Municipal Debt and Marginal Tax Rates: Is There a Tax Premium in Asset Prices?

FRANCIS A. LONGSTAFF*

ABSTRACT

We study the marginal tax rate incorporated into short-term municipal rates using municipal swap market data. Using an affine model, we identify the marginal tax rate and the credit/liquidity spread in 1-week tax-exempt rates, as well as their associated risk premia. The marginal tax rate averages 38.0% and is related to stock, bond, and commodity returns. The tax risk premium is negative, consistent with the strong countercyclical nature of after-tax fixed-income cash flows. These results demonstrate that tax risk is a systematic asset pricing factor and help resolve the muni-bond puzzle.

ONE OF THE MOST FUNDAMENTAL issues in financial economics is the question of how taxes affect security values. This important topic has been the focus of an extensive literature that now dates back nearly a century. Despite the many important contributions in this area, however, there is still much about the effects of taxation on investment values that is not yet fully understood.

The challenge is particularly evident in studying municipal debt markets. Many researchers document that the ratio of municipal bond yields to Treasury or corporate bond yields appears to imply marginal tax rates that are much smaller than would be expected given federal income tax rates. This perplexing relation between taxable and tax-exempt yields is often termed the muni-bond puzzle.[1]

This paper presents a new and fundamentally different approach to estimating the marginal tax rate τ_t incorporated into tax-exempt municipal debt rates. In doing so, we take advantage of an extensive new data set that

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[1] Key papers discussing the muni-bond puzzle include Trzcinka (1982), Livingston (1982), Arak and Gentner (1983), Stock and Schrems (1984), Ang, Peterson, and Peterson (1985), Buser and Hess (1986), Kochin and Parks (1988), and Green and Oedegaard (1997). A number of papers consider whether the puzzle can be explained by municipal credit risk, including Kidwell and Trzcinka (1982), Skelton (1983), Chalmers (1998), and Neis (2006). In an important paper, Green (1993) develops a simple model that takes into account the asymmetries between the taxation of capital gains and losses as well as the treatment of coupon income and shows that the resulting effect of these tax asymmetries may help explain the muni-bond puzzle.

includes both the yields of 1-week tax-exempt municipal debt as well as the term structure of rates for municipal swaps exchanging this tax-exempt yield for a percentage of the London Interbank Offering Rate (LIBOR). Using these data, we estimate an affine term structure model of the municipal swap curve via maximum likelihood and obtain estimates of both the marginal tax rate and the credit/liquidity spread embedded in municipal yields.

This new approach has a number of important advantages. First, by estimating the marginal tax rate from 1-week municipal yields, our results are free of the types of tax asymmetry or tax trading complications that Green (1993), Constantinides and Ingersoll (1982), and others show may affect yields on longer-term municipal bonds. Second, this approach allows us to estimate the market risk premia incorporated into the term structure as compensation to investors for bearing the risk of time variation in the marginal tax rate.[2] Thus, we can directly evaluate whether there is a tax premium embedded in asset prices stemming from tax risk. Third, our approach allows us to study directly how changes in marginal tax rates are related to financial and macroeconomic shocks.

The empirical results are very striking. We find that the average marginal tax rate during the 2001 to 2009 sample period is 38.0%. This value is very close to both the maximum Federal individual income tax rates during the sample period (39.1% during 2001, 38.6% during 2002, and 35.0% during the remainder of the sample period) and the maximum corporate income tax rate of 39.0% during the sample period. The estimated marginal tax rate, however, varies substantially over time and ranges from roughly 8% to 55% during the sample period. These estimates of the marginal tax rate are also consistent with the higher marginal rates identified by Ang, Bhansali, and Xing (2010) in a recent paper studying the cross-sectional pricing of discount municipal bonds. It is important to acknowledge the usual caveat, however, that our results are all conditional on the maintained assumption that our affine model is correctly specified.

The estimated values of the marginal tax rate are also significantly larger than those obtained by a naive comparison of the short-term tax-exempt rate to the corresponding fully taxable riskless rate. For example, the short-term tax-exempt rate has been higher than the riskless rate ever since the Lehman default in September 2008. A naive comparison might interpret this as evidence of a "negative" marginal tax rate. Intuitively, the reason our estimates of the marginal tax rate are higher is that we explicitly allow for the possibility of a credit/liquidity spread in short-term tax-exempt municipal yields. The empirical results show that there is a substantial credit/liquidity spread in these short-term tax-exempt yields. We find that the average value of this spread during the sample period is 56 basis points. The estimated spread,

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[2] Time variation in the marginal tax rate can occur as the marginal investor's income stream changes and is taxed via the progressive income tax schedule, as the marginal investor changes because of liquidity shocks or other reasons, or as tax laws change and affect the value of tax exemption. I am indebted to the referee for these insights.

however, increased dramatically during the early stages of the subprime credit crisis as monoline municipal bond insurers suffered major credit-related losses and auction failures in the short-term auction rate security markets became widespread.[3]

To explore how the marginal tax rate evolves over time, we regress changes in the marginal tax rate on a number of variables proxying for changes in investors' personal income and in the macroeconomic environment. We find that the marginal tax rate is significantly positively related to returns on the S&P 500 and U.S. Treasury bonds, and significantly negatively related to returns on an index of commodities. These results provide intriguing insights into the nature of the marginal investor in the municipal bond markets.

One of the most surprising empirical results is that the market risk premium for the marginal tax rate is negative in sign. In particular, the long-run expected marginal tax rate is 38.2% under the physical measure, but only 27.2% under the risk-neutral pricing measure. This implies that the market values a taxable bond coupon payment at a higher value than if there were no tax risk. To understand the intuition for this negative risk premium, observe that marginal tax rates are very procyclical because of the progressivity of the Federal income tax system. In good states of the economy, taxable income increases and investors move into higher marginal tax brackets, while the opposite is true in bad states of the economy. This means that $c(1 - \tau_t)$, where c is the coupon on a bond, is actually highly countercyclical. Thus, the risk premium for this cash flow can be negative because of its "negative consumption beta."

These results are important for a number of reasons. First, they provide clear evidence that taxation has first-order effects on the valuation of securities. Second, the marginal tax rate incorporated into the short-term tax-exempt rate makes sense from an economic perspective; the estimated marginal tax rate of 38.0% closely matches the top income tax rate during the sample period. Third, these results offer a possible resolution of the long-standing muni-bond puzzle that has perplexed financial researchers for nearly 30 years. Fourth, the evidence of a significant negative tax risk premium suggests that the market rationally takes into account the countercyclical nature of after-tax cash flows. For example, our results suggest that the negative risk premium may reduce the spread between longer-term Treasury and tax-exempt municipal yields by 50 basis points or more during the sample period. Finally, the evidence of a significant tax risk premium in the bond market raises the strong possibility that tax risk is a systematic factor that might affect asset prices in other markets such as the real estate, commodity, and stock markets.[4]

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[3] In an important recent paper, McConnell and Sarreto (2010) study the events in the auction rate markets.

[4] Other important research on municipal debt markets includes Yawitz, Maloney, and Ederington (1985), Green (1993), Green and Oedegaard (1997), Chalmers (1998), Downing and Zhang (2004), Nanda and Singh (2004), Green (2007), Green, Hollifield, and Schürhoff (2007a, 2007b), Green, Li, and Schürhoff (2007), Wang, Wu, and Zhang (2008), and Ang et al. (2010). Important papers addressing the impact of taxation on bond prices and trading strategies include Livingston (1979), Constantinides and Ingersoll (1982), Schaefer (1982), Litzenberger and Rolfo

The remainder of the paper is organized as follows. Section I provides an introduction to the municipal swap market. Section II describes the data. Section III presents the affine model of the term structure of municipal swap rates. Section IV describes the maximum likelihood estimation of the model. Section V presents the empirical results. Section VI discusses the implications of the results for the muni-bond puzzle. Section VII summarizes the results and presents concluding remarks.

I. The Municipal Swap Market

In this section, we provide a brief introduction to the municipal swap market. Because swaps in this market are tied to the Securities Industry and Financial Markets Association Municipal Swap Index (MSI, formerly known as the Bond Market Association (BMA) index), we first explain how this index is constructed. We then describe the various types of municipal swap contracts available in the over-the-counter financial markets.

A. The Municipal Swap Index

The MSI is a high-grade market index reflecting the yields on 7-day-resettable tax-exempt variable rate demand obligations (VRDOs). Thus, the MSI is effectively a 1-week tax-exempt rate. The index is produced by Municipal Market Data, which maintains an extensive database containing information for more than 15,000 active VRDOs. Municipal Market Data is a subsidiary of Thompson Financial Services.[5]

VRDOs are long-term tax-exempt floating rate notes issued by municipalities. Typically, the floating rate on the notes is reset at a weekly frequency, although both shorter and longer frequencies occur in the markets. Although the maturities of VRDOs are often 30 to 40 years, they are effectively shorter-term securities because they can be put back or tendered to the investment dealer or remarketing agent on a schedule coinciding with the weekly yield reset.

The remarketing agent, which is often the financial institution that originally issued the VRDO for the municipality, has two ongoing roles. First, the remarketing agent functions as a broker in that if VRDOs are tendered at the weekly yield reset, the remarketing agent attempts to find a buyer for the tendered VRDOs. Second, as part of this process, the remarketing agent sets the weekly yield to whatever level is required for the market to clear the tendered VRDOs (and which may also incorporate market information about market clearing rates for similar VRDO issues). In this respect, VRDOs have a number of features in common with auction rate securities, which also reset

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(1984), Jordan (1984), Dybvig and Ross (1986), Dammon and Green (1987), Graham (2003), and Dammon, Spatt, and Zhang (2004).

[5] This section is based on the description of the market provided by the Securities Industry and Financial Markets Association (www.sifma.org/capital_markets/swapindex.shtml).

frequently via a market clearing mechanism. Note, however, that the weekly reset for a VRDO is determined by the remarketing agent while the weekly reset for an auction rate note is determined via a constrained Dutch auction (which may fail in that the maximum allowable yield is below the rate needed to clear the market). VRDOs are typically issued at par. When they are put back to the remarketing agent, an investor receives par plus accrued interest.

