The Level, Slope and Curve Factor Model for Stocks

Charles Clarke*

January 2016

Abstract

I develop a method to extract only the priced factors from stock returns. First, I use multiple regression on anomaly characteristics to predict expected returns. Next, I form portfolios of stocks sorted by their expected returns. Then, I extract statistical factors from these sorts using principal components. The procedure isolates and emphasizes the comovement across assets that is related to expected returns as opposed to firm characteristics. The procedure produces level, slope and curve factors for stock returns. The factors perform better than the Fama and French (1993, 2014) three and five factor models and comparably to the four factor models of Carhart (1997), Novy-Marx (2013) and Hou, Xue, and Zhang (2012). Horse races show that other factors add little to the Level, Slope and Curve factors. The Level, Slope and Curve factors have macroeconomic interpretations. The factors capture strong variation in consumption growth across the sorted portfolios, and when embedded in an ICAPM, proxy for innovations to dividend yield, credit spread and stock volatility.

JEL classification: G12

*University of Connecticut, School of Business, Department of Finance, Storrs, CT 06269; charles.clarke@business.uconn.edu. I am grateful to Paul Borochin, John Campbell, Scott Cederburg, Kent Daniel, Assaf Eisdorfer, Joseph Golec, Shiyang Huang, Mihai Ion, Serhiy Kozak, Steven Kozlowski, Efdal Misirli, Stefan Nagel, Cesare Robotti, Mihail Velikov, Chu Zhang and conference participants at the American Finance Association 2016 Annual Meeting (San Francisco), European Finance Association 2015 Annual Meeting (Vienna) and Financial Management Association 2015 Annual Conference and Docotral Consortium (Orlando) for helpful comments and suggestions. A special thanks to John Cochrane for discussing the paper on his weblog and his many helpful comments.

1 Introduction

The number of potential new asset pricing factors is exploding. Some of these factors are priced and some are not. A priced factor exposes the investor to systematic risk and comes with a risk premium. An unpriced factor represents common movement across stocks unaccompanied by systematic risk or a risk premium. The standard approach is to first identify a common factor, and then to test whether this factor is priced.

In contrast, I describe a method to search specifically for the priced common factors in stock returns. First, sort stocks by expected returns using several different return predictors. Second, use principal components to extract the common factors. The first stage produces portfolios sorted in one dimension on expected returns. The second stage isolates comovement that is priced in the cross-section of stocks by searching for common factors in portfolios already sorted by expected returns.

In order to separate priced from unpriced common factors, the method relies on sorting expected returns using many different predictors. Sorting stocks by only book to market in step one produces a sort on expected returns, but it is unlikely to isolate priced risk factors, because portfolios sorted on book to market have both priced and unpriced common movement (Daniel and Titman (1997) and Gerakos and Linnainmaa (2014)). To overcome this obstacle, I sort on expected returns using multiple regressions to combine signals from many different predictor variables. The procedure isolates factors with nonzero prices of risk and separates these factors from their unpriced counterparts.

Using this method, I show that extracting common factors from these portfolios extracts three factors with the familiar level, slope and curve pattern that can be extracted from returns of bond portfolios sorted by maturity (Litterman and Scheinkman, 1991). The level factor is highly correlated with the market factor, but the slope and curvature factors are distinct from commonly used factors.

I find that the Level, Slope and Curve model outperforms the Fama and French three factor and five factor models and performs similarly to three other leading models: a four factor model with momentum of Carhart (1997), the four factor model of Hou, Xue, Zhang (2012), and the four factor model of Novy-Marx (2013).

In addition, I use "horse races" to test whether adding one of the other anomaly-based factors to the Level, Slope and Curve model adds any explanatory power. I find that additional factors add very little to the Level, Slope and Curve model.

Finally, I test whether the Level, Slope and Curve factors relate to deeper models. I find evidence that the three factors proxy for priced risk in accordance with the Intertemporal Capital Asset Pricing Model. Factor returns correspond to changes in the investment opportunity set. I also find that at an annual frequency, the cross-sectional dispersion across the one dimensionally sorted portfolios and the extracted Level, Slope and Curve factors can largely be explained by their exposure to changes in consumption growth. The portfolios sorted on expected return have large spreads in their covariance with consumption growth, a prediction of the Consumption Capital Asset Pricing Model (CCAPM). A standard linearized CCAPM explains almost all of the spread in average returns across the portfolios. Since the Level, Slope and Curve model explains almost all of the variance in the expected return sorted portfolios, another interpretation is that the model represents high frequency factors mimicking changes in expected future consumption growth.

This procedure to extract priced factors makes an important contribution to the literature on the cross-section of stocks. First, Cochrane (2011) calls for a reorganization of the factor structure of stock returns. Which factors are the most important? Which factors should we be writing consumption based asset pricing models to explain? The procedure searches for the most economically important risk factors that are *priced* in the cross-section, and finds that the cross-section can be summarized by three important factors. This result stands in stark contrast to Green et al. (2014) who find staggering multidimensionality in stock returns, and suggest a factor model with twenty-four factors. Their approach follows the more traditional procedure of grouping portfolios by characteristics, but this approach allows unpriced factors to creep into the results.

Second, this paper bridges a gap between empirically generated factor models and theoretically generated factor models. The models of Fama and French (1993), Carhart (1997), and Novy-Marx (2013) are all empirically motivated factor models. The prediction of Arbitrage Pricing Theory of Ross (1976) is that expected returns should be accompanied by common factors. Novy-Marx (2013) explains the motivation, "While I remain agnostic here with respect to whether these factors are associated with priced risks, they do appear to be useful in identifying underlying commonalities in seemingly disparate anomalies." Unfortunately, the general procedure is prone to concerns about data mining. It is not clear precisely when, after observing alphas on sorted portfolios in time-series, we should add another return factor.

The traditional response to data mining concerns is to lean more heavily on theoretical motivations. The five factor model of Fama and French (2014) and the Q-factor model of Hou, Xue and Zhang (2014) are both advocated on theoretical grounds. But the specific theoretical motivations are different and will likely continue to be contentious. Hou et al. (2015) criticizes the Fama and French (2014a) on theoretical grounds and Novy-Marx (2015) criticizes the Q-factor model on theoretical grounds. More importantly, models based on these specific theories are less useful to researchers developing distinct theoretical insights to deepen our understanding of the cross-section of returns. Cochrane (2011) calls not for a perfect theory to end all debate, but rather a synthesis of data, a parsimonious description of the important factors in the cross-section that theory is meant to explain.

This paper offers a theoretically motivated search for empirical factors. This procedure bridges theory and empirics to identify the salient facts that deeper models should be trying to explain. The Arbitrage Pricing Theory *predicts* that expected returns should be associated with a factor structure. Kozak et al. (2015) show that empirical factor models don't have the ability to distinguish between rational and behavioral explanations of the cross-section of returns. This highlights a great strength of this approach. By using only the law of one price and the absence of arbitrage, the procedure synthesizes the most important factors for both rational and behavioral models to explain without taking a stand on the explanations.

Third, a strength of this paper is its description of the factor structure of returns that is not centered around firm characteristics. While much has been learned by characteristic-based sorts and described by characteristic-based factors, it is at least conceivable that the common movement across stocks is not principally caused or described by firm characteristics. In this paper, characteristics are just useful signals for identifying latent factors. The Level, Slope and Curve Model offers a lens through which to view the factor structure of the cross-section without appealing to characteristics.

There are limitations to this approach. The procedure may not find all priced factors. There may be important priced factors that are only important to a small number of stocks or there may be factors that affect many stocks, but have very small prices of risk. The procedure is also limited by the predictors used in the first stage. A strong enough predictor could change the factors found in this paper. But the method has great resilience to these changes, since a new predictor would have to be strong *in the presence of other predictors* (Fama and French, 2014b). Thus, identifying priced factors using this procedure makes it less likely that future researchers will write consumption based asset pricing models explaining false factors. The method also provides some protection against the datamining concerns of Kogan and Tian (2013). Harvey et al. (2013) show datamining may be a concern in the first stage, linking characteristics to expected returns, but this datamining need not be associated with the strong factor structure produced in the second stage.

The benefit of this approach is that it narrows the factor space once again. A primary goal of reducing a set of portfolio returns to a much smaller set of common factors is data reduction (Cochrane, 2011). After all, we do not require a theory that explains all of the comovement of the hundreds of assets used in this paper; one that explains the common factors that price them would be very useful. Since this method shows some robustness to new anomalies, theoretical work to explain the statistically extracted factors is less likely to go off course. The underlying factor structure is more stable. A disciplined empirical approach to generating priced factors can help narrow the focus of new theoretical work from many disparate characteristics to a few priced factors. While unpriced common factors may be interesting in their own right, they are unlikely to be central components connecting asset price movements to business cycle movements. They are not likely to be central puzzles in the intersection of macroeconomics and finance.

2 Literature Review

I draw on the large literature of firm characteristics that predict returns (often called anomalies). There are several papers that have used multiple regression on many predictive characteristics. Haugen and Baker (1996) use regressions on many variables to sort stocks by predicted next period returns. Fama and French (2008) show that many characteristic variables contain separate and distinct information that varies across size groups. Lewellen (2014) shows that these variables also predict returns out of sample.

Each predictor individually has information about expected returns, and portfolios sorted on each individual predictor likely has unpriced common variation. But as long as the unpriced variation is not perfectly correlated across predictors, the common signal of expected return will be reinforced and the noise will be averaged out. Zhang (2009) demonstrates this logic using principal components analysis to find factors similar to the Fama and French (1993) small minus big (size) and high minus low (value). When using principal components to extract common factors from individual stocks, he finds no evidence of common variation due to differences in size and book to market (Connor and Korajczyk, 1986, 1988), but when using principal components analysis on a 10 by 10 portfolio sort on size and book to market, he recovers the common variation due to the size and book to market factors. Sorting on a common characteristic reinforces the common signal. The difference in my approach is that I use expected return as the common signal rather than a firm characteristic. By sorting on expected returns, I reinforce the priced factors and average out the unpriced factors, allowing me to uncover the priced factors through principal components analysis.

The method can both grow with and show robustness against the growth and inclusion of new anomaly variables. Including an additional anomaly with predictive power will help generate an even sharper estimate of the true underlying factors, but it would take a very strong new anomaly to dramatically alter the factors. The anomaly must have large explanatory power even after controlling for several other strong anomalies.

Often, authors choose factors in response to the observation that there is a spread in average returns across stocks. The spread is accompanied by common movement across stocks, and the authors test to see if the comovement is priced in the cross-section. Fama and French (1993) saw persistent differences in average returns created by a double sort on size and book to market, and reasoned that the spread in value versus growth stocks holding size constant and the spread in small minus large stocks holding book to market constant could explain returns by proxying for latent priced factors. Fama and French (1996) show the model successfully prices other anomalies such as sales growth and long-term reversal. In a general sense, the method and goal of this paper are the same. I start with anomaly variables that generate large spreads in return, then identify comovement across the portfolios caused by a factor structure. Finally, I determine whether these factors can price other assets as the Arbitrage Pricing Theory predicts. The difference is that I am looking only for common factors that explain expected returns. I am not looking for common factors related to firm characteristics that may be priced or may have a priced component.

While the three factor model of Fama and French (1993) provided a workable replacement for the Capital Asset Pricing Model of Sharpe (1964) and Lintner (1965), the number of anomalies has continued to grow. The response has traditionally been to add additional factors to describe portfolio spreads unexplained by the three factor model, such as the momentum factor of Carhart (1997), the liquidity factors of Pastor and Stambaugh (2003) and Sadka (2006), and the volatility factor of Ang et al. (2006). More recently, authors have questioned the three factor model more fundamentally and produced factor models combining different anomalies, such as the four factor models of Novy-Marx (2013) and Hou et al. (2012), and later the five factor model of Fama and French (2014a). All of these factors are formed from sorts on firm characteristics or firm betas. This paper is the first to extract factors from portfolios sorted by expected returns using many characteristics.

3 Factor Structure of Anomalies

The Arbitrage Pricing Theory of Ross (1976) posits that returns are generated by an asset's loadings on common factors and an idiosyncratic term. If the APT holds, the return on security i, X_i, is the sum of its expected return (E_i) and its loadings multiplied by priced and unpriced factors:

$$X_i = E_i + \beta_{1,i}F_1 + \beta_{2,i}F_2 + ... + \beta_{N,i}F_N + \phi_{1,i}G_1 + \phi_{2,i}G_2 + ... + \phi_{N,i}G_N + \epsilon_i$$

Without loss of generality, I separate the priced factors (F) from the unpriced factors (G). An unpriced factor has a zero risk premium, for example industry factors. Idiosyncratic risk is captured by a mean zero error term (ϵ).

The loadings on the unpriced factors (ϕ) do not enter into the expected return of the asset.

$$E[X_i] = E_i = \lambda_1\beta_{1,i} + \lambda_2\beta_{2,i} + ... + \lambda_N\beta_{N,i}$$

Only the risk premiums of the priced factors (λ) and an asset's loadings on those factors (β) determine its expected return.

In the Arbitrage Pricing Theory, expected return itself is the characteristic of interest. Using this as motivation, I sort stocks by their expected returns, in order to reinforce the priced comovement across stocks and wash out the unpriced comovement. The one dimensional sort strengthens patterns created by the comovement of the portfolios related to expected returns, and weakens the patterns created by common factors with zero risk premiums. If there is only one predictor of expected returns, such as book to market, then my ability to isolate priced movements from unpriced movements is minimal. But by utilizing the expansive anomaly literature, I can sort on expected returns from several different sources. This creates large spreads in expected returns both in and out of sample.

4 Data and Variables

The sample runs from July 1964 until December 2014. The variable definitions are identical to Fama and French (2008) with two exceptions. Reacting to Novy-Marx (2013) and Ball et al. (2015), Fama and French (2014a) argue that operating profit is a more robust predictor of average returns in the cross-section than return on book equity, and Aharoni et al. (2013) show that asset growth at the firm level is a better and more theoretically motivated predictor than asset growth per share. Thus, I slightly alter the Fama and French (2008) regressions to reflect these insights and to match the definitions used in Fama and French (2014a).

Returns are monthly holding period returns obtained from the Center for Research in Security Prices (CRSP) and adjusted for delisting return when available. The accounting data is from Compustat, which has survivor bias before 1962. Since I require two years of accounting data to form some characteristics, my sample begins with stock returns in 1964. The sample includes only common equity securities (share code 10 and 11) for firms traded on NYSE, NASDAQ or AMEX. Additionally, I drop financial firms (Standard Industry Classification codes of 6000 to 6999) and stocks trading below $1. All anomaly variables are measured at the end of June using the last fiscal year's accounting data, except for momentum, which is defined monthly. The precise variable definitions can be found in Fama and French (2008) and include: size, book to market, momentum, net stock issues, accruals, investment, and profitability.[1]

[1] Size is attributable to Banz (1981), book to market to Rosenberg et al. (1985), Chan et al. (1991), and Fama and French (1992), momentum to Jegadeesh and Titman (1993), and net stock issues to Daniel and Titman (2006) and Pontiff and Woodgate (2008) following earlier work by Ikenberry et al. (1995) and Loughran and Ritter (1995). Accruals is attributable to (Sloan, 1996), profitability to Haugen and Baker (1996), Cohen et al. (2002) and Novy-Marx (2013), and investment to Fairfield et al. (2003), Titman et al. (2004) and Cooper et al. (2008).

