DEVELOPMENT AND APPLICATION OF A
HIGHWAY NETWORK DESIGN MODEL

Volume 2 of 2
Appendices

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DEVELOPMENT AND APPLICATION OF A HIGHWAY NETWORK DESIGN MODEL


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Volume 2. Appendices

The Highway Network Design Model was developed to assist transportation planners in the initial stages of development of system plans. It considers a wide variety of improvements to the system and their economic, social and environmental impacts. By treating the network at a fairly aggregate level, it is possible to explore many alternatives quickly and with a minimum of resources. Only data which is readily available in urban transportation studies today is required.

The model is quite promising as a sketch planning tool. Furthermore, it is well-suited for use in community interaction, because of its efficiency in answering questions posed and because of the inclusion of relatively simple, easily understood, descriptors of system impacts and system characteristics. A program of testing and development in the context of an actual transportation planning effort is recommended. This could equally well be in urban, state-wide, regional, or national planning, for the model is readily adapted to any of these applications.
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APPENDIX A

PREPROCESSOR OPTIMA

This appendix describes the preprocessor for preparing input data to the OPTIMA code to solve the core network design model. PREPROCESSOR OPTIMA is a collection of programs designed to take the network data and generate the set of constraints of the network design problem in a form suitable to the X-OPTIMA input format.

OPTIMA is a commercial code which was developed by the Control Data Corporation to solve linear programs. X-OPTIMA is the modified form of the OPTIMA code designed to accept the input data of the linear programming problem in a simplified form. The input data should include the objective function and the constraints in equations form and/or inequalities as the case may be. If the number of constraints is large, then it would be desirable to have some mechanized process (computer subroutines) to generate these constraints. This is the problem which PREPROCESSOR OPTIMA is aimed to solve.

Structure of the Preprocessor

The set of programs which make up the PREPROCESSOR OPTIMA are as follows:

PROGRAM AB
PROGRAM BREAK
PROGRAM LINK
PROGRAM OBJFCN
PROGRAM DATA
PROGRAM CAPCTY

The programs listed above serve the following purpose.

PROGRAM AB This one reads the network data for one link
at a time and then calculates the parameters of the nonlinear travel-time versus volume cost function.

PROGRAM BREAK
This one linearly approximates the travel-time vs. volume cost function in two linear pieces for every link and determines the break volume point.

PROGRAM LINK
This one generates the new set of link data cards (one card for each direction). The output also includes the capacities of two arcs for every link.

PROGRAM OBJFCN
This one prepares the objective function for the X-OPTIMA program.

PROGRAM DATA
This program is used to generate the node flow conservation equations and the link-flow defining constraints in the X-OPTIMA FORMAT.

PROGRAM CAPCTY
Its function is similar to PROGRAM DATA. This generated the capacity constraints for all arcs in the network in the X-OPTIMA FORMAT.

Program Specifications
The format statements for each input, output statement are given in the listings of the programs. Also, each program includes comment statements to simplify the understanding of each program.

1) PROGRAM AB

INPUT:

READ 4, LINKS  one card
System Operation Flow-Chart

START
   ↓
NETWORK STRUCTURE
   ↓
NETWORK DATA
   ↓
PROGRAM AB
PROGRAM BREAK
PROGRAM LINK
PROGRAM OBJFCN
PROGRAM DATA
PROGRAM CAPCTY
   ↓
PREPROCESSOR
OPTIMA
   ↓
BUDGET
CONSTRAINT
   ↓
X-OPTIMA
   ←→ NETWORK DESIGN STOP

Figure A-1. Pre-processor Flow Chart
READ 1, NODE 1, NODE 2, PCAP  
where

LINKS  the number of roads in the network (existing and proposed.

NODE 1, NODE 2  end nodes of the link

A  mean free travel time on the road (link)

PCAP  the practical capacity of the road (link) (one-way)

OUTPUT:

PUNCH 2, NODE 1, NODE 2, A, B, PCAP, VMAX  one card per link

PRINT 12, NODE 1, NODE 2, A, B, PCAP, VMAX, FCN

where

B  the parameter of the travel time vs. volume cost function (T = A + B * V^4)

VMAX  maximum capacity of the road (one-way)

VMAX = 2. * PCAP

FCN  value of the total travel time at the maximum volume. (useful in the PROGRAM BREAK in choosing the vertical increment values)

2) PROGRAM BREAK

INPUT:

READ 7, LINKS  one card per link

READ 1, NODE 1, NODE 2, A, B, PCAP, VMAX  one card per link

OUTPUT:

PRINT 499, REALK, K, VMAX

PRINT 10, XXMIN, SAVVB, SAVHB

PRINT 99, NODE 1, NODE 2, A, B, PCAP, VMAX, SAVVB, SAVSLP1, SAVSLP2

A-4
PUNCH 101, NODE 1, NODE 2, A, B, PCAP, VMAX, SAVVB, SAVSLP1, SAVSLP2.

where:

REAL K maximum value of the FUNCTION \( F(VMAX) = A \times VMAX + (B \times VMAX \times 5)/5 \).

K scaled value of the REALK (to be used in determining the BREAK point)

XXMIN maximum deviation of the linearly approximating curve and the nonlinear cost function.

SAVVB break volume of the link (\(^*\) capacity of the lower cost arc) - one way

SAVHB value of the FUNCTION at the BREAK volume.

SAVSLP1 cost of travel on arc 1 (lower cost arc)

SAVSLP2 cost of travel on arc 2 (high cost arc)

3) **PROGRAM LINK**

**INPUT:**

READ 2, NODE 1 (I), NODE 2 (I), (REAL (I, J), J = 1, 7)

where

I subscript indicates the link number \( I = 1, ..., \text{LINKS} \)

(REAL (I,J), J=1,7) represents A, B, PCAP, VMAX, SAVVB, SAVHB, SAVSLP1, SAVSLP2.

**OUTPUT:**

PRINT AND PUNCH

NODE 1 (I), NODE 2 (I), (REAL (I, J), J=1, 7), I

for I=1,...,LINKS

one card per link
PRINT AND PUNCH

NODE 2 (I), NODE 1 (I), (REAL (I,J), J=1, 7), K

for I=1,...,LINKS one card per link

Here K= (LINKS + 1),..., (LINKS + LINKS)

PRINT AND PUNCH

CAPM1, CAPM2 one card per link

where:

CAPM1 the capacity of the link M=1 part \( K_{ij}^1 \)
CAPM2 the capacity of the link M=2 part \( K_{ij}^2 \)

4) PROGRAM OBJFCN

INPUT:

READ 1111, LINKS, NODES one card

READ 1112, SLOPE 1, J, SLOPE 2, K one card per link

where

SLOPE 1 is \( C_{ij}^1 \) (= SAVSLP1)
SLOPE 2 is \( C_{ij}^2 \) (= SAVSLP2)

J variable index number of \( x_{ij}^m \), m=1
K variable index number of \( x_{ij}^m \), m=2

5) PROGRAM DATA

INPUT:

READ 91, NODES, LINKS, ARCS, DEST one card

READ 2, NODE 1 (I), NODE 2 (I), (REAL (I,J), J=1,7) one card per link

READ 701, IORIG, IDEST, TRIP (I,J) one card per origin destination pair

OUTPUT:

Punched cards for constraints type (4-13) in the model.
Punched cards for constraints type (4-14) in the model.
where:

ARCS  the number of links in the ANALYSIS NETWORK.

(# arcs = m * # links, where m=2)

Note: the name arcs has also been used interchangeably
to represent the number of two-way roads (links) in the road
network.

DEST  the number of destinations in the network.

IORIG demand generating node i.e. origin node.

IDEST demand attracting node i.e. destination node.

TRIP (I,J) number of trips from node I to node J.

6) PROGRAM CAPCTY

INPUT:

READ 6, LINKS, ARCS, NODES  one card

READ 15, CAPM1, CAPM2  one card per link

OUTPUT:

Punched cards for constraints type (4-15) and (4-16) in the
model.

Print out of the constraints.

Solving the Continuous Network Design Model

(X-OPTIMA RUN)

Following program describes the basic structure of the source
deck to solve a linear program using the X-OPTIMA code. The set up of
the control cards is for the CDC 6400 computer and this may vary in
practice from one computer to another.

VJOB,...,MT1.

LIBRARY (X-OPTIMA)
LGO
E-Ø-R
FILE NETWORK
COLUMNS 1=ONE, 2=TWO, 3=ALMOST THE LAST ONE/
RHS RIGHT
MINIMIZE THIS OBJECTIVE FUNCTION
TOTAL TIME

OUTPUT OF PROGRAM
OBJFCN

CONSTRAINTS

OUTPUT OF PROGRAM DATA
OUTPUT OF PROGRAM CAPCTY

BUDGET CONSTRAINT

OPTIMIZE
E-Ø-R
ACL CARDS FOR OPTIMA
E-Ø-R
E-Ø-I

As an example of the use of the Preprocessor programs for OPTIMA, the following pages give the input and related output for these inter-related programs. Figure A-2 is a diagram of the network to be represented. Figures A-3, 4, 5, 6, 7, and 8 are listings of the various programs. Figures A-9, 10, 11, 12, and 13 give the resulting input and output for those programs as applied to the network shown in Figure A-2.
Figure A-2. Example Network Structure

A-9
program ab (input, output, punch)

program ab reads the following input parameters of the links:
1. HMAX=HMEP=TRAVEL TIME
2. PRACTICAL CAPACITY
3. END NODE NUMBERS OF THE LINK

and then calculates the a and p parameters of the calculated
travel time versus volume relationship.

vmax and pcap are in thousands of vehicles per day and are

calculated from an assumed peak hour factor.

12 format(1x,12.1x12.1xf5.4,1x,f13.12,1xf4.1,1xf4.1,1x,f15.5)
4 format(15)
1 format(12.1xf4.1)
2 format(12.1xf4.1,1xf5.4,1x,f13.12,1xf4.1,1xf4.1)

read4 links
do 3 j = 1, links
read 1 * node1,node2,a,pcap
r = 0.15*a/(pcap**4)
max = 2*pcap
fcn = a* v^max + r*(v^max**6)
punch 2 * node1, node2, a, r, pcap, v^max
print 12 * node1, node2, a, r, pcap, v^max, fcn
3 continue
stop
end

Figure A-3. Program AB
PROGRAM BREAK (INPUT, OUTPUT, PUNCH)

PROGRAM BREAK READS THE OUTPUT OF PROGRAM AR. THERE IS ONE CARD
FOR EVERY LINK CONTAINING NODE NUMBERS, PARAMETERS A AND B OF THE
TRAVEL TIME VERSUS VOLUME RELATIONSHIP, PRACTICAL CAPACITY AND
THE MAXIMUM CAPACITY OF THE LINK.
The program then linearly approximates the nonlinear cost function
in two parts and determines the break point which minimizes the
absolute deviation of the linear curve from the nonlinear
function.

This program then calculates:
1. The break point which is the capacity of the arc
having the lowest cost of travel
2. The cost of travel on both arcs for every link.

The values of the parameter AMULT, AINCNT, and the increment of
the DO loop for IVF will depend upon the characteristics of the
link and the accuracy required.

COMMON A, B

499 FORMAT (1XF10.5,15,F6.1)
1'1 FORMAT (1P1,1X,I7,1X,F4.4,1X,F13.12,1XF4.1,1XF4.1,1XF10.5)
99 FORMAT (1X,1P1,1X,I7,1X,F4.4,1X,F13.12,1XF4.1,1XF4.1,1XF10.5)
10 FORMAT (1X,FMAXIMUM DEVIATION 15#F10.2#COORDINATES(X,Y) ARE#2F6.3)
7 FORMAT (15)
1 FORMAT (12,1X,I7,1XF5.4,1X,F13.12,1XF4.1,1XF4.1)

RFAN 7, LINKS

DO 100 INDEX = 1, LINKS

LINKS = 38
RFAN 1, NODE1, NODE2, A, B, PCAP, VMAX
XXMIN = 0.00000000000
RFALK = F(VMAX)

RFALK IS THE MAXIMUM VALUE OF THE FUNCTION F(VMAX) = AMULT*VMAX + (B*VMAX**2)/2.
SCALING OF RFALK ON AN INTEGER SCALE

AMULT = 20.
AINCNT = .05

K = AMULT*RFALK
IVMAX = VMAX - 1.
PRINT 499, 499, RFALK, K, VMAX

DO 2 1VB = 1, IVMAX

DO 2 1HR = 1, K

NOW VR AND HR ARE FIXED
C THIS IS IN THOUSANDS OF VEHICLE HOURS PER DAY
HR = AINCNT*1HR
C
C AMULT = 1./AMULT
C SCALING DOWN
C
VR = IVR
XYMAX = 0.
SLP0 = HR/VR
SLP2 = (RFALK - HR) / (VMAX - VR)

C NON COUPLED MAXIMUM ABSOLUTE DEVIATION FOR CURRENT VALUE OF VR AND HB

Figure A-4. Program Break

A-11
DO 3 J = 1, IVR
   POINT = J
   DEV = ARS (F(POINT) - SLOPE1 * POINT)
   IF (DEV .GT. XXMAX) GOTO 5
   XXMAX = DEV
3 CONTINUE

C
DO 4 J = IVR, IVMAX
   POINT = J
   DEV = ARS (F(POINT) - (SLOPE2 * (POINT - VR) + HB))
   IF (DEV .GT. XXMAX) GOTO 6
   XXMAX = DEV
4 CONTINUE

C
IF (XXMAX .GE. XXMIN) GO TO 2
   XXMIN = XXMAX
C
SAVVR IS THE CAPACITY OF THE ARC OF LOWER COST OF TPAVEL
C
SAVSLP1 IS THE COST OF TPAVEL ON ARC 1
C
SAVSLP2 IS THE COST OF TPAVEL ON ARC 2
C
SAVSLP1 IS SMALLER THAN SAVSLP2
C
SAVVBN = VB
SAVVB = HB
SAVSLP1 = SLOPE1
SAVSLP2 = SLOPE2
2 CONTINUE

C
POINT 10 * XXMIN, SAVVR, SAVVB
POINT 99 * NODE1, NODE2, A, R, PCAP, VMAX, SAVVR, SAVSLP1, SAVSLP2
POINT 101 * NODE1, NODE2, A, R, PCAP, VMAX, SAVVR, SAVSLP1, SAVSLP2
1 CONTINUE
C
STOP
END
FUNCTION F(X)
COMMON A, R
   F = A + X + R * X ** 5
RETURN
END

Figure A-4. (continued)
PROGRAM LINK (INPUT, OUTPUT, PUNCH)

PROGRAM LINK reads the output of program BREAK, one card per link, and
then generates the new set of cards, one card per link for each direction.
This program also puts the serial number on each card. The program then
calculates the capacities of two arcs for each link. The output is in the
form of punched cards as well as in the print form.

OUTPUT FORM:

NODE1, NODE2, ARC, PCAP, VMAX, SAVVR, SAVSI, P1, SAVSLP2
CAPM1 (CAPACITY OF ARC 1), CAPM2 (CAPACITY OF ARC 2)

DIMENSION REAL (76, 7), NODE1 (76), NODE2 (76)

LINKS = 38
ARCS = 76

2 FORMAT (12, 1X, I7, 1XF5, 4, 1XF13, 12, 1XF4, 1, 1XF4, 1, 1XF8, 5, 4XF6, 5, 4XF6, 5)
3 FORMAT (12, 1X, I7, 1XF5, 4, 1XF13, 12, 1XF4, 1, 1XF4, 1, 1XF8, 5, 4XF6, 5, 4XF6, 5, 113X, I7)
5 FORMAT (2F6, 2)
11 FORMAT (13, 1X, I7, 1XF5, 4, 1XF13, 12, 1XF4, 1, 1XF5, 1, 1XF8, 5, 4XF6, 5, 4XF6, 5, 113X, I7)

DO 1 I=1, 138
READ 2, NODE1(I), NODE2(I), (REAL(I, J), J=1, 7)

MAKE THE TWO WAY CAPACITIES:

REAL (I, 3) = 0.9 * REAL (I, 3)
REAL (I, 4) = 0.9 * REAL (I, 4)
REAL (I, 5) = 0.9 * REAL (I, 4)
PRINT 11, NODE1(I), NODE2(I), (REAL(I, J), J=1, 7), I

PUNCH 7, NODE1(I), NODE2(I), (REAL(I, J), J=1, 7), I

DO 6 I = 1, 138
N = I + 38
ISAVE = NODE1(I)
NODE1(I) = NODE2(I)
NODE2(I) = ISAVE
PRINT 11, NODE1(I), NODE2(I), (REAL(I, J), J=1, 7), N

PUNCH 3, NODE1(I), NODE2(I), (REAL(I, J), J=1, 7), N

DO 4 I = 1, 138
CAPM1 = REAL(I, 5)
CAPM2 = REAL(I, 4) - REAL(I, 5)
PRINT 5, CAPM1, CAPM2
PUNCH 5, CAPM1, CAPM2

CONTINUE

STOP
END

Figure A-5. Program Link
PROGRAM DATA (INPUT, OUTPUT, PUNCH)

THIS PROGRAM READS THE LINK DATA CAPS AND GENERATES THE NODE FLOW
CONSERVATION EQUATIONS AND ALSO THE LINK FLOW DEFINING CONSTRAINTS.

INPUT TO THIS PROGRAM IS
1. NUMBER OF NODES, DESTINATIONS, LINKS AND ARCS.
   WHERE
   NUMBER OF LINKS IS THE NUMBER OF ROADS IN THE
   NETWORK AND NUMBER OF ARCS IS 2*NUMBER OF LINKS.
2. FOR EVERY ARC, ITS NODE NUMBERS, PARAMETER A
   AND PARAMETER B, PRACTICAL CAPACITY AND MAXIMUM
   CAPACITY, BREAK VOLUME AND COSTS OF TRAVEL FOR N=1 AND M=2.
3. TRIP DISTRIBUTION MATRIX.

THE OUTPUT IS IN THE FORM OF PUNCHED CARDS. ALSO THE PRINTOUT OF THE
CONSTRAINTS IS PRODUCED. THE FORM OF THE OUTPUT IS ACCORDING
TO THE XOPTIMA FORMAT.

DIMENSION REAL(76,7), A(24,76), A(1900), NODE(176), NODE2(76)
COMMON NODES, LINKS, M, TRIP(24,24), ARCS, DEST

INTEGER ARCS, DEST

702 FORMAT(*ERROR*12,1X,I2)
701 FORMAT(I2,1X,I2,1X,F5)
91 FORMAT(415)
2 FORMAT(I2,1X,I2,1X,F5,1,1X,F4,1,1X,F5,1,1X,F4,5,4X,F6,5,4X,F6,5)

READ 91, NODES, LINKS, ARCS, DEST

IN OUR EXAMPLE DEST = NODES = 24
NODES = 24
ARCS = 76
LINKS = 38

ARCS = NUMBER OF VARIABLES PER BLOCK.

DO 1 = 1, ARCS
1 READ 2, NODE(1), NODE(2) (REAL(I,J), J=1,7)

START WORKING ON NODE FLOW CONSERVATION EQUATIONS.

DO 301 I=1, NODES
   DO 301 J=1, NODES

   READ 701, IOTP(I), DEST, TRIP(I,J)
   TRIP(I,J) = TRIP(I,J)/1000.

   CHECKING THE ACCURACY OF THE DATA.
   IF((I,EQ.IOTP(IOTP(I)), AND, (J,EQ.IDEST))) GO TO 701
   PRINT 702, I, J
STOP

CONTINUE

DO 10 N = 1, NODES
   ROUTINE TO FORM APC - NODE INCIDENCE MATRIX FOR EACH DESTINATION. N
   CLEAR OUT A MATRIX FROM LAST BLOCKS DATA
   INITIALIZE
   DO 53 I=1, NODES
      DO 53 J=1, ARCS
         A(I,J) = 0.
   DO 10 J=1, ARCS
   FORM 1 COLUMN OF THE INCIDENCE MATRIX FOR DESTINATION N

Figure A-6. Program Data
A-14
A(NODEP(I),I) = -1.
A(NODEF(I),I) = 1.
10 CONTINUE
C CALL SUBROUTINE TO GENERATE THE NODE FLOW CONSERVATION EQUATIONS.
C THERE ARE 552 (= NODES*DESE TINATIONS) OF THESE.
C CALL GEN(A)
C INDEX = NODES*ARCS
C IN OUR PROBLEM INDEX = 1824
C START WORKING ON LINK FLOW DEFINING CONSTRAINTS.
C THERE ARE 38 (= NUMBER OF LINKS) OF THESE.
DO 600 IROW = 1, LINKS
   INUM = NODES*ARCS + 2*LINKS
C IN OUR PROBLEM INUM = 1900
C INITIALIZE
DO 503 J = 1, INUM
   503 R(J) = 0.
C DO 601 IDEST = 1, DESE T
C WORK ON THE FIRST HALF BLOCK X(1, I, S)
   R(IROW + 76, * (IDEST - 1)) = 1.
C NOW WORK ON THE OTHER HALF OF THE BLOCK X(1, I, S)
   601 R(IROW + 38, * (IDEST - 1)) = 1.
C FORMAT X(I, J, M)
   R(INDEX + 1) = -1.
   R(INDEX + 2) = -1.
   INDEX = INDEX + 2
C THE VARIABLES X(I, J, M) ARE AS FOLLOWS (FLOW ON LINK (I, J, M))
C LINK NUMBER 1, M = 1, X(I, J, M) = X1285
C LINK NUMBER 1, M = 2, X(I, J, M) = X1286
C THESE ARE X1285 THROUGH X1900
C NOW CALL THE SUBROUTINE TO GENERATE LINK FLOW DEFINING CONSTRAINTS,
C CALL GENB(R, 1900)
C 600 CONTINUE
C STOP
END
SUBROUTINE GEN(A)
 INTEGER ARCS, DESE T
 DIMENSION A(24, 76), LINE(160), ILPHA(10)
 COMMON NODES, LINKS, MTRIP(24, 24), ARCS, DESE T
 A FORMAT(1X, 80A1)
 C FORMAT(80A1)
 C ILPHA(1) = IH0
 ILPHA(2) = IH1
 ILPHA(3) = IH2
 ILPHA(4) = IH3
 ILPHA(5) = IH4
 ILPHA(6) = IH5
 ILPHA(7) = IH6
 ILPHA(8) = IH7
 ILPHA(9) = IH8
 ILPHA(10) = IH9

Figure A-6. (continued)
DO 10 P  ICOUNT=1,DEST

C DO 100 I = 1, NODES
IF (I.EQ. ICOUNT) GO TO 100
INDEX = 0

C DO 910 JJ = 1, 160

910 LINE(JJ) = 1H

C DO 101 J = 1, ARCS
IF ((INDEX.GE.70).AND. (INDEX.LE.90)) GO TO 551
GO TO 651

551 INDEX = A0

651 IF (A(I,J)) .GT. 101 + 3
1 LINE(INDEX + 1) = 1H-
4 LINE(INDEX + 2) = 1H1
INDEX = INDEX + 4 = 1H0
LINE(INDEX + 5) = 1HX
INDEX = INDEX + 5
JJ = J + (ICOUNT - 1) * ARCS

C COMPUTE NUMBER OF DIGITS IN JJ
NUMDIG = NUMFR(JJ)

C 870 DO 16 K = 1, NUMDIG
LINE(INDEX + K) = ILPHA(JJ/M + 1)
JJ = JJ - M * (JJ/M)
16 M = W/10

C INDEX = INDEX + NUMDIG
GO TO 101
3 LINE(INDEX + 1) = 1H+
GO TO 4

101 CONTINUE

C PUT IN RHS
IF ((INDEX.GE.73).AND. (INDEX.LE.90)) GO TO 752
GO TO 652

752 INDEX = A0

652 LINE(INDEX + 1) = 1H-
INDEX = INDEX + 1
JJ = TRIP(I, ICOUNT)

C THIS IS THE INTEGER PART
INCPT = 1

12700 NUMDIG = NUMFR(JJ)

C DO 17 K = 1, NUMDIG
LINE(INDEX + K) = ILPHA(JJ/M + 1)
JJ = JJ - M*(JJ/M)
17 M = W/10
INDEX = INDEX + NUMDIG
GO TO (10000, 11000), INCPT

C FINISHED WITH THE INTEGER PART

11500 LINE(INDEX + 1) = 1H-
INDEX = INDEX + 1
INCPT = 2
JJ = INT((TRIP(I, ICOUNT) - FLOAT(INT((TRIP(I, ICOUNT)))))*1000.)
GO TO 12000

C II = 1, 80
PUNCH 5*(LINE[I] + II = 1, 80)
PRINT 6*(LINE[I] + II = 1, 80)
IF (LINE(R1), F0, 1H ) GO TO 100
PUNCH 5*(LINE[I] + II = R1, 160)
PRINT 6*(LINE[I] + II = R1, 160)
100 CONTINUE

Figure A-6. (continued)
C CONTINUE
C
RETURN.
END

SUBROUTINE GENR(B,KK)

INTEGER ARCS,DEST
DIMENSION B(1900), LINE(560), ILPHA(10)
COMMON NODES, LINKS, M, TRIP(24,24), ARCS, DEST

6 FORMAT (1x,80A1)
5 FORMAT (A01)

ILPHA(1) = 1H0
ILPHA(2) = 1H1
ILPHA(3) = 1H2
ILPHA(4) = 1H3
ILPHA(5) = 1H4
ILPHA(6) = 1H5
ILPHA(7) = 1H6
ILPHA(8) = 1H7
ILPHA(9) = 1H8
ILPHA(10) = 1H9

910 LINE(I) = 1H

INDEX = 0

DO 911 J=1,KK
IFI(INDEX,GE,71).AND.(INDEX,LE,90) GO TO 664
IFI(INDEX,GE,151).AND.(INDEX,LE,160) GO TO 665
IFI(INDEX,GE,231).AND.(INDEX,LE,240) GO TO 666
IFI(INDEX,GE,311).AND.(INDEX,LE,320) GO TO 667
IFI(INDEX,GE,391).AND.(INDEX,LE,400) GO TO 668
IFI(INDEX,GE,471).AND.(INDEX,LE,480) GO TO 669
GO TO 200

664 INDEX = 80
GO TO 200

665 INDEX = 160
GO TO 200

666 INDEX = 240
GO TO 200

667 INDEX = 320
GO TO 200

668 INDEX = 400
GO TO 200

669 INDEX = 480

200 IF (M(JJ)) 1,911,3
1 LINE(INDEX+1) = 1H-
4 LINE(INDEX+2) = 1H1
LINE(INDEX+3) = 1H-
LINE(INDEX+4) = 1H0
LINE(INDEX+5) = 1H1
INDEX=INDEX+5
J=J

C COMPUTE NUMBER OF DIGITS IN JJ
NUMDIG = NUMBER(JJ)

C

RAA DO 16 K = 1,NUMDIG
LINE(INDEX+K) =ILPHA(JJ/M)*11
J=JJ -M* (JJ/M)
16 M = M/10

C
INDEX = INDEX + NUMDIG
GO TO 911

Figure A-6. (continued)
A-17
3 LINE(INDEX + 1) = 1H
GO TO 4

911 CONTINUE

C IF((INDEX .GE. 71) .AND. (INDEX .LE. 80)) GO TO 964
IF((INDEX .GE. 151) .AND. (INDEX .LE. 160)) GO TO 965
IF((INDEX .GE. 231) .AND. (INDEX .LE. 240)) GO TO 966
IF((INDEX .GE. 311) .AND. (INDEX .LE. 320)) GO TO 967
IF((INDEX .GE. 391) .AND. (INDEX .LE. 400)) GO TO 968
IF((INDEX .GE. 471) .AND. (INDEX .LE. 480)) GO TO 969
GO TO 300

964 INDEX = 80
GO TO 300

965 INDEX = 160
GO TO 300

966 INDEX = 240
GO TO 300

967 INDEX = 320
GO TO 300

968 INDEX = 400
GO TO 300

969 INDEX = 480
GO TO 300

300 LINE(INDEX+1)=1H
LINE(INDEX+2) = 1H0
LINE(INDEX+3) = 1H.

C

11700 PUNCH 5*(LINE(II), II = 1,80)
PRINT 6*(LINE(II), II = 1,80)
IF(LINE(11),F0,1H ) GO TO 100
PUNCH 5*(LINE(II),II=1,160)
PRINT 6*(LINE(II),II=1,160)
IF(LINE(161),F0,1H ) GO TO 100
PUNCH 5*(LINE(II),II = 161,240)
PRINT 6*(LINE(II),II = 161,240)
IF(LINE(241),F0,1H ) GO TO 100
PUNCH 5*(LINE(II),II = 241,320)
PRINT 6*(LINE(II),II = 241,320)
IF(LINE(321),F0,1H ) GO TO 100
PUNCH 5*(LINE(II),II = 321,400)
PRINT 6*(LINE(II),II = 321,400)
IF(LINE(401),F0,1H ) GO TO 100
PUNCH 5*(LINE(II),II = 401,480)
PRINT 6*(LINE(II),II = 401,480)
IF(LINE(481),F0,1H ) GO TO 100
PUNCH 5*(LINE(II),II = 481,560)
PRINT 6*(LINE(II),II = 481,560)

Figure A-6. (continued)
C
100 RETURN
END
FUNCTION NUMBER(JJ)
C
COMMON NODES, LINKS, M,TRIP(24,24),ARCS,DEST
C
1 IF(JJ/1000.GT.0)GO TO 14
2 IF(JJ/100.GT.0)GO TO 13
3 IF(JJ/10.GT.0)GO TO 12
4 NUMBER = 1
5 M = 1
6 RETURN
7 14 NUMBER = 4
8 M = 10**((NUMBER-1))
9 RETURN
10 13 NUMBER = 3
11 M = 10 ** (NUMBER -1 )
12 RETURN
13 12 NUMBER = 2
14 M = 10 ** (NUMBER -1 )
C
RETURN
END
PROGRAM OBJFCN (INPUT, OUTPUT, PUNCH)
C
C PROGRAM OBJFCN PUNCHES THE OBJECTIVE FUNCTION IN THE FORMAT
C REQUIRED BY OPTIMA. THE INPUT FOR THIS PROGRAM IS THE NUMBER OF
C LINKS, NUMBER OF NODES, AND THE COSTS OF TRAVEL ON BOTH ARCS FOR
C EVERY LINK IN THE NETWORK.
C
INTEGER ARCS
C
1 FORMAT(315)
1112 FORMAT(49X,F6.5*X,F6.5)
1113 FORMAT(10X,**F6.5*X*14*,*F6.5*X*14*)
C
READ 1,LINKS,NODES,ARCS
C
IN THE EXAMPLE PROBLEM LINKS=38
C IN THE EXAMPLE PROBLEM ARCS=76
C
INDEX=ARCS*NODES
C
DO 1111 I=1,ARCS*2
  J=INDFX*I
  K=J+1
  READ 1112,SLOPE1,SLOPE2
  PUNCH 1113,SLOPE1,J,SLOPE2,K
C
1111 CONTINUE
C
STOP
END

Figure A-7. Program Objfcn
PROGRAM CAPTY(INPUT,OUTPUT,PUNCH)

1. NUMBER OF LINKS, ARCS AND NODES IN THE NETWORK.
2. CAPACITIES OF BOTH ARCS FOR EVERY LINK.

THE OUTPUT OF THE PROGRAM IS THE PUNCHED CARDS OF CAPACITY CONSTRAINTS
ACCORDING TO THE OPTIMAL INPUT REQUIREMENTS (ONE CARD FOR EVERY ARC).

