Intermediated Surge Pricing

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Abstract

I study a market in which a profit-maximizing intermediary facilitates trade between buyers and sellers. The intermediary sets prices for buyers and sellers, and keeps the difference as her fee. Optimal prices increase when demand increases, i.e., shifts right. If a demand increase is due to an increase in the number of ex ante similar buyers, then the intermediary's optimal percent fee decreases. If, instead, a demand increase is due to a reduction in the elasticity of demand, then the intermediary's optimal percent fee increases. In either case, if the intermediary keeps a constant percent fee regardless of shifts in demand, as is the case for some intermediaries, then surge pricing (i.e., the ratio of price during high demand to price during low demand) is amplified on one side of the market and diminished on the other side.

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1 Introduction

Intermediaries abound in markets, facilitating trade between buyers and sellers. The services they provide include reduction in search costs, information exchange, access to inventory, and diversification of risk. While the digital economy has diminished the need for intermediaries in some areas, it has also created new markets intermediated by middlemen.

Some of these new intermediated markets are for goods or services for immediate delivery. Uber, the online transportation company, provides a motivating example. The firm is an intermediary between car drivers and passengers. Its superior matching technology reduces search costs on both sides of the market. Uber sets a price (per mile) that passengers pay and takes a fixed percent fee from each transaction it mediates. Uber's software system allows it to monitor local demand and supply conditions in real time. While Uber responds to sharp increases in demand by raising price, it keeps the same percent fee, usually 20% of revenue, at each demand level. Thus, the payment that a car driver receives is a fixed percent of the payment made by the passenger. Conceivably, if Uber were to reduce its percent fee when many passengers enter the market, such as during rush hour, it may increase profits by enticing more drivers to enter the supply pool. At other times when passengers are few but have a greater willingness-to-pay, such as late at night or after a severe snowstorm, it may be profitable for Uber to increase its percent fee while charging passengers more. In either case, the flexibility afforded by not having the payment by a passenger and the payment to a driver in lock step may have an impact on price increases during periods of high demand.

These questions are explored in a simple market model with a monopolist intermediary who sets prices for buyers and sellers. The focus is on how optimal prices and the intermediary's revenue share vary with changes in demand, especially when there is some inflexibility in the prices set by the intermediary.

In the model, an intermediary facilitates trade between a large number of buyers and sellers. Search costs for buyers and sellers are larger than the gains to trade; hence without the intermediary there is (essentially) no trade. This is the case in several markets that have experienced extraordinary growth after recent advances in digital technology enabled intermediaries to reduce search costs. In the model, each buyer's value and each seller's cost for the good are private information. Consequently, the intermediary does not have the ability to price discriminate between buyers or between sellers. It is optimal for the intermediary to set a price for all buyers and a price for all sellers. As there are a large number of buyers and sellers, it is optimal for each buyer and seller to act as a price-taker. The intermediary keeps the difference between the buyer price and the seller price in each trade.

The technological advances that have reduced search costs and enabled better matches between buyers and sellers have also made it easier for the intermediary to monitor demand and supply and adjust prices accordingly. Changes in optimal prices with changes in market conditions are a focus of this paper. Of particular interest is the change in the optimal revenue share[1] of the intermediary. To capture changes in market conditions in a tractable manner, in the model there is a continuum of buyers and a continuum of sellers rather than a large finite number of each.[2]

It is shown that there exists a unique set of optimal prices at which the intermediary's profit is maximized. As demand increases, the optimal price charged to buyers and paid to sellers increase – also known as *surge pricing*. However, the optimal share (i.e., percent fee) of the intermediary may increase or decrease depending on the nature of the increase in demand.

The intermediary's role consists of two interlinked parts: it acts as a monopolist in its interactions with buyers and as a monopsonist in its interactions with sellers. The marginal cost of the intermediary monopolist is determined by the intermediary monopsonist, while the marginal revenue of the intermediary monopsonist is determined by the intermediary monopolist. The ratio of the optimal buyer price to the optimal seller price charged by the intermediary equals the product of the gross mark-up of the intermediary acting as a monopolist and the gross mark-down of the intermediary acting as a monopsonist. Thus, the change in the intermediary's optimal percent fee after an increase in demand depends entirely on the elasticities

[1] The percent of buyer price that is taken by the intermediary as fees.

[2] Myerson and Satterthwaite [16] obtain the optimal mechanism for a profit-maximizing intermediary with one buyer and one seller.

of demand and supply through their effects on the gross mark-up and on the gross mark-down, respectively.

A constant percent fee for the intermediary that does not change with demand conditions is rarely optimal. Two types of changes in demand, somewhat orthogonal in nature, are considered. First, the demand curve may shift because a larger number (mass) of buyers enter the market; in effect, buyers are replicated without changing the distribution of buyer values.[3] This leads to an outward rotation of the demand curve about the vertical intercept. Second, the demand curve may shift because the distribution of buyer values becomes less elastic but the number of buyers does not change. This leads to an outward rotation of the demand curve about the horizontal intercept. Of course, a shift in demand may be due to both factors causing the new demand curve to lie completely above the old one. However, it is useful to analyze separately the impact of these two factors as they have opposite effects on the intermediary's optimal revenue share.

For a large increase in buyer mass, the optimal percent fee of the intermediary decreases. Reducing the intermediary's percent fee draws more sellers in, thereby increasing the number of trades and making it optimal for the intermediary to capture a smaller fraction of this larger pie.

If, instead, the positive change in demand is entirely due to a large reduction in demand elasticity, the optimal percent fee of the intermediary increases. It is optimal for the intermediary to charge buyers a higher price. While sellers also receive a higher price their share of the pie decreases; as there is no increase in buyer mass, there is less incentive to draw more sellers into the market.

However, firms such as Uber and Lyft extract the same fraction, charging a constant percent fee of 20% of buyer price, regardless of the level of demand. This inflexibility in pricing results in a match between demand and supply that is less than optimal for the intermediary. Under a constant percent fee, the number of transactions is less than optimal (for the intermediary's profit when compared to flexible

[3]This is one of the types of demand uncertainty considered by Klemperer and Meyer [14] in a model of differentiated duopolists. In [14], price or quantity decisions are taken before resolution of demand uncertainty whereas in this paper decisions are taken after resolution of demand uncertainty.

percent fees) when the buyer mass is high and it is greater than optimal when the buyer mass is low. In contrast, if the demand change is due to a change in elasticity, then under a constant percent fee the number of transactions is greater than optimal when demand is inelastic and less than optimal when demand is elastic.

Apart from being suboptimal, a constant percent fee amplifies the surge in prices on one side of the market and diminishes the surge in prices on the other side of the market. To see this, consider a scenario where high and low demand periods are determined by a change in the mass of buyers. In the absence of a constant percent fee constraint, it is optimal for the intermediary to reduce its percent fee when buyer mass increases. In this scenario, under a constant percent fee constraint the intermediary will charge a fee that is smaller than optimal during low demand and greater than optimal during high demand. This in turn leads to an even higher buyer price during high demand and an even lower buyer price during low demand, compared to optimal prices with a flexible percent fee. Consequently, the surge in buyer prices is amplified. A constant percent fee also diminishes the surge in seller prices in response to an increase in the mass of buyers.

While much of the analysis in the paper is centered on a monopolist intermediary, it is shown that the insights gained extend to two or more intermediaries in Cournot competition for buyers and sellers.

Thus, the reasons for the observed rigidity in the intermediary's share must lie outside the model. I discuss two possible explanations at the end of the paper. The first is behavioral. Sellers may be resistant to increases in the intermediary's share during some episodes of surge-pricing if there are reductions in other surge-pricing episodes. A second reason may be related to court cases filed against Uber and Lyft.

The question addressed in this paper – the impact of rigidity in pricing decisions on surge pricing – has not been asked in the literature. Several papers, notably Armstrong [1], Caillaud and Jullien [7], Hagiu [13], and Rochet and Tirole [18, 19], analyze two-sided markets, where two types of agents interact on a platform provided by a third party. Network externalities play a significant role in these papers. In particular, costs on one side of the market cannot be easily passed through to the other side. Consequently, the terms of trade depend not only on the total amount that the two parties to a transaction pay but also on the division of the payments.

An older literature investigates equilibrium and efficiency in markets with intermediaries. Rubinstein and Wolinsky [20] obtain a steady-state equilibrium in a market with buyers, sellers, and intermediaries, highlighting the relationship between trading rules and the endogenous terms of trade. Stahl [23], Spulber [22], and Yanelle [24] explore conditions under which competition between intermediaries leads to (in)efficient outcomes. The welfare-improving role of a monopolist intermediary in a market with search costs is examined in Yavas [25]. Biglaiser [4] shows that a fully-informed intermediary can overcome adverse selection between a buyer and a seller in a lemons market, while Glode and Opp [11] show that one or more moderately-informed intermediaries can also achieve efficiency in such a market.

Recent work in operations management investigates static and dynamic contracts in intermediated markets. See Banerjee, Johari, and Riquelme [2], Cachon, Daniels, and Lobel [6], and Bai et. al [3].

The paper is organized as follows. The model is presented in the next section. In Section 3, optimal prices are derived. Comparative statics for the intermediary's optimal fee with respect to changes in buyer mass and changes in demand elasticity are derived in Sections 3.1 and 3.2, respectively. Pricing under a constant-fee constraint is examined in Section 4. Cournot intermediaries are analyzed in Section 5. Section 6 concludes. All proofs are in an appendix.

2 The model

There are (potential) gains to trade between risk-neutral buyers and sellers. Each buyer has utility for one unit of a homogenous object and each seller has one unit of the object to sell. There is a continuum of buyers and a continuum of sellers, with each buyer and seller being infinitesimal. The mass of buyers is μ and the mass of sellers is one.[4] Each buyer's valuation v is an independent draw from cumulative distribution

[4]The assumption of a unit mass of sellers is without loss of generality as μ may be viewed as the mass of buyers per unit mass of sellers.

function F_b with strictly positive and continuous density function f_b on support $[0, a_\gamma]$. Each seller's cost c is an independent draw from cumulative distribution function F_s with strictly positive and continuous density function f_s on support $[0, 1]$.

An assumption that is maintained throughout the paper is that the distributions F_b and F_s are *regular* in the sense of Myerson [15]. That is, the virtual utility of buyers, $v - \frac{1-F_b(v)}{f_b(v)}$, is strictly increasing in v and the virtual cost of sellers, $c + \frac{F_s(c)}{f_s(c)}$, is strictly increasing in c.[5]

Changes in demand are captured either by changes in μ (Section 3.1) or by changes in F_b (Section 3.2). In the price-theoretic interpretation of mechanism design due to Bulow and Roberts [5], the demand curve is $q = \mu(1 - F_b(p_b))$ and the supply curve is $q = F_s(p_s)$, where p_b is the buyer price and p_s is the seller price.

The search costs for buyers and sellers are assumed to be prohibitively high. An intermediary with superior matching technology enables trade between the two sides. To simplify the notation, the intermediary's fixed cost and marginal costs per transaction are assumed to be zero.

The intermediary is a matchmaker who does not trade but simply matches buyers with sellers. She is not a market maker who buys and holds inventory. This role as a matchmaker but not a market maker is appropriate in markets for immediate delivery of perishable goods.[6]

There are no network externalities in the model. The analysis applies to relatively mature markets where large numbers of buyers and sellers are present and consequently any network effects have tapered off.

The intermediary knows μ, F_b, and F_s. However, a buyer's value or a seller's cost are not known to the intermediary. Thus, perfect price discrimination is not possible.

Assume that the intermediary can commit to a trading mechanism. From earlier work, it follows that the intermediary cannot increase her profits by using a random-

[5]Most of the results in the paper can be proved if the virtual utility and virtual cost functions are non-decreasing. The proofs are simpler when these functions are assumed to be strictly increasing.