5 One Dimensional Portfolio Sorting Procedure

In order to sort stocks by expected returns, I use a procedure that forms portfolios using many firm characteristics as predictors. Fama and French (2006) provide a logical three step procedure to do this. First, run Fama-MacBeth cross-sectional regressions of one month ahead firm-level returns on current values of the anomaly variables. Second, use the coefficient estimates from the regressions to predict the one month ahead return for each stock. Third, sort stocks into portfolios based on the predicted returns.

The goal of the procedure is to yield a portfolio sort that creates a wide spread in average returns using only information available to the investor at the time they form their portfolios. An economically significant predictor will account for a relatively large portion of the spread. Clearly, I must be explicit when I define an investor's information set. Fama and French (2006) use parameter estimates from the full sample in order to sort into portfolios. A rationale for this approach is that the whole time series best reflects the contribution of each anomaly to expected returns. Alternatively, I could use regressions only on past data to form sorts or rolling regressions that capture time varying betas as in Haugen and Baker (1996) and Lewellen (2014). Since my goal is to identify the factors, rather than trading on them, I use the full sample for my main tests, but I also show out of sample results and find that the level, slope and curve factors are not very sensitive to the choice of information set.

Each cross-sectional regression takes the following form:

$$\begin{aligned} XRet_{i,t+1} = \beta_0 + \beta_1 LogSize_{i,t} + \beta_2 LogB/M_{i,t} + \beta_3 Mom_{i,t} + \beta_4 zeroNS_{i,t} + \beta_5 NS_{i,t} \\ + \beta_6 negACC_{i,t} + \beta_7 posACC_{i,t} + \beta_8 dA/A_{i,t} + \beta_9 posOP_{i,t} + \beta_{10} negOP + e_{i,t+1} \end{aligned} \tag{1}$$

The stock return in excess of the risk free (XRet) rate for each stock in the following month is regressed on log firm size (LogSize), log book to market (LogB/M), momentum (Mom), a dummy if no stock was issued (zeroNS), net stock issues (NS), negative accruals (negACC), positive accruals (posACC), asset growth (dA/A), positive operating profit (posOP) and negative operating profit (negOP). Fama and French (2008) find that stocks of different size groups (micro, small and large) have different exposures to characteristic predictors, so I run the regression above separately for each size group allowing the parameter estimates to differ across these groups.

These sorts are very effective at generating a spread in portfolio returns. Figure 1 shows the results of the sort for each portfolio. Predicted returns, represented by the line, are produced from the fitted values of the regressions for each stock combined into a value-weighted portfolio. Average excess returns, represented by dots, are the average value-weighted returns for each portfolio.

Table 1 displays the summary statistics of the twenty-five sorted portfolios. Each portfolio characteristic is formed by the value-weighted average (using beginning of the month market equity) of each stock in the portfolio. Thus, the portfolios are not dominated by the plentiful, but tiny micro cap stocks. The sorting method creates a large spread in average excess returns (XRet) similar to the spread in predicted returns ($\widehat{\text{XRet}}$). All the multiple regression characteristics, except for size (JME), show monotonically increasing or decreasing patterns in expected returns with the sign predicted by previous research. Momentum (Mom) and investment (dA/A) show especially strong patterns, the difference between the extreme low return and extreme high return portfolios are two or more standard deviations. Net stock issues (NS) also shows a strong trend, but it is concentrated in the low return, high net issue portfolios. Accruals (A/BE) and book-to-market (B/M) both create spreads of less than one standard deviation between the high and low return portfolios. Lastly, size has a somewhat curved sort. The June month end market equity increases to a maximum at portfolio five and then decreases from portfolio five to the highest return portfolio twenty-five, consistent with the size effect. The extreme low return portfolios aren't especially dominated by small stocks. The value-weighted June market equity is still larger than half the other portfolios.

I also include two characteristics not included in the regression, return on equity (ROE) and idiosyncratic volatility (IVOL). There is not a clear trend for return on equity except to note that it is only negative at the extremes. Idiosyncratic volatility, the average daily realized volatility in the current (not preceding) month, shows a curved pattern related to, but not identical to size. Idiosyncratic volatility is largest at the each extreme and monotonically decreases from both extremes until it reaches the minimum value at portfolio nine.

In the last column, I show the sort on average excess returns from the "No Peeking" sorts (XRet-NP). I run the same Fama-MacBeth regressions using ten years to form an initial estimate and then re-estimating the regressions with an ever expanding window. The first month of portfolio formation is July 1974 using data on characteristics and returns from July 1964 to June 1974. The window expands to include each new month of data on returns and characterstics to form portfolios over the remaining sample. The spread in returns formed by the no peeking regressions is nearly as large as the full sample regressions, only 20 basis points smaller (7% smaller than the full sample spread).

6 Factor Structure of One Dimensional Sorts

The next step is to extract common factors from these portfolios. I use principal components analysis (PCA), which uses an eigenvalue decomposition to identify common factors across portfolios. By construction, the method extracts linear combinations of the test asset returns that explain the structure of the covariance matrix (Tsay, 2005). This approach translates the comovement between the test assets from a covariance matrix to uncorrelated factors. Each factor is formed as a set of weights on the test portfolios. The first factor explains the largest amount of the covariance between the portfolios. The second factor explains the next largest amount that is not captured by the first factor and so on. In total, the factors describe the entire covariance structure between test assets.

When used on a large sample of individual stocks, as in Connor and Korajczyk (1986, 1988), PCA has little power to extract useful factors from stock returns (Brennan et al., 1998), but Zhang (2009) uses portfolios to recover the underlying comovement across stocks related to their characteristics. His insight is that portfolios sorted on firm-level characteristics strengthen the patterns in stock returns related to the firm-specific pattern. Patterns in returns unrelated to the characteristics cancel out. This paper extends that insight by sorting the portfolios on expected returns, rather than the firm-level characteristics. Using PCA on these portfolios isolates the common factors that determine expected returns.

I use PCA on the twenty-five portfolios sorted from low to high by expected returns using the anomaly regressions. Table 2 shows the results of the principal components analysis. The table presents the first ten components, the respective eigenvalues and variance explained. The first three components explain 86% of the variance of the portfolios. The first component explains 74% of returns, the second explains 9% of returns, and the third explains 3% of returns. In Figure 2, I present the weightings of the first three components.

The first factor resembles a general market portfolio because it approximately equally weights all 25 portfolios. The factor has a correlation of .95 with the CRSP value weighted market index used in all popular factor models. This is a "level" factor as it represents comovement with the overall level of the market. Stocks tend to rise and fall together. The second factor is long low expected return stocks and short high expected return stocks. Weights decrease monotonically from long to short. This "slope" factor captures the feature that high expected return stocks all tend to move together and opposite low expected returns stocks, which symmetrically are also moving together. Since the slope factor is going long low expected return stocks and short high expected return stocks, on average it has a negative realization.

Most factors already identified in the finance literature are slope factors. The hml factor captures the tendency of growth stocks to move opposite of value stocks, while the smb factor captures the tendency for small stocks to move opposite of large stocks. Other examples include slope factors for momentum, profitability, investment, volatility, and liquidity.

My slope factor is different in that it captures common movement using all of the characteristics at once. The underlying characteristic of interest is expected returns and not a firm-level proxy for expected returns. While each firm-level characteristic offers some information about expected returns, portfolios built on characteristics alone may share a large degree of common movement that isn't related to expected returns.

The last "curve" factor is short the extreme low and high return portfolios and long the middle portfolios. The curvature factor shows that extreme stocks tend to move together. If the curvature factor has a positive realization, both very high and very low expected return stocks will have relatively low returns and the stocks with moderate expected returns will have relatively high returns. Altogether, the factors bear a striking resemblance to the bond factors found by Litterman and Scheinkman (1991).[2] Lord and Pelsser (2007) show that level, slope and curvature characterizes a robust fact about the variance-covariance matrix. Since any factor model can be written as a one factor model, the important point is not the number of factors that principal components produces, but that the factors yield a stable description of the variance-covariance matrix (Roll (1977) and Hansen and Richard (1987)).

Principal components are identified down to a scalar transformation of the factors, so without effecting any of the results I scale each factor to make the interpretation more intuitive. To reinforce

[2]Lustig et al. (2011) find level and slope factors in portfolios formed on the carry trade.

a portfolio interpretation, Campbell et al. (1997) suggest dividing by the sum of the loadings on each factor, so that the weights sum to one. That works well for the first factor, but creates a very unintuitive hedge fund for the slope factor. Since the factors are excess returns, the slope factor would represent borrowing $1 at the risk-free rate, and investing over $7 long and over $6 short. Since the slope factor is negative, the hedge portfolio is long low return stocks and short high return stocks and losing money at a very rapid pace. Instead I adopt a different definition of the slope and curve factors by limiting the factors to 100% short. This choice is made in the spirit of Fama and French (1993) who define their high low factors by investing $1 short and $1 long. The choice of scalar is somewhat arbitrary and made only to aide the interpretation of the factors.

In order to ascertain if the principal components analysis uncovers a true common signal, I also use the same method for sorts on ten portfolios and 100 portfolios. The results also show the level, slope and curve patterns. In Table 3, I show the correlations of the first five components using sorts on 10, 25 and 100 portfolios. The results show a very strong correlation among the first three components, regardless of the number of portfolios used. The lowest correlation is always between the component extracted from 100 portfolios and the component extracted from 10 portfolios, and for the first three components, the correlation is .993, -.955, and .842, respectively.[3] The fourth and fifth components are not nearly as correlated across sorts. For the fourth, the 100 and 10 portfolio sorts only share a correlation of .150. For the fifth component, the 100 and 25 sorts share the smallest correlation of .065. I exclude the fourth and higher factors from this study. While I make no attempt to rule out the possibility that the fourth and fifth factors represent some form of priced risk, there is at a minimum an issue with measuring that signal precisely. I only include the first three factors in this study in order to get strong common signals that are not dependent on the sorting procedure.

The factors are very stable across subsamples. Table 4 shows the results of splitting the sample into two halves and conducting principal components on each subsample. The first sample runs from July 1964 to March 1988, while the second sample runs from April 1988 to December 2012. Each subsample shows the level, slope and curve factors. Dividing the sample allows for two out of sample tests. Since I run the regression and PCA separately in each subsample, I can compare the

[3]Since the components are describing variance and are only estimated up to a scalar transformation, a sign change in the correlation coefficient has no content.

factors formed with first half weights with the factors formed with the second half weights. Thus, I have two out of sample tests. An investor forming a factor model in the first half can be taken out of sample to the second half. Alternatively, the factor model formed in the second half can be tested out of sample in the first half of the data. In the first half of the sample, when the end of sample components are out-of-sample, the in and out of sample level, slope and curve factors have correlations of 1.00, -0.92, and 0.73. In the second half of the sample, the correlations are 1.00, -0.96, and 0.82. Evidently, the factor structure of these portfolios is very stable.

These factors differ from Fama and French's three factors. Table 5 shows the correlation of the level, slope, and curvature factors with several other proposed factors. None of the three extracted factors has a correlation above 0.25 with HML. SMB is correlated at a moderate level of 0.46 with the level factor and 0.49 with the curve factor, echoing somewhat the shape in the characteristics in Table 1, but has correlations below 0.40 with slope and curve. The slope factor has a correlation of 0.77 with momentum, the strongest correlation in the table. The profitability factors, RMW, ROE and PMU have low correlations with the slope factor and with the curve factor. Investment shares a low correlation with level, slope, and curve. None of the three factors are very correlated with liquidity. The slope and curve factors are different than the factors already represented in these leading models.

7 Time-series Asset Pricing Tests

The Arbitrage Pricing Theory predicts that the wide spread of excess returns created by sorting stocks into portfolios based on their expected returns will be explained by each portfolio's loadings on common factors. Table 6 shows the results of time series regressions of the portfolio returns in excess of the risk free rate on the first one, two, three and four principal components. Since the factors are uncorrelated, the pattern in betas are captured by the loadings shown in Figure 2. The table shows the alphas, t-statistics and R-squareds from each of the four time series regressions on each of the twenty-five portfolios.

The third column α_1 shows that the large spread in returns is not captured by the first factor. This regression is almost identical to the traditional CAPM, so while it is not surprising that the alphas are not captured, it is interesting that much of the alpha shifts to the short leg. Over 60% of the alpha on the high return portfolio is explained by the first factor. The level loadings in Figure 2 actually mask a somewhat significant variation in market betas across porftolios. The extreme portfolios have loadings of 0.26 and the middle portfolios of 0.17, which form a barely perceptible curved pattern in the figure, but represent a 50% increase from the low beta middle portfolios to the high beta extremes. This curved pattern in the level beta creates the result in column three, helping the first factor capture the alpha on the high beta, high return portfolio, and increasing the alpha on the extreme low return portfolio.

The fourth column shows that the second factor explains a large portion of the one factor alpha. The alpha on the extreme high portfolio is slightly negative and insignificant. While the alpha on the extreme low portfolio has fallen over 50% from the one factor model, it remains statistically significant. The average R-squared of the twenty-five regressions rises from 76% to 83% with the addition of the slope factor, while the average alpha falls from 0.40% per month to 0.19% per month. Column five shows adding the curve factor increases the average R-squared to 86% and decreases the alphas to 0.17% per month. The fourth factor adds little additional R-squared, and while it seems to decrease some alphas, it follows a somewhat suspect zig-zag pattern that was shown earlier to be somewhat unstable.

The GRS Tests show that all four specifications are strongly rejected, not unlike Fama and French (1993). Importantly, the three factor model captures a large portion of the spread in average returns, and a large portion of the variance of the twenty-five portfolios. Perhaps unsurprisingly, the low return portfolio proves much more difficult to price than the high return portfolio given that an arbitrager must take a short position to profit off of these portfolios.

8 Time Series and Cross-sectional Asset Pricing Tests and Comparisons with Leading Models

If the APT holds and this method succeeds at extracting priced factors, the model predicts a relationship between expected returns and factor loadings. In this section, I perform a number of asset pricing tests in order to compare the Level, Slope and Curve model to leading factor models. Using a variety of test portfolios, I compare the model to the Fama and French three factor, four and five factor models, as well as the four factor models of Novy-Marx (2013) and Hou et al.

(2012).[4] I also compare the Level, Slope and Curve model to the no peeking expanding window version of itself, which is first available in July 1974.

The goal of this paper is to introduce a new method to summarize the cross-section of stocks. To this end, I will test the Level, Slope and Curve model against a large variety of benchmark models with both time series and cross-sectional test designs to try to give a comprehensive view of how well the model captures important features of the cross-section and how that relates to other leading models. Since all the factors are tradeable, the time series tests impose the requirement that the factors price themselves without error. I compare each model's ability to price a range of assets and ultimately summarize the results by calculating the Hansen and Jagannathan (1997) distance for each model.

