Partially Directed Search for Prices*

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Abstract

I analyse a model of partially directed search where buyers decide which firm to visit based on correct, but incomplete, information about firms’ prices. I show that the unique symmetric pure-strategy equilibrium is in price distributions. A firm’s equilibrium price distribution assigns positive mass to prices below the marginal cost (“deals”). If some buyers are “shoppers” (i.e., better at searching than the others), then a larger fraction of shoppers can make the best deals worse. This effect can be so large that the welfare of nonshoppers is reduced.

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1 Introduction

In many instances, buyers decide about which firm to visit first based on partial, but correct, price information. For example, a person may visit a specific supermarket first because his weekly shop was cheap there last week, despite knowing that this week he needs a different basket of groceries. Another might start looking for a laptop in a firm where she previously bought a cheap smartphone. And a third may start searching for flights to a destination on the website where her friend got a great deal. In all these cases, the buyer knows the price that a specific firm has charged (potentially for another product), but also knows that the price offer that she gets today can be different.

I model the idea that a buyer can get partial but correct information about firms’ prices prior to costly search by allowing firms to post (possibly degenerate) price distributions and buyers to direct their search based on a sample of price draws from the firms’ distributions. A price distribution can be interpreted as a

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distribution across different products, product baskets, or a short time interval. A buyer’s partial information about prices can be interpreted as stemming from his own past search, potentially for a different product, or from information from other people. I show that the unique symmetric pure-strategy equilibrium is in price distributions and, in contrast to standard search models with price dispersion, that shoppers can increase the lowest quoted prices.

In my main model, buyers sequentially search for a product at a low price. The product can be thought of as homogeneous or horizontally differentiated. A fixed number of homogeneous firms sell the product. Firms post, potentially degenerate, price distributions. Prior to search, each buyer gets a sample of price signals for free, one randomly drawn price from each firm’s distribution. By assumption, a buyer partially directs her search: she visits first the firm with the lowest price in her sample. When the buyer visits a firm, however, she cannot buy at the price based on which she directed search, but draws a new price offer from the firm’s price distribution. Each price offer costs a fixed amount for the buyer. In Section 4, I let some buyers be “shoppers”, i.e., better at searching than the others (“nonshoppers”). Shoppers can buy at any price in their free sample of price signals, but are otherwise exactly like nonshoppers.

The results of the model are as follows. First, in the unique symmetric pure-strategy equilibrium firms commit to nondegenerate price distributions (even in the absence of shoppers). A single-price equilibrium fails to exist because a firm benefits from setting at least two prices if others set one. A profitable deviation is to set one price marginally below the proposed equilibrium price and the other equal to the buyers’ continuation value (that exceeds the price by the search cost), with equal probabilities. The deviating firm attracts half of the buyers and makes on average higher profits on them than in the proposed equilibrium.

Second, I characterise the equilibrium price distribution. The highest posted price equals the buyers’ cutoff price: a buyer just accepts this price rather than continues to search. The lowest prices are (weakly) below the marginal cost, prices that I call “deals”. As the number of firms increases, as in Spiegler (2006), nonshoppers’ welfare is unaffected and price dispersion increases because the best deals get better. As search cost decreases, price dispersion decreases because the

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1 I discuss three extensions to the model in Section 5 (as described below).
2 This assumption has multiple interpretations. First, it is the natural extension to a model where firms are restricted to setting a single price each and buyers direct their search based on prices. An alternative interpretation is that buyers use the rule to break their indifference in a symmetric equilibrium. Finally, in case of a symmetric equilibrium, buyers can be thought of as boundedly rational and not realising that firms set the same price distributions in equilibrium.
3 I argue in Section 4 that this way of modelling shoppers is in the spirit of Stahl (1989), but the exact way that I model them is unimportant.
4 In the absence of shoppers, the proof relies heavily on Spiegler (2006).
buyers’ cutoff price increases and, to counteract this negative effect on profits, firms offer worse best deals. Nonshoppers’ welfare decreases in the search cost. I summarise the testable predictions implied by the comparative statics in Table 1.

Third, I show that an increase in the fraction of shoppers leads to worse deals in my model, in contrast to other search models with price dispersion that follow Stahl (1989). In the other models, shoppers make the market more competitive, so that the entire price distribution (including the lowest price) shifts down and nonshoppers’ welfare increases. In my model, shoppers are buyers who often execute deals, which are unprofitable for a firm. Thus, a larger fraction of shoppers can make deals worse. This effect can be so large that the average price increases, reducing nonshoppers’ welfare.

Finally, in Section 5 I show that the equilibrium in price distributions does not change much under three extensions. If only a fraction of nonshoppers directs search, the equilibrium exists as long as the fraction is large enough. If shoppers are replaced by supershoppers, i.e., buyers with zero search cost (who inspect an infinite number of prices), firms no longer offer deals in equilibrium. If products are horizontally differentiated, i.e., a buyer’s value from a product is positive with a probability less than one, the equilibrium distribution is unchanged except that the effective search cost, search cost divided by the match probability, matters.

Literature. My paper contributes to consumer search literature, by showing that if buyers partially direct search on prices, the only symmetric pure-strategy equilibrium is in price distributions. The equilibrium distribution puts a positive weight on below-marginal-cost prices. In contrast to results in earlier literature, these lowest prices can increase in the fraction of shoppers.

In most other sequential search models where firms sell a homogeneous product that feature price dispersion, the dispersion is across rather than within firms. An exception is Salop (1977), where a monopolist posts a price distribution to discriminate between buyers with different search costs. In the main example of Salop (1977), the distribution has two-point support and both prices exceed the marginal cost, whereas in my model the distribution is continuous and the lowest prices are below the marginal cost. In the models that follow Stahl (1989), nonshoppers search randomly across firms and price dispersion stems from firms using mixed strategies. Instead, I show that if nonshoppers partially direct search on prices, then in the symmetric pure-strategy equilibrium firms set price distributions. In the resulting market, in the above models no firm posts below-marginal-cost prices, whereas in my model firms post such prices with positive probability. In addition, in these models of across-firm price dispersion, adding more shoppers
leads to a decrease in the lowest price in the market, while in my model can lead to an increase in the lowest price.

Other papers have studied partially directed search. In the consumer search framework, the models assume that buyers direct their search based on non-price information (which is exogenous, as in ordered search, or endogenous) and/or full price information. In advertising literature, an ad from a seller reveals its product’s characteristics and/or price at no cost to the buyer. In other setups, Menzio (2007) allows job-seekers to direct search based on cheap-talk messages from firms. Bethune et al. (2018) and Lentz and Moen (2017) are mixture models where a person directs search perfectly with some probability and searches randomly with the remaining probability. In all these models, a firm is assumed to commit to a single price, which the buyer pays if she buys from this firm. In contrast, in my model a buyer may pay a different price than the one she directs search on.

In other search models where a single firm sets different prices for different units of the same product, a firm can discriminate between buyers based on their characteristics. For example, in one model in Armstrong and Zhou (2011), firms can charge different prices to new and repeat customers. Fabra and Montero (2017) and Fabra and Reguant (2018) explicitly study price discrimination. In contrast, in my model firms cannot discriminate between buyers.

Other models where firms charge a below-marginal-cost price to attract rational buyers and another price to make profits are Gerstner and Hess (1990), Weinstein and Ambrus (2008), and Chen and Rey (2012). In Gerstner and Hess (1990), firms advertise a low price for one product, that they strategically under-stock, and in its stead sell a substitute product at a higher price. In the other two papers, a firm sustains a below-marginal-cost price on one product because (some) buyers also buy other products at the firm. None of these paper analyses truly sequential search, including the effect of shoppers. In Spiegler (2006), a buyer gets price signals as in my model and visits first the firm with the lowest price signal. Firms set price distributions in equilibrium and the lowest prices are below marginal cost, as in my model. In Spiegler (2006), a buyer must purchase at the first firm she visits. In contrast, she can continue searching in my model, which allows me to analyse the effects of lower search cost and more shoppers.

