Production Externalities, Congruity of Aggregate Signals, and Optimal Task Assignments*

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1. Introduction

Production externalities pose interesting issues regarding task assignments in multi-task, multi-agent settings. The rationale for past practice of assigning agents similar tasks was the view that there are gains to repetition. However, there has been a trend in recent times toward assigning agents to diverse tasks in order to exploit complementarities. Illustrations of this trend include combining sales and service with the same person rather than assigning these duties to different individuals, organizing credit review and loan pricing by case rather than by stage in the credit granting process, bundling production and testing operations rather than separating those tasks, and having engineers contributing to the design of several different product components rather than having them specialize in particular components.[1] Although the prevailing view seems to favor diverse task assignments when it comes to production externalities, we remain agnostic on whether externalities for either task assignment choice are beneficial or dysfunctional.

Notwithstanding the benefits from externalities that are said to attach to diverse task assignments, such assignments also imply the prospect of higher expected costs in providing incentives when signals for contracting purposes take the common form of noisy aggregate signals of agent efforts. The aggregation of agent efforts across diverse tasks restricts the ability of the principal to fine-tune incentives to take account of differences in the marginal contributions of each task to the payoff from production and levels of noise in measuring agent efforts. Accordingly, in choosing between similar and diverse task assignments, the principal faces a potential tradeoff between production externalities and distortions of effort allocations due to aggregation, noise, and restrictions on the design of incentives.

Our purpose in this paper is to consider the principal's task assignment problem in the presence of production externalities of the nature described above. Holmstrom and Milgrom (1991) have characterized the higher expected cost of providing incentives under diverse task assignments, but without consideration of the impact of production externalities on those assignments. Milgrom and Roberts (1992) offer the conjecture that complementarities are necessary to justify diverse task assignments in light of the higher expected costs of incentives without formally characterizing the tradeoff. Close to our analysis, Zhang (2003) considers a task assignment problem in which a production externality is imbedded in the probability structure of payoffs from production conditional on agent efforts.[2] In her analysis, the payoffs

are contractible rather than signals of agent efforts *per se*. The tradeoff under this structure is between the gains to complementarities and a so-called heterogeneity loss from variations in the precisions of the noise components of the payoffs.

Our main proposition separates the principal's task assignment decision rule neatly into two components; one captures the relative effect of externalities on the principal's expected payoffs net of compensation, and the other captures the effect of distortions in effort allocations. Complementarities enter the rule only through the first component while the effects of repetition influence both components. From this representation of the principal's choice, we show that the greater the disparity in either the marginal productivities or the noisiness of signals of agent efforts across different types of tasks, then the greater opportunity costs of being unable to fine tune incentives under diverse task assignments with consequent distortion in effort allocations. The effects of a change in repetition are to exacerbate or ameliorate this distortion depending on marginal productivities, and noise levels.

Section 2 describes our model, Section 3 contains the results from our analysis, and Section 4 concludes the paper.

2. Model

A risk neutral principal (the firm) engages two identical risk-averse agents (workers) to perform four tasks indexed by $(i, j); i, j \in \{1, 2\}$. Tasks $(1,1)$ and $(1,2)$ are equivalent and $(2,1)$ and $(2,2)$ are equivalent in the sense of having the same marginal effects on production, $b_1 > 0$ and $b_2 > 0$, respectively.[3] The (hidden) effort that is supplied by an agent in performing task (i, j) is denoted a_{ij}. The payoff from the production that results from these tasks depends on whether each agent is assigned similar tasks or diverse tasks.

Under similar task assignments, the payoff from the production is

$$x_S = b_1(a_{11} + a_{12}) + b_2(a_{21} + a_{22}) + \gamma(a_{11}a_{12} + a_{21}a_{22}), \quad \gamma \leq 1 \quad . \tag{1}$$

Without loss of generality, we assume that $b_2 > b_1 > 1$. The third term on the right-hand side of (1) reflects gains (losses) due to repetition depending on whether high effort in one task increases (decreases) the marginal productivity of effort the agent applies to the other similar task. The upper bound on the marginal

effects of repetition per unit of effort applied to the other similar task, $\gamma \leq 1$, rules out pathological cases in which an agent's optimal effort may become negative. Whether the effect of repetition is positive or negative depends on the nature of the production process and its behavioral implications. For example, there may be economies to specialization. However, contemporary wisdom appears to be that negative behavioral effects of repetition dominate efficiency gains in modern production environments.[4]