The cross-sectional tests relax this assumption allowing the price of risk to deviate from the average in sample return of the factor. Lewellen et al. (2010) point out a number of problems with cross-sectional asset pricing tests, especially when only twenty-five portfolios of size and book to market are used for test assets. If the test portfolios have a strong factor structure, the cross-sectional asset pricing tests may not be informative. They show that including many diverse test assets relaxes the factor structure and creates more informative asset pricing tests.

In my tests, I include a diverse set of portfolios in order to relax the factor structure and attain more informative results. These cross-section tests have three testable implications. The R-squared of the cross-sectional regression should be close to 1, as the assets should be priced by the factors. The constant term should be close to zero, as the constant return represents the zero-beta rate, which should be near the risk free rate. The coefficients of the cross-sectional regressions should be near the average return on the factors, as the coefficient should equal the cross-sectional risk premium.

8.1 Factors and Test Assets

The Fama and French three factor model (Fama and French, 1993) uses the market portfolio and two hedge portfolios, one long high book to market stocks and short low book to market stocks (HML) and the other long small stocks and short large stocks (SMB). The Fama and French four

[4] I would like to give special thanks to each author for generously sharing their factors. I obtained the Fama and French factors (including momentum) from Ken French's data library. I obtained the Novy-Marx four factor model from his data library. Chen Xue shared the Q-Factor model through email correspondence.

factor model, also called the Carhart (1997) model, adds a momentum factor (MOM), long stocks that have risen over the last 12 months and short stocks that have fallen over the last 12 months. The Fama and French (2014a) five factor model excludes momentum and includes a factor long low investment stocks and short high investment stocks (CMA) and a factor long high profit stocks and short low profit stocks (RMW). Since both the five factor models and three factor models use multidimensional sorts to form factors, the SMB and HML differ across the two models. I use the appropriate version for each. The Novy-Marx four factor model uses the market portfolio combined with hedge portfolios of industry adjusted value (HML), momentum (UMD) and gross profitability (PMU). The Hou, Xue and Zhang four factor model uses the market portfolio combined with hedge portfolios on size (SIZE), investment (INV) and profitability measured by return on equity (ROE). In the asset pricing tests, the use of the Novy-Marx and Hou, Xue, Zhang model restrict the sample to end December of 2012.

For test assets, I use two groups, one consisting of 119 portfolios of stocks and bonds and the other consisting of 32 "hedge" portfolios formed as high minus low, long/short portfolios formed from the extreme deciles of stock characteristic sorts. For the 119 portfolios, I use ten portfolios formed by the results of the "No Peeking Dissecting Anomalies" regressions in Section 4, twenty-five portfolios sorted on size and book-to-market, 10 portfolios sorted on momentum, returns on five treasury bonds (1 year, 5 year, 10 year, 20 year and 30 year), forty-nine industry portfolios, ten portfolios formed on operating profit and ten portfolios formed on investment (asset growth).[5]

The second group of test assets consists of 32 hedge portfolios formed by decile sorts on 32 anomalies by Novy-Marx and Velikov (2013).[6] Each hedge portfolio is formed by sorting on a characteristic and then taking a long position in one extreme decile and an equal short position in the other extreme decile. The portfolios are formed on sorts on size, gross profitability, value, value-profitability (combined), accruals, asset growth, investment, Piotroski's f-score, net issuance, return on book equity, failure probability, value-profitability-momentum (combined), value-momentum (combined), idiosyncratic volatility, momentum, standardized unexpected earnings, earnings surprise, industry momentum, industry relative reversals, industry relative reversals combined with industry momentum, short-term reversals, and low volatility industry relative reversals. Detailed

[5]I obtain the portfolios formed on size and book to market, momentum, industries, operating profit, investmen and the Fama and French factors from Ken French's website. I obtain the bond portfolio returns from CRSP.

[6]I obtain these portfolios from Robert Novy-Marx's data library and I thank the authors for making them available.

descriptions of each hedge portfolio are available in the appendix of Novy-Marx and Velikov (2013). The diverse set of portfolios serve to give a holistic look at how well the models capture the cross-section of expected returns.

8.2 The Level, Slope and Curve Model vs. The Fama and French Three Factor Model

First, I compare the level, slope and curve model with the Fama and French three factor model using time series tests. The comparison with the three factor model serves to capture what Cochrane (2011) refers to as the world, “once again descending into chaos.” Figure 3 displays graphically the results of time series tests of each of 119 test portfolios on the respective three factor models. In each panel, X-axis is the predicted return of the model, the betas times the average return on the factor for each test asset. The Y-axis is the average return. The pricing error (alpha) is the vertical distance of each graphed test asset from the 45-degree line. Perfect pricing with zero alphas would put each test asset on the 45-degree line. The right panel, the results with the Fama and French three factor model, captures the chaos that has befallen the cross-section. The left panel shows that without increasing the number of factors the Level, Slope and Curve model captures a large amount of the spread in average returns, effectively reorganizing the cross-section. A summary statistic for the graphical relationships is the cross-sectional R-squared (while still imposing the time series restrictions), which is equivalent to the squared correlation of average returns and predicted returns. The R-squared for the Fama and French three factor model is 12%, while it is 56% for the Level, Slope and Curve model. Figures 4 through 10 break apart this picture, showing each group of assets individually.

Figure 4 shows the pricing errors of the two models tested against the 10 “No Peeking Dissecting Anomaly” portfolios formed using expanding window regressions of returns on seven firm characteristics. The Level, Slope and Curve model captures almost all the spread in average returns, another victory for the Arbitrage Pricing Theory.[7] The Fama and French three factor model shows fairly large pricing errors, with just a little ability to price some of the high expected return portfolios.

[7]Despite the seeming success of the model, the GRS test of Gibbons et al. (1989) rejects the model on these ten portfolios with a p-value on the order of 10^{-7}. With the understanding that all the models are to some degree misspecified, I for the most part omit the GRS tests.

Figure 5 shows the big success of the Fama and French three factor model, pricing twenty-five portfolios built on size and book to market. Each test asset is labeled by it's sorted portfolios with the size quintile listed first (smallest to largest) and the book to market quintile listed next (growth to value). The small growth portfolio (SBM11) clearly confounds both factor models. While the Level, Slope and Curve model captures a large spread in the average returns, it leaves alphas on the extreme value portfolios (SBM15, SBM35) and doesn't price the second to smallest growth portfolio (SBM21) as well as the Fama and French three factor model.

Figure 6 shows a stark contrast between the two models. The Level, Slope and Curve model captures all of the spread in momentum and then some, slightly reversing the momenum anomaly. The return to high past return stocks (MOM10) aren't quite high enough to justify their factor loadings. The Fama and French three factor model actually has pricing errors larger than the spread in average returns. Figure 7 shows the two models on portfolios formed on 49 industries. While there is less spread in average returns to price in the industry portfolios (notice the decreasing axes), the Level, Slope and Curve model has smaller pricing errors than the Fama and French three factor model. The average absolute alpha is 15 and 21 basis points for the two models, respectively.

Figure 8 shows the two models tested on U.S. Treasuries ranging from one to thirty years in duration. The Level, Slope and Curve model shows a clear spread in predicted returns that begins to line up with average returns. The spread comes from significant positive loadings on the curve factor for long-term bonds, suggesting that long-term bonds behave similarly to large, low volatility stocks. The Fama and French three factor model doesn't produce much of any spread in the predicted returns of treasury bonds.

Figure 9 shows the two models tested on ten portfolios formed on operating profit. Most of the alphas are actually created by the models as the spread in the average returns from the most profitable decile to least profitable decile is only 26 basis points. That the lowest profit stocks (OP1) has a high level and market beta serves to confound both models, and while it sticks out as an outlier in the left panel, average returns on the other nine deciles line up reasonable well with predicted returns. In the right panel, both low and high profit stocks confound the three factor model, such that the ranking of predicted returns is nearly the inverse of the ranking of average returns.

The ten investment portfolios in Figure 10 show similar results for both models. The average absolute alpha for both models is 10 basis points. The high investment portfolio (INV1) isn't quite captured by either model with the two models leaving -26 and 22 basis points unexplained, respectively. The horizontal spread between the high investment portfolio (INV1) and low investment portfolio (INV10) shows that the Level, Slope and Curve model is generating a spread in predicted returns. In this case, a slightly higher level beta for the high investment portfolio is being offset by a much larger slope beta.

8.3 Level, Slope and Curve Model Vs. Leading Factor Models

The Level, Slope and Curve Model performs very well versus the Fama and French three factor model. The comparison is important, because of the preeminence of the model's status as the default method for risk adjustment over the last two decades, and also, because the two models have the same number of factors. I also test the Level, Slope and Curve model versus more recent models that use additional factors to price assets. I find that the Level, Slope and Curve model, despite having fewer factors, performs comparably and often better than other leading models. The panel A of Table 7 shows the results of time series tests on the set of 119 test portfolios comparing the Level, Slope and Curve model (LSC) and the out of sample Level, Slope and Curve No Peeking model (LSC-NP) to the Capital Asset Pricing Model (CAPM), the Fama and French three factor model (FF3), the Carhart model (CAR), the Fama and French five factor model, the Novy-Marx four factor model (RNM) and the Hou, Xue, and Zhang four factor model (HXZ). The first row shows that the average absolute alpha of the Level, Slope and Curve model is the lowest of all models at 15 basis points and only equaled by the Carhart model. The No Peeking LSC model has only slightly higher alphas of 16 basis points. The CAPM has the highest average absolute alphas of 23 basis points, followed by the Carhart and Hou, Xue, Zhang models at 20 basis points.

The next row shows the number of t-statistics significant at the 5% level of confidence. While not a rigorous comparison of the models, this serves to capture what researchers mean by "model X explains Y anomaies," especially in the next set of assets formed on anomaly hedge portfolios. The Level, Slope and Curve model and its expanding window counterpart leave the least significant alphas. The next row shows the average R-squared on the time series regression. This isn't a test of the model. The correct model of expected returns need not explain time series variation, but it may be interesting none the less to know how well each model captures the time series variation.

The next row shows last the cross-sectional R-squared of the time series tests. That is, it is a summary statistic for the percent of variation in average returns captured by the models predicted returns. It acts as a summary statistic for the evidence presented graphically in Figures 3 through 10. The Level, Slope and Curve models both explain the largest spread in average returns with R-squared of 56% and 51%. The next highest are the Carhart model at 48% and the Novy-Marx model at 43%.

I summarize the results of the time series test by computing the Hansen and Jagannathan (1997) distance (HJ). The HJ distance is defined as:

$$HJ = \sqrt{\alpha'(E[RR']^{-1})\alpha}$$

The alphas are the the pricing errors in the time series regressions and the middle term is the inverse of the second moment matrix of the test asset returns, that will be estimated by its sample counterpart. Unlike the GRS test, the HJ distance doesn't use the covariance matrix of the estimated pricing errors for inference. A model can't lower it's HJ distance just by enlarging its standard errors. The GRS test often easily rejects very good models for not being perfect and fails to reject very bad models that have large pricing errors and estimates them with great uncertainty. Scaling the alphas by the same second moment matrix for all the models makes the HJ distance well suited to compare models, even acknowledging that all the models may be misspecified to some degree.

The bottom half of panel A shows the HJ distances for each model estimated on the 119 test portfolios. I report the HJ distance for all the assets, as well as for the 114 stock portfolios, omitting the bond portfolios. The Level, Slope and Curve model is a smaller HJ distance than five of the six leading models. Only the Novy-Marx model produces a lower HJ distance. The no peeking version performs better than four of the six models with only the Fama and French five factor model out performing it. The next two rows show that the superior performance of the other models is driven by the bond portfolios. Within the subset of stock portfolios, both the Level, Slope and Curve model and its expanding window counter part have lower HJ distance than all of the other leading models.

Panel B displays the results for the cross-sectional tests. For the cross-sectional tests, I follow the Black et al. (1972) two-step approach. First, I estimate the full-sample betas of each test asset on the level, slope and curve factors using time series regressions, then I regress the average returns on the estimated betas. The regression estimates the risk premium associated with each factor. If the risk premium is significantly different from zero, the factor is priced. A high R-squared indicates the spread in average returns is explained by the spread in the betas of the test assets on the common factors. I estimate the model with an intercept term. The model predicts the intercept term should be close to zero as the zero-beta rate should be close to the risk free rate. Since an arbitrager would have to borrow at the risk-free rate and buy a zero beta portfolio to profit from a spread in the risk-free and zero beta rate, the two rates will only be equal if the arbitrager can borrow at the risk free lending rate (Brennan, 1971). Because the error terms may be cross-sectionally correlated, I report coefficients and t-statistics using Fama and MacBeth (1973) regressions. I report the R-squared statistics from the ordinary least squares regressions.

The first two columns of Table 7, panel B show the results of the two step procedure for the Level, Slope and Curve models. For the Level, Slope and Curve model, each factor risk premium is large and statistically significant. The first factor, which is highly correlated with the market portfolio, generates a factor risk premium of 0.50% monthly, statistically significant at the 10% level and reasonably close to the historical excess return of the level factor over the time period of .89%. The second factor generates a risk premium of -0.93% per month, statistically significant at the 1% level and close to the factors average return of -1.35%. The third factor, curvature, is associated with a risk premium of 0.41% monthly, statistically significant at the 5% level and similar to the factor's historical average excess return of 0.52%. Since the test assets are excess returns, the APT implies that the constant term should be close to zero. The constant term estimate is 0.31% and also statistically significant. This implies a difference in the zero-beta rate and the risk free rate of 31 basis points monthly or 4.65% annually. The 56% R-squared implies that the model captures a large amount of the cross-sectional spread in risk. The no peeking model performs comparably with similar risk premiums measured with similar precision. The average returns on the factors in the no peeking model are 0.87%, -0.99% and 0.52%, differing slightly from the full sample version.

On several important dimensions, the Level, Slope and Curve models perform well compared to the other leading models in the cross-sectional tests. The factors always produce statistically significant risk premiums (at least at the 10% level), which no other model accomplishes. The level factor has a risk premium different than zero, which five of the size models don't accomplish. Even though all of the models have factors represented by test assets sorted on similar characteristics, only the momentum factor consistently produces a risk premium. HML only produces a risk premium in Novy-Marx's model, while the size factor has a positive price of risk in two of the four models that employ it. Investment is significant at the 1% level for the Fama and French five factor model, but not significant in Hou, Xue and Zhang's Q-factor model. Profitability is accompanied by a risk premium in the Hou, Xue, Zhang model, but not in Novy-Marx's model or the five factor model.