[6]If the intermediary were to act as a matchmaker and hold inventory, it would reduce price surges. However, a constant percent fee would remain suboptimal.

ized mechanism. In a general environment, Riley and Zeckhauser [17] show that a seller's best strategy is to charge a take-it-or-leave-it price to buyers. Myerson and Satterthwaite [16] show that it is optimal for a profit-maximizing intermediary between one buyer and one seller to charge take-it-or-leave-it prices. These papers imply that in the model with a continuum of buyers and sellers considered here, we may restrict attention to deterministic selling mechanisms, i.e., to prices.

Proposition A: (Myerson and Satterthwaite [16], Riley and Zeckhauser [17]) *The optimal strategy for the intermediary is to announce a take-it-or-leave-it price for buyers and a take-it-or-leave-it price for sellers.*

Hence, in order to maximize profits, the intermediary selects a price p_b for buyers and a price p_s for sellers. Any buyer with value greater than p_b will purchase a unit and any seller with cost less than p_s will sell a unit, through the intermediary. Optimal prices are derived in the next section.

3 Optimal intermediation

At price $p_b = v$, buyers demand $\mu(1 - F_b(v))$ units and at price $p_s = c$, sellers are willing to supply $F_s(c)$ units. Thus, $q = \min\{\mu(1-F_b(v), F_s(c)\}$ is the amount traded. If $\mu(1-F_b(v)) < F_s(c)$ then the intermediary can lower the seller price p_s slightly below c and still trade q units. Similarly, if $\mu(1 - F_b(v)) > F_s(c)$ then the intermediary can raise the buyer price p_b slightly above v and still trade q units. Hence, intermediary profit-maximization implies that optimal prices $p_b = v$ and $p_s = c$ are such that demand equals supply:

$$q \;=\; \mu[1 - F_b(v)] \;=\; F_s(c) \tag{1}$$

Thus, $v = F_b^{-1}(1 - \frac{q}{\mu})$, $c = F_s^{-1}(q)$ and the intermediary's profit as a function of q is

$$\begin{aligned}
\Pi_I(q) &= q\,[v - c] \\
&= q\,[F_b^{-1}(1 - \frac{q}{\mu}) - F_s^{-1}(q)] \\
\Longrightarrow \qquad \frac{d\Pi_I}{dq} &= F_b^{-1}(1 - \frac{q}{\mu}) - F_s^{-1}(q) + q\frac{dF_b^{-1}(1 - \frac{q}{\mu})}{dq} - q\frac{dF_s^{-1}(q)}{dq}
\end{aligned}$$

$$= F_b^{-1}(1-\frac{q}{\mu}) - F_s^{-1}(q) - \frac{q}{\mu f_b(F_b^{-1}(1-\frac{q}{\mu}))} - \frac{q}{f_s(F_s^{-1}(q))}$$

$$= v - c - \frac{1-F_b(v)}{f_b(v)} - \frac{F_s(c)}{f_s(c)} \tag{2}$$

The first-order condition, $\frac{d\Pi_I}{dq} = 0$, implies that at optimal prices $p_b = v$ and $p_s = c$,

$$v - \frac{1-F_b(v)}{f_b(v)} = c + \frac{F_s(c)}{f_s(c)} \tag{3}$$

Thus, a necessary condition for optimality is that the virtual utility of the marginal buyer equals the virtual cost of the marginal seller. Following Bulow and Roberts [5], this necessary condition may be interpreted as stating that marginal revenue equals marginal cost.

There are many solutions to (3). As shown below, the demand equals supply condition (1) pins down a unique solution.

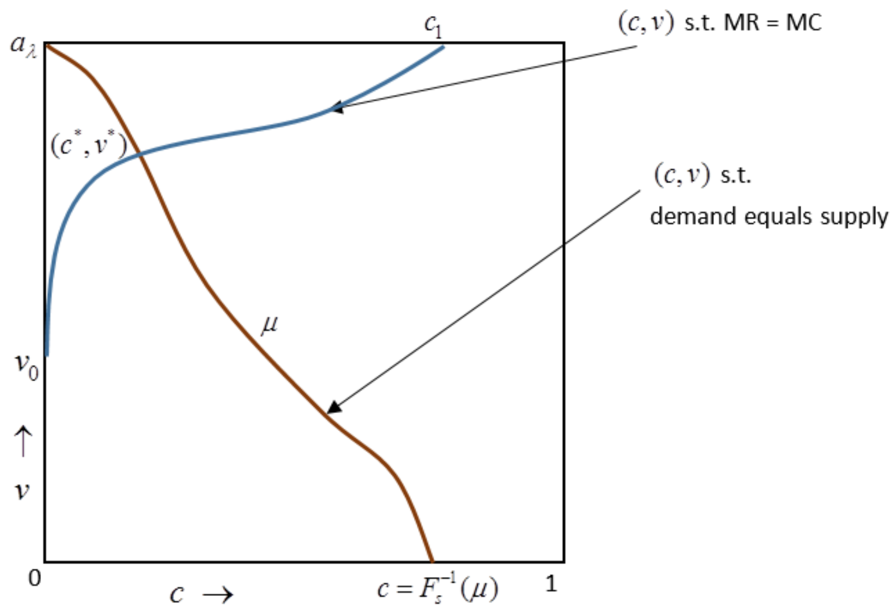

Figure 1

As the densities f_s and f_b are strictly positive and continuous on their support the locus of points (c, v) satisfying (3) is a continuous curve. This is the blue curve, labeled 'MR=MC,' in Figure 1. This curve is a positively-sloped function in the regular case. To see this, start at any point (c, v) on the 'MR=MC' curve. If the cost is increased from c to $c + \Delta c$, then the right-hand side of (3) increases by regularity. To restore equality in (3) the buyer's value must be increased from v, once again by regularity. Hence, there is a $\Delta v > 0$ such that $(c + \Delta c, v + \Delta v)$ is on the 'MR=MC' curve. The points $v_0 > 0$ and $c_1 < 1$ are obtained from $v_0 - \frac{1-F_b(v_0)}{f_b(v_0)} = 0$ and $c_1 + \frac{F_s(c_1)}{f_s(c_1)} = 1$, respectively. Buyers with $v < v_0$ and sellers with $c > c_1$ never trade. Note also that $MR = v - \frac{1-F_b(v)}{f_b(v)} > c + \frac{F_s(c)}{f_s(c)} = MC$ above the 'MR=MC' curve and $MR < MC$ below this curve.

The negatively-sloped brown curve, labeled demand equals supply, represents the points (c, v) that satisfy (1).[7] The intersection of the (blue) 'MR=MC' curve with the (brown) demand equals supply curve yields the optimal prices (c^*, v^*). This is proved next.

Proposition 1 *Assume that F_b and F_s are regular. There exists a unique pair of optimal prices (c^*, v^*) at which the intermediary's profit is maximized.*

The following example illustrates the equilibrium.

Example 1: Assume that F_b and F_s are uniformly distributed on $[0, \gamma]$ and $[0, 1]$ respectively. These distributions are regular. The two conditions for optimality (1) and (3) are

$$\mu(1 - \frac{v^*}{\gamma}) = c^*, \qquad 2v^* - \gamma = 2c^*$$
$$\Longrightarrow \qquad v^* = \frac{\gamma(\gamma + 2\mu)}{2(\gamma + \mu)} \qquad c^* = \frac{\mu\gamma}{2(\gamma + \mu)} \qquad \Pi_I = \frac{\mu\gamma^2}{4(\gamma + \mu)}$$

Optimal prices increase with μ:

$$\frac{\partial v^*}{\partial \mu} = \frac{\gamma^2}{2(\gamma + \mu)^2} > 0, \qquad \frac{\partial c^*}{\partial \mu} = \frac{\gamma^2}{2(\gamma + \mu)^2} > 0$$

The optimal percent fee of the intermediary (expressed as a fraction of the buyer price), α^*, is

$$\alpha^* \equiv \frac{v^* - c^*}{v^*} = \frac{\gamma + \mu}{\gamma + 2\mu}$$

[7] As the quantity supplied increases with c, this curve is a monotone transformation of the usual demand curve with (buyer) price on the vertical axis and quantity demanded on the horizontal axis.

Sections 3.1 and 3.2 examine the impact of two distinct types of shifts in demand: an increase in the number of buyers, which corresponds to an increase in μ, and a decrease in demand elasticity, which in this example corresponds to an increase in γ.

Note that for any μ, $v^* - c^* = \gamma/2$. Thus, as v^* increases with μ, α^* decreases

$$\frac{\partial \alpha^*}{\partial \mu} = -\frac{\gamma}{(\gamma + 2\mu)^2} < 0$$

As the mass of buyers μ increases, it is optimal for the intermediary to take a smaller share of a larger pie.

If, instead, demand becomes less elastic (γ increases) but there is no change in μ, it is optimal for the intermediary to charge higher prices and take a larger share:

$$\frac{\partial v^*}{\partial \gamma} = \frac{(\gamma + \mu)^2 + \mu^2}{2(\gamma + \mu)^2} > 0, \qquad \frac{\partial c^*}{\partial \gamma} = \frac{\mu^2}{2(\gamma + \mu)^2} > 0$$

$$\frac{\partial \alpha^*}{\partial \gamma} = \frac{\mu}{(\gamma + 2\mu)^2} > 0$$

□

In the example, as buyer mass μ increases, the intermediary's optimal percent fee, α^*, decreases while as γ increases (i.e., demand elasticity decreases) α^* increases. In general, α^* may increase or decrease with μ and γ, depending on the elasticities of demand and supply; however, as shown in Propositions 5 and 6, α^* decreases for large enough increases in μ and α^* increases for large enough increases in γ.

The intermediary is both a monopolist and a monopsonist. It acts as a monopolist in its interactions with buyers, with its marginal cost determined by the equilibrium in the sellers' market. The intermediary also acts as a monopsonist in its interactions with sellers, with its marginal revenue from a unit of input determined by the equilibrium in the buyers' market. With this in mind, define price elasticities of demand and supply

$$\eta_b(v) = \frac{v}{q}\frac{dq}{dv} = -v\frac{f_b(v)}{1 - F_b(v)} \tag{4}$$

$$\eta_s(c) = \frac{c}{q}\frac{dq}{dc} = c\frac{f_s(c)}{F_s(c)}$$

where we use $q = \mu(1 - F_b(v))$ and $q = F_s(c)$.

Consider the intermediary in its role as a monopolist with constant marginal cost c. From price theory we know that the price v charged by this monopolist satisfies

$$\frac{v}{c} = \frac{1}{1 + \frac{1}{\eta_b(v)}}$$

Call $\frac{1}{1+\frac{1}{\eta_b(v)}}$ the *gross mark-up* for our intermediary monopolist. Next, consider the intermediary in its role as a monopsonist with constant marginal revenue product from a unit of input equal to v. It will set price for input at c such that

$$\frac{v}{c} = 1 + \frac{1}{\eta_s(c)}$$

Call $1 + \frac{1}{\eta_s(c)}$ the *gross mark-down* for our intermediary monopsonist.

The main result of this section establishes that gross mark-up of the monopolist and the gross mark-down of the monopsonist defined above determine the optimal prices charged by the intermediary.

Proposition 2 *The ratio of the optimal buyer price, v^*, to the optimal seller price, c^*, is the product of the gross mark-up of the intermediary monopolist and the gross mark-down of the intermediary monopsonist*

$$\frac{v^*}{c^*} = \frac{1 + \frac{1}{\eta_s(c^*)}}{1 + \frac{1}{\eta_b(v^*)}} \tag{5}$$

Note that $\frac{v^*}{c^*} = \frac{1}{1-\alpha^*}$, where α^* is the intermediary's optimal percent fee. As $\frac{1}{1-\alpha^*}$ increases if and only if α^* increases, the following corollary is immediate.