Under diverse task assignments the payoff from production is

$$x_D = b_1(a_{11} + a_{12}) + b_2(a_{21} + a_{22}) + \alpha(a_{11}a_{21} + a_{12}a_{22}), \quad \alpha \leq 1. \tag{2}$$

The third term on the right-hand side of (2) reflects either gains to complementarities or losses when agents are assigned to diverse tasks. As above, the upper bound on the marginal effects of complementarities per unit of effort applied to the other diverse task, $\alpha \leq 1$, rules out pathological cases. The common rationale given for diverse task assignments in the recent literature that familiarity with one task facilitates marginal productivity of the same agent in performing the other suggests complementarities are a positive externality (e.g., economies of scope).

We assume that the payoff from production is not contractible. Explanations may be that payoffs are affected by too many other factors or surface too late to be of use in contracting with agents. These conditions appear to be commonplace in many work settings where employees are compensated before the full outcomes of their efforts become observable. Incentives are, we assume, based on imperfect aggregate signals of each agent's total efforts:

$$y_{S1} = a_{11} + a_{12} + \varepsilon_1, \quad y_{S2} = a_{21} + a_{22} + \varepsilon_2, \tag{3}$$

where $\varepsilon_1, \varepsilon_2$ are normal independent mean zero noise terms with variances σ_1^2, σ_2^2, respectively, if each agent is assigned to similar tasks, and

$$y_{D1} = a_{11} + a_{21} + \varepsilon_{12}, \quad y_{D2} = a_{12} + a_{22} + \varepsilon_{21}, \tag{4}$$

where $\varepsilon_{12}, \varepsilon_{21}$ are normal independent mean zero noise terms with variance $\sigma^2 = (\sigma_1^2 + \sigma_2^2)/2$ if each agent is assigned to diverse tasks. While this characterization has a nice interpretation of signals as unbiased estimates of total agent efforts, the restriction to equal (unit) weights on effort components is imposed as a technical convenience in reducing the number of parameters in the analysis that follows.[5]

Next, we assume that agents are identical with private monetary cost of effort

$$C(a_{ij}) = \frac{1}{2} a_{ij}^2 . \tag{5}$$

and utility for compensation, W, net of private monetary cost $a^2 / 2$,

$$U(W, a) = -e^{-\rho(W - a^2/2)} . \tag{6}$$

As in Feltham and Xie (1994), we restrict incentive compensation contracts to be linear in signals, implying contract payments for agent k of the form

$$w_k(y_k) = \beta_k + \nu_k y_{Sk}, \quad w_k(y_k) = \beta_k + \nu_k y_{Dk}, \quad k = 1, 2 , \tag{7}$$

when agents are assigned similar tasks and diverse tasks, respectively.[6]

We assume that the principal seeks to maximize the expected payoff from production less the expected wages paid to the agents. Because the agents require a premium to bear risk of $(1/2)\rho \nu_k^2 \sigma_k^2, k = 1, 2$, the expected wages must cover that premium as well as their private costs of effort and meet their reservation wage that (without loss of generality) we assume to be 0. Accordingly, the principal's problem under similar task assignments (i.e., $k = i$) is

$$\underset{\{a_{ij}\},\{\nu_k\}}{Max} \; V_S = x_S - \frac{1}{2} \sum_{i=1,2} \left(\sum_{j=1,2} a_{ij}^2 + \rho \nu_i^2 \sigma_i^2 \right) \tag{8}$$

subject to agents choosing actions that maximize certainty equivalents:[7]

$$\begin{aligned} a_{11}, a_{12} &\in \arg\max \; \nu_1 E(y_{S1}) - \frac{1}{2} \left(\sum_{j=1,2} a_{1j}^2 + \rho \nu_1^2 \sigma_1^2 \right) \\ a_{21}, a_{22} &\in \arg\max \; \nu_2 E(y_{S2}) - \frac{1}{2} \left(\sum_{j=1,2} a_{2j}^2 + \rho \nu_2^2 \sigma_2^2 \right) \end{aligned} . \tag{9}$$

