Information in equity markets with ambiguity averse investors

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November 7, 2006

[1]This paper is based on my dissertation at the University of Michigans Stephen M. Ross School of Business. I would like to thank my dissertation committee members Ilia Dichev, Raffi Indjejikian (chair), Russell Lundholm, M. P. Narayanan and Emre Ozdenoren. I also thank Brett Trueman, Jack Hughes and seminar participants at UCLA, University of Chicago, Dartmouth College, Emory University, University of Georgia and University of Michigan. I gratefully acknowledge financial support from the Deloitte Foundation, the Paton Fund, the Ross School of Business and the Anderson School of Management.

Abstract

Investors in equity markets tend to use highly aggregated financial information. For example, share prices react strongly to summary measures of performance such as reported earnings while financial analysts issue fairly coarse information in the form of buy-sell recommendations, price targets and earnings forecasts. The aggregation of signals increases equity premiums in an economy with subjective expected utility maximizers because it reduces information content in the sense of Blackwell. I show that information aggregation reduces the equity premium when the market includes ambiguity averse investors who face uncertainty about the joint distribution of payoffs and signals. The benefit to aggregation arises because a reduction in ambiguity compensates for the loss of information.

JEL Classification: D81, G11, G14

Conventional wisdom suggests that public information reduces equity premiums by increasing the precision of risk averse investors' beliefs about security payoffs. On the other hand, much of the publicly available information such as financial statements and buy-sell recommendations consists of aggregated information. Aggregate signals that are not sufficient statistics for their components entail an information loss relative to the component signals. In a market comprised of subjective expected utility maximizers (hereafter SEU) who obey the Savage (1954) axioms, this information loss increases equity premiums to the extent that it pertains to non-diversifiable uncertainty. If SEU investors have access to both aggregate information and its components, then aggregate signals are redundant. However, I show that aggregate information can reduce equity premiums when some investors are ambiguity averse (hereafter AA) in the sense that they dislike uncertainty about the joint distribution of signals and security payoffs.

In order for ambiguity aversion to affect equity premiums, there must be uncertainty about the probability distribution governing security payoffs (ambiguity) and some fraction of investors must be ambiguity averse. The first condition regarding ambiguity is typically met because, as Markowitz (1991) states, "none of us know probability distributions of security returns." SEU investors deal with ambiguity by forming a unique subjective prior probability distribution so that the issue is moot. In contrast, AA investors facing ambiguity behave as if they use different prior distributions to evaluate different courses of action. The classic example of this behavior is the Ellsberg (1961) paradox.

By showing that summary information reduces firms' cost of capital in markets that include AA investors, I offer an alternative to explanations for aggregation based on frictions or other forms of so-called investor irrationality. I conduct my analysis using a CARA/normal framework in which the conditional expectation of firm value provides a sufficient statistic for individual signals. Although SEU investors who face information

processing costs could view the conditional expectation in lieu of individual signals without a loss of information, I show that distorting an aggregate signal relative to the conditional expectation will result in a lower equity premium when the market includes AA investors.

Prior research on the role of information in equity markets has suggested that more information reduces capital costs. Easley and O'Hara (2004), Hughes, Liu, and Liu (2006) and Lambert, Leuz, and Verrecchia (2006) show that equity premiums in exchange economies are decreasing in investors' average information precision. This literature primarily assumes that all investors are SEU and therefore prefer as fine of an information set as possible by the Blackwell (1953) theorem. Even in situations where frictions create a demand for releasing aggregate rather than disaggregate information, it unclear that the optimal aggregation should be biased. For example, Dye and Sridhar (2004) show that releasing an aggregate signal can reduce a manager's incentives to manipulate earnings and distort production, but the aggregate signal is a conditional expectation that, in their normal random variable setting, entails no loss of information. Models in which investors are SEU therefore tend to predict either no aggregation or unbiased aggregation of signals. In contrast, investors make use of aggregate signals such as earnings and book values that are biased in that they exclude value relevant information through rules such as historical cost accounting and writing off investments in research.

Similar to Cao, Wang, and Zhang (2005) and Easley and O'Hara (2005), I analyze an exchange economy in which investors have heterogeneous ambiguity aversion. As in prior research on markets that include AA investors, the equity premium includes both risk and ambiguity components. I model heterogeneity in ambiguity aversion by assuming that some investors are SEU while others are AA. This modeling choice provides qualitatively similar results to assuming heterogeneity amongst ambiguity averse

investors and provides a benchmark of behavior when all investors are SEU. The inclusion of SEU investors dampens the benefits of providing summarized information and, in the case where all investors are SEU, eliminates the benefits. The inclusion of SEU investors also allows me to analyze the case in which investors individually choose whether to view aggregate or disaggregate information.[1]

I deviate from much of the prior research on ambiguity aversion by using Klibanoff, Marinacci, and Mukerji's (2005) characterization of smooth ambiguity aversion rather than Gilboa and Schmeidler's (1989) max-min expected utility ($MMEU$) characterization. The smooth characterization allows for a distinction between investors' beliefs and their attitude towards ambiguity whereas the $MMEU$ characterization does not. It also allows for Bayesian updating and provides a more tractable framework. Both characterizations represent ambiguity by multiple prior distributions.

I show that AA investors value summarized information because its interpretation varies less over their multiple prior distributions. Providing summary information therefore reduces the ambiguity premium. Because I assume joint normality of payoffs and signals, the conditional expectation of payoffs given the individual signals is sufficient for the signals themselves. I use the conditional expectation as a benchmark summary and show that distorted summary information can yield further reductions in the equity premium by trading increases in the risk premium for decreases in the ambiguity premium. When ambiguity primarily pertains to the interpretation of information, summary information that deemphasizes ambiguous information yields a lower equity premium than the benchmark conditional expectation. When ambiguity primarily pertains to prior beliefs of asset payoffs, overemphasizing signals that pertain to ambiguous payoffs yields a low equity premium. The optimality of distorted summary information contrasts with

[1] The Blackwell (1953) theorem implies that given SEU preferences, an individual will view as much information as possible without imposing any further assumptions. Safra and Sulganik (1995) show that the Blackwell theorem need not hold for other types of preferences. This creates the possibility that SEU investors view detailed information while AA may choose to view summary information.

prior research that has shown benefits to aggregate information. As I stated earlier, prior research has demonstrated benefits to unbiased summary information.

I further show the implications of summarizing information on the conglomerate discount and markets where investors can also view detailed information. The ability of conglomerates to issue summary information dampens the conglomerate discount found by Cao, Wang, and Zhang (2005) and in some cases may completely offset the discount. When investors can view the detailed signals that underlie summary information, SEU investors view the detailed information while AA investors will still view only summary information if it is not too distorted relative to the benchmark conditional expectation. This provides a possible explanation for why some investors might fail to incorporate publicly available information as indicated by empirical regularities such as the accrual anomaly (Sloan 1996).[2]

My characterization of how the packaging of information affects the equity premium and investor behavior contributes to the research on ambiguity aversion in capital markets and on information in capital markets. In a concurrent paper, Epstein and Schneider (2006) analyze how ambiguity averse investors process individual signals; however, their emphasis is on a model of learning when ambiguity aversion is characterized by Gilboa and Schmeidler's (1989) max-min expected utility ($MMEU$) and they do not analyze multiple signals. They show that ambiguity averse investors make worst-case interpretations of ambiguous information. For example, if they are uncertain of the noise in a signal of firm value, they interpret good news as being noisy and bad news as being precise. I take a different approach than (Epstein and Schneider 2006) by focusing on

[2]Sloan (1996) provides evidence that investors appear to properly value earnings, an aggregation of cash and accrual components, but do not properly value the individual cash and accrual components. Because cash and accruals have different valuation implications, a Bayesian aggregation scheme would not weigh them equally so that the equal weights on accruals and cash used when computing earnings can be interpreted as a biased aggregation scheme in the sense used in this paper. The accrual anomaly is therefore consistent with AA investors using a distorted summary signal, earnings, rather than the individual signals provided by the cash and accrual components of earnings.

how ambiguity aversion affects how the packaging of information impacts the equity premium.

This paper also relates to previous studies such as Dow and Werlang (1992) and Epstein and Wang (1994) that have shown that ambiguity aversion can affect the equity premium and may contribute to the high equity premium noted by Mehra and Prescott (1985). Anderson, Ghysels, and Juergens (2006) provide empirical evidence that ambiguity explains a significant portion of equity returns. Cao, Wang, and Zhang (2005) show that heterogeneous ambiguity aversion can lead to the limited market participation noted by Mankiw and Zeldes (1991). They also show that allowing the market to exclude highly ambiguity averse investors results in a lower equity premium than would obtain under full participation.[3] In addition, they show that heterogeneous ambiguity aversion can yield a conglomerate discount even without considering agency problems. Easley and O'Hara (2005) also show that heterogeneous ambiguity aversion can reduce participation in the stock market.

The paper proceeds as follows. Section 1 describes the model and derives equilibrium prices and holdings. Section 2 shows that releasing aggregate rather than individual signals reduces ambiguity and, in turn reduces the equity premium. Section 3 extends the model by showing how the market behaves when investors can individually choose whether to view aggregate or disaggregate information. Section 4 shows how aggregate signals affect the equity premium when ambiguity pertains to fundamental values rather than information. Section 5 concludes.

[3]More ambiguity averse investors demand a higher premium to own shares than less ambiguity averse. If the average demand curve sets prices, then prices reflect the higher premium. In equilibrium, investors with relatively low ambiguity aversion outbid investors with relatively high ambiguity aversion so that highly ambiguity averse investors may not hold any shares. This implies a lower equity premium than under full participation since outbidding implies higher prices and the equity premium is decreasing in price.

1 Model and Equilibrium

I analyze the impact of information on a one-period exchange economy that includes one firm and cash. The structure is similar to that in Easley and O'Hara (2005) in that the population of investors includes both AA and SEU individuals. I extend their setup by adding informative signals that investors observe prior to trading and analyze how the structure of information affects the equity premium on the firm's shares. After trade, the firm pays a terminal dividend. The investors then consume the dividend and their cash holdings. The remainder of this section describes the elements of the model and the resulting equilibrium.

I model ambiguity averse preferences using Klibanoff, Marinacci, and Mukerji's (2005) representation (KMM) of ambiguity aversion rather than Gilboa and Schmeidler's (1989) max-min expected utility (MMEU) model used by Easley and O'Hara. Decision makers' preferences over Savage acts in the KMM representation are given by:

$$\begin{aligned} f \succsim g \Leftrightarrow \int_{\mathscr{P}} h\left(\int_S u(f(s))\,\mathrm{d}P(s)\right)\mathrm{d}Q(P) &= \mathrm{E}\left[h\left(\mathrm{E}_P[u(f)]\right)\right] \\ &\geq \int_{\mathscr{P}} h\left(\int_S u(g(s))\,\mathrm{d}P(s)\right)\mathrm{d}Q(P) = \mathrm{E}\left[h\left(\mathrm{E}_P[u(g)]\right)\right] \quad (1) \end{aligned}$$

where $u(\cdot)$ is a utility function that represents preferences for pure risk, $h(\mathrm{E}_P[u])$ is a concave transform that represents preferences for ambiguity. A linear h gives SEU preferences that comply with the Savage (1954) axioms. The set $\mathscr{P}$ represents the set of probability distributions that the decision maker perceives to be relevant and Q is a subjective distribution over $\mathscr{P}$ representing the decision maker's beliefs.

The advantages of the KMM model include tractability due to smooth indifference curves and a separation of tastes and beliefs. This allows for Bayesian updating as in Klibanoff, Marinacci, and Mukerji (2006). Gollier (2005) shows that an increase in ambiguity aversion has a similar effect to a distortion in the prior Q that shifts mass to

distributions in $\mathscr{P}$ in a pessimistic fashion. In contrast, the MMEU characterization is equivalent to a prior Q that places all of its mass on the least favorable distribution in $\mathscr{P}$. The KMM model yields ambiguity aversion as in the Ellsberg (1961) paradox by using the transform h to ensure that preferences are not linear in probabilities whereas the MMEU model creates nonlinearity in probabilities by minimizing over $\mathscr{P}$.[4]

The firm's terminal per share dividend $v = v_1 + v_2$ where the two components represent different activities or accounts. All investors share the prior belief that each v_i is independent and normally distributed with mean μ_{vi} and variance σ^2_{vi}. Each component v_i of the terminal dividend generates a signal $s_i = v_i + e_i$. The noise terms e_i are independent of v_1 and v_2 and independent of each other. All investors share the prior belief that e_2 is normally distributed with mean zero and variance σ^2_{e2}. They also believe that e_1 is normally distributed and that it has an unknown mean b implying that it is ambiguous. Investors' subjective beliefs about b are given by a normal distribution with mean μ_b and variance σ^2_b. This implies that the noise term e_1 is equivalent to the sum of independent normal random variables ε_1 and b where ε_1 has a zero mean and variance $\sigma^2_{\varepsilon 1}$. SEU investors do not distinguish between types of uncertainty so that they form a unique prior in which e_1 is an independent variable with mean $\mu_{e1} = \mu_b$ and variance $\sigma^2_{e1} = \sigma^2_{\varepsilon 1} + \sigma^2_b$. The variance of s_i is then $\sigma^2_{si} = \sigma^2_{vi} + \sigma^2_{ei}$.