Corollary 1 *The intermediary's optimal percent fee α^* decreases after a shift in the demand curve if and only if $\frac{1+\frac{1}{\eta_s(c^*)}}{1+\frac{1}{\eta_b(v^*)}}$ decreases with the shift in demand.*

Thus, the direction of change of the intermediary's optimal share after a shift in demand depends entirely on the consequent changes in demand and supply elasticities. Estimates of supply elasticities of Uber drivers are available in Chen and Sheldon [8] and of New York city cab drivers in Farber [9]; however, these papers do not estimate how elasticity changes along the supply curve. I am not aware of any studies that estimate demand elasticities of passengers.

The impact of demand shifts on optimal pricing is examined in Sections 3.1 and 3.2. An outward shift of the demand curve can be decomposed into two orthogonal components: either the demand curve pivots outwards at the vertical intercept or the demand curve pivots outwards at the horizontal intercept. The former is associated with buyer mass increases and the latter with less elastic distributions of buyer values.

3.1 Surge pricing when mass of buyers increases

In this section, the impact of an increase in μ, the mass of buyers (per unit mass of sellers) is analyzed. The distribution of buyer valuations remains F_b. This represents a shift in the demand curve due to an increase in the number of buyers without any change in demand elasticity.[8]

Figure 2 depicts demand equals supply curves for two different values of buyer mass μ_ℓ and μ_h, with $\mu_\ell < \mu_h$. As buyer mass increases, more sellers are required to fulfill demand at any given buyer price $v < a_\gamma$. Thus, the demand equals supply curve for μ_h lies to the right of the demand equals supply curve for μ_ℓ except at $v = \mu_\ell$. The buyers' demand curve pivots outwards at the vertical intercept.

The 'MR=MC' curve is the same at μ_ℓ and at μ_h as (3) does not depend on μ. Consequently, as the 'MR=MC' curve has positive slope and the demand equals supply curve for μ_h is higher than the curve for μ_ℓ, optimal prices increase with μ: $c^*(\mu_h) > c^*(\mu_\ell)$ and $v^*(\mu_h) > v^*(\mu_\ell)$. It is reasonable that $|\eta_b(v)|$ increases (i.e., demand becomes more elastic) as v increases;[9] therefore the denominator of the expression in (5) increases with μ. If, in addition, $\eta_s(c)$ either increases or does not decrease too fast as c increases, then the numerator of (5) decreases with μ.[10] Thus, if $|\eta_b(v)|$ increases with v and $\eta_s(c)$ does not decrease too fast as c increases, then Corollary 1 implies that α^* decreases with μ.

[8]As F_b does not change, it is clear from (4) that the demand elasticity does not change with μ.

[9]This is true if the hazard rate $\frac{f_b(v)}{1-F_b(v)}$ of F_b increases with v, i.e., $1 - F_b(v)$ is log concave.

[10]Note that a decreasing reversed hazard rate $\frac{f_s(c)}{F_s(c)}$, i.e., log concave $F_s(c)$, limits the rate at which $\eta_s(c)$ might decrease as c increases.

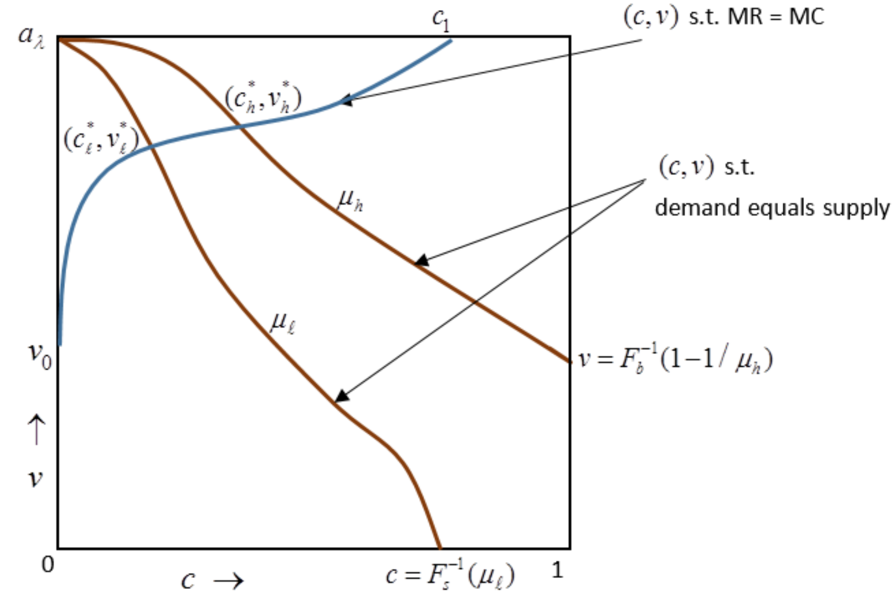

Figure 2

Another necessary and sufficient condition for $\frac{\partial \alpha^*}{\partial \mu} < 0$ is that along the 'MR=MC' curve the elasticity of v^* with respect to c^*, $\frac{c^*}{v^*}\frac{dv^*}{dc^*}$, is less than 1. This is proved next.

Proposition 3 *The intermediary's optimal percent fee, α^* decreases with μ if and only if along the locus of points satisfying equation (3)*

$$\frac{dv^*}{dc^*} < \frac{v^*}{c^*}$$

In particular, if $\frac{dv^*}{d\mu} \leq \frac{dc^*}{d\mu}$ then $\frac{d\alpha^*}{d\mu} < 0$.

The condition $\frac{dv^*}{dc^*} < \frac{v^*}{c^*}$ admits a geometric interpretation. It states that, at optimal prices (c^*, v^*) for buyer mass μ, the slope of the 'MR=MC' curve in Figure 2 is less than the slope of the straight line from the origin to (c^*, v^*).

While Proposition 3 is stated for local changes in buyer mass μ, it is readily adapted to large changes in μ: $\alpha^*(\mu_\ell) > \alpha^*(\mu_h)$ if and only if

$$\frac{v^*(\mu_h) - v^*(\mu_\ell)}{c^*(\mu_h) - c^*(\mu_\ell)} \leq \frac{v^*(\mu_\ell)}{c^*(\mu_\ell)}$$

The proof is omitted. Similarly, Corollary 1 implies that $\alpha^*(\mu_\ell) > \alpha^*(\mu_h)$ if and only if

$$\frac{1 + \frac{1}{\eta_s(c^*(\mu_\ell))}}{1 + \frac{1}{\eta_b(v^*(\mu_\ell))}} > \frac{1 + \frac{1}{\eta_s(c^*(\mu_h))}}{1 + \frac{1}{\eta_b(v^*(\mu_h))}}$$

A sufficient condition on F_b and F_s that implies $\frac{dv^*}{dc^*} < \frac{v^*}{c^*}$ and therefore $\frac{\partial \alpha^*}{\partial \mu} < 0$ is stated next. Note that the inverse hazard rate of F_b is $\frac{1-F_b(v)}{f_b(v)}$, the reciprocal of the hazard rate. The inverse reverse hazard rate of $F_s(c)$ is similarly defined.

Proposition 4 *Suppose that F_b has increasing hazard rate and F_s has decreasing reverse hazard rate. If, along the 'MR=MC' curve, the inverse hazard rate of F_b decreases at least as fast as the inverse reverse hazard rate of F_s increases, then α^* decreases with μ.*

As shown next, the intermediary's optimal share decreases after a sufficiently large increase in μ. This is regardless of the elasticities of supply and demand.

Proposition 5 *For any μ_h, there exists $M(\mu_h) \leq \mu_h$ such that for any $\mu_\ell \leq M(\mu_h)$, we have $\alpha^*(\mu_\ell) > \alpha^*(\mu_h)$.*

The intuition behind Proposition 5 is as follows. Observe that the intermediary will never trade with a buyer with negative marginal revenue, $v < v_0$. If μ is much smaller than 1, then the demand $\mu(1 - F_b(v_0))$ from buyers with valuation v_0 or more can be satisfied by a small fraction of the unit mass of sellers. Consequently, optimality conditions (1) and (3) are satisfied at $(c^*, v^*) = (\epsilon, v_0 + \epsilon')$ where ϵ, ϵ' are small, positive. Thus, as μ becomes smaller, elasticity of demand at the optimal buyer price $v_0 + \epsilon'$ increases to -1 (marginal revenue approaches zero). Hence, the denominator of (5) approaches zero while the numerator is at least 1. Consequently, $\frac{1}{1-\alpha^*} = \frac{v^*}{c^*} = \frac{v_0+\epsilon'}{\epsilon}$ becomes arbitrarily large and α^* approaches 1. The continuity of α^* with respect to μ and $\alpha^*(\mu_h) < 1$ for any μ_h imply the proposition.

That $M(\mu_h) = \mu_h$ for each μ_h is possible follows from Example 1 where it was directly established that $\frac{\partial \alpha^*}{\partial \mu} < 0$ for each μ. This can also be established by Proposition 3 or by Corollary 1. In this example, the 'MR=MC' curve is $v^* = c^* + \gamma/2$. Thus, $\frac{dv^*}{dc^*} = 1 < \frac{v^*}{c^*}$, satisfying the necessary and sufficient condition for $\frac{\partial \alpha^*}{\partial \mu} < 0$ in Proposition 3. Alternatively, note that in Example 1, $\eta_b(v) = -\frac{v}{\gamma - v}$, $\eta_s(c) = 1$. Thus,

$$\frac{1 + \frac{1}{\eta_s(c^*)}}{1 + \frac{1}{\eta_b(v^*)}} \quad = \quad \frac{2}{1 - \frac{\gamma - v^*}{v^*}} \quad = \quad \frac{2v^*}{2v^* - \gamma}$$

which decreases with μ because v^* increases with μ. Corollary 1 implies $\frac{\partial \alpha^*}{\partial \mu} < 0$.

3.2 Surge pricing when demand elasticity decreases

Next, a different type of demand shift is considered, one in which the demand curve becomes less elastic but the mass of buyers does not change. This is effected by changing the distribution of buyer valuations so that at each price $v > 0$, the quantity demanded is greater and demand is less elastic.

Let $F_b^1(v)$ be a cumulative distribution function with strictly postive and continuous density f_b^1 on support $[0, 1]$. For each γ, define a cumulative distribution function with support $[0, a_\gamma]$

$$F_b^\gamma(v) \;\equiv\; F_b^1(v/a_\gamma), \qquad \forall v \tag{6}$$

where $a^1 = 1$ and a_γ is strictly increasing in γ with $\lim_\gamma a_\gamma = \infty$.[11] It is clear from (6) that if $\gamma_h > \gamma_\ell$ then $F_b^{\gamma_h}$ dominates $F_b^{\gamma_\ell}$ by first-order stochastic dominance.

The following assumption is maintained throughout this section:

Assumption IHR: *The hazard rate of* $F_b^1(v)$, $\frac{f_b^1(v)}{1-F_b^1(v)}$, *is increasing in* v.

An immediate consequence of Assumption IHR is that for each γ, $F_b^\gamma(v)$ has increasing hazard rate. Therefore, demand elasticity $|\eta_b^\gamma(v)|$ is increasing in v (as is clear from equation 4) and marginal revenue $MR^\gamma(v) = v - \frac{1-F_b^\gamma(v)}{f_b^\gamma(v)}$ is increasing in v.