Analogously, the principal's problem under diverse task assignments is

$$\underset{\{a_{ij}\},\{\nu_k\}}{Max} \; V_D = x_D - \frac{1}{2} \sum_{j=1,2} \left(\sum_{i=1,2} a_{ij}^2 + \rho \nu_j^2 \sigma^2 \right) \tag{10}$$

subject to:

$$a_{11}, a_{21} \in \arg\max \ \ v_1 E(y_{D1}) - \frac{1}{2}(\sum_{i=1,2} a_{i1}^2 + \rho v_1^2 \sigma^2)$$
$$a_{12}, a_{22} \in \arg\max \ \ v_2 E(y_{D2}) - \frac{1}{2}(\sum_{i=1,2} a_{i2}^2 + \rho v_2^2 \sigma^2) \tag{11}$$

3. Analysis

Having defined the principal's problem for similar and diverse task assignments, we characterize the solution for each before turning to the choice between them.

Similar Task Assignments

From the constraints given by (9), we can solve for the optimal actions as functions of an agent's marginal wage payment (recall that $k = i$):

$$a_{kj} = v_k, \quad k, \ j = 1, 2 . \tag{12}$$

Substituting from (12) into (8), differentiating with respect to the marginal wage payments, and solving for those payments,[8] we obtain

$$v_k = \frac{b_k}{M_k - \gamma}, \ \ M_k = 1 + \rho \frac{\sigma_k^{\ 2}}{2}, k = 1, 2 . \tag{13}$$

Observe that under similar task assignments, the principal can fine tune incentives to provide stronger incentives for tasks with higher marginal productivity of effort and weaker incentives for tasks involving signals with greater noise in order to reduce the risk premium required.

The principal's expected net payoff under similar task assignments is obtained by substituting for effort levels in (8) from (12) and (13):

$$V_S = \frac{b_1^{\ 2}}{M_1 - \gamma} + \frac{b_2^{\ 2}}{M_2 - \gamma} . \tag{14}$$

Note that the effect of repetition is to scale up expected payoffs if $\gamma > 0$ and scale down those payoffs if $\gamma < 0$, while complementarities are, of course, absent in the case.

Diverse Task Assignments

From the constraints given by (11), we can solve for the optimal actions as functions of an agent's marginal wage payment (recall that now $k = j$):

$$a_{ik} = v_k\,,\ k, i = 1, 2\,. \tag{15}$$

Substituting from (15) into (10), differentiating with respect to the marginal wage payments, and solving for those payments, we obtain

$$v_k = \frac{b_1 + b_2}{M_1 + M_2 - 2\alpha}\,,\ \ M_k = 1 + \rho\frac{\sigma_k^{\ 2}}{2}\,,\ k = 1, 2. \tag{16}$$

Observe that now the marginal wage is constructively restricted to be the same across different types of tasks, thereby eliminating the principal's ability to fine tune incentives. Consequently, a larger disparity in marginal productivities or levels of noise across tasks implies a greater distortion of incentives for each type of task.

The principal's expected net payoff under diverse task assignments is obtained by substituting for effort levels in (10) from (15) and (16):

$$V_D = \frac{(b_1 + b_2)^2}{M_1 + M_2 - 2\alpha}\,. \tag{17}$$

Similar to the effects of repetition under similar task assignments, the effects of complementarities on expected net payoffs under diverse task assignments are to scale expected payoffs up or down depending on the sign of α . However, the implications of complementarities and repetition vary with respect to effort allocations across diverse tasks.