DATA FILE

INTEGER ARCS

8 FORMAT(*1,0*E14.8-0.75*E14.8*LE*F4.1)
9 FORMAT(*1,0*E14.8-0.25*E14.8*LE*F4.1)
4 FORMAT(*1,0*E14.8-0.75*E14.8*LE*F4.1)
5 FORMAT(*1,0*E14.8-0.25*E14.8*LE*F4.1)
6 FORMAT(315)
15 FORMAT(2F6.2)

READ 6, LINKS, ARCS, NODES
LAT = ARCS * NODES + 2 * LINKS

LINKS = 38

ARCS = 76

INDEX = ARCS * NODES
DO 1 I = 1, ARCS + 2
READ 15, CAPM1, CAPM2
MAT = INDEX * I
LAT = LAT + 1
PUNCH 2 * MAT * LAT * CAPM1
PRINT 4 * MAT * LAT * CAPM1
MAT = MAT + 1
PUNCH 3 * MAT * LAT * CAPM2
PRINT 5 * MAT * LAT * CAPM2
1 CONTINUE

STOP
END

Figure A-8. Program Capty
number of links, format - (I5)

One card for each Link, format (I2, Ix, I2, Ix, F5.4, Ix, F4.1)

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<th>Node 2</th>
<th>A</th>
<th>PCAP</th>
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Figure A-9. Input Data For Program AB
38

no. of links, format (I5)

One card for each link,

format (I2, L2, I2, L2, F4.5, L2, F13.12, L2, F4.1, L2, F4.1)

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<th>Node 2</th>
<th>A</th>
<th>B</th>
<th>Pract. max. cap.</th>
<th>cap.</th>
</tr>
</thead>
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Figure A-10. Output of Program AB, Input to Program Break
One card per Link, format

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Figure A-11. Output of Program Break, Input to Program Link
One card for each Arc, format

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Figure A-12 (continued)
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38 24  76

One card for each link, format (49X, F6.5, 4X, F6.5)

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Figure A-13. Input to Program Objfcn
Figure A-14. Identification Indices for Linear Programming Formulation

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Example Problem Solution

The problem solved for the purposes of this example is identical to the problem formulated in Chapter 4.

The input data is generated through the previously discussed preprocessor programs, in the format required by OPTIMA, a general linear programming code developed by Control Data Corporation.

The formulation of the problem requires the definition of 78 variables as given in Table A-14. X1 thru X60 correspond to the $x_{ij}$ variables representing the destination specific-arc flow. Each of the twelve arcs has five variables of this type associated with it. X61 thru X72 are the individual link-segment flows, $x_{ij}^m$. X73 thru X78 represent the link improvements or increased link capacity, $k_{ij}^m$.

Figure A-15 presents the data in the form required by the linear program. The objective function in this problem is expressed so as to minimize the total travel time on the network, so the coefficients represent the total travel time in hours per thousand vehicles for each segment and the $x_{ij}^m$ variables the flow assigned to that segment.

The six traffic assignment constraints force the sum of the destination specific arc flows for each link to equal the sum of the associated link segment flows. A constraint of this type is required for each link.

There are twenty node-flow constraints, one for each of the twenty elements in the demand matrix. The first constraint is for flow generated at node 2 destined for node 1, with X1 and X3 representing flow out of node 2 destined for node 1 ($X_{21}$ and $X_{23}$), X7 and X9 the flow into node 2 destined for node 1 ($X_{12}$ and $X_{32}$), and the right hand side of the
equation, 7.6875, the demand in thousands of vehicles from node 2 to node 1.

A capacity constraint is necessary for each segment in the same form as discussed in Chapter 4. The capacity is expressed in thousands of vehicles per day for this problem.

The budget constraint represents the cost in dollars $10^5$ to increase the total capacity of each link by 1000 vehicles per day. The budget amount, or right hand side, is in dollars $10^5$, or in this case, five million dollars. The problem was solved in the CDC 6400 computer at Northwestern University requiring approximately eighteen seconds for solution.

Figure A-16 is a listing of the initial constraint-related output relating to the optimal solution. "Basic", printed in the Logical Indicator column, indicates that the optimum solution contains a program-generated slack variable (L-value) for that constraint. In this case, 17.248 is the value of the objective function (total travel time on the network in thousands of hours). The L-values for rows 29, 30, 36, and 38 define the unused capacity on link 2, segment 1; link 2, segment 2; link 5, segment 2; and link 6, segment 2, respectively.

The values of the dual variables, shown in the PI column, indicate the change in the objective function, initially at 17,248 hours, which would result from an increase of 1.0 in the right hand side of the associated constraint. For example, increasing the travel demand from node 2 to node 1 (Row 7) by one unit (up 1000 vehicles to 8,685 thousand) would increase the network travel time by 1738 hours, or 1.738 hours per additional vehicle.
Increasing the budget by $100,000 would lower the total travel-time by 241 hours for that time period. Thus, for this solution to the problem, the marginal cost of reducing travel time by one hour is $419 (annually) per daily hour saved.

The values of the defined variables are shown in Figure A-17. Again, "basic" in the structural indicator column means the variable is part of the optimal solution. The DJ column represents the "trade-off" or cost associated with forcing that particular variable into solution. When applied to X7, this would mean that the total network travel time would increase by 3,476 hours if traffic to node 1 were forced onto the southbound lane of link 3. Looking at variable X78, an increase in total network travel time of 1,128 hours would result if the capacity of link 6 were increased by 1000 vehicles through a reallocation of resources.

This is the output of the basic model. The planner or decision maker, however, may wish to further check the range of the solution subject to changes in the input data. This may be accomplished through the use of ranging analysis, shown in Figure A-18, applied to the right-hand sides of the constraints.

The budget constraint has an associated dual value of .2407 at the 5 million dollar value. As shown in the Figure, if all other inputs remain the same, the budget limit could vary from 4.98 million to 5.13 million dollars, without changing the optimal solution defined at the $5 million level. Similar interpretations can be developed for the remainder of the constraint right-hand sides.
C OBJECTIVE FUNCTION TO MINIMIZE TRAVEL TIME ON THE NETWORK

COST \[ 1.713X61 + 0.0530X62 \]
\[ 0.003X61 + 0.746X64 \]
\[ 0.042X64 + 3.9627X66 \]
\[ 0.05714X67 + 3.7667X66 \]
\[ 10.00X61 + 0.6774X70 \]
\[ 0.2163X71 + 1.3927X72 + 0X78 \]

CONSTRAINTS

C ASSIGNMENT OF TRAFFIC CONSTRAINTS
\[ x_1 + x_7 + x_{13} + x_{19} + x_{25} + x_{31} + x_{37} + x_{43} + x_{49} + x_{55} - x_{61} - x_{62} = 0 \]
\[ x_1 + x_8 + x_{14} + x_{20} + x_{26} + x_{32} + x_{38} + x_{44} + x_{50} + x_{56} - x_{62} = 0 \]
\[ x_1 + x_9 + x_{15} + x_{21} + x_{27} + x_{33} + x_{39} + x_{45} + x_{51} + x_{57} - x_{63} - x_{64} = 0 \]
\[ x_1 + x_{10} + x_{16} + x_{22} + x_{28} + x_{34} + x_{40} + x_{46} + x_{52} + x_{58} - x_{66} = 0 \]
\[ x_1 + x_{11} + x_{17} + x_{23} + x_{29} + x_{35} + x_{41} + x_{47} + x_{53} + x_{59} - x_{67} - x_{68} = 0 \]
\[ x_1 + x_{12} + x_{18} + x_{24} + x_{30} + x_{36} + x_{42} + x_{48} + x_{54} + x_{60} - x_{71} + x_{72} = 0 \]

C DEMAND OF NODE-FLOW CONSTRAINTS
\[ x_1 + x_3 + x_7 - x_9 = 7.685 \]
\[ x_3 + x_4 + x_6 + x_7 + 1.10 - 1.12 = 5.517 \]
\[ x_7 + x_4 + x_5 + x_8 + 1.10 - 1.11 = 6.244 \]
\[ x_5 + x_6 + x_11 + 1.12 = 1.333 \]
\[ x_1 + x_4 + x_5 + x_9 + 1.10 + 1.22 = 3.229 \]
\[ x_1 + x_6 + 1.16 + 1.17 + 1.18 + 1.22 = 3.429 \]
\[ x_1 + x_7 - x_23 - 1.24 + 1.29 \]
\[ x_1 + x_5 + x_26 + x_31 - 1.32 = 4.516 \]
\[ x_1 + x_5 + x_27 - 1.31 - 1.33 = 6.218 \]
\[ x_1 + x_24 + 1.28 + 1.29 - 1.34 - 1.35 = 1.704 \]
\[ x_1 + x_3 + x_30 + 1.35 - 1.35 = 3.200 \]
\[ x_1 + x_37 + 1.36 + 1.37 = 4.222 \]
\[ x_1 + x_37 + 1.36 + 1.37 = 4.14 \]
\[ x_1 + x_39 + 1.40 - 1.42 + 1.42 = 1.711 \]
\[ x_1 + x_41 + 1.42 + 1.42 = 5.817 \]
\[ x_1 + x_49 + 1.50 + 1.51 = 7.000 \]
\[ x_1 + x_49 + 1.51 + 1.52 = 1.12 \]
\[ x_1 + x_51 + x_52 + x_53 + 1.55 = 2.217 \]
\[ x_1 + x_51 + x_52 + x_53 + 1.55 = 0.858 \]

C CAPACITY CONSTRAINTS

X61 = \[ 7.5X73 \text{ LE } 15 \]
X62 = \[ 2.5X73 \text{ LE } 5 \]
X63 = \[ 7.5X74 \text{ LE } 40 \]
X64 = \[ 2.5X74 \text{ LE } 12 \]
X65 = \[ 7.5X75 \text{ LE } 15 \]
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X67 = \[ 7.5X76 \text{ LE } 15 \]
X68 = \[ 2.5X76 \text{ LE } 5 \]
X69 = \[ 7.5X77 \text{ LE } 15 \]
X71 = \[ 2.5X77 \text{ LE } 5 \]
X71 = \[ 7.5X78 \text{ LE } 15 \]
X72 = \[ 2.5X78 \text{ LE } 5 \]

C BUDGET CONSTRAINT
\[ 6.064X73 + 2.52X74 + 3.762X75 + 3.476X76 + 1.264X77 + 5.056X78 \text{ LE } 50 \]

Figure A-15. Problem Formulation

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Parentheses on L or X=VALUE denote infeasibilities. On PI or DJ, non-optimalities.

Figure A-17. (continued)

APPENDIX B

EVALUATION OF TRAFFIC ASSIGNMENTS IN THE NETWORK DESIGN MODEL

Introduction

In this appendix we will compare the modeling of flow on networks in such a manner that total cost is minimized with the modeling of flow so that an equilibrium is achieved. An analysis and comparison of the output from these two models applied to the Sioux Falls network (described in Appendix A) is presented. This analysis justifies the use of a simplifying assumption in the network design problem regarding the assignment of traffic, namely that total user cost is minimized, which makes the model significantly more tractable from a computational point of view.

One of the most common behavioral assumptions made in urban transportation planning is that each vehicle on a highway network will take the path of least resistance between his origin and destination. It is known that travel time is a significant factor to the majority of travelers when they choose their routes in the network, and most urban studies have utilized travel time as the basic measure of travel, (Comsis, 1972)

The assumption that each driver takes the path of least resistance gives rise to the concept of network equilibrium. A set of flows along the arcs of a network is said to be at equilibrium (Wardrop, 1952) if the following two conditions are satisfied for every origin-destination pair, r-s:

1. If two or more routes between node r and node s are actually travelled, then the cost to each traveler between r and s must be the same for each of these routes.

2. There does not exist an alternative unused route between nodes r and s with less cost than that of the routes which are traveled.
The assumption is made that each user of the network seeks to minimize his own travel cost, and that he experiments with different routes, eventually finding the least cost one. Although this may not be completely true in reality, it is assumed that those drivers using more costly routes constitute a negligible portion of the total. It is clear that if (1) or (2) were not true, some drivers would switch to the cheaper routes, congesting them, and causing a new flow pattern to evolve. An equilibrium is the aggregate result of individual decisions; at an equilibrium, no single driver can reduce his own cost by choosing an alternative route in the network.

The two equilibrium conditions above are equivalent to Wardrop's first principle, the principle of equal travel times for all users. His second principle, that of overall minimization, leads to a different assumption of driver behavior. Wardrop's second principle states that flows are distributed over the arcs of the network in such a manner that the sum of the travel times for all users is minimized. Models based on the assumption of equilibrium are often referred to as "user-optimal" models, while models based on Wardrop's second principle are referred to as "system optimal" models.

Using the FHWA user time functions, we can find the system optimal flows on a network by solving the problem

\[
\begin{align*}
\text{minimize} & \quad \sum_{ij \in A} (A_{ij} x_{ij} + B_{ij} x_{ij}^5) \\
\text{subject to} & \quad x_{ij} = \sum_s x_{ij}^s, \quad \forall \ ij \\
& \quad \sum_k x_{jk} - \sum_i x_{ij} = \phi_j - D_j, \quad \forall \ j \\
& \quad x_{ij} \geq 0, x_{ij}^s \geq 0
\end{align*}
\]
As is shown in Beckman (1956), the equilibrium flows can be found by solving the problem

$$\text{minimize} \sum_{ij \in A} \left[ A_{ij} x_{ij} + \frac{B_{ij}}{5} (x_{ij})^5 \right]$$

subject to the constraints (B-2), (B-3), (B-4).

Now the only difference in the problems is in the coefficient of the second term for each link in objective function. Therefore it is intuitive that the equilibrium flows will not be significantly different from the system optimal flows; this is borne out in the comparison which follows.

An important characteristic of the network design model is that the flow is distributed over the arcs of the network in such a manner that the linearized total user cost is minimized. Although the assumption of equilibrium is more common in the literature, a study of the Sioux Falls network indicated that the difference between the system optimal flows and the equilibrium flows is not significant. In (LeBlanc, 1973), the Frank-Wolfe algorithm is presented as a very efficient technique for finding the equilibrium flows and the system optimal flows. In Tables B-1 through B-4 below, a detailed comparison of three sets of flows is given: the equilibrium and system optimal flows given by the Frank-Wolfe algorithm, and the system optimal flows obtained by linearizing the user cost functions. The linearization of the user cost functions is analogous to that in the core model of Chapter 4. For each of the thirty-eight arcs in the network, the table gives the type (E-expressway, A-arterial, C-CBD, F-fringe), flow, total vehicle hours and total vehicle miles for each set of flows. The units are respectively thousands of vehicles/day, thousands of vehicle hours/day, and thousands of vehicle miles/day. For example, in Tables B-1 and B-2
### Table B-1. A Comparison of Flows, Total Vehicle Hours and Total Vehicle Miles.

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Data for Table B-2. A Comparison of Flows, Total Vehicle Hours and Total Vehicle Miles.

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Expressway 3.4 191.9 4.1 225.2
CBD 15.9 122.6 15.4 121.7
Fringe 33.2 511.6 32.7 546.7

Number and Types of Congested Arcs

| Arterial | 14 |
| Expressway | 0 |
| CBD | 5 |
| Fringe | 9 |

B-4
### Table B-2 A Comparison of Flows, Total Vehicle Hours, and Total Vehicle Miles

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Totals   48.1 668.4                            48.7 641.9

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Number and Types of Congested Arcs

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B-6
Table B-4. A Comparison of Congested Arcs (More than 1300 Vehicle Hours).

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<td>3.4</td>
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<tr>
<td>12 A, C</td>
<td>9.0 1.9 14.4</td>
<td>9.4</td>
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<tr>
<td>15 A, F</td>
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<td>4</td>
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<tr>
<td>17 A, F</td>
<td>17.9 1.6 21.5</td>
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<td>1.8</td>
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<tr>
<td>20 A, F</td>
<td>11.7 2.4 17.5</td>
<td>12.0</td>
<td>2</td>
<td>2.7</td>
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<tr>
<td>21 A, F</td>
<td>10.6 1.4 10.6</td>
<td>9.9</td>
<td>6</td>
<td>1.4</td>
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<tr>
<td>24 A, F</td>
<td>13.6 2.7 12.2</td>
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<td>11</td>
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<td>11.4 2.6 12.5</td>
<td>11.5</td>
<td>0</td>
<td>2.7</td>
</tr>
</tbody>
</table>

B-7
arc 1 is an expressway on the fringe of the network. From the Frank-Wolfe algorithm applied to the equilibrium problem, we see that arc 1 has a flow of 17,400 vehicles/day at equilibrium. The system optimal flow level on arc 1 is 18,700, a 7% change from the equilibrium flow value. Using the linear approximations to the cost functions yields a flow of 17,300, a 7% change from the flow value of 18,700.

Looking again at the table, we see that there are 400 vehicle hours on arc 1 at equilibrium. The vehicle hours at the system optimal flows on arc 1 are 400, an 8% difference from the equilibrium vehicle hours (the apparent discrepancy is due to rounding in the printout). The vehicle hours on arc 1 using the linearized cost functions are 600, a 36% change from the system optimal vehicle hours. In the "Totals" row, we see that there are 49,200 vehicle hours in the network at equilibrium, 48,100 vehicle hours at system optimal flows, and 48,700 vehicle hours at the system optimal flows using linearized travel time functions. Furthermore, there are 45,700 vehicle hours at equilibrium on arterial arcs in the network and 15,900 vehicle hours in the CBD at equilibrium.

Next, the table compares vehicle miles on each arc for each set of flows. On arc 1, there are 20,900 vehicle miles/day at equilibrium, 22,400 vehicle miles at system optimal flows, and 20,800 vehicle miles using linearized cost functions. In the "Totals" row, there are 634,200 vehicle miles/day in the network at equilibrium. The number of vehicle miles on arterial streets at equilibrium is 442,300.

Finally, the number and types of congested arcs (more than 1,300 vehicle hours) are shown. For example, there are 14 congested arterial arcs and five congested CBD arcs in the network at equilibrium. In Table B-3
and B-4, these same basic comparisons are presented for each congested arc in the network.

Observe that total vehicle hours and miles in the network using linearized cost functions differ by only 1% from the equilibrium values. The difference in vehicle hours and miles in the CBD for the two algorithms is 2 or 3%, and the average difference in flows along the arcs of the network from the three algorithms is insignificant. At the bottom of Table B-1 and B-2, we see that the number of congested arcs at equilibrium is approximately the same as the number of congested arcs when we minimize the linear user costs. In conclusion, the assumption that flows will be distributed over the arcs of the network such that the linearized user costs are minimized yields results very close to those of the assumption of equilibrium.

The network design model can be easily generalized to include limitations on air and noise pollution and general impact constraints such as restriction on traffic flows through certain neighborhoods or the Central Business District. Including capacities on the flow level on specified arcs in the network might result in more dispersed traffic and thus reduce the concentration of pollutants. Alternatively, we could include a cost function of the level of pollutants in the objective function, and minimize a weighted combination of driving time plus pollution costs. However, using these more general criteria would not necessarily produce an accurate prediction of driver behavior. Undoubtedly, drivers would have to be forced to divert from their preferred routes by use of tolls, signals, etc.
References


APPENDIX C

NOISE ESTIMATING RELATIONSHIPS

FOR USE IN THE

DESIGN OF URBAN HIGHWAY NETWORKS

Introduction

The impact of traffic noise upon the urban environment is becoming increasingly more important as a design criterion for transportation facilities. In particular, decisions concerning the location, types of facilities to be constructed, and design characteristics of those facilities must be made with appropriate consideration given to the effects of noise upon adjoining land uses. Recently promulgated Federal Highway Administration Standards specify the limiting relationships between proposed highway transportation facilities and their expected noise outputs.

In an effort to improve the systematic basis for design of entire urban transportation networks, the Department of Civil Engineering at Northwestern University is currently conducting a study for the Federal Highway Administration which attempts to apply mathematical optimization techniques to the development of good network designs. A major objective of this project is to discover effective ways in which to introduce consideration of the environmental impacts of these networks, including the effects of noise, into the design optimization process.

The objective of this report is to utilize the existing body of knowledge on highway traffic noise as a basis for considering noise impacts in the network design process. It has been well established that measured traffic noise levels are a function of such variables
as traffic volume, speed, and distance of the observer from the roadway. This report proposes a simplified methodology for utilizing these known functional relationships, along with limiting noise levels at various distances from the traffic stream, to derive traffic volume constraints for use in an optimization model. These constraints would then influence the solution to the problem of finding a network which is in some sense optimal, either by restricting flows on links or by limiting the capacity of links in order to meet noise standards.

This report provides a highway noise model applicable in this context, along with documentation of this model and its derivation, and a computer program devised to investigate noise-traffic volume constraint relationships.

Review of Existing Highway Noise Models

This section presents a summary of all noise models which were considered, including the assumptions and limitations of each model. The objective is to select the best noise model in light of the needs: simplicity, consistency, and easy incorporation into the network design model.

There are basically four models which were considered. The first model is given in (1).

(1) NCHRP No. 78, p. 7.
A. Model 1:

\[ \bar{L} = 20 - 10 \log_{10} D_E + 20 \log_{10} S + 10 \log_{10} V \]  

(C-1)

where \( \bar{L} \) = mean noise level in dBA (the noise distribution is nearly normal so that the median \( L_{50} \) is essentially equal to the mean),

\( D_E \) = distance in feet from the center of the pseudo-lane (also the single lane equivalent) to the observer,

\( S \) = average vehicle speed in mph

\( V \) = volume of the traffic (aggregated over all lanes) in vehicles per hour (vph).

The assumptions are as follows:

(a) Total flow (volume) on the highway is assumed to be a continuous line noise source with flows in excess of 1000 vph. It is not clear precisely how critical this assumption is. Below 1000 vph, however, it is likely that the assumption of a line noise source becomes poor, and that one must examine the noise output of isolated vehicles. In terms of mean noise level, isolated vehicles may produce less noise than the assumed line source, and so this assumption may be conservative.

(b) Only automobile noise (power train and pavement-tire interaction) is considered. The coefficient 20 of \( 20 \log_{10} S \) is related to the assertion that noise level varies with the square of the speed.

(c) For purposes of data gathering, noise is assumed to emanate from the single lane equivalent, approximated by the "lane" whose centerline is \( 12 (\sqrt{m} -1) \) feet away from the centerline of the lane nearest the observer, where \( m \) is the number of lanes (a 12 foot length width is assumed).
The limitations of this model are:

(a) It is unclear how total flow \( V \) is related to the separate flows per lane. For example, whether the total aggregate flow is the simple sum of all lane flows or a weighted sum of lane flows is not known. It is assumed, because of lack of further documentation, that the former is true. Furthermore, the precise meaning of average vehicle speed is also unclear. It is assumed that it is the statistical mean of the speeds of all autos in all lanes.

(b) It is unclear how the coefficients of each term in the above expression were obtained. If they were, in fact, obtained empirically, then the database from which they were determined differs from those used to calibrate the subsequent models. It is most likely that they were obtained theoretically, with the term 20 obtained by empirical means.

An addition to the above model can be obtained by considering the effect of trucks in the traffic flow. Toward that end, one can use Table 1 of the same report (2).

B. Model 2.

This model is taken from (3).

\[
L_{50} = 10 \log_{10} V - 15 \log_{10} D_e + 30 \log_{10} S + 10 \log_{10} \left[ \tanh \left( 1.19 \times 10^{-3} \frac{VD_e}{S} \right) \right] + 29
\]  

(C-2)

where \( V \) and \( S \) are as defined in A,

(2) MCHRP No. 78, p. 9

(3) MCHRP No. 117, p. 9, equation 20.
Table C-1.

Effect Of Adding Trucks To The Vehicle Mix

<table>
<thead>
<tr>
<th>% trucks in traffic stream</th>
<th>Additional dBA</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
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<tr>
<td>2.5</td>
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<td>5</td>
<td>2</td>
</tr>
<tr>
<td>10</td>
<td>4</td>
</tr>
<tr>
<td>20</td>
<td>8</td>
</tr>
</tbody>
</table>
\( D_e = \) distance from the observer to the edge of the nearest lane to the observer,

\( L_{50} = \) median noise level of autos (nearly equal to the mean).