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8Other models where firms use such strategies, but where the lowest price exceeds the marginal cost, are, for example, Salop and Stiglitz (1977), Varian (1980), and Lazear (1995).
The rest of the paper is organised as follows. Section 2 introduces the model. Section 3 shows that if buyers partially direct search, then a single-price equilibrium fails to exist and characterises the equilibrium in price distributions. Section 4 introduces shoppers to the model and shows that increasing their fraction can make the best offers worse, and that to the extent that the welfare of nonshoppers decreases. Section 5 analyses a few extensions. All proofs are in the appendix.

2 Model

A number $n \geq 2$ of firms produce a homogeneous product at zero marginal costs. Firm $i$ posts a price distribution $F_i(p)$, potentially degenerate, that it commits to.

Firms aim to maximise profits.

A unit mass of homogeneous buyers with a unit demand are looking for the product at a cheap price. A buyer’s valuation for the unit of the product is $v = 1$. Buyers can always get zero utility by not buying. Buyers are partially informed about the firms’ prices. In particular, a buyer gets a sample of $n$ price signals, one signal per firm. Signal $k$ in a buyer’s sample is a random draw from firm $k$’s price distribution. The samples of signals are independent across buyers. One interpretation of this information is that the buyer hears about prices at which her friends bought at the different firms. Another interpretation is that she remembers the prices at the different firms from a time in the past when she sought a different product. Buyers direct their search based on the price sample: a buyer visits first the cheapest firm in her sample. If she finds an acceptable price at the firm, she buys and leaves the market. If she finds a price that she does not like at that firm, she can continue to search for a better price at that firm or at a different firm. Getting a price offer costs $c > 0$ for a buyer. If a buyer’s expected value from starting the search process is negative, she does not start searching.

I look for symmetric pure-strategy equilibria where firms post the same price distribution $F(p)$ and buyers use the same cutoff rule. For the rest of the paper, instead of prices, I let firms to set distributions $G(u)$ over offered net utilities $u := v - p = 1 - p$. Then buyers accept all offers that exceed their optimal cutoff $\bar{u}$. I assume that a buyer buys if she is indifferent between buying and continuing

\footnote{An interpretation of the price distribution is that the firm sells product baskets or different varieties of the same product at different prices. Another interpretation is that the firm commits to prices for a short time interval, say, a week, and buyers arrive at different days of the week. Infrequent price changes could be the result of a high cost of reoptimising or changing prices.}

\footnote{In a symmetric equilibrium each firm posts the same price distribution so there is no financial reason for a buyer to visit any particular firm before another. One interpretation of the assumption that these buyers nevertheless direct search is that they use the sample of prices to break their indifference, another is that they are boundedly rational, and, finally, it suffices if firms expect these buyers to direct search in this manner.}
to search.

3 Equilibrium in price distributions

I first describe a buyer’s and then a firm’s problem. Then I describe the equilibrium and provide the comparative statics’ results.

A buyer can continue to search if she gets a low net utility offer. Her continuation value is

\[ V = (1 - G(\bar{u}))E[u | u \geq \bar{u}] + G(\bar{u})V - c. \]

A buyer accepts an offer \( u \) if it exceeds her cutoff \( \bar{u} \) and rejects the offer otherwise. In the latter case, she continues to search (at the same or a different firm). The buyer has to pay the search cost \( c \) per offer. The optimal cutoff of a buyer is equal to her continuation value.

Firm \( i \)'s expected profit is

\[ \pi = E[D(u)|u \geq \bar{u}]E[1 - u|u \geq \bar{u}], \]

where \( D(u) = \prod_{j \neq i} G_j(u) \) is the probability that the signal \( u \) from firm \( i \) is the best one received by a buyer. \( D(u) \) can be interpreted as a firm’s expected demand at net utility \( u \). Offering net utilities below \( \bar{u} \) is strictly dominated for the firms: shifting the mass from utilities below \( \bar{u} \) to \( \bar{u} \) would increase demand. Thus, a firm’s expected profit simplifies to

\[ \pi = E[D(u)]E[1 - u], \quad (1) \]

and a buyer’s problem to

\[ \bar{u} = E[u] - c. \quad (2) \]

The expectations and \( D(u) \) in equations (1) and (2) depend on the equilibrium distribution \( G \). The derivation of the distribution differs from the standard method used in papers of search with price dispersion where an individual firm mixes across prices, but posts a single price. In these models, a firm is indifferent across all the prices in the support of the equilibrium price distribution. Here, a firm is not necessarily indifferent across the different prices in the support of its distribution. I use the methods in Spiegler (2006) to solve for the equilibrium.

I first show that a pure-strategy equilibrium where firms post the same price cannot be an equilibrium. This result is present in Spiegler (2006), but is worth a greater emphasis in the context of search literature.

**Proposition 1.** Suppose that firms post a single net utility \( u^* \) in equilibrium.
Then a firm has an incentive to deviate to a distribution of net utilities that has at least two-point support.

One profitable deviation for a firm is to use a “bait-and-switch” strategy: with equal probabilities, to set one price lower and the other higher than the proposed equilibrium price. The low price attracts half of the buyers, but once there, half of these buyers pay the higher price that leaves them just indifferent between buying and continuing. Such a deviation is always profitable because the firm can choose the low price arbitrarily close to the proposed equilibrium price, whereas the higher price is strictly above the proposed equilibrium price. In the context of search literature, this means that if (firms expect that) buyers direct their search based on price information, the single-price pure-strategy equilibrium breaks down if firms are allowed to set nondegenerate price distributions. In other words, price dispersion is generated in a search model with homogeneous buyers and sellers.

The unique symmetric pure-strategy equilibrium is summarised in

**Proposition 2.** A firm’s equilibrium net utility distribution is $G(u) = \left( \frac{u - u_{\min}}{u_{\max} - u_{\min}} \right)^{\frac{1}{n-1}}$ with support $[u_{\min}, u_{\max}]$ where $u_{\min} = \bar{u} = 1 - 2c$ and $u_{\max} = 1 + (n - 2)c$. The equilibrium exists if $c \leq \frac{1}{2}$.

In equilibrium, sellers post a smooth distribution of net utilities, which is illustrated in Figure 1. The lowest utility in the support of the distribution leaves
a buyer just indifferent between accepting the offer and continuing to search. The firms offer “deals”, i.e., prices below the marginal cost (or $u > 1$), with positive probability if there are at least three firms. These low prices attract buyers and make losses, but the firm knows that the buyer ends up paying such prices rarely. The best deals are the better the more firms there are (i.e., $u_{\text{max}}$ increases in $n$) because of stiffer competition. Interestingly, the probability that a deal is offered (i.e., $1 - G(1)$) is nonmonotone in $n$: if the number of firms is small, a lot of buyers can be attracted by offering deals. But as competition increases, the additional demand is so small and the best deals so costly that a firm is better off by reducing the probability of deals. The nonmonotonicity is illustrated in Figure 2.

The comparative statics’ results are summarised in

**Corollary 1.** As the search cost, $c$, decreases,

(i) the dispersion of net utilities decreases: the lower bound of the distribution shifts up and the upper bound down.

(ii) buyers’ welfare increases.

As the number of firms, $n$, increases,

(i) the dispersion of net utilities increases: the lower bound of the distribution does not change, but the upper bound shifts up.