Optimal Task Assignments

Our first proposition characterizes the principal's choice between similar and diverse task assignments:[9]

PROPOSITION 1. *The principal strictly prefers diverse task assignments if, and only if, the following condition holds:*

$$\left(b_1\left(\frac{M_2 - \gamma}{M_1 - \gamma}\right)^{\frac{1}{2}} - b_2\left(\frac{M_1 - \gamma}{M_2 - \gamma}\right)^{\frac{1}{2}}\right)^2 + 2\left(\gamma - \alpha\right)V_S < 0\,. \tag{18}$$

The squared term in the above condition is a measure of the relative effort allocations across diverse tasks when agents are assigned to similar tasks as compared to diverse tasks.[10] As we elaborate below, the larger this term, the greater the benefit from an ability to fine tune incentives under similar task assignments or, equivalently, the more costly the distortion of effort allocations that ensue from an inability to fine tune incentives under diverse task assignments. The remaining term on the left-hand side of (18) is a measure of the relative effects of repetition associated with similar task assignments, γ, and complementarities, α, associated with diverse task assignments. The fact that the condition only depends on complementarities through its presence in this term implies a convenient separation of the relative benefits of complementarities from the implicit costs of distortions in effort allocations under diverse task assignments. The effects of repetition on (18) are more complex. Because the benefits or costs of repetition modify the total marginal productivity of effort applied to diverse tasks under similar task assignments, they also affect marginal wages and, hence, effort allocations in that case.

Dealing first with the impact of complementarities on the principal's choice of task assignments, we have the following obvious corollary to Proposition 1:

COROLLARY 1. *For a fixed level of repetition effects, γ, there exists a critical level of complementarities, α^*, such that the principal strictly prefers diverse task assignments to similar task assignments if, and only if, $1 \geq \alpha > \alpha^*$ where*

$$\alpha^* = \gamma + \left(b_1 \left(\frac{M_2 - \gamma}{M_1 - \gamma} \right)^{\frac{1}{2}} - b_2 \left(\frac{M_1 - \gamma}{M_2 - \gamma} \right)^{\frac{1}{2}} \right)^2 / (2V_S) \tag{19}$$

Thus, for any given set of parameters other than α such that the right-hand side of (19) is strictly less than 1, there is a non-empty region in which diverse task assignments are optimal. It is readily apparent from (18) that as we allow the benefits of complementarities to become large in comparison to the benefits (if any) of repetition, then given the implicit costs of distortions in effort allocations are unaffected by complementarities we would expect diverse task assignments to dominate. Alternatively, if the benefits of repetition are sufficiently high, there may be no scope for diverse task assignments to dominate.

Alternatively, we can characterize a critical level of repetition effects such that similar task assignments are preferred to diverse task assignments:

COROLLARY 2. *For a fixed level of complementarities,* α *, there exists a critical level of marginal repetition effects,* γ^* *, such that the principal strictly prefers similar task assignments to diverse task assignments if, and only if,* $1 \geq \gamma > \gamma^*$, where

$$\gamma^* = \alpha + \frac{b_1 b_2}{V_D} - \frac{\sqrt{[(b_1^2 - b_2^2) - V_D(M_1 - M_2)]^2 + 4b_1^2 b_2^2}}{2V_D} < \alpha \tag{20}$$

The fact that the third term on the right hand side of (20) exceeds the second implies the same ordering of externalities; i.e., benefits from complementarities must exceed those from repetition for diverse task assignments to be preferred.

Incentives and Effort Allocations

We now consider the effects of task assignments and externalities on the efficient design of incentives and relative allocation of efforts. The following table depicts all combinations of effort allocation ratios for similar and diverse task assignments assuming that either agent efforts are publicly observable (the *first-best* case) or only noisy signals of each agent's total efforts are contractible (the *second-best* case):

(Insert Table 1 Here)

Table 1 confirms the obvious intuition that effort allocation ratios will always be equal across similar tasks no matter whether efforts *per* se are publicly observable or only imperfect aggregate signals are contractible, or how agents are assigned.

Feltham and Xie (1994) provide a useful characterization of performance measures for examining distortions in effort allocations when an agent is assigned to diverse tasks. In particular, they define a measure of *non-congruity* for an aggregate signal relative to the principal's expected payoff before compensation as $[(b_1 + \alpha b_2) - (b_2 + \alpha b_1)]^2$.[11] They define *perfect congruity* as the case in which the above measure is zero, equivalently:

$$\frac{b_1 + \alpha b_2}{b_2 + \alpha b_1} = 1. \tag{21}$$

From Table 1 it is evident that when each agent is assigned diverse tasks and aggregate signals of an agent's efforts are employed in setting incentives, then second-best effort allocation ratios will correspond to first-best effort allocation ratios if and only if those signals are perfectly congruent; i.e., equation (21) holds. Note further that as α increases from 0 to 1 the distortion in effort allocations from perfect congruity is reduced, implying less inefficiency from restricting incentives (marginal wages) to be the same for diverse tasks when performed by the same agent. In other words, complementarities between efforts applied to diverse tasks tend to *reduce* differences in their (total) marginal effects on expected payoffs from production, thereby making it less important whether incentives can be fine-tuned.