The assumptions here are as follows:

(a) automobiles only,

(b) the distance from the observer to the edge of the nearest lane is the determining distance,

(c) the coefficient of \(30 \log_{10} S\) is 30 instead of 20 as in model 1, related to the assertion that noise level varies according to the cube of the speed rather than the square.

Limitations:

(a) It is unclear in this model, as in model 1, how one interprets aggregate volume, whether as a simple or weighted sum. Neither is it obvious how to interpret average speed.

The model is an attempt to improve upon model 1 and to glean the best results from a traffic noise simulation model documented in (4). One can account for trucks in this model by using the following formula:

\[
L_{50} = 10 \log_{10} V - 10 \log_{10} S - 15 \log_{10} D_e + 10 \log_{10} \left[ \tanh \left( 1.19 \times 10^{-3} \frac{VD_e}{S} \right) \right] + 95
\]

where \( V, S \) and \( D_e \) are as defined in B.

\( L_{50} = \) median noise level for trucks.

Notice that this relationship for trucks shows decreasing noise level with increasing speed. The authors in (4) point out that this

\((4)\) NCHRP No. 117, p. 6
relationship has not been verified empirically. In addition, one finds the computation forbiddingly difficult when one combines the two equations (C-2) and (C-3):

\[
\frac{L_{50a}}{10} = \frac{V_a S^3 \tanh (1.19 \times 10^{-3} \frac{V_{ae}}{S})}{D_e^{1.5}} 10^{2.9} \tag{C-4}
\]

\[
\frac{L_{50t}}{10} = \frac{V_t \tanh (1.19 \times 10^{-3} \frac{V_{De}}{S})}{S \cdot D_e^{1.5}} 10^{9.5} \tag{C-5}
\]

where \( L_{50a} \) = median noise level for autos in dBA

\( L_{50t} \) = median noise level for trucks in dBA

\( V_a \) = volume of autos in vph

\( V_t \) = volume of trucks in vph

A combined value, \( \bar{L} \), can be defined where:

\[
\frac{\bar{L}}{10} = \frac{L_{50a}}{10} + \frac{L_{50t}}{10} = \frac{V_a S^3 \tanh (1.19 \times 10^{-3} \frac{V_{ae}}{S})}{D_e^{1.5}} 10^{2.9} \]

\[
+ \frac{V_t \tanh (1.19 \times 10^{-3} \frac{V_{De}}{S})}{S \cdot D_e^{1.5}} 10^{9.5} \tag{C-6}
\]

for fixed \( D_e \).

One solves then for \( \bar{L} \) to get the resulting noise level in dBA.
C. Model 3.

This model is based upon the work of Johnson and Saunders (5).

While this model is the best documented of all four models and serves as a benchmark for model 2, it is based on data taken from rural British roadways.

\[ L_{50} = 3.5 + 10 \log_{10} V - 10 \log_{10} D_N + 30 \log_{10} S \]  \hspace{1cm} (C-7)

where \( V, S, \) and \( L_{50} \) are defined in A and B but

\[ D_N = \text{distance in feet from the observer to the center of the nearest lane to the observer.} \]

The assumptions are:

(a) The distance from the observer to the centerline of the nearest lane is the determining distance,

(b) Traffic in both streams is aggregated according to the following rule:

\[ V_T = V_1 + \left( \frac{d_1}{d_2} \right) \left( \frac{S_2}{S_1} \right)^3 V_2 \]  \hspace{1cm} (C-8)

where \( V_T = \text{total flow in vph} \)

\[ V_1 = \text{aggregated flow on the nearside stream} \]

\[ V_2 = \text{aggregated flow on the farside stream} \]

\[ d_1 = \text{distance from the observer to the edge of the nearest lane in feet} \]

\[ d_2 = \text{distance from the observer to the nearest edge of the farthest lane in feet} \]

\( S_1 \) = average vehicle speed in mph on the nearside stream
\( S_2 \) = average vehicle speed on mph on the farside stream

(c) There is a 20% mix of "heavy vehicles" in the traffic.

Notice that the noise level is dependent upon the cube of the speed as in model 2, rather than the square of the speed in model 1. It is unclear from reference (5) what \( S \) actually means. Most likely it is the statistical mean of all speeds of all autos and trucks in all lanes.

D. Model 4.

This model is given in (6), but parts of it are taken directly from (1) and (3).

\[
\bar{L}_a = 20 - 10 \log_{10} D + 20 \log_{10} S + 10 \log_{10} V \tag{C-9}
\]

where \( \bar{L}_a \) = mean noise level from autos in dBA
\( D \) = distance from the stream of vehicles in feet
\( S \) = average vehicle speed in mph
\( V \) = traffic volume in vph.

The assumptions here are apparently the same as in model 1. There is an equation for mean truck noise \( \bar{L}_t \), given by

\[
\bar{L}_t = 117 - 20 \log_{10} D \tag{C-10}
\]

which is not very meaningful in light of the statement accompanying equation 9: "...diesel trucks are occasionally in the traffic stream..."

(6) "Methods of Evaluation of the Effects of Transportation Systems on Community Values" by the Stanford Research Institute, April 1971, prepared for HUD, Contract H-1122, SRI Project MU-8493, p. 204.
There is apparently a difference of opinion on the definition of D. The SRI report defines D as given above, but cites model 1 as the source, in which D is defined differently.

This model is so poorly documented that it seems impossible to discern whether equation 10 is derived empirically or theoretically. A further difficulty arises in attempting to justify the relationship given in (7).

By rearranging terms in equations 9 and 10, one finds

\[ \frac{\bar{L}}{10} = 2 + \log_{10} \frac{S^2V}{D} \]  

(C-11)

from which one gets

\[ \frac{\bar{L}}{10} = \frac{100 S^2V}{D} \]  

(C-12)

Also

\[ \frac{\bar{L}}{10} = 11.7 - \log_{10} D^2 \]  

(C-13)

from which

\[ \frac{\bar{L}}{10^{10}} = \frac{10^{12}}{D^2} \]  

(C-14)

where "\approx" means "is nearly equal to".

Therefore, if \( L \) is defined as the cumulative noise level, one finds

\[ \frac{L}{10^{10}} = \frac{\bar{L}}{10^{10}} + \frac{\bar{L}}{10^{10}} + \frac{100 S^2V}{D} + \frac{10^{12}}{D^2} \]  

(C-15)

(7) SRI report, p. 210
from which
\[ D^2 C - 100 S^2 VD - 10^{12} = 0 \] (C-16)
where \( C = 10^{L/10} \). The only real solution to this equation is
\[ D = \frac{100 S^2 V + \sqrt{10^4 S^4 V^2 + 4C \cdot 10^{12}}}{2C} \] (C-17)
which differs from the result on p. 210 of (6) (the term is \( 4C \cdot 10^{12} \) instead of \( 2C \cdot 10^{12} \)). Furthermore, it is moot whether the combination of the two expressions was valid. The variable \( V \) refers to the aggregate volume (trucks and cars) and by combining the two relationships, it appears that one assumes that there are 100% autos in the stream with only an "occasional" truck. In short, the model given above is open to many criticisms and will not be used.

Difficulties Inherent in the Models

There are a number of questionable aspects of each model, not the least of which is inconsistency in the documentation. For example, let us consider the definition of a single lane equivalent. We find contradictory definitions in the body of the same report (8), and we find even another one at variance with these in (9). The intent is to provide an approximation to the location on the highway which one may assume to be the acoustical source of all noise on the highway.

The following relationship was derived in order to arrive at an acceptable definition. It is assumed initially that the distance \( D_E - D_N \)

(8) NCHRP No. 78, pp. 7 and 13.
(9) NCHRP No. 117, p. 11.
from the single lane equivalent to the centerline of the nearest lane is given by

\[ D_E - D_N = 12 \left( \sqrt{m - 1} \right) \text{ (feet)} \]  \hspace{1cm} (C-18)

where \( m \) = the number of lanes. See the accompanying diagram for an explanation. It is assumed that the lanes are 12' wide and there are \( m \) of them. One assumes also that the distance \( D - D_N \) from the centerline of the highway to the centerline of the nearest lane is

\[ D - D_N = \frac{m}{2} \cdot 12 - 6 + \frac{W}{2} = 6(m - 1) + \frac{W}{2} \]  \hspace{1cm} (C-19)

where \( W \) = width of the highway median (includes the inside shoulders).

When Equations (C-18) and (C-19) are combined, one gets

\[ D - D_E = 6m - 6 + \frac{W}{2} - 12(\sqrt{m - 1}) = 6(m + 1) - 12\sqrt{m + \frac{W}{2}} \]  \hspace{1cm} (C-20)

It is important that \( D \) and \( D_E \) aren't confused. For instance, if \( D = 50' \), \( m = 4 \), \( W = 40' \), then \( D - D_E = 26' \) and \( D_E = 24' \). The difference in noise resulting from confusing these two in Equation 1 can be found by considering

\[ 10 \left[ \log_{10} 50 - \log_{10} 24 \right] = 10 \log_{10} \left( \frac{50}{24} \right) \approx 3.2 \text{ dBA.} \]

This difference is large enough to encourage the user to be careful in specification of approximate distances.

Further difficulties arise when one begins to ask what volume \( V \) really means. Does it, for example, really mean aggregate volume from both directions and from all lanes or does it imply some weighting before
Figure C-1.

Schematic Representation

Of Observer And Highway
aggregation? The Johnson-Saunders model 3 indicates a method in Equation 8, but the other references do not.

Finally, all the references cited fail to mention the speed-volume relationship which is so crucial to traffic flow theory. It is apparently assumed that speed and volume are independent over the ranges considered, yet both empirical and theoretical studies show that speed and volume are functionally related. Therefore, a quantitative speed volume relationship has been introduced into the noise model selected for further use. Two such relationships have been applied, one derived empirically in (10) based on urban expressway flow and one established heuristically for use in traffic assignment models (11).

Suggested Model

Model 1 has been selected for use because of its simplicity and the apparent close agreement with model 3 (12). The other models are not, for reasons enumerated below, as easily amenable to the kinds of applications anticipated in the development of network design models. More specifically, model 3 is most attractive because of its good documentation and internal consistency, yet is still a model of noise from rural British roadways. Differences in vehicle types and driver behavior may limit the applicability of model 3 to U.S. cities. Furthermore, the effect of diesel trucks in the traffic stream cannot be predicted with this model. Model 2 is more sophisticated than model 1 but the combination of auto-truck noise in Equation (C-6) is too cumbersome for easy


(12) NCHRP No. 117, p. 4.
computation. Model 4 is so poorly documented that consideration was not even given to its use.

Application of this model for highway noise level estimation must be made for short time periods only, on the order of one hour or less. This is because the traffic stream flow relationships, on which the model is built, become meaningless when one attempts to aggregate them over longer time periods, and because the same problem arises with efforts to aggregate noise levels over time. Within a short time period, traffic volumes, and thus noise levels, can be expected to vary over only a relatively small range, and it becomes meaningful to talk about average flows and average noise levels. Over longer time periods, the variation increases and the concept of the average value loses its meaning. Initially, this model will be applied to peak period flows, during which the noise levels can be expected to be the highest observed over the day. Of course, sensitivity to noise of adjacent land uses may be much higher during off-peak hours of the day. Modifying this model to treat off-peak flows is a simple matter. For use in a network design model where overall flows are described on a 24-hour basis, it would even be possible to write some link noise constraints for the peak periods, and others for off-peak periods. This assumes, of course, that appropriate hourly volume factors are available for relating 24-hour flow to the flow during any particular hour of the day.

The initial noise model will not include gradient, obstruction and element effects but will be a level, on-grade model including the effects of truck noise plus the effects (if any) of the number of lanes. Effects
of curves and grades requires a level of precision greater than that which is compatible with other elements of the network design model. Effects of elevated and depressed highways probably should be considered at a later time, and should be relatively simple, since such design details seem only to result in simple, constant shifts in noise outputs of facilities.

Because the available models operate on aggregated volumes on each side of the roadway, this effort shall not be concerned with the lane distribution of volumes on the same side of a highway. As will be shown later, the error introduced by ignoring lane distribution of volumes is probably acceptably small.

When the model under consideration was devised, it was apparently calibrated for a small number of lanes with a small median. Therefore, a correction factor needs to be applied so that, for instance, one could adequately model the noise from a large highway (8 lanes with a wide median). Specifically, when one considers Figure 5 of (13), the following table may be extrapolated. The entries represent number of dBA to be subtracted from the noise level output from the model for a given combination of distance \( D_N \), and equivalent number of lanes \( n \) (which includes the width \( W \) of the median).

For the purposes of this project, the equivalent number of lanes, \( n \), will be around 8 or less and in any case, \( D_N \) will normally be in excess of 100 feet. Thus the required correction is small, and will not be considered. In cases where highways having more than 8 lanes are to be evaluated, it will be necessary to apply this correction factor externally.

(13) NCHRP No. 117, p. 12.
Table C-2.

Noise Level Corrections

<table>
<thead>
<tr>
<th>Dn</th>
<th>4</th>
<th>8</th>
<th>16</th>
<th>32</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>2</td>
<td>5</td>
<td>8</td>
<td>10</td>
</tr>
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</table>
Added to Equation (C-1) is the term \( L_t \), the additional dBA due to trucks in the traffic stream, obtainable from Table 1. Therefore, the estimating relationship becomes

\[
\overline{L} = 10 \log_{10} V + 20 \log_{10} S - 10 \log_{10} D_E + 20 + L_t \quad (C-21)
\]

where \( \overline{L} \) = mean (or median, \( L_{50} \)) noise level of all traffic in dBA,

\( V \) = aggregated volume in vph,

\( S \) = average vehicle speed in mph,

\( D_E \) = distance from the observer to the centerline of the single lane equivalent.

\( D_E \) is defined by Equation (C-20), and rewritten, it becomes

\[
D_E = D - 6(m + 1) + 12\sqrt{m} - \frac{W}{2} \quad (C-22)
\]

where \( D \) = distance in feet from the observer to the centerline of the roadway,

\( m \) = number of lanes,

\( W \) = median width in feet.

To reflect the effect of speed and volume interaction, the simple linear speed-density relationship proposed by Greenshields and fitted to urban expressway data in (10) was utilized:

\[
S = 58.6 - 0.468 \rho \quad (C-23)
\]

\[
\frac{V}{m} = 126.02 S - 2.15 S^2 \quad (C-24)
\]

where \( \rho \) = density of traffic in vehicles per mile per lane

\( V \) = aggregate volume in vph

\( S \) = average vehicle speed in mph.
The coefficients in Equations (C-23) and (C-24) were obtained from peak hour data on an urban expressway using single lane volumes and speeds. Notice that Equation (C-24) applies this relationship to the **average** volume per lane to the average vehicle speed. When one plots Equation (C-24) one obtains a parabola as in Figure (C-2). If in application it becomes appropriate to utilize the Greenshields speed-volume relationship calibrated for other types of facilities, adoption of this function becomes simple since the parabolic speed-volume curve requires only two data points for determination of its parameters. One might, for example, measure volume and average vehicle speed during an off-peak hour when traffic is light and during a period of heavier flow at another time of day. Then suppose one assumes the following functional relationship:

\[
\frac{V}{m} = \alpha S + \beta S^2 \tag{C-25}
\]

Suppose further the two data points are \((V_1, S_1)\) and \((V_2, S_2)\). Then

\[
\frac{V_1}{m} = \alpha S_1 + \beta S_1^2 \tag{C-26}
\]

\[
\frac{V_2}{m} = \alpha S_2^2 + \beta S_2^2
\]

Since two equations in two unknowns (\(\alpha\) and \(\beta\)) have a unique solution, the coefficients \(\alpha\) and \(\beta\) are determined and may be substituted in Equation (C-25).
Figure C-2.
Parabolic Speed-Volume Relationship

SPEED IN MPH (s)

VOLUME IN VEHICLES PER LANE-HOUR (V/m)
From Figure (C-2), it can be seen that for low volume traffic, the average vehicle speed is high, and as more vehicles are added to the traffic stream, the average speed drops to the point where volume reaches its maximum (in this case representing urban expressway flow of around 1846 vph per lane). When more vehicles are added, the situation grows worse to the point where congestion and delay increase and both speed and volume drop towards zero. Notice that it is assumed in dividing V by m that traffic is uniformly distributed over the lanes. To test the sensitivity of noise level to this assumption, consider a four lane highway with no median and assume a typical peak period imbalance as might be observed on an urban expressway. Suppose there are two lanes carrying 1500 vph per lane and two carrying 500 vph per lane. If an observer is stationed 100 feet from the dividing line between the two busiest lanes, the noise emanating from the two nearest (and busiest) lanes is 68 dBA. The noise from the two far lanes is 64 dBA. By Table 3 of (14), the difference is 4 dBA and therefore the cumulative noise level is 70 dBA. If one assumes, instead, that the aggregate volume of 4000 vph is spread uniformly over the four lanes to get an average volume of 1000 vph per lane, the total noise the observer detects is between 70 and 71 dBA. Therefore, the difference between assuming uniformity and inequality among the lanes appears to be negligible.

By considering Equation (C-24) one finds two roots \( s \), given by

\begin{equation}
(14) \quad \text{NCHRP No. 78, p. 13.}
\end{equation}
\[ S = \frac{126.02 \pm \sqrt{(126.02)^2 - 8.6 \text{ V/m}}}{4.3} \quad (C-27) \]

Note that if \( \frac{V}{m} < \frac{(126.02)^2}{8.6} = 1846 \text{ vehicles per lane} \)

then the two roots to Equation 24 are real and unequal. Furthermore, the speed \( S_1 \) always exceeds the speed \( S_2 \) where

\[ S_1 = \frac{126.02 + \sqrt{(126.02)^2 - 8.6 \text{ V/m}}}{4.3} \quad (C-28) \]

\[ S_2 = \frac{126.02 - \sqrt{(126.02)^2 - 8.6 \text{ V/m}}}{4.3} \]

In physical terms, for all feasible volumes \( V (< 1846 \text{ vph per lane}) \), two speeds are possible, one representing congested flow and one representing uncongested flow. For example if \( V/m = 1500 \text{ vph per lane} \) then \( S = 16 \text{ or } 42 \text{ mph} \). It is obvious that the combination \( (V, S_1) \) has a higher associated noise level than \( (V, S_2) \). To see this, consider only the difference 20 \( \log_{10} 42 - \log_{10} 16 = 8.4 \text{ dBA} \). In other words, at the given volume, the higher speed produces a noise level in excess of 8 dBA greater than traffic flowing at the lower speed. In order to predict the worst situation, then, it is necessary to relate speed to volume as follows:

\[ S = \frac{126.02 + \sqrt{(126.02)^2 - 8.6 \text{ V/m}}}{4.3} \quad (C-29) \]

Graphically, this is equivalent to selecting the upper half of the
parabola in Figure (C-2). Therefore, given this relationship, version 1 of Model 1 is given by Equations (C-21), (C-22) and (C-29).

By considering a different speed-volume relationship, version 2 of Model 1 is devised. The Federal Highway Administration (FHWA) normally uses the volume-travel time relationship given in reference (11) in the capacity-restraint component of its traffic assignment model.

\[ t = t_0 \left[ 1 + 0.15 \left( \frac{V}{mC} \right)^4 \right] \]  

where \( t = \) travel time over the link in question in minutes,
\( t_0 = \) mean free flow travel time over the link in minutes,
\( V = \) aggregated volume over the link in vph,
\( m = \) number of lanes,
\( C = \) practical capacity of the link in vph/lane.

If the link in question has length \( \ell \), then

\[ \ell = St = S_0 t_0 \]  

where \( S = \) average vehicle speed in mph
\( S_0 = \) mean free speed in mph.

Therefore,

\[ St_0 \left[ 1 + 0.15 \left( \frac{V}{mC} \right)^4 \right] = S_0 t_0 \]  

which becomes

\[ S = \frac{S_0}{1 + 0.15 (V/mC)^4} \]  

which is the speed-volume relationship obtained from the FHWA travel
time equation. Notice again the assumption that the volume $V$ is uniformly spread over $m$ lanes and $S$ is average vehicle speed per lane. This relationship, when graphed, appears as in Figure (C-3). The volumes in Figure (C-3) for which the speed approaches zero asymptotically are of no concern since the $V/C$ ratio exceeds 2. See Figures III-2 and III-15 in (11). So version 2 of Model 1 is given by Equations (C-21), (C-22), and (C-33).

Now, given either version 1 or version 2 of Model 1, suppose the problem is to determine all these speed-volume-distance interactions. If the roadway has $m$ lanes, then the first step is to aggregate traffic flows in each direction to get flows $V_1$ and $V_2$. (Figure (C-4))

For lack of better information, it is assumed that the total aggregate flow $V$ is given by

$$V = V_1 + V_2 \quad \text{(C-34)}$$

The other method discussed previously was given in Equation (C-8). Corresponding to $V$, there is an average lane flow of $V/m$ vehicles per hour per lane. From this, as estimate of $S$ is obtained from either speed-volume relationship. By specifying the truck mix, the median width $W$, and the noise level criterion $\bar{L}$, a distance $D$ from the center of the roadway is determined beyond which the mean noise level does not exceed $\bar{L}$. As given in Figure (C-5), $(D - DR)$ is the width in feet of the noise polluted area (area where the mean noise level exceeds $\bar{L}$). If the link in question has length $L$ in miles, then the total noise polluted area in square feet along both sides of the highway is given by

C-24
Figure C-3.

Quartic FHWA Speed-Volume Relationship
Figure C-4.
Generalized Description Of Flows
On Multi-lane Highways
Figure C-5.

Generalized Description Of Noise-Polluted Areas Near Highways
Total Area in square feet = 2(5280 ft) (D - DR) \quad (C-35)

If both sides of Equation (C-35) are divided by \( (5280 \frac{ft.}{mi.})^2 \), then the total noise polluted area in square miles is given by

\[
\text{Total Area in square miles} = \frac{\lambda (D - DR)}{2640} \quad (C-36)
\]

If \( \bar{\rho} \) is the average density of affected population in persons/mi\(^2\), then

\[
N = \frac{\bar{\rho} \lambda (D - DR)}{2640} \quad (C-37)
\]

is the expected number of people along the link affected by noise emanating from the roadway in excess of the criterion level. If one divides both sides of Equation (C-37) by \( \lambda \), then the expected number of people \( M \) per mile of roadway affected by noise exceeding the criterion level is given by

\[
M = \frac{\bar{\rho} (D - DR)}{2640} \quad (C-38)
\]

Noise Level Standards

Interim noise standards and procedures pursuant to Section 109 (i) Title 23 U.S.C., are shown in reference (15), in which Table (C-3) appears. The table lists, for each land use category (except for category D which is for undeveloped land), the recommended noise levels not to be exceeded over 10% of the time \( (L_{10}) \). For category E, an interior noise

(15) PPM 90-2, FHWA, 26 April, 1972.
Table C-3.

Design Noise Levels/Land Use Relationships

<table>
<thead>
<tr>
<th>Land Use Category</th>
<th>Design Noise Level $L_{10}$</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>60 dBA (Exterior)</td>
<td>Quiet parks, etc.</td>
</tr>
<tr>
<td>B</td>
<td>70 dBA (Exterior)</td>
<td>Residences, motels, schools, churches, libraries, playgrounds, etc.</td>
</tr>
<tr>
<td>C</td>
<td>75 dBA (Exterior)</td>
<td>Developed lands not in A or B</td>
</tr>
<tr>
<td>D</td>
<td>---</td>
<td>Undeveloped land</td>
</tr>
<tr>
<td>E</td>
<td>55 dBA (Interior)</td>
<td>Residences, hotels, motels, meeting rooms, etc.</td>
</tr>
</tbody>
</table>
level is specified which together with the type of structure (frame, masonry, etc.) would determine the maximum exterior noise level.

Because the mean noise level $L$ in Model 1 corresponds to the median noise level $L_{50}$, these standards are not compatible with the model unless the following correction is made (16):

$$L_{10} = L_{50} + 10 \log_{10} \left[ \frac{\cosh (1.19 \times 10^{-3} \text{VD/s})}{\cosh (1.19 \times 10^{-3} \text{VD/s}) - .951} \right] \quad (C-39)$$

where $L_{10}$ = noise level in dBA exceeded 10% of the time,

$L_{50}$ = median noise level in dBA (because the noise distribution is nearly normal, the median is nearly equal to the mean),

$V$, $D$ and $S$ are defined in I.A.

From this relationship it can be seen that it is not possible to convert the $L_{10}$ values in Table (C-3) to more directly applicable $L_{50}$ values without consideration of $V$, $D$, and $S$.

Another set of noise level criteria may be obtained by considering (17), in which mean ambient noise levels for various residential/land use categories are given. One might aggregate these categories and get Table (C-4).

In the same report (18), the authors indicate that intruding noise levels must not exceed the mean ambient level plus 9 dBA if widespread complaints are to be avoided. Therefore, if 9 dBA are added to each of the values in Table (C-4), maximum mean noise levels for each of the three

(16) NCHRP No. 117, p. 12, equation 27.

(17) NCHRP No. 117, p. 29, Table 10.

(18) NCHRP No. 117, p. 23, Table 5.
<table>
<thead>
<tr>
<th>Category</th>
<th>Mean Ambient Level</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>60 dBA</td>
<td>Single occupancy dwelling in low or medium density areas</td>
</tr>
<tr>
<td>2</td>
<td>65 dBA</td>
<td>Multiple occupancy dwellings in low or medium density areas</td>
</tr>
<tr>
<td>3</td>
<td>70 dBA</td>
<td>Single or multiple dwellings, hotels, hospitals in high density area</td>
</tr>
</tbody>
</table>
categories are obtained.

Controlling noise levels in commercial areas should, on the other hand, be more directed toward preserving the quality of the existing land uses. In reference (19), it is suggested that 75 dBA is an important level insofar as noise levels exceeding it produce temporary physiological change. In the same report, 75 dBA is given as a typical mean ambient noise level in downtown commercial areas with heavy traffic (20). By combining all this information, one could arrive at some definitive mean ($L_{50}$) noise level criteria, based on adding 9 dBA to the values in Table (C-4), as summarized in Table (C-5). Of course, the values in Table (C-5) should be considered only if local noise ordinances prohibit noise levels in excess of those above.

**Implementing the Model in the Network Design Process**

It has been shown that once one specifies a maximum allowable mean noise level $\bar{L}$, the percentage of truck traffic, $m$, $W$, and the speed-volume relationship, then distance $D$ within which noise levels exceed the standards can be directly related to volume $V$. The question arises whether this relationship is unique. If the answer is yes, then it seems plausible to ask one of two questions:

1. What is the maximum volume $V$ which can be tolerated on the link if no more than $M$ persons are to be affected by the excessive noise along the highway? or

2. What is the maximum volume $V$ to be tolerated if, beyond a

---


(20) NCHRP No. 117, p. 22, Figure 15.

C-32
Table C-5.
Maximum Mean Noise Levels ($L_{50}$)

<table>
<thead>
<tr>
<th>Category</th>
<th>Maximum $L_{50}$</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>69</td>
<td>Single occupancy dwelling in low or medium density areas</td>
</tr>
<tr>
<td>2</td>
<td>74</td>
<td>Multiple occupancy dwellings in low or medium density areas</td>
</tr>
<tr>
<td>3</td>
<td>79</td>
<td>Single or multiple dwellings, hotels, hospitals in high density areas</td>
</tr>
<tr>
<td>4</td>
<td>84</td>
<td>Downtown commercial areas with heavy traffic</td>
</tr>
</tbody>
</table>
distance D, the mean noise level from the roadway is to be less than some prespecified value \( L \)?