Criscuolo and Faloon (2007) estimate that 70% of VRDOs are held by money market funds, 15% by corporations, 7% by bond funds, and 8% by trust departments. Thus, the marginal tax rate applied to interest received by a VRDO investor is likely to reflect that of an individual. However, it is also possible that the marginal tax rate could reflect a marginal corporate tax rate or the marginal rate faced by a taxable trust. The VRDO market presents a large and rapidly growing segment of the $2.6 trillion municipal debt market. In particular, the Securities Industry and Financial Markets Association reports that $63.3 billion of variable rate municipal bond obligations were issued during 2007, $109.2 billion were issued during 2008, and $32.0 billion were issued through October of 2009.

There are a number of criteria that a VRDO must satisfy for its yield to be included in the MSI. First, the VRDO must have a weekly reset, effective on Wednesday. Second, the VRDO must not be subject to alternative minimum tax. Third, the VRDO must have an outstanding amount of at least $10 million . Fourth, the VRDO must have the highest short-term rating, which is VMIG1 by Moody's or A-1+ by Standard and Poor's. Historically, a municipal issuer of VRDOs would need to obtain some sort of credit enhancement (such as a letter of credit from a highly rated bank) to obtain the highest short-term rating.[6] Fifth, the VRDO must pay interest on a monthly basis. Finally, only one quote per obligor per remarketing agent can be included in the MSI. The MSI can include issues from any state. The MSI is calculated weekly on Wednesday and officially released on Thursday.[7]

The underlying data for the index come from Municipal Market Data's Variable Rate Demand Note Network. This network collects market data from over 80 remarketing agents who download daily rate change information to Municipal Market Data's network. The actual number of VRDOs included in the weekly index fluctuates, but is estimated to include roughly 650 issues in any given week.

B. *The Municipal Swap Market*

The primary type of municipal swap contract available in the financial markets is the percentage-of-LIBOR contract. This contract is very similar to a

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[6] For a discussion of the role of credit enhancement in VRDO issuance, see Criscuolo and Faloon (2007).

[7] Market participants, however, are easily able to infer the index value by the end of Wednesday because the VRDO resets are posted throughout the day and remarketing agents provide transparency.

standard floating-for-floating basis swap contract. Specifically, one counterparty to the municipal swap contract agrees to pay the other the numerical value of the MSI at some frequency, say, monthly. In exchange, the other counterparty commits to pay the first counterparty a fixed percentage P of the numerical value of the LIBOR rate. Both payments are made relative to a specific notional amount. For example, if payments are exchanged monthly, the first counterparty would pay the second the average value of the 1-week MSI rate during the month on the swap notional amount. The second counterparty would pay the first P times the 1-month LIBOR rate set at the beginning of the month on the swap notional.

It is important to stress that the cash flows from both the MSI and LIBOR legs of a municipal swap contract will typically be fully taxable to the swap counterparties. The tax-exempt status of the interest from the VRDOs included in the MSI does not carry over to financial contracts with cash flows that are tied to the numerical value of the index. Thus, the marginal tax rate enters into the pricing of a municipal swap only through its effect on the 1-week MSI rate. It is this feature that enables us to abstract completely from the types of tax asymmetries that affect the valuation of longer-maturity municipal bonds as described by Green (1993). Furthermore, it allows us to model and price municipal swap contracts using a standard term-structure framework.[8]

In this market, municipal swaps are quoted in terms of the percentage P required to make both legs of the swap have equal value. Intuitively, the reason for the percentage P is easily seen. Because the MSI is a tax-exempt rate, its numerical value will likely be substantially lower than the numerical value of the fully taxable LIBOR rate. Thus, the counterparty paying LIBOR would generally not be willing to pay LIBOR flat in exchange for the MSI rate. Typically, the market clearing value of P is significantly lower than 100%. Like conventional interest rate swaps, municipal swaps are traded in the OTC markets. Market quotations for municipal swaps with 1-, 2-, 3-, 4-, 5-, 7-, 10-, 12-, 15-, 20-, 25-, and 30-year maturities are currently readily available in the Bloomberg system and from other market data sources.

A popular alternative type of municipal swap contract is given by combining a percentage-of-LIBOR contract with a standard fixed-for-floating LIBOR interest rate swap. To illustrate, imagine that municipal swap market participants are willing to pay 70% of LIBOR to receive the MSI rate over the next 10 years. Furthermore, imagine that swap market participants are also willing to pay LIBOR to receive a fixed rate of 6% over the next 10 years in a standard swap. Then a simple arbitrage argument implies that market participants should be willing to pay a fixed rate of $0.70 \times 0.0600 = 0.0420$ to receive the MSI rate over the next 10 years. Thus, there is a simple equivalence between percentage-of-LIBOR swaps and these fixed-for-MSI-rate swaps.

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[8] For example, this allows us to abstract from the issues surrounding the existence of a unique pricing measure in a market populated with agents who face different marginal tax rates. For a discussion of these issues, see Ross (1985, 1987) and Dybvig and Ross (1986).

Table I
Summary Statistics for the Municipal Index and Municipal Swaps

This table reports summary statistics for the indicated variables. The 1-week MSI rate is expressed as a percentage. The municipal swap rates are expressed as percentages of LIBOR. The sample consists of weekly (Wednesday) observations for the August 1, 2001 to October 7, 2009 period.

Index	Mean	Standard Deviation	Minimum	Median	Maximum	Serial Correlation	N
1-week MSI rate	2.017	1.157	0.240	1.675	7.960	0.963	428
1-year municipal swap	76.769	8.544	66.500	73.380	104.500	0.968	428
2-year municipal swap	75.876	6.863	67.250	73.563	98.000	0.964	428
3-year municipal swap	75.583	6.093	67.625	73.750	98.000	0.961	428
4-year municipal swap	75.627	5.727	68.125	74.380	98.000	0.963	428
5-year municipal swap	75.844	5.584	68.500	74.880	98.000	0.969	428
7-year municipal swap	76.392	5.310	69.563	75.750	98.500	0.962	428
10-year municipal swap	77.239	5.154	70.563	76.630	97.750	0.973	428
12-year municipal swap	77.901	5.314	71.125	77.380	101.750	0.971	428
15-year municipal swap	78.744	5.488	71.813	78.250	104.000	0.975	428
20-year municipal swap	79.820	5.672	72.813	79.130	106.000	0.975	428

II. The Data

The data for the study include the 1-week tax-exempt MSI rate; market rates for percentage-of-LIBOR municipal swaps; as well as Treasury, repo, and swap market rates. The different categories of data are described individually below.

A. *Municipal Swap Index Data*

We obtain weekly observations of the 1-week tax-exempt MSI rate directly from the Securities Industry and Financial Markets Association website for the period from August 1, 2001 to October 7, 2009; see http://archives.sifma.org/swapdata.html. We choose this time period because municipal swap data are only available for this horizon. The time period provides a total of 428 weekly observations. The vast majority of these weekly observations are for Wednesday.[9] Table I provides summary statistics for the data.

B. *Treasury Repo Rate Data*

In solving for the marginal tax rate incorporated into the 1-week tax-exempt MSI, it will be helpful to have a fully taxable 1-week riskless rate to use as a benchmark. While 1-, 3-, and 6-month Treasury bill yield data are readily available in the financial markets, data for shorter maturities are difficult to obtain and are likely to be less reliable. To circumvent this difficulty, we use the 1-week Treasury repo rate as a proxy for the 1-week riskless

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[9] In a few instances, the MSI is reported for an alternative day of the week such as Thursday.

rate.[10] We obtain midmarket data for the 1-week Treasury repo data from the Bloomberg system for the same dates as the MSI data.

There are a number of justifications for the use of the Treasury repo rate as a proxy for the riskless rate. First, as argued in Longstaff (2000), repo rates reflect the actual cost of capital to government bond dealers for their positions in Treasury bonds. Second, Treasury repo contracts are fully collateralized, or more generally overcollateralized, by the underlying Treasury bonds associated with the transaction. Thus, there is little default risk associated with a short-term government repo contract. Third, as Duffee (1998) and others discuss, Treasury bill yields display a significant amount of idiosyncratic variation that may not be related to movements in the economic riskless rate. For example, Longstaff (2004) shows that Treasury yields can be affected by flights to quality or flights to liquidity.

Finally, Treasury securities may not actually be default free. In particular, the 10-year credit default swap premium for the U.S. Treasury has been quoted at levels as high as 100 basis points.[11]

To provide some preliminary perspective on the relation between taxable and tax-exempt rates, Figure 1 plots the MSI and repo rates in the upper panel and the difference between the repo rate and the MSI rate in the lower panel. As illustrated, the relation between the taxable and tax-exempt rates is fairly complex. During the sample period, the average MSI rate is 84.1% of the average repo rate. At first glance, this seems to suggest that the average marginal tax rate is only $100 - 84.1 = 15.9\%$. In reality, however, this simplistic measure of the marginal tax rate fails to take into account the credit/liquidity risk incorporated into the tax-exempt curve. While the MSI rate is based on yields for VRDOs with the highest short-term credit rating, the MSI rate may still reflect the default risk inherent in the municipal bond issuers (as well as the illiquidity of the securities they offer) and/or financial institutions providing credit enhancement for the VRDOs. Thus, if the MSI rate contains a credit risk spread, the simple ratio of the MSI rate to the repo rate would give a downward-biased measure of the marginal tax rate.

In fact, Figure 1 shows that the tax-exempt rate has frequently exceeded the repo rate. For example, the MSI rate on September 24, 2008 (the week after the Lehman default) was 7.96% while the repo rate was only 1.75%. Thus, the premium of the tax-exempt rate over the taxable rate was very likely due to the perceived increase in systemic credit risk in the debt markets, or, equivalently, the concurrent flight to quality that occurred in the Treasury markets. A key advantage of the empirical approach we adopt in this paper is that it allows us to identify the marginal tax rate separately from the credit/illiquidity spread incorporated into the tax-exempt curve.

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[10] The empirical results of this study are virtually the same when the 1-month Treasury bill rate is used as a proxy for the 1-week riskless rate.