Also, from Table 1, we can see that given similar task assignments and aggregate signals of each agent's total effort, second-best effort allocations will correspond to first-best allocations when the following condition is met:

$$\frac{b_1}{b_2} = \frac{\dfrac{b_1}{M_1 - \gamma}}{\dfrac{b_2}{M_2 - \gamma}} = \frac{b_1}{b_2}\left(\frac{1 + \dfrac{\rho\sigma_2}{2} - \gamma}{1 + \dfrac{\rho\sigma_1}{2} - \gamma}\right). \tag{22}$$

Under similar task assignments, signals represent total efforts by each agent and those efforts are applied to similar tasks. As a result, incentives can be fine-tuned to fit the type of task by setting a different marginal wage for each agent. Equation (22) can be interpreted as perfect congruity with respect to diverse tasks in the case of similar task assignments. Notwithstanding that first-best effort allocations are feasible under similar task assignments, it is evident that second-best effort allocations will correspond to first-best effort allocations if and only if $\sigma_1 = \sigma_2$. As the disparity in noisiness of signals across diverse tasks under similar task assignments becomes larger, so too does the distortion of effort levels from those in the first-best case. Note further that the effects of repetition may either exacerbate or diminish distortions in effort allocations depending on the sign of γ and the orderings of marginal productivity parameters, b_1, b_2, and noise levels, σ_1^2, σ_2^2. We say more about the effects of repetition later.

In order to focus more closely on the role of congruity, we put aside externalities by assuming for the moment that $\alpha = \gamma = 0$. We can isolate the effects of signal congruity *per se* by further assuming equivalent noise across the two types of tasks, $\sigma_1 = \sigma_2$. It is apparent from equation (22) that similar task assignments induce the first-best effort allocations, and from expression (18) that such assignments dominate diverse task assignments by an amount proportional to Feltham and Xie's (1994) non-congruity measure; i.e., $(b_1 - b_2)^2 / 2M, M = M_1 = M_2$. In this case the advantage of similar task assignments lies entirely with benefits of bundling like tasks so as to avoid under (over) production of the more (less) valuable tasks in terms of their contribution to expected payoffs. Alternatively, we can isolate the effects of differential noise by assuming that $b_1 = b_2$, but allow $\sigma_1 \neq \sigma_2$. While aggregate signals of each agent's total efforts are now congruent under diverse task assignments, the expected payoffs net of compensation are still greater under similar task assignments because of more efficient risk sharing. For example, in the case where $\sigma_2 > \sigma_1 = 0$, it is optimal to have one agent bear all the risk which is only feasible under similar task assignments. More generally, we observe from the squared term in (22) that, as the distortion in effort allocations across diverse tasks under diverse task assignments from those allocations under similar task assignments becomes greater, the critical level of complementarities required for the principal to prefer diverse task assignments becomes higher as well.

Although comparisons of second-best effort allocation ratios with those in the first-best case are insightful, a different perspective can be obtained by comparing second-best effort allocation ratios across diverse tasks under similar and diverse task assignments. Effort allocation ratios will be the same under diverse and similar task assignments if and only if

$$\frac{\dfrac{b_1}{M_1 - \gamma}}{\dfrac{b_2}{M_2 - \gamma}} = \frac{\dfrac{b_1}{1 + \dfrac{\rho\sigma_1}{2} - \gamma}}{\dfrac{b_2}{1 + \dfrac{\rho\sigma_2}{2} - \gamma}} = 1 . \tag{23}$$

Note that the above condition implies that the squared term in (18) equals zero. In other words, given that effort allocations are unaffected by task assignments, the only thing that matters is whether more benefits

are derived from complementarities under diverse task assignments than from repetition under similar task assignments. It is evident from (23) that the disparity between effort allocations under diverse and similar task assignments in the second-best case depends the effects of repetition. The following proposition provides a comparative static descriptive of these effects:

PROPOSITION 2: *If the aggregate signal of total efforts for the agent assigned to the task with the higher marginal productivity parameter under similar task assignments is less noisy than the aggregate signal of total efforts for the other agent,* $\sigma_2^2 < \sigma_1^2$*, then a marginal increase in the repetition parameter,* γ*, results in effort allocations under diverse task assignments that deviate further from those under similar task assignments*[12]*;i.e., for j=1,2*

$$\frac{\partial}{\partial\gamma}\left(\left[a_{1j} - a_{2j}\right]^2\right) = 2\left(\frac{b_1}{(M_1-\gamma)} - \frac{b_2}{(M_2-\gamma)}\right)\left(\frac{b_1}{(M_1-\gamma)^2} - \frac{b_2}{(M_2-\gamma)^2}\right) > 0 \quad . \tag{24}$$

Intuitively, if the task with the greater marginal productivity is accompanied by a signal with a lower level of noise, then increasing the beneficial effect of repetition will cause the principal to strengthen the incentive placed on that task relative to the incentive placed on the other task. In turn, this change in incentives implies a higher opportunity cost of an inability to fine tune incentives under diverse task assignments.

4. Conclusion

In this paper, we characterize task assignment decisions in the presence of production externalities. The tension the principal faces in making such decisions is between the relative effects of complementarities and repetition and distortions in effort allocations when each agent is assigned diverse tasks rather than similar tasks.

Summarizing our results, the beneficial effects of complementarities necessary to justify diverse task assignments must exceed the opportunity cost of an inability to fine tune incentives. These costs become more severe as disparities in marginal productivities or noise levels increase across diverse tasks. When one type task dominates in the sense of both higher marginal productivity and less noise in signals,

then an increase in the beneficial effects of repetition (or reduction in the dysfunctional effects) leads to greater deviations in effort allocations under diverse task assignments from those under similar task assignments and, hence, higher opportunity costs.

Like many model ours is intended as a metaphor. At the core of our analysis is the efficient resolution of the multi-agent multi-task assignment problem in the presence of production externalities when only noisy aggregate signals of agent efforts are available for contracting purposes. While we streamline the problem, the essential conflict between exploiting complementarities by bundling diverse tasks and tailoring incentives to induce desired actions seems likely to be present in a wide range of settings. A fairly literal illustration of a setting where our model might apply is the assignment of workers either to one or more stages of an assembly line where wages are based on total hours worked as a proxy for effort and the effort applied to one task impacts on the marginal productivity of effort applied to another. Such an application can be likened to the pin factory as a metaphor for economies of specialization. However, the forces in play within our analysis would not appear to depend on restricting either the numbers of tasks to just two types or signals available for contracting to just one.[13] The key issue is whether the exploitation of positive production externalities more than offsets the opportunity costs of less efficient compensation commonly associated with assigning agents to diverse tasks.

Appendix

PROOF OF PROPOSITION 1AND COROLLARY 1.

$$V_D = \frac{(b_1+b_2)^2}{M_1+M_2-2\alpha},\ V_S = \frac{b_1^{\ 2}}{M_1-\gamma}+\frac{b_2^{\ 2}}{M_2-\gamma} \text{ where } M_k = 1+\rho\frac{\sigma_k^{\ 2}}{2}, k=1,2$$