Results have been obtained from Model 1, version 1 for the following set of conditions: noise levels \( L \) are given for distances D from 100 to 500 feet, for 2-8 lanes, for \( w = 20 \) feet, and for volumes from 100 - 1800 vph per lane. Recall on page (C-3) that \( L \) is probably inaccurate for volumes per lane less than 1000 vph. Sample results are given in Figure (C-6).

The relationship between noise level and traffic volume can be summarized as follows: \( L \) is an increasing function of \( V \) for fixed \( D \), \( w \) and \( m \) until \( V = 1400 \) vph. At this point, the maximum noise level is attained (in this case, 69 dBA) and the noise level begins to decrease. Beyond \( V = 1400 \), the noise level decreases due to the severe decrease in the average vehicle speed. By varying the distance \( D \), a family of curves is generated, given in Figure (C-7). In this figure, \( D_1 < D_2 < D_3 \).

Corresponding to a noise level \( L = \), there would be six points \((V_1, L), (V_2, L), (V_3, L), (V_4, L), (V_5, L), \) and \((V_6, L)\) from which a new graph can be drawn as follows: Corresponding uniquely to the six points above are six points \((V_1, D_1), (V_2, D_2), (V_3, D_3), (V_4, D_3), (V_5, D_2), \) and \((V_6, D_1)\) from which Figure (C-8) can be constructed. As given in Figure (C-8), \( D \) is uniquely related to \( V \), provided one considers only the increasing portion of the curve.

Therefore, in light of this argument, whether one poses question 1 or 2 is irrelevant. Each is tantamount to prespecifying distance \( D \). Now what specifically are the equations relating \( D \) to \( V \)?

C-34
Figure C-6. Relationship Between Noise At Given Distance
And Traffic Volumes (Parabolic Speed-Volume Assumption)

70 dBA Criterion: Constraint = none

65 dBA Criterion: Constraint = 300 vph/lane

60 dBA Criterion: Constraint = 75 vph/lane

Bolt Beranek Newman Model Version 1;
Parabolic Speed-Volume Model; 4 lanes;
200' from centerline; 20' median;
1846 vph absolute capacity; 58.6 mph
mean free speed; 5% trucks

VOLUME PER LANE (VPH)
Figure C-7

MEAN NOISE LEVEL IN DBA (\(\bar{L} = L\))

VOLUME IN VEHICLES PER LANE HOUR (V/m)

C-36
Figure C-8.

Distance At Which Noise Level Equals Noise Standard

As A Function Of Volume

DISTANCE FROM OBSERVER TO CENTERLINE IN FEET (D)

VOLUME IN VEHICLES PER LANE HOUR (V/m)
By rearranging Equation (C-21), one finds

\[
\frac{\bar{L} - L_t}{10^{10}} \cdot D_E \cdot \frac{10^2}{10^2} = VS^2
\]

(C-40)

Define

\[
k = \frac{\bar{L} - L_t}{10^{10}}
\]

(C-41)

\[
K = \frac{k \cdot D_E}{10^2}
\]

Then

\[
K = VS^2
\]

(C-42)

This is the equation which when given \( L, W, m, \) and \( D, \) allows solution for \( V. \) Substituting in Equation (C-24) for \( V \) results in

\[
2.15 \ S^4 - 126.02 \ S^3 + K/m = 0.
\]

(C-43)

Another way of solving Equation (C-42) for \( V \) is to substitute Equation (C-29) for \( S, \) and by solving for \( V, \) but the form of the resulting equation is far more complicated than Equation (C-43). Now one may solve Equation (C-43) numerically, obtaining four roots. At least two of the roots will be complex conjugates and two may be either (a) real and unequal, or (b) complex conjugates. If one finds (a) is true, then given the criterion level \( L_1, \) two speeds \( S_1 \) and \( S_2 \) are found, corresponding uniquely to two volumes \( V_1 \) and \( V_2, \) respectively, where \( V_1 \) and \( V_2 \) appear in Figure (C-9).

Obviously \( V_1 \) is the root to Equation (C-42) which is sought, since the noise level above \( V_1 \) violates the criterion \( L_1. \) That is, the higher speed
Figure C-9.
Traffic Volumes Exceeding
Given Noise Standards

\[ \bar{L} = L_2 \]

\[ \bar{L} = L_1 \]

Volumes at which criterion level is violated

Distance D

VOLUME PER LANE IN VEHICLES PER HOUR (V/m)

C-39
S is selected, since it corresponds to the lower uncongested volume. Now if (b) is true, the criterion level $L_2$ exceeds the mean noise level $\bar{L}$ for all feasible volumes $V$. Thus, practically speaking, no feasible volume would exceed the specified noise standards, and link design can proceed with no further consideration of noise.

In summary, given $\bar{L}$, $m$, $W$, $D$ and the speed-volume relationship given by version 1, a volume constraint $V_1$ is selected for the link in question. (If there are no real roots to Equation (C-43), then the constraint will be the practical capacity of the roadway.)

Given the volume constraint which is output from the above procedure, a constraint on the volume of flow in each direction can be shown by the following relationship:

$$V' + V'' \leq V$$  \hspace{1cm} (C-44)

where $V'$ is the volume in vph on the nearside stream and $V''$ is the volume in vph on the farside stream.

The procedure for finding $V$, given $\bar{L}$, $m$, $W$ and $D$ is very similar when the speed-volume relationship is given by version 2, the FHWA form. As in Figure (C-6), a sample result for version 2 is given in Figure (C-10).

Similar to version 1, one finds $\bar{L}$ to be a convex function of $V$, first increasing to a maximum at 1200 vph (which can be shown analytically by solving $\frac{dL}{dv} = 0$ for $V$), then decreasing with increasing volume $V$.

The task at hand now is to solve Equation (C-42) where $S$ is given by Equation (C-33). By combining these two equations, one gets

$$(b^2K) V^8 + (2bK) V^4 - a^2V + K = 0$$  \hspace{1cm} (C-45)
Figure C-10.
Relationship Between Noise At
Given Distance And Traffic Volumes
(FHWA Speed-Volume Assumption)

70 dBA Criterion: Constraint = 950 vph/lane

65 dBA Criterion: Constraint = 270 vph/lane

60 dBA Criterion: Constraint = 70 vph/lane

Bolt Beranek Newman Model Version 2:
FHWA Speed-Volume Model; 4 lanes;
200' from centerline; 20' median;
1200 vph practical capacity; 58.6 mph
mean free speed; 5% trucks
where \( a = S_0 \),
\[
b = 0.15/(mC)^4
\]
and \( K \) is given in Equation (C-41).

This equation can be solved by numerical methods. In general, one finds three pairs of complex conjugate roots and either (a) two real roots which are unequal, or (b) two complex conjugate roots. If case (a) is true, then two volumes \( V_1 \) and \( V_2 \) are found where \( V_1 < V_2 \). An illustrative example is found in Figure (C-10). When \( \bar{L} = 70 \text{ dBA} \) is imposed and when \( m = 4, D = 200 \text{ feet}, 5\% \text{ trucks}, C = 1200 \text{ vph}, S_0 = 58.6 \text{ mph}, \)
then \( V_1 = 950 \text{ vph/lane} \) and \( V_2 = 1400 \text{ vph/lane} \). The lower value would be selected as the noise-determined volume constraint. If case (b) is true, then for all volumes \( V \), the mean noise level \( \bar{L} \) is less than the criterion level \( \bar{L} = \bar{L} \) which has been prespecified. See Figure (C-9) for further explanation.

**Strategies for Implementation**

The model described in the previous sections of this report is specifically designed for the development of appropriate volume constraints which can be utilized in a network design optimization model. The requisite steps for its use are explained below.

1. Select appropriate speed-volume relationships.
   a. If the parabolic Greenshields form is acceptable, its parameters must be calibrated through the use of at least two observations of speed-volume pairs.
   b. If flow on urban expressways is being evaluated, the parameters included in this report can be used.
   c. If the user desires to apply the FHWA speed-volume...
relationship, he may similarly utilize the functional forms reported above.

d. If the user wishes to introduce an alternative speed-volume function, the equations must be re-solved.

2. Specify noise level standards in dBA, number of lanes, percent trucks, median width, and criterion distance D. If alternate values for lane width are to be used, the relationships presented must be modified.

3. The model described above can now be used to compute individual link volume constraints, which can then be introduced into the optimization model.

A computer program has been prepared to perform the computations described above. It provides the user with the ability to select one of the two speed-volume relationships discussed previously, as well as to specify the values of the other variables in the model. A listing of this program, along with its typical output, is provided in the Appendix. Since the program itself has been carefully annotated, it serves as its own users manual, and no further documentation is provided.

Additional applications of this model, beyond the scope of the network design process, are also possible. Because this approach includes in the noise model the effect of the relationship between speed and volume, it offers the planner an alternative, simplified approach to the estimation of the noise outputs of proposed or existing highways. The computer program provided can be used directly for this task. Furthermore, the tables provided offer typical values of volumes, number of lanes, distance of the observer, and noise levels which the planner can use with no further computations.
For example, the description of the effect of volumes on noise levels can be used to evaluate the impact of any traffic improvements proposed for existing highways which serve to increase capacity. As shown in the Addendum, and in Figures (C-6) and (C-10), under most conditions, any increase in the volume carried by a highway will encourage an increase in noise level. Thus, capacity improvement programs may actually influence the degree to which transportation facilities meet noise standards. Alternatively, to meet given noise standards, it may be necessary to restrict the volumes on some links through traffic control procedures.

Finally, both the tabular data and the model may be useful in the evaluation of the feasibility of proposed noise standards. In particular, it is possible to anticipate the feasibility of meeting given standards in specific locations, where land use and the relevant distance from the roadway at which the standards will be effective are known.
Addendum

Incorporating Considerations of Noise Impacts into the Continuous System Requirements Model

In the interest of developing a network design model which can efficiently handle large scale networks, a revised formulation has been proposed which avoids the requirement for an integer solution. In this new formulation, capacity can be added in continuous quantities, as opposed to the previous approach, in which capacity was added in integer increments amounting to the addition of extra lanes to existing or proposed roadways.

The model originally developed for including noise impacts in the network design process assumed that the range of possible numbers of added lanes was given at the outset. In the continuous network design model, however, this information will not be available.

The noise model proposed in the body of this report is considerably more sensitive to the volume of flow on a link than it is to the number of lanes. As a consequence, only a rough estimate of the number of lanes which may ultimately exist on a link will be sufficient to produce noise estimates which are accurate within the normally expected range of measurement errors.

Thus, the use of the approach proposed for treating noise within the integer network design model may still be utilized. This requires that the user provide the noise modeling system with the noise standard level, the distance from the link at which that standard is to be enforced, and the number of lanes. In case of the continuous model, the number of lanes should be estimated as follows:
1. For existing links on which capacity may be added, estimate the likely number of lanes to be added, and supply the noise model with \( n = \text{existing} + \text{expected number of lanes} \);

2. For new links where capacity addition is permitted, estimate the likely number of lanes to be added and supply this information to the model;

3. On existing links where capacity is not to be added, supply the model with the number of lanes on the existing facility.
Computer Program Listing

For Defining Noise Emission Constraints
PROGRAM PUNT(TAPE6,TAPE7)

DIMENSION CCE(10),CRICK,ROCTII(10)

C THIS PROGRAM WAS WRITTEN AS A THEORETICAL TOOL, WHICH CAN BE USED
C IN DETERMINING, FOR ROADWAYS AT GRADE, THE MAXIMUM VOLUME TO BE
C TOLERATED IN ORDER THAT AN OBSERVER BEYOND A DISTANCE D FROM THE
C ROADWAY, WILL EXPERIENCE A NOISE LEVEL (EXPRESSED IN UNITS OF DBA)
C WHICH IS LESS THAN SOME PRESPECIFIED MEAN NOISE LEVEL VALUE.
C
C THE DBA EQUATION IS BASED ON GALLOWAY'S LINEARIZED APPROXIMATION
C TO THE MONTE CARLO SIMULATION OF TRAFFIC NOISE; REF. NCHRP NO. 717
C IN THE PROGRAM DESIGN THE USER MAY VARY CERTAIN INPUT PARAMETERS
C WHICH WILL MAKE THIS PROGRAM BETTER MEET HIS NEEDS. HE
C ALSO HAS THE OPTION OF RUNNING THE PROGRAM FOR VARIOUS DISTANCES,
C NUMBER OF LANES, AND MEAN NOISE LEVELS BY VARYING THE INCREMENT
C VALUE AND THE NUMBER OF TIMES IT IS TO BE INCREMENTED.

C INPUT REQUIREMENTS/ VARIABLE NAME - UNITS
C
C MEAN FREE SPEED=ISO = IN MPH
C INITIAL NUMBER OF LANES=AM = LANE INCREMENT VALUE(AIP)
C NUMBER OF TIMES INCREMENTED(IIP)
C INITIAL DISTANCE FROM CENTERLINE OF ROADWAY TO OBSERVER (D) = FEET
C DISTANCE INCREMENT VALUE(AII)
C NUMBER OF TIMES INCREMENTED(II)
C WIDTH OF MEDIAN=INFEET
C ADJUSTMENT FACTOR FOR TRUCK MIX=ALT = IN DBA
C THESE ADJ. FACTORS CAN BE FOUND IN TABLE 1 NCHRP NO. 78, PAGE 9
C INITIAL PRESPECIFIED MEAN NOISE LEVEL(DBAL) = IN DBA
C NOISE LEVEL INCREMENT VALUE(AID)
C NUMBER OF TIMES INCREMENTED(ID)
C PRACTICAL CAPACITY(CC) = IN VPH/LANE

C INPUT FORMATTED ON ONE CARD IN THE ORDER LISTED BELOW
C
C 95 FORMAT(F4.1,F3.0,F12,F4.0,F6.0,F12,F6.0,F3.0,F5.1,F12,F6.2,F6.0)

C TAK=AM
C TD=0
C NNP=I
C TDBAL=DBAL
C DO 3 M=1,IIP
C NNP=I
C D=CC*AM**4
C DO 2 K=1,II
C DE=DISTANCE FROM OBSERVER TO SINGLE LANE EQUIVALENT
C D2=1.5*(AK+1.1)*(12*SQRF(AM))**4*(M/2)
C DO 1 J=1,10
C E=(DBAL-ALT)/10.*
C AK=(10.*D2)**(-DE/100)
C
C THIS PART OF THE PROGRAM SEeks TO FIND THE INTERSECTION OF THE
C PRESPECIFIED DBA LEVEL WITH THE CURVE DEFINED BY
C MEAN NOISE LEVEL=10LOG(V)+20LOG(E)=10LOG(DBA)+20.*ALT,
C WHERE S=SC/II*(1.5)/V/(AM*CC)**4=1PR MODEL), FOR THESE PAR-
C TICULAR SET OF EQUATIONS THE INTERSECTION POINTS WILL REPRESENT
C VOLUMES AT WHICH PRESPECIFIED MEAN NOISE LEVEL. WHEN SOLVED, THE ROOTS ARE; IN GENERAL, COMPLEX. IF
C THE IMAGINARY PART OF EACH ROOT IS NOT EQUAL TO ZERO (IE..GT..0001)
C THIS SIGNIFIES THAT ALL ROOTS ARE COMPLEX, HENCE ANY VOLUME PLACED
C ON THAT LINK WILL NOT CAUSE THE ROADWAY TO PRODUCE A MEAN NOISE
LEVEL THAT IS ABOVE THE PRESPECIFIED DBA. IF THE IMAGINARY
PART OF THE I TH ROOT (WHOSE REAL PART IS A), IS EQUAL TO ZERO
(1E+6<LT.0001 BUT >LT.-0001) we CONSIDER ROOT(i) TO BE A ROOT
VII. TO THE ABOVE EQUATION, THE MINIMUM OF THE REAL ROOTS VII
BECOMES OUR MAX. VOLUME ALLOWED. AS A DOUBLE CHECK, we COMPARE
THE MAX. VOLUME TO THE MAX. CAPACITY FOR THAT ROADWAY AND IF IT
IS GREATER THAN THAT VALUE, WE AGAIN PLACE A NO VOLUME CONSTRAINT
(NVC) ON THAT LINK.

C (NVC) ON THAT LINK
CCE(1)=(B*B)*AK
CCE(2)=0.
CCE(3)=0.
CCE(4)=0.
CCE(5)=2.*B*AK
CCE(6)=0.
CCE(7)=0.
CCE(8)=-(50*50)
CCE(9)=AK
N=8
NMAX=9

C FROM COMBINING THE ABOVE EQUATIONS AND SOLVING FOR V/THE INTERSECTION
C POINT) WE OBTAIN THE FOLLOWING RELATIONSHIP:
C CCE(1)*V=0. CCE(5)*V=0. CCE(8)*V+CCE(9)=0;
C WHICH IS THEN SOLVED BY USING THE LIBRARY FUNCTION MULLER.
CALL MULLER(CCE,X,ROOTXROOTT,NMAX)
NII=2
GO TO (57,15)*NPP
C THIS INPUT DATA TABLE ALLOWS THE USER TO CHECK THAT THE PROGRAM HAS
C READ IN HIS INPUT CORRECTLY.
57 WRITE(6,260)
200 FORMAT(* USER INPUT DATA TABLE*)
WRITE(6,200) SG
201 FORMAT(5X* MEAN FREE SPEED=,F4.1*, (MPH)*
WRITE(6,201) AM*IP*AI
202 FORMAT(5X* INITIAL NUMBER OF LANES=,F3.0*, INCREMENTED=,I2*)
WRITE(6,202) DII,AIL
203 FORMATE(5X* INITIAL DISTANCE=,F6.0*, INCREMENTED=,I2*)
WRITE(6,203) DII,AIL
204 FORMAT(5X* WIDTH OF MEDIAN=,F6.0*, (FEET)*
WRITE(6,204) W
205 FORMAT(5X* TRUCK MIX ADJUSTMENT FACTOR=,F3.0*, (DBA)*
WRITE(6,205) O:DAL,AD
206 FORMAT(5X* INITIAL PRESPECIFIED DBAL LEVEL=,F5.1*)
WRITE(6,206) CC
207 FORMAT(5X* PRATICAL CAPACITY=,F6.0*, (VPH PER LANE)*
WRITE(6,207) NPP=2
15 GO TO (5,9),NP
10 WRITE(6,250)
7 FORMAT(/5X* N LANES=,5X* DISTANCE=,5X* NOISE LEVEL=,5X*)
WRITE(6,260)
8 FORMAT(18X* (FEET)=,11X*, (DBA)=,6X*, (6PR-MODEL)=,11X*)
C PROCEDURE TO CALCULATE MIN. (*) ROOT
9 THIN=999999.9
DC 26 I=1,8
 IF(RCRS1(I).LT.0.) GO TO 26
 IF(ABS(RCOT1(I)-10000.6.1.0) GO TO 26
 IF(RCOT1(I).GT.THIN) GO TO 26
THIN=RCOT1(I)
26 CONTINUE
C VOLUME CONSTRAINT TEST WHERE SS IS THE MAX. VPH FOR ALL AM LANES
120 SS=2400.*AM
NKN=2
 IF(INTHN.GT.SS) NNN=1
 GO TO (30,31)**NNN
30 WRITE(6,22) AM,D,DELAL
22 FORMAT(8X,F2.0,8X,F6.0,10X,F6.1,14X,** NVN*)
 GO TO 18
C VV CONVERTS TOTAL VOLUME (THIN) TO VOLUME PER LANE
31 VV=THIN/AM
WRITE(6,24) AM,D,DEAL,VV
24 FORMAT(8X,F2.0,8X,F6.0,10X,F6.1,12X,F6.0)
C THIS PART OF THE PROGRAM SEEKS TO FIND THE INTERSECTION OF THE
C PRESPECIFIED DEA LEVEL WITH THE CURVE GIVEN BY
C MEAN NOISE LEVEL=10LOG(V)+20*ALT
C BUT THIS TIME OUR SPEED-VOLUME RELATIONSHIP IS
C S=126.*V+4.7(ISCHOFER MODEL). FOR THESE
C PARTICULAR SET OF EQUATIONS, THE INTERSECTION POINTS WILL
C REPRESENT SPEEDS AT WHICH THE PRESPECIFIED DEA LEVEL IS IDENTICAL
C TO THE MEAN NOISE LEVEL WE HAVE SOLVED FOR DUE TO THE EASE IN
C MATHEMATICAL MANIPULATION, THE ROOTS AGAIN ARE COMPLEX. IF THE
C IMAGINARY PART OF EACH ROOT IS NOT EQUAL TO ZERO (IE., GT. 0.0001)
C THIS MEANS THAT ALL ROOTS ARE COMPLEX; HENCE ANY VOLUME PLACED ON
C THAT LAN WILL NOT CAUSE THE ROADWAY TO PRODUCE A MEAN NOISE
C LEVEL THAT IS ABOVE THE PRESPECIFIED DEA LEVEL, IF THE IMAGINARY
C PART OF THE ITH ROOT (WHOSE REAL PART IS +) IS EQUAL TO ZERO
C (IE., LT. 0.0001 BUT.GT.-0.0001), WE CONSIDER ROOT (I) TO BE A ROOT
C AT THE ABOVE EQUATION. THE MAX. OF THE REAL ROOTS 5(I) WILL
C BE THE SPEED CORRESPONDING TO THE MAX. VOLUME ALLOWED, AND AS
C BEFORE WE COMPARE THE MAX. VOLUME TO THE MAX. CAPACITY AS A DOUBLE
C CHECK FOR (NVN).
140 C
145 C CHECK FOR (NVN).
150 18 CCE(1)=2.15
CCE(2)=-126.02
CCE(3)=0.0
CCE(4)=0.0
CCE(5)=AK/AM
N=4
NMAX=5
C FROM COMBINING THE ABOVE EQUATIONS AND SOLVING FOR S (THE INTERSECTION
C POINT) WE OBTAIN THE FOLLOWING RELATIONSHIP:
C CCE(1)*S+CCE(2)*S*S+CCE(3)=0.
C WHICH IS THEN SOLVED BY THE LIBRARY PROGRAM MULLER.
CALL MULLER(CCE,NROOTS+1,NMAX)
96 NP=2
C PROCEDURE TO CALCULATE MAX. (*) ROOT
MAX=999.
DC 10 I=1,4
IF(RCCRT(I) .LT. 0.0) GO TO 10
IF(AABS(RCCRT(I)) .LT. 1000.) GO TO 10
IF(RCCRT(I) .LT. TMAX) GO TO 10
THAX = RCCRT(I)

170 CONTINUE
IF(TMAX .EQ.*999.* ) GO TO 20
C V CONVERTS TMAX(MAX. SPEED) TO VOLUME PER LANE.
V = AK/TMAX**(2/AM)

C VOLUME CONSTRAINT TEST WHERE SS = THE MAX. VPH PER LANE.
175 SS = 2400.
    NN = 2
    IF(V .GT. SS) NN = 1
78 GO TO (20, 21) *NN
20 WRITE(6, 32)
180 FORMAT(1H*, 71X*, NVN*)
    DBAL = DBAL + AID
    GO TO 1
21 WRITE(6, 34) V
34 FORMAT(1H*, 69X*, F6.0)
185 DBAL = DBAL + AID
1 CONTINUE
    DBAL = DBAL
    D = D + AII
190 CONTINUE
    DBAL = DBAL
    D = TD
195 AM = AM + AIP
3 CONTINUE
STOP
END
### USER INPUT DATA TABLE:

**Mean Free Speed = 58.6 (MPH)**

**Initial Number of Lanes = 2**, **Incremented 4 Times by 2 Lanes**  
**Initial Distance = 100 Feet**, **Incremented 4 Times by 100 Feet**  
**Width of Median = 20 Feet (FEET)**  
**Truck Mix Adjustment Factor = 2 (DBA)**  
**Initial Specified DBA Level = 55.0 DBA**, **Incremented 7 Times by 5.00 DBA**  
**Practical Capacity = 1200 VPH Per Lane**

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<th>Noise Level (DBA)</th>
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<td>Max. Vol./Lane (Schofer)</td>
</tr>
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APPENDIX D

AIR POLLUTION IMPACTS

Introduction

Within the past few decades, the management of air resources has become one of the most rapidly growing environmental problems. As concern over the quality of the urban environment intensifies, transportation planners, and those who contribute to the design, planning, and operation of related public systems, are being compelled to contribute to the abatement of air pollution emissions caused by transportation systems. This network design study has been conducted in an effort to improve the systematic basis for design of entire transportation networks. A major objective of this project is to discover effective ways in which to introduce consideration of the environmental impacts of these networks, including the effects of air pollution, into the design optimization process.

Prediction and control of transportation-caused air pollution is very complex, involving considerations of topography, sources and types of pollutants, and physical-chemical processes in the environment. The nature of pollutants and their impacts is not well known; furthermore, there is no readily established market value which can be placed on the consequences of degraded air quality.

The objective of this Appendix is to develop, within the existing body of knowledge on air pollution, a simple method for relating the contribution to air pollution of transportation emissions to the network design process.
Air Quality

Motor vehicles are a major source of carbon monoxide, hydrocarbons, and nitrogen oxides in the atmosphere. The primary mobile source of these emissions is the gasoline-powered motor vehicle, while other significant sources include diesel-powered trucks and buses. From the perspective of transportation network planning, it would be desirable to be able to relate the effect of these emissions to air quality for different network configurations. To accomplish this, it is necessary to predict air quality as a function of emissions associated with each transportation facility and the manner in which it is used. This task is quite complex, for air quality is dependent not only on emissions, but also the atmospheric dispersion process. To model the latter, we must account for meteorological conditions, including winds, atmospheric stability, temperature, and mixing heights, as well as characteristics of local terrain and buildings.

Efforts to model the urban diffusion process have been under way since 1955, when a combined estimate of pollution concentrations from both fixed and vehicle sources in the Los Angeles area was made (Nieburger, 1955). Others have contributed a number of refinements and advances to the state-of-the-art in this area. Kenneth Uherka (1972) described a model that relates meteorological conditions (wind, speed, wind direction, height, eddy diffusion coefficient), traffic characteristics (volume, length of roadway), and vehicular emission rates, to estimate concentrations of vehicular pollutants.

In its present state, however, this model has some inherent difficulties which both limit its usefulness and also help to define the complexity of this general problem. For example, Uherka's model assumes
that the prevailing wind direction is perpendicular to a given roadway, while obviously the wind is not limited to any particular orientation. But attempts to generalize the assumptions regarding wind direction quickly become stalled by the complexity of the resulting mathematical relationships.

A further deficiency of the Uherka model is in the evaluation of the eddy diffusion coefficient. This coefficient measures the diffusivity of the atmosphere, and is a function of height, location, air turbulence, etc. Because these meteorological characteristics change with great rapidity, the eddy diffusion coefficient is a necessarily dynamic factor. This results in a model which is not only complex to develop and apply, but also to test and validate. Other, emerging, diffusion models tend to be even more complex, making their use within the already complex network design model infeasible at this time.

When appropriate diffusion models become operational, the link between the transportation network design and the concomitant air pollution impacts could be used as an effective plan evaluation tool. This might be accomplished by introducing constraint equations into the design model based directly on air quality standards, such as those promulgated by the Environmental Protection Agency as required by the Clean Air Act.

While federal standards are likely to change in the future, and while state and local standards may vary somewhat from these national levels, it is apparent that air quality standards are available for use in transportation planning. Still, the problem remains to establish an efficient and effective link between network structure, traffic flows, and air quality. Until that is accomplished, a more simplistic strategy
for linking transportation planning and air quality must be relied upon. At present, the direct inclusion of air quality measures is impossible, due to complexity of atmospheric dispersion models.