While I model e_1 as ambiguous and the remaining variables as purely risky, real economic situations rarely involve pure risk. This stark contrast provides a stark representation of varying degrees of ambiguity so that the results pertaining to the ambiguous signal can be interpreted as the relatively more ambiguous signal. Varying degrees of ambiguity might arise based on the nature of a given situation. For example, investors might have access to econometric models that provide an accurate distribution of life insurance claims or credit card receivables but may not have as much of a basis for

[4]Klibanoff, Marinacci, and Mukerji (2005) show that the MMEU model is equivalent to theirs with infinite ambiguity aversion.

determining a prior distribution for the payoffs from a new technology.

Prior to trade, the firm releases either the vector of underlying signals $S = \begin{bmatrix} s_1 & s_2 \end{bmatrix}'$ or an aggregate report s that is a linear aggregation of S. The information content of the aggregate signal s only depends on the relative coefficients on s_1 and s_2. I define the aggregate signal relative to $\mathrm{E}[v|S]$ without loss of generality. The independence of the v_i and e_i terms implies that $\mathrm{E}[v|S] = \mathrm{E}[v] + \frac{\sigma_{v1}^2}{\sigma_{s1}^2}(s_1 - \mu_b) + \frac{\sigma_{v2}^2}{\sigma_{s2}^2}s_2$. I index linear aggregation schemes by the parameter λ:

$$s = \lambda \frac{\sigma_{v1}^2}{\sigma_{s1}^2} s_1 + \frac{\sigma_{v2}^2}{\sigma_{s2}^2} s_2$$

The parameter λ determines the degree to which the aggregate signal s incorporates ambiguous information s_1 relative to the benchmark aggregation $\mathrm{E}[v|S]$. Setting $\lambda = 1$ yields the same relative coefficients as $\mathrm{E}[v|S]$ so that it yields the same conditional expectation. The conditional variances $\mathrm{var}(v|s; \lambda = 1)$ and $\mathrm{var}(v|S)$ are equal as well because $s = \mathrm{E}[v|S]$ is a sufficient statistic for S with respect to inferring v.[5] Setting $\lambda \in [0, 1)$ denotes an aggregate signal s that deemphasizes ambiguous information s_1.

The firm has a known per capita supply of $x > 0$ shares. The model's use of CARA utility abstracts from wealth effects so that prices and holdings do not depend on whether supply x represents a share issuance or an endowment. Investors are represented by a continuum on $[0, 1]$ so that individual investors do not impact prices and therefore behave competitively. Fraction $\alpha \in [0, 1]$ of investors are AA with homogeneous preferences and the rest are SEU with homogeneous preferences. The utility function $u(w) = -\exp\{-rw\}$ represents all investor types' preferences over certain wealth and purely risky bets where $r > 0$ specifies their constant absolute risk aversion. The AA investors' preferences over ambiguity are as in (1) with $h(\mathrm{E}_P[u]) = -(-\mathrm{E}_P[u])^a$ giving constant relative ambiguity aversion. The parameter $a > 1$ indexes ambiguity aversion and $a = 1$ yields SEU preferences.

[5] To see this, compute $\mathrm{var}(v|s; \lambda = 1) = \mathrm{var}(v) - \mathrm{var}(\mathrm{E}[v|s; \lambda = 1]) = \mathrm{var}(v) - \mathrm{var}(\mathrm{E}[v|S]) = \mathrm{var}(v|S)$.

The portrayal of investors as being either AA or SEU provides a clear and tractable representation of heterogeneity in investors' attitudes toward ambiguity. It also allows me to contrast behavior under SEU preferences and AA preferences. An alternative approach would be to maintain the assumption that fraction $1-\alpha$ of investors are SEU while the fraction α of AA investors have an ambiguity parameter distributed over $(1, \bar{a})$ with distribution $F(a)$. I do not take this approach because the results are qualitatively similar while the additional notation only complicates the analysis.

The SEU and AA investors demand per capita quantities q_S and q_A, respectively, of the firm's shares given information $J \in \{S, s\}$ and price p. The market clearing price p^* solves:

$$(1-\alpha)q_S(J, p^*) + \alpha q_A(J, p^*) = x \tag{2}$$

The investors' objective functions in terms of certainty equivalents are as follows where I've used the fact that CARA utility allows me to ignore the investors' initial endowments (See the Appendix derivation):

$$SEU: \ \max_q \ q(\mathrm{E}[v|J] - p) - \frac{r}{2}\, q^2 \,\mathrm{var}(v|J) \tag{3a}$$

$$AA: \ \max_q \ q(\mathrm{E}[v|J] - p) - \frac{r}{2}\, q^2 \,\mathrm{var}(v|J)[1 + (a-1)R(v, b|J)] \tag{3b}$$

The SEU objective function in (3a) is standard while the AA objective function includes an additional term $R(v, b|J) = \mathrm{corr}(v, b|J)^2 \in [0, 1]$. $R(v, b|J)$ represents the degree to which residual uncertainty about v depends on the ambiguous component b of the signal s_1. If information J eliminates all ambiguity, then $R(v, b|J) = 0$ while $R(v, b|J) = 1$ implies that all residual uncertainty about v pertains to ambiguity. SEU investors price uncertainty about b through its impact on $\mathrm{var}(v|J)$; however, they do not price ambiguity b any differently than the pure risk given by $\mathrm{var}(v|s, b)$. The $R(v, b|J)$ term appears in (3b) because the AA investors require a greater premium to bear ambiguity than they require to bear pure risk.

The objective functions in (3) are concave in q so that the investors' first-order conditions yield the optimal demands:

$$q_S(J,p) = \frac{\mathrm{E}[v|J] - p}{r\,\mathrm{var}(v|J)} \qquad q_A(J,p) = \frac{\mathrm{E}[v|J] - p}{r\,\mathrm{var}(v|J)[1 + (a-1)R(v,b|J)]} \tag{4}$$

Substituting (4) into the market clearing condition (2) then gives the equilibrium price p^*:

$$p^*(x,J) = \mathrm{E}[v|J] - rx\,\mathrm{var}(v|J)\left(1 + \frac{\alpha(a-1)R(v,b|J)}{1 + (1-\alpha)(a-1)R(v,b|J)}\right) \tag{5a}$$

$$q_S^*(x,J) = x\left(1 + \frac{\alpha(a-1)R(v,b|J)}{1 + (1-\alpha)(a-1)R(v,b|J)}\right) \tag{5b}$$

$$\begin{aligned} q_A^*(x,J) &= x\left(1 - \frac{(1-\alpha)(a-1)R(v,b|J)}{1 + (1-\alpha)(a-1)R(v,b|J)}\right) \\ &= \frac{1}{1 + (a-1)R(v,b|J)} q_S(x,J) \end{aligned} \tag{5c}$$

The AA investors take smaller positions than SEU investors as shown in (5c); however, they trade in the same direction because they have the same information set. The lower relative holdings of AA investors resembles lower relative holdings of a set of investors with greater risk aversion. Indeed, the same equilibrium price and holdings result from an assumption that fraction α of investors are SEU with risk aversion $\bar{r} = r[1 + (a-1)R(v,b|J)]$ while fraction $1-\alpha$ are SEU with risk aversion r.

Despite the similarity between ambiguity aversion and high risk aversion, the two differ in how the nature of information affects them. In the case where there are no AA investors but instead investors with high risk aversion, changes in information do not affect the relative holdings of the two investor types, which in the previous example would be the ratio $r/\bar{r}$ of their risk aversion coefficients. Any reduction or distortion of information increases the equity premium in this case, as well. Section 2 shows that neither of these results hold when some investors are AA. Equation (5c) implies that when some investors are AA, the relative holdings of the two investor types depends

on the information set J. Also, while reductions in information can increase the equity premium by increasing $\text{var}(v|J)$, reductions in information can also yield an offsetting reduction in the ambiguity term $\frac{\alpha(a-1)R(v,b|J)}{1+(1-\alpha)(a-1)R(v,b|J)}$ in price (5a).

2 Aggregate signals and the equity premium

In this section, I analyze the equilibrium derived in Section 1. I first show how aggregating information reduces ambiguity. I then compare the equity premium when investors observe disaggregate information S to the equity premium when they observe aggregate information s. If investors observe only one type of public information, then an aggregate signal that deemphasizes ambiguous information minimizes the equity premium. The deemphasizing of ambiguous information resembles the tendency of accounting reports to exclude difficult to value transactions or deemphasize them by disclosing them in footnotes rather than recognizing them in earnings.[6]

When the market includes AA investors, the equity premium reflects the degree to which residual uncertainty about firm value is ambiguous. The term $R(v,b|J) = \text{corr}(v,b|J)^2$ represents the extent of ambiguity. In order to gain the intuition behind the $R(v,b|J)$ term, we can rewrite the expected value $\text{E}[v|J]$ to explicitly capture its dependence on estimates $\text{E}[b|J]$ of the ambiguous component of s_1:

$$\begin{aligned}\text{E}[v|J] &= \text{E}\left[\text{E}[v|J,b]|J\right] \\ &= \text{E}[v] + \text{cov}(v,J|b)\,\text{var}(J|b)^{-1}(J-\text{E}[J]) + \frac{\text{cov}(v,b|J)}{\text{var}(b|J)}(\text{E}[b|J]-\text{E}[b]) \quad (6)\end{aligned}$$

One can think of the expectation over parameters in (1) as reflecting the AA investors'

[6] A clear example is the accounting for credit sales. The typical method is to record amounts owed by customers on the balance sheet less the expected amount of default; however, companies use the cash-based installment method of recognizing sales when there is no reasonable method for estimating defaults.

sensitivity the robustness of their estimates to different parameters. The AA investors second-guess their estimate $\mathrm{E}[b|J]$ in (6). In order to see how this impacts the ambiguity term $R(v, b|J)$, rewrite $R(v, b|J)$ as:

$$R(v, b|J) = \frac{\mathrm{cov}(v, b|J)^2}{\mathrm{var}(v|J)\,\mathrm{var}(b|J)} = \left(\frac{\mathrm{cov}(v, b|J)}{\mathrm{var}(b|J)}\right)^2 \frac{\mathrm{var}(b|J)}{\mathrm{var}(v|J)} \tag{7}$$

The first factor is the squared coefficient on $\mathrm{E}[b|J]$ in (6) and reflects the degree to which $\mathrm{E}[v|J]$ depends on having an accurate estimate of the ambiguous parameter b. The second factor depends on residual ambiguity $\mathrm{var}(b|J)$.

Summary information reduces ambiguity $R(v, b|J)$ in two ways. First, aggregation masks the effect of the ambiguous signal s_1 and therefore increases $\mathrm{var}(b|J)$. This increase reduces the sensitivity of the estimate $\mathrm{E}[v|J]$ to the ambiguous parameter as reflected by a reduction in the coefficient $\frac{\mathrm{cov}(v,b|J)}{\mathrm{var}(b|J)}$ in (6). The increase in residual ambiguity $\mathrm{var}(b|J)$, reflected by the second factor in (7), dampens the effect of the reduction in $\frac{\mathrm{cov}(v,b|J)}{\mathrm{var}(b|J)}$ but does not dominate since $\mathrm{var}(b|J)$ also appears in the denominator of the coefficient, which is then squared. The dominance of the impact on the coefficient makes intuitive sense because investors are only concerned about ambiguity to the extent that it impacts their estimates of firm value.

Summary information reduces ambiguity in a second way through its impact on $\mathrm{cov}(v, b|J)$ in the numerator of the coefficient on $\mathrm{E}[b|J]$ in (6). Aggregation schemes that deemphasize ambiguous information reduce the sensitivity of the summary signal s to ambiguity b. The assumption $\mathrm{cov}(v, b) = 0$ implies that $\mathrm{cov}(v, b|J) = -\mathrm{cov}(\mathrm{E}[v|J], b)$ since $\mathrm{cov}(v, b|J) = \mathrm{cov}(v, b) - \mathrm{cov}(\mathrm{E}[v|J], b)$. Intuitively, a low coefficient on the ambiguous signal s_1 suggests a low covariance $\mathrm{cov}(\mathrm{E}[v|s], b)$. While, in principle, the effect of a low coefficient on s_1 in the summary signal s might be offset by a high coefficient

$\frac{\text{cov}(v,s)}{\text{var}(s)}$ in $\text{E}[v|s]$, direct computation shows that this is not the case:

$$\text{cov}(v,b|S) = -\frac{\sigma_{v1}^2}{\sigma_{s1}^2}\sigma_b^2 \qquad \text{cov}(v,b|s;\lambda) = -\frac{\lambda\,\text{cov}(v,s)}{\text{var}(s)}\frac{\sigma_{v1}^2}{\sigma_{s1}^2}\sigma_b^2 = \frac{\lambda\,\text{cov}(v,s)}{\text{var}(s)}\text{cov}(v,b|S)$$

The proof of the following proposition shows that $\frac{\lambda\,\text{cov}(v,s)}{\text{var}(s)} \in [0,1)$ for all $\lambda \in [0,1)$. This and the above computation imply that summary signals that deemphasize s_1 (i.e. $\lambda \in [0,1)$) attenuate $\text{cov}(v,b|s)$ relative to the covariance $\text{cov}(v,b|S)$ with disaggregate information. The following proposition formally states this result:

Proposition 1 (Reduction in ambiguity)**.** *Given aggregation schemes of the form* $s = \lambda\frac{\sigma_{v1}^2}{\sigma_{s1}^2}s_1 + \frac{\sigma_{v2}^2}{\sigma_{s2}^2}s_2$*, any* $\lambda \in [0,1]$ *yields lower ambiguity* $R(v,b|s) < R(v,b|S)$ *than the disaggregate information set* S*. Furthermore,* $R(v,b|s)$ *is strictly increasing in* λ *for* $\lambda \in [0,1]$*.*

Proof. See Appendix. ■

Proposition 1 implies that summary signals that weakly deemphasize ambiguous information (i.e. $\lambda \in [0,1]$) increase the relative holdings of AA investors. In equilibrium, AA investors' relative holdings equal $\frac{q_A^*(x,J)}{q_S^*(x,J)} = \frac{1}{1+(a-1)R(v,b|J)}$ as given by (5c). Proposition 1 states that $R(v,b|s) < R(v,b|S)$ for all $\lambda \in [0,1]$ giving the following corollary:

Corollary 1.1. *Given aggregation schemes of the form* $s = \lambda\frac{\sigma_{v1}^2}{\sigma_{s1}^2}s_1 + \frac{\sigma_{v2}^2}{\sigma_{s2}^2}s_2$*, any* $\lambda \in [0,1]$ *yields greater relative holdings of* AA *investors than disaggregate information and the relative holdings of* AA *investors are strictly decreasing in* λ *for* $\lambda \in [0,1]$*.*

Corollary 1.1 demonstrates that information can be packaged in a way that mitigates the non-participation of AA investors predicted by Cao, Wang, and Zhang (2005) and Easley and O'Hara (2005). The market never totally excludes AA investors in my model due to their smooth indifference curves versus the kinked indifference curves in the $MMEU$ characterization of ambiguity aversion used by Cao, Wang, and Zhang and

Easley and O'Hara. The relative holdings in my model resemble the participation of AA investors in their models.