(ii) buyers’ welfare is unaffected.
The offered net utilities are less dispersed (i.e., \( u_{\text{max}} - u_{\text{min}} \) decreases) as search cost decreases. If the equilibrium distribution did not change, buyers would increase their optimal cutoff (which shifts up the lower bound of net utilities). But if sellers have to improve the worst offers, then they optimally also reduce the best offers (which shifts the upper bound down). In line with intuition, buyers’ welfare, measured by \( \bar{u} \), increases as search cost decreases. These comparative statics generate testable predictions that differ from those in [Spiegler (2006)].

The comparative statics with respect to the number of firms \( n \) are as in [Spiegler (2006)]. Firms offer better deals as the number of firms increases due to stiffer competition. The number of firms does not affect buyers’ welfare. If there are more firms, they offer better deals in equilibrium, but at the same time shift more weight to the lowest net utilities in the support of the distribution. The optimal way of doing this happens to be such that buyers’ welfare is unaffected.

4 Adding shoppers

To compare my model’s results to the class of search models with price dispersion that follows [Stahl (1989)], in this Section I let some buyers be “shoppers”. Shoppers are better at searching than the rest of the buyers (“nonshoppers”).

In [Stahl (1989)], the search cost of shoppers is zero, which means in his model that shoppers inspect the price that each firm charges; altogether \( n \) prices. In my model, if a buyer’s search cost is zero, she can search through all prices of all \( n \) firms. Since a firm charges a distribution of prices, this means that a buyer of this sort (who I call a “supershopper”) inspects infinitely many prices.

Supershoppers are buyers with an extreme (and perhaps unrealistic) knack for shopping.\(^{11}\) Therefore, I model shoppers as buyers who see one price from each firm at no cost, as in [Stahl (1989)], but have to pay the same search cost \( c \) as nonshoppers to get further price offers. I show that as the fraction of shoppers increases, firms can offer worse deals. This effect can be so strong that the offered expected utility falls and nonshoppers’ welfare decreases.

The details of how I model shoppers, i.e., buyers who are better at searching than nonshoppers, is unimportant. Shoppers’ effect on the equilibrium distribution is qualitatively the same as long as they are more likely to buy at low prices than nonshoppers. Without shoppers, the lowest prices are below marginal cost (i.e., unprofitable in expectation). If the likelihood that buyers actually buy at such prices grows, a firm raises these prices. That is, adding shoppers makes the best deals worse. The effect can be large enough to reduce nonshoppers’ welfare.

\(^{11}\)I analyse my model with a positive fraction of supershoppers in Section 5.2. Their presence has a discrete effect on the equilibrium price distribution: firms no longer offer deals.
4.1 Model
Consider a model where a fraction $1 - \mu$ of the buyers are as before (nonshoppers) and a fraction $\mu < 1$ are shoppers. Shoppers are otherwise like nonshoppers, except that a shopper can also purchase at prices in her sample of price signals (that nonshoppers can only use for directing search). In other words, a shopper first gets $n$ price offers, one from each firm, for free. She can buy at any of these prices or continue searching for a lower one. Further price offers cost $c$ each, just as for nonshoppers.

4.2 Equilibrium in price distributions
I first describe a shopper’s and then a firm’s problem. A nonshopper’s problem is exactly as before. Then I describe the equilibrium.

After receiving the $n$ free price offers, a shopper can continue to search if she wishes. Suppose that she uses a cutoff $\tilde{u}$ if she continues. Her continuation value $\tilde{V}$ is

$$\tilde{V} = (1 - G(\tilde{u}))\mathbb{E}[u|u \geq \tilde{u}] + G(\tilde{u})\tilde{V} - c,$$

because she has to pay the search cost $c$ for any further offer. But then her continuation problem is identical to a nonshopper’s problem and her optimal cutoff must also be the same: $\tilde{u} = \bar{u}$. As a result, offering utilities below $\bar{u}$ is again dominated for a firm. This in turn means that all offers in the set of a shopper’s free offers exceed her cutoff. A shopper, thus, optimally accepts the best offer among her set of $n$ free offers. A buyer’s optimal cutoff solves

$$\bar{u} = \mathbb{E}[u] - c. \quad (3)$$

Given the optimal behaviour of nonshoppers and shoppers, firm $i$’s expected profit when posting net utility distribution $G(u)$ is

$$\pi = (1 - \mu)\mathbb{E}[D(u)]\mathbb{E}[1 - u] + \mu\mathbb{E}[D(u)(1 - u)], \quad (4)$$

where $D(u) = \Pi_{j \neq i} G_j(u)$ is the probability both that the signal $u$ from firm $i$ is the best one received by a nonshopper and that the offer $u$ from firm $i$ is the best among the free offers received by a shopper. All expectations are taken with respect to the endogenous distribution $G(u)$. The profit from nonshoppers is as before: if firm $i$’s signal is the best among the nonshopper’s signals, she visits firm $i$ first and then accepts the new net utility draw. The profit from shoppers is different. A shopper buys from firm $i$ if its offer is the best among the shopper’s free offers, in which case she gets the same offered net utility. Thus, there are
two expectation operators in the expected profit associated with nonshoppers and only one in the part associated with shoppers.

All the expectations and $D(u)$ in equations (3) and (4) depend on the endogenous equilibrium distribution $G$. I can no longer use the methods in Spiegler (2006) to solve for the equilibrium because the profit function has a different form, which implies that $D(u)$ is no longer linear.

**Proposition 3.** A firm’s equilibrium net utility distribution is

$$G(u) = \left( \frac{u - u_{\min}}{u_{\max} - u_{\min} + \gamma(u_{\max} - u)} \right)^{\frac{1}{n-1}}$$

with support $[u_{\min}, u_{\max}]$ where $u_{\min} = \bar{u} = 1 - \frac{c}{\gamma + n}[(1 + \gamma)K + n]$ and $u_{\max} = 1 + (1 + \gamma)^{-1}[(n - 1)(1 - \bar{u}) - nc]$, with $\gamma = n\mu/(1 - \mu)$ and $K = [1 - \int_0^1 \frac{1 - z^{n-1}}{1 + \gamma z^{n-1}} \, dz]^{-1}$. The equilibrium exists if $c \leq \frac{\gamma + n}{(1 + \gamma)K + n}$.

The base within the exponentiation of $G(u)$ (which is a firm’s expected demand from buyers who draw $u$, $D(u)$) looks like a combination of the equivalent terms in Proposition 2 and in the unit-demand version of Stahl (1989) (see, for example, Proposition 2 in Janssen and Moraga González (2004)). I use a clever trick in Janssen and Moraga González (2004) to get the closed-form expression for the buyers’ cutoff $\bar{u}$. Three or more firms is no longer a sufficient guarantee that firms offer deals because shoppers make deals less attractive for firms.

The comparative statics with respect to the fraction of shoppers are summarised in

**Proposition 4.** As the fraction of shoppers, $\mu$, increases,

(i) the dispersion of net utilities decreases.

(ii) a sufficient condition for the best deal to become worse is that $n \geq 3$ and $\mu \leq \bar{\mu}(n)$.

(iii) as $\mu \to 0$, nonshoppers’ welfare decreases if $n \geq 4$.

An increase in the fraction of shoppers has a similar effect on the dispersion of utilities, $u_{\max} - u_{\min}$, as a decrease in the search cost: (most of the time) the cutoff utility increases, which firms counteract by lowering $u_{\max}$. The best deal $u_{\max}$ can get worse because firms do not want to make unprofitable offers if these offers are frequently realised. The region of the parameters for which the best deals gets worse as the fraction of shoppers increases is not small: for example, $\bar{\mu}(3) = 0.78$ and $\bar{\mu}(4) = 0.96$.