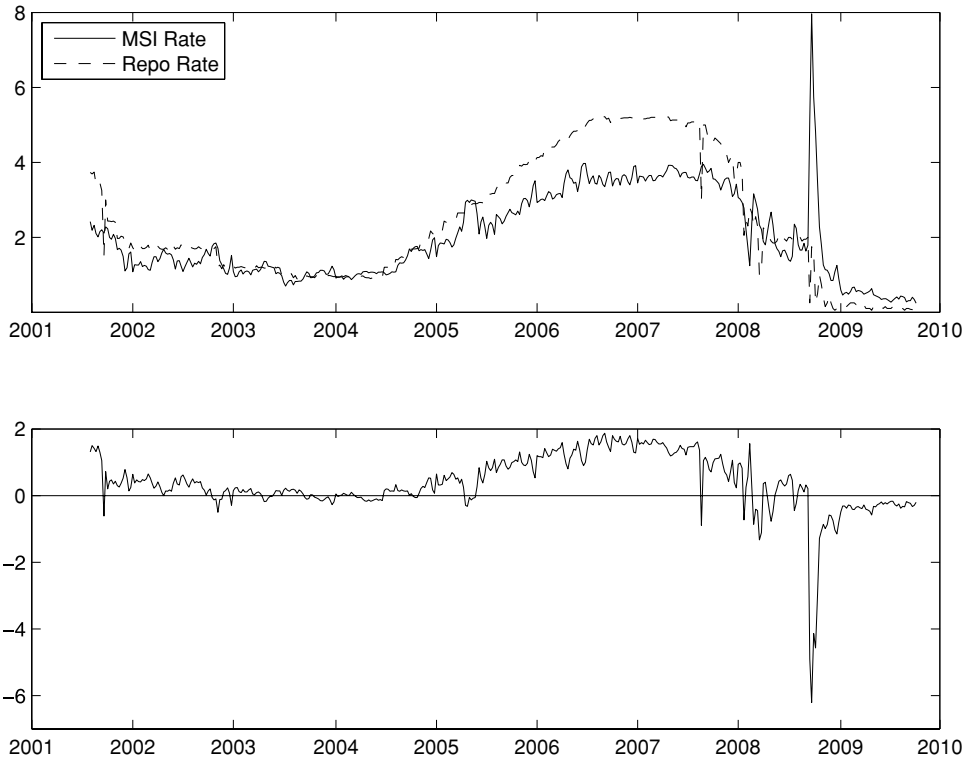[11] Based on intraday Bloomberg quotations on February 23, 2009.

Figure 1. The MSI and repo rates. The upper panel plots the MSI rate and the repo rate. The lower panel plots the difference between the repo rate and the MSI rate.

C. Municipal Swap Data

We obtain midmarket rates for the term structure of percentage-of-LIBOR municipal swaps from the Bloomberg system for the same dates as described above. Recall that these municipal swap rates are quoted as percentages.[12]

Table I provides summary statistics for these municipal swap rates. As shown, the average percentage swap rate is not monotonic in the maturity of the swap. The average percentage is 76.77 for the 1-year swap, declines to 75.58% for the 3-year swap, and then increases to a maximum of 79.82% for the 20-year swap. Although the average percentage swap rates are not monotonic, we observe that there are many dates during the sample period when the percentage swap rates are either monotonically increasing or decreasing with swap maturity. Table I also shows that there is considerable time-series variation in the percentage swap rates. In particular, the standard deviation of the percentage swap rate ranges from 8.54% for the 1-year swap to 5.15% for the

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[12] We do not include the 25- and 30-year maturities in the study because data for these swaps are not available for much of the sample period.

10-year swap and 5.67% for the 20-year swap. Thus, longer-term percentage swap rates are less volatile than are shorter-maturity percentage swap rates. This suggests the possibility that there could be a mean-reverting nature to the relation between tax-exempt and taxable rates.

D. *Treasury Term Structure and Interest Rate Swap Data*

In the analysis later in the paper, we discount cash flows using a riskless discount function bootstrapped from the Treasury yield curve. Specifically, we obtain constant maturity Treasury (CMT) rates from the Federal Reserve Board's historical H.15 data for 1-month, 3-month, 6-month, 1-year, 2-year, 3-year, 5-year, 7-year, 10-year, and 20-year maturities for the same dates as for the other time series. Using a standard cubic spline algorithm, we then solve for the riskless discount function for weekly maturities up to 20 years for each date during the sample period. This algorithm is described in Longstaff, Mithal, and Neis (2005).

We also use midmarket data for conventional fixed-for-floating LIBOR interest rate swaps in the analysis. In particular, we collect midmarket rates for interest rate swaps from the Bloomberg system for the same maturities and dates as above.[13]

III. The Marginal Tax Rate Model

In this section, we describe the approach used to model the marginal tax rate incorporated into the tax-exempt MSI rate. In doing so, it is important to allow for the possibility that the MSI rate may include a spread reflecting the higher credit risk of even highly rated VRDOs relative to the riskless rate. Our approach will also address the possibility that VRDO yields may include a component reflecting the lower liquidity of municipal securities relative to Treasury securities.

Let M_t denote the tax-exempt 1-week MSI rate. We assume that this rate can be expressed in the following way,

$$M_t = r_t (1 - \tau_t) + \lambda_t, \tag{1}$$

where r_t is the riskless pre-tax interest rate. In this expression, τ_t designates the marginal tax rate of the marginal investor in VRDOs, and λ_t is a spread reflecting either the credit risk of the tax-exempt index, the illiquidity of the VRDOs incorporated in the index, or some combination of both. Note that inherent in this model specification are the assumptions that marginal tax rates

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[13] These swap data represent the market rate for exchanging fixed coupons for 3-month LIBOR. In contrast, the LIBOR leg of the municipal swaps involves 1-month LIBOR. During most of the sample period, however, the midmarket value of the basis swap for exchanging 1-month LIBOR for 3-month LIBOR is within a fraction of a basis point of zero. Thus, there is little or no loss of accuracy in treating the LIBOR legs of the municipal and conventional interest rate swaps as if they were on the same underlying LIBOR index.

affect income multiplicatively and that the credit/liquidity component is not multiplicative in r_t. Both of these assumptions are standard in the literature. The second assumption, however, is what allows us to identify the marginal tax rate and the credit/liquidity spread separately. Thus, it is important to acknowledge that our estimates of these two variables are not model-free; the estimates of the marginal tax rate and the credit/liquidity spread are conditional on our model specification. An implication of this, of course, is that if we were to use a different model specification, then our results might be different. For example, if we were to assume that the credit/liquidity spread were of the form $r_t \lambda_t$, then we might not be able to separately identify τ_t and λ_t without additional assumptions.[14]

In light of this, it is important to explain why we choose the model in equation (1) rather than an alternative model in which the credit spread is of the form $r_t \lambda_t$. First, our specification of the credit spread as an additive process is a standard one in the literature. Examples of this modeling approach include Duffie and Singleton (1997, 1999), Duffee (1999), Duffie, Pedersen, and Singleton (2003), Driessen (2005), Longstaff et al. (2005), Pan and Singleton (2008), and many others. Second, to our knowledge, the only paper that considers a credit spread specification that is multiplicative in r_t is Liu, Longstaff, and Mandell (2006). Applying their model to the interest rate swap curve and estimating it via maximum likelihood, they find that the portion of the credit spread that is proportional to r_t is not statistically significant, while the opposite is true for the additive component (see Liu et al. (2006, p. 2352)). Finally, the empirical literature provides little support for the view that the credit spread is proportional to r_t. In particular, Giesecke et al. (2010) find that the riskless rate has no relation to corporate bond default rates over the 1866 to 2008 period. Similar results are documented by Collin-Dufresne, Goldstein, and Martin (2001) and many others.

We also assume that the taxable 1-month LIBOR rate L_t can be expressed as

$$L_t = r_t + \mu_t, \tag{2}$$

where μ_t also represents a credit/liquidity spread incorporated into the LIBOR rate. Furthermore, we make the simplifying assumption that r_t is uncorrelated with τ_t and λ_t. This assumption has little effect on the results and could easily be relaxed. By making this assumption, however, we avoid the need to specify the dynamics of the riskless rate r_t and the LIBOR credit/liquidity spread μ_t.

The dynamics of the VRDO credit/liquidity spread λ_t are given by

$$d\lambda_t = (a - b\,\lambda_t)\,dt + c\,dZ_{\lambda t}, \tag{3}$$

$$d\lambda_t = (\hat{a} - \hat{b}\,\lambda_t)\,dt + c\,d\hat{Z}_{\lambda t}, \tag{4}$$

under the risk-neutral Q measure and the actual P measure, respectively. Thus, we allow both of the constant parameters in the drift of the above processes to

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[14] I am grateful to the referee for these insights.

differ between the risk-neutral and actual measures. This simple but general specification has the advantage of allowing the market price of risk for λ_t to be time varying. The processes $Z_{\lambda t}$ and $\hat{Z}_{\lambda t}$ are standard Brownian motions. These dynamics allow the credit/liquidity spread to be mean reverting and to take on negative values. This latter feature is important because it is at least theoretically possible that under some extreme scenarios, the liquidity of the highest-rated municipal securities might equal or even exceed that of Treasury securities; these dynamics allow us to address this possibility.

Similarly, the dynamics of the marginal tax rate τ_t are assumed to follow

$$d\tau_t \; = \; (\alpha \; - \; \beta \; \tau_t) \, dt \; + \; \sigma \; dZ_{\tau t}, \tag{5}$$

$$d\tau_t \; = \; (\hat{\alpha} \; - \; \hat{\beta} \; \tau_t) \, dt \; + \; \sigma \; d\hat{Z}_{\tau t}, \tag{6}$$

under the Q and P measures, respectively. These dynamics again imply that τ_t follows a mean-reverting Gaussian or Ornstein-Uhlenbeck process.[15] The motivation for allowing for mean reversion in these dynamics comes from the observation that the volatility of longer-term municipal swap rates is a decreasing function of maturity. The motivation for assuming Gaussian dynamics, which can allow τ_t to take on negative values, is to allow for the fact that an investor's marginal tax rate can actually be negative under some circumstances.[16]

Turning now to the valuation of percentage-of-LIBOR municipal swap contracts, observe that all of the cash flows associated with the swap will typically be taxable; the tax-exempt status of the VRDOs underlying the MSI rate does not transfer to swaps even though these swaps have cash flows tied to the tax-exempt rate. Thus, in discounting swap cash flows, it is appropriate to use the usual pre-tax riskless discount function applied in standard valuation problems in finance.