Finally, I summarize the cross-sectional tests with Hansen and Jagannathan distances for each model. Kan and Robotti (2009) develop a method to compare models using the HJ distances and produce asymptotically valid confidence intervals under the assumption that factors and returns are multivariate elliptically distributed.[8]

The last six results for panel B show the results for the tests of cross-sectional HJ distance. Again, I report results for the whole sample and the subset of stocks. The first row shows the HJ distance for each model when the prices of risk are chosen to minimize the HJ distance. The next row is the difference between the Level, Slope and Curve model and the comparison models. Positive squared distances mean the Level, Slope and Curve model has a lower HJ distance. The third row gives a p-value for the hypothesis test that the two models are equal.

The CAPM, Fama French three factor model and the Carhart model all have higher HJ distances than the Level, Slope and Curve model, statistically significant at the 1% level. The Fama French five factor model, which benefits substantially from allowing its risk premiums to differ from the average returns on the factor is not statistically distinguishable from the Level, Slope and Curve model. The Novy-Marx model is the only model outperforming the Level, Slope and Curve model in the whole sample, with a statistically significantly lower HJ distance, but again the distance evaporates in the stock subsample. The Hou, Xue and Zhang model shows the opposite pattern, under performing the Level, Slope and Curve model in the full sample, but not statistically distinguishable in the subsample.

Broadly, the results for the 32 hedge portfolios presented in Table 8 are similar. In panel A,

[8] The method calls for estimating the factor risk premiums to minimize the HJ distance, which in general, differs from the ordinary least squares estimate (Kan et al., 2013). I thank the authors for making the code available on Cesare Robotti's and Raymond Kan's website for the related empirical papers Gospodinov et al. (2015) and Gospodinov et al. (2014) that (among many other contributions) demonstrate the method.

the Level, Slope and Curve model and its no peeking counterpart have the lowest average absolute alphas with an average of 28 basis points. The two models also have the lowest number of anomalies left "unexplained" with only 8 and 10 statistically significant t statistics. The low number is not just due to explaining large average returns on hedge portfolios, sometimes it is due to not creating large alphas where there is little to no average returns to explain. The Novy-Marx model seems particularly susceptible to this with average absolute alphas even higher than the CAPM and more significant anomalies. The hedge portfolios of asset turnover and idiosyncratic volatility make good examples. The hedge portfolio on asset turnover returns over the sample -1 basis point, but due to it's high loading on PMU has a predicted return of 72 basis points for an alpha of -73 basis points. A 26 basis point average return on portfolios sorted by idiosyncratic volatility is turned into an -88 basis point alpha due to high loadings on HML and PMU that aren't offset by a market beta of -0.5.

The average R-squared of the time series tests are lowest for the Level, Slope and Curve model showing that unlike a pure APT model, it doesn't need to explain time variation to effectively price assets. The next row shows the two models capture by far the most spread in average returns with their predicted returns. At the bottom of panel A, the time series HJ distances are smallest for the Level, Slope and Curve models. The last row shows the squared differences in the HJ distances. That all the numbers are positive demonstrates all six benchmark models have higher HJ distances than the Level, Slope and Curve model.

Panel B shows that again the three factors in the Level, Slope and Curve models all generate risk premium estimates different than zero at a confidence level of at least 5%. Similar inconsistencies arise as before with the factor risk premiums in the other models. Market beta only caries a positive risk premium statistically different than zero in two of the benchmark models. Value and investment never generate risk premiums and profitability only carries a risk premium in one of the three models its used. The zero beta rates are lowest for the Level, Slope and Curve models, but peaking at 28 basis points are a reasonable magnitude for all the models. The cross-sectional R-squared is highest for the full sample Level, Slope and Curve model, but the no peeking version is bested by the Carhart and Novy-Marx model. The cross-sectional test effectively strips both models down to the market factor and momentum. Since the cross-sectional R-squared is adjusted for the number of factors, three of the models end up with negative values.

The HJ distances, differences in squared differences and p-values summarize the cross-sectional results. Only the HJ distance for the Novy-Marx model is ever so slightly lower than the Level, Slope and Curve model, a difference possibly due to sample error. The no peeking version has a smaller HJ distance than the remaining models. A hypothesis test on the differences rejects the hypothesis that the differences in the Level, Slope and Curve model and the Fama and French three factor model, the Carhart model and the Fama and French five factor model is due to sampling error at the 5% level of significance.

8.4 Horse Races of All Factors

A central question remains, which factors are important in explaining the cross-section of returns? Which factors provide marginal explanatory power in the presence of other factors? The goal of this paper is to organize the many disparate characteristics and factors into a parsimonious factor model of expected returns. If all or many of the factors in the literature can be boiled down to a much more parsimonious representation, the space left to explain with theory is dramatically reduced. If the Level, Slope, and Curve model is a better representation of the latent factor structure in returns, then it should drive out other factors. Other factors may just be some combination of level, slope and curve and potentially several unpriced factors.

I follow the procedure in Cochrane (2005) to conduct factor horse races. I run ordinary least squares regressions with returns on each individual asset pricing factor. When the estimated coefficient is added to a cross-sectional asset pricing test with other factors, the resulting coefficient estimate yields the marginal significance of the factor. If a factor is insignificant, it adds little explanatory power to the model. In the spirit of a fair race, I use as the benchmark the no peeking version of the Level, Slope and Curve model, though the results change very little using the full sample version of the model. Tables 9 and 10 present the results starting with the three factor Level, Slope and Curve model.

Table 9 shows the resulting horse race using the 119 test portfolios. The leftmost column shows the results with the Level, Slope and Curve model. First, I add the four non-market factors from the Fama and French five factor model HML, SMB, CMA and RMW. When univariate betas of the factors are added to the model, none of the factor risk premiums are significantly different than zero. While the HML and SMB factors from the three factor model differ somewhat than their five factor counterparts, the same results hold (not shown). The momentum factor added next is also driven out by Level, Slope and Curve. The next three factors from the Novy-Marx model UMD, PMU and HML similarly don't help price the 119 test portfolios. Lastly, neither ROE nor INV from the Hou, Xue and Zhang model add to the Level, Slope and Curve model.

Examining the Level, Slope and Curve risk premiums shows some interesting patterns. First, in all specifications, the slope beta has a risk premium significantly different than zero at the 1% level. The curve factor seems to compete with RMW and INV as it no longer has a significant price of risk when those factors are added. Lastly, market beta (level) is driven out when SMB is added and when ROE is added. No factor adds much to the reported adjusted R-squared, adding no more than 1%.

Table 10 shows the same horse race using the 32 hedge portfolios of the extreme deciles of characteristic sorts. Again, the addition of the four Fama and French factors change the results very little as all three factors remain significant and no Fama and French factor is significant. The pattern persists for momentum, the Novy-Marx factors and the Q-factors. The level risk premium is fairly resilient, only competing with the two investment factors CMA and INV. The slope factor remains significant in eight of the ten tests, but the factor risk premium is no longer significant when the two momentum factors are added. The curve factor competes with the profitability factors and SMB. None of the additional factor increase the adjusted R-squared more than 1%. Eight of the ten additional factors decrease the adjusted R-squared.

Across both tests, the Level, Slope and Curve model performs very well. The three factors are consistently priced in both tests. Additional factors add little explanatory power to the Level, Slope and Curve model. The slope factor is clearly related to momentum and the curve factor somewhat related to sorts on profitability and size as was evidenced in Tables 1 and 5. Overall, the horse races suggest that the Level, Slope and Curve model largely succeeds at the goal of summarizing the key features of the cross-section of stock returns.

9 ICAPM Interpretation

While the methodology in this paper is general enough to find pricing factors consistent with a wide range of pricing models, the asset pricing literature has stressed interpretations of empirical factor models through the lens of the Intertemporal Capital Asset Pricing Model of Merton (1973), at least since Fama and French (1996) suggest this interpretation for their three factor model. While I don't stress an ICAPM interpretation above any other, the relationship between the level, slope and curve factors to well established state variables is still of great interest. Petkova (2006) shows a simple way to embed pricing factors into an ICAPM in the style of Campbell (1996). First, she sets up a Vector Autoregression model to capture the relationship between the state variables and the market return, as well as the predictability of each state variable. Then, she tests whether changes in the pricing factors proxy for innovations in the economic state variables.

I specify the following VAR model:

$$\begin{bmatrix} R_{M,t} \\ DIV_t \\ TERM_t \\ DEF_t \\ RF_t \\ SVAR_t \\ Lev_t \\ Slp_t \\ Cur_t \end{bmatrix} = A \begin{bmatrix} R_{M,t-1} \\ DIV_{t-1} \\ TERM_{t-1} \\ DEF_{t-1} \\ RF_{t-1} \\ SVAR_{t-1} \\ Lev_{t-1} \\ Slp_{t-1} \\ Cur_{t-1} \end{bmatrix} + u_t$$

The first term $R_{M,t}$ is the excess return on the market defined as the value-weighted return on the CRSP index less the risk-free rate from Ken French's data library. The remaining state variables are dividend to price, the term spread, the default yield and stock variance. These state variables are obtained from the Goyal and Welch data library on Amit Goyal's website.[9]

The dividend to price is defined as the log of the trailing sum of the 12 month dividends minus the log month end value of the CRSP index. The term spread is the *U.S. Yield on Long-term United States Bonds* series from NBER's Macrohistory database minus the *3-Month Treasury Bill: Secondary Market Rate* from the research database at the Federal Reserve Bank at St.Louis (FRED). The default spread is the difference between the yield on BAA- from FRED and the

[9]Special thanks to Amit Goyal and Ivo Welch for making this data available and keeping it updated. The data is available at [http://www.hec.unil.ch/agoyal/].

long-term U.S. Yield (defined identically to the term spread). The stock variance variable is the sum of squared daily returns on the S&P 500 for the month.[10] Additionally, the excess returns on the Level, Slope and Curve factors are included in the VAR system as potential state variables. The error term u_t is a vector of innovations, unpredicted changes in state variables. The question is whether the Level, Slope and Curve factors are good proxies for these unexpected innovations in state variables.

Following Petkova (2006), I orthogonalize each innovation to the excess return on the market, and scale the innovation so the variance is equal to the market. Table 11 shows the results of the innovations in predictive variables regressed on the Level, Slope and Curve factors. The t-statistics are corrected for heteroskedasticity and autocorrelation with Newey-West regression using five lags. Table 11 shows that Level, Slope and Curve are all significantly correlated with innovations in dividend yield. The slope factor and curve factors are negatively associated with innovations in dividend yield. Since a decrease in the dividend yield is associated with lower future returns, the slope factor does best (other things equal) when expected future returns are low. Recall the slope factor is long low return stocks and short high return stocks, so that an increase in the slope factor means low return stocks are doing better versus high return stocks. Low return stocks respond positively to decreases in the dividend yield (deterioration in the investment opportunity set). This suggests a good beta, bad beta interpretation for the slope factor. After controlling for the market return, low return stocks do well when future expected returns are relatively low (good beta). High return stocks do relatively better, when future expected returns are relatively high (bad beta).

None of the factors seem to be associated with large moves in the term spread or risk-free rate. The curve factor is associated with an increase in the default spread. The curve factor has greater returns when the default spread increases. The curve factor is long larger, often more profitable and less volatile stocks. These stocks do relatively better, when the default spread is higher. Thus, this finding is similar to Petkova (2006) that SMB has a negative association with the default spread.

Lastly, increases in the slope and curve factors are associated with increases in monthly stock variance. Low return stocks do relatively well when future stock variance is higher than expected. Since high stock variance is relatively bad for the investment opportunity set, low return stocks

[10]Detailed definitions are available at Amit Goyal's website [http://www.hec.unil.ch/agoyal/docs/AllTables2013.pdf].

again act as a hedge for negative shocks to investors.[11] The curve factor's exposure implies that large, low volatility stocks outperform small volatile stocks when there is shock to market volatility. Taken together, the slope and curve factors seem to capture risk relative to the marginal investor's opportunity set consistent with an ICAPM interpretation.

10 Consumption Based Asset Pricing

A considerable body of theoretical work argues that returns across assets should be explained by an asset's covariance with investor consumption. Assets that covary positively with consumption increase the investor's consumption volatility and therefore should have lower prices and higher returns than assets covarying negatively with consumption that act as insurance for the investor.[12] Empirical research has found little support for the consumption based approach.[13] In contrast to the thrust of the literature, Jagannathan and Wang (2007) find that annual returns matched with annual consumption growth measured in the fourth quarter provides surprisingly successful evidence in favor of the linearized consumption based model (CCAPM). The Fama and French 25 size and book to market portfolio show a spread in average returns largely explained by each portfolio's covariance with consumption.

I follow their methodology for the ten Dissecting Anomaly portfolios and the Level, Slope and Curve factors. Figure 11 displays the results. Average returns, on the y-axis, are regressed on covariance with consumption growth on the x-axis. The resulting regression line is displayed, with most of the portfolios and factors fitting tightly around the line. The adjusted R^2 is 89%, matching the smooth linear pattern in the graph.

The x-intercept is not statistically significant -4.3% (t-statistic of -0.85). The coefficient is statistically different from zero suggesting that consumption is priced in the cross-section. The equity premium puzzle remains as the coefficient on consumption growth is 154 (t-statistic 3.29). But the important point is that the line passing near the origin and near the level factor (a proxy for the market), also crosses through nearly all the other test assets. The cross-sectional premium is largely explained by each asset's covariance with consumption. From this perspective, the level,

[11] (Campbell et al., 2012) report a similar finding in a different setting.
[12] Rubinstein (1976), Lucas Jr (1978) and Breeden (1979)
[13] Hansen and Singleton (1982), Hansen and Singleton (1983) and Hansen and Jagannathan (1997)

slope and curve factors are consumption mimicking portfolios. The sorts on expected returns also create large spreads in consumption betas. Level, slope and curve then act as summary statistics for these sorted portfolios.

Much of the failure of the CCAPM may be related to a small number of stocks. The one dimensional characteristic sorts pre-dominantly used in the literature may be poor proxies of expected returns with much less stable covariance with consumption. The success of the CCAPM suggests that the Level, Slope and Curve model not only provides factors consistent with Arbitrage Pricing Theory, the resulting factors show a strong relationship between risk and return as measured by macroeconomic data.

11 Conclusion

This paper develops a new method for extracting the priced factors in the cross-section of stock returns. The first step is using cross-sectional regressions on many predictive variables to sort stocks into portfolios from high return to low return. The second step is using principal components to extract factors from these portfolios. The goal of this approach is to sort portfolios on expected returns and then extract factors related to expected returns. The resulting factors are level, slope and curve as the loadings form this familiar pattern over the test assets. I perform asset pricing tests using the Level, Slope and Curve model compared to several leading models. I find that the model performs very well, despite having only three factors. Horse races show that adding additional factors adds very little additional explanatory power. The factors have compelling relationships with the ICAPM and CCAPM, suggesting a deeper relationship with the Level, Slope and Curve Model and systematic risk.

References

Aharoni, G., Grundy, B., and Zeng, Q. (2013). Stock returns and the miller modigliani valuation formula: Revisiting the fama french analysis. *Journal of Financial Economics*, 110(2):347–357.

Ang, A., Hodrick, R. J., Xing, Y., and Zhang, X. (2006). The cross-section of volatility and expected returns. *The Journal of Finance*, 61(1):259–299.