Let $\gamma_\ell < \gamma_h$ and, to simplify notation, let $F_b^\ell \equiv F_b^{\gamma_\ell}$, $F_b^h \equiv F_b^{\gamma_h}$, $a_\ell \equiv a_{\gamma_\ell}$ and $a_h \equiv a_{\gamma_h}$. Another consequence of Assumption IHR is that F_b^h dominates F_b^ℓ by the hazard rate order[12]

$$\frac{f_b^h(v)}{1 - F_b^h(v)} \;=\; \frac{1}{a_h}\frac{f_b^1(\frac{v}{a_h})}{1 - F_b^1(\frac{v}{a_h})} \;<\; \frac{1}{a_\ell}\frac{f_b^1(\frac{v}{a_\ell})}{1 - F_b^1(\frac{v}{a_\ell})} \;=\; \frac{f_b^\ell(v)}{1 - F_b^\ell(v)}, \qquad \forall v$$

Consequently

$$\begin{aligned}
|\eta_b^h(v)| \;&=\; v\frac{f_b^h(v)}{1 - F_b^h(v)} \;<\; v\frac{f_b^\ell(v)}{1 - F_b^\ell(v)} \;=\; |\eta_b^\ell(v)|, && \forall v \\
MR^h(v) = v\Big(1 + \frac{1}{\eta_b^h(v)}\Big) \;&<\; v\Big(1 + \frac{1}{\eta_b^h(v)}\Big) \;=\; MR^\ell(v), && \forall v
\end{aligned} \tag{7}$$

[11] In Example 1, $a_\gamma = \gamma$.

[12] See Shaked and Shanthikumar [21] for properties of the hazard rate order.

Thus, an increase in γ represents an increase in demand in that there is a reduction in demand elasticity and, as $1 - F_b^h(v) > 1 - F_b^\ell(v)$ for $v > 0$ by first-order stochastic dominance, greater demand at each strictly positive buyer price.

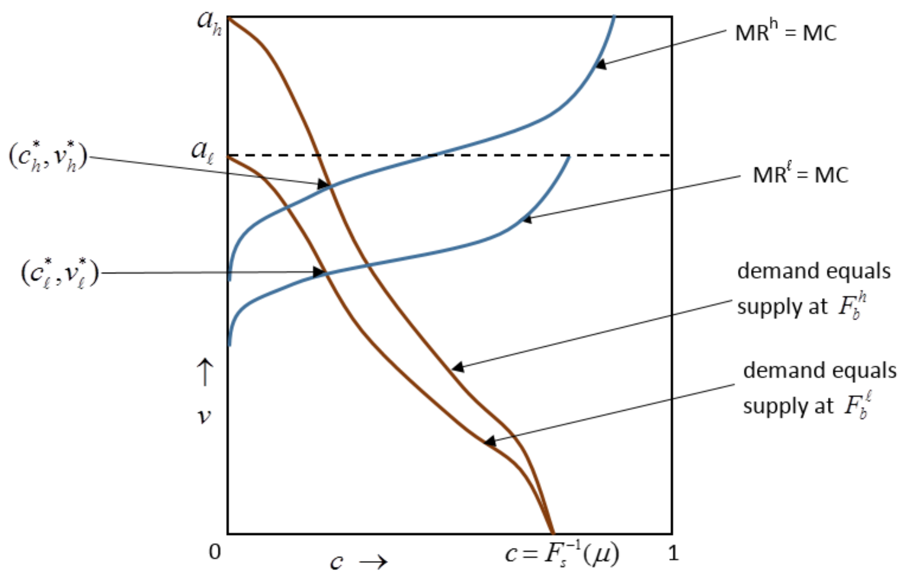

Figure 3

Figure 3 depicts optimal prices for the two distributions, $F_b^\ell(v)$ and $F_b^h(v)$, with $\gamma_\ell < \gamma_h$. As $MR^h < MR^\ell$, the 'MR^h=MC' curve lies above the 'MR$^\ell$=MC'. Further, as γ increases the demand equals supply curve pivots outwards at the horizontal intercept. Too see this, first note that at price $v = 0$ demand is μ for both $F_b^\ell(v)$ and $F_b^h(v)$. Therefore, the horizontal intercept of the two demand equals supply curves is identical at $c = F_s^{-1}(\mu)$.[13] Second, the vertical intercept of the two curves are a_ℓ and a_h – the prices at which demand is zero at $F_b^\ell(v)$ and $F_b^h(v)$, respectively. Third, because $1 - F_b^h(v) > 1 - F_b^\ell(v)$ for all v by first-order stochastic dominance, the demand equals supply curve at F_b^h lies above the demand equals supply curve at F_b^ℓ for all $v > 0$.

The intersection of the respective demand equals supply curve and 'MR=MC' curves at F_b^ℓ and F_b^h yield the optimal prices at γ_ℓ and γ_h. As one may expect, the optimal prices increase with γ.

[13]As depicted in the Figure 3, we have $\mu < 1$. If, instead, $\mu > 1$ then an extension of the two demand equals supply curves would coincide at $v = 0$ and a value of c greater than 1.

Lemma 1 *Suppose that* F_b^1 *has increasing hazard rate. Let* $\gamma_\ell < \gamma_h$ *and let* (c_ℓ^*, v_ℓ^*) *and* (c_h^*, v_h^*) *be the optimal prices at* F_b^ℓ *and* F_b^h*, respectively. Then,* $c_h^* \geq c_\ell^*$ *and* $v_h^* > v_\ell^*$.

As noted earlier, Assumption IHR implies that for any γ demand becomes more elastic as v increases and for any v demand becomes less elastic as γ increases. Therefore, the denominator of the expression in (5) may decrease or increase with γ. The numerator of (5) may also increase or decrease as c increases with γ. Hence, Corollary 1 is not directly applicable in determining the direction of change of the intermediary's optimal share as γ increases.

However, as shown next, the intermediary's optimal share increases after a sufficiently large decrease in demand elasticity (increase in γ). This is regardless of the elasticities of supply and demand.

Proposition 6 *Suppose that* F_b^1 *has increasing hazard rate. For any* γ_ℓ*, there exists* $K(\gamma_\ell) \geq \gamma_\ell$ *such that for any* $\gamma_h > K(\gamma_\ell)$*, we have* $\alpha^*(\gamma_\ell) < \alpha^*(\gamma_h)$.[14]

Propositions 5 and 6 provide an interesting contrast. The optimal share of the intermediary decreases when the increase in demand is due to an increase in the number of buyers without a change in demand elasticity (Proposition 5). The optimal share increases when the increase in demand is due to a reduction in demand elasticity without an increase in the number of buyers.

3.3 Comparison with a benchmark monopolist

It is useful to compare pricing by the intermediary with pricing by a monopolist who acquires the productive assets and services of all the sellers who contract with the intermediary. The comparison is made under the assumption that the monopolist's total cost of providing goods is the same as the sellers' cost.[15]

[14] In Example 1, $K(\gamma_\ell) \equiv \gamma_\ell$ as $\frac{\partial \alpha^*}{\partial \gamma} > 0$.

[15] From the resistance of some intermediaries to classify sellers as employees rather than as independent contractors it appears that this may not be a tenable assumption. That is, total costs may be higher if the intermediary acquired the assets and services of the sellers. Nevertheless, it is

Thus, the marginal cost of the monopolist at $q = F_s(c)$ units is c, which is less than $c + \frac{F_s(c)}{f_s(c)}$, the marginal cost at q units for the intermediary. The marginal revenue of the monopolist is the same as that of the intermediary. Consequently, the benchmark monopolist's optimal buyer price is lower and the quantity higher than the corresponding optimal values for the intermediary. Monopoly prices, (c_m, v_m), satisfy

$$v_m - c_m \;=\; \frac{1 - F_b(v_m)}{f_b(v_m)} \;<\; \frac{1 - F_b(v_m)}{f_b(v_m)} + \frac{F_s(c_m)}{f_s(c_m)}$$

Both (c^*, v^*) and (c_m, v_m) lie on the demand equals supply curve in Figure 1. However, (c^*, v^*) is on the 'MR=MC' curve, while (c_m, v_m) is below the 'MR=MC' curve. Consequently, $v^* > v_m$, $c_* < c_m$, and $v^* - c^* > v_m - c_m > 0$. As the distortion due to monopsonistic behavior is absent at the monopoly outcome, it is more efficient than the intermediary outcome

4 Surge pricing under constrained intermediation

As mentioned earlier, intermediaries such as Uber and Lyft keep a constant percent of the buyer price regardless of demand conditions. I show below that if an intermediary operates under the constraint that its percent fee is constant then as demand increases the magnitude of the price surge is amplified on one side of the market and diminished on the other side, compared to the surge in unconstrained optimal prices. In particular, if an increase in demand is due to a substantial increase in buyer mass, then the buyer-price surge is amplified and the seller-price surge is muted under a constant percent fee.

It is sufficient to consider two possible levels of demand, D_h or high demand, and D_ℓ or low demand. Demand changes arise from one of two scenarios described below.

> Change in buyer mass: The mass of buyers is μ_h during high demand and μ_ℓ during low demand, with $\mu_h > \mu_\ell$. The distribution of buyer valuations is F_b^γ (as defined in equation 6) at high and at low demand.

instructive to consider this benchmark monopolist with the same cost of providing service as that of the sellers collectively.

Change in demand elasticity: The distribution of buyer valuations is F_b^h during high demand and F_b^ℓ during low demand, with $\gamma_h > \gamma_\ell$. The mass of buyers is μ at high demand and at low demand.

The fraction of time that demand is high is r. Alternatively, r may be viewed as the probability that demand is high at any given moment.

The constraint is that the intermediary keeps the same fraction α of the buyer price regardless of the level of demand.[16] Thus, rather than choose any prices v_h, c_h during high demand and any prices v_ℓ, c_ℓ during low demand, the intermediary is constrained to choose v_h, v_ℓ and α and set $c_h = (1-\alpha)v_h$, $c_\ell = (1-\alpha)v_\ell$. That is, prices must satisfy

$$\frac{1}{1-\alpha} = \frac{v_h}{c_h} = \frac{v_\ell}{c_\ell} \tag{8}$$

for some $\alpha \in (0,1)$. I refer to (8) as the constant-fee constraint.

Let $(\hat{c}_\ell, \hat{v}_\ell)$ and $(\hat{c}_h, \hat{v}_h)$ be the optimal prices at D_ℓ and at D_h respectively under the constant-fee constraint. Let

$$\hat{\alpha} = \frac{\hat{v}_\ell - \hat{c}_\ell}{\hat{v}_\ell} = \frac{\hat{v}_h - \hat{c}_h}{\hat{v}_h}$$

be the optimal fee of the intermediary under this constraint.

As already mentioned, the object being sold is perishable and cannot be stored. Thus, optimality implies that demand must equal supply, i.e., (1) is satisfied at D_ℓ and at D_h. Therefore, $(\hat{c}_k, \hat{v}_k)$ lies on the demand equals supply curve for D_k, $k = \ell, h$. A consequence is that

$$\hat{c}_h > \hat{c}_\ell \quad \text{and} \quad \hat{v}_h > \hat{v}_\ell \tag{9}$$

To see this, note that the constant-fee constraint states that the straight line through $(\hat{c}_\ell, \hat{v}_\ell)$ and $(\hat{c}_h, \hat{v}_h)$ passes through the origin and has slope $\frac{1}{1-\hat{\alpha}} > 0$. In each of the two demand change scenarios, the demand equals supply curve for D_h lies above the demand equals supply curve for D_ℓ (see Figures 2 and 3). Therefore, (9) is implied.

Let v_k^*, c_k^*, and α_k^* be the (unconstrained) optimal prices and percent fee at demand

[16] Equivalently, the intermediary adds a constant mark-up of $\frac{\alpha}{1-\alpha}\%$ to the seller price to obtain the buyer price.

level D_k, $k = \ell, h$. The next proposition says that the constrained optimal fee for the intermediary is in between the unconstrained optimal fees at D_ℓ and at D_h.