Recall from Section 1 that this result fundamentally differs from what would obtain given heterogeneous risk aversion. For example, if the market includes proportion $1-\alpha$ of SEU investors with risk aversion $\underline{r}$ and proportion α of SEU investors with risk aversion $\bar{r} > \underline{r}$, then the relative holdings of the high risk aversion investors equals $\underline{r}/\bar{r}$ regardless of the packaging of information. In this case with SEU investors with heterogeneous risk aversion, there would be no benefit to increasing the relative holdings of the highly risk averse investors. Doing so would require granting them private information, but publicly releasing that information would yield a lower equity premium than giving it to a subset of investors.

I will now show that when the market includes AA investors, summarizing information can reduce the equity premium while providing an accompanying increase the relative holdings of AA investors. The equity premium equals:

$$\mathrm{E}[v - p^*(x, J)] = rx\,\mathrm{var}(v|J)\left(1 + \frac{\alpha(a-1)R(v,b|J)}{1 + (1-\alpha)(a-1)R(v,b|J)}\right) \tag{8}$$

where (5a) gives the equilibrium price p^*. Information affects the equity premium through the posterior variance $\mathrm{var}(v|J)$ and ambiguity $R(v,b|J)$. As noted earlier, the unbiased summary with $\lambda = 1$ puts $\mathrm{var}(v|s) = \mathrm{var}(v|S)$ and $R(v,b|s) < R(v,b|S)$. This demonstrates that releasing a summary signal reduces the equity premium relative to a disaggregate signal.

A summary signal that deemphasizes ambiguous information (i.e. $\lambda < 1$) will further reduce the equity premium and increase the relative holdings of AA investors thereby improving risk sharing. While releasing a summary signal s that totally ignores the ambiguous signal will put $R(v,b|s) = 0$ and ensure full participation of AA investors, the associated cost in terms of increasing the posterior variance $\mathrm{var}(v|s)$ would result

in an increase in the equity premium. In general, as the summary signal s excludes more of the information in s_1, the posterior variance $\text{var}(v|s)$ increases. The following proposition formalizes this notion that the effect of summary information on the equity premium reflects a tradeoff between providing information and exposing AA investors to ambiguity.

Proposition 2 (Minimum equity premium)**.** *Given aggregation schemes of the form* $s = \lambda \frac{\sigma_{v1}^2}{\sigma_{s1}^2} s_1 + \frac{\sigma_{v2}^2}{\sigma_{s2}^2} s_2$*, the aggregation scheme that minimizes the equity premium deemphasizes but does not exclude ambiguous information. That is, it sets* λ *strictly between zero and one. The resulting equity premium is lower than when investors view disaggregate information.*

Proof. See Appendix. ■

The proof of Proposition 2 makes use of the fact that the equity premium is inversely proportional to the weighted average precision of the two investor types, where I use the term 'precision' loosely in the case of AA investors to refer to $1/\{\text{var}(v|s)[1 + (a - 1)R(v, b|s)]\}$. The change in precision with respect to the summary parameter λ is:

$$\frac{\partial}{\partial \lambda} \begin{array}{c}\text{Average}\\ \text{precision}\end{array} = \left(1 - \frac{\alpha(a-1)R(v,b|s)}{1+(a-1)R(v,b|s)}\right) \frac{\partial}{\partial \lambda}\left(\frac{1}{\text{var}(v|s)}\right) + \frac{\alpha}{\text{var}(v|s)} \frac{\partial}{\partial \lambda}\left(\frac{1}{1+(a-1)R(v,b|s)}\right)$$

The above implies that reductions in ambiguity tend to be valuable when the posterior precision $\text{var}(v|s)$ is high and the fixed proportion of AA investors is high.[7] Clearly the equity premium does not depend on ambiguity $R(v, b|s)$ when there are no AA investors ($\alpha = 0$) so that $\lambda = 1$ minimizes the equity premium by minimizing $\text{var}(v|s)$.

[7] Also note that $\frac{\partial}{\partial \lambda}\left(\frac{1}{\text{var}(v|s)}\right)$ is small when precision is high.

When the market includes only AA investors ($\alpha = 1$), the minimizing aggregation still includes some ambiguous information due to its impact on the posterior precision $\text{var}(v|s)$ because AA investors do not wish to totally disregard ambiguous information.

Figure 1 illustrates Proposition 2. The aggregate signal that minimizes the equity premium incorporates the ambiguous signal s_1 by setting $\lambda > 0$ because doing otherwise ignores value relevant information. Increasing the aggregation parameter λ from zero initially reduces the equity premium because the value of the information in the ambiguous signal s_1 outweighs the cost of exposing AA investors to ambiguity. Eventually as λ increases, the aggregate signal incorporates a sufficiently large portion of the value relevant information in s_1 that further increases in λ do not compensate for the additional exposure of AA investors to ambiguity.

The following proposition summarizes some of the attributes of the minimizing aggregation parameter λ^*:

Proposition 3 (Attributes of aggregation parameter)**.** *The aggregation parameter λ^* that minimizes the equity premium is:*

(a) Invariant to changes in per capita supply x

(b) Decreasing in the proportion α of AA investors

(c) Decreasing in ambiguity aversion for low levels of ambiguity aversion ($a < 1 + \frac{1}{R(v,b|s)\sqrt{1-\alpha}}$), but is increasing in ambiguity aversion for large levels of ambiguity aversion ($a > 1 + \frac{1}{R(v,b|s)\sqrt{1-\alpha}}$)

Proof. See Appendix. ■

Part (a) of Proposition 3 suggests that the optimal aggregation parameter does not depend on firm size because the per capita supply x scales the mean and variance of the

firm's payout. The result is due to the investors' constant absolute risk aversion. Large per capita supply x increases per capita risk, but that risk is a linear function of supply x. Changes in the aggregation parameter affect the term that multiplies x to determine per capita risk and the minimizing equity premium will minimize that term. Part (b) follows from the fact that the equity premium depends on a weighted average of the two investor types' posterior precisions. An increase in the proportion α of AA investors increases their relative contribution to the average precision therefore increasing the cost of ambiguity relative to pure risk. This makes a reduction in λ valuable even though it reduces ambiguity at the expense of increasing risk. The first part of (c) follows from an increase in ambiguity aversion a increasing the relative cost of ambiguity. The second part of (c) obtains because increases in ambiguity aversion also reduce the size of AA investors' holdings of the firm's shares. When the a is high, AA investors contribute little to the weighted average precision because they take small positions in the firm's shares. At high levels of ambiguity aversion, this effect dominates so that the optimal aggregation parameter increases as ambiguity aversion increases. Stated differently, a high sensitivity to ambiguity makes AA investors demand large reductions in ambiguity that entail sacrificing value relevant information. In this case, firms find it less costly to satisfy SEU investors' demand for a more informative signal because it is too costly to satisfy AA investors with further reductions in λ.

Cao, Wang, and Zhang (2005) show that ambiguity aversion can induce a conglomerate discount whereby the value of a combined firm falls below the sum of the values that would obtain if investors could separately trade its components. While I intend for the two components of firm value in this model to represent non-separable components of a firm, thinking of them as two firms provides some insight into how providing summary information affects the conglomerate discount. When investors can trade the two components separately, the SEU investors hold relatively large portions of the component v_1 that has ambiguous information while AA investors hold relatively more of the

component v_2. Both investors hold some of each because they provide portfolio diversification. My model predicts a conglomerate discount if the two components were to merge and issue disaggregate information. This case produces no information benefit because investors' information does not change from when they could separately trade v_1 and v_2. On the other hand, conglomeration creates a cost to investors by restricting their portfolios to equal proportions of v_1 and v_2. If the components merge and release a summary signal they may realize a conglomerate premium if the reduction in ambiguity offsets the elimination of the benefits of allowing investors to hold different proportions of v_1 and v_2. Figure 2 shows an example in which summary information yields a conglomerate premium. The lower equity premium for summary information shown in the figure depends on the parameters of the model. Providing summary information at least dampens the conglomerate discount and may, in some cases, yield a conglomerate premium.

3 Endogenous choice of information

The preceding discussion assumed that all investors view the same information set. I now relax that assumption and allow investors to individually choose what information to view. I assume that the firm makes both disaggregate and summary information available to investors without charge. This setting resembles one in which investors might choose between viewing a tear sheet summarizing the company's performance versus viewing a comprehensive report that includes a filing such as a 10-K. Investors who fail to view the disaggregate information face an adverse selection problem because they may be trading against other investors who have superior information. Indeed, the Blackwell theorem implies that we can assume without loss of generality that the SEU view the disaggregate information so that any AA investor who views only summary

information will face an adverse selection problem.[8] Note, however, that in the setting of this model the competition amongst SEU investors mitigates the adverse selection problem faced by an AA investor who chooses to view only summary information.

In order to frame the discussion of information choice, first consider the information collection problem faced by a SEU investor. At the time of trade, an SEU investor will demand $q_S(J,p) = \frac{\mathrm{E}[v|J,p]-p}{r\,\mathrm{var}(v|J,p)}$ as given by (4). The SEU investor can then compute the *ex ante* expected utility given information (J,p) as follows under the assumption that J and p are joint normally distributed. The SEU investors' preferences imply that they will use all costless information so that including price p in the information set is without loss of generality. I include price p in the information set for this illustration because it simplifies the expression.[9]:

$$\begin{aligned}\mathrm{E}\left[u\left(q_S(v-p)\right)\right] &= \mathrm{E}\left[\mathrm{E}\left[u\left(q_S(v-p)\right)|J,p\right]\right] \\ &= u\left(\frac{\mathrm{E}[v-p]^2}{2r\,\mathrm{var}(v-p)} + \frac{1}{2r}\log\frac{\mathrm{var}(v-p)}{\mathrm{var}(v|J,p)}\right) \quad (9)\end{aligned}$$

where I have used $\mathrm{E}[\exp\{tX\}] = \frac{1}{1-2t\,\mathrm{var}(X)}\exp\left\{\frac{t\,\mathrm{E}[X]^2}{1-2t\,\mathrm{var}(X)}\right\}$ for normally distributed X and the fact that $v-\mathrm{E}[v|J,p]$ is orthogonal to p since $\mathrm{E}[v|J,p]$ is an orthogonal projection of v onto (J,p). The ratio $\frac{\mathrm{var}(v-p)}{\mathrm{var}(v|J,p)}$ exceeds one so that the log term is positive.[10] Only the denominator of $\frac{\mathrm{var}(v-p)}{\mathrm{var}(v|J,p)}$ in (9) depends on information J so that SEU investors will indeed view as much information as possible in order to minimize $\mathrm{var}(v|J,p)$. Also note that if all investors view the same information set J, then the price equals $\mathrm{E}[v|J]$ less a constant equity premium so that $\mathrm{var}(v-p) = \mathrm{var}(v-\mathrm{E}[v|J]) = \mathrm{var}(v|J)$ and the log term is zero.

The AA expression corresponding to (9) adds the complication that AA investors

[8] In the special case of the unbiased summary signal ($\lambda = 1$), $\mathrm{E}[v|s] = \mathrm{E}[v|S]$ so that there is no cost to adverse selection.

[9] Also see Verrecchia (1982) for a similar expression.

[10] $\mathrm{var}(v-p) - \mathrm{var}(v|J,p) > \mathrm{var}(v-p) - \mathrm{var}(v|p) = \mathrm{var}(p)\left(1-\frac{\mathrm{cov}(v,p)}{\mathrm{var}(p)}\right)^2 > 0.$

consider the impact their choice of information set has on their exposure to ambiguity. Consistent with Safra and Sulganik (1995), this consideration can cause them to forgo costless, non-redundant information. In contrast, the Blackwell theorem implies that an SEU investor will look at any costless, non-redundant information.