$$V_D > V_S \Leftrightarrow \frac{(b_1+b_2)^2}{M_1+M_2-2\alpha} > \frac{b_1^{\ 2}}{M_1-\gamma}+\frac{b_2^{\ 2}}{M_2-\gamma}$$
$$\Leftrightarrow (b_1+b_2)^2 > b_1^{\ 2}\frac{M_1+M_2-2\alpha}{M_1-\gamma}+b_2^{\ 2}\frac{M_1+M_2-2\alpha}{M_2-\gamma}$$
$$\Leftrightarrow (b_1+b_2)^2 > b_1^{\ 2}\frac{M_1-\gamma+M_2-\gamma+2(\gamma-\alpha)}{M_1-\gamma}+b_2^{\ 2}\frac{M_1-\gamma+M_2-\gamma+2(\gamma-\alpha)}{M_2-\gamma}$$
$$\Leftrightarrow 2b_1b_2 > b_1^{\ 2}\frac{M_2-\gamma+2(\gamma-\alpha)}{M_1-\gamma}+b_2^{\ 2}\frac{M_1-\gamma+2(\gamma-\alpha)}{M_2-\gamma}$$
$$\Leftrightarrow b_1^{\ 2}\frac{M_2-\gamma}{M_1-\gamma}+b_2^{\ 2}\frac{M_1-\gamma}{M_2-\gamma}-2b_1b_2+2(\gamma-\alpha)(\frac{b_1^{\ 2}}{M_1-\gamma}+\frac{b_2^{\ 2}}{M_2-\gamma})<0$$
$$\Leftrightarrow [b_1\sqrt{\frac{M_2-\gamma}{M_1-\gamma}}-b_2\sqrt{\frac{M_1-\gamma}{M_2-\gamma}}]^2+2(\gamma-\alpha)V_S<0$$

The above inequality holds if and only if

$$\alpha > \alpha^* \text{ where } \alpha^* = \gamma+[b_1\sqrt{\frac{M_2-\gamma}{M_1-\gamma}}-b_2\sqrt{\frac{M_1-\gamma}{M_2-\gamma}}]^2/(2V_S)$$ □

PROOF OF COROLLARY 2.

$$V_D < V_S \Leftrightarrow V_D < \frac{b_1^{\ 2}}{M_1-\gamma}+\frac{b_2^{\ 2}}{M_2-\gamma}$$
$$\Leftrightarrow V_D(M_1-\gamma)(M_2-\gamma) < b_1^{\ 2}(M_2-\gamma)+b_2^{\ 2}(M_1-\gamma)$$
$$\Leftrightarrow V_D\gamma^2+[b_1^{\ 2}+b_2^{\ 2}-(M_1+M_2)V_D]\gamma+V_DM_1M_2-b_1^{\ 2}M_2-b_2^{\ 2}M_1<0$$

Observe that the left-hand-side of the above inequality is in quadratic form, the corresponding equality has two real roots γ^*, γ^{**} because

$$[b_1^{\ 2}+b_2^{\ 2}-(M_1+M_2)V_D]^2-4V_D(V_DM_1M_2-b_1^{\ 2}M_2-b_2^{\ 2}M_1)$$
$$=(b_1^{\ 2}+b_2^{\ 2})^2-2V_D(b_1^{\ 2}-b_2^{\ 2})(M_1-M_2)+V_D^{\ 2}(M_1-M_2)^2 \quad .$$
$$=[(b_1^{\ 2}-b_2^{\ 2})-V_D(M_1-M_2)]^2+4b_1^{\ 2}b_2^{\ 2}>0$$

With some algebraic manipulation, we obtain

$$\gamma^{**},\ \gamma^{*} = \alpha + \frac{b_1 b_2}{V_D} \pm \frac{\sqrt{[(b_1^2 - b_2^2) - V_D(M_1 - M_2)]^2 + 4b_1^2 b_2^2}}{2V_D}.$$

Hence, the inequality

$V_D\gamma^2 + [b_1^2 + b_2^2 - (M_1 + M_2)V_D]\gamma + V_D M_1 M_2 - b_1^2 M_2 - b_2^2 M_1 < 0$ holds if and only if $\gamma^* < \gamma < \gamma^{**}$. The above inequality holds when $\gamma = 1$, implying $\gamma^* < 1 < \gamma^{**}$. Note that we have set $\gamma \le 1$ and $\alpha \le 1$ to keep optimal efforts positive. Therefore, $V_D < V_S$ if and only if $\gamma > \gamma^*$, where

$$\gamma^* = \alpha + \frac{b_1 b_2}{V_D} - \frac{\sqrt{[(b_1^2 - b_2^2) - V_D(M_1 - M_2)]^2 + 4b_1^2 b_2^2}}{2V_D}$$

□

PROOF OF PROPOSITION 2.