Proposition 7 *Under the constant-fee constraint, (8), the optimal fee for the intermediary, $\hat{\alpha}$, satisfies*

$$\min\{\alpha_\ell^*, \alpha_h^*\} \;\leq\; \hat{\alpha} \;\leq\; \max\{\alpha_\ell^*, \alpha_h^*\}, \tag{10}$$

with strict inequalities if $\alpha_\ell^* \neq \alpha_h^*$. Further, optimal prices $(\hat{c}_\ell, \hat{v}_\ell)$, $(\hat{c}_h, \hat{v}_h)$ and an optimal fee $\hat{\alpha} = \frac{\hat{v}_k - \hat{c}_k}{\hat{v}_k}$ exist.

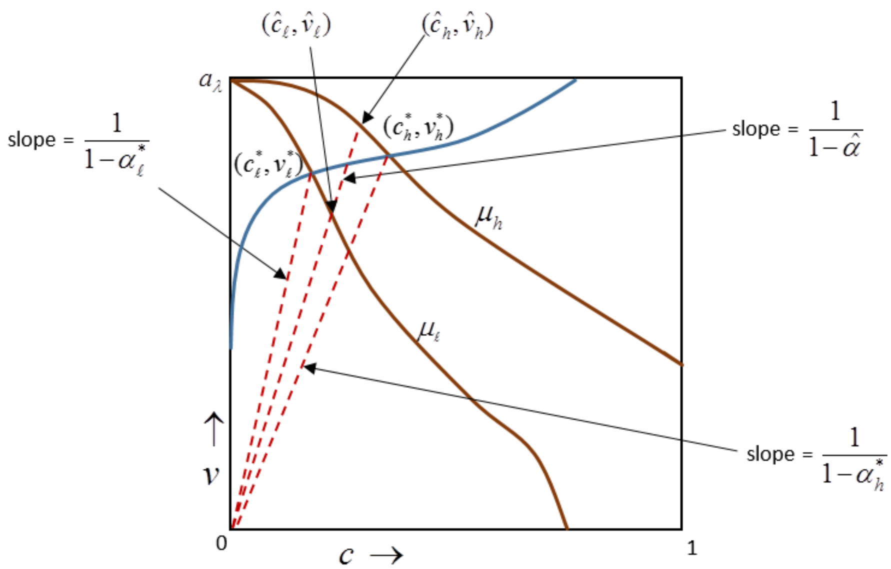

Figure 4

The necessity of (10) for the case when demand changes due to a change in buyer mass can be seen from Figure 4.[17] As $\alpha_\ell^* > \alpha_h^*$ in the figure, we have $\frac{1}{1-\alpha_\ell^*} > \frac{1}{1-\alpha_h^*}$. That is, the line from the origin to (c_ℓ^*, v_ℓ^*) is steeper than the line from the origin to (c_h^*, v_h^*). According to Proposition 7, $\alpha_\ell^* > \hat{\alpha} > \alpha_h^*$. That is, as illustrated in Figure 4, the line from the origin to $(\hat{c}_\ell, \hat{v}_\ell)$ and $(\hat{c}_h, \hat{v}_h)$[18] is less steep [steeper] than the line from the origin to (c_ℓ^*, v_ℓ^*) [(c_h^*, v_h^*)]. Suppose to the contrary that $\hat{\alpha} > \alpha_\ell^* (> \alpha_h^*)$; the line from the origin to the purported constrained optimal prices is steeper than the line from the origin to (c_ℓ^*, v_ℓ^*). As a result, the constrained optimal prices lie on

[17] The necessity of (10) for the scenario where demand changes due to a change in elasticity follows from a similar argument.

[18] Both points lie on the same line due to the constant-fee constraint.

the portions of the demand equals supply curves for μ_ℓ and for μ_h that is above the 'MR=MC' curve. In this region, marginal revenue exceeds marginal cost at μ_ℓ and at μ_h. Decreasing $\hat{\alpha}$ by lowering buyer prices and raising seller prices, while staying on the demand equals supply curves, would increase the intermediary's profit at μ_ℓ and at μ_h. Thus, $\hat{\alpha} > \max\{\alpha_\ell^*, \alpha_h^*\}$ cannot be optimal. A symmetric argument shows that $\hat{\alpha} < \min\{\alpha_\ell^*, \alpha_h^*\}$ is not optimal.

The preceding analysis establishes that if $\alpha_\ell^* > \alpha_h^*$, then $(\hat{c}_h, \hat{v}_h)$ is to the northwest of (c_h^*, v_h^*) and $(\hat{c}_\ell, \hat{v}_\ell)$ is to the southeast of (c_h^*, v_h^*). Hence, the interval $[\hat{v}_\ell, \hat{v}_h]$ brackets $[v_\ell^*, v_h^*]$ amplifying surge pricing for buyers while $[c_\ell^*, c_h^*]$ brackets $[\hat{c}_\ell, \hat{c}_h]$ diminishing surge pricing for sellers. The opposite is true if $\alpha_\ell^* < \alpha_h^*$.

Proposition 8 *If $\alpha_\ell^* > \alpha_h^*$, then as demand increases from D_ℓ to D_h the surge in buyer prices is greater and the surge in seller prices is smaller under the constant-fee constraint than at unconstrained optimal prices. That is,*

$$\begin{array}{ccccccc} \hat{v}_h & > & v_h^* & > & v_\ell^* & > & \hat{v}_\ell \\ c_h^* & > & \hat{c}_h & > & \hat{c}_\ell & > & c_\ell^* \end{array}$$

If, instead $\alpha_\ell^* < \alpha_h^*$, then

$$\begin{array}{ccccccc} v_h^* & > & \hat{v}_h & > & \hat{v}_\ell & > & v_\ell^* \\ \hat{c}_h & > & c_h^* & > & c_\ell^* & > & \hat{c}_\ell \end{array}$$

The quantity traded increases as seller price increases. Thus, Proposition 8 implies the following. If $\alpha_\ell^* > \alpha^*$, then the quantity traded decreases at D_h under a constant percent fee (compared to the quantity traded at D_h with flexible percent fees) and it increases at D_ℓ. In contrast, if $\alpha_\ell^* < \alpha^*$, then the quantity traded increases at D_h under a constant percent fee and decreases at D_ℓ.

Proposition 5 together with Proposition 8 imply

Corollary 2 *Suppose that an increase in demand is due to an increase in buyer mass. For any μ_h, there exists $M(\mu_h) \leq \mu_h$ such that for any $\mu_\ell \leq M(\mu_h)$, the surge in buyer prices is greater and the surge in seller prices is smaller under the constant-fee constraint than at unconstrained optimal prices.*

Similarly, Proposition 6 together with Proposition 8 imply

Corollary 3 *Suppose that an increase in demand is due to less elastic demand and that F_b^1 has increasing hazard rate. Suppose further that F_b^1 has increasing hazard rate and F_b^γ is as defined in (6). For any γ_ℓ, there exists $K(\gamma_\ell) \geq \gamma_\ell$ such that for any $\gamma_h > K(\gamma_\ell)$, the surge in buyer prices is smaller and the surge in seller prices is greater under the constant-fee constraint than at unconstrained optimal prices.*

The impact of a constant fee on efficiency is mixed. Under the constant-fee constraint, efficiency deteriorates (improves) in the demand condition with the lower (higher) unconstrained percent fee. To see this, consider a change in buyer mass from μ_ℓ to μ_h and suppose $\alpha^*(\mu_\ell) > \alpha^*(\mu_h)$. Observe that at the efficient outcome the intermediary's profit is zero: the efficient outcome with buyer mass μ_k, $k = \ell, h$ is $v_k^e = c_k^e$ obtained by the intersection in Figure 4 of the demand equals supply curve for μ_k with the positive-sloped diagonal (not shown in the figure). The unconstrained optimal prices are inefficient as $v_k^* > c_k^*$, i.e., (c_k^*, v_k^*) is above the diagonal. Any movement from (c_k^*, v_k^*) towards (away from) the diagonal along the demand equals supply curve increases (decreases) the gains from trade. As $\alpha_h^* < \alpha_\ell^*$, we have $\alpha_h^* < \hat{\alpha} < \alpha_\ell^*$ by Proposition 7. Consequently, the constrained optimal prices $(\hat{c}_h, \hat{v}_h)$ are further away from the diagonal than (c_h^*, v_h^*) and the constant-fee constraint decreases efficiency in the high demand setting. Similarly, $\alpha_h^* < \alpha_\ell^*$ implies that efficiency increases in the low demand setting under the constant-fee constraint.

A necessary condition for optimality under the constant-fee constraint is presented below. The lemma applies to increases in buyer mass or decreases in demand elasticity or both.

Lemma 2 *Let $\mu_\ell < \mu_h$ and $\gamma_\ell < \gamma_h$. Under the constant-fee constraint, (8), the optimal prices $(\hat{c}_\ell, \hat{v}_\ell)$ at μ_ℓ, F_b^ℓ and $(\hat{c}_h, \hat{v}_h)$ at μ_h, F_b^h satisfy*

$$\left[\hat{v}_h - \frac{1 - F_b^h(\hat{v}_h)}{f_b^h(\hat{v}_h)}\right] - \left[\hat{c}_h + \frac{F_s(\hat{c}_h)}{f_s(\hat{c}_h)}\right] = \frac{\lambda(1-\hat{\alpha})\hat{v}_\ell\hat{v}_h}{r}\left[\frac{1}{|\eta_b^h(\hat{v}_h)|} + \frac{1}{\eta_s(\hat{c}_h)}\right] \quad (11)$$

$$\left[\hat{v}_\ell - \frac{1 - F_b^\ell(\hat{v}_\ell)}{f_b^\ell(\hat{v}_\ell)}\right] - \left[\hat{c}_\ell + \frac{F_s(\hat{c}_\ell)}{f_s(\hat{c}_\ell)}\right] = -\frac{\lambda(1-\hat{\alpha})\hat{v}_\ell\hat{v}_h}{1-r}\left[\frac{1}{|\eta_b^\ell(\hat{v}_\ell))|} + \frac{1}{\eta_s(\hat{c}_\ell)}\right] \quad (12)$$

where λ is a Lagrangian multiplier.

As noted earlier, if $\alpha_\ell^* > \alpha_h^*$ then MR>MC at $(\hat{c}_h, \hat{v}_h)$ and MR<MC at $(\hat{c}_\ell, \hat{v}_\ell)$. That is,

$$\hat{v}_h - \frac{1 - F_b^h(\hat{v}_h)}{f_b^h(\hat{v}_h)} \;>\; \hat{c}_h + \frac{F_s(\hat{c}_h)}{f_s(\hat{c}_h)} \quad \text{and} \quad \hat{v}_\ell - \frac{1 - F_b^\ell(\hat{v}_\ell)}{f_b^\ell(\hat{v}_\ell)} \;<\; \hat{c}_\ell + \frac{F_s(\hat{c}_\ell)}{f_s(\hat{c}_\ell)}$$

Thus, the Lagrange multiplier λ in (11) and (12) is positive when $\alpha_\ell^* > \alpha_h^*$. Similarly. λ is negative if $\alpha_\ell^* < \alpha_h^*$. In either event, the divergence between the marginal revenue and marginal cost at $(\hat{c}_k, \hat{v}_k)$, $k = \ell, h$ increases as (i) the absolute value of the elasticity of demand at $\hat{v}_k$ decreases or (ii) the elasticity of supply at $\hat{c}_k$ decreases or (ii) the fraction of time that buyer mass is μ_k decreases.

In the example below, constrained optimal prices are easily computed with the necessary conditions in Lemma 2.