The effect of the best deals getting worse may be so large that nonshoppers’ welfare, $\bar{u}$, decreases.

\[12\] For very large $\mu$, the comparative statics are similar to those in the unit-demand version of Stahl (1989), where both the best offer and expected utility increase in $\mu$. 

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The net utilities’ distribution for two different fractions of shoppers $\mu$ is depicted in Figure 3. The lower bounds of the distributions for these two values of $\mu$ happen to be close. Figure 3 illustrates two features. First, as in other models a la Stahl (1989), firms place more mass on utilities close to $u_{max}$ if $\mu$ is larger. A firm has an incentive to set high utilities more frequently because these attract a lot of the shoppers. Second, the best offers are worse if $\mu$ is larger. The intuition is, as described above, that if a firm knows that it has to pay out the unprofitable deals to a considerable fraction of buyers, then it offers worse deals.

4.3 Testable predictions

Table 1 summarises for $n \geq 4$ how the testable predictions of my model differ from Spiegler (2006) and the models following Stahl (1989). I list the results in terms of prices (instead of net utilities) because these are easier to observe in markets. The testable predictions distinguishing my model from Spiegler (2006) are with respect to the search cost. The most notable prediction is that the best offers get worse (or, equivalently, the lowest prices increase) as the fraction of shoppers increases unless the fraction of shoppers is very large. This is in direct contrast to the results of other models that build on Stahl (1989).

My model, thus predicts that if we compare a market with many shoppers to one with few shoppers, the market with many shoppers should exhibit worse deals. For example, a market with a price-comparison website (where buyers using the
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<td>Lower search cost: $c \downarrow$</td>
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<td>More shoppers: $\mu \uparrow$</td>
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<td>$p_{\text{min}} \uparrow$ for $\mu \leq \bar{\mu}$</td>
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Table 1: Testable predictions of this model differ from those of Stahl (1989) and Spiegler (2006); $n \geq 4$.

A website could be interpreted as shoppers) should have worse best deals than a market without such a website.

5 Extensions

I discuss three extensions to the main model. The main results remain unchanged. I assume that there are no shoppers ($\mu = 0$) throughout this section for clarity.

5.1 Some buyers do not receive price signals

A fraction $\lambda > 0$ of the buyers are partially informed about the firms’ prices as before. The rest of the buyers, fraction $1 - \lambda$, are uninformed buyers who do not receive price information. The uninformed buyers have the same search cost $c$ as the partially informed buyers. I assume that the uninformed buyers visit each firm first with an equal probability. They can continue to search just like the partially informed buyers.

After visiting the first firm, the continuation problem of a partially informed and an uninformed buyer looks identical. Thus, they optimally use the same cutoff $\bar{u}$, that is equal to the continuation value $V$. A buyer’s continuation value is, as before,

\[ V = (1 - G(\bar{u}))\mathbb{E}[u|u \geq \bar{u}] + G(\bar{u})V = c. \]

Firm $i$’s expected profit is now

\[ \pi = \lambda \mathbb{E}[D(u)|u \geq \bar{u}]\mathbb{E}[1 - u|u \geq \bar{u}] + \frac{1 - \lambda}{n} \mathbb{E}[1 - u|u \geq \bar{u}], \]

where $D(u) = \prod_{j \neq i} G_j(u)$ is the probability that the signal $u$ from firm $i$ is the
best one received by a buyer who partially directs search. Offering net utilities below \( \bar{u} \) is strictly dominated for the firms. The reason is twofold. First, no buyer buys if the net utility that is offered to her is below \( \bar{u} \) so shifting the mass from utilities below \( \bar{u} \) to \( \bar{u} \) does not affect profits. Second, a fraction \( \lambda \) of buyers direct their search based on the offer distribution and shifting the mass from utilities below \( \bar{u} \) to \( \bar{u} \) would increase demand from them. Thus, a firm’s expected profit simplifies to

\[
\pi = \lambda \mathbb{E}[D(u)]\mathbb{E}[1 - u] + \frac{1 - \lambda}{n}\mathbb{E}[1 - u],
\]

and a buyer’s problem to

\[
\bar{u} = \mathbb{E}[u] - c.
\]

The equilibrium is summarised in

**Proposition 5.** Suppose that a fraction \( 1 - \lambda \) of the buyers are uninformed. A firm’s equilibrium net utility distribution is \( G(u) = \left( \frac{u - u_{\min}}{u_{\max} - u_{\min}} \right)^{\frac{1}{1+\lambda}} \) with support \([u_{\min}, u_{\max}]\) where \( u_{\min} = \bar{u} = 1 - \frac{1+\lambda}{\lambda} c \) and \( u_{\max} = 1 + \left( n - \frac{1+\lambda}{\lambda} \right) c \). The equilibrium exists if \( c \leq \frac{\lambda}{1+\lambda} \).

Firms do not offer deals in equilibrium if partially informed buyers are rare. If partially informed buyers are rare, then firms compete less fiercely for them and instead focus on making higher profits on the uninformed buyers.

**Corollary 2.** As the fraction of partially informed buyers, \( \lambda \), increases,

(i) the dispersion of net utilities stays the same, but the support shifts up: the lower and upper bound of the distribution shift up by the same amount.

(ii) buyers’ welfare increases.

The comparative statics’ results with respect to the fraction of partially informed buyers are intuitive because these buyers are price-sensitive: their demand is determined by the firms’ prices. Thus, if their amount increases, competition increases, leading to a higher cutoff for buyers and better best offers.

### 5.2 Supershoppers

Let a fraction \( 1 - \sigma \) of the buyers be nonshoppers with positive search cost \( c \) (who partially direct search) as before and a fraction \( \sigma \) be supershoppers with zero search costs. Supershoppers get as many draws as they like from each firm’s price distribution. As a result, they always get the maximum net utility available in
the market. Without loss of generality, assume that supershoppers start searching across firms in a random order.

A firm’s profit in this version of the model is

\[ \pi = (1 - \sigma) \mathbb{E}[D(u)|u \geq \bar{u}] \mathbb{E}[1 - u|u \geq \bar{u}】 + \sigma (1 - u_{\text{max}}), \]

if the highest utility in the firm’s support \( u_{\text{max}} \) is above the market’s maximum,

\[ \pi = (1 - \sigma) \mathbb{E}[D(u)|u \geq \bar{u}] \mathbb{E}[1 - u|u \geq \bar{u}】 + \frac{\sigma}{n} (1 - u_{\text{max}}), \]

if \( u_{\text{max}} \) is equal to the market’s maximum, and

\[ \pi = (1 - \sigma) \mathbb{E}[D(u)|u \geq \bar{u}] \mathbb{E}[1 - u|u \geq \bar{u}], \]

otherwise.

In a symmetric equilibrium, all firms post the same price distribution. The next Lemma shows that the maximum net utility offered in equilibrium must be equal to the buyers’ valuation \( v = 1 \).

Lemma 1. The maximum net utility that firms post in equilibrium is \( u_{\text{max}} = 1 \).

No firm wants to post the highest utility in the market if it exceeds one (or, equivalently, a price below the marginal cost) because then a deviation to a slightly lower maximum utility would be profitable. All supershoppers (who generate losses) would buy from a different firm after the deviation. Conversely, if the highest utility in the market is below one, a deviation to a slightly higher maximum utility would be profitable. All supershoppers (who now generate profits) would buy from the deviating firm. Thus, in a symmetric equilibrium firms post \( u_{\text{max}} = 1 \) and make zero profits on supershoppers.

