} - (p_1 + c_G)q_G\right],$$

and his optimal price and quantity decisions satisfy the following optimality equations

$$\frac{(\delta p_2 - 2p_G) + p_1 + c_G}{\delta(1-\delta)} b_2 + z_G(p_G) - \int_{L_G}^{z_G(p_G)} F_G(x)dx = 0,$$

$$q_G = \frac{\delta p_2 - p_G}{\delta(1-\delta)} b_2 + z_G(p_G), \qquad z_G(p_G) = F_G^{-1}\left(1 - \frac{p_1 + c_G}{p_G}\right).$$

Since the optimal decisions of the parallel importer do not have a closed form, we are not able to characterize the solution to the SSG analytically. We repeated our numerical experiments assuming that the parallel importer also faces uncertain demand. Our experiments indicate that although uncertainty in parallel importer's demand changes the values of equilibrium decisions, it does not impact the qualitative nature of our observations and managerial insights. Interestingly, we observe that uncertainty in the parallel importer's demand reduces the manufacturer's profit. This is because the parallel importer acts conservatively and sets a lower price when his demand is uncertain, which in turn creates a stronger competition for the manufacturer in market 2. Nevertheless, the profit loss for the manufacturer was on average 3% and at most 5% in our experiments.