EXAMPLE 2: Consider Example 1 with $\gamma = 1$, i.e., the support of F_b is [0,1]. Suppose that the intermediary operates under a constant-fee constraint. The buyer mass is $\mu_\ell = 1$ half the time and $\mu_h = 2$ half the time, i.e., $r = 0.5$. As F_b and F_s are uniformly distributed on [0,1], $\eta_b(v) = -\frac{v}{1-v}$ and $\eta_s(c) = 1$. Substituting in the first-order conditions above, we obtain

$$\begin{aligned} 2\hat{\alpha}\hat{v}_h - 1 &= 2\gamma(1-\hat{\alpha})\hat{v}_\ell \\ 1 - 2\hat{\alpha}\hat{v}_\ell &= 2\gamma(1-\hat{\alpha})\hat{v}_h \end{aligned}$$

Moreover, the demand equals supply condition implies that

$$\hat{v}_k \;=\; \frac{\mu_k}{1 - \hat{\alpha} + \mu_k}, \quad k = \ell, h$$

Substituting $\hat{v}_k$ in the first-order conditions we have two equations in two unknowns: $\hat{\alpha}$ and γ. For $\mu_\ell = 1$, $\mu_h = 2$, the constrained optimal solution is

$$\hat{\alpha} \;=\; 0.6316, \quad (\hat{c}_\ell, \hat{v}_\ell) = (0.2692, 0.7308), \quad (\hat{c}_h, \hat{v}_h) = (0.3111, 0.8444)$$

The unconstrained optimal solutions at high and low demand are

$$\alpha_\ell^* = 0.6667,\ (c_\ell^*, v_\ell^*) = (0.25, 0.75), \qquad \alpha_h^* = 0.6,\ (c_h^*, v_h^*) = (0.3333, 0.8333)$$

Note the amplification of the surge in buyer prices and the reduction of the surge in seller prices under the constant-fee constraint, as established in Proposition 8.

A direct computation shows that change (increase) in efficiency (at constrained optimal prices in comparison to unconstrained optimal prices) during low demand is

$$\Delta E_\ell \;=\; \frac{1}{2}(\hat{c}_\ell - c_\ell^*)(\hat{v}_\ell - \hat{c}_\ell + v_\ell^* - c_\ell^*) \;=\; +0.00923$$

while the change (decrease) in efficiency during high demand is

$$\Delta E_h \;=\; \frac{1}{2}(\hat{c}_h - c_h^*)(\hat{v}_h - \hat{c}_h + v_h^* - c_h^*) \;=\; -0.01146$$

As $r = 0.5$, the net effect of a constant-fee is a decrease in expected efficiency. □

5 Cournot intermediaries

The insights from the monopoly model generalize to a model of Cournot competition. Consider two identical intermediaries who compete on both sides of the market. That is, buyers and sellers are free to trade through either intermediary. It does not pay an intermediary to have an imbalance in supply and demand, for instance by purchasing more units from sellers than it sells to buyers. Therefore, with q_i as the number units that intermediary i decides to trade, we have

$$q_1 + q_2 \;=\; \mu[1 - F_b(v)] \;=\; F_s(c) \tag{13}$$

The next result generalizes the optimal gross markup for a monopolist intermediary in (5) to Cournot competition.

Proposition 9 *Assume that F_b and F_s are regular. The optimal prices (c_2^*, v_2^*) at the unique Cournot equilibrium with two intermediaries satisfy*

$$\left[v_2^* - \frac{1 - F_b(v_2^*)}{f_b(v_2^*)}\right] - \left[c_2^* + \frac{F_s(c_2^*)}{f_s(c_2^*)}\right] \;=\; -(v_2^* - c_2^*) \tag{14}$$

which is equivalent to

$$\frac{1}{1 - \alpha_2^*} \;=\; \frac{v_2^*}{c_2^*} \;=\; \frac{2 + \frac{1}{\eta_s(c_2^*)}}{2 + \frac{1}{\eta_b(v_2^*)}} \tag{15}$$

The first-order condition (14) together with (13) and the symmetry of the intermediaries pin down the Cournot equilibrium. A comparison of (14) with (3) implies that the buyer price is lower and the seller price is higher at the Cournot equilibrium than the optimal buyer and seller prices for the monopolist intermediary. The Cournot prices (c_2^*, v_2^*) lie further down the demand equals supply curve from (c^*, v^*) in Figure 1, closer to the diagonal. Therefore, $\frac{v_2^*}{c_2^*} < \frac{v^*}{c^*}$ which implies that Cournot intermediaries' percent fee is lower, and the outcome more efficient, than that of the monopolist intermediary.

Once again, after a shift in the demand curve there is no reason to expect the elasticities of supply and demand at the new optimal prices to adjust such that the ratio on the right-hand side of (15), and therefore α_2^*, remains constant. As in the monopolist intermediary case, when either the buyer mass μ goes to zero or demand elasticity reduces sharply (i.e., γ becomes large), the optimal total share of the two intermediaries approaches one. Consequently, Propositions 5 and 6 generalize to Cournot competition.

The proof of Proposition 9 easily extends to I intermediaries in Cournot competition. Optimal prices (c_I^*, v_I^*) satisfy

$$\begin{aligned}\left[v_I^* - \frac{1-F_b(v_I^*)}{f_b(v_I^*)}\right] - \left[c_I^* + \frac{F_s(c_I^*)}{f_s(c_I^*)}\right] &= -(I-1)(v_I^* - c_I^*) \\ \implies \qquad \frac{1}{1-\alpha_I^*} = \frac{v_I^*}{c_I^*} &= \frac{I + \frac{1}{\eta_s(c_I^*)}}{I + \frac{1}{\eta_b(v_I^*)}}\end{aligned}$$

6 Discussion

Optimal prices set by an intermediary are obtained and related to the intermediary's behavior as a monopsonist and as a monopolist in its interactions with sellers and buyers, respectively. The optimal percent fee of the intermediary changes with buyer demand. If the demand curve shifts out primarily due to a large increase in the number of buyers, then a smaller percent fee is optimal for the intermediary; the reduced fee per trade is more than compensated by the larger number of trades facilitated by a smaller fee. If, instead, the demand curve shifts out primarily due to a large decrease in the elasticity of demand, then a larger percent fee is optimal for the intermediary; there is less incentive to complete more trades by raising the seller price substantially.

Both types of changes may be present in any actual increase in demand which may reduce the intermediary's incentives for adjusting the percent fee. For instance, the increase in intermediary profits from lowering the percent fee in response to an increase in buyer mass is moderated if, concurrently, demand becomes somewhat less elastic. Nevertheless, a fixed percent fee at all times is suboptimal.

I discuss two reasons, both outside the model, that might explain the observed rigidity in the intermediary's share.

The first possible explanation is behavioral. If, as is optimal, Uber were to reduce its share of revenue during surge-pricing periods when buyer mass is high then sellers (Uber car drivers) may expect Uber to reduce its share whenever there is surge pricing. In particular, drivers may become aggrieved if Uber were to optimally increase its share when the surge is mainly due to a sharp decrease in elasticity. A fixed fee gets around this problem.

There may also be a legal rationale for the constant percent fee charged by Uber. Currently, the firm is a defendant in lawsuits that question its standing as an intermediary. The plaintiffs are sellers who claim that they should be considered employees of Uber and are entitled to various benefits under labor regulations. An important consideration in the lawsuit is who controls the prices and fees.[19] Plausibly, if Uber were to exercise greater flexibility in pricing, such as fine tune its percent fee to changes in demand, then Uber's legal claim that it acts as an intermediary may be weakened.

Thus, while a constant percent fee may strengthen Uber's position in pending litigation, it is costly for two reasons. First, it reduces profits. Second, in instances where lowering the fee during a surge is optimal, a constant fee exacerbates surge pricing for buyers leading to a loss in goodwill.[20]

The model in this paper is static. A natural extension of the analysis is to a

[19] See Douglas O'Connor et al. v. Uber Technologies et al., 2015 [10]. Similar lawsuits have been filed against other intermediaries including Lyft, Postmates, Instacart, GrubHub, and Shyp.

[20] See Grubb [12], for instance.

dynamic model in which, at some cost, buyers can delay purchasing during a surge in prices and sellers can withhold selling until there is a surge. Lower-value buyers (higher-cost sellers) would benefit from postponing a trade during a surge (non-surge). However, the factors in the static model that lead to changes in the intermediary's optimal share would also operate in a dynamic model as long there is a cost of delay.

That the intermediary has market power in its interactions with buyers and sellers has implications for anti-trust policy. One's initial intuition might be that if an intermediary hires its sellers, an issue in pending litigation, then its marginal costs would decrease, thereby reducing the distortions of an intermediary monopsonist. The comparison with a benchmark monopolist in Section 3 supports this view. However, this argument assumes that the total cost and the value proposition to buyers would remain unchanged if sellers were to become employees. This assumption may not be tenable. First, many car drivers sell their services to both Uber and Lyft, switching between the two depending on which intermediary provides a closer next passenger pick-up. This practice reduces idle time thereby increasing the efficiency of the sellers and leads to greater competition on the supply side; it would likely be curtailed if each seller were to become an employee of one of the two firms. Second, the intermediary's marginal cost might decrease only if it bought the productive assets of the sellers along with their services; this would reduce the scale of operations of the intermediary due to capital constraints which in turn may reduce the value of the service to buyers. These considerations are also germane to anti-trust policy.

7 Appendix: Proofs

Proof of Proposition 1: First, it is shown that the necessary conditions for optimality, (1) and (3), are satisfied at exactly one set of prices.

If (c, v), (c', v'), satisfy (1) then $v < v'$ if and only if $c > c'$. That is, a demand equals supply curve in Figure 1 has strictly negative slope. To see this, note that $v < v'$ implies $F_b(v) < F_b(v')$, as f_b is strictly positive on its support. Thus, $F_s(c) = \mu(1 - F_b(v)) > \mu(1 - F_b(v')) = F_s(c')$. Hence, $c > c'$. Reversing this argument we have $c > c'$ implies $v < v'$.

It was argued earlier that the locus of points (c, v) that satisfy (3) has strictly positive slope. Thus, as the locus of points that satisfy (1) has strictly negative slope, there is a unique (c^*, v^*) that satisfies the two necessary conditions for optimality: the demand equals supply condition (1) and the marginal revenue equals marginal cost condition (3).

As the intermediary's profit function is maximized on a compact domain $\{0 \leq c \leq 1,\ 0 \leq v \leq 1,\ v \geq c\}$, a maximum exists. Further, this maximum must occur in the interior of the domain because the intermediary profit is zero at any point on the boundary of the domain while it is strictly positive at any point in the interior. As the two necessary conditions are satisfied at this interior maximum, this maximum must occur at the unique (c^*, v^*) that satisfies (1) and (3). □

Proof of Proposition 2: Equation (3) may be written as

$$\begin{aligned}
v^*\Big[1 - \frac{1}{v^*}\frac{1 - F_b(v^*)}{f_b(v^*)}\Big] &= c^*\Big[1 + \frac{1}{c^*}\frac{F_s(c^*)}{f_s(c^*)}\Big] \\
\iff \qquad v^*\Big[1 + \frac{1}{\eta_b(v^*)}\Big] &= c^*\Big[1 + \frac{1}{\eta_s(c^*)}\Big] \\
\iff \qquad \frac{v^*}{c^*} &= \frac{1 + \frac{1}{\eta_s(c^*)}}{1 + \frac{1}{\eta_b(v^*)}} \qquad \square
\end{aligned}$$

Proof of Proposition 3: We have

$$\frac{d\alpha^*}{d\mu} = \frac{d}{d\mu}\Big[\frac{v^* - c^*}{v^*}\Big] < 0$$

$$
\begin{aligned}
\iff \quad & v^*\Big[\frac{dv^*}{d\mu} - \frac{dc^*}{d\mu}\Big] - \frac{dv^*}{d\mu}[v^* - c^*] &=& \; c^*\frac{dv^*}{d\mu} - v^*\frac{dc^*}{d\mu} \; < \; 0 \\
\iff \quad & \frac{c^*}{v^*}\frac{dv^*/d\mu}{dc^*/d\mu} &<& \; 1 \\
\iff \quad & \frac{c^*}{v^*}\frac{dv^*}{dc^*} &<& \; 1 \\
\iff \quad & \frac{dv^*}{dc^*} &<& \; \frac{v^*}{c^*}
\end{aligned}
$$

As $c^* < v^*$, this inequality is satisfied if $\frac{dv^*}{d\mu} \leq \frac{dc^*}{d\mu}$. □

Proof of Proposition 4: Let $IHR_b(v) = \frac{1-F_b(v)}{f_b(v)}$ be the inverse hazard rate of F_b and $IRHR_s(c) = \frac{F_s(c)}{f_s(c)}$ be the inverse reverse hazard rate of F_s. By assumption, $IHR_b(v)$ is decreasing and $RHR_s(c)$ is increasing.