Given Lemma 1 and the fact that no firm offers utilities below \( \bar{u} \), a firm’s expected equilibrium profit is

\[ \pi = (1 - \sigma) \mathbb{E}[D(u)] \mathbb{E}[1 - u]. \]

Thus, the equilibrium distribution of utilities can be derived in the same way as in the model without supershoppers because firms effectively compete only over nonshoppers. The equilibrium is summarised in

Proposition 6. Suppose that a positive fraction \( \sigma \) of the buyers are supershoppers. The equilibrium distribution of net utilities is given by \( G(u) = \left( \frac{u - u_{\text{min}}}{u_{\text{max}} - u_{\text{min}}} \right)^{\frac{1}{\sigma - 1}} \) with support \([u_{\text{min}}, u_{\text{max}}]\), where \( u_{\text{min}} = \bar{u} = 1 - 2c \) and \( u_{\text{max}} = 1 \). The equilibrium exists if \( c \leq \frac{1}{2} \) and the characterisation is the same for all \( \sigma < 1 \).
Supershoppers have a discrete effect on the equilibrium distribution of net utilities: the support is capped at \( u_{\text{max}} = 1 \). However, an increase in the amount of supershoppers has no further effect on the equilibrium.

5.3 Horizontally differentiated products

Suppose that a buyer has a match with a firm’s product with probability \( \beta \in (0, 1] \). If she has a match with the product, she gets utility \( v = 1 \) from purchasing the product and utility zero otherwise. The buyer observes if she has a match with a product upon visiting the firm. This modification has no effect on the equilibrium except that a buyer’s effective search cost becomes \( c/\beta \).

A firm offers positive value to a fraction \( \beta \) of the buyers so its expected profit can be written as

\[
\pi = \beta \mathbb{E}[D(u)] \mathbb{E}[1 - u | u \geq \bar{u}].
\]

A buyer accepts a firm’s offer only if she has a match with the firm’s product and gets utility at least \( \bar{u} \) from it. Otherwise, she continues to search. Her continuation value is

\[
V = \beta (1 - G(\bar{u})) \mathbb{E}[u | u \geq \bar{u}] + [1 - \beta + \beta G(\bar{u})]V - c,
\]

and her optimal cutoff \( \bar{u} \) is equal to the continuation value.

By the same argument as in the main model, a firm never offers net utilities below the buyer’s cutoff \( \bar{u} \). A firm’s problem simplifies to

\[
\pi = \beta \mathbb{E}[D(u)] \mathbb{E}[1 - u],
\]

and a buyer’s problem to

\[
\bar{u} = \mathbb{E}[u] - \frac{c}{\beta}.
\]

Thus, the problem looks exactly like in the main model except that the effective search cost of the buyer is \( c/\beta \). The equilibrium is described in

**Corollary 3.** Suppose that a buyer has a match with a firm’s product with probability \( \beta \in (0, 1] \). A firm’s equilibrium net utility distribution is

\[
G(u) = \left( \frac{u - u_{\text{min}}}{u_{\text{max}} - u_{\text{min}}} \right)^{\frac{1}{n-1}}
\]

with support \([u_{\text{min}}, u_{\text{max}}]\) where \( u_{\text{min}} = \bar{u} = 1 - \frac{2}{\beta} \) and \( u_{\text{max}} = 1 + (n - 2) \frac{\bar{u}}{\beta} \). The equilibrium exists if \( c \leq \frac{\beta}{2} \).

The comparative statics with respect to the probability of a match \( \beta \) are opposite to those with respect to the search cost \( c \), summarised in Corollary 1.
A  Proofs

Here are the proofs omitted from the paper.

A.1  Main model

Proof. (Proposition 1) Suppose that all firms but i post a single net utility $u^* \geq 0$ in equilibrium. In order for firms to make positive profits, $u^* \leq 1$ must hold in such an equilibrium. If all firms post this utility, then a buyer’s cutoff is $\bar{u} = \max\{u^* - c, 0\}$. A firm’s expected equilibrium profit is $\pi^* = \frac{1 - u^*}{n}$.

A profitable deviation for a firm is to set a distribution $G'$ with $P_{G'}(u = u^* + \varepsilon) = \frac{1}{2}$, $\varepsilon > 0$ small, and $P_{G'}(u = u^* - c) = \frac{1}{2}$. The deviating firm would attract half of the buyers, those who get signal $u^* + \varepsilon$ from it. Half of them pay $1 - (u^* + \varepsilon)$ and half $1 - (u^* - c)$ to the firm. The profit from deviating is thus $\pi' = \frac{1}{2} - \frac{1}{2}(u^* + c - \varepsilon)$. The deviation is profitable if $u^* \leq 1$ and $\varepsilon < c$.

Proof. (Proposition 2) The proof is a special case of the proof of Proposition 5 on page 22, where $\lambda = 1$.

A.2  Adding shoppers

Proof. (Proposition 3) I can no longer use the method developed in Spiegler (2006) because, as I show below, $D(u)$ is not linear in $u$. In Step 1, I show that a single-$u$ equilibrium never exists. In Step 2, I derive the properties that $G_i(u)$ must have in any equilibrium and in Step 3, the properties of a symmetric-equilibrium $G(u)$. In Step 4 I argue that such an equilibrium exists.

Step 1: An equilibrium in degenerate distributions $G(u)$ does not exist.

Suppose that all firms set $u = \hat{u}$ in equilibrium with probability one. The proposed equilibrium profits are $\hat{\pi} = \frac{1 - \hat{u}}{n}$, where $\hat{u} \leq 1$ must hold for weakly positive profits. If all firms set $u = \hat{u}$ in equilibrium, then a buyer’s expected value is $E[u] - c = \hat{u} - c$ so she accepts any first offer.

I show that firm i has an incentive to deviate to a dispersed distribution $G'_i$ such that $P'(u = \hat{u} + \varepsilon) = \frac{1}{2}$ and $P'(u = \hat{u} - c) = \frac{1}{2}$ for $\varepsilon > 0$ small.

Firm i’s profit from this deviation is

$$\pi' = \frac{\mu}{2}(1 - \hat{u} - \varepsilon) + \frac{1 - \mu}{2} \left[ \frac{1}{2}(1 - \hat{u} - \varepsilon) + \frac{1}{2}(1 - \hat{u} + c) \right].$$

Firm i sells to half of the shoppers (those, who get the offer $u = \hat{u} + \varepsilon$ from it) and gets revenue $1 - \hat{u} - \varepsilon$ from each of them. It also attracts half of
the nonshoppers (those, who get the signal \( u = \hat{u} + \varepsilon \) from it), but gets revenue \( 1 - \hat{u} - \varepsilon \) from half of them and \( 1 - \hat{u} + c \) from the rest of them. This deviation is profitable if \( \pi' > \hat{\pi} \), or,

\[
2n(1 - \hat{u}) + n[(1 - \mu)c - (1 + \mu)\varepsilon] > 4(1 - \hat{u})
\]

which holds for all \( n \geq 2 \) as long as \( \varepsilon < \frac{1 - \mu}{1 + \mu}c \).

**Step 2:** Let \( T_i \) denote the support of \( G_i \), \( u_{\min} := \inf(T_i) \) and \( u_{\max} := \sup(T_i) \). I already established that \( u_{\min} \geq \bar{u} \). I need to derive the rest of \( G(u) \). Recall that a firm’s expected profit is

\[
\pi = (1 - \mu)\mathbb{E}[D(u)\mathbb{E}[1 - u]] + \mu\mathbb{E}[D(u)(1 - u)],
\]

and a buyer’s cutoff solves is

\[
\bar{u} = \mathbb{E}[u] - c.
\]

**Step 2a:** I derive \( D_i(u) \) by using the property that if \( G_i(u) \) is optimal, it must be unprofitable to reallocate mass within its support \( T_i \).