Ball, R., Gerakos, J., Linnainmaa, J. T., and Nikolaev, V. (2015). Deflating profitability. *Journal of Financial Economics*.

Banz, R. W. (1981). The relationship between return and market value of common stocks. *Journal of financial economics*, 9(1):3–18.

Black, F., Jensen, M., and Scholes, M. (1972). The capital asset pricing model: Some empirical tests.

Breeden, D. T. (1979). An intertemporal asset pricing model with stochastic consumption and investment opportunities. *Journal of financial Economics*, 7(3):265–296.

Brennan, M. J. (1971). Capital market equilibrium with divergent borrowing and lending rates. *Journal of Financial and Quantitative Analysis*, 6(05):1197–1205.

Brennan, M. J., Chordia, T., and Subrahmanyam, A. (1998). Alternative factor specifications, security characteristics, and the cross-section of expected stock returns. *Journal of Financial Economics*, 49(3):345–373.

Campbell, J. Y. (1996). Understanding risk and return. *Journal of Political Economy*, pages 298–345.

Campbell, J. Y., Giglio, S., Polk, C., and Turley, R. (2012). An intertemporal capm with stochastic volatility. Technical report, National Bureau of Economic Research.

Campbell, J. Y., Lo, A. W.-C., MacKinlay, A. C., et al. (1997). *The econometrics of financial markets*, volume 2. princeton University press Princeton, NJ.

Carhart, M. M. (1997). On persistence in mutual fund performance. 52(1):57–82.

Chan, L. K., Hamao, Y., and Lakonishok, J. (1991). Fundamentals and stock returns in japan. *The Journal of Finance*, 46(5):1739–1764.

Cochrane, J. H. (2005). *Asset pricing*, volume 1. Princeton university press Princeton.

Cochrane, J. H. (2011). Discount rates. 66:1047–1108.

Cohen, R. B., Gompers, P. A., and Vuolteenaho, T. (2002). Who underreacts to cash-flow news? evidence from trading between individuals and institutions. *Journal of Financial Economics*, 66(2):409–462.

Connor, G. and Korajczyk, R. A. (1986). Performance measurement with the arbitrage pricing theory: A new framework for analysis. *Journal of financial economics*, 15(3):373–394.

Connor, G. and Korajczyk, R. A. (1988). Risk and return in an equilibrium apt: Application of a new test methodology. *Journal of Financial Economics*, 21(2):255–289.

Cooper, M. J., Gulen, H., and Schill, M. J. (2008). Asset growth and the cross-section of stock returns. *The Journal of Finance*, 63(4):1609–1651.

Daniel, K. and Titman, S. (1997). Evidence on the characteristics of cross sectional variation in stock returns. *The Journal of Finance*, 52(1):1–33.

Daniel, K. and Titman, S. (2006). Market reactions to tangible and intangible information. *The Journal of Finance*, 61(4):1605–1643.

Fairfield, P. M., Whisenant, J. S., and Yohn, T. L. (2003). Accrued earnings and growth: Implications for future profitability and market mispricing. *The Accounting Review*, 78(1):353–371.

Fama, E. F. and French, K. R. (1992). The cross-section of expected stock returns. 47:427–465.

Fama, E. F. and French, K. R. (1993). Common risk factors in the returns on stocks and bonds. 19:3–29.

Fama, E. F. and French, K. R. (1996). Multifactor explanations of asset pricing anomalies. *The journal of finance*, 51(1):55–84.

Fama, E. F. and French, K. R. (2006). Profitability, investment and average returns. 82:491–518.

Fama, E. F. and French, K. R. (2008). Dissecting anomalies. 63:1653–1678.

Fama, E. F. and French, K. R. (2014a). A five-factor asset pricing model. *Journal of Financial Economics*.

Fama, E. F. and French, K. R. (2014b). Incremental variables and the investment opportunity set.

Fama, E. F. and MacBeth, J. D. (1973). Risk, return, and equilibrium: Empirical tests. *The Journal of Political Economy*, pages 607–636.

Gerakos, J. and Linnainmaa, J. T. (2014). Dissecting factors. *Fama-Miller Working Paper*, pages 12–18.

Gibbons, M. R., Ross, S. A., and Shanken, J. (1989). A test of the efficiency of a given portfolio. *Econometrica: Journal of the Econometric Society*, pages 1121–1152.

Gospodinov, N., Kan, R., and Robotti, C. (2014). Misspecification-robust inference in linear asset-pricing models with irrelevant risk factors. *Review of Financial Studies*, 27(7):2139–2170.

Gospodinov, N., Kan, R., and Robotti, C. (2015). On the properties of the constrained hansen–jagannathan distance. *Journal of Empirical Finance*.

Green, J., Hand, J. R., and Zhang, F. (2014). The remarkable multidimensionality in the cross section of expected us stock returns. *Available at SSRN 2262374*.

Hansen, L. P. and Jagannathan, R. (1997). Assessing specification errors in stochastic discount factor models. *The Journal of Finance*, 52(2):557–590.

Hansen, L. P. and Richard, S. F. (1987). The role of conditioning information in deducing testable restrictions implied by dynamic asset pricing models. *Econometrica: Journal of the Econometric Society*, pages 587–613.

Hansen, L. P. and Singleton, K. J. (1982). Generalized instrumental variables estimation of non-linear rational expectations models. *Econometrica: Journal of the Econometric Society*, pages 1269–1286.

Hansen, L. P. and Singleton, K. J. (1983). Stochastic consumption, risk aversion, and the temporal behavior of asset returns. *The Journal of Political Economy*, pages 249–265.

Harvey, C., Liu, Y., and Zhu, H. (2013). ... and the cross-section of expected returns. *Available at SSRN 2249314*.

Haugen, R. A. and Baker, N. L. (1996). Commonality in the determinants of expected stock returns. *Journal of Financial Economics*, 41(3):401–439.

Hou, K., Xue, C., and Zhang, L. (2012). Digesting anomalies: An investment approach. Technical report, National Bureau of Economic Research.

Hou, K., Xue, C., and Zhang, L. (2015). A comparison of new factor models.

Ikenberry, D., Lakonishok, J., and Vermaelen, T. (1995). Market underreaction to open market share repurchases. *Journal of financial economics*, 39(2):181–208.

Jagannathan, R. and Wang, Y. (2007). Lazy investors, discretionary consumption, and the cross-section of stock returns. *The Journal of Finance*, 62(4):1623–1661.

Jegadeesh, N. and Titman, S. (1993). Returns to buying winners and selling losers: Implications for stock market efficiency. *The Journal of Finance*, 48(1):65–91.

Kan, R. and Robotti, C. (2009). Model comparison using the hansen-jagannathan distance. *Review of Financial Studies*, 22(9):3449–3490.

Kan, R., Robotti, C., and Shanken, J. (2013). Pricing model performance and the two-pass cross-sectional regression methodology. *The Journal of Finance*, 68(6):2617–2649.

Kogan, L. and Tian, M. (2013). Firm characteristics and empirical factor models: a data-mining experiment. *Working Paper*.

Kozak, S., Nagel, S., Santosh, S., and Daniel, K. (2015). Interpreting factor models.

Lewellen, J. (2014). The cross section of expected stock returns. *Forthcoming in Critical Finance Review*.

Lewellen, J., Nagel, S., and Shanken, J. (2010). A skeptical appraisal of asset pricing tests. *Journal of Financial Economics*, 96(2):175–194.

Lintner, J. (1965). The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets. *The review of economics and statistics*, pages 13–37.

Litterman, R. B. and Scheinkman, J. (1991). Common factors affecting bond returns. *The Journal of Fixed Income*, 1(1):54–61.

Lord, R. and Pelsser, A. (2007). Level–slope–curvature–fact or artefact? *Applied Mathematical Finance*, 14(2):105–130.

Loughran, T. and Ritter, J. R. (1995). The new issues puzzle. *The Journal of Finance*, 50(1):23–51.

Lucas Jr, R. E. (1978). Asset prices in an exchange economy. *Econometrica: Journal of the Econometric Society*, pages 1429–1445.

Lustig, H., Roussanov, N., and Verdelhan, A. (2011). Common risk factors in currency markets. *Review of Financial Studies*, page hhr068.

Merton, R. C. (1973). An intertemporal capital asset pricing model. *Econometrica: Journal of the Econometric Society*, pages 867–887.

Novy-Marx, R. (2013). The other side of value: The gross profitability premium. *Journal of Financial Economics*, 108(1):1–28.

Novy-Marx, R. (2015). How can a q-theoretic model price momentum?

Novy-Marx, R. and Velikov, M. (2013). Anomalies and their trading costs. *Unpublished working paper*.

Pastor, L. and Stambaugh, R. F. (2003). Liquidity risk and expected stock returns. *Journal of Political Economy*, 111(3).

Petkova, R. (2006). Do the fama–french factors proxy for innovations in predictive variables? *The Journal of Finance*, 61(2):581–612.

Pontiff, J. and Woodgate, A. (2008). Share issuance and cross-sectional returns. *The Journal of Finance*, 63(2):921–945.

Roll, R. (1977). A critique of the asset pricing theory's tests part i: On past and potential testability of the theory. *Journal of financial economics*, 4(2):129–176.

Rosenberg, B., Reid, K., and Lanstein, R. (1985). Persuasive evidence of market inefficiency. *The Journal of Portfolio Management*, 11(3):9–16.

Ross, S. A. (1976). The arbitrage theory of capital asset pricing. *Journal of Economic Theory*, 13(3):341–360.

Rubinstein, M. (1976). The valuation of uncertain income streams and the pricing of options. *The Bell Journal of Economics*, pages 407–425.

Sadka, R. (2006). Momentum and post-earnings-announcement drift anomalies: The role of liquidity risk. *Journal of Financial Economics*, 80(2):309–349.

Sharpe, W. F. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. *The journal of finance*, 19(3):425–442.

Sloan, R. G. (1996). Do stock prices fully reflect information in accruals and cash flows about future earnings? *Accounting review*, pages 289–315.

Titman, S., Wei, K.-C., and Xie, F. (2004). Capital investments and stock returns. *Journal of Financial and Quantitative Analysis*, 39(04):677–700.

Tsay, R. S. (2005). *Analysis of financial time series*, volume 543. John Wiley & Sons.

Zhang, C. (2009). On the explanatory power of firm-specific variables in cross-sections of expected returns. 16:306–317.

Table 1: 25 Portfolios Sorted by Expected Returns

The table shows the result of first running cross-sectional Fama-MacBeth regressions with each firm's return in excess of the risk-free rate on size, book to market, momentum, net stock issues (and a dummy for 0), accruals split into positive and negative, asset growth and operating profit. The regression is run each month, separately for big, small and mirco cap stocks defined with size breakpoints of 50% and 20% of NYSE market equity. Firms are then sorted into twenty-five value weighted portfolios based on the predicted returns from the regressions. The table shows the excess returns, predicted returns and characteristics, all value weighted. The last column shows the excess returns from portfolios formed using the same methodology performed in an expanding window fashion.

Portfolios	XRet	$\widehat{\text{XRet}}$	JME	B/M	Mom	dA/A	A/BE	NS	OP	ROE	IVol	Xret NP
1	-1.12	-0.76	9786	0.44	-0.17	0.69	0.10	0.32	-0.08	-0.30	3.04	-0.90
2	-0.18	-0.26	16519	0.44	-0.08	0.44	0.05	0.19	0.15	-0.08	2.42	-0.14
3	0.25	-0.03	20735	0.46	-0.04	0.30	0.05	0.10	0.23	0.04	2.17	0.31
4	0.25	0.12	24073	0.47	0.00	0.21	0.03	0.05	0.27	0.08	1.97	0.42
5	0.48	0.23	24614	0.51	0.05	0.17	0.02	0.04	0.30	0.12	1.85	0.43
6	0.57	0.31	23686	0.55	0.09	0.14	0.02	0.02	0.32	0.14	1.78	0.52
7	0.53	0.38	21601	0.58	0.13	0.12	0.02	0.01	0.33	0.14	1.75	0.51
8	0.77	0.44	20122	0.61	0.18	0.10	0.01	0.01	0.33	0.15	1.73	0.74
9	0.71	0.50	17535	0.64	0.21	0.10	0.01	0.01	0.33	0.14	1.75	0.69
10	0.87	0.55	16133	0.67	0.25	0.09	0.01	0.01	0.33	0.14	1.77	0.82
11	0.88	0.60	14062	0.69	0.29	0.09	0.01	0.00	0.34	0.15	1.81	0.76
12	0.85	0.65	11715	0.71	0.32	0.08	0.01	0.00	0.34	0.14	1.84	0.88
13	0.87	0.70	9923	0.74	0.36	0.08	0.01	0.00	0.35	0.15	1.89	0.70
14	0.97	0.75	8487	0.76	0.39	0.08	0.01	0.00	0.41	0.08	1.95	0.77
15	1.07	0.80	7054	0.77	0.42	0.07	0.00	0.00	0.36	0.15	2.01	1.02
16	0.90	0.86	6954	0.78	0.45	0.07	0.03	0.00	0.49	0.12	2.06	0.88
17	0.94	0.91	5929	0.79	0.49	0.07	0.07	0.00	0.62	0.22	2.13	0.91
18	1.07	0.97	5140	0.81	0.53	0.07	0.00	0.00	0.41	0.17	2.19	1.01
19	1.39	1.03	3591	0.84	0.56	0.06	-0.02	0.00	0.37	0.11	2.24	1.14
20	1.23	1.10	2955	0.86	0.59	0.06	0.11	0.00	0.82	0.20	2.29	1.05
21	1.17	1.18	2260	0.89	0.65	0.05	0.00	-0.01	0.52	0.03	2.39	1.21
22	1.32	1.26	1859	0.92	0.73	0.05	-0.03	-0.01	0.46	0.15	2.48	1.22
23	1.33	1.37	1615	0.97	0.86	0.04	-0.05	-0.01	0.47	0.12	2.61	1.30
24	1.68	1.53	2128	1.05	1.07	0.02	-0.09	-0.01	0.50	0.12	2.80	1.41
25	1.73	1.89	3418	1.16	1.52	-0.03	-0.37	-0.02	0.84	-0.04	3.14	1.67

Table 2: Principal Components Analysis on Twenty-Five Anomaly Sorted Portfolios

The table shows principal components analysis of the realized returns of 25 anomaly portfolios. I form anomaly portfolios using Fama-MacBeth regression on seven anomaly variables with separate regressions for each size group.

Component	Eigenvalue	Variance Explained	Cumulative
Component 1	668.16	74.18%	74.18 %
Component 2	81.19	9.01%	83.19 %
Component 3	27.42	3.04%	86.23 %
Component 4	13.99	1.55%	87.79 %
Component 5	11.44	1.27%	89.06 %
Component 6	10.41	1.16%	90.21 %
Component 7	8.47	0.94%	91.15 %
Component 8	8.04	0.89%	92.05 %
Component 9	7.93	0.88%	92.93 %
Component 10	6.48	0.72%	93.65 %

Table 3: Cross-Correlation Table for the First Five Components

In each panel, the table shows the correlation of each of the first five principal components with the identical principal component formed using a different number of sorted portfolios.