Writing v and c for v^* and c^*, equation (3) may be written as

$$
\begin{aligned}
& v - c &=& \; IHR_b(v) + IRHR_s(c) \\
\implies \quad & \frac{dv}{dc} - 1 &=& \; \frac{d\,IHR_b(v)}{dv}\frac{dv}{dc} + \frac{d\,IRHR_s(c)}{dc} \\
\implies \quad & \Big[1 - \frac{d\,IHR_b(v)}{dv}\Big]\frac{dv}{dc} &=& \; 1 + \frac{d\,IRHR_s(c)}{dc} \\
\implies \quad & \frac{dv}{dc} &=& \; \frac{1 + \frac{d\,IRHR_s(c)}{dc}}{1 - \frac{d\,IHR_b(v)}{dv}} \\
& &\leq& \; 1
\end{aligned}
$$

where the inequality is implies by the hypothesis of the proposition that

$$
-\frac{d\,IHR_b(v)}{dv} \geq \frac{d\,IRHR_s(c)}{dc}
$$

As $v > c$ at an optimal solution, we have $\frac{dv}{dc} \leq 1 < \frac{v}{c}$ which, by Proposition 3, completes the proof. □

Proof of Proposition 5: For any μ_h, we have $v^*(\mu_h) > c^*(\mu_h)$. Hence, $\alpha^*(\mu_h) < 1$. As $\mu \to 0$, we have $v^*(\mu) \to v_0 > 0$, $c^*(\mu) \to 0$ and thus, $\alpha^*(\mu) \to 1$. By continuity of the 'MR=MC' curve, there exists $M(\mu_h) \in (0, \mu_h]$ such that $\alpha^*(\mu) \in (\alpha^*(\mu_h), 1)$ for all $\mu \in (0, M(\mu_h))$. □

Proof of Lemma 1: Suppose that $c_h^* < c_\ell^*$. Then

$$\begin{aligned}
\mu\Big(1 - F_b^\ell\Big(\frac{a_\ell}{a_h} v_h^*\Big)\Big) &= \mu\Big(1 - F_b^1\Big(\frac{v_h^*}{a_h}\Big)\Big) \\
&= \mu(1 - F_b^h(v_h^*)) \\
&= F_s(c_h^*) \\
&< F_s(c_\ell^*) \\
&= \mu(1 - F_b^\ell(v_\ell^*))
\end{aligned}$$

which implies that $\frac{a_\ell}{a_h} v_h^* > v_\ell^*$. Next, note that

$$\begin{aligned}
\eta_b^h(v) &= -v \frac{f_b^h(v)}{1 - F_b^h(v)} &&= -\frac{1}{a_h} v \frac{f_b^1(\frac{v}{a_h})}{1 - F_b^1(\frac{v}{a_h})} &&= \eta_b^1(\frac{v}{a_h}) \\
\implies \quad MR^h(v) &= v\Big(1 + \frac{1}{\eta_b^h(v)}\Big) &&= a_h\Big[\frac{1}{a_h} v\Big(1 + \frac{1}{\eta_b^1(\frac{v}{a_h})}\Big)\Big] &&= a_h MR^1(\frac{v}{a_h}) \\
\implies \quad MR^h(v) &= \frac{a_h}{a_\ell} MR^\ell\Big(\frac{a_\ell}{a_h} v\Big)
\end{aligned}$$

Thus, using the fact that $MR^k = MC$ at optimal prices (c_k^*, v_k^*), $k = \ell, h$, we have

$$MC(c_h^*) = MR^h(v_h^*) = \frac{a_h}{a_\ell} MR^\ell\Big(\frac{a_\ell}{a_h} v_h^*\Big) > \frac{a_h}{a_\ell} MR^\ell(v_\ell^*) = \frac{a_h}{a_\ell} MC(c_\ell^*) > MC(c_\ell^*) > MC(c_h^*)$$

where the first inequality follows from $\frac{a_\ell}{a_h} v_h^* > v_\ell^*$ and increasing marginal value, the second inequality from $a_h > a_\ell$ and the last inequality from $c_\ell^* > c_h^*$. The contradiction implies $c_h^* \geq c_\ell^*$.

Next,

$$MR^\ell(v_h^*) > MR^h(v_h^*) = MC(c_h^*) \geq MC(c_\ell^*) = MR^\ell(v_\ell^*)$$

where the first inequality follows from (7) and the second from $c_h^* \geq c_\ell^*$. Thus, $v_h^* > v_\ell^*$.
□

Proof of Proposition 6: Let c_γ be the optimal seller price at F_b^γ. By Lemma 1, $\{c_\gamma\}_\gamma$ is a non-decreasing series with values in a compact set $[0, 1]$. Therefore, it has a limit c^*. Define v^* by

$$\mu(1 - F_b^1(v^*)) = F_s(c^*)$$

Note that $\mu(1 - F_b^\gamma(a_\gamma v^*)) = \mu(1 - F_b^1(v^*))$. Therefore, as $c_\gamma \to c^*$, we have $\frac{v_\gamma}{a_\gamma} \to v^*$. Thus, as each $v_\gamma > 0$, we have

$$\alpha_\gamma \equiv 1 - \frac{c_\gamma}{v_\gamma} = 1 - \frac{a_\gamma}{a_\gamma}\frac{c_\gamma}{v_\gamma} \to 1 - \frac{1}{a_\gamma}\frac{c^*}{v^*} \to 1$$

as a_γ increases without bound with γ.

For any γ_ℓ, we have $v_\ell^* > c_\ell^*$. Hence, $\alpha_\ell^* = 1 - \frac{c_\ell^*}{v_\ell^*} < 1$. As $\alpha_\gamma \to 1$ as γ increases, there exists $K(\gamma_\ell)$ such that $\alpha_\gamma > \alpha^*(\gamma_\ell)$ for all $\gamma > K(\gamma_\ell)$. □

Proof of Proposition 7: A detailed proof for the case when the surge in demand is due to an increase in buyer mass is provided below.

If quantity q_k is sold at buyer mass μ_k then the seller price is $c_k = F_s^{-1}(q_k)$ and the buyer price is $v_k = F_b^{-1}(1 - \frac{q_k}{\mu_k})$, $k = \ell, h$, where the superscript γ on F_b is dropped to simplify notation. The intermediary's expected profit under the constant-fee constraint is

$$\Pi_I(q_h, q_\ell) = r\, q_h\Big[F_b^{-1}(1 - \frac{q_h}{\mu_h}) - F_s^{-1}(q_h)\Big] + (1-r)\, q_\ell\Big[F_b^{-1}(1 - \frac{q_\ell}{\mu_\ell}) - F_s^{-1}(q_\ell)\Big]$$

s.t.

$$F_s^{-1}(q_\ell)F_b^{-1}(1 - \frac{q_h}{\mu_h}) = F_s^{-1}(q_h)F_b^{-1}(1 - \frac{q_\ell}{\mu_\ell})$$

The profit may be written as the integral of its derivative obtained in (2)

$$\begin{aligned} & r\int_{v_h}^{a_\gamma}\int_0^{c_h}\Big(v - \frac{1-F_b(v)}{f_b(v)} - \Big[c + \frac{F_s(c)}{f_s(c)}\Big]\Big) f_s(c) f_b(v)\, dc\, dv \qquad (16) \\ & +(1-r)\int_{v_\ell}^{a_\gamma}\int_0^{c_\ell}\Big(v - \frac{1-F_b(v)}{f_b(v)} - \Big[c + \frac{F_s(c)}{f_s(c)}\Big]\Big) f_s(c) f_b(v)\, dc\, dv \end{aligned}$$

The argument below is followed in Figure 4. Note that if $\hat{\alpha}$ is the optimal fee, then the optimal prices $(\hat{c}_k, \hat{v}_k)$ are at the intersection of a straight line through the origin with slope $\frac{1}{1-\hat{\alpha}}$ and the demand equals supply curve for D_k, $k = \ell, h$.

If $\hat{\alpha} < \min\{\alpha_\ell^*, \alpha_h^*\}$ then, as $\frac{1}{1-\hat{\alpha}} < \frac{1}{1-\alpha_k^*}$, the straight line from the origin through $(\hat{c}_k, \hat{v}_k)$ is less steep than each of the two straight lines from the origin to (c_k^*, v_k^*), $k = \ell, h$. Each $(\hat{c}_k, \hat{v}_k)$ lies below the 'MR=MC' curve, the region where $MR = v - \frac{1-F_b(v)}{f_b(v)} < c + \frac{F_s(c)}{f_s(c)} = MC$. Thus, each of the two integrands in (16) is negative at $(\hat{c}_k, \hat{v}_k)$. Selling a little less by decreasing seller price from $\hat{c}_k$ and increasing buyer price from $\hat{v}_k$, while maintaining the constant-fee constraint and the two demand equals supply conditions, will increase profit in each of the two states ℓ and h and thereby increase $\Pi_I(q_h, q_\ell)$. This contradicts the optimality of $\hat{\alpha}$.

Similarly, if $\hat{\alpha} > \min\{\alpha^*_\ell, \alpha^*_h\}$ then each $(\hat{c}_k, \hat{v}_k)$ is above the 'MR=MC' curve, the region where $MR > MC$. Selling a little more will increase profit in each the two states. Thus, (10) must hold.

Suppose that $\alpha^*_\ell \neq \alpha^*_h$ and that it is optimal to set $\hat{\alpha} = \alpha^*_\ell$. Therefore, $(\hat{c}_\ell, \hat{v}_\ell) = (c^*_\ell, v^*_\ell)$. Thus, at low demand the prices are unconstrained optimal but not at high demand (as $\alpha^*_\ell \neq \alpha^*_h$). Consequently, marginal revenue equals marginal cost at low demand but not at high demand. Because marginal revenue and marginal cost are continuous functions, the intermediary's profit is greater if the constant percent fee $\hat{\alpha} + \epsilon$ rather than at $\hat{\alpha}$, where $|\epsilon|$ is small with $\epsilon > 0$ if $\alpha^*_h > \alpha^*_\ell$ and $\epsilon < 0$ if $\alpha^*_h < \alpha^*_\ell$. This contradicts the assumption that $\hat{\alpha}$ is optimal. An identical argument implies that $\hat{\alpha} \neq \alpha^*_h$. Thus, each of the two inequalities in (10) must be strict when $\alpha^*_\ell \neq \alpha^*_h$.

To prove that an optimal fee and prices exist first note that for each α there exist a unique set of prices $(c_k(\alpha), v_k(\alpha))$ that satisfies the demand equals supply constraint:

$$\mu_k(1 - F_b(v_k)) \;=\; F_s(c_k) \;= F_s((1-\alpha)v_k)$$

As v_k is increased from 0, the left-hand side decreases from μ_k and the right-hand side increases from 0, with equality at a unique point $(c_k(\alpha), v_k(\alpha))$ where $c_k(\alpha) = (1-\alpha)v_k(\alpha)$. Hence, the profit function may be written as a function of α:

$$\begin{aligned}\Pi_I(\alpha) \;=\; & r\int_{v_h(\alpha)}^{a_\gamma}\int_0^{c_h(\alpha)} \Big(v - \frac{1-F_b(v)}{f_b(v)} - [c + \frac{F_s(c)}{f_s(c)}]\Big) f_s(c) f_b(v)\, dc\, dv \\ & +(1-r)\int_{v_\ell(\alpha)}^{a_\gamma}\int_0^{c_\ell(\alpha)} \Big(v - \frac{1-F_b(v)}{f_b(v)} - [c + \frac{F_s(c)}{f_s(c)}]\Big) f_s(c) f_b(v)\, dc\, dv\end{aligned}$$

where the constant-fee constraint is satisfied as $c_k(\alpha) = (1-\alpha)v_k(\alpha)$. The domain for continuous function $\Pi_I(\alpha)$ is a compact set $[0, 1]$. Hence, there exists an $\hat{\alpha}$ at which $\Pi_I(\alpha)$ is maximized.