To keep the notation as simple as possible, we will generally omit time subscripts for current variables and assume that we are valuing contracts as of time zero. Let $D(T)$ denote the current value of a riskless zero-coupon bond with a maturity of T years.[17] Under the risk-neutral pricing measure, the present value of the floating MSI leg of a percentage-of-LIBOR municipal swap contract with maturity T can be expressed formally as

$$E_Q \left[\int_0^T \exp \left(- \int_0^t r_s \, ds \right) \left(r_t \, (\, 1 \, - \, \tau_t \,) \, + \, \lambda_t \, \right) dt \right]. \tag{7}$$

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[15] Practitioners are cognizant of the fact that tax rate and credit risk changes can affect the valuation of securities and contracts. For example, in a recent National Association of Bond Lawyers conference presentation, John Lutz, Doug Youngman, and Jeffrey Klein stated "Using a percentage of LIBOR leaves the VRDN issuer exposed to changes in tax rates, credit enhancement quality, and remarketer performance" (see www.nabl.org). I am grateful to the referee for this insight.

[16] Feldstein and Samwick (1992) discuss the situations under which negative marginal tax rates occur.

[17] Throughout this section, we assume that swap cash flows are paid continuously. In actuality, however, cash flows from swaps are paid discretely. This assumption greatly simplifies the exposition and has virtually no effect on the empirical results.

Similarly, the present value of the LIBOR leg of this swap can be expressed as

$$P(T)\, E_Q\left[\int_0^T \exp\left(-\int_0^t r_s\, ds\right)(r_t + \mu_t)\, dt\right],\tag{8}$$

where $P(T)$ designates the fraction of LIBOR paid in this percentage-of-LIBOR swap.

This latter expression depends on the LIBOR credit/liquidity spread μ_t. This spread, however, can be substituted out of the model by noting that in a standard interest rate swap, the present value of receiving 100% of LIBOR is just the present value of receiving the current market swap rate, which we designate $S(T)$. Specifically, the present value of the LIBOR leg in a standard interest rate swap,

$$E_Q\left[\int_0^T \exp\left(-\int_0^t r_s\, ds\right)(r_t + \mu_t)\, dt\right],\tag{9}$$

equals the present value of receiving an annuity of $S(T)$ from the fixed leg of the swap,

$$S(T)\, E_Q\left[\int_0^T \exp\left(-\int_0^t r_s\, ds\right) dt\right],\tag{10}$$

which can also be expressed as

$$S(T) \int_0^T D(t)\, dt.\tag{11}$$

Combining these results implies that the present value of the percentage-of-LIBOR leg of the municipal swap is given by

$$P(T)\, S(T) \int_0^T D(t)\, dt.\tag{12}$$

To solve for the percentage swap rate $P(T)$, we observe that,

$$-D'(T) = E_Q\left[\exp\left(-\int_0^T r_s\, ds\right) r_T\right].\tag{13}$$

Setting the present values in equations (7) and (12) equal to each other and solving for $P(T)$ gives

$$P(T) = \frac{-\int_0^T D'(t)\, E_Q[\,1 - \tau_t\,]\, dt + \int_0^T D(t)\, E_Q[\,\lambda_t\,]\, dt}{S(T) \int_0^T D(t)\, dt},\tag{14}$$

$$= \frac{-\int_0^T D'(t)\, dt\; +\int_0^T D'(t)\, E_Q[\,\tau_t\,]\, dt\; +\int_0^T D(t)\, E_Q[\,\lambda_t\,]\, dt}{S(T)\int_0^T D(t)\, dt}. \qquad (15)$$

From equations (3) and (5),

$$E_Q[\,\tau_t\,] = \tau\, e^{-\beta t}\; +\frac{\alpha}{\beta}\, (1\, -\, e^{-\beta t}), \qquad (16)$$

$$E_Q[\,\lambda_t\,] = \lambda\, e^{-bt}\; +\frac{a}{b}\, (1\, -\, e^{-bt}). \qquad (17)$$

The next step is to substitute these last two expressions into the integrals in the numerator of equation (15) and evaluate them.[18]

To simplify notation, let us define the weighted annuity factor (which is a weighted sum of observable discount factors):

$$F(u, T) = \int_0^T e^{-ut}\, D(t)\, dt. \qquad (18)$$

The first integral in the numerator reduces to $1 - D(T)$. The second integral becomes

$$-\frac{\alpha}{\beta}(1 - D(T)) + \left(\tau - \frac{\alpha}{\beta}\right)(e^{-\beta T} D(T) - 1 + \beta\, F(\beta, T)), \qquad (19)$$

after integration by parts. The third integral can be expressed as

$$\frac{a}{b}\, F(0, T) + \left(\lambda - \frac{a}{b}\right)\, F(b, T). \qquad (20)$$

Substituting these expressions back into equation (15) and collecting terms gives the following solution for $P(T)$:

$$P(T) = A(T)\; +\; B(T)\, \tau\; +\; C(T)\, \lambda, \qquad (21)$$

where

$$A(T) = \frac{1 - \left(1 - \frac{\alpha}{\beta}(1 - e^{-\beta T})\right)D(T) - \alpha F(\beta, T) + \frac{a}{b}F(0, T) - \frac{a}{b}F(b, T)}{S(T)F(0, T)}, \qquad (22)$$

[18] As a check on our model specification, we also solve the model under the assumption that λ_t and τ_t follow Cox, Ingersoll, and Ross (1985; CIR) square-root processes. Because the conditional expected values of λ_t and τ_t in the CIR model have exactly the same form as in equations (16) and (17), the closed-form solution for the muni-swap using the CIR model is exactly the same as given in this section. This follows because only first moments appear in the numerator of equation (15). Note that the same would be true in much more general specifications; the closed-form solution for muni-swaps is robust to the assumption about the functional form of the diffusion term in the dynamics of λ_t and τ_t. We will use the Gaussian or Ornstein–Uhlenbeck specification in the empirical work rather than the CIR specification (or a more general specification) because it appears much more consistent with the data.

$$B(T) = \frac{-1 + e^{-\beta T}\ D(T) + \beta\ F(\beta, T)}{S(T)F(0, T)}, \tag{23}$$

$$C(T) = \frac{F(b, T)}{S(T)F(0, T)}. \tag{24}$$

From this equation, we see that, given the discount function $D(T)$, the percentage swap rate $P(T)$ is simply a linear function of the current values of τ and λ.

IV. Maximum Likelihood Estimation

To estimate the model, we use a maximum likelihood approach similar to that often used in estimating term structure models. Important examples of the applications of this methodology to term structure estimation include Duffie and Singleton (1997), Duffee (2002), and Liu et al. (2006).

Paralleling Duffie and Singleton (1997), we assume that the MSI rate and the 10-year percentage swap rates are measured without error. Industry sources suggest that the 10-year rate is one of the most liquid points on the curve. Thus, given the repo rate r and the discount function $D(T)$, and conditional on the parameter vector θ, equations (1) and (21) provide two linear equations in the two state variables λ and τ, and can be solved directly.[19] Specifically, the closed-form solutions for λ and τ are given by

$$\lambda = -r(1 - \tau)\ +\ M, \tag{25}$$

$$\tau = \frac{r\ C(T)\ -\ A(T)\ -\ C(T)\ M\ +\ P(T)}{B(T)\ +\ r\ C(T)}. \tag{26}$$

Thus, λ and τ can be expressed as explicit linear functions of M and $P(T)$. It is this simple two-equations-in-two-unknowns structure that allows us to identify the values of λ and τ for each date in the sample period from the observed values of M and $P(T)$. Let J denote the Jacobian of the mapping from M and $P(10)$ to λ and τ.

At time t, we can now solve for the percentage swap rate implied by the model for any maturity from the values of λ_t, τ_t, and the parameter vector θ. Let ϵ_t denote the vector of differences between the market and model values of $P_t(T)$ implied by the values of τ_t, λ_t, and θ for the remaining municipal swaps. Under the assumption that ϵ_t is conditionally multivariate normal with mean vector zero and a diagonal covariance matrix Σ with diagonal values $v_1^2, v_2^2, v_3^2, v_4^2, v_5^2, v_7^2, v_{12}^2, v_{15}^2$, and v_{20}^2 (where the subscripts denote the maturities of the corresponding municipal swaps), the log likelihood function for $M_{t+\Delta t}$,

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[19] This assumes, however, that $\beta \neq b$. I am grateful to the referee for pointing this out.

Table II
Maximum Likelihood Estimates of the Model Parameters

This table reports the maximum likelihood parameters of the model along with their asymptotic standard errors.

Parameter	Value	Std. Error
a	0.01062	0.00013
$\hat{a}$	0.06373	0.01025
b	1.33729	0.01505
$\hat{b}$	11.20705	1.22727
c	0.02933	0.00100
α	0.04808	0.00028
$\hat{\alpha}$	4.31606	0.11271
β	0.17689	0.00091
$\hat{\beta}$	11.30725	0.29209
σ	0.32333	0.01103
v_1	0.09831	0.00336
v_2	0.05187	0.00178
v_3	0.03417	0.00117
v_4	0.03364	0.00115
v_5	0.02953	0.00101
v_7	0.02232	0.00076
v_{12}	0.02057	0.00070
v_{15}	0.04362	0.00149
v_{20}	0.03049	0.00104
Log Likelihood	-10299.0872	

$P_{t+\Delta t}$, and $\epsilon_{t+\Delta t}$ conditional on M_t, $P_t(10)$, and the term structure information is

$$
\begin{aligned}
LLK_t = {} & -\frac{11}{2}\ln(2\pi) + \ln|J_{t+\Delta t}| - \frac{1}{2}\ln|\Sigma| - \frac{1}{2}\epsilon'_{t+\Delta t}\,\Sigma^{-1}\,\epsilon_{t+\Delta t} \\
& -\frac{1}{2}\ln\left(\frac{\sigma^2(1-e^{-2\hat{\beta}\Delta t})}{2\hat{\beta}}\right) - \left(\frac{\hat{\beta}\big(\tau_{t+\Delta t} - \tau_t e^{-\hat{\beta}\Delta t} - \frac{\hat{\alpha}}{\hat{\beta}}(1-e^{-\hat{\beta}\Delta t})\big)^2}{\sigma^2(1-e^{-2\hat{\beta}\Delta t})}\right) \\
& -\frac{1}{2}\ln\left(\frac{c^2(1-e^{-2\hat{b}\Delta t})}{2\hat{b}}\right) - \left(\frac{\hat{b}\big(\lambda_{t+\Delta t} - \lambda_t e^{-\hat{b}\Delta t} - \frac{\hat{a}}{\hat{b}}(1-e^{-\hat{b}\Delta t})\big)^2}{c^2(1-e^{-2\hat{b}\Delta t})}\right). \quad (27)
\end{aligned}
$$

The total log likelihood function is then given by summing LLK_t over all of the weekly observations.