The Appendix uses Klibanoff, Marinacci, and Mukerji's (2006) dynamic model of smooth ambiguity aversion to derive the following expression for the AA investors' *ex ante* value of viewing information set J:

$$\begin{aligned} g(J;\lambda) = & \frac{\mathrm{E}[v-p]^2}{2r\left(\mathrm{var}(v|J)[1+(a-1)R(v,b|J)]+\mathrm{var}(z)[1+(a-1)R(z,b)]\right)} \\ & + \frac{1}{2r}\log\left(1+\frac{\mathrm{var}(z|b)}{\mathrm{var}(v|J)[1+(a-1)R(v,b|J)]}\right) \\ & + \frac{1}{2ra}\log\left(1+\frac{a[\mathrm{var}(z)-\mathrm{var}(z|b)]}{\mathrm{var}(v|J)[1+(a-1)R(v,b|J)]+\mathrm{var}(z|b)}\right) \end{aligned} \tag{10}$$

where $z = \mathrm{E}[v|J] - p$. This expression simplifies to (9) for an SEU investor ($a = 1$) who conditions on (J, p). The AA investors consider the impact of ambiguity at the trading period, given by $R(v, b|J)$, and the *ex ante* ambiguity $R(z, b) = \mathrm{corr}(\mathrm{E}[v|J] - p, b)^2$. The evaluation of (10) required a conjecture of price. If all SEU investors view S and essentially all AA investors (all except a set of measure zero) view J_0 that is joint normally distributed with (S, b), then price equals:

$$p = \frac{\frac{1-\alpha}{\mathrm{var}(v|S)}\,\mathrm{E}[v|S] + \frac{\alpha}{\mathrm{var}(v|J_0)[1+(a-1)R(v,b|J_0)]}\,\mathrm{E}[v|J_0] - rx}{\frac{1-\alpha}{\mathrm{var}(v|S)} + \frac{\alpha}{\mathrm{var}(v|J_0)[1+(a-1)R(v,b|J_0)]}} \tag{11}$$

Now turning to the impact of summary signals on information choice and the equity premium, first consider the case of an unbiased summary signal ($\lambda = 1$). In this case, $\mathrm{E}[v|s] = \mathrm{E}[v|S]$ and investors who view the summary do not face adverse selection. This also implies that $z = \mathrm{E}[v|s] - p$ is constant so that the variances of z in (10) are zero:

$$g(s;\lambda=1) = \frac{\mathrm{E}[v-p]^2}{2r\,\mathrm{var}(v|s)[1+(a-1)R(v,b|s)]} = \frac{\mathrm{E}[v-p]^2}{2r\,\mathrm{var}(v|S)[1+(a-1)R(v,b|s)]}$$

The AA investors clearly have no incentive to view disaggregate information since $R(v, b|s) < R(v, b|S)$ as given by Proposition 1. Also note that both investor types have the same trade direction (long versus short) when investors observe an unbiased summary signal. The unbiased signal therefore cannot explain phenomenon in which some investors ignore information in ways that provide opportunities for others to profit.

If the summary signal deemphasizes ambiguous information ($\lambda < 1$), then AA investors face a tradeoff between exposure to ambiguity via the disaggregate signals S and adverse selection due to SEU investors viewing disaggregate signals. I only consider comparisons between summary s and disaggregate S. I do not evaluate the choice to condition beliefs on (s, p) because, when $\lambda \neq 1$, the choice between (s, p) and s is identical to the choice between S and s due to the known per capita supply x. To see this, assume that AA investors only view the aggregate signal s while SEU investors view the disaggregate signal S. Market clearing then gives the price as in (11) with $J_0 = s$. Because $\mathrm{E}[v|S]$ and $\mathrm{E}[v|s]$ are linear in the signal vector $S = (s_1, s_2)$ price equals $p = \beta_0 + \boldsymbol{\beta}' S - \beta_x x$. The summary signal $s = \boldsymbol{\gamma}' S$. If $\lambda = 1$, then $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ are linearly dependent, otherwise they are not. Thus, if $\lambda \neq 1$ then investors can infer S from (s, p) using:

$$S = \begin{bmatrix} \boldsymbol{\beta}' \\ \boldsymbol{\gamma}' \end{bmatrix}^{-1} \begin{bmatrix} p - \beta_0 + \beta_x x \\ s \end{bmatrix}$$

Because observing (s, p) exposes AA investors to same ambiguity as S, firms may wish to withhold S if investors cannot avoid processing information in price. Proposition 2 demonstrates that doing so will yield a lower equity premium than providing only disaggregate information.

In general, the AA investors will evaluate the *ex ante* value of information J using $g(J; \lambda)$ from (10). They will view only the summary signal s when $g(s; \lambda) > g(S; \lambda)$. We have already seen that they strictly prefer an unbiased summary signal ($\lambda = 1$) to

disaggregate information S. The following proposition states that there is a range of values in which the AA investors will prefer a summary that deemphasizes ambiguous information ($\lambda < 1$) to disaggregate information despite the adverse selection due to SEU investors viewing S.

Proposition 4 (Minimum equity premium with choice of information)**.** *Subject to the constraint that AA investors choose to ignore disaggregate information, the aggregate signal of the form* $\lambda \frac{\sigma_{v1}^2}{\sigma_{s1}^2} s_1 + \frac{\sigma_{v2}^2}{\sigma_{s2}^2} s_2$ *that minimizes the equity premium uses* $\lambda < 1$ *so that the aggregate signal deemphasizes ambiguous information. The constraint on the AA choice of information is not binding for sufficiently large per capita supply* x *if there are sufficiently many AA investors and it is not binding for sufficiently small per capita supply* x *if there are sufficiently few AA investors.*

Proof. See Appendix. ■

Proposition 4 states that releasing both summary and disaggregate information can, in some cases, yield a lower equity premium than the minimizing equity premium discussed in Proposition 2. Figures 3 through 5 illustrate Proposition 4. Figure 3 illustrates a case in which the constraint on AA choice of information never binds so that AA investors always prefer summary information. All of the figures show that releasing an unbiased summary signal ($\lambda = 1$) yields the same equity premium as releasing both disaggregate information and an unbiased summary signal. As the summary signal deemphasizes ambiguous information, declines in the aggregation parameter λ from the value $\lambda = 1$ yield greater reductions in the equity premium than when all investors observe only the summary signal as in Figure 1. This occurs because the release of both information types allows the AA investors to avoid ambiguity without taking any information away from the SEU investors who can now view disaggregate information S.

Figure 4 illustrates the case when AA investors comprise a small portion of the market. Proposition 4 states that they will prefer disaggregate information when there is a large per capita supply of shares. The AA investors have little to gain from viewing S since prices already largely reflect the information in S due to the large proportion of SEU investors. The constraint on the AA choice of information only binds when λ is so low that it excludes nearly all of the ambiguous information. This kink in the figure shows this point at which the AA investors choose to view disaggregate information. When per capita supply is large, the AA investors' concern with price risk causes them to view disaggregate information.

Figure 5 illustrates the case when AA investors comprise a large portion of the market. Proposition 4 states that they will prefer disaggregate information when there is a small per capita supply of shares. The AA investors have much to gain from viewing S in this case because the small proportion of SEU investors implies that prices do not reflect the information in S. In this example, the firm can obtain a lower equity premium by releasing both types of information than by releasing only summary information. The summary signal reflects more of the ambiguous information than the summary that minimizes the equity premium when all investors observe only the summary. The benefit to providing S to the SEU investors compensates for the cost of exposing AA investors to ambiguity with a larger λ than in the minimizing summary signal when all investors view only the summary.

The behavior of AA investors when they have the choice to view disaggregate information suggests that ambiguity aversion may play a role in causing investors to ignore public information. The choice to ignore disaggregate information arises from optimizing behavior rather than stupidity or naïveté. The resulting profits to SEU investors who view S represent a premium for bearing ambiguity. This behavior suggests that investors with the highest tolerance for ambiguity (e.g. fund managers) will view the

most information while those with a low tolerance for ambiguity will focus on summary information.

4 Ambiguity in fundamentals

The previous sections focus on situations in which investors do not know how information relates to payoffs, as represented by ambiguity in the noise term of their information. I now consider an alternative case in which investors face uncertainty regarding the distribution of payoffs rather than the distribution of information. I maintain the same setup as the previous sections but alter the information structure so that the distribution of the component v_1 of the terminal dividend is ambiguous. The distribution of the noise term e_1 in the signal $s_1 = v_1 + e_1$ is unambiguous. I assume that e_1 is normally distributed and independent of the other variables in the model. The first component of firm value v_1 is ambiguous and has an unknown mean $\mu_{v1} + c$ for which, as before, investors' have a subjective prior belief for c represented by a normal distribution with mean μ_c and variance σ_c^2. This implies that the v_1 is equivalent to the sum of independent normal random variables u_1 and c where u_1 has mean μ_{v1} and variance σ_{u1}^2 giving $\sigma_{v1}^2 = \sigma_{u1}^2 + \sigma_c^2$.

This setting corresponds to one in which investors have ambiguous priors about future payoffs while the preceding sections correspond to one in which investors have ambiguous priors about how information they receive relates to future payoffs. Whether ambiguity pertains more or less to fundamentals or information depends on the relative expertise of the investor. For example, a situation with relatively low ambiguity in fundamentals but high ambiguity in information could correspond to a person that has a great deal of industry knowledge so that he has a well-defined prior belief about the firm's value. He may not have expertise in interpreting accounting information or discerning information provided by analysts. A situation with relatively low ambiguity in information could

correspond to a person who has deep knowledge of how accounting information derives from firm value but lacks expertise to arrive at a well-defined prior belief about the firm's value.

An aggregate report still yields a lower equity premium than a disaggregate report in this setting; however, in contrast to Proposition 2, the aggregation scheme that minimizes the equity premium *overemphasizes* ambiguous information with an aggregation parameter of $\lambda > 1$. As before, the minimizing aggregation scheme reflects a tradeoff between providing information and reducing the number of potential interpretations of that information. Disaggregate information reduces the posterior uncertainty about the ambiguous parameter c more than aggregate information. This is because $\text{var}(c|S) \leq \text{var}(c|s)$; however, that benefit is partially offset by increasing alternative interpretations of the information because the expected value given disaggregate information $\text{E}[v|S, c]$ varies more than $\text{E}[v|s, c]$. It is reducing the number of interpretations that makes a distorted aggregation scheme ($\lambda \neq 1$) more valuable than disaggregate information. I summarize this result in the following proposition.

Proposition 5 (Ambiguity in fundamentals)**.** *Given aggregation schemes of the form* $s = \lambda \frac{\sigma_{v1}^2}{\sigma_{s1}^2} s_1 + \frac{\sigma_{v2}^2}{\sigma_{s2}^2} s_2$. *When the noise* e_1 *in the signal* s_1 *is unambiguous but the component of value* v_1 *is ambiguous, the aggregation scheme that minimizes the equity premium overemphasizes information about the ambiguous component of firm value. That is, it sets* λ *strictly greater than one. The resulting equity premium is lower than when investors view disaggregate information.*

Proof. See Appendix. ■

More generally, an unbiased aggregate signal of the form $s = \text{E}[v|S]$ will yield a lower equity premium than the disaggregate report S regardless of the number of ambiguous components of firm value or whether ambiguity pertains to fundamentals and

signals. This follows from the fact that $s = \mathrm{E}[v|s]$ implies $\mathrm{var}(v|s) = \mathrm{var}(v|S)$ and $\mathrm{var}(v|s,c) > \mathrm{var}(v|S,c)$. The posterior variances therefore equal one another for aggregate and disaggregate signals while ambiguity is lower with the summary signal because $R(v,c|s) = 1 - \frac{\mathrm{var}(v|s,c)}{\mathrm{var}(v|s)} = 1 - \frac{\mathrm{var}(v|s,c)}{\mathrm{var}(v|S)} < 1 - \frac{\mathrm{var}(v|S,c)}{\mathrm{var}(v|S)} = R(v,c|S)$.

5 Conclusion

Previous research on the effect of information on the equity premium has typically supported the view that the equity premium declines as investors view more public information. I have shown that providing investors with summary signals can reduce the equity premium when the market includes ambiguity averse investors. The reduction in the equity premium obtains despite the fact that a summary signal contains less information than its components. The reduction arises because summary signals can reduce the distribution uncertainty that ambiguity averse investors penalize heavily when making their equity purchase decisions. In contrast, when the market includes only subjective expected utility maximizers (SEU), replacing detailed information with a summary signal increases the equity premium.

The benefit to summary information obtains even when investors can observe either summary or detailed information, as they can in actual equity markets. For example, investors have many sources of free information at the summary level, such as tear sheets provided by some information providers, and detailed information such as annual reports. Sloan (1996) suggests that investors, on average, properly price earnings but not the separate accrual and cash components of earnings. Earnings represent a distorted aggregation scheme in the sense used in this paper because net income adds earnings and cash flow with equal weight rather than a Bayesian weighting scheme. The behavior of investors in regard to earnings resembles the behavior in my model in which ambiguity

averse investors view a summary signal, earnings in this case, even though they can also observe detailed information, the accrual and cash components of earnings. The profit opportunities that arise in my model for SEU investors who view disaggregate information represent a premium for bearing ambiguity that might explain how investors such as hedge funds earn profits on an accruals strategy even though it relies only on public information. This profit opportunity resembles Knight's (1921) claim that entrepreneurs make profits for bearing ambiguity rather than risk.