First Component			
	10 Portfolios	25 Portfolios	100 Portfolios
10 Portfolios	1.000		
25 Portfolios	0.998	1.000	
100 Portfolios	0.993	0.997	1.000
Second Component			
	10 Portfolios	25 Portfolios	100 Portfolios
10 Portfolios	1.000		
25 Portfolios	0.976	1.000	
100 Portfolios	-0.955	0.980	1.000
Third Component			
	10 Portfolios	25 Portfolios	100 Portfolios
10 Portfolios	1.000		
25 Portfolios	0.908	1.000	
100 Portfolios	0.842	0.946	1.000
Fourth Component			
	10 Portfolios	25 Portfolios	100 Portfolios
10 Portfolios	1.000		
25 Portfolios	0.487	1.000	
100 Portfolios	0.150	0.619	1.000
Fifth Component			
	10 Portfolios	25 Portfolios	100 Portfolios
10 Portfolios	1.000		
25 Portfolios	0.065	1.000	
100 Portfolios	0.124	0.578	1.000

Table 4: Cross-Correlation Table Separating Beginning and End of Sample

In each panel, the table shows the correlation of each of the first three principal components formed in the first half of the sample and in the second half of the sample. The first panel shows the beginning of the sample, thus the beginning sample principal components are formed in sample and compared with the out of sample principal components formed using principal components on the second half of the sample. The second panel shows the correlations of components in the latter half of the sample. The End components are in sample and compared with the out of sample Beg components that were formed using only data from the first half of the sample.

Beginning of Sample						
	Beg 1	Beg 2	Beg 3	End 1	End 2	End 3
Beg 1	1.00					
Beg 2	0.00	1.00				
Beg 3	0.00	-0.00	1.00			
End 1	1.00	0.03	-0.02	1.00		
End 2	0.40	-0.92	-0.04	0.37	1.00	
End 3	0.64	-0.08	0.73	0.62	0.29	1.00
End of Sample						
	Beg 1	Beg 2	Beg 3	End 1	End 2	End 3
Beg 1	1.00					
Beg 2	0.25	1.00				
Beg 3	-0.48	-0.06	1.00			
End 1	1.00	0.30	-0.50	1.00		
End 2	0.05	-0.95	0.12	0.00	1.00	
End 3	0.03	-0.03	0.82	-0.00	-0.00	1.00

Table 5: Cross-correlation table

The table shows cross-correlation of the Level, Slope and Curve factor to the market factor, HML, SMB, Momentum, Profitability (PMU), and Liquidity.

Variables	PC 1	PC 2	PC 3	PC 4	PC 5
Mkt-RF	0.95	0.20	0.14	0.01	0.06
SMB	0.46	-0.30	-0.49	-0.09	-0.19
HML	-0.35	-0.17	0.22	-0.04	0.02
RMW	-0.31	0.00	0.41	0.11	0.04
CMA	-0.38	-0.30	0.16	-0.05	0.00
MOM	-0.05	-0.77	0.14	-0.02	0.04
HML*	-0.17	-0.23	0.13	-0.01	-0.06
UMD*	-0.11	-0.70	0.09	0.00	0.05
PMU*	-0.33	-0.11	0.25	0.08	-0.02
ROE	-0.22	-0.26	0.36	0.01	0.10
INV	-0.37	-0.28	0.26	-0.05	0.08
Liq-T	-0.03	0.04	-0.01	-0.01	-0.01

Table 6: Time Series Regressions of 25 Expected Return Sorted Portfolios on the Extracted Principal Components

The table shows regression results for the 25 portfolios sorted by expected returns on the extracted principal components. Each portfolio is regressed on the first one, two, three and four components. The alphas, t-statistics and R-squared for each regression are displayed. Portfolios are formed based on each stock's predicted return based on regressions on seven anomaly variables.

Port	Ret	α_1	α_2	α_3	α_4	t_1	t_2	t_3	t_4	R_1^2	R_2^2	R_3^2	R_4^2
1	-1.12	-2.23	-0.94	-0.65	-0.26	-9.72	-6.39	-5.66	-3.01	0.62	0.87	0.92	0.96
2	-0.18	-1.07	-0.18	-0.02	-0.08	-6.34	-1.32	-0.15	-0.67	0.65	0.84	0.87	0.87
3	0.25	-0.58	0.39	0.48	0.14	-3.27	2.65	3.21	1.19	0.59	0.82	0.83	0.88
4	0.25	-0.47	0.30	0.22	0.19	-3.25	2.39	1.74	1.42	0.62	0.83	0.83	0.84
5	0.48	-0.18	0.37	0.23	0.19	-1.48	3.37	2.38	1.78	0.65	0.78	0.82	0.82
6	0.57	-0.11	0.35	0.22	0.11	-1.02	3.65	2.55	1.17	0.72	0.82	0.85	0.86
7	0.53	-0.15	0.17	0.02	0.00	-1.42	1.77	0.18	-0.06	0.75	0.80	0.85	0.85
8	0.77	0.08	0.32	0.15	0.07	0.88	3.18	1.96	0.93	0.78	0.81	0.87	0.88
9	0.71	0.05	0.23	0.05	0.05	0.57	2.37	0.63	0.66	0.78	0.79	0.87	0.87
10	0.87	0.19	0.16	0.00	0.01	2.16	1.75	-0.06	0.13	0.80	0.81	0.87	0.87
11	0.88	0.19	0.09	-0.07	-0.04	2.16	1.05	-0.98	-0.53	0.81	0.82	0.87	0.88
12	0.85	0.15	0.02	-0.13	-0.13	1.67	0.19	-1.76	-1.71	0.83	0.83	0.88	0.88
13	0.87	0.15	0.00	-0.14	-0.06	1.72	-0.05	-1.66	-0.69	0.81	0.82	0.86	0.86
14	0.97	0.21	-0.05	-0.13	-0.19	2.39	-0.57	-1.67	-2.02	0.82	0.85	0.86	0.87
15	1.07	0.33	0.08	-0.01	-0.02	3.65	0.95	-0.10	-0.30	0.82	0.85	0.86	0.86
16	0.90	0.11	-0.19	-0.26	-0.21	1.16	-2.08	-2.95	-2.20	0.82	0.85	0.86	0.86
17	0.94	0.14	-0.14	-0.20	-0.18	1.42	-1.40	-2.17	-1.91	0.82	0.85	0.85	0.85
18	1.07	0.22	-0.18	-0.18	-0.04	2.10	-1.93	-2.02	-0.44	0.81	0.86	0.86	0.87
19	1.39	0.52	0.10	0.10	0.20	4.74	0.99	1.04	1.91	0.81	0.86	0.86	0.86
20	1.23	0.37	-0.03	-0.06	0.04	3.52	-0.36	-0.67	0.41	0.81	0.87	0.87	0.87
21	1.17	0.30	-0.22	-0.18	-0.12	2.69	-2.23	-1.94	-1.27	0.78	0.87	0.87	0.87
22	1.32	0.41	-0.07	-0.02	0.13	3.35	-0.62	-0.21	1.16	0.77	0.84	0.84	0.85
23	1.33	0.44	0.02	0.13	0.22	3.57	0.20	1.14	2.07	0.78	0.83	0.84	0.84
24	1.68	0.65	0.09	0.30	-0.25	4.53	0.76	2.58	-2.92	0.74	0.80	0.84	0.94
25	1.73	0.66	-0.03	0.31	0.30	3.80	-0.22	2.66	2.48	0.71	0.80	0.89	0.89
$\lvert\alpha\rvert$		0.40	0.19	0.17	0.13					0.76	0.83	0.86	0.87
GRS		5.43	3.20	2.74	2.67								
p-val		[0.00]	[0.00]	[0.00]	[0.02]								

Table 7: Comparing Level, Slope and Curve to Leading Factor Models With 119 Test Portfolios

Panel A: Time Series Results

	LSC	LSC-NP	CAPM	FF3	CAR	FF5	RNM	HXZ
Avg $\|\alpha\|$	0.15	0.16	0.23	0.20	0.15	0.20	0.20	0.18
$\|t\|$>1.96	19	18	40	38	24	38	27	26
Avg R^2	0.70	0.69	0.66	0.72	0.74	0.74	0.70	0.72
CS (TS) R^2	0.56	0.51	0.04	0.12	0.48	0.26	0.43	0.39
HJ - All	0.892	0.902	0.945	0.927	0.909	0.900	0.879	0.914
Diff HJ^2	-	[0.018]	[0.098]	[0.063]	[0.031]	[0.014]	[-0.023]	[0.039]
HJ - Stocks	0.824	0.828	0.856	0.846	0.838	0.833	0.834	0.846
Diff HJ^2	-	[0.008]	[0.055]	[0.037]	[0.023]	[0.016]	[0.017]	[0.038]

Panel B: Cross-Sectional Test

Factor	LSC	LSC-NP	CAPM	FF3	CAR	FF5	RNM	HXZ
	b/t	b/t	b/t	b/t	b/t	b/t	b/t	b/t
β Level	0.50*	0.49*						
	(1.82)	(1.79)						
β Slope	-0.93***	-0.83***						
	(-3.88)	(-3.51)						
β Curve	0.41**	0.51**						
	(2.02)	(2.47)						
β Market			0.19	0.10	0.36	0.21	0.47*	0.26
			(0.72)	(0.39)	(1.48)	(0.87)	(1.86)	(1.05)
β HML				0.24	0.20	0.01	0.19**	
				(1.56)	(1.28)	(0.04)	(2.03)	
β SMB				0.18	0.14	0.27*		0.34**
				(1.20)	(0.92)	(1.83)		(2.18)
β MOM					0.78***		0.49***	
					(3.50)		(3.33)	
β INV						0.42***		0.14
						(3.27)		(1.12)
β PROF						0.03	0.05	0.37**
						(0.27)	(0.63)	(2.39)
Cons	0.31***	0.31***	0.51***	0.53***	0.30***	0.39***	0.20*	0.35***
	(2.96)	(2.90)	(3.65)	(4.42)	(2.75)	(3.56)	(1.82)	(3.26)
R^2	0.56	0.55	0.03	0.17	0.53	0.38	0.58	0.42
HJ - All	0.891	0.902	0.945	0.927	0.909	0.897	0.869	0.906
Diff HJ^2		[0.019]	[0.099]	[0.064]	[0.032]	[0.011]	[-0.039]	[0.025]
p-val		(0.00)	(0.00)	(0.00)	(0.00)	(0.17)	(0.01)	(0.01)
HJ - Stocks	0.795	0.801	0.828	0.813	0.805	0.796	0.789	0.799
Diff HJ^2	-	[0.009]	[0.054]	[0.030]	[0.017]	[0.002]	[-0.008]	[0.007]
p-val	-	(0.02)	(0.00)	(0.02)	(0.01)	(0.54)	(0.44)	(0.22)

t statistics in parentheses

* p<0.10, ** p<0.05, *** p<0.01

Table 8: Comparing Level, Slope and Curve to Leading Factor Models With 32 Hedge Portfolios

Panel A: Time Series Results

	LSC	LSC-NP	CAPM	FF3	CAR	FF5	RNM	HXZ
Avg $\|\alpha\|$	0.28	0.28	0.42	0.47	0.32	0.36	0.47	0.30
$\|t\|$>1.96	8	10	15	18	17	13	20	13
Avg R^2	0.23	0.21	0.06	0.25	0.35	0.36	0.30	0.34
CS (TS) R^2	0.28	0.25	0.00	0.00	0.17	0.00	0.11	0.12
HJ	0.621	0.624	0.627	0.632	0.625	0.653	0.676	0.644
Diff HJ^2		[0.004]	[0.008]	[0.014]	[0.006]	[0.041]	[0.072]	[0.030]

Panel B: Cross-Sectional Test

Factor	LSC	LSC-NP	CAPM	FF3	CAR	FF5	RNM	HXZ
	b/t	b/t	b/t	b/t	b/t	b/t	b/t	b/t
β Level	1.04***	0.80**						
	(3.06)	(2.28)						
β Slope	-0.63**	-0.53**						
	(-2.41)	(-2.02)						
β Curve	0.78**	0.61***						
	(2.59)	(2.07)						
β Market			-0.02	-0.14	0.87**	-0.21	0.70*	0.53
			(-0.05)	(-0.36)	(2.57)	(-0.56)	(1.91)	(1.55)
β HML				0.05	0.08	-0.08	0.07	
				(0.30)	(0.50)	(-0.45)	(0.73)	
β SMB				0.03	-0.01	0.11		0.24
				(0.19)	(-0.05)	(0.69)		(1.46)
β MOM					0.64***		0.36**	
					(2.78)		(2.42)	
β INV						-0.06		0.08
						(-0.47)		(0.67)
β PROF						0.00	0.01	0.31**
						(0.02)	(0.13)	(2.15)
Cons	0.12**	0.14***	0.28***	0.26***	0.17***	0.28***	0.17***	0.17***
	(2.09)	(2.68)	(5.36)	(4.99)	(3.43)	(5.37)	(3.70)	(3.53)
R^2	0.28	0.19	-0.03	-0.09	0.23	-0.16	0.20	0.05
HJ	0.601	0.608	0.622	0.619	0.610	0.617	0.601	0.609
Diff HJ^2		[0.009]	[0.026]	[0.022]	[0.012]	[0.020]	[-0.000]	[0.010]
p-val		(0.03)	(0.06)	(0.05)	(0.05)	(0.02)	(0.68)	(0.11)

t statistics in parentheses

* p<0.10, ** p<0.05, *** p<0.01

Table 9: Horse Race Using 119 Portfolios

I regress monthly excess returns of 119 test assets on each factor in time series regressions from July 1974 to December 2012. I then regress the average excess return on each test asset on the estimated betas from the time series regressions in a cross-sectional regression with Fama-MacBeth standard errors.