First suppose \( G_i(u) \) assigns positive mass to utilities \( u \in T_i \), \( u_{\min} \) and \( u_{\max} \). I derive (a necessary condition that has to be satisfied by) \( D_i(u) \) by finding a particular number \( \alpha(u) \) for any \( u \) such that firm \( i \) does not want to reallocate a small amount of mass \( \varepsilon \neq 0 \) from \( u \) and put mass \( \alpha(u)\varepsilon \) on \( u_{\min} \) and mass \( (1 - \alpha(u))\varepsilon \) on \( u_{\max} \) (where the movement of mass is from \( u \) if \( \varepsilon > 0 \) and to \( u \) if \( \varepsilon < 0 \)).

Moving a small mass \( \varepsilon \neq 0 \) from \( u \) and put mass \( \alpha\varepsilon \) on \( u_{\min} \) and mass \( (1 - \alpha)\varepsilon \) on \( u_{\max} \) changes firm \( i \)’s profit by

\[
\Delta\pi = (1 - \mu)\{\Delta\mathbb{E}[D(u)]\mathbb{E}[1 - u] + \mathbb{E}[D(u)]\Delta\mathbb{E}[1 - u]\} + \mu\Delta\mathbb{E}[D(u)(1 - u)]
\]

\[
= (1 - \mu)\varepsilon\{[\alpha D(u_{\min}) + (1 - \alpha)D(u_{\max}) - D(u)](1 - m) + d[\alpha(1 - \bar{u}) + (1 - \alpha)(1 - u_{\max}) - (1 - u)]\}
\]

\[
+ \mu\varepsilon[\alpha D(u_{\min})(1 - u_{\min}) + (1 - \alpha)D(u_{\max})(1 - u_{\max}) - D(u)(1 - u)],
\]

where \( m := \mathbb{E}[u] \) and \( d := \mathbb{E}[D(u)] \) for brevity. If \( G(u) \) is optimal, this change has to be zero for any \( \varepsilon \neq 0 \). Noting that \( D(u_{\min}) = 0 \) and \( D(u_{\max}) = 1 \), we can simplify \( \Delta\pi = 0 \) to

\[
0 = (1 - \mu)\{[(1 - \alpha) - D(u)](1 - m) + d[u - \alpha u_{\min} - (1 - \alpha)u_{\max}]\}
\]
\[ + \mu[(1 - \alpha)(1 - u_{\text{max}}) - D(u)(1 - u)], \]  

which can be rearranged to

\[ D(u) = 1 - \alpha + \frac{(1 - \mu)d[u - u_{\text{min}} - (1 - \alpha)(u_{\text{max}} - u_{\text{min}})] - \mu(1 - \alpha)(u_{\text{max}} - u)}{\mu(1 - u) + (1 - \mu)(1 - m)}. \]

Then \( D(u) = 1 - \alpha(u) \) iff the numerator of the second term is equal to zero (I verify later that the denominator is positive for all \( u \), i.e., that \( u \leq 1 + \frac{(1 - \mu)(1 - m)}{\mu} \)), which can be rearranged to give

\[ 1 - \alpha(u) = \frac{(1 - \mu)d(u - u_{\text{min}})}{(1 - \mu)d(u_{\text{max}} - u_{\text{min}}) + \mu(u_{\text{max}} - u)}. \]

Thus, a necessary condition that \( D(u) \) has to satisfy in equilibrium is that

\[ D(u) = \frac{(1 - \mu)d(u - u_{\text{min}})}{(1 - \mu)d(u_{\text{max}} - u_{\text{min}}) + \mu(u_{\text{max}} - u)}, \]  

where \( d := E[D(u)] \).

The generalisation to a smooth \( G(u) \) follows when one considers moving mass from the close neighbourhood of any \( u \in (u_{\text{min}}, u_{\text{max}}) \) to the neighbourhood of \( u_{\text{min}} \) and \( u_{\text{max}} \).

Step 2a’: I derive \( u_{\text{max}} \) by using again the property that if \( G_i(u) \) is optimal, it must be unprofitable to reallocate mass within its support \( T_i \), and the necessary condition (6) on \( D(u) \).

In particular, I derive (a necessary condition that has to be satisfied by) \( u_{\text{max}} \) by showing that then, for any \( \alpha \in [0, 1] \), firm \( i \) does not want to reallocate a small amount of mass \( \varepsilon \neq 0 \) from \( u \in (u_{\text{min}}, u_{\text{max}}) \), \( u \in T_i \), and put mass \( \alpha \varepsilon \) on \( u_{\text{min}} \) and mass \( (1 - \alpha) \varepsilon \) on \( u_{\text{max}} \) (where the movement of mass is from \( u \) if \( \varepsilon > 0 \) and to \( u \) if \( \varepsilon < 0 \)). Plugging equation (6) into (5) gives

\[ 0 = [(1 - \mu)(1 - m) + \mu(1 - u_{\text{max}}) - d(1 - \mu)(u_{\text{max}} - u_{\text{min}})] \times \\
\{ -d(1 - \mu)(u - u_{\text{min}}) + (1 - \alpha)[(1 - \mu)d(u_{\text{max}} - u_{\text{min}}) + \mu(u_{\text{max}} - u)] \}, \]

which holds iff either the first term in the squared brackets or the term in the curly brackets is equal to zero. The latter, however, depends on \( \alpha \). Thus, the equality holds for all \( \alpha \) if the first term in the squared brackets
is equal to zero. This can be rearranged to give
\[ u_{\text{max}} = 1 + (1 - \mu) \frac{1 - m - d(1 - u_{\text{min}})}{(1 - \mu)d + \mu}, \]  
where \( d := \mathbb{E}[D(u)] \) and \( m := \mathbb{E}[u] \). Note that the condition that I assumed when deriving \( D(u) \), i.e., that \( u \leq 1 + \frac{(1 - \mu)(1 - m)}{\mu} \), is satisfied because \( u_{\text{max}} < 1 + \frac{(1 - \mu)(1 - m)}{\mu} \) (since \( m < 1 \) must hold for weakly positive profits in equilibrium).

If \( D_i(u) \) satisfies condition (6) and \( u_{\text{max}} \) condition (7), then moving mass from any \( u_1 \in T_i \) to another \( u_2 \in [u_{\text{min}}, u_{\text{max}}] \) is unprofitable. This is because I can choose \( u = u_1 \) and rewrite \( u_2 \) as \( u_2 = \alpha u_{\text{min}} + (1 - \alpha)u_{\text{max}} \) for some \( \alpha \in [0, 1] \) so the above analysis applies. Thus, if \( D_i(u) \) satisfies (6) and \( u_{\text{max}} \) satisfies (7), then \( G_i(u) \) is a best response by firm \( i \).

Steps 2b to 3a are almost exactly the same as in the proof of Proposition 5 on page 22 (where \( \lambda = 1 \)) so I skip them here to save space. The only difference is that if changes in \( \mathbb{E}[u] \) (or \( \mathbb{E}[D(u)] \)) are infinitesimal, then so are changes in \( \mathbb{E}[D(u)u] \) so the arguments in Spiegler (2006) apply.

**Step 3:** Let \( G \) be the symmetric equilibrium strategy and \( T \) be the support of \( G \). Let \( u_{\text{max}} := \text{sup}(T) \).

Step 3b: I derive \( \bar{u} \) by using a clever trick from Janssen and Moraga González (2004) (see p. 1097).