Recall the measure of the relative effort allocations across diverse tasks when agents are assigned to similar tasks as compared to diverse tasks is

$$\left[a_{1j} - a_{2j}\right]^2 = \left(\frac{b_1}{M_1 - \gamma} - \frac{b_2}{M_2 - \gamma}\right)^2.$$

Hence, $\frac{\partial}{\partial\gamma}\left(\left[a_{1j} - a_{2j}\right]^2\right) = \frac{\partial}{\partial\gamma}\left(\left(\frac{b_1}{M_1 - \gamma} - \frac{b_2}{M_2 - \gamma}\right)^2\right)$

$$= 2\left(\frac{b_1}{(M_1 - \gamma)} - \frac{b_2}{(M_2 - \gamma)}\right)\left(\frac{b_1}{(M_1 - \gamma)^2} - \frac{b_2}{(M_2 - \gamma)^2}\right)$$

$\sigma_2^2 < \sigma_1^2$ implies $M_2 < M_1$. Since $M_2 > \gamma$ and $b_1 < b_2$, it follows that

□

$$2\left(\frac{b_1}{(M_1 - \gamma)} - \frac{b_2}{(M_2 - \gamma)}\right)\left(\frac{b_1}{(M_1 - \gamma)^2} - \frac{b_2}{(M_2 - \gamma)^2}\right) > 0.$$

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TABLE 1

Effort Allocations

First-best allocations			
		Agents assigned similar tasks	Agents assigned diverse tasks
Effort allocation between diverse tasks	$\frac{a_{1j}}{a_{2j}}$	$\frac{b_1}{b_2}$	$\frac{b_1+\alpha b_2}{b_2+\alpha b_1}$
Effort allocation between similar tasks	$\frac{a_{i1}}{a_{i2}}$	1	1
Second-best allocations			
Effort allocation between diverse tasks	$\frac{a_{1j}}{a_{2j}}$	$\frac{\frac{b_1}{M_1-\gamma}}{\frac{b_2}{M_2-\gamma}}=\frac{\frac{b_1}{1+\frac{\rho\sigma_1}{2}-\gamma}}{\frac{b_2}{1+\frac{\rho\sigma_2}{2}-\gamma}}$	1
Effort allocation between similar tasks	$\frac{a_{i1}}{a_{i2}}$	1	1

Endnotes

1. See Brickley, Smith, and Zimmerman (1996) for further descriptions of these illustrations.
2. Zhang's (2003) analysis is predicated on an example by Demski (1994) that depicts inefficiencies in compensation from diverse task assignments.
3. While exactly the same marginal effects for each pair of tasks is convenient, the essential aspect is that tasks within pairs be more similar in those effects than tasks across pairs.
4. See Hackman and Oldham (1980) for further discussion.
5. The assumption of equal marginal effects of efforts on signals is not crucial to our later analysis of the effects of incongruities of signals relative to expected payoffs given that marginal productivities of efforts across diverse tasks are assumed to be arbitrary constants.
6. Linear contracts can be viewed as an approximation of optimal contracts useful in characterizing the strength of incentives as weights applied to signals available for contracting.
7. The risk premiums in (9) below and later in (11) are independent of efforts and can be deleted.
8. The first-order approach to solving the principal's problem is appropriate except in knife-edge cases where the denominator to equation (13) is zero.
9. Proofs for all propositions and corollaries are provided in the appendix.
10. To see this, note that under similar task assignments

$$\left[a_{1j} - a_{2j}\right]^2 = \left(\frac{1}{M_1 - \gamma}\right)\left(\frac{1}{M_2 - \gamma}\right)\left(b_1\left(\frac{M_2 - \gamma}{M_1 - \gamma}\right)^{1/2} - b_2\left(\frac{M_1 - \gamma}{M_2 - \gamma}\right)^{1/2}\right)^2, J = 1, 2.$$

11. This expression differs from that in Feltham and Xie (1994) due to our assumption of signal coefficients of 1 and the effect of complementarities on expected payoffs.
12. Recall the measure of the disparity in efforts across different types of tasks under similar task assignments below from endnote 10 following the discussion of Proposition 1.
13. Additional signals may reduce the opportunity costs of an inability to fine tune incentives under diverse task assignments, but need not recover the properties of similar task assignments in that respect.