We maximize the log likelihood function over the 19-dimensional parameter vector $\theta = \{a, \hat{a}, b, \hat{b}, c, \alpha, \hat{\alpha}, \beta, \hat{\beta}, \sigma, v_1, v_2, v_3, v_4, v_5, v_7, v_{12}, v_{15}, v_{20}\}$ with a standard quasi-Newton algorithm using a finite-difference gradient. As a robustness check that the algorithm achieves the global maximum, we repeat the estimation using a variety of different starting values for the parameter vector. Table II reports the maximum likelihood estimates of the parameters along with their asymptotic standard errors.[20]

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[20] To provide additional perspective, we also conducted likelihood ratio tests to examine whether

Table III

Summary Statistics for the Credit/Liquidity Spread and the Marginal Tax Rate

This table reports summary statistics of the estimated credit/liquidity spread λ_t and the marginal tax rate τ_t.

Variable	Mean	Standard Deviation	Minimum	Median	Maximum	Serial Correlation	N
λ_t	0.00565	0.00621	−0.00714	0.00435	0.07178	0.807	428
τ_t	0.38008	0.06742	0.07950	0.38194	0.55312	0.802	428

V. The Empirical Results

In this section, we focus first on the estimated municipal default/liquidity spread λ_t and its risk premium. We then report the results for the estimated marginal tax rate τ_t and examine the implications for asset prices and financial markets. Finally, we address the issue of the efficiency of prices in the municipal swap market and the relative valuation of municipal swap contracts.

A. The Credit/Liquidity Spread

Table III provides summary statistics for the estimated values of the municipal credit/liquidity spread λ_t. Figure 2 plots the time series of the estimated values of λ_t. As shown, there is a substantial credit/liquidity spread incorporated into the MSI rate. The average value of λ_t during the sample period is 56.5 basis points. The value of λ_t, however, has varied significantly throughout the sample period, ranging from −71.4 basis points to 717.8 basis points. The standard deviation of λ_t is 62.1 basis points.[21]

Figure 2 shows that the value of λ_t is generally positive. Of the 428 weeks in the sample period, the estimated value of λ_t is positive for 414 weeks, or equivalently, for 96.7% of the sample. For most of the first two-thirds of the sample period, the credit/liquidity spread hovers between roughly 20 basis points and 100 basis points. Beginning around mid-2007, however, the value of λ_t starts to increase, often reaching levels of 150 basis points or more as the global financial crisis began to unfold. The largest value of 717.8 basis points occurred on September 24, 2008 in the week following the Lehman default. The largest negative value of λ_t occurs on February 13, 2008, which was close to the height of the period during which auction failures in the auction rate security markets became widespread. Thus, the quality of market data in the closely

we could reject the hypotheses that there is no tax risk ($\sigma = 0$), that tax risk is unpriced ($\alpha = \hat{\alpha}$, $\beta = \hat{\beta}$), that the market price of tax risk is constant ($\beta = \hat{\beta}$), and that the market prices of tax risk and liquidity risk are both constant ($b = \hat{b}$, $\beta = \hat{\beta}$). All four of these hypotheses are strongly rejected by the data. I am grateful to the referee for suggesting these tests.

[21] As a robustness check, we estimated the model using a specification in which λ_t is correlated with the riskless rate r_t, and r_t also follows an Ornstein–Uhlenbeck (Vasicek) process. The estimated values of λ_t for this specification are virtually the same as those reported in the paper.
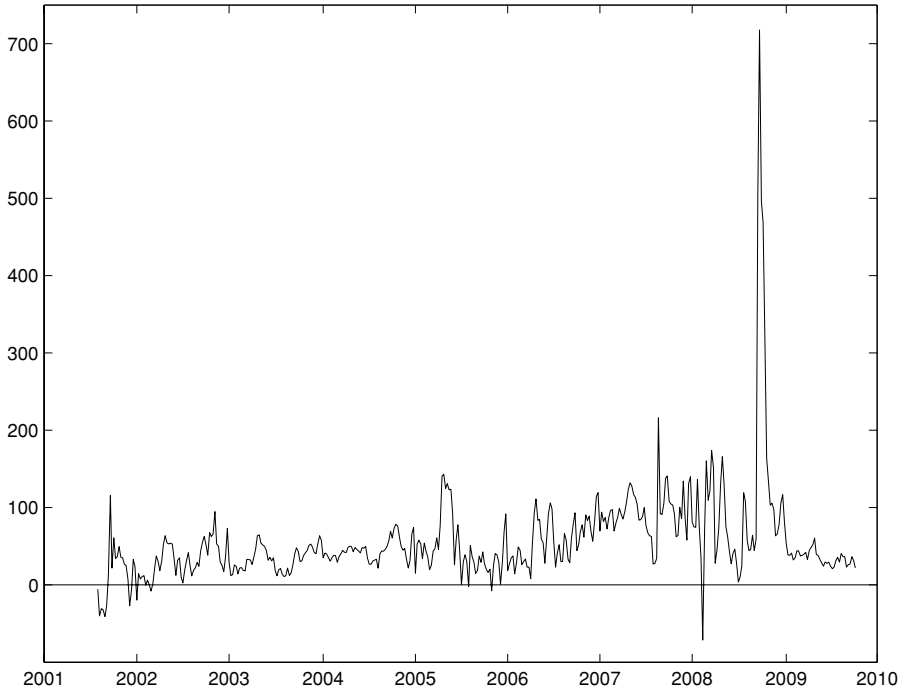
Figure 2. The credit/liquidity spread. This plot shows the estimated credit/liquidity spread λ_t during the sample period.

related VRDO market could easily have been adversely impacted during this period.

B. The Credit/Liquidity Risk Premium

The maximum likelihood estimates of $\hat{a}$ and $\hat{b}$ in Table II imply that the long-run mean of λ_t under the actual measure is 56.9 basis points. This is in close agreement with the average value of λ_t reported in Table III. In contrast, the maximum likelihood estimates of a and b imply that the long-run mean of λ_t under the risk-neutral measure is 79.4 basis points. Thus, there is clearly a significant risk premium associated with λ_t; the market prices securities as if the long-run value of λ_t were about 22.5 basis points higher than its actual long-run value.

To put these results into asset pricing terms, Table IV reports summary statistics for the difference between the expected value of λ_t under the risk-neutral and actual measures, $E_Q[\lambda_T] - E_P[\lambda_T]$. Recall that the expected value of λ_T under the risk-neutral measure Q is just the no-arbitrage price for a futures or forward contract that settles to λ_T. Thus, these differences capture the spread between the forward value of λ_T and the expected spot value of λ_T. As such, the spread directly measures the risk premium that a hedger would

Table IV
Risk Premia

This table reports the mean, minimum, and maximum values for the credit/liquidity and marginal tax rate risk premia for the indicated horizons (in years). The risk premium is defined as the difference between the forward value of the variable and its expected value, where the forward value represents the expected value of the variable under the risk-neutral measure.

		1	2	3	5	10	∞
λ_t risk premium	Mean	0.00165	0.00210	0.00221	0.00225	0.00226	0.00226
	Minimum	−0.00170	0.00122	0.00198	0.00224	0.00226	0.00226
	Maximum	0.01902	0.00665	0.00341	0.00234	0.00226	0.00226
τ_t risk premium	Mean	−0.01918	−0.03389	−0.04621	−0.06519	−0.09143	−0.10989
	Minimum	−0.27102	−0.24490	−0.22301	−0.18931	−0.14268	−0.10989
	Maximum	0.12578	0.08759	0.05557	0.00627	−0.06193	−0.10989

be willing to pay to lock in the future value of λ_T via a futures or forward contract.

As shown, the average risk premium is an increasing function of the horizon. The average risk premium is 16.5 basis points for a 1-year horizon, 21.0 basis points for a 2-year horizon, and 22.6 basis points for a 10-year horizon. Table IV also shows that there is considerable variation in the risk premium, at least for some of the shorter horizons. For longer horizons, the risk premium is less volatile, which is not surprising given the rapid estimated speeds of mean reversion for λ_t under both measures.

C. *The Marginal Tax Rate*

Table III also reports summary statistics for the estimated marginal tax rate τ_t. Figure 3 plots the time series of the estimated values of τ_t. The average value of τ_t during the sample period is 38.0%.[22] This average value is very similar to the highest federal income tax rates during the sample period. Specifically, the highest federal income tax rate was 39.1% during 2001, 38.6% during 2002, and 35.0% during 2003 to 2009. Note that top marginal corporate tax rate during the sample period is 39.0% and the top trust tax rate is 35.0%.[23]

It is important to recognize, however, that the MSI rate is an average of yields on VRDOs from a broad collection of municipal issuers from virtually every state. Thus, the marginal tax rate incorporated into the index may in fact reflect federal, state, and possibly county, city, or other local income taxes as well. For example, a resident of New York City faces a maximum

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[22] This estimated marginal tax rate is significantly larger than values that have been estimated in other markets. For example, Ang et al. (1985) estimate a marginal tax rate of 24% to 26% from corporate bond prices. Graham (2003) uses data from Engle, Erickson, and Maydew (1999) to infer a marginal tax rate of 13% for monthly income preferred stock.