First note that in asymmetric equilibrium, \( \mathbb{E}[D(u)] = \frac{1}{n} \) and let \( \gamma := n \frac{\mu}{1 - \mu} \). Then rewrite equation (6) as
\[ u = u_{\text{max}} - (u_{\text{max}} - u_{\text{min}}) \frac{1 - D}{1 + \gamma D}, \]  
where \( D := D(u) \). Then I define a new variable \( z := G(u) \) so that I can write
\[ \mathbb{E}[u] = \int_{u_{\text{min}}}^{u_{\text{max}}} u \, dG(u) = \int_0^1 u \, dz. \]  
From the buyer’s optimisation problem, we know that \( \mathbb{E}[u] = \bar{u} + c \) and in a symmetric equilibrium, \( D = z^{n-1} \). Thus, I can rewrite (9) using (8) as
\[ \bar{u} + c = \int_0^1 \left[ u_{\text{max}} - (u_{\text{max}} - u_{\text{min}}) \frac{1 - z^{n-1}}{1 + \gamma z^{n-1}} \right] \, dz, \]  
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and because $u_{\text{min}} = \bar{u}$,

$$u_{\text{max}} - u_{\text{min}} = c \left[ 1 - \int_0^1 \frac{1 - z^{n-1}}{1 + \gamma z^{n-1}} \, dz \right]^{-1}, \quad (10)$$

where the right-hand side of (10) depends only on exogenous variables. It is straightforward to see that $u_{\text{max}} - u_{\text{min}}$ decreases in $\gamma$ (equivalently, $\mu$).

Using equations (7) and (10), I can solve for $\bar{u}$:

$$\bar{u} = 1 - \frac{c}{\gamma + n} [K(1 + \gamma) + n], \quad (11)$$

where $K := \left[ 1 - \int_0^1 \frac{1 - z^{n-1}}{1 + \gamma z^{n-1}} \, dz \right]^{-1}$. Note that $K$ decreases in $\gamma$ and $K > 1$ for all $\gamma$. The equilibrium exists if the nonshoppers’ value from searching is positive, i.e., if $\bar{u} > 0$, or $c < \frac{\gamma + n}{K(1 + \gamma) + n}$.

I use the explicit form for $\bar{u}$ in (7) to get an explicit form for $u_{\text{max}}$:

$$u_{\text{max}} = 1 + (\gamma + n)^{-1} c [(n - 1)K - n],$$

Firms offer deals (or $u_{\text{max}} > 1$) if $(n - 1)K > n$, which is the hardest to satisfy for the smallest $K$, i.e., the largest $\gamma = n\frac{\mu - \mu}{1 - \mu}$. As $\gamma \to +\infty$, $\lim_{\gamma \to n} K = 1$ so there exists $\bar{\gamma}$ such that for $\gamma > \bar{\gamma}(n)$, $u_{\text{max}} < 1$. That is, firms stop offering deals if there are many shoppers on the market. Note that $\bar{\gamma}(n)$ increases in $n$: $(n - 1)K(\gamma = \bar{\gamma}) = n$ can be rearranged as $1 = n \int_0^1 \frac{1 - z^{n-1}}{1 + \gamma z^{n-1}} \, dz$, where the right-hand side increases in $n$ and decreases in $\gamma$. For example, for $n = 3$, $\bar{\gamma}(n)$ corresponds to $\mu = 0.78$ and for $n = 4$, to $\mu = 0.96$.

**Step 4:** An equilibrium as described in Step 3 exists.

Suppose all firms but $i$ use $G(u)$ as described in Step 3. Then we know that $D_i(u)$ satisfies (6) so firm $i$ cannot improve its profits by using a different distribution than $G(u)$. Thus, all firms using $G(u)$ is an equilibrium. \(\square\)

**Proof.** (Proposition 4) The result with respect to the range of the distribution, $u_{\text{max}} - \bar{u}$, follows from inspecting equation (10). The result with respect to nonshoppers’ welfare follows from differentiating equation (11) with respect to $\gamma = n\frac{\mu - \mu}{1 - \mu}$. The result is

$$\frac{\partial \bar{u}}{\partial \gamma} = \frac{c}{\gamma + n} [n - (n - 1)K - (\gamma + n)(1 + \gamma)K']$$

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As $\mu \to 0, \gamma \to 0$ with $\lim_{\gamma \to 0} K = n$ and $\lim_{\gamma \to 0} K' = -\frac{n(n-1)}{2n-1}$ so that

$$\lim_{\gamma \to 0} \frac{c}{2n - 1} [(4-n)n - 2],$$

which is negative for all $n \geq 4$.

The result with respect to $u_{\text{max}}$ follows from differentiating the equation for $u_{\text{max}}$ with respect to $\gamma = n \frac{\mu}{1-\mu}$:

$$\frac{\partial u_{\text{max}}}{\partial \gamma} = \frac{c}{\gamma + n} [n - (n-1)K - \gamma K'],$$

where $K' < 0$ so that a sufficient condition for $\frac{\partial u_{\text{max}}}{\partial \gamma} < 0$ is that $u_{\text{max}} \geq 1$, i.e., if $\gamma \leq \bar{\gamma}(n)$ (where from above, we know $\gamma < \bar{\gamma}(n)$ corresponds to approximately $\mu \leq 0.78$ for $n = 3$ and to $\mu \leq 0.96$ for $n = 4$). Let $\bar{\mu}(n)$ satisfy $\bar{\gamma}(n) = \frac{n \bar{\mu}(n)}{1 - \bar{\mu}(n)}$. \( \square \)

### A.3 Some buyers do not receive price signals

**Proof.** (Proposition 5) I solve for the equilibrium using the method developed in [Spiegler (2006)]. In Step 1, I show that a single-$u$ equilibrium never exists. In Step 2, I derive the properties that $G_i(u)$ must have in any equilibrium and in Step 3, the properties of a symmetric-equilibrium $G(u)$. In Step 4 I argue that such an equilibrium exists.

**Step 1:** An equilibrium in degenerate distributions $G(u)$ does not exist.

Suppose that all firms set $u = \hat{u}$ in equilibrium with probability one. The proposed equilibrium profits are $\hat{\pi} = \frac{1-\hat{u}}{n}$. For weakly positive profits in equilibrium, it must be that $\hat{u} \leq 1$. If all firms set $u = \hat{u}$ in equilibrium, then a buyer’s expected value is $E[u] - c = \hat{u} - c$ so she accepts any first offer.

I show that firm $i$ has an incentive to deviate to a dispersed distribution $G'_i$ such that $P'(u = \hat{u} + \varepsilon) = \frac{1}{2}$ and $P'(u = \hat{u} - c) = \frac{1}{2}$ for $\varepsilon > 0$ small. Firm $i$’s profit from this deviation is

$$\pi' = \left[ \frac{\lambda}{2} + \frac{1-\lambda}{n} \right] \left[ \frac{1}{2} (1 - \hat{u} - \varepsilon) + \frac{1}{2} (1 - \hat{u} + c) \right]$$

because it attracts half of the buyers who partially direct search (those, who get the signal $u = \hat{u} + \varepsilon$ from it) and its fair share of uninformed buyers. This deviation is profitable if $\pi' > \hat{\pi}$, or,

$$[2 + \lambda(n-2)] [2(1-\hat{u}) + c - \varepsilon] > 4(1-\hat{u}),$$

which holds for all $n \geq 2$ as long as $\varepsilon < c$. 