More broadly, this paper addresses the value of providing summary information to investors even though a summary signal may appear to be redundant if it aggregates public information. The nature of a summary signal that reduces the equity premium depends on the nature of the information environment. Unbiased signals that use a Bayesian weighting scheme to aggregate underlying detailed signals always reduce the equity premium compared to releasing the detailed signals. If investors face ambiguity primarily in the interpretation of information, summary signals that deemphasize ambiguous information reduce the equity premium versus an unbiased aggregation. If investors face ambiguity primarily in the prior beliefs about security payoffs, ambiguous signals that overemphasize the ambiguous components of payoffs reduce the equity premium relative to an unbiased aggregation.

These results contribute both to the literature on information in capital markets and the literature on ambiguity aversion. The capital markets literature has largely assumed that all investors are SEU, which leaves no role for the packaging of information. The impact of information on markets with ambiguity averse investors depends on both substance and form. The literature on ambiguity aversion has recently begun to address how ambiguity averse decision makers process information. This paper contributes to the ambiguity aversion literature by showing how ambiguity averse investors differ fundamentally from SEU investors by preferring summary information.

Figure legends

Figure 1 displays the equity premium $\mathrm{E}[v-p]$ from (8) as a function of the aggregation parameter $\lambda \in [0,1]$. The horizontal dashed line is the equity premium when investors observe disaggregate information $S = \begin{bmatrix} s_1 & s_2 \end{bmatrix}$. The solid line shows the equity premium when investors observe aggregate signals of the form $s = \lambda \frac{\sigma_{v1}^2}{\sigma_{s1}^2} s_1 + \frac{\sigma_{v2}^2}{\sigma_{s2}^2} s_2$. Setting $\lambda = 1$ yields $\mathrm{E}[v|s] = \mathrm{E}[v|S]$ while $\lambda = 0$ totally disregards the ambiguous signal s_1. This figure uses the parameters $x = \alpha = 0.5, r = \sigma_{v1}^2 = \sigma_{v2}^2 = \sigma_{e1}^2 = \sigma_{e2}^2 = \sigma_b^2 = 1, a = 5$.

Figure 2 displays the equity premium $\mathrm{E}[v-p]$ from (8) as a function of the aggregation parameter $\lambda \in [0,1]$. The thin horizontal dashed line is the equity premium when investors observe disaggregate information $S = \begin{bmatrix} s_1 & s_2 \end{bmatrix}$. The thick horizontal dashed line shows the total equity premium when investors can separately trade the two components of firm value v_1 and v_2. The solid line shows the equity premium when investors observe aggregate signals of the form $s = \lambda \frac{\sigma_{v1}^2}{\sigma_{s1}^2} s_1 + \frac{\sigma_{v2}^2}{\sigma_{s2}^2} s_2$. Setting $\lambda = 1$ yields $\mathrm{E}[v|s] = \mathrm{E}[v|S]$ while $\lambda = 0$ totally disregards the ambiguous signal s_1. The relative position of the line for separate trading (thick dashed line) to that for summary information (solid line) is particular to the parameters in this example. The relative position of disaggregate information (thin dashed line) to summary information (solid line) is general as is the position of the line for disaggregate information (thin dashed line) relative to separate trading (thick dashed line). This figure uses the parameters $x = \alpha = 0.5, r = \sigma_{v1}^2 = \sigma_{v2}^2 = \sigma_{e1}^2 = \sigma_{e2}^2 = \sigma_b^2 = 1, a = 5$.

Figure 3 displays the equity premium $\mathrm{E}[v-p]$ from (8) as a function of the aggregation parameter $\lambda \in [0,1]$. The horizontal dashed line is the equity premium when investors observe disaggregate information $S = \begin{bmatrix} s_1 & s_2 \end{bmatrix}$. The solid line shows the equity premium when investors observe aggregate signals of the form $s = \lambda \frac{\sigma_{v1}^2}{\sigma_{s1}^2} s_1 + \frac{\sigma_{v2}^2}{\sigma_{s2}^2} s_2$. The thick line shows the equity premium when the firm releases both aggregate and disaggregate information. Setting $\lambda = 1$ yields $\mathrm{E}[v|s] = \mathrm{E}[v|S]$ while $\lambda = 0$ totally disregards the ambiguous signal s_1. This figure uses the parameters $x = \alpha = 0.5, r = \sigma_{v1}^2 = \sigma_{v2}^2 = \sigma_{e1}^2 = \sigma_{e2}^2 = \sigma_b^2 = 1, a = 5$.

Figure 4 displays the equity premium $\mathrm{E}[v-p]$ from (8) as a function of the aggregation parameter $\lambda \in [0,1]$. The horizontal dashed line is the equity premium when investors observe disaggregate information $S = \begin{bmatrix} s_1 & s_2 \end{bmatrix}$. The solid line shows the equity premium when investors observe aggregate signals of the form $s = \lambda \frac{\sigma_{v1}^2}{\sigma_{s1}^2} s_1 + \frac{\sigma_{v2}^2}{\sigma_{s2}^2} s_2$. The thick line shows the equity premium when the firm releases both aggregate and disaggregate information. The kink in this line occurs where AA investors find disaggregate information more valuable *ex ante* than aggregate information. Setting $\lambda = 1$ yields $\mathrm{E}[v|s] = \mathrm{E}[v|S]$ while $\lambda = 0$ totally disregards the ambiguous signal s_1. This figure uses the parameters $x = 100, \alpha = 0.25, r = \sigma_{v1}^2 = \sigma_{v2}^2 = \sigma_{e1}^2 = \sigma_{e2}^2 = \sigma_b^2 = 1, a = 5$.

Figure 5 displays the equity premium $\mathrm{E}[v-p]$ from (8) as a function of the aggregation parameter $\lambda \in [0,1]$. The horizontal dashed line is the equity premium when investors observe disaggregate information $S = \begin{bmatrix} s_1 & s_2 \end{bmatrix}$. The solid line shows the equity premium when investors observe aggregate signals of the form $s = \lambda \frac{\sigma_{v1}^2}{\sigma_{s1}^2} s_1 + \frac{\sigma_{v2}^2}{\sigma_{s2}^2} s_2$. The thick line shows the equity premium when the firm releases both aggregate and disaggregate information. The kink in this line occurs where AA investors find disaggregate information more valuable *ex ante* than aggregate information. Setting $\lambda = 1$ yields $\mathrm{E}[v|s] = \mathrm{E}[v|S]$ while $\lambda = 0$ totally disregards the ambiguous signal s_1. This figure uses the parameters $x = 0.2, \alpha = 0.75, r = \sigma_{v1}^2 = \sigma_{v2}^2 = \sigma_{e1}^2 = \sigma_{e2}^2 = \sigma_b^2 = 1, a = 5$.

Appendix

Derivation of (3)

The AA investor's objective function is:

$$-\operatorname{E}\left[\left(\operatorname{E}\left[\exp\left\{-r\left(q(v-p)\right)\right\}|J,b\right]\right)^{a}|J\right]$$

The joint normality of v, J and b implies that the interior expectation is:

$$\operatorname{E}\left[\exp\left\{-r\left(q(v-p)\right)\right\}|J,b\right] = \exp\left\{-r\left(q(\operatorname{E}[v|J,b]-p) - \frac{r}{2}q^2\operatorname{var}(v|J,b)\right)\right\}$$

This gives:

$$\begin{aligned}&-\operatorname{E}\left[\left(\operatorname{E}\left[\exp\left\{-r\left(q(v-p)\right)\right\}|J,b\right]\right)|J\right]\\&\qquad= -\operatorname{E}\left[\exp\left\{-ra\left(q(\operatorname{E}[v|J,b]-p) - \frac{r}{2}q^2\operatorname{var}(v|J,b)\right)\right\}|J\right]\end{aligned}$$

The joint normality of $\operatorname{E}[v|J,b]$ and J then gives:

$$\begin{aligned}&-\operatorname{E}\left[\left(\operatorname{E}\left[\exp\left\{-r\left(q(v-p)\right)\right\}|J,b\right]\right)|J\right]\\&\qquad= -\exp\left\{-ra\left(q(\operatorname{E}[v|J]-p) - \frac{r}{2}q^2\operatorname{var}(v|J,b) - \frac{ra}{2}q^2\operatorname{var}(\operatorname{E}[v|J,b]|J)\right)\right\}\end{aligned}$$

The identity $\operatorname{var}(\boldsymbol{x}|\boldsymbol{y}) = \operatorname{var}(\boldsymbol{x}|\boldsymbol{y},\boldsymbol{z}) + \operatorname{var}(\operatorname{E}[\boldsymbol{x}|\boldsymbol{y},\boldsymbol{z}]|\boldsymbol{y})$ for jointly normally distributed random vectors $\boldsymbol{x}, \boldsymbol{y}$ and $\boldsymbol{z}$ implies that the above expression simplifies to:

$$\begin{aligned}&-\operatorname{E}\left[\left(\operatorname{E}\left[\exp\left\{-r\left(q(v-p)\right)\right\}|J,b\right]\right)|J\right]\\&\qquad= -\exp\left\{-ra\left(q(\operatorname{E}[v|J]-p) - \frac{r}{2}q^2\operatorname{var}(v|J)\left(1+(a-1)R(v,b|J)\right)\right)\right\}\end{aligned}$$

where $R(v,b|J) = 1 - \frac{\operatorname{var}(v|J,b)}{\operatorname{var}(v|J)} = \frac{\operatorname{cov}(v,b|J)^2}{\operatorname{var}(v|J)\operatorname{var}(b|J)} = \operatorname{corr}(v,b|J)^2 \in [0,1]$ represents the degree of ambiguity faced by investors. Setting $a = 1$ gives the objective function of SEU investors, who are unconcerned with ambiguity so that the $R(v,b|J)$ term does

not enter their objective function. ■

Proof of Proposition 1

The ambiguity term is:

$$R(v, b|J) = \frac{\text{cov}(\text{E}[v|J], b)^2}{\text{var}(v|J)\,\text{var}(b|J)}$$

The proof proceeds by showing that the two denominator terms are weakly greater for aggregate s than for disaggregate S and that both terms are decreasing in λ for $\lambda \in [0, 1]$. I then complete the proof by showing that the opposite is true for the numerator term.

It is clear that the first denominator term $\text{var}(v|s) = \text{var}(v) - \text{var}(\text{E}[v|s]) \leq \text{var}(v|S) = \text{var}(v) - \text{var}(\text{E}[v|S])$ with equality only for $\lambda = 1$. Direct computation gives the change in $\text{var}(v|s)$ with respect to λ:

$$\frac{\partial}{\partial \lambda}\,\text{var}(v|s) = -2\frac{\sigma_{v1}^4}{\sigma_{s1}^2}\frac{\sigma_{v2}^4}{\sigma_{s2}^2}\frac{\text{cov}(v, s)}{\text{var}(s)^2}(1 - \lambda)$$

The covariance $\text{cov}(v, s)$ is positive for all $\lambda > -\frac{\sigma_{v2}^4/\sigma_{s2}^2}{\sigma_{v1}^4/\sigma_{s1}^2}$ giving $\frac{\partial}{\partial \lambda}\,\text{var}(v|s) \leq 0$ for $\lambda \in [0, 1]$ with equality for $\lambda = 1$.

The second denominator term $\text{var}(b|s) > \text{var}(b|S)$ with strict inequality. To see this, note that s is equivalent to $\hat{s}_1 = \frac{\sigma_{s1}^2}{\lambda\sigma_{v1}^2}s = s_1 + \frac{\sigma_{v2}^2/\sigma_{s2}^2}{\lambda\sigma_{v1}^2/\sigma_{s1}^2}s_2$ with $\text{var}(\hat{s}_1) > \sigma_{s1}^2$ so that $\text{var}(b|s) = \text{var}(b|\hat{s}_1) = \sigma_b^2 - \frac{\text{cov}(b,\hat{s}_1)^2}{\text{var}(\hat{s}_1)} = \sigma_b^2 - \frac{\text{cov}(b,s_1)^2}{\text{var}(\hat{s}_1)} > \text{var}(b|S) = \text{var}(b|s_1) = \sigma_b^2 - \frac{\text{cov}(b,s_1)^2}{\sigma_{s1}^2}$. The parameter λ only affects $\text{var}(b|\hat{s}_1)$ through its affect on $\text{var}(\hat{s}_1) = \sigma_{s1}^2 + \left(\frac{\sigma_{v2}^2/\sigma_{s2}^2}{\lambda\sigma_{v1}^2/\sigma_{s1}^2}\right)^2\sigma_{s2}^2$ which is clearly decreasing in the magnitude of λ.