**Recommended Approach**

The simplified approach suggested for use with the design model is to limit consideration only to the emission of various types of pollutants. It is not possible, however, to create internal constraint equations for such emissions for use in the network design model. This is because no reasonable standards can be established for total emissions for a single link or the entire network, since it is only logical to apply rigid standards to overall air quality. The significance of a given emission level will necessarily depend on ambient air quality, adjacent land uses, and prevailing weather conditions.

These factors preclude direct inclusion of pollution emissions in the design model. Instead, it is appropriate only to examine the emission properties of the optimal networks which have been produced by the model. The planner and decision maker can use emission estimates prepared by a Post-Processor as a basis for subjective evaluation of networks, and as a foundation for preparing revised inputs to subsequent runs of the design model. The combination of an efficient network design model and a simple emissions post-processor should facilitate rapid search of the trade-off space between air pollution emissions and other network characteristics.

The remainder of this appendix focuses on the development of such a processor based on currently available emissions models.

**Background**

For the purposes of this investigation, carbon monoxide, hydrocarbons, and oxides of nitrogen are regarded as the three major air pollutants.
released by motor vehicles. Although vehicular emissions also contain two other important pollutants—sulfur oxides and suspended particulates—these constitute only a small percentage of total vehicle emissions. It is expected that, in the near future, most of the lead salts which are released in suspended particulate form will be eliminated with the use of lead-free gasolines (Wolsko, 1972).

Because any given transportation system comprises many types of vehicles of different ages and operating efficiencies, emissions will vary from vehicle to vehicle. Also, when determining the emission characteristics of a system, the effects of the operating cycles of individual vehicles ("cold" or "hot" starts, acceleration, speed, deceleration, stops, etc.) upon emission rates should be taken into account (Wolsko, 1972). This dependence of air pollutant emissions upon vehicle type and operating cycles requires that these characteristics of the transportation system be identified in order to assess the air pollution consequences of alternative networks.

Because of the lack of available data relating acceleration, deceleration, and "cold" and "hot" starts to emission rates for a given vehicle, and because specific driving cycle characteristics are not captured in any operational travel demand models, this report can consider only the effects of average travel speed on vehicle emission rates. The functional relationship between speed and vehicle emissions is referred to as the speed-emission relationship.

Review of Existing Speed-Emission Models

This section summarizes the four most salient of the existing models of vehicle speed-emission relationships, including the assumptions and limitations of each. The purpose is to select that model which is simple,
consistent with current knowledge, and can easily be incorporated into the network design model.

Most of the initial research relating emission rates to network parameters was done by A.H. Rose, Jr., et al. (1965), and N.P. Curnansky, et al. (1970). Curnansky formulated carbon monoxide, hydrocarbon, and oxide of nitrogen emission tables (on a total vehicle-mile basis) for vehicle populations of various years. Of the four models discussed below, the third and fourth are related to Curnansky's work, while the first two are based on the work of Rose, et al.

The earliest significant research on the relationship between emission rates and transportation network parameters was done by Rose, et al. (1965). It was concluded that, for automobiles, (1) regardless of the characteristics of the specific route, exhaust emissions can best be described as a relationship between the logarithm of pounds of containment per vehicle mile and average route speed; and (2) exhaust emissions expressed as concentration (parts per million--ppm) provide a less valid measure of atmospheric contamination than emissions expressed as pounds per vehicle mile travelled, since the latter is related to units of travel consumed. Logarithmic functions were developed by combining the emission data for individual gasoline-powered test vehicles, and fitting the data to a power function \( e = as^b \) by the least-squares technique*. Correlation coefficients were determined to establish the degree of fit for each of the following relationships:

*where \( e \) is the emission rate in grams per vehicle mile, \( s \) the speed in miles per hour, and \( a \) and \( b \) constants that depend on a variety of factors, such as model year, engine displacement, condition of vehicle, and engine operating modes.
<table>
<thead>
<tr>
<th>Relationship</th>
<th>Highest correlation coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydrocarbons vs. speed</td>
<td>.686</td>
</tr>
<tr>
<td>CO vs. speed</td>
<td>.630</td>
</tr>
<tr>
<td>NOx vs. air/fuel ratio</td>
<td>.675</td>
</tr>
</tbody>
</table>

The correlation coefficients for the regression of emissions vs. speed are not very good. This is an important point to consider when assessing the accuracy of methods using this speed-emission model.

**Model 1**

The Stanford Research Institute (1971) made use of the Rose, et al., carbon monoxide and hydrocarbon speed-emission curves as estimated emission factors for pre-1968 vehicles. They then developed speed-emission curves for hydrocarbons and carbon monoxide, for the years from 1968 on.

These relationships were derived by using test results published for a single vehicle, and applying the results to driving runs where steady speed, stops, and speed change cycles were observed. However, these expressions are only approximate, and are based on limited data. These data are not given in the report, nor are the correlation coefficients for the fitted expressions. For the model years 1974 and beyond, the carbon monoxide and hydrocarbon emission rates were estimated by assuming that the slope of the speed emission-vs. -year curve is constant after 1968, but that the level is proportional to some type of technological change. No significant statistical relationship was found between average speed and oxides of nitrogen emitted; therefore, an average NOx emission value for each model year was used (see Table D-1). No indication was given as to the origin of these figures.
### Table D-1 Exhaust Emission Control Requirements

**For Passenger Vehicles**

<table>
<thead>
<tr>
<th>Model Year</th>
<th>Hydrocarbons (gm/mi or ppm)</th>
<th>Carbon Monoxide (gm mi or percent)</th>
<th>Oxides of Nitrogen (gm/mi)</th>
<th>Particulates (gm/mi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-1968 (estimated emissions from uncontrolled cars)</td>
<td>828 ppm</td>
<td>2.94%</td>
<td>1500 ppm</td>
<td>No data</td>
</tr>
<tr>
<td>1968-69</td>
<td>275 ppm</td>
<td>1.5%</td>
<td>NR</td>
<td>NR</td>
</tr>
<tr>
<td>1970-72</td>
<td>2.2 gm/mi (180 ppm)</td>
<td>23.0 gm/mi (1.0%)</td>
<td>NR</td>
<td>NR</td>
</tr>
<tr>
<td>1973-74</td>
<td>2.2 gm/mi</td>
<td>23.0 gm/mi</td>
<td>3.0 gm/mi</td>
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<tr>
<td></td>
<td></td>
<td>39.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1975-79</td>
<td>0.5 gm/mi</td>
<td>11.0 gm/mi</td>
<td>0.9 gm/mi</td>
<td>0.1 gm/mi</td>
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<tr>
<td>1980- and on</td>
<td>0.25 gm/mi</td>
<td>4.7 gm/mi</td>
<td>0.4 gm/mi</td>
<td>0.03 gm/mi</td>
</tr>
</tbody>
</table>

NR = No requirement.

**SOURCE:** Stanford Research Institute, *Methods of Evaluation of The Effects of Transportation Systems on Community Values.*
Model 2

Bellomo, et al. (1971) summarized the relationship between emission per mile and average network speed in miles per hour for carbon monoxide, hydrocarbons, and oxides of nitrogen (Figs. D-1, 2, and 3). For carbon monoxide and hydrocarbons, it can be seen that an increase in average network speed decreases emission rates per vehicle mile; however, in the case of NOx, increases in average speed were found to increase emissions of oxides of nitrogen. These speed-emission curves were apparently constructed from the values shown in Table D-2, taken from the 2nd report of H.E.W. to the U.S. Congress (1965). Bellomo apparently connected the speed-emission rates for the four given speeds as listed above. Furthermore, the emission values for the speeds given in the HEW report were extrapolated from the poorly fitted curves produced by Rose. Therefore, the validity of the curves produced by Bellomo is not particularly strong. It should be noted that the Tennessee Department of Transportation (1972), in documenting methods for predicting highway air pollution has used these same speed-emission curves with no additional validation.

Model 3

The U.S. Environmental Protection Agency (1971) stratified gasoline-powered vehicles into classes, and differentiated between gasoline and diesel emissions. The gasoline-powered motor vehicles were classified as passenger cars, light-duty trucks, and gasoline-powered heavy-duty vehicles. An overall emission factor was developed for all gasoline-powered vehicles, by weighting each class according to urban or rural travel, allowing for incorporation of new vehicles, deterioration of vehicles with increasing age and mileage, and scrapping of older vehicles. These emission factors are presented in Table D-3; they are for the vehicle
Figure D-1. Carbon monoxide vehicular emission versus speed.

SOURCE: Bellomo (1971)
* Source: 2nd Report Secretary of H.E.W. to U.S. Congress pursuant to P.L. 88-206 Clean Air Act, 2/19/65, Table 1.

** Based on 1975 Emission Standards Set by U.S. Congress.

Figure D-2. Hydrocarbon vehicular emission versus speed.
* Source: 2nd Report Secretary of H.E.W. to U.S. Congress pursuant to P.L. 88-206 Clean Air Act, 2/19/65, Table 1.

** Based on 1975 Emission Standards Set by U.S. Congress.

Figure D-3. Nitrogen oxide vehicular emission versus speed.

SOURCE: Bellomo (1971)
<table>
<thead>
<tr>
<th>Location</th>
<th>10 miles per hour, business</th>
<th>18 miles per hour, residential</th>
<th>24 miles per hour, arterial</th>
<th>45 miles per hour, rapid transit</th>
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</thead>
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<tr>
<td>Denver</td>
<td>0.040</td>
<td>0.023</td>
<td>0.017</td>
<td>0.0094</td>
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<td>Cincinnati and Los Angeles</td>
<td>0.023</td>
<td>0.015</td>
<td>0.013</td>
<td>0.0085</td>
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<td>Denver values as percent of Cincinnati and Los Angeles</td>
<td>170</td>
<td>150</td>
<td>130</td>
<td>110</td>
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</table>

**Carbon Monoxide, Pounds per Mile**

<table>
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<tr>
<th>Location</th>
<th>10 miles per hour, business</th>
<th>18 miles per hour, residential</th>
<th>24 miles per hour, arterial</th>
<th>45 miles per hour, rapid transit</th>
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<tr>
<td>Denver</td>
<td>0.00</td>
<td>0.35</td>
<td>0.27</td>
<td>0.15</td>
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<tr>
<td>Cincinnati and Los Angeles</td>
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<td>0.21</td>
<td>0.17</td>
<td>0.10</td>
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<td>Denver values as percent of Cincinnati and Los Angeles</td>
<td>170</td>
<td>170</td>
<td>100</td>
<td>150</td>
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Table D-2. (continued)

OXIDES OF NITROGEN, POUNDS PER MILE

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<tr>
<td>Denver</td>
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<tr>
<td>Cincinnati and Los Angeles</td>
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<td>Denver values as percent of Cincinnati and Los Angeles</td>
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<td>Carbon Monoxide^b</td>
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<td>265</td>
<td>220</td>
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<td>160</td>
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<tr>
<td>Rural (45)</td>
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<td>110</td>
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<td>90</td>
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<td>3</td>
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<td>1</td>
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<td>24</td>
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<td>Nitrogen Oxides (NO₂)^b</td>
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<td></td>
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<tr>
<td>Urban</td>
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<td>20</td>
<td>18</td>
<td>17</td>
<td>15</td>
<td>11</td>
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<tr>
<td>Rural</td>
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<td>18</td>
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<tr>
<td>Particulates^d,e</td>
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<td>1.3</td>
<td>1.3</td>
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<td>Sulfur Oxides (SO₂)^f</td>
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<td>Aldehydes (HCHO)^g</td>
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<tr>
<td>Organic Acids (Acetic)^h</td>
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<td></td>
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</tr>
</tbody>
</table>

No legislation is in effect or has been proposed for these pollutants and thus only one factor is presented.

a - To convert emission factors to lb/1000 gal, assume the average gasoline-powered engine gets 12.5 miles per gal.
b - Uherka (1972)
c - Crankcase emissions for vehicles after 1962 are negligible. These factors are based on pre-1962 vehicles left in the vehicle population.
d - Wolsko (1972)
e - Urban Factor = Rural Factor
f - Based on sulfur content of 0.04% and a density of 6.17 lb/gal.
g - Rose (1965)
h - Rose (1965) and Bellomo (1971)

SOURCE: Transportation Air Pollutant Emissions Handbook, Argonne National Laboratory
population mix for the calendar year given, and not for vehicles of that model year only. The two types of operating conditions are urban travel, assumed to be at an average speed of 25 mph and beginning from a "cold" start, and rural travel, assumed to be at an average speed of 45 mph beginning from a "hot" start. Emission factors for speeds other than the assumed average speeds for urban and rural travel are available from Figures D-4 and D-5; factors for diesel engines, trucks, and buses are presented in Table D-4.

The report does not document the original emission values, nor is the weighting procedure explained. Furthermore, the speed-adjustment graphs for carbon monoxide and hydrocarbons (Figure D-5), seems to demonstrate a discontinuity at 35 mph between urban and rural travel. Although this discrepancy may be partly explained by the difference between "hot" and "cold" starts, there is still the problem of dealing with urban vehicles at speeds greater than 35 mph, and those rural vehicles at speeds less than 35 mph. No suggestions for resolving this problem are given in the EPA report; neither is there any indication as to where these speed curves originated, and what data were used to formulate them.

Model 4

T.D. Wolsko and M.T. Matthies (1972) have taken the EPA emission factors for gasoline-powered vehicles (see Table D-4), and have derived relationships for hydrocarbon (urban and rural driving), carbon monoxide (urban and rural driving), and nitrogen oxide (urban and rural driving are considered the same) emission rates.

Urban exhaust emissions factors of hydrocarbons (H0 in any year y (1960-1990) for cars of age i are calculated using equation (D-1).
Figure D-4. Speed adjustment graphs for hydrocarbon exhaust emission factors.

SOURCE: EPA (1971)
Figure D-5. Speed adjustment graphs for carbon monoxide emission factors.

SOURCE: EPA (1971)
Table D-4. Emission Factors for Diesel Engines

<table>
<thead>
<tr>
<th>Pollutant</th>
<th>Heavy-duty truck and bus engines</th>
<th>Locomotives</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>lb/10^3 gal</td>
<td>kg/10^3 liters</td>
</tr>
<tr>
<td>Particulates</td>
<td>13</td>
<td>1.56</td>
</tr>
<tr>
<td>Oxides of sulfur (SO_x as SO_2)</td>
<td>27</td>
<td>3.24</td>
</tr>
<tr>
<td>Carbon monoxide</td>
<td>225</td>
<td>27.0</td>
</tr>
<tr>
<td>Hydrocarbons</td>
<td>37</td>
<td>4.44</td>
</tr>
<tr>
<td>Oxides of nitrogen (NO_x as NO_2)</td>
<td>370</td>
<td>44.4</td>
</tr>
<tr>
<td>Aldehydes (as HCHO)</td>
<td>3</td>
<td>0.36</td>
</tr>
<tr>
<td>Organic acids</td>
<td>3</td>
<td>0.36</td>
</tr>
</tbody>
</table>

Data presented in this table are based on weighting factors applied to actual tests conducted at various load and idle conditions with an average gross vehicle weight of 30 tons (27.2 MT) and fuel consumption of 5.0 mi/gal (2.2 km/liter).

Data for trucks and buses based on average sulfur content of 0.20 percent, and for locomotives, on average sulfur content of 0.5 percent.

SOURCE: Environmental Protection Agency (1971)
\[ UEHC_i = (HCER_i) \cdot (SA) \cdot (THS) \cdot (DET_i) \] \hspace{1cm} (D-1) \\

where \\

\[ UEHC_i = \text{urban exhaust emission factor (gms/mi) in year } y \text{ for cars} \]
\[ \text{built in the year } (y-i) \]

\[ HCER_i = \text{exhaust factor (gms/mi) for HC for year } (y-i) \] 
\[ \text{(obtained from Table D-3)} \]

\[ SA = \text{a conversion factor for summer (e.g. test conditions) to} \]
\[ \text{annual temperature (factors are given in the context of} \]
\[ \text{this reference)} \]

\[ THS = \text{a conversion factor (.825) which converts the emission} \]
\[ \text{factor from a test speed of 18 mph to an average urban} \]
\[ \text{speed of 25 mph.} \]

\[ DET_i = \text{deterioration factor for vehicle of age } i \text{ (factors found in} \]
\[ \text{the context of this reference)} \]

Rural emissions factors for hydrocarbons are calculated using equation (D-2):

\[ REHC_i = (HCER_i) \cdot (SA) \cdot (THS) \cdot (HUR) \cdot (DET_i) \] \hspace{1cm} (D-2) \\

where \\

\[ REHC_i = \text{rural exhaust emission factor (gms/mi) in year } y \text{ for cars} \]
\[ \text{built in the year } (y-i) \]

\[ HUR = \text{a conversion factor (.67) which converts the emission factor} \]
\[ \text{from the average urban speed of 25 mph to the average rural} \]
\[ \text{speed of 45 mph.} \]

The urban exhaust emissions factors of carbon monoxide (CO) are calculated using equation D-3

\[ UECO_i = (COER_i) \cdot (SA) \cdot (TCS) \cdot (DET_i) \] \hspace{1cm} (D-3) \\

where \\

\[ UECO_i = \text{the urban emission factor (gms/mi) for a vehicle of age } i \]
\[ \text{in year } y. \]
\( \text{COER}_i \) = the emission factor (gms/mi) for cars of year \((y-i)\)

( obtained from Table D-3 )

\( \text{TCS} \) = a conversion factor (.766) for CO from the test speed of 18 mph to the average urban speed of 25 mph.

Rural carbon monoxide factors are calculated using equation D-4:

\[
\text{RECO}_i = \left( \text{COER}_i \right) \left( \text{SA} \right) \left( \text{TCS} \right) \left( \text{CUR} \right) \left( \text{DET}_i \right)
\]

(D-4)

where

\( \text{RECO}_i \) = rural emission factor (gms/mi) for a vehicle of age \(i\) in year \(y\)

\( \text{CUR} \) = a conversion factor (.61) for CO from the average urban speed of 25 mph to the average rural speed of 45 mph.

Urban and rural nitrogen oxide emission factors can be calculated using equation D-5:

\[
\text{UENO}_i = \left( \text{NOER}_i \right) \left( \text{SA} \right)
\]

where

\( \text{UENO}_i \) = the urban emission factors (gms/mi) for a vehicle of age \(i\) in the year \(y\) (obtained from Table 6)

\( \text{NOER}_i \) = the emission factor for cars of year \((y-i)\).

Wolsko and Matthies also give the basic emission factors for light-duty gasoline trucks (less than 6000 lbs. GVW), heavy-duty gasoline trucks (more than 6000 lbs. GVW), and diesel trucks and buses. The emission factors for diesel trucks and buses distinguish between pre-1970 and post-1970 engines. This change in emissions came as a result of the addition of a new needle-valve injector, which causes a reduction in the amount of fuel burned. The result of the engine modification was to decrease the level of hydrocarbon and carbon monoxide emissions, but the improved engine combustion efficiency is reflected in the increase in nitrogen oxide emission.
emissions (Wolsko, 1972).

The emission factors previously described are calculated at a single vehicle speed, either 25 mph for urban or 45 mph for rural travel. Since the average vehicle speed on roadways can vary quite widely in both urban and rural environments, speed adjustment factors are given in this report; they are the same as those in Model 3.

The Wolsko and Matthies report seems to be the most comprehensive, in that it incorporates not only speed adjustment factors, but also vehicle deterioration factors, test speed conversion factors, and an annual temperature conversion factor in the vehicle emission rates. However, no reference is given in the report as to the origin of the vehicle deterioration factors or the annual temperature conversion factors. Thus, the use of these factors in the network design model may not be appropriate.

Similarity of Model Foundations

It is interesting to note that all of these air pollution model formulations are directly or indirectly related to the initial research of Rose and Curnansky. Although this may increase the consistency of the several models, it does place a heavy burden on those two early studies. If the initial results of Rose and Curnansky were conclusive, it would be justified to use their results without further validation of their work. However, as pointed out earlier, Rose's conclusions provided general concepts and possible relationships between speed and emission rates, and not conclusive formulations.

In the subsequent years of model development, two classes of formulations have emerged, those based on Rose's findings, and those based
on Curnansky's. Although it appears that there is reasonable correlation between the work of Rose and Curnansky, it was not possible in this investigation to verify this match.

It appears that, through the years, the general concepts developed by Rose and Curnansky have been taken to be concrete relationships. Referring to Fig. D-6, the SRI report used the regression relationships of emissions as a function of speed from Rose's findings to formulate pre-1968 and post-1968 speed-emission models. In the second Report of the Secretary of HEW to Congress, regression relationships developed from Rose's work were used to formulate speed-emission tables. These tables were then used by Bellomo, to produce curves relating speed to emission rates. Similarly, the Tennessee Dept. of Transportation used the Bellomo curves to produce speed-emission rates.

Along the same lines (Fig. D-7) Curnansky's work provided carbon monoxide, hydrocarbon, and oxides of nitrogen emission tables for vehicle population mixes of various years. The National Technical Information Service (1971) guide for reducing auto air pollution used the emission tables given by Curnansky and applied a speed-emission conversion graph to produce an emission model. In the EPA report (1971), Curnansky's emission tables and a speed-emission relationship are also used. Argonne National Laboratory obtained the emission factors for gasoline-powered vehicles directly from the EPA report, and used these factors to formulate still another emission model.

Although a number of models are available for estimating air pollution emissions, the fact that the basis for most of them can be traced back to only two pieces of basic research makes it difficult to assess the validity of research in this area, and to evaluate alternative model
Rose: Comparison of Auto Exhaust Emissions

SRI Report

2nd Report of the Secretary of HEW to Congress

Bellomo, et al.

Tennessee Dept. of Transp.

Figure 6. Air Pollution Emission Models Based on the Work of Rose.

Gurnansky: Estimating Motor Vehicle Emissions

EPA: Compilation of Air Pollutant Emissions Factors

A Guide for Reducing Auto Air Pollution (PB-204-820)

Argonne: Transp. Air Pollution Handbook

Figure 7. Air Pollution Emission Models Based on the Work of Gurnansky.
formulations for use with the network design model.

With this cautionary note, it was decided to base a sample emissions model compatible with the network design model on the work of the Environmental Protection Agency (Model 3). While this model is not clearly superior to the others, it reflects a federally accepted approach to motor-vehicle emission estimation. Since all policy decisions concerning motor vehicle emission standards are likely to be compatible with the EPA model, it appears reasonable to base this effort on that model.

Computer Model Description

A program was written to serve as a post-processor for predicting, for a given network and set of flows, the exhaust emissions produced as a function of volumes, levels-of-service, and vehicle-truck mix. This program does not treat atmospheric diffusion processes, and thus does not provide direct information about air quality impacts.

Based on a given emission factor table for both gasoline and diesel powered vehicles, and on a speed adjustment factor based on level-of-service estimates, vehicle emissions of CO, HC, and NO are computed for each link. These emissions are then aggregated by volume at each level-of-service for each link, in order to determine the total emissions by type for each link, and for network as a whole. Thus, the output allows the user to see both the link emission characteristics and the total network emission characteristics.

The program input requirements are:
1. network design year (NYEAR);
2. number of links in the network (k);
3. starting node for each link M (NS(M));
4. finishing node for each link M (NF(M));
5. mean free travel time for each link M (TO(M));
6. practical capacity for each link M (VPH(M));
7. length of each link M (DIS(M)).

The program automatically determines the vehicle age mix; however, these data are only stored for the design years 1970-75 at the present time. The emission model operates on a peak period basis only. Since the network design model uses average daily traffic, it is necessary to convert these figures to a peak period volume estimate. This is accomplished using methods specified in the Highway Capacity Manual for estimating 30th highest hour volume. Link speeds are then calculated using the speed-volume function used by the Federal Highway Administration for capacity restrained traffic assignment; an alternative model has also been developed using a more realistic parabolic speed-volume curve.

In its present form, the emission model estimates the production of carbon monoxide, hydrocarbons, and oxides of nitrogen, in pounds per (peak) hour, produced by each link. These are reported in both absolute numbers and percentages. In addition, the model provides network-wide summaries of these emissions in terms of total pounds produced. Following this appendix an example of the output of this program is given.

A complete listing of the program, along with its typical output, is provided at the end of this appendix. Since the program is carefully annotated, it serves as its own users manual, and no further documentation is provided.

Additional applications of this model, beyond the scope of the network design process, are also possible. Because this approach includes in
the air pollution model the effect of the relationship between speed and volume, it offers the planner an alternative, simplified post-processor approach to the estimation of vehicle emissions from any proposed or existing highways. The computer program provided can be used directly for this task.
References


Stanford Research Institute, (Aug., 1971), Methods of Evaluation of The Effects of Transportation Systems on Community Values, DOT - FHWA.


Uherka, K.L., Zuu-Chang Hong, (1972), A Diffusion Model For Metropolitan Expressways, Working paper at the University of Illinois, Chicago Circle Campus, unpublished - UICC.

PROGRAM TOYS(TAPE5,TAPE6,TAPE7)

5 THIS PROGRAM WAS WRITTEN AS A THEORETICAL POST PROCESS TOOL, WHICH
CAN BE USED IN PREDICTING, FOR A GIVEN NETWORK, THE EXHAUST EMIS-
SION RATES PRODUCED FOR A GIVEN TRAFFIC VOLUME AND VEHICLE TRUCK
MIX. DUE TO THE PRESENT STATE OF THE ART, THIS PROGRAM IN NO WAY
DEALS WITH THE DISPERSSION CHARACTERISTICS CAUSED BY THE DIFFUSION
PROCESS. RATHER IT USES A GIVEN EMISSION FACTOR TABLE FOR BOTH
GASOLINE AND DIESEL POWERED VEHICLES AND CALCULATES A SPEED
ADJUSTMENT FACTOR TO THE VEHICLE EMISSION RATE BASED ON THE
TRAFFIC CHARACTERISTICS OF THE LINK. THE EMISSION RATE IS THEN
AGGREGATED BY VOLUME PER LINK TWICE, FIRST TO DETERMINE THE
EMISSION RATE FOR THE PARTICULAR LINK, AND SECOND TO DETERMINE THE
EMISSION RATE FOR THE NETWORK. THIS THE OUTPUT ALLOWS THE USER TO
SEE BOTH THE LINK EMISSION CHARACTERISTICS AND THE TOTAL NETWORK
EMISSION CHARACTERISTICS.

10 NOTE: THIS PROGRAM HAS INITIALLY BEEN DIMENSIONED FOR 50 LINKS, THIS
IS ONLY AN ARBITRARY NUMBER AND IS NO WHERE NEAR THE CAPACITY
OF THE COMPUTER. IF ONE HAS A NETWORK OF MORE THAN 50 LINKS,
ALL HE NEEDS TO DO IS TO REDIMENSION THE PROGRAM.