[23] The top marginal corporate tax rate of 39.0% applies to income between $100,000 and $335,000. For income levels between $15,000,000 and $18,333,333, the corporate tax rate is 38.0%. For income in excess of $18,333,333, the corporate tax rate is 35.0%.
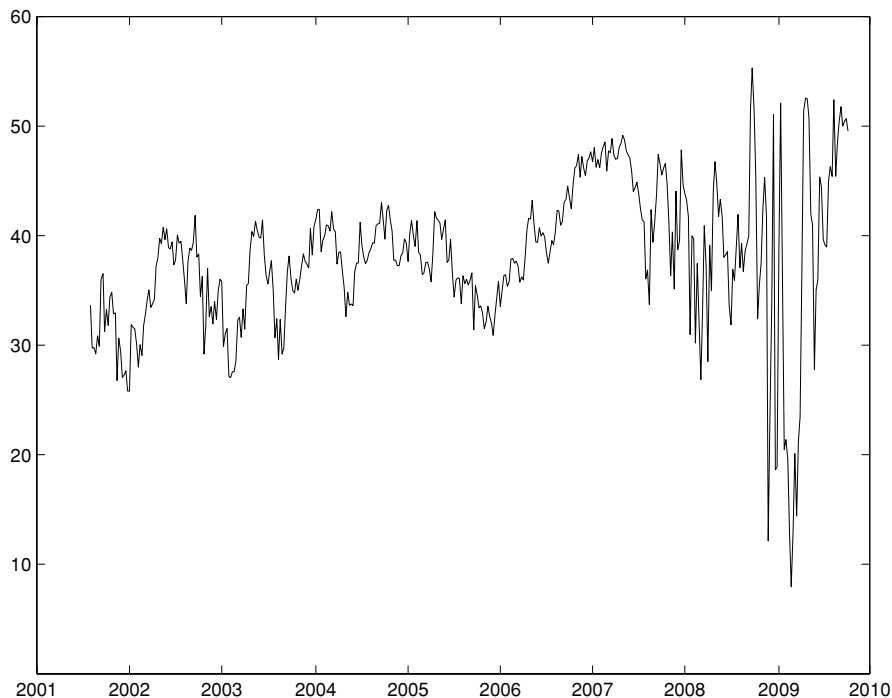
Figure 3. The marginal tax rate. This plot shows the estimated marginal tax rate τ_t during the sample period.

federal income tax rate of 35%, a maximum New York State income tax rate of 8.14%, and a maximum New York City income tax rate of 4.00%. The overall maximum tax rate, however, is not just the sum of these rates because state and local income taxes may be deductible from federal income taxes (subject to limitations such as those imposed by the alternative minimum tax; see Feenberg and Poterba (2004)). Assuming that the New York State and New York City income taxes were fully deductible, the maximum income tax rate faced by a New York City taxpayer would be $35.00 + 0.65 \times (8.14 + 4.00) = 42.89\%$. Similarly, California taxpayers face a maximum state income tax rate of 10.3%. Again assuming full deductibility, this implies that the maximum income tax rate faced by a California taxpayer would be $35.00 + 0.65 \times 10.3 = 41.695\%$.[24]

The estimated value of τ_t varies throughout the sample period. During the first half of the sample period, τ_t hovers between 30% and 40%. During the

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[24] Note that this discussion abstracts from many other tax complexities that could significantly increase the effective marginal tax rate such as the double or triple taxation that shareholders of corporations might face on interest income received and then paid out as dividends. Furthermore, self-employment taxes, Medicare taxes, alternative minimum taxes, etc. could also complicate the marginal tax rate.

early part of 2007, τ_t begins to increase and reaches about 50%. Once the 2008 recession begins, however, the marginal tax rate becomes very volatile. The marginal tax rate reaches a low of about 8% on February 25, 2009, coinciding with the lows in the stock market and concerns about the economy sinking into a depression. The highest value of the marginal tax rate of 55.3% occurs on September 24, 2008.[25]

D. The Tax Risk Premium

As with the credit/liquidity spread, we can also examine whether there is a tax risk premium embedded into security prices to compensate investors for being exposed to changes in the marginal tax rate. Turning again to Table II, we see that the maximum likelihood estimates of $\hat{\alpha}$ and $\hat{\beta}$ imply that the long-run mean of τ_t under the actual measure is 38.17%, which is very close to the average value reported in Table III.

Surprisingly, however, the maximum likelihood estimates of α and β imply that the long-run mean of τ_t under the risk-neutral pricing measure is only 27.18%. Thus, these results indicate that there is a tax risk premium. This tax risk premium, however, actually has a negative sign. This suggests that an investor would require a lower expected return to hold a security with cash flows that are sensitive to changes in the tax rate. In other words, investors view tax risk as being countercyclical.

To make this latter result more intuitive, let us consider the case of a taxable investor who holds a Treasury bond. For concreteness, assume that the bond has a market value of 100 and a fixed coupon of 6%. From an after-tax perspective, the actual cash flow received by the investor each year is $6(1 - \tau_t)$. Because federal marginal tax rates are progressive, this means that the investor's tax rate τ_t generally declines when his income decreases, and vice versa (we are abstracting from the discreteness of the federal income tax schedule). Thus, the after-tax cash flows received from the Treasury bond increase when the investor's income and marginal tax rate decline, and vice versa. Thus, the after-tax cash flows from the Treasury bond have almost a perfect negative correlation with the investor's income, which in turn maps into a strong negative consumption beta. Thus, in the same way that, for example, gold mining stocks have negative market betas and therefore lower required expected returns, Treasury bonds should have lower yields or expected returns because of their negative consumption betas.

To illustrate the size of the risk premium, Table IV also reports summary statistics for the difference between the expected values of τ_T under the Q and P measures for various values of T. As shown, the average difference between the forward and expected values of τ_T ranges from about -0.019 for a 1-year

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[25] It is interesting to note that the Tax Foundation (www.TaxFoundation.org) estimates that the top marginal tax rates for individuals given the 5.4% surtax proposed under the House Health Care Bill would be 56.92% for New York taxpayers, and 56.58% for California taxpayers (see www.TaxFoundation.org/UserFiles/Image/maps/health˙surtax˙display.jpg).

horizon to about −0.091 for a horizon of 10 years. The table also shows that there is significant time variation in the tax risk premium. For example, the tax risk premium for the 1-year horizon ranges from −0.271 to 0.126. Thus, the tax premium can sometimes take on positive values. These results are consistent with Sialm (2006), who finds that tax risk premia can be difficult to sign in the context of a general equilibrium model.[26]

E. What Drives the Marginal Tax Rate?

To explore the nature of the marginal tax rate in more detail, we regress changes in the estimated marginal tax rate on a number of measures potentially affecting the taxable income of the marginal municipal bond participant. In doing so, we first compute changes in the marginal tax rate over a monthly horizon (rather than over a weekly horizon as in the previous analysis). Specifically, we calculate the monthly change in τ_t using the first estimated value of τ_t for each month.

As explanatory variables, we use a variety of measures. First, we include the monthly return on the S&P 500 index (omitting dividends). Second, we use the monthly return on a broad portfolio of Treasury bonds with maturities ranging from 2 to 30 years. The data for this return series are reported by Bloomberg. Third, we use the monthly return on a broad index of commodity prices, also calculated and reported by Bloomberg.[27]

These three measures attempt to proxy for the components of the marginal municipal bondholder's income that may be based on financial market value.

To provide a macroeconomic perspective on the determinants of the marginal tax rate, we also include several variables reflecting changes in the economic environment. First, we include the monthly change in per capita personal disposable income as reported by the Bureau of Economic Analysis. Second, we include the monthly Consumer Price Index-Urban (CPI-U) inflation rate. Third, we include the monthly change in the (seasonally adjusted) national unemployment rate as reported by the Bureau of Labor Statistics. Finally, we include the monthly percentage change in industrial production (seasonally adjusted) as reported by the Federal Reserve Board.

Table V reports the results from this regression. As shown, changes in the marginal tax rate are significantly and positively related to stock market returns. Similarly, changes in the marginal tax rate are significantly positively related (at the 10% level) to returns on Treasury bonds. These results provide support for the hypothesis that the marginal tax rate is procyclical, and therefore that cash flows that are multiplied by $(1 − \tau_t)$ are countercyclical. These results also support the interpretation of the negative risk premium embedded in long-term municipal swap rates as a premium for the countercyclical

[26] See also Ross (1985, 1987), Constantinides (1983), and Sialm (2009).

[27] This index is the UBS Bloomberg Constant Maturity Commodity Index and consists of a diversified basket of commodities including energy, industrial metals, precious metals, agriculture, and livestock.

Table V
Regression Results

This table reports the slope coefficients and associated Newey–West t-statistics from the regression of monthly changes in the estimated marginal tax rate on the indicated explanatory variables.

Explanatory Variable	Coefficient	t-Statistic
Intercept	−0.00257	−0.44
S&P 500 Return	0.70723	3.01
Treasury bond return	1.00414	1.85
Commodity index return	−0.20760	−2.03
Personal income growth	−0.02583	−1.64
Inflation rate	0.29538	0.37
Change in unemployment	0.04108	0.78
Industrial production growth	−0.24283	−0.29
R^2		0.1880
Number of observations		98

behavior of after-tax fixed income cash flows. In addition, the relation between the marginal tax rate and the financial markets provides interesting insights into the nature of the marginal investor. In particular, they argue that a significant proportion of the marginal investor's personal income comes in the form of investment income.[28]

Table V also shows that the return on the commodity index is negative and significant. This curious result may be due to the particular role that commodities have played in the markets during the recent financial crisis. For example, the commodities index includes metals, such as gold, that have traditionally been viewed as countercyclical investments. In addition, many view commodities as a hedge against inflation risk. Thus, the negative coefficient for this variable may be a reflection of how commodities rallied during the depths of the financial crisis while the marginal tax rate dropped precipitously. Finally, Table V shows that none of the macroeconomic variables is statistically significant at conventional levels.

F. The Relative Valuation of Municipal Swaps

Because only the short-term tax-exempt rate M_t and the 10-year municipal swap percentages $P_t(10)$ are fitted exactly, the other municipal swap percentages implied by the model will typically not match the corresponding market values exactly. To examine whether there are systematic differences between model and market values, we report summary statistics for these differences. Specifically, the pricing difference is defined as the model-implied municipal swap rate minus the market municipal swap rate. Thus, the pricing differences are in the same units as the values of $P_t(T)$. Table VI reports these summary statistics.

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[28] See Feenberg and Poterba (1991).

Table VI
Municipal Swap Pricing Errors

This table reports summary statistics for the difference between the model-implied values of the indicated municipal swap rate and the market municipal swap rate. Municipal swap rates are expressed as percentages of LIBOR. Pricing errors for the 10-year swap are not reported because the 10-year swap rate is fitted exactly in the estimation algorithm.

Swap contract	Mean	Standard Deviation	t-Statistic	Serial Correlation	N
1-year municipal swap	4.140	8.917	3.36	0.784	428
2-year municipal swap	2.020	4.779	3.34	0.748	428
3-year municipal swap	0.158	3.415	0.33	0.785	428
4-year municipal swap	−0.322	3.349	−0.81	0.716	428
5-year municipal swap	−0.097	2.953	−0.27	0.736	428
7-year municipal swap	0.007	2.233	0.25	0.746	428
10-year municipal swap	−	−	−	−	−
12-year municipal swap	−0.567	1.978	−1.57	0.871	428
15-year municipal swap	−1.093	4.222	−1.25	0.899	428
20-year municipal swap	1.662	2.556	3.66	0.863	428

As shown, the mean pricing errors range from a minimum of −1.093 for the 15-year municipal swap contract to a maximum of 4.140 for the 1-year contract. To test for statistical significance, we calculate the *t*-statistics for the mean, where the standard deviation of each mean is adjusted for serial correlation of the pricing errors. In general, these mean values are not significant. The exceptions are the 1-, 2-, and 20-year contracts, which are all significantly positive.