The numerator term is the square of $\text{cov}(\text{E}[v|J], b)$ which for S equals $\text{cov}(\text{E}[v|S], b) = -\frac{\sigma_{v1}^2}{\sigma_{s1}^2}\sigma_b^2$. In the case of aggregate information s it equals:

$$\text{cov}(\text{E}[v|s], b) = -\frac{\lambda\,\text{cov}(v, s)}{\text{var}(s)}\frac{\sigma_{v1}^2}{\sigma_{s1}^2}\sigma_b^2 = \frac{\lambda\,\text{cov}(v, s)}{\text{var}(s)}\,\text{cov}(\text{E}[v|S], b)$$

which gives:

$$\frac{\partial}{\partial\lambda}\operatorname{cov}(\mathrm{E}[v|s],b)^2 = 2\frac{\lambda\operatorname{cov}(v,s)}{\operatorname{var}(s)}\operatorname{cov}(\mathrm{E}[v|S],b)^2\frac{\partial}{\partial\lambda}\left(\frac{\lambda\operatorname{cov}(v,s)}{\operatorname{var}(s)}\right)$$

The term $\frac{\lambda\operatorname{cov}(v,s)}{\operatorname{var}(s)} = \frac{\lambda^2\frac{\sigma_{v1}^4}{\sigma_{s1}^2}+\lambda\frac{\sigma_{v2}^4}{\sigma_{s2}^2}}{\lambda^2\frac{\sigma_{v1}^4}{\sigma_{s1}^2}+\frac{\sigma_{v2}^4}{\sigma_{s2}^2}} \in [0,1]$ for $\lambda \in [0,1]$. It equals one at $\lambda = 1$ and equals zero at $\lambda = 0$. Direct computation shows that:

$$\frac{\partial}{\partial\lambda}\left(\frac{\lambda\operatorname{cov}(v,s)}{\operatorname{var}(s)}\right) = \frac{\sigma_{v2}^4}{\sigma_{s2}^2\operatorname{var}(s)^2}\left(\lambda(2-\lambda)\frac{\sigma_{v1}^4}{\sigma_{s1}^2}+\frac{\sigma_{v2}^4}{\sigma_{s2}^2}\right)$$

which is positive for any $\lambda \in [0,1]$.

Summarizing, $\lambda \in [0,1]$ implies $\operatorname{var}(v|s) \geq \operatorname{var}(v|S), \operatorname{var}(b|s) > \operatorname{var}(b|S)$ and $\operatorname{cov}(\mathrm{E}[v|s],b)^2 \leq \operatorname{cov}(\mathrm{E}[v|S],b)^2$ which implies that $R(v,b|s) < R(v,b|S)$ for all $\lambda \in [0,1]$. Also, $\lambda \in [0,1]$ implies $\frac{\partial}{\partial\lambda}\operatorname{var}(v|s) \leq 0$, $\frac{\partial}{\partial\lambda}\operatorname{var}(b|s) < 0$ and $\frac{\partial}{\partial\lambda}\operatorname{cov}(\mathrm{E}[v|s],b)^2 \geq 0$ for all $\lambda \in [0,1]$ so that $R(v,b|s) > 0$ is increasing in λ for all $\lambda \in [0,1]$. ■

Proof of Proposition 2

First note that the equity premium has a local minimum in λ between zero and one. In order to see this, first note that the equity premium equals:

$$\begin{matrix}\text{Equity}\\ \text{premium}\end{matrix} = rx\operatorname{var}(v|s)\left(1+\frac{\alpha(a-1)R(v,b|s)}{1+(1-\alpha)(a-1)R(v,b|s)}\right) = \frac{rx}{\frac{1-\alpha}{\operatorname{var}(v|s)}+\frac{\alpha}{\operatorname{var}(v|s)[1+(a-1)R(v,b|s)]}}$$

The above shows that the equity premium is decreasing in the weighted average precision of the two investor types:

$$\frac{1-\alpha}{\operatorname{var}(v|s)}+\frac{\alpha}{\operatorname{var}(v|s)[1+(a-1)R(v,b|s)]}$$

where I use the term 'precision' loosely in the case of AA investors to refer to $\frac{1}{\mathrm{var}(v|s)[1+(a-1)R(v,b|s)}$. This gives the change in average precision with respect to the aggregation parameter λ:

$$\frac{\partial}{\partial\lambda}\begin{array}{c}\text{Average}\\ \text{precision}\end{array} = \left(1-\frac{\alpha(a-1)R(v,b|s)}{1+(a-1)R(v,b|s)}\right)\frac{\partial}{\partial\lambda}\left(\frac{1}{\mathrm{var}(v|s)}\right) + \frac{\alpha}{\mathrm{var}(v|s)}\frac{\partial}{\partial\lambda}\left(\frac{1}{1+(a-1)R(v,b|s)}\right)$$

The terms that multiply $\frac{\partial}{\partial\lambda}\left(\frac{1}{\mathrm{var}(v|s)}\right)$ and $\frac{\partial}{\partial\lambda}\left(\frac{1}{1+(a-1)R(v,b|s)}\right)$ are both positive because $R(v,b|s)\in[0,1]$ and variances are positive. Proposition 1 gives $\frac{\partial}{\partial\lambda}R(v,b|s)\geq 0$ for $\lambda\in[0,1]$ with equality only at $\lambda=0$. This implies that $\frac{\partial}{\partial\lambda}\left(\frac{1}{1+(a-1)R(v,b|s)}\right)\leq 0$ for $\lambda\in[0,1]$ with equality only at $\lambda=0$. On the other hand $\frac{\partial}{\partial\lambda}\left(\frac{1}{\mathrm{var}(v|s)}\right)\geq 0$ for $\lambda\in[0,1]$ with equality only at $\lambda=1$, corresponding to the minimum variance $\mathrm{var}(v|s;\lambda=1)=\mathrm{var}(v|S)$. These two results imply:

$$\frac{\partial}{\partial\lambda}\begin{array}{c}\text{Average}\\ \text{precision}\end{array}\Big|_{\lambda=0} = \left(1-\frac{\alpha(a-1)R(v,b|s)}{1+(a-1)R(v,b|s)}\right)\frac{\partial}{\partial\lambda}\left(\frac{1}{\mathrm{var}(v|s)}\right)\Big|_{\lambda=0} > 0$$

$$\frac{\partial}{\partial\lambda}\begin{array}{c}\text{Average}\\ \text{precision}\end{array}\Big|_{\lambda=1} = \frac{\alpha}{\mathrm{var}(v|s)}\frac{\partial}{\partial\lambda}\left(\frac{1}{1+(a-1)R(v,b|s)}\right)\Big|_{\lambda=1} < 0$$

The above inequalities imply that the average precision reaches a local maximum for $\lambda\in(0,1)$ corresponding to a local minimum of the equity premium.

The details showing that this local minimum is also a global minimum are available from the author. A global minimum for $\lambda\in[1,\infty)$ can be excluded because $\mathrm{var}(v|s)$ is strictly increasing in that range while $R(v,b|s)$ is initially increasing but then asymptotically approaches $R(v,b|s_1)>R(v,b|s;\lambda=1)$. A global minimum for $\lambda\in(-\infty,0]$ can be excluded because any value $R(v,b|s)$ with $\lambda<0$ can also be achieved by a $\lambda>0$ that also yields a lower variance $\mathrm{var}(v|s)$. ■

Proof of Proposition 3

This proof applies the implicit function theorem to the first-order condition for the minimizing aggregation parameter λ in Proposition 2:

$$\left(1 - \frac{\alpha(a-1)R(v,b|s)}{1+(a-1)R(v,b|s)}\right)\frac{\partial}{\partial\lambda}\left(\frac{1}{\mathrm{var}(v|s)}\right) + \frac{\alpha}{\mathrm{var}(v|s)}\frac{\partial}{\partial\lambda}\left(\frac{1}{1+(a-1)R(v,b|s)}\right) = 0$$

For the remainder of this proof, I use the notation g to refer to the above expression.

Given a parameter β, $\frac{\mathrm{d}\lambda^*}{\mathrm{d}\beta} = -\frac{\partial g/\partial\beta}{\partial g/\partial\lambda}$. Proposition 2 states that λ^* is an interior maximum in $(0,1)$ so that the second order condition implies $\frac{\partial g}{\partial\lambda}|_{\lambda=\lambda^*} < 0$ and $\mathrm{sign}\left(\frac{\mathrm{d}\lambda^*}{\mathrm{d}\beta}|_{\lambda=\lambda^*}\right) = \mathrm{sign}\left(\frac{\partial g}{\partial\beta}|_{\lambda=\lambda^*}\right)$.

Proof of part (a)

The per capita supply x factors out of $g(\lambda) = 0$ so that it does not affect the first-order condition.

Proof of part (b)

Direct computation gives:

$$\frac{\partial g}{\partial\alpha} = \frac{1}{\alpha}\left[g - \frac{\partial}{\partial\lambda}\left(\frac{1}{\mathrm{var}(v|s)}\right)\right]$$

Now use $g = 0$ at $\lambda = \lambda^*$ to get:

$$\frac{\partial g}{\partial\alpha}|_{\lambda=\lambda^*} = -\frac{1}{\alpha}\frac{\partial}{\partial\lambda}\left(\frac{1}{\mathrm{var}(v|s)}\right) < 0$$

The inequality is follows from the precision $1/\mathrm{var}(v|s)$ increasing in λ for $\lambda \in [0,1)$ until it reaches its maximum at $\lambda = 1$. This gives $\frac{\mathrm{d}\lambda^*}{\mathrm{d}\alpha}|_{\lambda=\lambda^*} < 0$ so that, at its optimum, λ^* is decreasing in the proportion α of AA investors.

Proof of part (c)

Direct computation gives:

$$\frac{\partial g}{\partial a} = \frac{1-(a-1)R(v,b|s)}{(a-1)[1+(a-1)R(v,b|s)]}\left[g - \frac{1-(1-\alpha)(a-1)^2R(v,b|s)^2}{1-(a-1)^2R(v,b|s)^2}\frac{\partial}{\partial\lambda}\left(\frac{1}{\mathrm{var}(v|s)}\right)\right]$$

At $\lambda = \lambda^*$:

$$\frac{\partial g}{\partial a}|_{\lambda=\lambda^*} = -\frac{1-(1-\alpha)(a-1)^2R(v,b|s)^2}{(a-1)[1+(a-1)R(v,b|s)]^2}\frac{\partial}{\partial\lambda}\left(\frac{1}{\mathrm{var}(v|s)}\right)$$

The precision $1/\mathrm{var}(v|s)$ is increasing in λ for $\lambda \in [0,1)$ so that $\frac{\partial}{\partial\lambda}\left(\frac{1}{\mathrm{var}(v|s)}\right)|_{\lambda=\lambda^*} > 0$. This then gives: $\mathrm{sign}\left(\frac{\mathrm{d}\lambda^*}{\mathrm{d}\alpha}|_{\lambda=\lambda^*}\right) = -\,\mathrm{sign}\left((1-\alpha)(a-1)R(v,b|s)^2 - \frac{1}{a-1}\right)$ so that:

$$\frac{\mathrm{d}\lambda^*}{\mathrm{d}\alpha}|_{\lambda=\lambda^*}\begin{cases} < 0 & \text{if } a < 1 + \frac{1}{R(v,b|s)\sqrt{1-\alpha}} \\ > 0 & \text{if } a > 1 + \frac{1}{R(v,b|s)\sqrt{1-\alpha}} \end{cases}$$

■

Derivation of (10)

Klibanoff, Marinacci, and Mukerji (2006) define a recursive representation over Savage acts f and g as $f \succeq g$ if and only if $V_{\omega^t}(f) \geq V_{\omega^t}(g)$ where ω^t denotes the history of a sequence of observations of a random variable, ω_t denotes the time t realization of the variable, β is a discount factor and:

$$V_{\omega^t}(f) = u(f(\omega^t)) + \beta h^{-1}\left(\int_{P\in\mathscr{P}} h\left(\int_{\Omega_{t+1}} V_{(\omega^t,\omega_{t+1})}\,\mathrm{d}P(\omega_{t+1}|\omega^t)\right)\mathrm{d}Q(P|\omega^t)\right)$$

Dividing the investor's actions over time where consumption occurs at t_2, trade occurs

at t_1 and the choice of information sets occurs at t_0 gives:

$$V_2(J) = u(q(v-p))$$
$$V_1(J) = u(0) + h^{-1}\left(\mathrm{E}\left[h\left(\mathrm{E}[u(q(J)(v-p)|J,b)]\right)|J\right]\right)$$

where at t_1 the investor chooses $q(J) = \frac{\mathrm{E}[v|J]-p}{r\,\mathrm{var}(v|J)[1+(a-1)R(v,b|J)]}$ giving the certainty equivalent $\frac{(\mathrm{E}[v|J]-p)^2}{2r\,\mathrm{var}(v|J)[1+(a-1)R(v,b|J)]}$ and:

$$V_1(J) = u(0) + u\left(\frac{(\mathrm{E}[v|J]-p)^2}{2r\,\mathrm{var}(v|J)[1+(a-1)R(v,b|J)]}\right)$$

giving:

$$\begin{aligned} V_0(J) &= u(0) + h^{-1}\left(\mathrm{E}\left[h\left(\mathrm{E}[u(V_1(J))|b]\right)\right]\right) \\ &= u(0) + h^{-1}\left(\mathrm{E}\left[h\left(\mathrm{E}\left[u\left(\frac{(\mathrm{E}[v|J]-p)^2}{2r\,\mathrm{var}(v|J)[1+(a-1)R(v,b|J)]}\right)\Big|b\right]\right)\right]\right) \end{aligned}$$

Evaluating the above expression requires a conjecture of price. Under the assumption that all SEU investors view disaggregate information S and AA investors view aggregate information s, the equilibrium price is given by (11) which implies that the random variable $\mathrm{E}[v|J]-p$ is normally distributed conditional on b. For normally distributed X, $\mathrm{E}[\exp\{tX^2\}] = \exp\left\{t\left(\frac{\mathrm{E}[X]^2}{1-2t\,\mathrm{var}(X)} - \frac{1}{2t}\log\left(1-2t\,\mathrm{var}(X)\right)\right)\right\}$. Using z to denote $\mathrm{E}[v|J]-p$ this gives the interior expectation:

$$\begin{aligned} &\mathrm{E}\left[u\left(\frac{(\mathrm{E}[v|J]-p)^2}{2r\,\mathrm{var}(v|J)[1+(a-1)R(v,b|J)]}\right)\Big|b\right] \\ &\qquad = u\Bigg(\frac{\mathrm{E}[z|b]^2}{2r\left[\mathrm{var}(v|J)[1+(a-1)R(v,b|J)]+\mathrm{var}(z|b)\right]} \\ &\qquad\qquad + \frac{1}{2r}\log\left(1+\frac{\mathrm{var}(z|b)}{\mathrm{var}(v|J)[1+(a-1)R(v,b|J)]}\right)\Bigg) \end{aligned}$$

The random variable $\mathrm{E}[z|b]$ is normally distributed, as well, so that a similar computation gives (10).