Factor	LSC	+HML	+SMB	+CMA	+RMW	+MOM	+UMD	+PMU	+HML*	+ROE	+INV
	b/t	b/t	b/t	b/t	b/t	b/t	b/t	b/t	b/t	b/t	b/t
β Level	0.49*	0.54*	0.44	0.60*	0.55*	0.45*	0.44	0.57*	0.51*	0.43	0.59*
	(1.79)	(1.78)	(1.37)	(1.78)	(1.84)	(1.68)	(1.60)	(1.87)	(1.81)	(1.52)	(1.76)
β Slope	-0.83***	-0.83***	-0.79***	-0.77***	-0.84***	-1.10***	-1.02***	-0.81***	-0.81***	-0.90***	-0.78***
	(-3.51)	(-3.47)	(-2.90)	(-3.02)	(-3.54)	(-2.22)	(-2.36)	(-3.33)	(-3.39)	(-3.45)	(-3.08)
β Curve	0.51**	0.43*	0.54**	0.41*	0.40	0.55**	0.54**	0.46*	0.44**	0.61**	0.37
	(2.47)	(1.91)	(2.13)	(1.84)	(1.57)	(2.38)	(2.42)	(1.86)	(2.10)	(2.15)	(1.48)
β HML		0.11									
		(0.56)									
β SMB			0.06								
			(0.26)								
β CMA				0.12							
				(0.70)							
β RMW					0.09						
					(0.52)						
β MOM						-0.28					
						(-0.59)					
β UMD							-0.14				
							(-0.52)				
β PMU								0.05			
								(0.05)			
β HML*									0.07		
									(0.68)		
β ROE										-0.12	
										(-0.53)	
β INV											0.12
											(0.68)
Cons	0.31***	0.32***	0.32***	0.32***	0.32***	0.33***	0.33***	0.31***	0.33***	0.32***	0.33***
	(2.90)	(3.07)	(3.01)	(3.05)	(2.99)	(3.20)	(3.19)	(2.90)	(3.14)	(3.00)	(3.19)
R^2	0.55	0.55	0.55	0.56	0.55	0.55	0.55	0.55	0.56	0.55	0.56

t statistics in parentheses

* p<0.10, ** p<0.05, *** p<0.01

Table 10: Horse Race Using 32 Hedge Portfolios

I regress monthly excess returns of 119 test assets on each factor in time series regressions from July 1974 to December 2012. I then regress the average excess return on each test asset on the estimated betas from the time series regressions in a cross-sectional regression with Fama-MacBeth standard errors.

Factor	LSC	+HML	+SMB	+CMA	+RMW	+MOM	+UMD	+PMU	+HML*	+ROE	+INV
	b/t	b/t	b/t	b/t	b/t	b/t	b/t	b/t	b/t	b/t	b/t
β Level	0.8**	0.77**	1.04**	0.66	0.84**	0.81**	0.83**	0.81**	0.78**	0.84**	0.66
	(2.28)	(2.03)	(2.48)	(1.60)	(2.22)	(2.33)	(2.36)	(2.09)	(2.12)	(2.35)	(1.55)
β Slope	-0.53**	-0.53**	-0.66**	-0.58**	-0.54**	-0.23	-0.30	-0.52**	-0.53**	-0.48*	-0.57**
	(-2.02)	(-2.02)	(-2.13)	(-2.07)	(-2.03)	(-0.41)	(-0.62)	(-2.00)	(-2.02)	(-1.75)	(-2.06)
β Curve	0.61**	0.61**	0.34	0.57**	0.50	0.46	0.47*	0.60**	0.59**	0.43	0.61**
	(2.07)	(2.06)	(1.09)	(2.12)	(1.45)	(1.54)	(1.66)	(2.06)	(2.21)	(1.31)	(2.08)
β HML		-0.05									
		(-0.22)									
β SMB			-0.26								
			(-0.93)								
β CMA				-0.1							
				(-0.61)							
β RMW					0.06						
					(0.28)						
β MOM						0.29					
						(0.57)					
β UMD							0.15				
							(0.53)				
β PMU								0.00			
								(0.05)			
β HML*									0.03		
									(-0.25)		
β ROE										0.10	
										(0.53)	
β INV											-0.10
											(-0.55)
Cons	0.14***	0.15***	0.17***	0.16***	0.14***	0.16***	0.16***	0.14***	0.15***	0.16***	0.16**
	(2.68)	(3.15)	(3.68)	(3.38)	(2.74)	(3.46)	(3.43)	(2.75)	(3.23)	(3.29)	(3.38)
R^2	0.19	0.17	0.20	0.18	0.17	0.18	0.19	0.16	0.17	0.18	0.18

t statistics in parentheses

* p<0.10, ** p<0.05, *** p<0.01

Table 11: Innovations in State Variables Regressed on Level, Slope and Curve

I regress the innovation from each state variable in the VAR model on the Level, Slope and Curve factors. The state variables are dividend to price, term spread, default spread, the risk-free rate and one month stock variance.

Dep. Variable	a_0	Level	Slope	Curve
u_{DIV}	-0.20	0.10**	-0.22***	-0.41***
	(-0.66)	(2.58)	(-5.30)	(-6.20)
u_{TERM}	0.01	0.01	-0.01	-0.07
	(-0.05)	(0.30)	(-0.13)	(-0.91)
u_{DEF}	-0.04	-0.04	0.01	0.21***
	(-0.23)	(-0.78)	(0.27)	(3.22)
u_{RF}	0.08	-0.00	0.04	-0.07
	(0.45)	(-0.08)	(0.82)	(-1.00)
u_{SVAR}	0.09	-0.03	0.09**	0.12***
	(0.34)	(-0.19)	(2.18)	(2.68)

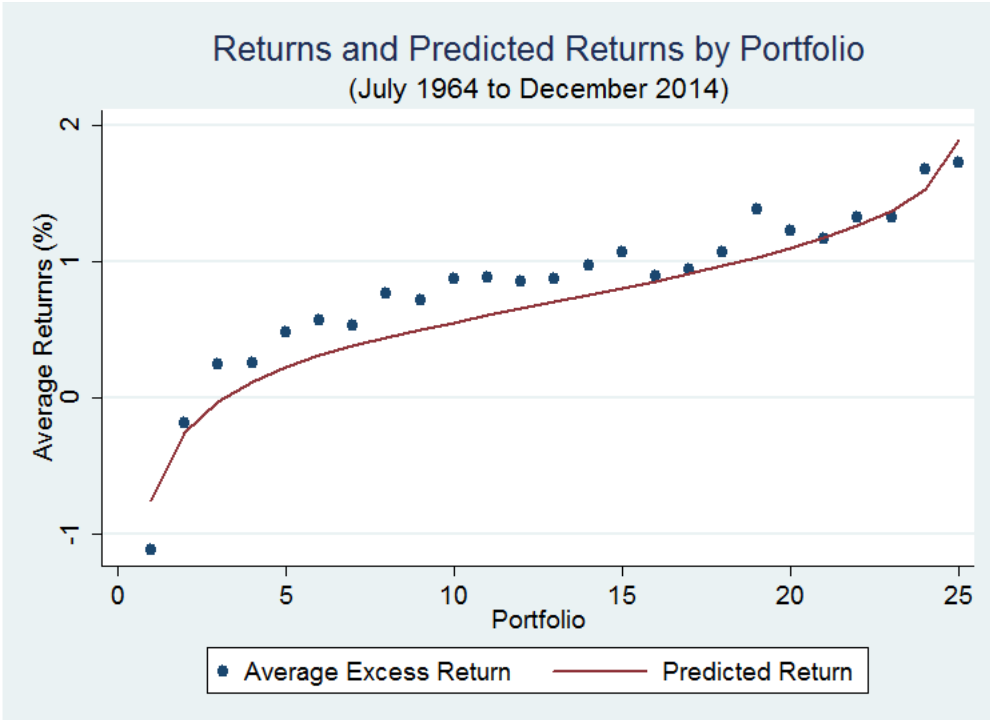


Figure 1: Twenty-Five Dissecting Anomaly Portfolios

The figure shows average returns and predicted returns for twenty-five portfolios built on seven asset pricing anomalies. I form anomaly portfolios using Fama-MacBeth regression on seven anomaly variables with separate cross sectional regressions for each size group. Stocks are sorted into portfolios based on the fitted value from each cross sectional regression. Predicted return is the value-weighted average fitted value in each regression. Average return is the value-weighted average return on each portfolio.

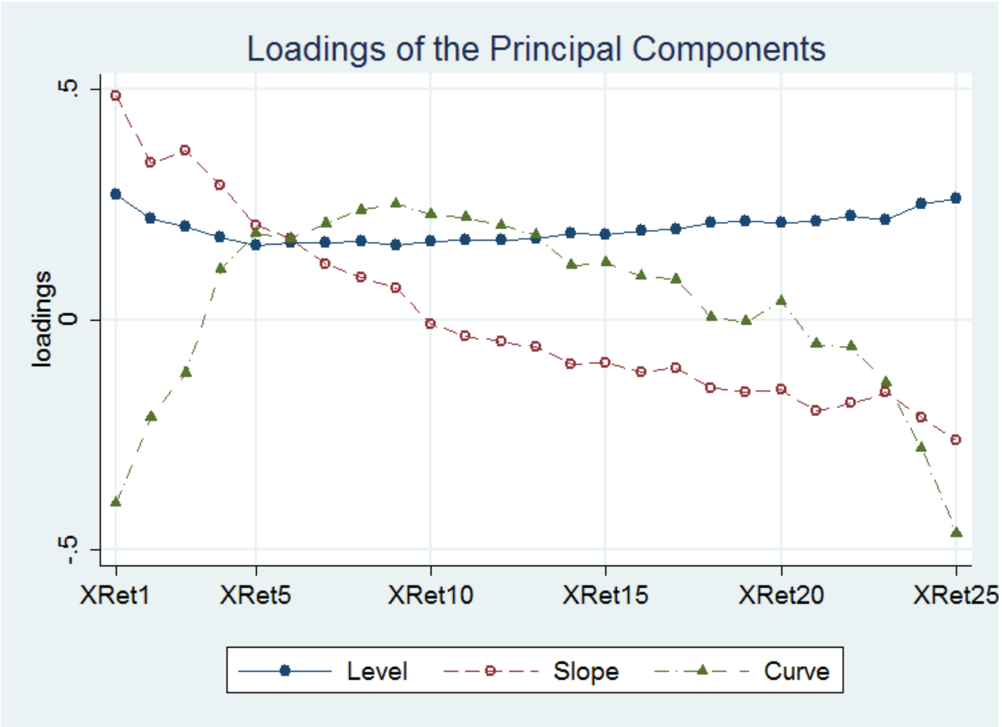
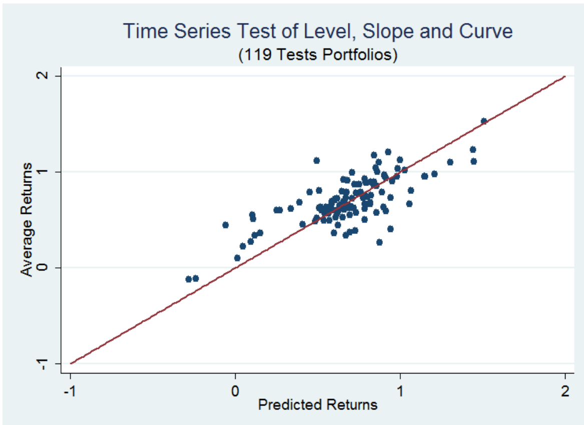
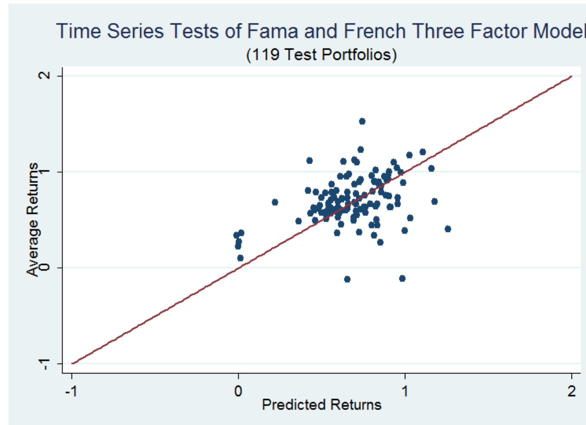


Figure 2: PCA Weights

The figure shows the loadings of each of the first three principal components of twenty-five anomaly portfolios.



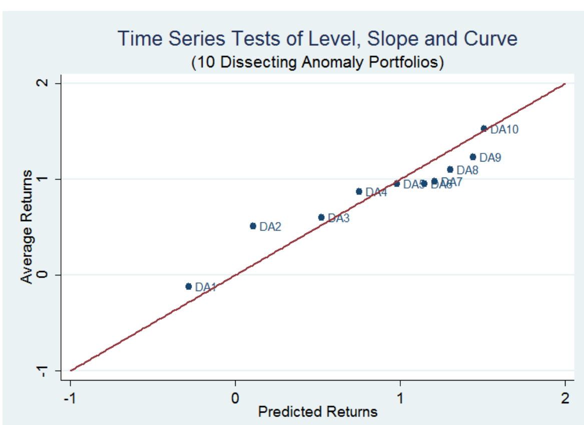
(a) LSC



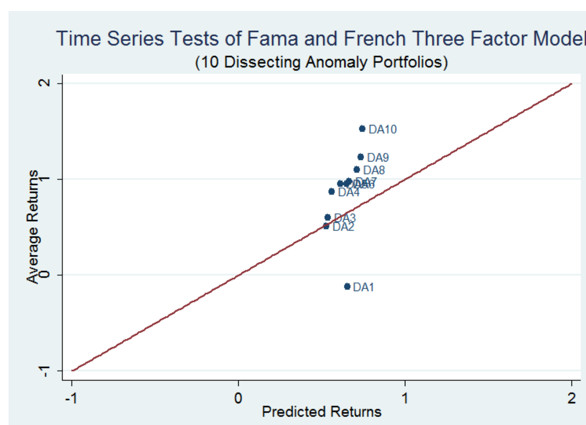
(b) FF3

Figure 3: Level, Slope and Curve vs. Fama and French Three Factor Model on 119 Test Portfolios

The figure shows the results of the times series regressions of the Level, Slope and Curve model and the Fama and French three factor model on 119 portfolios formed on seven anomaly variables. The X axis is the model predicted excess return. The Y axis is the average return of the portfolio over the sample. The vertical distance from the observation to the 45 degree line is the estimated pricing error (alpha).



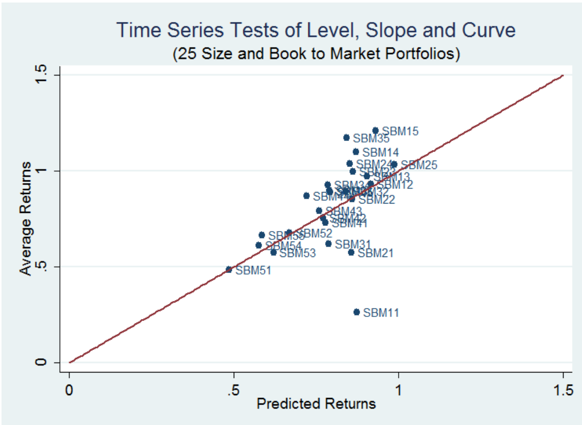
(a) LSC



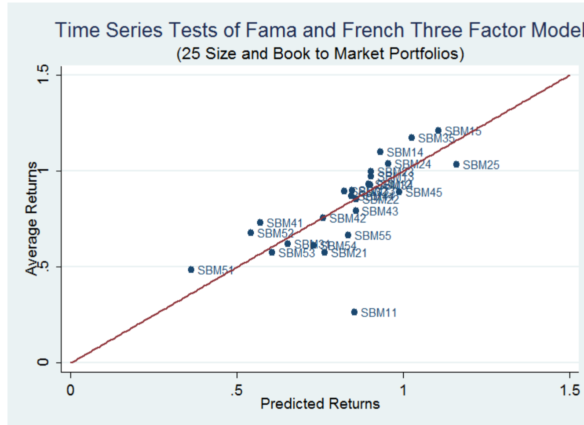
(b) FF3

Figure 4: Models vs. 10 Dissecting Anomaly Portfolios

The figure shows the results of the times series regressions of the Level, Slope and Curve model and the Fama and French three factor model on 10 portfolios formed on seven anomaly variables using the predictions on multivariate regressions. The X axis is the model predicted excess return. The Y axis is the average return of the portfolio over the sample. The vertical distance from the observation to the 45 degree line is the estimated pricing error (alpha).