A proof for the case when the surge in demand is due to a decrease in elasticity is similar. The only difference is that, instead of (16), the intermediary's expected profit is

$$
\begin{aligned}
& r\int_{v_h}^{a_h}\int_0^{c_h}\Big(v-\frac{1-F_b^h(v)}{f_b^h(v)}-\Big[c+\frac{F_s(c)}{f_s(c)}\Big]\Big)f_s(c)f_b(v)\,dc\,dv \\
& +(1-r)\int_{v_\ell}^{a_\ell}\int_0^{c_\ell}\Big(v-\frac{1-F_b^\ell(v)}{f_b^\ell(v)}-\Big[c+\frac{F_s(c)}{f_s(c)}\Big]\Big)f_s(c)f_b(v)\,dc\,dv \qquad \square
\end{aligned}
$$

Proof of Proposition 8: As $\alpha_\ell^* > \alpha_h^*$, Proposition 7 implies that $\hat{\alpha}$ satisfies $\alpha_h^* < \hat{\alpha} < \alpha_\ell^*$. Thus, the constrained optimal prices satisfy

$$
\frac{1}{1-\alpha_h^*}=\frac{v_h^*}{c_h^*} \quad < \quad \frac{\hat{v}_h}{\hat{c}_h}=\frac{1}{1-\hat{\alpha}}=\frac{\hat{v}_\ell}{\hat{c}_\ell} \quad < \quad \frac{v_\ell^*}{c_\ell^*}=\frac{1}{1-\alpha_\ell^*}
$$

That is, $(\hat{c}_\ell, \hat{v}_\ell)$ is below the 'MR=MC' curve (and on the demand equals supply line for μ_ℓ) and, similarly, $(\hat{c}_h, \hat{v}_h)$ is above 'MR=MC' curve; this can be seen in Figure 2 where $\alpha_\ell^* > \alpha_h^*$. As the demand equals supply curve is negatively sloped, it follows immediately that $\hat{v}_h > v_h^*$ and $v_\ell^* > \hat{v}_\ell$. That $v_h^* > v_\ell^*$ was noted in Section 3.1.

A symmetric argument implies $c_h^* > \hat{c}_h$, $\hat{c}_\ell > c_\ell^*$; $\hat{c}_h > \hat{c}_\ell$ follows from (9). $\square$

Proof of Lemma 2: The Lagrangian for intermediary profit-maximization is

$$
\begin{aligned}
Ł_I(q_h,q_\ell,\lambda) \;=\;& r\,q_h\Big[F_b^{h^{-1}}(1-\frac{q_h}{\mu_h})-F_s^{-1}(q_h)\Big]+(1-r)\,q_\ell\Big[F_b^{\ell^{-1}}(1-\frac{q_\ell}{\mu_\ell})-F_s^{-1}(q_\ell)\Big] \\
& +\lambda\Big[F_s^{-1}(q_\ell)F_b^{h^{-1}}(1-\frac{q_h}{\mu_h})-F_s^{-1}(q_h)F_b^{\ell^{-1}}(1-\frac{q_\ell}{\mu_\ell})\Big]
\end{aligned}
$$

$$
\begin{aligned}
\Longrightarrow \qquad \frac{\partial Ł_I}{\partial q_h} \;=\;& r\Big[F_b^{h^{-1}}(1-\frac{q_h}{\mu_h})-F_s^{-1}(q_h)\Big]+rq_h\frac{dF_b^{h^{-1}}(1-\frac{q_h}{\mu_h})}{dq_h}-rq_h\frac{dF_s^{-1}(q_h)}{dq_h} \\
& +\lambda\Big[F_s^{-1}(q_\ell)\frac{dF_b^{h^{-1}}(1-\frac{q_h}{\mu_h})}{dq_h}-F_b^{\ell^{-1}}(1-\frac{q_\ell}{\mu_\ell})\frac{dF_s^{-1}(q_h)}{dq_h}\Big] \\
=\;& r\Big[F_b^{h^{-1}}(1-\frac{q_h}{\mu_h})-F_s^{-1}(q_h)\Big]-r\frac{q_h}{\mu_h f_b^h(F_b^{h^{-1}}(1-\frac{q_h}{\mu_h}))}-r\frac{q_h}{f_s(F_s^{-1}(q_h))} \\
& -\lambda F_s^{-1}(q_\ell)\frac{q_h}{\mu_h f_b^h(F_b^{h^{-1}}(1-\frac{q_h}{\mu_h}))}-\lambda F_b^{\ell^{-1}}(1-\frac{q_\ell}{\mu_\ell})\frac{q_h}{f_s(F_s^{-1}(q_h))} \\
=\;& r(v_h-c_h)-(r+\lambda c_\ell)\frac{1-F_b^h(v_h)}{f_b^h(v_h)}-(r+\lambda v_\ell)\frac{F_s(c_h)}{f_s(c_h)}
\end{aligned}
$$

As $\frac{\partial \mathrm{L}_I}{\partial q_h} = 0$ at optimal prices $(\hat{c}_\ell, \hat{v}_\ell)$ and $(\hat{c}_h, \hat{v}_h)$, we have

$$\begin{aligned}
\hat{v}_h - \frac{1 - F_b^h(\hat{v}_h)}{f_b^h(\hat{v}_h)} - \frac{\lambda \hat{c}_\ell}{r} \frac{1 - F_b^h(\hat{v}_h)}{f_b^h(\hat{v}_h)} &= \hat{c}_h + \frac{F_s(\hat{c}_h)}{f_s(\hat{c}_h)} + \frac{\lambda \hat{v}_\ell}{r} \frac{F_s(\hat{c}_h)}{f_s(\hat{c}_h)} \\
\Longrightarrow \quad \left[\hat{v}_h - \frac{1 - F_b^h(\hat{v}_h)}{f_b^h(\hat{v}_h)}\right] - \left[\hat{c}_h + \frac{F_s(\hat{c}_h)}{f_s(\hat{c}_h)}\right] &= \frac{\lambda \hat{c}_\ell}{r} \frac{1 - F_b^h(\hat{v}_h)}{f_b^h(\hat{v}_h)} + \frac{\lambda \hat{v}_\ell}{r} \frac{F_s(\hat{c}_h)}{f_s(\hat{c}_h)}
\end{aligned}$$

which implies (11).

Similarly, $\frac{\partial \mathrm{L}_I}{\partial q_\ell} = 0$ at optimal prices $(\hat{c}_\ell, \hat{v}_\ell)$ and $(\hat{c}_h, \hat{v}_h)$ implies

$$\begin{aligned}
\hat{v}_\ell - \frac{1 - F_b^\ell(\hat{v}_\ell)}{f_b^\ell(\hat{v}_\ell)} + \frac{\lambda \hat{c}_h}{1-r} \frac{1 - F_b^\ell(\hat{v}_\ell)}{f_b^\ell(\hat{v}_\ell)} &= \hat{c}_\ell + \frac{F_s(\hat{c}_\ell)}{f_s(\hat{c}_\ell)} - \frac{\lambda \hat{v}_h}{1-r} \frac{F_s(\hat{c}_\ell)}{f_s(\hat{c}_\ell)} \\
\Longrightarrow \quad \left[\hat{v}_\ell - \frac{1 - F_b^\ell(\hat{v}_\ell)}{f_b^\ell(\hat{v}_\ell)}\right] - \left[\hat{c}_\ell + \frac{F_s(\hat{c}_\ell)}{f_s(\hat{c}_\ell)}\right] &= -\frac{\lambda \hat{c}_h}{1-r} \frac{1 - F_b^\ell(\hat{v}_\ell)}{f_b^\ell(\hat{v}_\ell)} - \frac{\lambda \hat{v}_h}{1-r} \frac{F_s(\hat{c}_\ell)}{f_s(\hat{c}_\ell)}
\end{aligned}$$

which implies (12). □

Proof of Proposition 9: Equation (13) implies that if intermediary i trades q_i units then the buyer price is $v = F_b^{-1}(1 - \frac{q_1+q_2}{\mu})$ and the seller price is $c = F_s^{-1}(q_1+q_2)$. Thus intermediary 1's profit is

$$\Pi_1(q_1, q_2) = q_1 [v - c] = q_1 [F_b^{-1}(1 - \frac{q_1+q_2}{\mu}) - F_s^{-1}(q_1+q_2)]$$

$$\begin{aligned}
\Longrightarrow \quad \frac{d\Pi_1}{dq_1} &= F_b^{-1}(1 - \frac{q_1+q_2}{\mu}) - F_s^{-1}(q_1+q_2) + q_1 \frac{\partial F_b^{-1}(1 - \frac{q_1+q_2}{\mu})}{\partial q_1} - q_1 \frac{\partial F_s^{-1}(q_1+q_2)}{\partial q_1} \\
&= F_b^{-1}(1 - \frac{q_1+q_2}{\mu}) - F_s^{-1}(q_1+q_2) - \frac{q_1}{\mu f_b(F_b^{-1}(1 - \frac{q_1+q_2}{\mu}))} - \frac{q_1}{f_s(F_s^{-1}(q_1+q_2))} \\
&= v - c - \frac{1 - F_b(v)}{f_b(v)} - \frac{F_s(c)}{f_s(c)} + \frac{q_2}{\mu f_b(F_b^{-1}(1 - \frac{q_1+q_2}{\mu}))} + \frac{q_2}{f_s(F_s^{-1}(q_1+q_2))}
\end{aligned}$$

The first-order condition, $\frac{d\Pi_1}{dq_1} = 0$, implies that along the reaction curve,

$$v - \frac{1 - F_b(v)}{f_b(v)} < c + \frac{F_s(c)}{f_s(c)}$$

We may add the two intermediaries' first-order conditions to obtain

$$\begin{aligned}
& 2v - 2c - 2\frac{1 - F_b(v)}{f_b(v)} - 2\frac{F_s(c)}{f_s(c)} + \frac{q_1+q_2}{\mu f_b(F_b^{-1}(1 - \frac{q_1+q_2}{\mu}))} + \frac{q_1+q_2}{f_s(F_s^{-1}(q_1+q_2))} \\
= \; & 2v - 2c - \frac{1 - F_b(v)}{f_b(v)} - \frac{F_s(c)}{f_s(c)} = 0
\end{aligned}$$

which implies (14). The above equation may be written as

$$
\begin{aligned}
& v_2^* \Big[2 - \frac{1}{v_2^*} \frac{1 - F_b(v_2^*)}{f_b(v_2^*)} \Big] &=& \; c_2^* \Big[2 + \frac{1}{c_2^*} \frac{F_s(c_2^*)}{f_s(c_2^*)} \Big] \\
\Longleftrightarrow \quad & v_2^* \Big[2 + \frac{1}{\eta_b(v_2^*)} \Big] &=& \; c_2^* \Big[2 + \frac{1}{\eta_s(c_2^*)} \Big] \\
\Longleftrightarrow \quad & \frac{1}{1 - \alpha_2^*} = \frac{v_2^*}{c_2^*} &=& \; \frac{2 + \frac{1}{\eta_s(c_2^*)}}{2 + \frac{1}{\eta_b(v_2^*)}}
\end{aligned}
\qquad \square
$$

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