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Step 2: Let $T_i$ denote the support of $G_i$ and $u_{min} := \inf(T_i)$. I already established that $u_{min} \geq \bar{u}$. I need to derive the rest of $G(u)$. Recall that a firm’s expected profit is

$$
\pi = \left( \lambda \mathbb{E}[D(u)] + \frac{1-\lambda}{n} \right) \mathbb{E}[1-u],
$$

and a buyer’s problem is

$$
\bar{u} = \mathbb{E}[u] - c.
$$

Step 2a: All the points $\{(u,D_i(u))| u \in T_i\}$ lie on a straight line, except possibly for a zero-measure subset.

The proof follows directly from the proof of Lemma 1 in Spiegler (2006).

A Corollary to Step 2a, which I use below, is

**Corollary 4.** Given $G_{-i}$, firm $i$ is indifferent between $G_i(u)$ and $G^*_i(u)$ such that $\mathbb{E}_{G}[u] = \mathbb{E}_{G^*}[u]$ and $T^*_i \subseteq T_i$.

**Proof.** Because $D_i(u)$ is linear, firm $i$’s expected profit is the same under $G_i$ and $G^*_i$ if $\mathbb{E}_{G}[u] = \mathbb{E}_{G^*}[u]$. \(\square\)

Step 2b: For any $\lambda > 0$, in any Nash equilibrium $G(u)$ is continuous on $\left(\bar{u}, \infty\right)$. The proof follows directly from the proof of Lemma 2 in Spiegler (2006), where zero is replaced by $\bar{u}$.

Step 2c: In any Nash equilibrium, $u_{min} = \bar{u}$.

I only need to show that $u_{min} > \bar{u}$ cannot hold in equilibrium. The proof follows directly from the proof of Lemma 3 in Spiegler (2006), where zero is replaced by $\bar{u}$.

Step 3: Let $G$ be the symmetric equilibrium strategy and $T$ be the support of $G$. Let $u_{max} := \sup(T)$.

Step 3a: For any $\lambda > 0$, $T = [u_{min}, u_{max}]$ and $G$ is continuous over $[u_{min}, u_{max}]$.

The proof follows directly from Steps 1 and 2 in the proof of Proposition 1 in Spiegler (2006), where zero is replaced by $\bar{u}$.

Step 3b: If $\frac{1+\lambda}{\lambda} c < 1$, then $\bar{u} = 1 - \frac{1+\lambda}{\lambda} c$, $u_{max} = 1 + \left(n - \frac{1+\lambda}{\lambda}\right) c$, and $G(u) = \left(\frac{u-\bar{u}}{n-1}\right)^{\frac{1}{n-1}}$.

The proof follows from Step 3 in the proof of Proposition 1 in Spiegler (2006). I use Corollary 4 to derive the equilibrium. Let $G^*$ be such that
\(T^* = \{\bar{u}, u_{\text{max}}\}\) with \(P^*(u = \bar{u}) = \alpha\) and \(P^*(u = u_{\text{max}}) = 1 - \alpha\) with \(E_G[u] = E^*[u]\), where \(E^* := E_{G^*}\). Then the expected profit under \(G^*, \pi^*,\) is

\[
\pi^* = \left(\lambda E^*[D(u)] + \frac{1 - \lambda}{n}\right) E^*[1 - u]
\]

\[= \left[\lambda(1 - \alpha) + \frac{1 - \lambda}{n}\right] [1 - u_{\text{max}} + \alpha(u_{\text{max}} - \bar{u})],\]

where the latter equality follows from writing out the expectations explicitly and from the fact that \(D(\bar{u}) = 0\) and \(D(u_{\text{max}}) = 1\). The first-order condition with respect to \(\alpha\) yields

\[
2\lambda(u_{\text{max}} - \bar{u})(1 - \alpha^*) = \lambda(1 - \bar{u}) - (u_{\text{max}} - \bar{u})\frac{1 - \lambda}{n}.
\]  

(12)

Since \(E^*[D(u)] = 1 - \alpha^*\) and, in a symmetric equilibrium, \(E[D(u)] = \frac{1}{n}\), equation (12) can be rearranged to give

\[
\hat{u} = \frac{1 + \alpha}{\lambda} (u_{\text{max}} - \bar{u}).
\]  

(13)

From the buyer’s optimisation problem, we know that \(E[u] = \bar{u} + c\). But \(E_G[u] = E^*[u] = \bar{u} + (1 - \alpha^*)(u_{\text{max}} - \bar{u})\) so that

\[
\bar{u} = 1 - \frac{1 + \lambda}{\lambda} c.
\]

Plugging this back to (13) yields \(u_{\text{max}} = 1 + (n - \frac{n + \lambda}{\lambda}) c\). Note that \(u_{\text{max}} > 1\) if \(n > \frac{1 + \lambda}{\lambda}\) (which collapses to \(n > 2\) if \(\lambda = 1\)).

Since \(D(u)\) is linear and continuous, \(D(u) = \frac{u_{\text{max}} - u}{u_{\text{max}} - \bar{u}}\) and in a symmetric equilibrium, \(G(u) = (D(u))^{\frac{1}{n-1}}\).

Step 3c: If \(\frac{1 + \lambda}{\lambda} c \geq 1\), then \(\bar{u} = 0\) and buyers choose not to search.

If buyers’ optimal cutoff is zero, a distribution \(G^*\) as described in Step 3a still exists. Thus, \(u_{\text{max}}\) can be derived in the same way, which gives \(u_{\text{max}} = \frac{\lambda}{1 + \lambda}\). Note that \(E[u] = (1 - \alpha^*)u_{\text{max}} = \frac{\lambda}{1 + \lambda}\) so that \(E[u] - c \leq 0\) (and \(\bar{u} = 0\)) if \(\frac{1 + \lambda}{\lambda} c \geq 1\).

Step 4: An equilibrium as described in Step 3a-b exists.

Suppose all firms but \(i\) use \(G(u)\) as described in Step 3a-b. Then any cdf \(G_i(u)\) with expectation equal to \(E_G[u]\), including \(G(u)\), is a best reply for firm \(i\) because \(D_i(u)\) is linear. Thus, all firms using \(G(u)\) is an equilibrium.

\[\Box\]
A.4 Supershoppers

Proof. (Lemma 1) The proof is by contradiction. Suppose first that all firms set \( u_{\text{max}} > 1 \) so that firms are making losses on their sales to supershoppers. Consider a deviation by firm \( i \) to \( u'_{\text{max}} = u_{\text{max}} - \varepsilon \), for some small \( \varepsilon > 0 \). The firm loses a negligible fraction of searchers from this deviation (while increasing the expected price from these buyers). But the firm saves the loss associated with supershoppers because all of them will now buy at some other firm. Thus, the deviation is profitable and in a symmetric equilibrium, \( u_{\text{max}} \leq 1 \).

Suppose instead that all firms set \( u_{\text{max}} < 1 \) in equilibrium. Consider a deviation by firm \( i \) to \( u'_{\text{max}} = u_{\text{max}} + \varepsilon \), for some small \( \varepsilon > 0 \) such that \( u'_{\text{max}} < 1 \). The firm gains a negligible fraction of searchers from this deviation, but gets a slightly lower expected price from them. But the firm gains the custom of all supershoppers and they all yield a profit. Thus, the deviation is profitable. Altogether, in a symmetric equilibrium, \( u_{\text{max}} = 1 \).

Proof. (Proposition 6) A firm’s equilibrium profit is

\[
\pi = (1 - \sigma) \mathbb{E}[D(u)] \mathbb{E}[1 - u].
\]

Note that the profit looks like in the model without supershoppers, except that the profit is multiplied by a constant. Thus, the derivations follow closely those in the proof of Proposition 5 on page 22, where \( \lambda = 1 \) and \( u_{\text{max}} = 1 \). These yield \( \alpha^* = \frac{1}{2} \) and \( \bar{u} = 1 - 2c \).

References