In general, the shorter-maturity municipal swap contracts tend to have more volatile fitting errors than longer-maturity contracts. This feature is very similar to the patterns found in other swap markets. For example, Duffie and Singleton (1997) and Liu et al. (2006) model the term structure of interest rate swaps using an affine framework and also find that the errors of shorter-maturity contracts are more volatile.

Although not significant on average, Table VI also shows that many of the pricing errors display a substantial amount of serial correlation. For example, the first-order serial correlation coefficients for the pricing errors are all in excess of 0.70. Because we do not have transaction cost estimates for trading these municipal swap contracts, we cannot evaluate whether the persistence in these pricing errors could be the basis for a trading strategy. Nevertheless, the results raise interesting questions about relative valuation in the municipal swap market.

G. Alternative Explanations

While the results of the analysis suggest that the municipal swap market values cash flows in a way that is remarkably consistent with the highest marginal federal tax rates, it is important to consider whether there might be

alternative explanations for the results. In particular, one alternative explanation might simply be that municipal swap rates are artificially inflated for reasons that have nothing to do with expected tax rates or tax risk premia and that the average estimated marginal tax rate of 38% is purely coincidental.[29]

For example, a recent paper by Gârleanu, Pedersen, and Poteshman (2009) documents that there are demand-related pricing effects in options markets. Specifically, they show that the aggregate positions of option dealers are related to market option values. If the same types of effects are present in the municipal swap market, then municipal swap rates might reflect demand imbalances between different types of counterparties. In general, municipalities are typically net demanders of swaps while banks are net suppliers. Intuitively, these types of institutional differences in the nature of the supply and demand of municipal swap contracts might translate into market prices that deviate from the values implied by no-arbitrage considerations.

Another influence on municipal swap rates might be the possibility of tax arbitrage. In particular, if municipalities face a different marginal tax rate on their swap-related cash flows than the institutions on the other side of the transaction, then observed municipal swap rates might be influenced by changes in the viability of the tax arbitrage over time as well as shifts in the relative bargaining position of the parties to the swap.

Another consideration is the presence of transaction and shorting costs associated with hedging municipal swap transactions. In particular, if the municipal swap dealers or banks providing liquidity to the market face significant obstacles in hedging their positions because of the transaction, illiquidity, and short-selling costs of hedging vehicles, then these factors might be reflected in the market prices of municipal swaps.

As discussed in Section I, the MSI index is based on the rates provided by the remarketing agents who are part of the market clearing process for VRDOs. In concept, there might be potential agency conflicts affecting the behavior of remarketing agents because of their ongoing incentive to cater to institutional investors. This concern is relevant given that McConnell and Saretto (2010) show that there have been periods in which VRDO rates have been above the market clearing rates for auction-rate securities.

While each of these alternatives might possibly result in municipal swap rates P_t that exceeded theoretical no-arbitrage values, several considerations argue against our results being due entirely to these factors. For example, the muni-bond puzzle itself suggests that the market values other types of tax-exempt securities in a way similar to that observed in the municipal swap market. In particular, the fact that yields on longer-term municipal bonds tend to be on the order of 80% or more of the yields on taxable bonds lends support to the notion that the high average municipal swap rates reported in Table I are not entirely due to municipal swap market microstructure effects of the type described above (see Green (1993), Graham (2003), and others). Furthermore,

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[29] I am indebted to the referee for raising this issue. This section draws heavily on the many insightful comments and specific examples provided by the referee.

while the considerations described above provide a narrative for why municipal swaps might exceed their fair values, they do not provide a rationale for why these apparent arbitrages might persist in the municipal swap market. Also, it is not apparent from a theoretical perspective why these effects would go in one direction rather than the other, for example, why the relative bargaining power of banks vs. municipalities would result in equilibrium swap rates being higher rather than lower than fair value.

Finally, VRDOs often have explicit backstop liquidity guarantees that allow investors to put the bonds back to liquidity providers (typically banks) if the securities cannot be remarketed to other investors. VRDOs that are put back to liquidity providers may carry higher "penalty" rates and be subject to accelerated amortization. To the extent that the MSI index includes these "bank bonds," the index might include a component due to the illiquidity of these securities. Note, however, that this should be captured in the liquidity component λ_t.

VI. The Muni-Bond Puzzle

Although the focus of this paper is primarily on the marginal tax rate incorporated into short-term municipal yields, our results may also have implications for the muni-bond puzzle. In particular, it is possible that our results might help explain why the ratio of tax-exempt bond yields to taxable bond yields is typically much higher than $1 - \tau_t$.

As shown by Green (1993), modeling tax-exempt bond yields and their relation to taxable yields is a very complex problem. One major reason for this is that the relative values of tax-exempt and taxable bonds depend on the tax trading strategies followed by market participants. In light of this, our approach in this section will simply be to gauge whether the dynamics of τ_t and λ_t estimated in the previous section can generate back-of-the-envelope estimates of tax-exempt rates that are roughly comparable to those observed. Thus, our objective is far less ambitious than providing a full-fledged model of tax-exempt bond yields.

Specifically, we solve for the coupon rate for a tax-exempt bond that equates the value of its cash flows to the value of the after-tax cash flows of a Treasury bond. Let w denote the coupon rate of a tax-exempt bond with maturity T that is subject to a credit and liquidity spread of λ_t. The value of this bond to a buy-and-hold investor (who does not follow tax-timing strategies) can be expressed as

$$w \int_0^\infty D(t) E\left[\exp\left(- \int_0^t \lambda_s \, ds \right) \right] dt + D(T) E\left[\exp\left(- \int_0^T \lambda_t \, dt \right) \right]. \quad (28)$$

Evaluating the expectations gives

$$w \int_0^\infty D(t) \, \Phi(t) \, e^{-\Psi(t)\lambda} \, dt + D(T) \, \Phi(T) \, e^{-\Psi(T)\lambda}, \quad (29)$$

where

$$\Phi(T) = \exp\left(\left(\frac{c^2}{2b^2} - \frac{a}{b}\right)T + \left(\frac{a}{b^2} - \frac{c^2}{b^3}\right)\left(1 - e^{-bT}\right) + \frac{c^2}{4b^3}\left(1 - e^{-2bT}\right)\right), \quad (30)$$

$$\Psi(T) = \frac{1}{b}\left(1 - e^{-bT}\right). \quad (31)$$

In contrast, the value to the same buy-and-hold investor of the after-tax cash flows from a par Treasury bond with coupon rate u can be expressed as

$$u \int_0^\infty D(t)(1 - E[\tau_t])\, dt + D(T). \quad (32)$$

Evaluating the expectation gives

$$u(1 - \alpha/\beta)\, F(0, T) - u(\tau - \alpha/\beta)\, F(\beta, T) + D(T). \quad (33)$$

Given the observed par rates for Treasury bonds as well as the maximum likelihood estimates of the parameters in Table II, it is straightforward to solve for the value of w that sets the two bond values equal.[30]

To compare the model's pricing implications to the market prices of tax-exempt bonds, we obtain Bloomberg indexes for the yields on tax-exempt general obligation municipal bonds with ratings ranging from AAA to A−. Table VII reports the ratio of the average yields for these indexes to the average Treasury CMT rates, where the averages are computed over the August 2001 to October 2009 period. Table VII also reports the ratio of the average estimated value of w to the average Treasury CMT rate over the same period.

As shown, the muni-bond puzzle is definitely present in the market tax-exempt data. In particular, the ratios of tax-exempt yields to Treasury yields range from 79% to nearly 94%. Thus, the simple "implied" marginal tax rates range from 21% to about 6%. These values are clearly significantly less than the maximum federal income tax rates during the sample period.

Turning to the ratios implied by the model, Table VII shows that these are generally in the same "ballpark" as those observed in the markets. In particular, the model's ratios range from 87% for the 1- and 2-year maturities to about 82% for the 20-year maturity. Thus, on average, the model seems to capture the level of the muni-bond puzzle. However, Table VII shows that the model tends to overestimate the ratio for shorter maturities, while it underestimates the ratio for longer maturities. Taken together, these results suggest that a model fitted

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[30] In doing so, we are implicitly assuming that after-tax cash flows can be discounted using $D(T)$ (rather than an after-tax discount factor). In actuality, this assumption has very little effect on the estimated values of w. This is because the same discount factors are used for both the taxable and tax-exempt bonds; when the two bond values are set equal to each other, the choice of discount function largely washes out. We also computed the values of w using $D^\gamma(T)$ to discount cash flows, where γ is some value such as $1 - \alpha/\beta$. The results are very similar to those reported in this section.

<div align="center">

Table VII

Ratios of Tax-Exempt to Taxable Bond Yields

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This table reports the ratio of tax-exempt to taxable bond yields. The first column reports the ratio of the average yield on an index of general obligation municipal bonds with ratings from AAA to A− to the average constant maturity Treasury yield of the indicated maturity (measured in years). The second column reports the ratio of the average yield on tax-exempt bonds implied by the fitted municipal swap model to the average constant maturity Treasury yield of the indicated maturity (measured in years). The averages are based on data for the August 2001 to October 2009 period.

Maturity	Empirical Ratio	Model Implied Ratio
1	0.790	0.870
2	0.807	0.870
3	0.819	0.864
5	0.834	0.850
7	0.853	0.842
10	0.894	0.823
20	0.936	0.819

to municipal swap data may resolve much of the muni-bond puzzle, although the simple tax-exempt bond model we use in this section falls short of fully explaining the slope of the tax-exempt yield curve. Future research could focus on whether combining the municipal swap model of this paper with a more in-depth model of tax-exempt bond valuation such as Green (1993) is able to fully resolve the muni-bond puzzle.

VII. Conclusion

This paper uses a unique data set of municipal swap rates to identify both the marginal tax rate and the credit/liquidity spread embedded in 1-week tax-exempt municipal yields. By inferring these values from the 1-week rate, our approach has the important advantage of completely avoiding the complexities of the tax treatment of long-term municipal bonds, which Green (1993) illustrates can be very formidable.