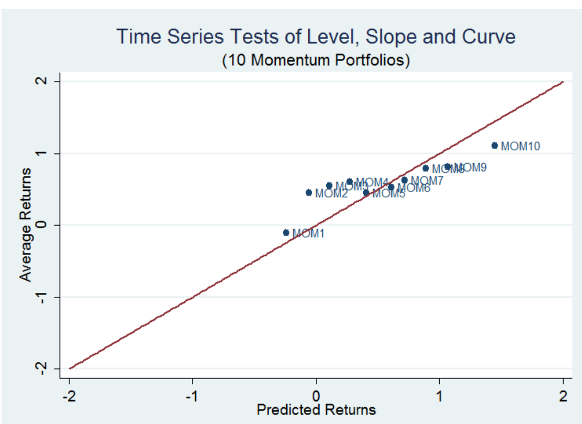
(a) LSC



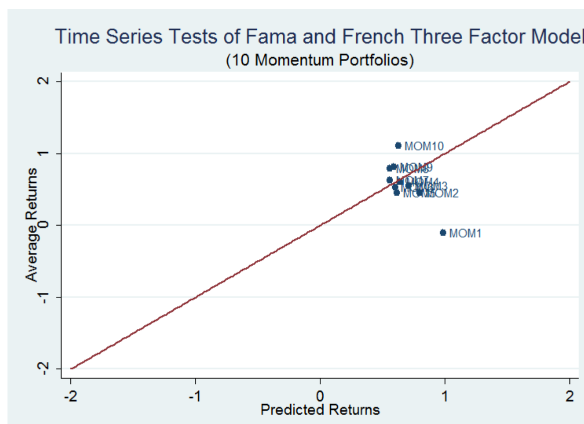
(b) FF3

Figure 5: Models vs. 25 Size and Book to Market

The figure shows the results of the times series regressions of the Level, Slope and Curve model and the Fama and French three factor model on 25 portfolios formed on size and book to market. The X axis is the model predicted excess return. The Y axis is the average return of the portfolio over the sample. The vertical distance from the observation to the 45 degree line is the estimated pricing error (alpha).



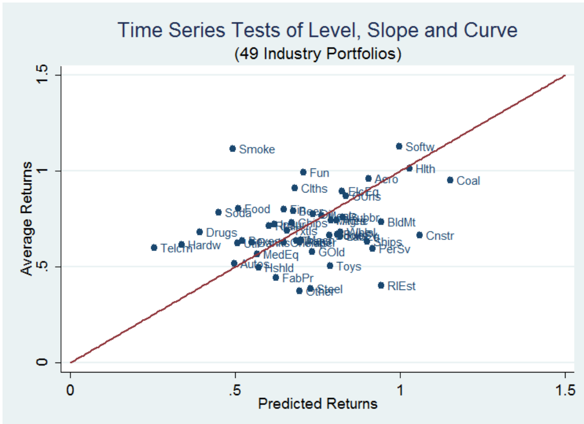
(a) LSC

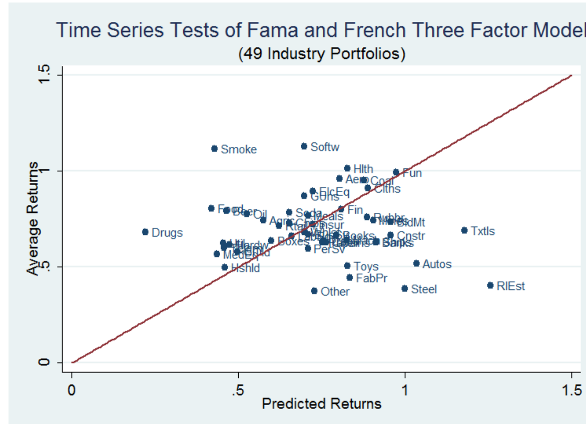


(b) FF3

Figure 6: Models vs. 10 Momentum Portfolios

The figure shows the results of the time series regressions of the Level, Slope and Curve model and the Fama and French three factor model on ten portfolios formed on momentum. The X axis is the model predicted excess return. The Y axis is the average return of the portfolio over the sample. The vertical distance from the observation to the 45 degree line is the estimated pricing error (alpha).



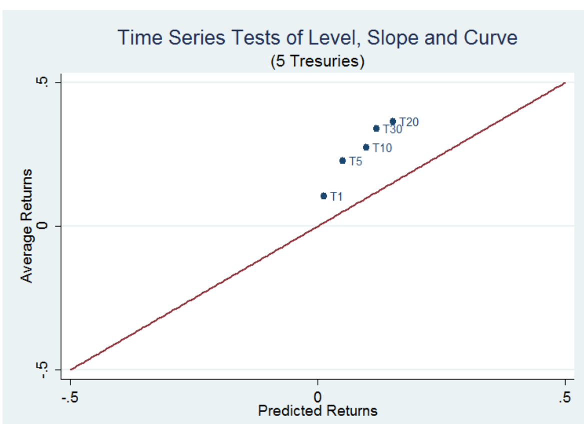


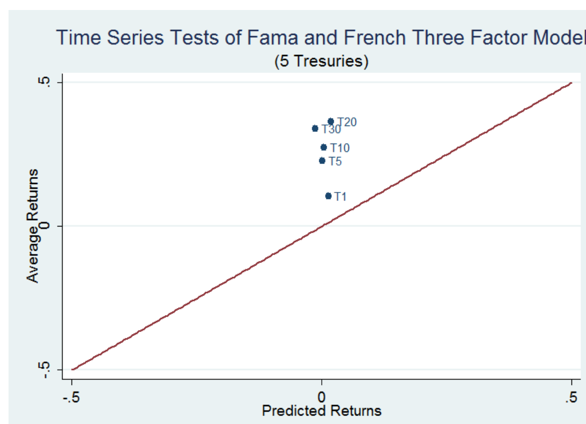
(a) LSC

(b) FF3

Figure 7: Models vs. 49 Industry Portfolios

The figure shows the results of the times series regressions of the Level, Slope and Curve model and the Fama and French three factor model on 49 industry portfolios formed on the Fama and French industry definitions. The X axis is the model predicted excess return. The Y axis is the average return of the portfolio over the sample. The vertical distance from the observation to the 45 degree line is the estimated pricing error (alpha).



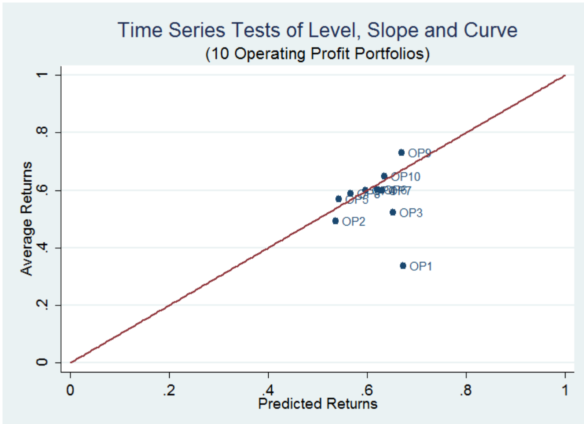


(a) LSC

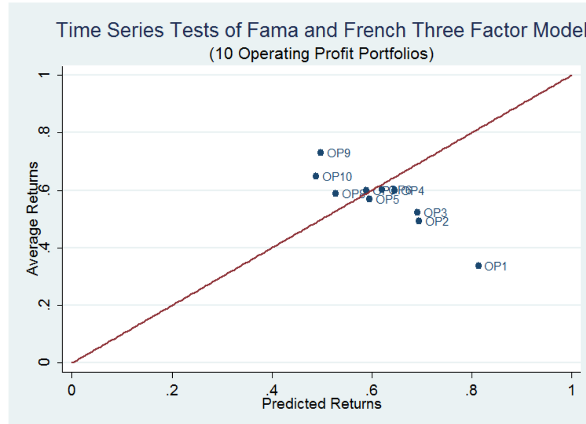
(b) FF3

Figure 8: Models vs. 5 Bond Portfolios

The figure shows the results of the time series regressions of the Level, Slope and Curve model and the Fama and French three factor model on five treasury bond returns of different maturities, 1 year, 5 year, 10 year, 20 year and 30 year. The X axis is the model predicted excess return. The Y axis is the average return of the portfolio over the sample. The vertical distance from the observation to the 45 degree line is the estimated pricing error (alpha).



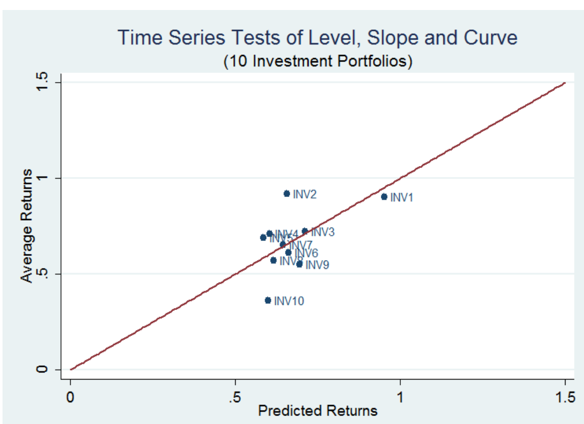
(a) LSC



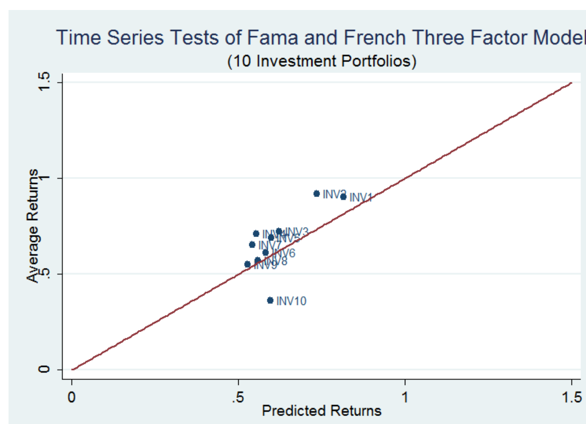
(b) FF3

Figure 9: Models vs. 10 Operating Profit Portfolios

The figure shows the results of the time series regressions of the Level, Slope and Curve model and the Fama and French three factor model on five treasury bond returns of different maturities, 1 year, 5 year, 10 year, 20 year and 30 year. The X axis is the model predicted excess return. The Y axis is the average return of the portfolio over the sample. The vertical distance from the observation to the 45 degree line is the estimated pricing error (alpha).



(a) LSC



(b) FF3

Figure 10: Models vs. 10 Investment Portfolios

The figure shows the results of the time series regressions of the Level, Slope and Curve model and the Fama and French three factor model on five treasury bond returns of different maturities, 1 year, 5 year, 10 year, 20 year and 30 year. The X axis is the model predicted excess return. The Y axis is the average return of the portfolio over the sample. The vertical distance from the observation to the 45 degree line is the estimated pricing error (alpha).

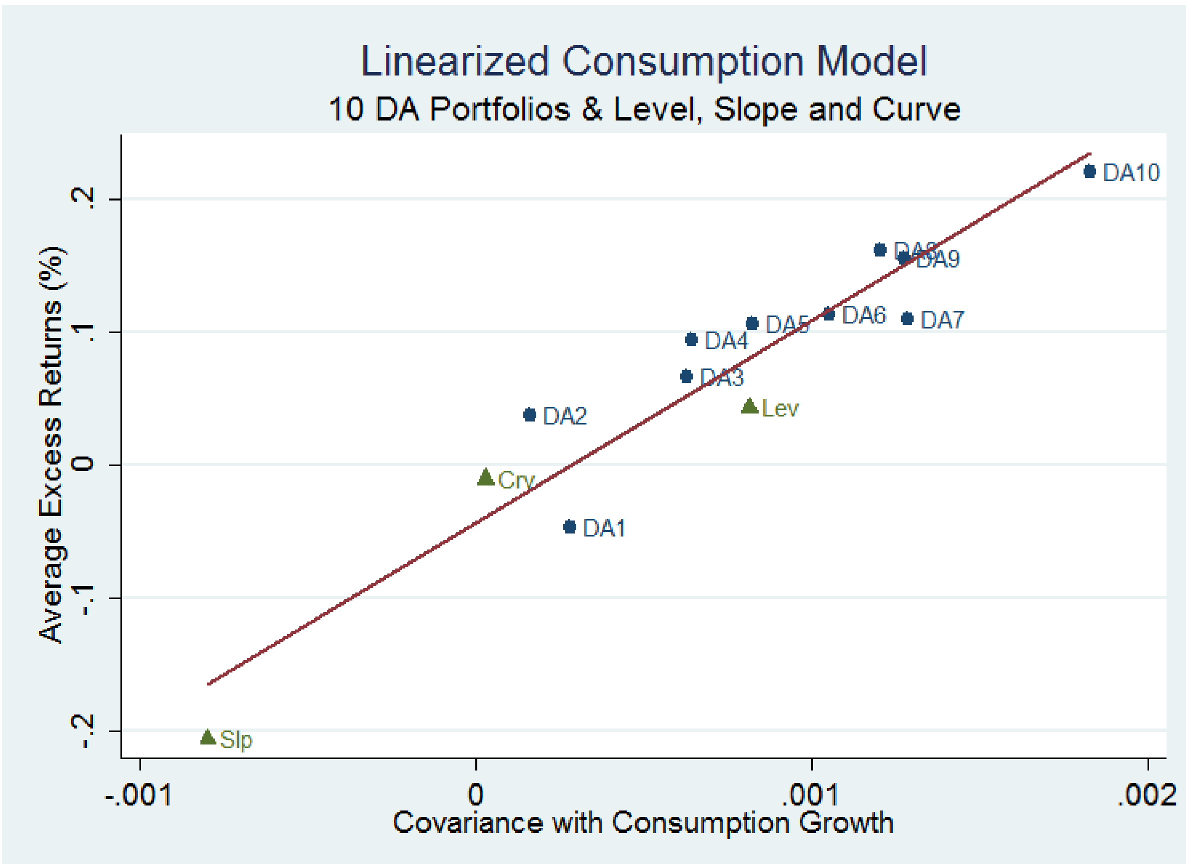


Figure 11: Consumption Based Asset Pricing

The figure shows the Linearized Consumption Model. The test assets are 10 portfolios of annual excess returns sorted from high to low returns using the Dissecting Anomalies predictors as well as annual returns on the Level, Slope and Curve Factors. Consumption is measured as real nondurable consumption plus services per capita, and consumption growth is the change from Q4 to Q4 of the calendar year, which matches the January to December annual returns.