The expression for the value of information (9) for SEU investors where price $p \in J$ follows by setting $a = 1$ in (10) and using $\text{var}(v|J)+\text{var}(z) = \text{var}(v|J)+\text{var}(\text{E}[v|J]-p) = \text{var}(v) - \text{var}(\text{E}[v|J]) + \text{var}(\text{E}[v|J]) + \text{var}(p) - 2\,\text{cov}(\text{E}[v|J], p)$ and, because $p \in J$ implies $\text{cov}(v - \text{E}[v|J], p) = 0$, $\text{cov}(\text{E}[v|J], p) = \text{cov}(\text{E}[v|J] - v, p) + \text{cov}(v, p) = \text{cov}(v, p)$. This gives $\text{var}(v|J) + \text{var}(z) = \text{var}(v - p)$. ■

Proof of Proposition 4

I use the notation $\text{var}(z; J), \text{var}(z|b; J)$ to denote $\text{var}(\text{E}[v|J] - p)$ and $\text{var}(\text{E}[v|J] - p|b)$, respectively.

The constrained minimization of the equity premium is:

$$\min_{\lambda} \text{E}[v - p]$$

subject to:

$$g(s; \lambda) > g(S; \lambda)$$

where the constraint reflects the AA investors' choice to view only summary information. If they view disaggregate information, the equity premium does not depend on the aggregation parameter λ. Given that essentially all AA investors view the same information set J_0, the equity premium is:

$$\text{E}[v - p] = \frac{rx}{\frac{1-\alpha}{\text{var}(v|S)} + \frac{\alpha}{\text{var}(v|J_0)[1+(a-1)R(v,b|J_0)]}}$$

In order to prove the statement that the minimizing $\lambda < 1$, the discussion preceding the proposition gives $g(s; \lambda = 1) > g(S; \lambda = 1)$ so that, at $\lambda = 1$, the constraint on AA choice does not bind. A lower λ will not cause them to view disaggregate information, but it will reduce the equity premium because, as shown in the proof of Proposition 2,

the equity premium is increasing in λ at $\lambda = 1$.

In order to prove the second part of the proposition regarding when the constraint on AA choice of information does not bind, I first introduce some notation. Define the variable γ as:

$$\gamma = \frac{1-\alpha}{\alpha} \frac{\text{var}(v|s)[1 + (a-1)R(v,b|s)]}{\text{var}(v|S)}$$

We can then rewrite the equity premium under the assumption that essentially all AA view the summary signal s as:

$$\text{E}[v - p] = rx \frac{\text{var}(v|S)}{1-\alpha} \frac{\gamma}{1+\gamma}$$

The only term that depends on λ is γ. The equity premium is strictly increasing in γ so that minimizing the equity premium with respect to λ is equivalent to minimizing γ with respect to λ.

Now compute the variance terms involving $z = \text{E}[v|J] - p$. The computations model the choice of an individual AA investor so I maintain the assumption that essentially all AA view s. The notation $\text{var}(z|\cdot; J)$ denotes $\text{var}(\text{E}[v|J] - p|\cdot)$.

$$\begin{aligned}
\text{var}(z; S) &= \left(\frac{1}{1+\gamma}\right)^2 \text{var}(\text{E}[v|S] - \text{E}[v|s]) \\
\text{var}(z|b; S) &= \left(\frac{1}{1+\gamma}\right)^2 \text{var}(\text{E}[v|S] - \text{E}[v|s]|b) \\
\text{var}(z; s) &= \left(\frac{\gamma}{1+\gamma}\right)^2 \text{var}(\text{E}[v|S] - \text{E}[v|s]) \\
\text{var}(z|b; s) &= \left(\frac{\gamma}{1+\gamma}\right)^2 \text{var}(\text{E}[v|S] - \text{E}[v|s]|b) \\
R(z, b; s) &= 1 - \frac{\text{var}(z|b; s)}{\text{var}(z; s)} = 1 - \frac{\text{var}(\text{E}[v|S] - \text{E}[v|s]|b)}{\text{var}(\text{E}[v|S] - \text{E}[v|s])} = R(z, b; S)
\end{aligned}$$

The variance terms given aggregate information s are increasing in γ while the variance terms given disaggregate information S are decreasing in γ. Because unconstrained minimization of the equity premium is equivalent to minimizing γ, minimization of the

equity premium maximizes the difference between the variances due to s and S.[11] Also note that the *ex ante* ambiguity $R(z, b; \cdot)$ does not differ between the information types.

Changes in the information set have two counteracting effects. First, the log terms in $g(J; \lambda)$ reflect an informational effect. Second, the term that divides $\mathrm{E}[v-p]^2$ reflects the risk effect of taking large position when the investor has precise information. Direct computation shows that the information advantage, given by sum of the log terms in $g(J; \lambda)$, are increasing in the variances $\mathrm{var}(z|b; J)$ and $\mathrm{var}(\mathrm{E}[z|b]; J)$. The risk term, given by term with $\mathrm{E}[v-p]^2$ in $g(J; \lambda)$, is decreasing in $\mathrm{var}(z|b; J)$. Also note that all three terms are decreasing in $\mathrm{var}(v|J)[1 + (a-1)R(v, b|J)]$, which is higher for disaggregate information S than for any summary signal s that one would release in order to minimize the equity premium.

In order to show that AA will view only summary information when there are sufficiently few AA investors, first note that we can see from the above variance computations that $\mathrm{var}(z; b|S) < \mathrm{var}(z; b|s), \mathrm{var}(\mathrm{E}[z|b]; S) < \mathrm{var}(\mathrm{E}[z|b]; s)$ whenever $\gamma > 1$ and vice versa. The effect of γ on the variances involving z, combined with $\mathrm{var}(v|s)[1 + (a-1)R(v, b|s)] < \mathrm{var}(v|S)[1 + (a-1)R(v, b|S)]$ implies that if the minimizing γ, $\underline{\gamma} = \frac{1-\alpha}{\alpha} \frac{\min_\lambda \mathrm{var}(v|s)[1+(a-1)R(v,b|s)]}{\mathrm{var}(v|S)} > 1$ then the information effect given by the log terms is greater for summary information. The term $\frac{\min_\lambda \mathrm{var}(v|s)[1+(a-1)R(v,b|s)]}{\mathrm{var}(v|S)} > 1$ for all λ because $R(v, b|s) \geq 0$ and $\mathrm{var}(v|s) \geq \mathrm{var}(v|S)$ for all λ. The condition $\underline{\gamma} > 1$ is therefore satisfied so long as the proportion α of AA investors is sufficiently small.

A potentially offsetting effect may arise in the risk effect. If $\mathrm{var}(z|b; s)$ is not so much larger than $\mathrm{var}(z|b; S)$ as to maintain:

$$\mathrm{var}(v|s)[1 + (a-1)R(v, b|s)] + \mathrm{var}(z|b; s) < \mathrm{var}(v|S)[1 + (a-1)R(v, b|S)] + \mathrm{var}(z|b; S)$$

then the risk effect is also greater for summary information and the constraint $g(s; \lambda) >$

[11] Changes in γ due to reducing λ from 1 also increase $\mathrm{var}(\mathrm{E}[v|S] - \mathrm{E}[v|s])$ and $\mathrm{var}(\mathrm{E}[v|S] - \mathrm{E}[v|s]|b)$.

$g(S;\lambda)$ on AA choosing summary information does not bind. On the other hand, if $\text{var}(z|b;s)$ is sufficiently larger than $\text{var}(z|b;S)$ then we may have:

$$\text{var}(v|s)[1+(a-1)R(v,b|s)]+\text{var}(z|b;s) > \text{var}(v|S)[1+(a-1)R(v,b|S)]+\text{var}(z|b;S)$$

even though $\text{var}(v|s)[1+(a-1)R(v,b|s)] < \text{var}(v|S)[1+(a-1)R(v,b|S)]$. In this case the risk effect may dominate the information effect giving $g(s;\lambda) < g(S;\lambda)$ for large per capita supply x because $\text{E}[v-p]$ in the numerator of the risk effect is increasing in per capita supply x.

In the case where the minimizing γ is less than one, then $\text{var}(z|b;S) > \text{var}(z;b|s)$ and $\text{var}(\text{E}[z|b];S) > \text{var}(\text{E}[z|b];s)$. We can only have $\underline{\gamma} < 1$ if the proportion α of AA investors is large enough to compensate for $\frac{\min_\lambda \text{var}(v|s)[1+(a-1)R(v,b|s)]}{\text{var}(v|S)} > 1$.[12] If the differences in variances are sufficiently large to compensate for the lower value $\text{var}(v|s)[1+(a-1)R(v,b|s)] < \text{var}(v|S)[1+(a-1)R(v,b|S)]$ for summary information then the information effect given by the log terms in $g(J;\lambda)$ is larger for disaggregate information S. On the other hand, this case implies:

$$\text{var}(v|S)[1+(a-1)R(v,b|S)]+\text{var}(z|b;S) > \text{var}(v|s)[1+(a-1)R(v,b|s)]+\text{var}(z|b;s)$$

so that the risk effect is smaller for disaggregate information. If per capita supply x is sufficiently large, then it can increase $\text{E}[v-p]$ so that the risk effect will dominate and make AA investors prefer summary information. ■

Proof of Proposition 5

The benefit from aggregation can be seen by setting $\lambda = 1$ so that $\text{E}[v|s] = \text{E}[v|S]$, $\text{var}(v|s) = \text{var}(v|S)$ and $\text{var}(v|s,c) > \text{var}(v|S,c)$ so that $R(v,c|s) = 1 - \frac{\text{var}(v|s,c)}{\text{var}(v|s)} <$

[12] For example, they must comprise at least half of the market.

$R(v, c|S)$. The equity premium is:

$$\mathrm{E}[v-p] = \frac{rx}{\frac{1-\alpha}{\mathrm{var}(v|s)} + \frac{\alpha}{\mathrm{var}(v|s)[1+(a-1)R(v,c|s)]}}$$

so that an aggregate report with $\lambda = 1$ yields a lower equity premium than disaggregate information.