15 MODEL ASSUMPTION 1: CARBON MONOXIDE, HYDROCARBONS, AND OXIDES OF
NITROGEN ARE THE THREE MAJOR AIR POLLUTANTS RE-
LEASED BY MOTORED VEHICLES. THESE VEHICLES EMIT
ONLY A SMALL PERCENTAGE OF TWO OTHER POLLUTANTS
USUALLY CONSIDERED IN OVERALL AIR POLLUTION
PROBLEMS: SULFUR OXIDES AND SUSPENDED PART-
TICULATES. MOST OF THE LEAD SALTS WHICH ARE
RELEASED IN SUSPENDED PARTICULATE FORM WILL BE
ELIMINATED WITH THE USE OF LEAD FREE GASOLINES.
(REF: TRANSPORTATION AIR POLLUTANT EMISSION HAN-
DBOOK ARGONNE NATIONAL LABORATORY)

20 MODEL ASSUMPTION 2: SINCE THIS MODEL WILL MAINLY BE USED AS A PRED-
ICTIVE TOOL FOR PRESENT AND FUTURE NETWORK
SYSTEMS, AND SINCE FEDERAL LEGISLATION HAS ONLY
BEEN PROPOSED FOR VEHICLES UP TO 1975 (REF: EPA
COMPLIANCE OF AIR POLLUTANT EMISSION FACTORS) WE
ARE ONLY CONCERNED WITH PREDICTIVE NETWORKS FOR

25 DIMENSION PCO(50),PCH(50),PNOX(50)
DIMENSION AS(50),S(50),NX(50)
DIMENSION TACO(50),TAHC(50),TANOX(50)
DIMENSION TC0(50),THC(50),TN0X(50)
DIMENSION ACO(6,2),AHCE(6,2),ANOX(6,2)
DIMENSION DCO(1),DHCE(1),DNOX(1)
DIMENSION NS(50),NF(50),AL(50),VPH(50),DIS(50),T0(50),C(50),S0(50)

30 DO 1 I=1,6
DO 2 K=1,2

35 THE CARBON DIOXIDE(ACO), HYDROCARBON(ACH), AND OXIDES OF
NITROGEN(ANOX) EMISSION FACTORS FOR GASOLINE POWERED VEHICLES
WERE OBTAINED FROM TABLE 3-1, REF: EPA COMPILATION OF AIR POL-
LUTANT EMISSION FACTORS. THE EMISSION FACTORS ARE GIVEN BY YEAR I
GRAMS/VEH. MILE, THE GASOLINE POWERED MOTOR VEHICLE CATEGORY CONSISTS OF THREE MAJOR TYPES OF VEHICLES: PASSENGER CARS, LIGHT DUTY TRUCKS, AND HEAVY DUTY GASOLINE POWERED VEHICLES.

MODEL ASSUMPTION 3: SINCE THE FACTORS GIVEN IN TABLE 3-1 ARE FOR VEHICLE POPULATION MIX FOR THE CALENDAR YEAR GIVEN AND NOT FOR VEHICLES OF THAT MODEL YEAR ONLY, WE ARE ASSUMING THAT THEIR VEHICLE POPULATION IS OF SUBSTANTIAL BASE.

READ(5,3) ACO(I,K), AHC(I,K), ANOX(I,K)
3 FORMAT(F7.2,3X,F7.2,3X,F7.2)
2 CONTINUE
1 CONTINUE

THE CARBON DIOXIDE (DCO), HYDROCARBON (DHC), AND OXIDES OF NITROGEN (DNOX) EMISSION FACTORS FOR DIESEL POWERED VEHICLES WERE OBTAINED FROM TABLE 6, p.28, REF. TRANSPORTATION AIR POLLUTANT EMISSION HANDBOOK, ARGONNE NATIONAL LAB. THE EMISSION FACTORS ARE GIVEN FOR A SINGULAR YEAR SPAN ONLY, (GT. 1970) IN UNITS OF GRAM/VEH.

READ(5,3) DCO(I), DHC(I), DNOX(I)
READ(5,8) TRUCK, NYEAR, K
8 FORMAT(F3,2X,2X,12,2X,12)
DO 7 M=1,K

INPUT REQUIREMENTS/ VARIABLE (VARIABLE NAME) — UNITS

PERCENTAGE OF DIESEL TRUCKS IN VEHICLE MIX (TRUCK) — DECIMAL FORM

NUMBER OF LINKS IN THE NETWORK (K)
STARTING NODE FOR A LINK M (NS(M))
FINISHING NODE FOR A LINK M (NF(M))
MEAN FREE TRAVEL TIME FOR LINK M (TO(M)) — HRS.
PRACTICAL CAPACITY FOR LINK M (C(M)) — VEHICLES PER DAY
VOLUME FOR A LINK M (VPH(M)) — VEHICLES PER DAY
LENGTH OF LINK M (DIS(M)) — MILES

READ(5,9) NS(M), NF(M), TO(M), C(M)
9 FORMAT(I2,1X,I2,1X,F5.4,2X,F4,1)
7 CONTINUE
DO 77 M=1,K
READ(5,78) VPH(M), DIS(M)
78 FORMAT(F6.0,2X,F4,1)
77 CONTINUE
TOTCO=0.0
X=0.0
TOTHC=0.0
TOTNOX=0.0

PEAK HOUR ADJUSTMENT FACTORS, THE PROGRAM IS DESIGNED TO TAKE BOTH PRACTICAL CAPACITY (C(M)) AND INPUT VOLUMES (VPH(M)) FOR EACH LINK M IN TERMS OF VEHICLES PER 24-HR DAY; BUT SINCE WE ARE INTERESTED IN PEAK HOUR EMISSION RATES, NOT DAILY AVERAGES, WE NOW
CONVERT OUR DAILY VOLUMES ON VEHICLES PER DAY TO PEAK VOLUMES
IN VEHICLES PER HOUR BASED ON THE DAILY PRACTICAL CAPACITY OF THE
ROAD (REF. HIGHWAY CAPACITY MANUAL, 1965, P40*) AND CONVERT OUR
DAILY PRACTICAL CAPACITY ALSO TO VEHICLES PER HOUR FOR THE PEAK
HOUR. (REF. WILBUR SMITH AND ASS., SIoux FALLS METROPOLITAN AREA
TRANSPORTATION STUDY, AUG. 1965). THESE CHANGES ARE ESSENTIAL IN
ORDER TO OBTAIN THE PEAK HOUR SPEED ON EACH LINK.

MODEL ASSUMPTION 4* WE ARE AT THIS TIME CONSIDERING THE 30TH HIGHEST
HOUR TO BE OUR REPRESENTATIVE PEAK HOUR FLOW.

                   DO 11 L=1,K
                 IF(C(L).GT.10.) GO TO 50
                     VPH(L)=VPH(L)*.106
                     GO TO 55
                  50 IF(C(L).GT.20.) GO TO 51
                     VPH(L)=VPH(L)*.100
                     GO TO 55
                  51 IF(C(L).GT.35.) GO TO 52
                     VPH(L)=VPH(L)*.095
                     GO TO 55
                  52 VPH(L)=VPH(L)*.12
                     C(L)=C(L)*.12
                     S0(L)=DIS(L)/TO(L)
                     AS(L)=(VPH(L)/C(L)*1000.)**4

THE SPEED ON EACH LINK S(L) IS CALCULATED FROM THE SPEED VOLUME
RELATIONSHIP, S=SO/(1+1.15(V/C)**4) (MAR MODEL) WHERE SO IS THE
MEAN FREE SPEED (DIS/TO) AND V IS THE PEAK VOLUME. C THE PEAK
PRACTICAL CAPACITY.

                   S(L)=SO(L)/(1+1.15*AS(L))
                 IF(S(L).GE.0.0) NX(L)=1
                 IF(S(L).GT.35.) NX(L)=2

CONTINUE

THE SUBROUTINE AUTO ADJUSTS THE CARBON MONOXIDE AND HYDROCARBON
EMISSION RATES FOR GASOLINE POWERED VEHICLES AS A FUNCTION OF THE
SPEED OF THE VEHICLE. THE SPEED ADJUSTMENT EQUATION TO THE
EMISSION RATES, GIVEN IN THE SUBROUTINE, ARE LINEARIZED AP-
PROXIMATIONS OF THE GRAPHICAL RELATIONSHIPS GIVEN IN FIGS 3-1 AND
3-2. (REF. EPA COMPILATION OF AIR POLLUTANT EMISSION FACTORS, P3-3,
3-4). THE GRAPHS ARE APPROXIMATED IN INCREMENTS OF 5 MPH STARTING
FROM 10 MPH TO 65 MPH. OXIDES OF NITROGEN EMISSION RATES ARE
INDEPENDENT OF AVERAGE VELOCITY SPEED. (REF. SAME AS ABOVE)

MODEL ASSUMPTION 5* THE SPEED ADJUSTMENT GRAPHS ARE FOR TWO TYPES OF
VEHICLE OPERATION CONDITIONS. URBAN TRAVEL (10-35
MPH) WHICH BEGINS FROM A #COLD START#, AND RURAL
TRAVEL (35-65 MPH) WHICH BEGINS FROM A #HOT START#.
* BECAUSE OF THESE TWO STARTING CONDITIONS THERE
IS A DISCONTINUOUS JUMP IN THE GRAPHS, AT 35 MPH.
* BUT DUE TO THE FACT THAT ON URBAN FREEWAYS, EVEN
AT PEAK HOURS, WE MAY OCCUR SPEEDS GREATER THAN
35 MPH, WE ARE USING THE RURAL GRAPHS FOR
SPRINGS GREATER THAN 35 MPH IGNUING THE EFFECTS
CAUSED BY THE DISCONTINUOUS JUMP, WE ARE ALSO
CONSIDERING ALL SPEED LESS THAN 10MPH TO EMIT
AT A RATE WHICH IS EQUAL TO 10 MPH.

CALL AUTO(X,NYEAR,S,A,ACG,AH,C,ANOX,TACO,TAHC,TANOX)
DO 33 L=1,K

TCO(L),THC(L),TNOX(L) ARE THE TOTAL CARBON MONOXIDE, HYDROCARBON,
AND OXIDES OF NITROGEN EMISSION RATES (IN UNITS OF LR/HR) FOR
EACH LINK L IN THE NETWORK. THE FIRST HALF OF THE EQUATION IS THE
CONTRIBUTION OF GASOLINE POWERED VEHICLES TO THE EMISSION RATE
TOTAL AND THE SECOND HALF IS THE DIESEL POWERED VEHICLE CONT-
RIBUTION. .002205 IS A CONVERSION FACTOR WHICH CONVERTS GW/HR
TO LB/HR.

TCO(L)=(1.0-TRUCK)*VPH(L)*TACO(L)+TRUCK*VPH(L)*DCO(1))*0.002205
THC(L)=(1.0-TRUCK)*VPH(L)*TAHC(L)+TRUCK*VPH(L)*DHC(1))*0.002205
TNOX(L)=(1.0-TRUCK)*VPH(L)*TANOX(L)+TRUCK*VPH(L)*DNIX(1))*0.002205

TCO(L)=TCO(L)*DIS(L)
THC(L)=THC(L)*DIS(L)
TNOX(L)=TNOX(L)*DIS(L)

TOTO,TOTHC=TOTNOX ARE TOTAL CARBON MONOXIDE, HYDROCARBON, AND
OXIDES OF NITROGEN EMISSION RATES FOR THE ENTIRE NETWORK (IN
UNITS OF LR/HR).

TOTO=TOTO+TCO(L)
TOTHC=TOTHC+THC(L)
TOTNOX=TOTNOX+TNOX(L)

X=(VPH(L)*DIS(L))/1000.*X

33 CONTINUE

Y,Z,W ARE TOTAL CARBON MONOXIDE, HYDROCARBON AND OXIDES OF
NITROGEN EMISSION RATES FOR THE ENTIRE NETWORK IN UNITS OF
LB/1000 VEH MILES, WHERE X IS THE CONVERSION FACTOR FROM LR/HR TO
LB/1000 VEH MILES.

Y=TOTCO/X
Z=TOTHC/X
W=TOTNOX/X
DO 12 N=1,K

PCO(N),PHC(N),PNOX(N) ARE THE CARBON MONOXIDE, HYDROCARBON, AND
OXIDES OF NITROGEN PERCENTAGE CONTRIBUTION OF EACH LINK (N) TO THE
TOTAL NETWORK EMISSION RATES FOR EACH POLLUTANT.

PCO(N)=(TCO(N)/TOTO)*100.
PHC(N)=(THC(N)/TOTHC)*100.
PNOX(N)=(TNOX(N)/TOTNOX)*100.

12 CONTINUE
IYEAR=1969+NYEAR
WRITE(6,13) IYEAR
13 FORMAT(*5X,I4,2X,* EXHAUST EMISSION TABLE*)
WRITE(6,14)
14 FORMAT(2X*, LINK*, 6X*, TRAFFIC CHARACTERISTICS*, 16X*,
   1* CARBON MONOXIDE*, 10X*, HYDROCARBONS*, 9X*, OXIDES OF NITROGEN*)
WRITE(6,15)
15 FORMAT(10X*, (VPH)*, 2X*, (WPH)*, 2X*, (LANES)*, 2X*, (LENGTH)*,
   17X*, (LB/HR)*, 4X*, (0/0 TOT)*, 5X*, (LR/HR)*,
   12X*, (0/0 TOT)*, 4X*, (LB/HR)*, 3X*, (0/0 TOT)*)
DO 20 I=1,K
WRITE(6,16) 'NS(I), NF(I), VPH(I), S(I), AL(I), DIS(I), TCO(I), PCO(I),
   1THC(I), PNC(I), TNOX(I), PNCX(I)
20 CONTINUE
WRITE(6,17) TOTCO
17 FORMAT(/5X*, TOTAL CARBON MONOXIDE EMISSION BY NETWORK=*, F12.1
   1* (LB/HR)*)
WRITE(6,18) TOTHRC
18 FORMAT(/5X*, TOTAL HYDROCARBON EMISSION BY NETWORK=*, F12.1
   1* (LB/HR)*)
WRITE(6,19) TOTNOX
19 FORMAT(/5X*, TOTAL OXIDES OF NITROGEN EMISSION BY NETWORK=*, F12.1
   1* (LB/HR)*)
WRITE(6,40) Y
40 FORMAT(/5X*, TOTAL CARBON MONOXIDE EMISSION BY NETWORK=*, F8.1
   1* (LB PER 1000 VEHICLE MI)*)
WRITE(6,41) Z
41 FORMAT(/5X*, TOTAL HYDROCARBON EMISSION BY NETWORK=*, F8.1
   1* (LB PER 1000 VEHICLE MI)*)
WRITE(6,42) W
42 FORMAT(/5X*, TOTAL OXIDES OF NITROGEN EMISSION BY NETWORK=*, F8.1
   1* (LB PER 1000 VEHICLE MI)*)
STOP
END
SUBROUTINE AUTO

READ ACO(6,2), AMC(6,2), ANOX(6,2)
REAL S(50), TACO(50), TAHC(50), TANOX(50)
INTEGER NX(50)

DO 50 I=1,K
  GO TO (1,2,3,4,5,6)
1 IF(S(I).GE.15.) GO TO 10
  CO=0.12*S(I)+3.28
  HC=0.07*S(I)+2.4
  GO TO 10
4 IF(S(I).GE.20.) GO TO 5
  CO=0.07*S(I)+2.65
  HC=0.04*S(I)+1.95
  GO TO 10
5 IF(S(I).GE.25.) GO TO 6
  CO=0.05*S(I)+2.25
  HC=0.03*S(I)+1.75
  GO TO 10
6 IF(S(I).GE.30.) GO TO 7
  CO=0.02*S(I)+1.75
  HC=0.05*S(I)+1.45
  GO TO 10
7 IF(S(I).GE.35.) GO TO 2
  CO=0.02*S(I)+1.5
  HC=0.02*S(I)+1.5
  GO TO 10
2 IF(S(I).GE.40.) GO TO 8
  CO=0.01*S(I)+1.81
  HC=0.01*S(I)+1.75
  GO TO 10
8 IF(S(I).GE.45.) GO TO 9
  CO=0.01*S(I)+1.81
  HC=0.01*S(I)+1.675
  GO TO 10
9 IF(S(I).GE.50.) GO TO 12
  CO=0.01*S(I)+1.72
  HC=0.02*S(I)+1.54
  GO TO 10
12 IF(S(I).GE.55.) GO TO 13
  CO=0.01*S(I)+1.62
  HC=0.02*S(I)+2.54
  GO TO 10
13 IF(S(I).GE.60.) GO TO 14
  CO=0.01*S(I)+1.565
  HC=0.01*S(I)+1.485
  GO TO 10
14 CO=0.01*S(I)+1.485
  HC=0.009*S(I)+1.365
10 TACO(I)=CO*ACO(NYEAR,NX(I))
  TAHC(I)=HC*AHC(NYEAR,NX(I))
  TANOX(I)=ANOX(NYEAR,NX(I))
50 CONTINUE
RETURN
END
### Table D-6. Sample Emissions Program Output.

<table>
<thead>
<tr>
<th>Link</th>
<th>Traffic Characteristics</th>
<th>1970 Exhaust Emission Table</th>
</tr>
</thead>
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<td>(VPH)</td>
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<tr>
<td>10-16</td>
<td>343</td>
<td>24.4</td>
</tr>
</tbody>
</table>

**Total Carbon Monoxide Emission by Network = 2351.2 (LR/HR)**

**Total Hydrocarbon Emission by Network = 2736.2 (LB/HR)**

**Total Oxides of Nitrogen Emission by Network = 1879.6 (LB/HR)**

**Total Carbon Monoxide Emission by Network = 235.1 (LB per 1000 Vehicle Mi)**

**Total Hydrocarbon Emission by Network = 27.3 (LB per 1000 Vehicle Mi)**

**Total Oxides of Nitrogen Emission by Network = 18.8 (LB per 1000 Vehicle Mi)**
APPENDIX E

ACCESSIBILITY MEASURES

Introduction

The essential purpose behind the construction and operation of urban transportation systems is to provide for the movement of people and goods within a region. In the construction of a network design model, then, it will be important to include considerations of the degree to which alternative systems provide this capability. The service provided by a transportation system to a particular part of the region, or to a particular group of residents residing in a given location, is often termed accessibility.

It is the objective of this discussion to explore alternative measures of accessibility, to evaluate those measures with special emphasis on their compatibility with current network design model formulations, and to propose a strategy for treatment of accessibility in that model. Toward this end, a brief review is given in the next section of some current approaches to the measurement of accessibility.

A Summary of Current Measures of Accessibility

Among all factors included in measures of accessibility found in this study, three are most salient by virtue of their importance:

a. Travel friction
b. willingness to travel
c. Factors affecting spatial distribution of opportunities

These factors purport to indicate respectively the difficulty in traveling given the opportunity and the inclination to do so, the propensity to travel and the attractions which are prospective ends of travel desires.

Not all of the references cited in the body of this report attempt to account completely for all three of these factors. Therefore each candidate measure will be presented along with a brief description of the basic assumptions
and a comment on its compatibility with the current network design models.

Hansen defines accessibility as a measure of the intensity of the possibility of interaction. He formulates the measure as follows: accessibility at area \( i \) to a particular type of activity at area \( j \) (e.g., employment) is directly proportional to the size of the activity at area \( j \) (e.g., number of jobs) and inversely proportional to some function of the distance separating the two areas. He then formulates the relative accessibility of \( i \) to \( j \) as

\[
A_{ij} = \alpha_j / d_{ij}^b
\]  

(E-1)

whereas accessibility at \( i \) is

\[
A_i = \sum_{j=1}^{n} A_{ij}
\]  

(E-2)

where \( \alpha_j \) = measure of the size of activity at area \( j \).

\( d_{ij} \) = distance function from \( i \) to \( j \) (e.g., airline distance).

\( b \) = exponent to which \( d_{ij} \) is raised reflecting propensity to travel (the smaller the value of \( b \), the greater the propensity to travel).

\( A_i \) = accessibility at area \( i \).

\( n \) = number of areas \( j \) to which \( i \) is attracted.

A candidate measure is termed compatible with current network design models if the measure is readily inserted into the constraint matrix or the objective function of the model. Because current models are linear programs, and the relationship

\[
A_i = \sum_{j=1}^{n} \left[ \frac{\alpha_j}{d_{ij}^b} \right]
\]  

(E-3)

is non-linear in \( d_{ij} \), this candidate measure is not compatible with the current network design models.
Olsen (1972) defines accessibility in terms of quality of transportation service provided. In his formulation, two super districts (zones) i and j are selected and the transportation mode k is determined. Then based upon existing data, travel times (costs) \( Y_{ijk} \) for all i, j, and k between the centroids are regressed on airline distances between the centroids. In this manner, a "norm" (the linear regression equation) is selected for travel between i and j via mode k as a function of airline distance. The actual travel time \( \Lambda Y_{ijk} \) from the centroid in i to the centroid in j via mode k is then compared to the expected travel time \( Y_{ijk} \). A measure of accessibility, therefore, is given by the sum of the actual deviations from the "norm"

\[
SAD_{ik} = \sum_{j \neq i} (Y_{ijk} - \Lambda Y_{ijk})
\]  

(E-4)

at area i via mode k. Another candidate measure is given by the sum of the fractional deviations

\[
SFD_{ik} = \sum_{j \neq i} \frac{Y_{ijk} - \Lambda Y_{ijk}}{\Lambda Y_{ijk}}
\]  

(E-5)

Notice that in each case, a negative value of the measure indicates relatively good service, while a positive value indicates relatively poor service.

Two points must be made concerning these proposed measures. The first is that each measure depends not at all on attractions at other zones, the second that accessibility for a particular zone is too dependent upon the spatial interconnections of all other zones. In other words, because the expected travel times (costs) are found by the least squares criterion, the accessibility of the "average" zone will tend to be near zero, indicating neither poor nor good accessibility. This measure would not be particularly difficult to operationalize in the context of the network design model if one were to either obtain expected
travel times (costs) \( Y_{ijk} \) or set desirable standards. However, because of the obvious conceptual difficulties mentioned before, it isn't particularly useful.

In their paper, Cohen and Basner (1972) define accessibility as the ease with which one can travel to various areas of interest. The measure is essentially the same as in Hansen and is given by

\[
A_i = \sum_{j=1}^{n} \alpha_j F_{ij}
\]

(E-6)

where

\[
F_{ij} = \frac{1}{t_{ij}^b}
\]

(E-7)

\( \alpha_j \) = the number of attractions in \( j \) (e.g., jobs)

\( F_{ij} \) = the friction factor from \( i \) to \( j \)

\( t_{ij} \) = distance function from \( i \) to \( j \) (e.g., time or cost in a minimum time path between \( i \) and \( j \))

\( b \) = exponent to which \( t_{ij} \) is raised reflecting willingness to travel.

Because of those reasons given by Hansen, this formulation of accessibility is not compatible with present model formulations.

Ingram (1971) defines accessibility in terms of the ability of a transportation system to provide a low cost and/or a quick method of overcoming distance between different locations. Furthermore, he distinguishes between relative accessibility and integral accessibility, the former being the degree to which two points (places) on the same surface are connected, the latter being the degree of interconnection with all other points on the same surface. The measure is given as

\[
A_i = \sum_{j=1}^{n} a_{ij}
\]

(E-8)

where

\( A_i \) = integral accessibility of zone (or point) \( i \)
\[ a_{ij} = \text{relative accessibility of zone } j \text{ at } i \]

Two candidate measures of \( a_{ij} \) are given by

\[ a_{ij} = 100 \, d_{ij}^{-k} \quad \text{(E-9)} \]

\[ a_{ij} = 100 \, e^{-d_{ij}} \quad \text{(E-10)} \]

where

\[ d_{ij} = \text{straight line distance between zones } i \text{ and } j \]

\[ k = \text{a constant to be determined reflecting propensity to travel.} \]

Notice that these measures are functionally the same as in Hansen, therefore incompatible with the current network design models.

Kissling (1966) desires to relate his measure of accessibility to time and cost of traveling rather than in units of distance. He adopts the measure given by Shimbel:

\[ A(i) = \sum_{j=1}^{n} d(i,j) \quad \text{(E-11)} \]

where

\[ A(i) = \text{accessibility at zone (point) } i \]

\[ d(i,j) = \text{distance function (time or cost) associated with minimum paths from } i \text{ to } j. \]

\[ n = \text{number of nodes.} \]

Furthermore, similar to Shimbel's treatment, the dispersion \( D(N) \) of the network \( N \) is defined as

\[ D(N) = \sum_{i=1}^{n} A(i) \quad \text{(E-12)} \]

Two things are obvious in these measures of accessibility: each link is weighted equally with any other link and the peripheral nodes are penalized by not being centrally located. Kissling suggests a method by which one can measure the importance of various links in the network. The procedure is to analyze
D(N) for the network with all links intact. Then a link is removed and the minimum cost (time) paths are re-calculated and C(N) is computed for the altered network. The increase in D(N) is an indication of the importance of the link removed. A small increase means the link is unimportant, but a large increase means the link is crucial.

The above measure D(N) of the dispersion of the network can be interpreted as follows: if one were to assign one trip to each link ij then D(N) would be the total minimum user cost for the network N. However, since in the present network design model links are weighted differently (by volume), this measure of accessibility is of little use.

Nakkash (1966) defines accessibility as contact with relatively little friction. The measure which he uses is essentially the same as in Cohen and Basner, with one exception: he defines relative accessibility as follows:

\[
RA_{ikm} = \frac{A_{ikm}}{\sum_{i=1}^{R} A_{ikm}}
\]

(E-13)

where

\[
A_{ikm} = \sum_{j=1}^{N} S_{jk} F_{ijm}
\]

(E-14)

and

\[
RA_{ikm} = \text{relative accessibility of zone } i \text{ to activity } k \text{ for purpose } m
\]

\[
F_{ijm} = \text{friction factor as a function of travel time from zone } i \text{ to zone } j \text{ for purpose } m.
\]

\[
S_{jk} = \text{size of activity } k, \text{ zone } j.
\]

Because the friction factor \( F_{ijm} \) is usually defined as the reciprocal of travel time \( t_{ij} \) raised to some power \( b \), this measure is not compatible with the current network design model.

Gur (1971) defines accessibility as a property of an area which measures the benefits, costs and availability of substitutes for activities involving trips. He begins by defining the near domain as the location of a trip
generator (e.g., a person) and the far domain as a goal. A trip will be defined as movement from a near to a far domain. Hence he is concerned mainly with far accessibility (near accessibility is defined for travel within the near domain). Now given the proportion \( P_j \) of trips originating in near domain (zone) \( i \) and ending in far domain (zone) \( j \), he defines

\[
A_i = \frac{n}{\sum_{j=1}^{n} F_{ij} P_j}
\]  

(E-15)

where

\( A_i \) = accessibility of zone \( i \)
\( F_{ij} \) = friction factor of travel from \( i \) to \( j \)
\( P_j \) = proportion of trips originating in \( i \) and terminating in \( j \)
\( n \) = the number of zones

Notice if one defines

\[
P_j = \frac{\alpha_j}{\sum_{j=1}^{n} \alpha_j}
\]  

(E-16)

and

\[
\sum_{j=1}^{n} \alpha_j = M
\]  

(E-17)

then

\[
A_i = \frac{\sum_{j=1}^{n} F_{ij} P_j}{\sum_{j=1}^{n} \alpha_j} = \frac{n}{\sum_{j=1}^{n} \alpha_j} \cdot \frac{\sum_{j=1}^{n} F_{ij} \alpha_j}{M} = A_i(C)
\]  

(E-18)

where \( A_i(C) \) stands for the index defined in \( C \). Therefore the index of accessibility in this case is equal to a fraction \( \left( \frac{1}{M} \right) \) of the index given in \( C \).

Gur lists the properties of attraction: (1) it is a property of a zone with respect to a trip type independent of the origin zone. (2) it is dependent upon the intensity of activities within a zone which relate to the trip type being considered.

He quantifies attraction by the following:

\[
P_j = \sum_{k=1}^{M} r_k P_{E_{kj}}
\]  

(E-19)

E-7
where

\[ O_j^p = \text{attractions of zone } j \text{ to trip type } p \]
\[ r_k^p = \text{rate of attraction of a unit activity of type } k \text{ to trip type } p \]
\[ E_{kj} = \text{amount of activity type } k \text{ in zone } j. \]
\[ m = \text{number of activity types} \]

The current measures of activity (for work trips) are (1) land area (2) floor space and (3) employment.

Travel friction is a measure of the separation between two points, and he uses different measures for different systems: for highway networks, daily average travel time plus parking times are used. For transit networks, zonal average approach time (time elapsed on access portions at both ends of the trip), plus travel time are used.