We find that the average marginal tax rate incorporated into the 1-week MSI rate is 38.0% during the 2001 to 2009 sample period. This average corresponds very closely to the actual maximum marginal federal income tax rate. Furthermore, the marginal tax rate incorporated into tax-exempt rates is significantly related to Treasury bond, stock market, and commodity returns.

Most surprisingly, we find that market prices imply a negative risk premium for bearing the risk of time-varying marginal tax rates. This result, however, is fully consistent with the countercyclical behavior of after-tax cash flows. This follows simply from the fact that the marginal tax rate is higher in good states of the economy and vice versa. Thus, after-tax fixed income cash flows, which are multiplied by $(1 - \tau_t)$, are negatively correlated with the state of the economy. This implies that after-tax cash flows essentially have negative consumption betas and therefore negative risk premia.

These findings have a number of important implications. For example, our results largely resolve the long-standing muni-bond puzzle. Specifically, after fitting the model to 1-week tax-exempt rates and the term structure of municipal swaps, the model implies long-term municipal bond yields that approximate those in the market. Further, finding that the variation in the implied marginal tax rate is significantly related to stock, bond, and commodity returns suggests that the marginal investor derives a substantial portion of his income from capital sources. Finally, the presence of a significant negative risk premium in taxable bond yields argues that tax risk may be an important systematic factor affecting returns in other financial markets.

REFERENCES

Ang, Andrew, Vineer Bhansali, and Yuhang Xing, 2010, Taxes on tax-exempt bonds, *Journal of Finance* 65, 565–601.

Ang, James, David Peterson, and Pamela Peterson, 1985, Marginal tax rates: Evidence from nontaxable corporate bonds: A note, *Journal of Finance* 40, 327–332.

Arak, M., and K. Guentner, 1983, The market for tax-exempt issues: Why are the yields so high? *National Tax Journal* 36, 145–161.

Buser, Stephen A., and Patrick J. Hess, 1986, Empirical determinants of the relative yields on taxable and tax-exempt securities, *Journal of Financial Economics* 17, 335–355.

Chalmers, John M. R., 1998, Default risk cannot explain the muni puzzle: Evidence from municipal bonds that are secured by U.S. Treasury obligations, *Review of Financial Studies* 11, 281–308.

Collin-Dufresne, Pierre, Robert S. Goldstein, and J. Spencer Martin, 2001, The determinants of credit spread changes, *Journal of Finance* 56, 2177–2207.

Constantinides, George M., 1983, Capital market equilibrium with personal tax, *Econometrica* 51, 611–636.

Constantinides, George M., and Jonathan E. Ingersoll, Jr, 1982, Optimal bond trading with personal tax: Implications for bond prices and estimated tax brackets and yield curves, *Journal of Finance* 37, 349–352.

Cox, John C., Jonathan E. Ingersoll, and Stephen A. Ross, 1985, A theory of the term structure of interest rates, *Econometrica* 53, 385–407.

Criscuolo, Anthony C., and Michael Faloon, 2007, Variable rate demand obligations, Research report, Standish Mellon Asset Management.

Dammon, Robert M., and Richard C. Green, 1987, Tax arbitrage and the existence of equilibrium prices for financial assets, *Journal of Finance* 42, 1143–1166.

Dammon, Robert M., Chester S. Spatt, and Harold H. Zhang, 2004, Optimal asset location and allocation with taxable and tax-deferred investing, *Journal of Finance* 59, 999–1037.

Downing, Chris, and Frank Zhang, 2004, Trading activity and price volatility in the municipal bond market, *Journal of Finance* 59, 899–931.

Driessen, Joost, 2005, Is default event risk priced in corporate bonds? *Review of Financial Studies* 18, 165–195.

Duffee, Gregory R., 1998, Idiosyncratic variation in Treasury bill returns, *Journal of Finance* 51, 527–551.

Duffee, Gregory R., 1999, Estimating the price of default risk, *Review of Financial Studies* 12, 197–226.

Duffee, Gregory R., 2002, Term premia and interest rate forecasts in affine models, *Journal of Finance* 57, 405–443.

Duffie, Darrell, Lasse H. Pedersen, and Kenneth J. Singleton, 2003, Modeling sovereign yield spreads: A case study of Russian debt, *Journal of Finance* 58, 119–159.

Duffie, Darrell, and Kenneth J. Singleton, 1997, An econometric model of the term structure of interest rate swap yields, *Journal of Finance* 52, 1287–1323.

Duffie, Darrell, and Kenneth J. Singleton, 1999, Modeling term structures of defaultable bonds, *Review of Financial Studies* 12, 687–720.

Dybvig, Philip H., and Stephen A. Ross, 1986, Tax clienteles and asset pricing, *Journal of Finance* 41, 751–762.

Engel, Ellen, Merle Erickson, and Edward Maydew, 1999, Debt-equity hybrid securities, *Journal of Accounting Research* 37, 249–274.

Feenberg, Daniel, and James M. Poterba, 1991, Which households own municipal bonds? Evidence from tax returns, *National Tax Journal* 44, 93–103.

Feenberg, Daniel, and James M. Poterba, 2004, The alternative minimum tax and effective marginal tax rates, *National Tax Journal* 57, 407–427.

Feldstein, Martin, and Andrew Samwick, 1992, Social security rules and marginal tax rates, *National Tax Journal* 45, 1–22.

Gârleanu, Nicolae, Lasse H. Pedersen, and Allen Poteshman, 2009, Demand-based option pricing, *Review of Financial Studies* 10, 4259–4299.

Giesecke, Kay, Francis A. Longstaff, Stephen Schaefer, and Ilya Strebulaev, 2010, Corporate bond default risk: A 150-year perspective, Working paper, UCLA.

Graham, John R., 2003, Taxes and corporate finance: A review, *Review of Financial Studies* 16, 1075–1129.

Green, Richard C., 1993, A simple model of the taxable and tax-exempt yield curves, *Review of Financial Studies* 6, 233–264.

Green, Richard C., 2007, Issuers, underwriter syndicates, and aftermarket transparency, *Journal of Finance* 62, 1529–1550.

Green, Richard C., Burton Hollifield, and Norman Schürhoff, 2007a, Financial intermediation in opaque markets, *Review of Financial Studies* 20, 275–314.

Green, Richard C., Burton Hollifield, and Norman Schürhoff, 2007b, Dealer intermediation and price behavior in the aftermarket for new bond issues, *Journal of Financial Economics* 86, 643–682.

Green, Richard C., Dan Li, and Norman Schürhoff, 2007, Price discovery in illiquid markets, Working paper, Carnegie-Mellon University.

Green, Richard C., and Bernt A. Oedegaard, 1997, Are there tax effects in the relative pricing of U.S. government bonds? *Journal of Finance* 52, 609–633.

Jordan, James V., 1984, Tax effects in term structure estimation, *Journal of Finance* 39, 393–406.

Kidwell, David S., and Charles A. Trzcinka, 1982, Municipal bond pricing and the New York City fiscal crisis, *Journal of Finance* 37, 1239–1246.

Kochin, Levis A., and Richard W. Parks, 1988, Was the tax-exempt bond market inefficient or were future expected tax rates negative? *Journal of Finance* 43, 913–931.

Litzenberger, Robert H., and Jacques Rolfo, 1984, An international study of tax effects on government bonds, *Journal of Finance* 39, 1–22.

Liu, Jun, Francis A. Longstaff, and Ravit E. Mandell, 2006, The market price of risk in interest rate swaps: The roles of default and liquidity risks, *Journal of Business* 79, 2337–2359.

Livingson, Miles, 1979, Bond taxation and the shape of the yield-to-maturity curve, *Journal of Finance* 34, 189–196.

Livingston, Miles, 1982, The pricing of municipal bonds, *Journal of Financial and Quantitative Analysis* 17, 179–193.

Longstaff, Francis A., 2000, The term structure of very short term rates: New evidence for the expectations hypothesis, *Journal of Financial Economics* 58, 397–415.

Longstaff, Francis A., 2004, The flight-to-liquidity premium in U.S. Treasury bond prices, *Journal of Business* 77, 511–526.

Longstaff, Francis A., Sanjay Mithal, and Eric Neis, 2005, Corporate yield spreads: Default risk or liquidity? New evidence from the credit-default swap market, *Journal of Finance* 60, 2213–2253.

McConnell, John, and Alessio Saretto, 2010, Auction failures and the market for auction rate securities, *Journal of Financial Economics* 97, 451–469.

Nanda, Vikram, and Rajdeep Singh, 2004, Bond insurance: What is special about munis? *Journal of Finance* 59, 2253–2279.

Neis, Eric, 2006, Liquidity and municipal bonds, Working paper, UCLA.

Pan, Jun, and Kenneth J. Singleton, 2008, Default and recovery implicit in the term structure of sovereign CDS spreads, *Journal of Finance* 63, 2345–2384.

Ross, Stephen A., 1985, Debt and taxes and uncertainty, *Journal of Finance* 40, 637–657.

Ross, Stephen A., 1987, Arbitrage and martingales with taxation, *Journal of Political Economy* 95, 371–393.

Schaefer, Stephen M., 1982, Tax-induced clientele effects in the market for British Government securities, *Journal of Financial Economics* 10, 121–159.

Sialm, Clemens, 2006, Stochastic taxation and asset pricing in dynamic general equilibrium, *Journal of Economic Dynamics and Control* 30, 511–540.

Sialm, Clemens, 2009, Tax changes and asset pricing, *American Economic Review* 99, 1356–1383.

Skelton, Jeffrey L., 1983, Relative risk in municipal and corporate debt, *Journal of Finance* 38, 625–634.

Stock, Duane, and Edward L. Schrems, 1984, Municipal bond demand premiums and bond price volatility: A note, *Journal of Finance* 39, 535–539.

Trzcinka, Charles, 1982, The pricing of tax-exempt bonds and the Miller Hypothesis, *Journal of Finance* 37, 907–923.

Wang, Junbo, Chunchi Wu, and Frank X. Zhang, 2008, Liquidity, default, taxes, and yields on municipal bonds, *Journal of Banking and Finance* 32, 1133–1149.

Yawitz, Jess B., Kevin J. Maloney, and Louis H. Ederington, 1985, Taxes, default risk, and yield spreads, *Journal of Finance* 30, 1127–1140.