The equity premium is locally decreasing in λ because aggregation reduces investors' ability to infer c and an increase in λ will reveal more information about c. Formally, the same first-order condition is the same as in the proof of Proposition 2:

$$\left(1 - \frac{\alpha(a-1)R(v,c|s)}{1+(a-1)R(v,c|s)}\right)\frac{\partial}{\partial\lambda}\left(\frac{1}{\mathrm{var}(v|s)}\right) + \frac{\alpha}{\mathrm{var}(v|s)}\frac{\partial}{\partial\lambda}\left(\frac{1}{1+(a-1)R(v,c|s)}\right)$$

At $\lambda = 1$, $\frac{\partial}{\partial\lambda}\frac{1}{\mathrm{var}(v|s)} = 0$. Also:

$$\frac{\partial}{\partial\lambda}\frac{1}{1+(a-1)R(v,c|s)} = -\frac{1}{[1+(a-1)R(v,c|s)]^2}\frac{\partial}{\partial\lambda}R(v,c|s)$$
$$= -\frac{1}{\mathrm{var}(v|s)[1+(a-1)R(v,c|s)]^2}\left[\frac{\partial}{\partial\lambda}\left(\frac{\mathrm{cov}(v,c|s)^2}{\mathrm{var}(c|s)}\right) - \frac{\mathrm{var}(v|s)-\mathrm{var}(v|s,c)}{\mathrm{var}(v|s)}\frac{\partial}{\partial\lambda}\left(\frac{1}{\mathrm{var}(v|s)}\right)\right]$$

which, at $\lambda = 1$, equals:

$$\frac{\partial}{\partial\lambda}\left(\frac{1}{1+(a-1)R(v,c|s)}\right) = -\frac{1}{\mathrm{var}(v|s)[1+(a-1)R(v,c|s)]^2}\frac{\partial}{\partial\lambda}\left(\frac{\mathrm{cov}(v,c|s)^2}{\mathrm{var}(c|s)}\right)$$

Direct computation shows that at $\lambda = 1$ the sign of $\frac{\partial}{\partial\lambda}\left(\frac{\mathrm{cov}(v,c|s)^2}{\mathrm{var}(c|s)}\right)$ is the same as the sign of:

$$\frac{\mathrm{cov}(v,c|s)}{\mathrm{var}(c|s)}\frac{\frac{\sigma_{v1}^2}{\sigma_{s1}^2}\sigma_c^2}{\mathrm{var}(s)} - 1 < 0$$

The inequality follows from, at $\lambda = 1$:

$$\frac{\mathrm{cov}(v,c|s)}{\mathrm{var}(c|s)}\frac{\frac{\sigma_{v1}^2}{\sigma_{s1}^2}\sigma_c^2}{\mathrm{var}(s)} = \frac{1-\frac{\sigma_{v1}^2}{\sigma_{s1}^2}}{1-\frac{\sigma_c^2}{\sigma_{s1}^2}+\frac{\sigma_{v2}^4/\sigma_{s2}^2}{\sigma_{v1}^4/\sigma_{s1}^2}}\frac{\sigma_c^2}{\sigma_{v1}^2} < 1$$

where the inequality follows from $\mathrm{var}(s) > \sigma_c^2$. The average precision is therefore increasing in λ at $\lambda = 1$ so that the equity premium is decreasing in λ. Reductions in λ

increase the equity premium because they both increase posterior variance $\text{var}(v|s)$ and increase ambiguity $R(v, c|s)$. ■

Notes

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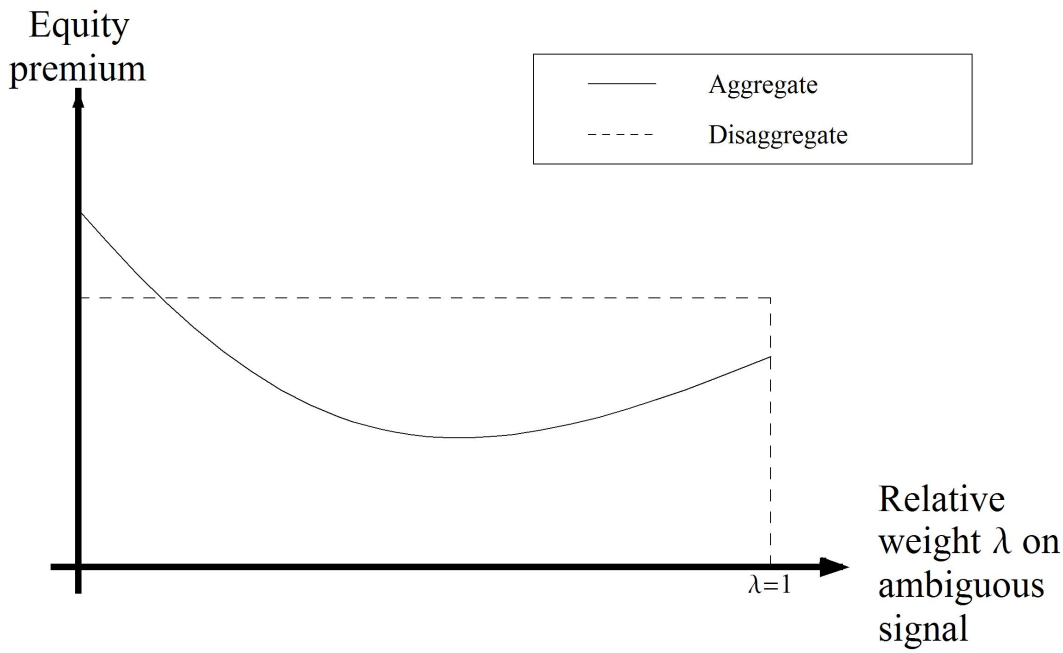
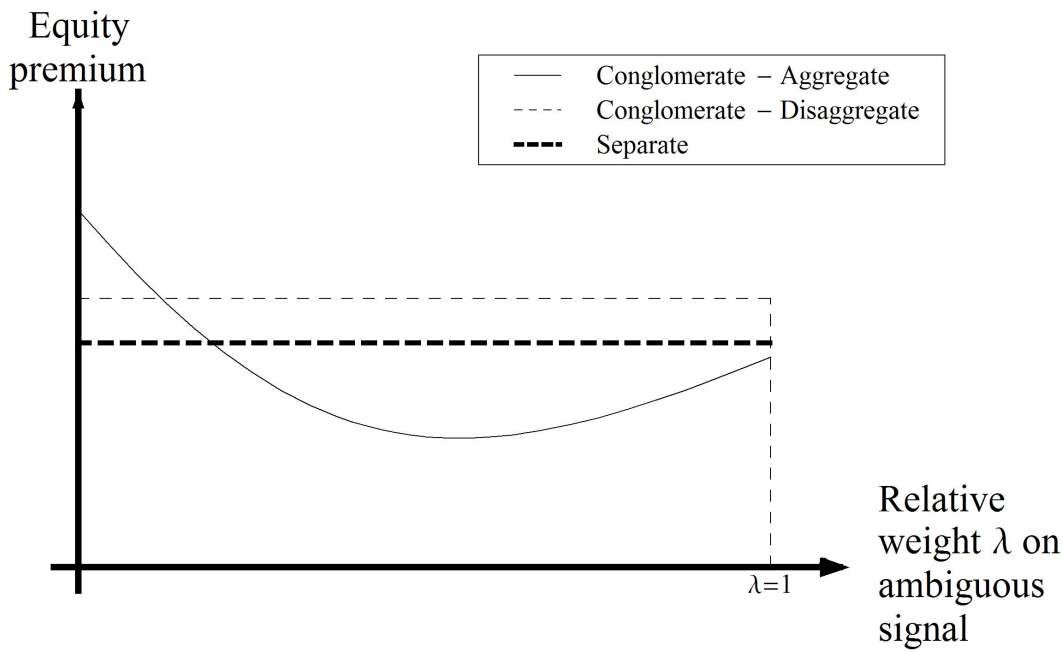
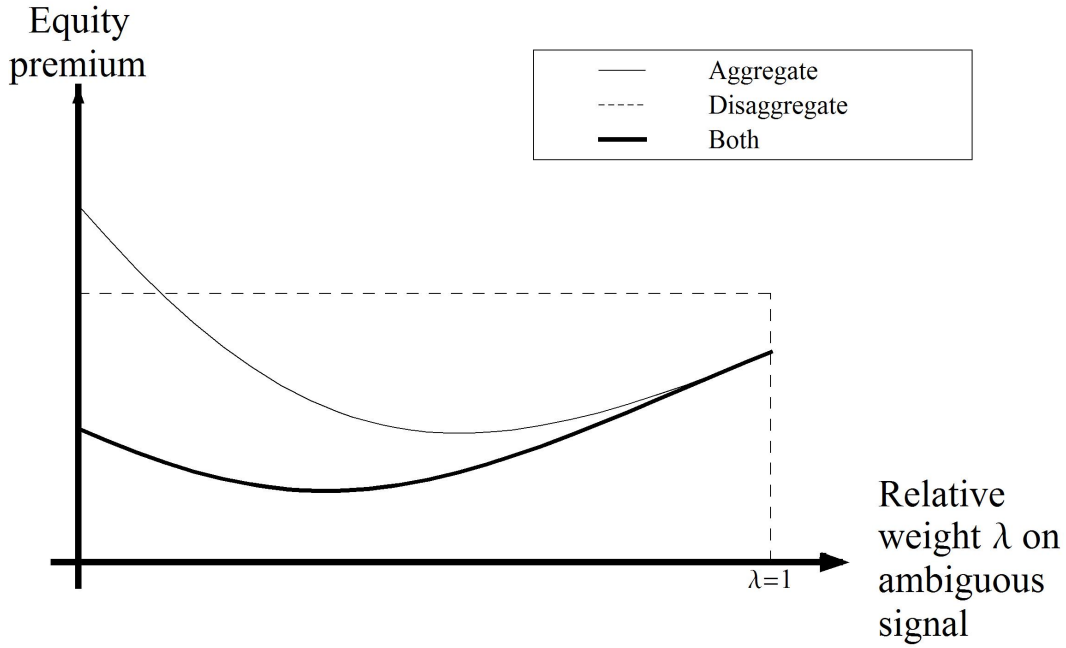
Figures



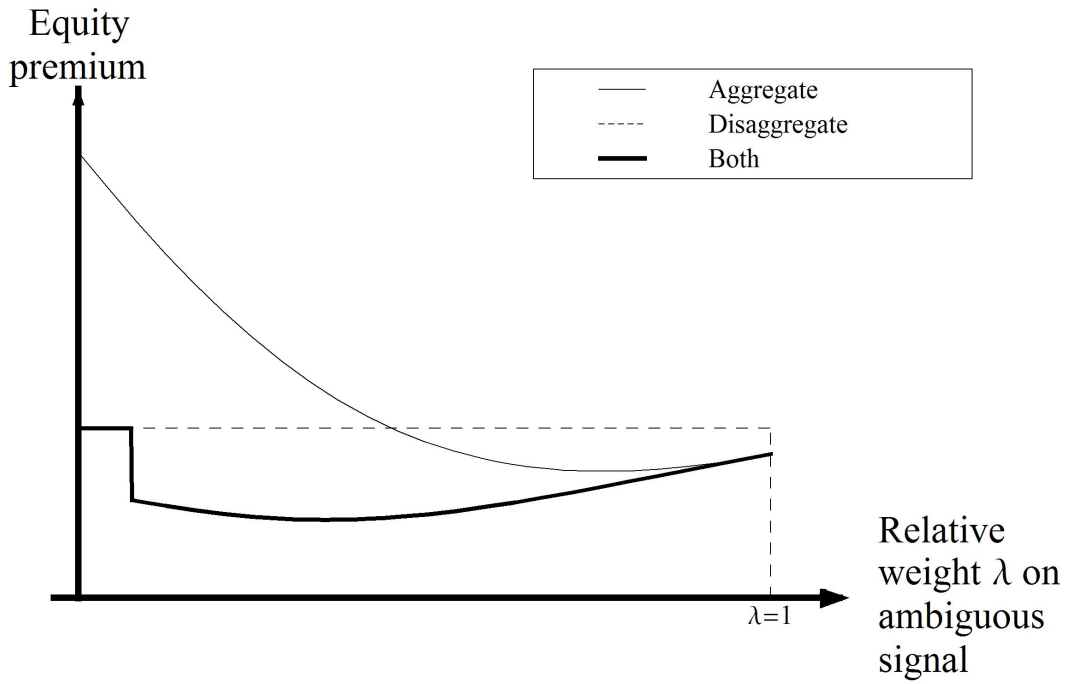
(Figure 1)


Equity premium
Conglomerate – Aggregate
Conglomerate – Disaggregate
Separate
Relative weight λ on ambiguous signal
λ=1

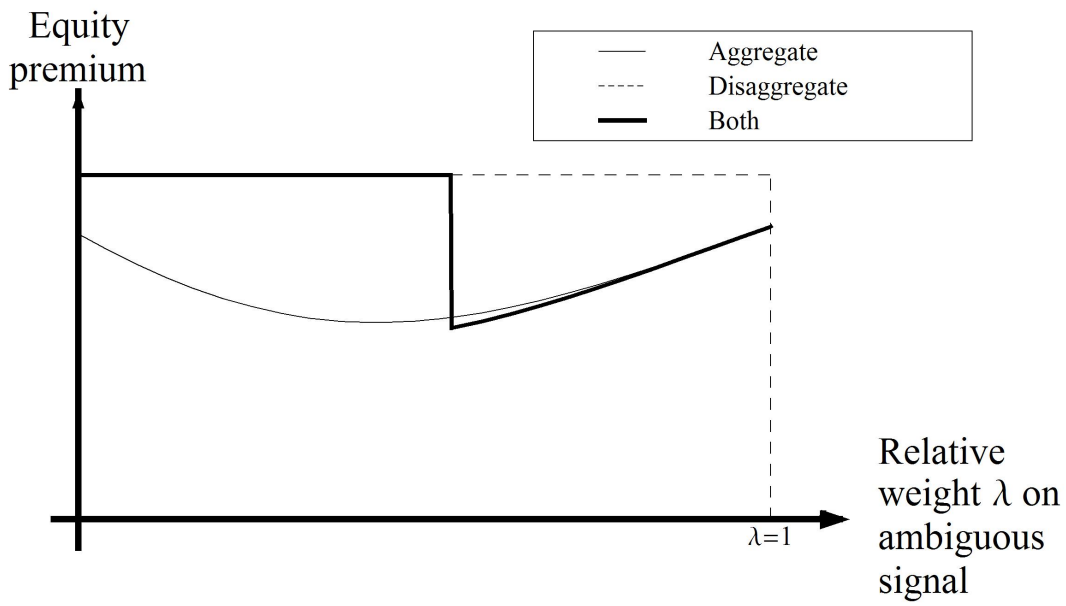
(Figure 2)


Equity premium
Aggregate
Disaggregate
Both
λ=1
Relative weight λ on ambiguous signal

(Figure 3)


Equity premium
Aggregate
Disaggregate
Both
Relative weight λ on ambiguous signal
λ=1

(Figure 4)


Equity premium
Aggregate
Disaggregate
Both
Relative weight λ on ambiguous signal
λ=1

(Figure 5)