Given the measures of attraction and friction, he defines four different measures of far accessibility:

\[ A_{i1}^w = f(0^w, F_i^1, L^w) \]
\[ A_{i2}^w = f(0^w, F_i^2, L^w) \]
\[ A_{i1}^n = f(0^n, F_i^1, L^n) \]
\[ A_{i2}^n = f(0^n, F_i^2, L^n) \]  \hspace{1cm} (E-20)

where

\[ A_i = \text{accessibility of zone } i \]
\[ w,n = \text{respectively work, non-work} \]
\[ 1,2 = \text{respectively car available, no car} \]
\[ 0 = \text{vector of attractions of all zones in the region} \]
\[ F_i = \text{vector of travel friction from zone } i \text{ to all other zones} \]
L = probability that a trip is selected if it is considered

The measure of joint far accessibility \( A_i \) is defined as

\[
A_i = A_i^1 W_i^1 + A_i^2 W_i^2
\]  
(E-21)

where

\( W_i^1, W_i^2 \) are the weights of the \( A_i \)s

\( A_i^1, A_i^2 \) are respectively the accessibilities of zone \( i \) modes 1 and 2.

The measure of accessibility given in equation (E-15) is quite compatible with the present network design model, since the friction factor \( F_{ij} \) is defined to be travel time. More will be said about this in a subsequent section.

In the SRI report (1971), accessibility is defined as the ease with which one can reach desired opportunities. The authors go on to say that ideally, an index or measure should measure

(1) latent demand.

(2) the opportunities for satisfying demand that can be reached more easily under a new system.

(3) the matching of needs and opportunities.

(4) competition for these opportunities from other zones.

(5) the amount access is improved and whether trips to it are likely to be taken.

Since pertinent data is generally not available upon which to base measures of those items above, the authors suggest the consideration of average cost/trip per zone as the measure of the zonal accessibility. Thus one finds

\[
A_i = \sum_{j=1}^{N} P_{ij} F_{ij}
\]  
(E-22)
where

\[ A_i = \text{accessibility of zone } i \]
\[ P_{ij} = \text{proportion of trips from } i \text{ to } j \]
\[ F_{ij} = \text{friction factor from } i \text{ to } j \text{ and specifically, travel cost (time) from } i \text{ to } j. \]

Notice that this is identical to \( G_u \) hence the measure is compatible with the current network design model.

In conclusion, the last two measures of zonal accessibility (i.e. accessibility made zone-specific) are the most compatible with the current network design model formulations. In both cases, the measure of accessibility for zone \( i \) is average travel friction per trip in zone \( i \). If travel friction is taken to be travel cost (time), then the measure is average cost per trip. Therefore, to maximize accessibility measured in this way, it will be necessary to minimize average travel time.

In the next section, the relationship between this objective and the current formulations of the network design model is explored.

**Suggested Version of the Network Design Model**

In either version of the network design model (mixed integer or continuous), total user cost is minimized subject to a body of constraints such as meeting demand and budget requirements and maintaining conservation of flow in the network. Regardless of the measure of accessibility to be used, only three ways of incorporating the measure in the model seem feasible. The first way is to allow the model to assign volumes to the links in order to minimize total user cost and subsequently (in a post-processing manner) measure the accessibility for each zone \( i \) of the optimal network. This would, however, not allow for finding the optimal network in terms of accessibility and total user cost since
the evaluation does not allow for feedback into the traffic assignment model. The second way is to include the measure in the objective function together with total user cost and somehow weight the two measures in a meaningful manner so as to minimize total user cost and maximize accessibility. This obviously has a host of associated difficulties. The third and final way is to include the measure in the body of constraints so that, for instance, total user cost would be minimized subject to a certain level of accessibility.

Only the last two ways seem feasible since the first method would be costly in terms of finding the "optimally accessible" network. Therefore since the measure must be included in either the objective function or the constraint matrix, the measure must be a linear function of the decision variables (in either case, volumes assigned to links). In light of these requirements, only the measure defined in Gur or the SRI report would be compatible with the current network models.

The following treatment is intended to explain the suggested measure of accessibility and how it is incorporated into the current Systems Requirements model.

Let

\[ \begin{align*}
X &= \text{the set of all nodes in the network} \\
T_{ij} &= \text{number of trips from zone } i \text{ to zone } j \\
N_i &= \text{total number of trips from } i \\
P_{ij} &= \text{proportion of trips beginning in } i \text{ and terminating in } j \\
A_i &= \text{accessibility in zone } i \\
F_{ij} &= \text{friction factor in traveling from } i \text{ to } j
\end{align*} \]

Now

\[ P_{ij} = \frac{T_{ij}}{\sum_{j \in X} T_{ij}} \tag{E-23} \]

Recalling the definition of \(A_i\) given by Gur,
\[ A_i = \sum_{j \in X} P_{ij} F_{ij} \]  \hspace{1cm} (E-24)

whence
\[ A_i = \frac{\sum_{j \in x} T_{ij} F_{ij}}{\sum_{j \in x} T_{ij}} = \frac{1}{N_i} \sum_{j \in x} T_{ij} F_{ij} \]  \hspace{1cm} (E-25)

Therefore
\[ N_i A_i = \sum_{j \in x} T_{ij} F_{ij} \]  \hspace{1cm} (E-26)

from which
\[ \sum_{i \in x} N_i A_i = \sum_{i \in x} \sum_{j \in x} T_{ij} F_{ij} \]  \hspace{1cm} (E-27)

If \( F_{ij} \) is the minimum travel cost incurred from \( i \) to \( j \), then equation (27) gives the total user cost over the network. Therefore the problem

\[
\text{MIN} \sum_{i \in x} \sum_{j \in x} T_{ij} F_{ij}
\]

should be identical to

the problem
\[
\text{MIN} \sum_{i \in x} \sum_{j \in x} C_{ij} X_{ij}
\]

where
\[ C_{ij} \] is the user cost incurred over the link \( ij \) and \( X_{ij} \) is the volume in vph over the same link. In other words, the current network design model formulation in which total demand is fixed will automatically maximize accessibility (by the definition of Gur) by minimizing total user cost. Notice also that by dividing both sides of equation (E-27) by \( \sum_{i \in x} N_i \), the total number of trips in the network, one gets the average cost per trip. Since \( \sum_{i \in x} N_i \) is a constant, minimizing average cost or total cost will yield the same traffic assignment.

**Consideration of Special Accessibility Needs of Particular Travelers**

It may be desirable in the design of transportation systems to favor one segment of the using public over another. That segment may be a group of people, perhaps identified by their socio-economic characteristics, who have special needs for transportation, or whose needs are not well served by the
existing transportation system. Methods which are compatible with the existing network models and which give special consideration to accessibility of those groups might be particularly useful.

If one considers all zones where the groups with special needs are located, then the nodes from which their trips originate may be identified by the set \( G \). Let \( S \) be the set of sources (nodes) so that the set \( S - G \) is the set of sources not corresponding to the "special needs groups." Now if one defines the decision variable \( sX_{ij}^m \) to be the flow over link \((i, j)\) originating from source \( s \) at cost level \( m \) and \( sC_{ij}^m \) to be the corresponding unit cost, then one may consider the following formulation

\[
\text{MIN} \sum_{s \in S} \sum_{m=1}^{M} \left( \sum_{i \in A}^{M} sC_{ij}^m - sX_{ij}^m + \sum_{s \in S-G} sC_{ij}^m \cdot sX_{ij}^m \right) + \sum_{i \in A}^{M} 0 \cdot k_{ij} \quad (E-28)
\]

subject to

\[
\sum_{s \in S} sX_{ij}^m \leq k_{ij}^m + k_{ij}^m \quad \forall s, j \quad (E-29)
\]

\[
\sum_{i \in A}^{M} B_{ij}^m \cdot k_{ij}^m \leq B
\]

\[
\text{ALL} sX_{ij}^m, k_{ij}^m \geq 0
\]

where

\( A = \text{set of all links} \)

\( M = \text{number of cost levels} \)

\( k_{ij}^m = \text{amount of added capacity to link } ij \text{ at cost level } m \)

\( B_j = \text{set of links terminating on node } j \)

\( A_j = \text{set of links emanating from } j \)

\( s\emptyset_j = \text{supply at node } j \text{ from source } s \) (equal to zero unless \( j = s \))

\( sD_j = \text{demand at node } j \text{ from source } s \).

\( K_{ij}^m = \text{existing capacity of link } ij \text{ at cost level } m \)

\( B_{ij}^m = \text{budget for link } ij \text{ at cost level } m \)

\( B = \text{total budget allocated to the network} \)
Now for those costs $s_{C_{ij}^m}$ where $s \in G$, a multiplicative factor $s^m \mu_{ij}$ can be applied giving a higher cost $s^m \mu_{ij} s_{C_{ij}^m}$ for those persons coming from G over link ij at cost level m, thereby enhancing the improvement over the link ij. In essence what one does is to declare that the inherent costs of travelling for a person of the "special needs group" are higher than those who aren't in the group. These higher costs will then be interpreted by the network design model as a requirement to exaggerate improvements in network facilities serving the "special needs groups" over and above what might be done if their costs were the real costs $s_{C_{ij}^m}$.

As for the changes in the Network Design Model, it appears that instead of $M + S$ decision variables $s_{X_{ij}^m}$ and $s_{X_{ij}^m}$ for each link, this model would have $MS$ associated variables $s_{X_{ij}^m}$. For purposes of tying this to reality, $m$ will generally be 3 or 4, thereby increasing the number of variables by at least threefold. However, one can dispense with the constraint equations,

$$\frac{M}{\sum_{m=1}^{M} X_{ij}^m} = \frac{S}{\sum_{s=1}^{S} X_{ij}^s} \forall i,j$$

thus reducing the number of equations by (No. of Destinations) : (Number of Nodes-1).


Stanford Research Institute, (April, 1971), "Methods of Evaluation of the Effects of Transportation Systems on Community Values."
Appendix F
SOLUTIONS TECHNIQUES FOR THE NETWORK DESIGN MODEL

In Chapter 4, a network design model was formulated. The potential use of any mathematical model is at least partly determined by the efficiency with which the solution of the model can be obtained. In this chapter, different approaches to solve the continuous form of the network design model are presented. The simplest approach to the solution of this model is to treat it as a regular linear program. Other approaches involve exploiting the special structure of the model.

A transportation network design model is first expressed as a large linear program with block angular structure. Solution of this model, then, is attempted by three methods: (a) regular linear program using the OPTIMA code, (b) Dantzig-Wolfe decomposition, and (c) a new price-directive algorithm called BOXSTEP method.

Transportation Network Design Model

Using the model developed in Chapter 4, the transportation network design model can be described as follows:

\[
\text{(NDM) Minimize } \sum_{\ell=1}^{L} \left( c_{\ell}^{1} y_{\ell}^{1} + c_{\ell}^{2} y_{\ell}^{2} \right)
\]

subject to:

\[
\sum_{k \in A_j} x_{jk}^{S} - \sum_{k \in A_j} x_{kj}^{S} = \delta_{j}^{S}
\quad \text{for } j = 1, \ldots, S \quad \text{and } s = 1, \ldots, S
\]

\[
y_{\ell}^{1} \leq k_{\ell}^{1} + a_{\ell}^{1} z_{\ell}
\quad \text{for } \ell = 1, \ldots, L
\]

\[
y_{\ell}^{2} \leq k_{\ell}^{2} + a_{\ell}^{2} z_{\ell}
\quad \text{for } \ell = 1, \ldots, L
\]
\[ \sum_{k=1}^{L} B_{k} z_{k} \leq B \]  

(d)

\[ y_{k}^{1} + y_{k}^{2} = \sum_{s=1}^{S} (x_{k}^{s} + x_{k+s}^{s}) \quad \text{for } k = 1, \ldots, L \]  

(e)

\[ x_{k}^{s}, x_{k+s}^{s}, y_{k}^{i} \geq 0 \quad \text{for } k = 1, \ldots, L; s = 1, \ldots, S; \]  

\[ \text{and } i = 1, 2. \]  

(f)

\[ x_{jk}^{s} \geq 0 \quad \text{for } s = 1, \ldots, S; j = 1, \ldots, N; \]  

\[ \text{and } k = 1, \ldots, N. \]  

(g)

Figure F-1 shows this linear program in the form of a block angular structure.

**Regular Linear Programming Solution**

To solve the network design model as a large linear program, OPTIMA linear programming code was used. The example test problem was an aggregated highway network for Sioux Falls, South Dakota, having 24 nodes and 38 links. The linear program for this network had 667 rows and 1,938 variables. OPTIMA code solved this problem on a CDC 6400 computer in 14 minutes, 16 seconds (cp time).

This computer time was felt to be excessive, and therefore a search was initiated for more efficient ways to solve network design model. An obvious approach was to exploit the block angular structure of the model as shown in Figure 6.1 using Dantzig-Wolfe Decomposition algorithm.

**Dantzig-Wolfe Decomposition**

To solve transportation network design problem (NDM) by Dantzig-Wolfe decomposition, let us make the following definitions.
Figure P-1. Graphic Structure of the Basic Model
Let $x$ denote the complete set of $x$ variables,

$$x = [x^1 \mid \ldots \mid x^S \mid \ldots \mid x^S]$$  \hspace{1cm} (F-1)

where

$$x^S = [x^S_1, \ldots, x^S_L, x^S_{L+1}, \ldots, x^S_{2L}]$$  \hspace{1cm} (F-2)

for $s = 1, \ldots, S$.

Now define $X$ to be the feasible region for $x$.

$$X = \{ x \geq 0 \mid x \text{ satisfies constraints (a)} \}$$  \hspace{1cm} (F-3)

Suppose that we have $\hat{T}_1$ of its extreme points. Then $X$ can be approximated by the convex hull of these extreme points. ($X$ is a bounded polyhedral set and hence has no extreme rays.) Let

$$\hat{X} = \{ x \mid x = \sum_{t=1}^{\hat{T}_1} \lambda_t x^t, \lambda \geq 0, \sum_{t=1}^{\hat{T}_1} \ lambda_t = 1 \}$$  \hspace{1cm} (F-4)

where $\hat{T}_1$ is a subset of the extreme points.

Then $\hat{X} \subseteq X$.

Now let $(y,z)$ denote the complete set of $y$ and $z$ variables,

$$(y,z) = (y^1_1, y^2_1, \ldots, y^1_L, y^2_L, z_1, \ldots, z_L)$$  \hspace{1cm} (F-5)

and let $YZ$ denote the feasible region for $y$ and $z$,

$$YZ = \{(y,z) \geq 0 \mid (y,z) \text{ satisfies (b), (c), (d) and (g)} \}.$$  

Note that $YZ$ has no extreme rays (i.e., $YZ$ is a compact polyhedral set) and if we have $T_2$ of its extreme points, then its approximation can be constructed as follows.
\( \hat{YZ} = \{(y,z) \mid (y,z) \text{ satisfies (F-7)-(F-10) for some } \mu \geq 0\} \) \hspace{1cm} (F-6)

\[
\hat{T}_2 \\
y^1 = \sum_{t=1}^{\hat{T}_2} \mu^t y^1 t \\
y^2 = \sum_{t=1}^{\hat{T}_2} \mu^t y^2 t \\
z = \sum_{t=1}^{\hat{T}_2} \mu^t z t
\]

where \((y^t, z^t)\) are the extreme points of \(YZ\) for \(t = 1, \ldots, \hat{T}_2\).

Given these extreme points, the approximation of \((\text{NDM})\) can be written as

\[
(\text{NDM}) \quad \text{Minimize } \sum_{\ell=1}^{L} (c_{\ell}^1 y_{\ell}^1 + c_{\ell}^2 y_{\ell}^2) \\
\text{subject to } \sum_{s=1}^{S} [x_{\ell s}^s + x_{\ell s+L}^s] \text{ for } \ell = 1, \ldots, L \hspace{1cm} (e)
\]

\[x \in \hat{X}\]

and \((y,z) \in \hat{YZ}\).

Now using (F-4) and (F-6) this problem can be rewritten in terms of the \(\lambda's\) and \(\mu's\) as follows:

\[
\sum_{\ell=1}^{L} (c_{\ell}^1 y_{\ell}^1 + c_{\ell}^2 y_{\ell}^2) = \sum_{\ell=1}^{\hat{T}_2} \sum_{t=1}^{\hat{T}_2} \mu^t y_{\ell}^1 t + \sum_{\ell=1}^{\hat{T}_2} \sum_{t=1}^{\hat{T}_2} \mu^t y_{\ell}^2 t
\]

or

\[F-5\]
\[
\sum_{t=1}^{T_2} \left( c_1 y_{\ell t}^1 + c_2 y_{\ell t}^2 \right) = \sum_{t=1}^{T_2} \mu t^t
\]

where

\[
w_t = \sum_{\ell=1}^L \left( c_1 y_{\ell t}^1 + c_2 y_{\ell t}^2 \right) \text{ for } t = 1, \ldots, T_2.
\] (F-11)

Now

\[
y_{\ell}^1 + y_{\ell}^2 = \sum_{t=1}^{T_2} \mu t y_{\ell t}^1 + \sum_{t=1}^{T_2} \mu t y_{\ell t}^2 = \sum_{t=1}^{T_2} \mu t a_{\ell t}
\]

where

\[
a_{\ell t} = y_{\ell t}^1 + y_{\ell t}^2 \text{ for } t = 1, \ldots, T_2;
\] (F-12)

and \(\ell = 1, \ldots, L\).

Also

\[
\sum_{s=1}^{S} \left[ x_{\ell+s}^s + x_{\ell+s+L}^s \right] = \sum_{s=1}^{S} \sum_{t=1}^{T_1} \left( \lambda t x_{\ell+s}^s + \lambda t x_{\ell+s+L}^s \right) = \sum_{t=1}^{T_1} \lambda t b_{\ell t}
\]

where

\[
b_{\ell t} = \sum_{s=1}^{S} \left( x_{\ell+s}^{st} + x_{\ell+s+L}^{st} \right) \text{ for } t = 1, \ldots, T_1;
\] (F-13)

and \(\ell = 1, \ldots, L\).

Now (NDM) model can be written in its approximate form as follows:

\[
(NDM) \ \ \text{Minimize} \ \ \sum_{t=1}^{T_2} \mu t^t
\]

\[
\lambda, \mu \geq 0
\]

subject to

\[
\sum_{t=1}^{T_2} \mu t a^t = \sum_{t=1}^{T_1} \lambda t b_t^t
\] (A)
\[
\begin{align*}
\hat{T}_1 & \quad \sum_{t=1}^{T} \lambda^t = 1 \\
\hat{T}_2 & \quad \sum_{t=1}^{T} \mu^t = 1 \\
\end{align*}
\]

where

\[
\begin{bmatrix}
a_1^t \\ \vdots \\ a_t \\ \vdots \\ a_L^t \\
\end{bmatrix}
\quad \text{and} \quad
\begin{bmatrix}
b_1^t \\ \vdots \\ b_t \\ \vdots \\ b_L^t \\
\end{bmatrix}
\]

Thus (NDM) has two convexity constraints and L regular constraints.

This problem is a linear program and can be solved by a suitable LP algorithm. Doing so produces an approximate solution of the original problem (NDM).

Note that the constraints set (A) is equivalent to

\[
\begin{align*}
\hat{T}_2 & \quad \sum_{t=1}^{T} \mu^t a_t - \sum_{t=1}^{T} \lambda^t b_t \geq 0 \\
\end{align*}
\]

(B)

since the master problem seeks to min \( \sum_{t=1}^{T} \mu^t w^t \) where \( \mu^t \geq 0 \) and \( w^t > 0 \) is given from (F-11).

To see this we must return to the original problem which is equivalent to the master problem. That is, the constraint

\[
\begin{align*}
\sum_{s} (x_{ls}^s + x_{ls+L}^s) &= y_{l1}^s + y_{l2}^s \\
\end{align*}
\]

(C)

can be replaced by

\[
\begin{align*}
\sum_{s} (x_{ls}^s + x_{ls+L}^s) &\leq y_{l1}^s + y_{l2}^s \\
\end{align*}
\]

(D)
since any feasible solution with

\[ \sum_{s} (x_{s}^L + x_{s+L}^L) < y_{1}^L + y_{2}^L \text{ for some } L \]

cannot be optimal since \( c_{1}^L > 0 \) and \( c_{2}^L > 0 \).

But this replacement of (D) for (C) is equivalent to replacing (B) for (A).

The result of this whole analysis is that although \( \pi \) (the dual variables for the problem (NDM)) is nominally unconstrained, it is in effect non-negative by (B).

Now if we wish to improve the approximation we can do so by generating additional extreme points of \( X \), thus making \( \hat{X} \) a closer approximation of \( X \), or by generating additional extreme points of \( YZ \) to get a better \( \hat{YZ} \). Consider first the question of generating one more extreme point of \( X \). Each extreme point, \( x^t \) for \( t = 1, \ldots, T_1 \), constitutes one column, \( b^t \), to the approximation (NDM). If we are going to generate a new extreme point, and hence a new column, then we would like to get the column that has the most negative "reduced cost" with respect to the optimal dual variables for (NDM). Let \( \pi^L \) be the dual variable for the \( L \)th constraint. Let \( \gamma_1 \) be the dual variable for the convexity constraint on the \( \lambda \)'s and \( \gamma_2 \) be the one for the \( \mu \)'s. Then the reduced cost for any new column \( b \) is just

\[ \text{reduced cost} = \sum_{L=1}^{L} \pi^L b^L - \gamma_1 \]  

(\text{F-14})

We want to find the extreme point \( x \) for which this reduced cost is as negative as possible. (Note that \( \pi^L > 0 \) for \( L = 1, \ldots, L \)). By (F-13) we have
\[ \sum_{l=1}^{L} \pi_l b_l = \sum_{l=1}^{L} \sum_{s=1}^{S} \left[ x_{ls}^s + x_{l+L}^s \right] = \sum_{s=1}^{S} \sum_{l=1}^{L} \pi_l \left[ x_{ls}^s + x_{l+L}^s \right] \]

Minimizing this expression over \( x \in X \) separates into the following subproblems

\[(P1^s) \text{ Minimize } \sum_{l=1}^{L} \pi_l \left[ x_{ls}^s + x_{l+L}^s \right] \]

subject to \( x^s \in X^s \)

for \( s = 1, \ldots, S \) where \( x^s \) is defined by the constraints that insure the conservation of the flows \( x^s \) (to destination \( s \)). Thus \((P1^s)\) contains only the flows of "commodity \( s\)" which is "traffic to destination \( s\)".

Explicitly, \( x^s \) is defined by \( x^s \geq 0 \) and

\[ \sum_{k \in A_j} x_{jk}^s - \sum_{k \in A_j} x_{kj}^s = \theta_j^s \text{ for } j = 1, \ldots, S \quad (P-15) \]

We notice that problems \((P1^1), \ldots, (P1^S)\) are linear programs with identical constraint matrices and identical objective functions. In fact, they differ only in their right hand sides, \( \theta^1, \ldots, \theta^S \).

The dual variable for link \( l, \pi_l \), plays the role of a length for link \( l \).

Problem \((P1^s)\) is simply to find the shortest routes for the flow from each node to node \( s \), using the \( \pi_l \) as the arc lengths. Notice that \((P1^s)\) deals only with flow (traffic) to node \( s \) and does not contain any capacity constraints.\(^1\)

\(^1\) A consequence of this is the fact that \( x_{ls}^s \cdot x_{l+L}^s = 0 \) for \( l = 1, \ldots, L \) in any optimal solution of \((P1^s)\). That is, there cannot be flow in both directions on the same link, e.g., \( x_{ij}^s \cdot x_{ji}^s = 0 \) if \( l \) corresponds to \((i,j)\). This is also because \( x_{ls}^s \) and \( x_{l+L}^s \) are dependent columns in the linear program \((P1^s)\).
Therefore problem (Pl) for s = 1, . . . , S can be solved by any efficient shortest route algorithm. One of the most efficient algorithms is one due to Hu (1968) which determines the shortest chains between all pairs of nodes in a network. Since the constraint matrix and objective function are the same for each subproblem s = 1, . . . , S, it is only necessary to store the constraint matrix and objective function once.

Now suppose that we solve all of these problems and get optimal solutions x^s* for s = 1, . . . , S. Then the minimal value of (Pl) is given by

\[ v(Pl) = \sum_{l=1}^{L} \pi_{l} [x_{l}^{s*} + x_{l+L}^{s*}] \quad \text{(F-16)} \]

The column generated for (NDM) is

\[ b_{l}^{s*} = \sum_{s=1}^{S} [x_{l}^{s*} + x_{l+L}^{s*}] \text{ for } l = 1, . . . , L \quad \text{(F-17)} \]

and using (F-14), (F-16), and (F-17) we see that it has a reduced cost of

\[ \sum_{l=1}^{L} \pi_{l} b_{l}^{s*} - \gamma_{1} = \sum_{s=1}^{S} v(Pl) - \gamma_{1} \quad \text{(F-18)} \]

If this reduced cost is negative then we can get an improved solution to our original problem (NDM). Therefore if

\[ \sum_{s=1}^{S} v(Pl) < \gamma_{1} \quad \text{(F-19)} \]

then we set \( \hat{T}_{1} = \hat{T}_{1} + 1 \) and \( \hat{T}_{1} = x = [x_{1}^{1*}, \ldots, x_{S}^{S*}] \) and \( \hat{T}_{1} = b^{*} \).
We then have one more extreme point defining our approximation of \( \hat{X} \) of \( X \). The relation (F-19) insures that the new column will enter the basis in (NDM).

Now consider generating a new extreme point \((y, z)\) of \(YZ\). In this case we would get a new column \(a\) with

\[
\text{reduced cost} = w - \sum_{k=1}^{L} \pi_k a_k - \gamma_2.
\]

Using (F-11) and (F-12) we have

\[
w - \sum_{k=1}^{L} \pi_k a_k = \sum_{k=1}^{L} \Sigma_{m=1}^{2} c_{k} y_{k} m - \sum_{k=1}^{L} \Sigma_{m=1}^{2} y_{k} m = \sum_{k=1}^{L} \Sigma_{m=1}^{2} (c_{k} m - \pi_k) y_{k} m.
\]

Minimizing this expression over \(YZ\) means

(P2) Minimize \( \sum_{k=1}^{L} \Sigma_{m=1}^{2} (c_{k} m - \pi_k) y_{k} m \)

subject to

\[
\begin{align*}
y_1^1 & = 0 \\
y_2^1 & = 0 \\
y_1^2 & = 0 \\
y_2^2 & = 0 \quad \text{for} \quad k = 1, \ldots, L
\end{align*}
\]

\[
\begin{align*}
y_1^1 & \leq k_1^1 \quad \text{for} \quad k = 1, \ldots, L (h) \\
y_2^2 & \leq k_2^2 \quad \text{for} \quad k = 1, \ldots, L (i) \\
\sum_{k=1}^{L} B_k z_k & \leq B (j) \\
0 & \leq z_k \leq 1 \quad \text{for} \quad k = 1, \ldots, L (k)
\end{align*}
\]

This problem will be referred to as the "capacity problem" since it contains all of the capacity constraints. Note that it also contains the budget constraint. Note further that the \(z\)-variables appear only in (P2). Although this problem is a linear program, it can be formulated
as a knapsack problem and therefore has a closed-form solution.

Let

\[ d^m_k = \min \{ c^m_k - \nu^m_k, 0 \} \text{ for } m = 1, 2; \]

and \( k = 1, \ldots, L. \) \hfill (F-21)

Now

if \( z_k = 0 \) we can improve the objective function by

\[ d^1_k k^1_k + d^2_k k^2_k, \text{ and} \]

if \( z_k = 1 \) we can get

\[ d^1_k (k^1_k + \alpha^1_k) + d^2_k (k^2_k + \alpha^2_k). \]

So define the gain as

\[ g_k = d^1_k \alpha^1_k + d^2_k \alpha^2_k. \] \hfill (F-22)

Note that \( g_k \leq 0. \) Then, the problem (P2) is equivalent to the following knapsack problem:

\[
\text{Minimize } \sum_{k=1}^{L} g_k z_k \\
\text{subject to } \sum_{k=1}^{L} B_k z_k \leq B \\
\text{and } 0 \leq z_k \leq 1 \text{ for } k = 1, \ldots, L. \] \hfill (j) \hfill (k)

Since \( g_k \leq 0 \) for all \( k, \) this is equivalent to

\[
\text{Maximize } \sum_{k=1}^{L} (-g_k) z_k \\
\text{subject to } \sum_{k=1}^{L} B_k z_k \leq B \\
\text{and } 0 \leq z_k \leq 1 \text{ for } k = 1, \ldots, L. \] \hfill (j) \hfill (k)
Let \( h_\lambda = -g_\lambda \geq 0 \).

Then we have

\[
\begin{align*}
\text{(KS)} \quad \text{Maximize} & \quad \sum_{\lambda=1}^{L} h_\lambda z_\lambda \\
\text{subject to} & \quad \sum_{\lambda=1}^{L} B_\lambda z_\lambda \leq B \\
\text{and} & \quad 0 \leq z_\lambda \leq 1 \quad \text{for} \quad \lambda = 1, \ldots, L.
\end{align*}
\]

To solve (KS), rank \( \frac{h_\lambda}{B_\lambda} \) with subscript \( \nu \) from largest to smallest as follows:

\[
\begin{align*}
\frac{h_{\lambda_1}}{B_{\lambda_1}} \geq \frac{h_{\lambda_2}}{B_{\lambda_2}} \geq \cdots \geq \frac{h_{\lambda_L}}{B_{\lambda_L}}.
\end{align*}
\]  

(F-23)

Let \( r = \max\{i \mid \sum_{\nu=1}^{i} B_{\lambda_\nu} < B\} \)  

(F-24)

The optimal solution is

\[
\begin{align*}
\mathbf{z}^* &= (z_1^*, \ldots, z_r^*, z_{r+1}^*, \ldots, z_L^*) \\
&= (1, \ldots, 1, 0, \ldots, 0)
\end{align*}
\]

(F-25)

and the optimal values of the \( y \) variables are given by
\[
\begin{align*}
y_{\ell}^{m^*} &= \begin{cases} 
  k^m_{\ell} + a^m_{\ell} & \text{if } z^*_\ell = 1 \text{ and } d^m_{\ell} < 0 \\
  0 & \text{if } z^*_\ell = 1 \text{ and } d^m_{\ell} = 0 \\
  k^m_{\ell} & \text{if } z^*_\ell = 0 \text{ and } d^m_{\ell} < 0 \\
  0 & \text{if } z^*_\ell = 0 \text{ and } d^m_{\ell} = 0 \\
  k^m_{\ell} + a^m_{\ell} z^*_\ell & \text{if } 0 < z^*_\ell < 1 \text{ and } d^m_{\ell} < 0 \\
  0 & \text{if } 0 < z^*_\ell < 1 \text{ and } d^m_{\ell} = 0 
\end{cases}
\end{align*}
\]

The optimal value of (P2) is given by

\[
v(P2) = \sum_{\ell=1}^{L} \sum_{m=1}^{2} \left( c^m_{\ell} - \Pi_{\ell} \right) y_{\ell}^{m^*} = \sum_{\ell=1}^{L} \sum_{m=1}^{2} d^m_{\ell} y_{\ell}^{m^*}
\]

The column generated is (by (F-12))

\[
a^*_\ell = y^1_{\ell} + y^2_{\ell} \text{ for } \ell = 1, \ldots, L
\]

and it has its cost coefficient (by (F-11))

\[
w^* = \sum_{\ell=1}^{L} \sum_{m=1}^{2} c^m_{\ell} y_{\ell}^{m^*}
\]

and reduced cost (by (F-20) and (F-27))

\[
w^* - \sum_{\ell=1}^{L} \Pi_{\ell} a^*_\ell - \gamma_2 = v(P2) - \gamma_2^*
\]

So if

\[
v(P2) < \gamma_2
\]

then we set \( T_2 = T_2^* + 1 \) and \( y^{1T} = y^{1*} \), \( y^{2T} = y^{2*} \), and \( z^T = z^* \). This gives one more extreme point for the approximation \( \hat{Y_Z} \) of \( Y_Z \). Conditions (F-31) insures that \( * \) will be pivoted into the basis of the new (NDM).
We have now derived all of the ingredients necessary for solving problem \( \hat{\text{NDM}} \) by Dantzig-Wolfe decomposition. A stopping criterion (Lasdon, 1970) is provided by the following lower bound

\[
\sum_{s=1}^{s} v(P) - \gamma_{2} \geq v(\hat{\text{NDM}}) + [v(P2) - \gamma_{2}] + \{ \sum_{s=1}^{s} v(P1) - \gamma_{1} \},
\]

or

\[
v(\hat{\text{NDM}}) \geq v(\hat{\text{NDM}}) + [v(P2) - \gamma_{2}] + [v(P1) - \gamma_{1}] \tag{F-32}
\]

One final question that must be addressed is how to get the procedure started. This is done as follows. Choose arbitrary values for \( \pi_{1}, \ldots, \pi_{L} \). Taking

\[
\pi_{\ell} = \frac{c_{\ell}^{a}}{k_{\ell}} = (c_{\ell}^{\frac{1}{k_{\ell}}} + c_{\ell}^{\frac{2}{k_{\ell}}})/(k_{\ell}^{1} + k_{\ell}^{2}) \text{ for } \ell = 1, \ldots, L
\]

has turned to work very nicely. (P1) and (P2) can be solved using these \( \pi \) values. Find the optimal solutions of these problems, and compute \( b^{1} \) and \( a^{1} \) according to (F-13) and (F-12) respectively. Determine \( w^{1} \) by (F-11) also.

At this stage, problem \( \hat{\text{NDM}} \) is simply

\[
\hat{\text{NDM}} \text{ Minimize } \mu w^{1}
\]

subject to

\[
\mu a^{1} = \lambda^{1} b^{1}
\]

\[
\lambda^{1} = 1
\]

\[
\mu^{1} = 1
\]

If \( a^{1} = b^{1} \), then we have an optimal solution of \( \hat{\text{NDM}} \) and we are done. Otherwise, put artificial variables, \( n_{\ell} \), into the problem as follows:

F-15
Set

\[
 g_j^\ell = \begin{cases} 
 b_j^1 - a_j^1 & \text{if } j = \ell \\
 0 & \text{if } j \neq \ell
\end{cases}
\]  

(\text{F-33})

\[
g^\ell = (g_1^\ell, \ldots, g_L^\ell)
\]

Then \( \sum_{\ell=1}^L g^\ell = b^1 - a^1 \) and the following linear program is feasible.

\[
(\text{NDM}) \quad \text{Minimize } \mu^1 w^1 + \sum_{\ell=1}^L \eta^\ell v^\ell
\]

subject to \( \mu^1 a^1 - \lambda^1 b^1 + \sum_{\ell=1}^L \eta^\ell g^\ell = 0 \)

\[
\lambda^1 = 1 \\
\mu^1 = 1
\]

where \( v^\ell \) is a large position cost.

Solving this problem will give a non-zero value for every \( \pi^\ell \), and problems (P1) and (P2) should be solved again. Since the \( v^\ell \)'s are large and positive, the artificial variables (\( \eta^\ell \)'s) will be driven out of the basis of \( (\text{NDM}) \) after a few passes.

**Computational Experience with Dantzig-Wolfe Decomposition**

The test problem used to apply Dantzig-Wolfe decomposition algorithm is the same 24 node, 58 link network which was solved using the OPTIMA code. The computational experience, while quite limited, has been very discouraging. The starting \( \pi \)'s were set equal to \( c^1 \).
In the first run both subproblems were solved as linear programs. The computer run was stopped after 300 seconds. The total number of iterations performed in 300 seconds was 28. The value of the objective function for each iteration is given in Table F-1. The best value reached in 300 seconds was 325.71. The optimum is known to be 86.67 from OPTIMA.

In the second run, subproblem 1 was solved as a shortest route problem using the Hu algorithm (1968). Moreover, to reduce the number of artificial variables in the initial simplex tableau of the master problem, subproblem 2 (capacity problem) was solved heuristically in the first iteration. Thus a feasible heuristic column was generated from subproblem 2 which matched the column from the shortest route problem for as many rows as possible. The number of artificial variables was 14 for this case. This time the run was made for 400 seconds. Again the time limit of 400 seconds was selected arbitrarily. The best value of the objective function reached was 307.26 at the 40th iteration. The value of the objective function at each iteration is given in Table F-2.

The final run was made by modifying the heuristic technique to generate the starting solution of subproblem 2. This was achieved by renumbering the links in the ascending order of construction costs. It reduced the number of artificial variables in the master problem to zero. This time the run was made for 150 seconds. But the results were again very discouraging. There was no change in the value of the objective function even after 23 iterations (150 seconds.)
Table F-1. Dantzig-Wolfe Solution of 24 Node 38 Link Network
(First Run)

<table>
<thead>
<tr>
<th>Iteration Number</th>
<th>Objective Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7400.000000</td>
</tr>
<tr>
<td>2</td>
<td>1210.326229</td>
</tr>
<tr>
<td>3</td>
<td>1160.016015</td>
</tr>
<tr>
<td>4</td>
<td>1088.057648</td>
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<tr>
<td>5</td>
<td>943.041778</td>
</tr>
<tr>
<td>6</td>
<td>821.789317</td>
</tr>
<tr>
<td>7</td>
<td>770.651116</td>
</tr>
<tr>
<td>8</td>
<td>754.298926</td>
</tr>
<tr>
<td>9</td>
<td>737.174855</td>
</tr>
<tr>
<td>10</td>
<td>721.193376</td>
</tr>
<tr>
<td>11</td>
<td>703.274887</td>
</tr>
<tr>
<td>12</td>
<td>677.370424</td>
</tr>
<tr>
<td>13</td>
<td>660.190020</td>
</tr>
<tr>
<td>14</td>
<td>642.673250</td>
</tr>
<tr>
<td>15</td>
<td>636.153311</td>
</tr>
<tr>
<td>16</td>
<td>621.886082</td>
</tr>
<tr>
<td>17</td>
<td>608.391024</td>
</tr>
<tr>
<td>18</td>
<td>582.212026</td>
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<tr>
<td>19</td>
<td>556.420137</td>
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<td>20</td>
<td>525.365322</td>
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<td>21</td>
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<td>26</td>
<td>379.098549</td>
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<tr>
<td>27</td>
<td>357.468809</td>
</tr>
<tr>
<td>28</td>
<td>325.710596</td>
</tr>
</tbody>
</table>
Table F-2. Dantzig-Wolfe Solution of 24 Node 38 Link Network  
(Second Run)

<table>
<thead>
<tr>
<th>Iteration Number</th>
<th>Objective Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2822.474925</td>
</tr>
<tr>
<td>2</td>
<td>2822.474925</td>
</tr>
<tr>
<td>3</td>
<td>2822.474925</td>
</tr>
<tr>
<td>4</td>
<td>2822.474925</td>
</tr>
<tr>
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<td>6</td>
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<td>13</td>
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<td>15</td>
<td>2822.474925</td>
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<tr>
<td>16</td>
<td>1082.496308</td>
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<tr>
<td>17</td>
<td>1080.704869</td>
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<tr>
<td>18</td>
<td>752.926409</td>
</tr>
<tr>
<td>19</td>
<td>577.772088</td>
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<tr>
<td>20</td>
<td>537.674482</td>
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<tr>
<td>21</td>
<td>518.564836</td>
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<td>22</td>
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<tr>
<td>23</td>
<td>488.943342</td>
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<tr>
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<td>466.073270</td>
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<tr>
<td>25</td>
<td>456.831726</td>
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<tr>
<td>26</td>
<td>444.295776</td>
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<td>27</td>
<td>434.669787</td>
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<tr>
<td>28</td>
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<td>29</td>
<td>377.376215</td>
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<tr>
<td>30</td>
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<td>31</td>
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<td>33</td>
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<tr>
<td>34</td>
<td>330.013483</td>
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<tr>
<td>35</td>
<td>323.989028</td>
</tr>
<tr>
<td>36</td>
<td>316.139012</td>
</tr>
<tr>
<td>37</td>
<td>312.719761</td>
</tr>
<tr>
<td>38</td>
<td>308.496195</td>
</tr>
<tr>
<td>39</td>
<td>307.723872</td>
</tr>
<tr>
<td>40</td>
<td>307.264883</td>
</tr>
</tbody>
</table>
Therefore we started looking for other ways to solve the network design problem efficiently. This led to the development of a new strategy called BOXSTEP for large scale mathematical programs (Hogan, Marsten and Blankenship, 1973).

**Lagrangian Viewpoint of Dantzig-Wolfe Decomposition**

Let us consider the following linear program

\[(P) \min cx \]
\[\text{s.t. } Ax \leq b \]
\[\overline{Ax} \leq \overline{b} \]
\[x \geq 0 \]

where \(\overline{Ax} \leq \overline{b}\) is a subproblem and \(Ax \leq b\) are coupling constraints.

Using the regular Dantzig-Wolfe approach, let

\[X = \{x \geq 0 \mid \overline{Ax} \leq \overline{b}\} \]

\(X\) has extreme points \(x^1, \ldots, x^P\).

Then

\[x \in X \text{ if and only if } x = \sum_{k=1}^{P} \lambda_k x^k \]

where

\[\lambda \geq 0 \text{ and } \sum_{k=1}^{P} \lambda_k = 1 \]

Now

\[cx = \sum_{k=1}^{P} \lambda_k \cdot (cx^k) \]

\[Ax = \sum_{k=1}^{P} \lambda_k \cdot (Ax^k) \]
so that (P) becomes

\[
(DW) \min_{\lambda} \sum_{k=1}^{p} \lambda_k \cdot (c_k x^k)
\]

subject to

\[
\sum_{k=1}^{p} \lambda_k \cdot (A_k x^k) \leq b
\]

\[
\sum_{k=1}^{p} \lambda_k = 1 \quad \text{and} \quad \lambda \geq 0
\]

Sometimes it is much easier to evaluate the Lagrangian function \(L(u)\) for a given values of \(u\) than to solve (DW). Defining \(X\) as above

\[
L(u) = \min_{x \in X} cx + u(Ax - b) = \min_{1 \leq k \leq p} cx^k + u(Ax^k - b)
\]

where \(x^1, \ldots, x^p\) are as above.

The dual problem of (DW) is to maximize the Lagrangian \(L(u)\) and can be written as

\[
(D) \max_{u \geq 0} L(u) = \max_{u \geq 0} \min_{1 \leq k \leq p} cx^k + u(Ax^k - b) = \max_{u \geq 0, 1 \leq k \leq p} cx^k + uAx^k - ub
\]

or, equivalently, as

\[
(D) \max \sigma - ub \\
\text{s.t. } \sigma \leq cx^k + uAx^k \text{ for } k = 1, \ldots, p.
\]

Thus (D) is expressed as a linear program where the number of constraints equals the number of extreme points of \(X\). Rewriting (D) we obtain

\[
(D) \max \sigma - ub \\
\text{s.t. } \sigma - u(Ax^k) \leq cx^k \text{ for } k = 1, \ldots, p.
\]

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Now it is easy to see that this final form for (D) is just the linear programming dual problem for (DW); hence there is no advantage to maximizing the Lagrangian over the original problem (DW), because we are in effect just solving the linear programming dual problem. In fact, as rows are added to (D) the size of the basis increases; hence (D) may indeed be harder to solve.

A "steepest ascent" approach to maximizing $L(u)$ was tried but did not work well. The test problem was again a network design problem but of smaller size. The number of nodes and links were 12 and 18 respectively. A Grinold-type steepest ascent algorithm (1972) "jammed" at about 50.96. With Grinold's primal/dual step-size rule the steps became very short very quickly. The optimal step-size rule climbed higher but eventually suffered the same fate--it appeared to be converging to the value 50.96. The maximum was at 56.65 as described later. Therefore, the procedure was terminated after several hundred seconds.

**BOXSTEP Strategy for Network Design Problem**

It was discovered that a local version of (D) could be solved much more quickly than (D) itself. The local problem at $u^0$ is defined to be

$$D(u^0; \beta) \max L(u)$$

subject to

$$u^0_i - \beta \leq u_i \leq u^0_i + \beta \text{ for } i = 1, \ldots, m$$

$$u \geq 0$$

where $\beta$ specifies the size of a "box" or hypercube centered at $u_0$. Rewriting
\[ D(u^0; \beta) \quad \text{max} \quad \sigma - ub \]
\[ \text{s.t.} \quad \sigma - u(Ax^k) \leq cx^k \quad \text{for} \ k = 1, \ldots, p. \]
\[ u_i^0 - \beta \leq u_i \leq u_i^0 + \beta \quad \text{for} \ i = 1, \ldots, m. \]
\[ u \geq 0. \]

If \( u^0 = 0 \) and \( \beta = +\infty \), then \( D(u^0; \beta) \) is equivalent to \( (D) \). Suppose that \( D(u^0; \beta) \) is solved by Outer Linearization/Relaxation (Geoffrion, 1972) and that \( N \) denotes the number of extreme points of \( X \) that have to be generated. Thus \( N \) is the number of extreme points of \( X \) that have to be generated. \( N \) is an increasing function of \( \beta \) for given \( u^0 \). When \( \beta \) is very large, this is equivalent to the Dantzig-Wolfe method and \( N \) is typically very large. When \( \beta \) is small, however, \( N \) can become quite reasonable. For the network design problem, with \( \beta \) fixed, the value of \( N \) does not appear to depend on the proximity of \( u^0 \) to the global optimum.

This suggests the following algorithm. First choose the box size \( \beta \). Then, starting from any \( u^0 \geq 0 \), solve \( D(u^0; \beta) \). If the solution, \( u^* \), lies in the interior of the box, then \( u^* \) is the globally optimal dual solution since \( L \) is concave. If \( u^* \) lies on the boundary of the box, then a new box of size \( \beta \) can be placed at \( u^* \) (i.e., \( u^0 = u^* \)) and the process repeated. The main drawback to this algorithm is the overlap between successive boxes. This can be reduced by determining an optimal step-size in the direction \( (u^*-u^0) \). That is, determine \( \alpha^* \) by the one dimensional maximization

\[ \max_{\alpha} L(u^*+\alpha(u^*-u^0)) \]

and then center the next box at \( u^* + \alpha^*(u^*-u^0) \). Thus this is an ascent method for \( (D) \) and this ascent has an insured minimum step-size at
every step, since  \[ \|u^*-u^0\| \geq \beta. \]

The BOXSTEP approach can now be formally stated as follows.

Step 1. Choose \( u^0 \) and \( \beta \).

Step 2. Solve \( D(u^0;\beta) \) by Outer Linearization/Relaxation to obtain an optimal solution \( u^* \).

Step 3. If \( u^* \) is in the interior of the box, stop.

Otherwise, continue.

Step 4. Choose \( \alpha^* \) so as to maximize \( L \) along the ray \( u^* + \alpha(u^*-u^0) \).

Step 5. Set \( u^0 = u^* + \alpha^*(u^*-u^0) \) and go to Step 2.

The direction \( (u^*-u^0) \) is determined by the point \( u^* \) on the boundary of the box where \( L \) has its maximum. Thus \( (u^*-u^0) \) is not, in general, a direction of steepest ascent from \( u^0 \). BOXSTEP chooses directions so as to maximize the actual increase in \( L \) rather than the initial rate of increase, and this appears to be a major reason for its computational success.

**Computational Experience with BOXSTEP**

The test problem is a network design problem with 12 nodes and 18 links. The first subproblem was solved using a shortest route algorithm. Subproblem 2 was solved as a continuous knapsack problem in closed form.

For this problem the best performance was obtained by solving each local problem from scratch. Thus constraints from previous local problems were not saved. A line search (one-dimensional maximization) was performed between successive boxes. This was done with an adaptation
of Fisher and Shapiro's efficient method for concave piecewise linear functions (Fisher and Shapiro, 1972). BOXSTEP was implemented using the SEXOP linear programming package (Marsten, 1972).

Table F-3 summarizes the results of the test problem. The problem was run with several different box sizes. In all cases the problem was run until the optimum was achieved. The optimum value of the objective function was 56.65. Each run was started at the same point—a heuristically determined solution using dual values $u^0 = c^a$ (where the $c^a$ are the average costs of travel on network links). For each box size $\beta$ the column headed $N(\beta)$ gives the average number of constraints generated per box. Notice that this number increases monotonically as the box size increases. For a fixed box size, the number of constraints generated per box did not appear to increase systematically as we approached the global optimum. The column headed time gives the total computation time (CP), in seconds, for a CDC 6400 computer.

The large box ($\beta = 1000$) represents the Dantzig-Wolfe end of the scale. The smallest box ($\beta = .1$) produces an ascent that is close to being steepest ascent. A pure steepest ascent algorithm, as proposed by Grinold [35], was tried on this problem as described earlier. The poor performance of steepest ascent was consistent with our poor results for very small boxes.

The BOXSTEP algorithm was also used to solve the 24 node and 38 link network problem but the results were very discouraging. It took about 150 seconds to solve a box of very small size. Therefore the approach was dropped for this network after solving several boxes.

Note that the computational time of the algorithm BOXSTEP depends on the starting dual values $u$ associated with the constraints $Ax \leq b$. 

F-25
Table F-3. Solution of Test Problem (12 Nodes, 18 Links) by BOXSTEP with Varying Box Sizes

<table>
<thead>
<tr>
<th>$\delta$ (box size)</th>
<th>No. of boxes required</th>
<th>$\bar{N}(\delta)$</th>
<th>Time (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>34</td>
<td>12.7</td>
<td>172</td>
</tr>
<tr>
<td>0.5</td>
<td>18</td>
<td>14.2</td>
<td>118</td>
</tr>
<tr>
<td>1.0</td>
<td>13</td>
<td>17.1</td>
<td>104</td>
</tr>
<tr>
<td>2.0</td>
<td>9</td>
<td>17.7</td>
<td>88</td>
</tr>
<tr>
<td>3.0</td>
<td>6</td>
<td>25.0</td>
<td>99</td>
</tr>
<tr>
<td>4.0</td>
<td>4</td>
<td>26.8</td>
<td>76</td>
</tr>
<tr>
<td>5.0</td>
<td>5</td>
<td>33.4</td>
<td>134</td>
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<tr>
<td>6.0</td>
<td>4</td>
<td>34.3</td>
<td>115</td>
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<tr>
<td>7.0</td>
<td>3</td>
<td>38.0</td>
<td>119</td>
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<td>2</td>
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<td>203</td>
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<td>2</td>
<td>74.0</td>
<td>243</td>
</tr>
<tr>
<td>30.0</td>
<td>1</td>
<td>74.0</td>
<td>128</td>
</tr>
<tr>
<td>1000.0</td>
<td>1</td>
<td>97.0</td>
<td>217</td>
</tr>
</tbody>
</table>
In BOXSTEP, the total computational burden to solve a local problem involves the functional evaluations of the Lagrangian function, and re-optimizing the linear programming problem \( N \) times, where \( N \) is the number of constraints generated within the box. In cases where evaluations of \( L(u) \) can be made easily but solving the linear program is computationally expensive, an initial good approximation to the optimal dual variables is desired. This will reduce the number of boxes to be solved. This was the case with the network design problem, as evaluating \( L(u) \) involves solving the shortest route and continuous knapsack problem, both having closed form solution.

The approach used here is based upon a technique, "the relaxation method" for the problem of finding a feasible solution for a system of linear inequalities. The basic idea is following.

\[(D) \quad \max \sigma \]
\[
s.t. \ \sigma \leq cx^k + u(Ax^k - b) \text{ for } k = 1, \ldots, p.
\]
\[
u \geq 0
\]
Define \( \gamma^k = Ax^k - b \)
Assume \( \sigma \) is feasible for \( D \) and we wish to find a \( \overline{u} \) for which \( \sigma = L(\overline{u}) \).
That is, we need a solution to
\[
\overline{\sigma} \leq cx^k + u \gamma^k \text{ for } k = 1, \ldots, p.
\]
or
\[
\gamma^k u + (cx^k - \sigma) \geq 0 \text{ for } k = 1, \ldots, p.
\]

Mottzin-Schoenberg (1954) give the following iterative process for finding a solution to problem \( D \) with \( \sigma = \overline{\sigma} \).

Given \( u^0 \), construct.
\[
u^{v+1} = u^v + \lambda \left[ \frac{\sigma - L(u^v)}{\| \gamma^v \|^2} \right] \gamma^v \quad (m)
\]
for \( v = 0, 1, 2, \ldots \) where \( y^v = Ax^v - b \) and \( x^v \) is an optimal solution of the subproblem for \( u = u^v \). This process will converge to a feasible solution of (D) with \( \sigma = \sigma^* \) if \( \lambda \) is in the interval \( 0.2 \). The details of this are given in Blankenship (1973). The obvious drawback to this method is that one does not know, in general, a feasible target \( \sigma^* \), or more appropriately, a maximal feasible value \( \sigma^* \). Moreover, convergence of the procedure for \( \lambda = 1 \) is infinite, while the procedure terminates finitely for \( \lambda = 2 \).

This approach using (m) was exploited to obtain a good initial approximation to the optimal dual variables of the network design problem. This was done using following heuristic procedure:

Step 0. Establish error criteria \( \delta \).

Step 1. Begin with a lower and upper bound \((L_B, U_B)\). This is not difficult. For the network design problem \( L_B \) can be obtained by assigning traffic to the network using \( c^1 \) cost without any capacity constraints. \( U_B \) can be obtained, approximately, by assigning traffic using \( c^2 \) cost.

Step 2. Establish a target value, \( \overline{\sigma} = (L_B + U_B)/2 \).

Step 3. Take 20 steps with step-sizes and directions determined by (m) with \( \lambda = 1 \) when \( u \geq 0 \), otherwise use \( \lambda = \lambda^* \) which keeps \( u \geq 0 \).

Step 4. If convergence to within 30 percent of \( \overline{\sigma} \) did not occur, replace \( U_B \) by \( \overline{\sigma} \).

Step 5. Replace \( L_B \) by maximum value of \( L(u) \) achieved at this point.
Step 6. If \(|L_B - U_B| \leq \delta\), stop (enter BOXSTEP), otherwise go to Step 2.

Note that this method does not guarantee that the optimal value of \(L(u)\) lies between the final values of \(L_B\) and \(U_B\). We solved the network design problem of 12 nodes and 18 links, described earlier, using this method and our results have been encouraging. We were able to reduce the total computational time for this problem by 50 percent. Box size of 2.0 was used for this run. The results are summarized below.

1. Previous Solution

Start BOXSTEP algorithm at \(u = c^a\).

<table>
<thead>
<tr>
<th>(\beta) (box size)</th>
<th>no. of boxes required</th>
<th>(\bar{N}(\beta))</th>
<th>total time (Sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>9</td>
<td>17.7</td>
<td>88</td>
</tr>
</tbody>
</table>

2. BOXSTEP with initial approximation of the optimal dual variables using the relaxation method.

Starting \(L_B = 5.99\), \(U_B = 60.00\), \(\delta = 1.0\)

Start heuristic method at \(u = c^a\).

\(L(u)\) before entering BOXSTEP = 28.84

<table>
<thead>
<tr>
<th>(\beta) (box size)</th>
<th>no. of boxes required</th>
<th>(\bar{N}(\beta))</th>
<th>total time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>4</td>
<td>17.5</td>
<td>44</td>
</tr>
</tbody>
</table>

Therefore this appears to be useful heuristic to be employed before entering BOXSTEP. For a smaller network design problem of 8 nodes and 11 links, the computational time was 11.524 seconds while regular linear programming code (SEXOP) took 26 seconds to solve it.

The efficiency of this method depends critically on the ease of evaluating \(L(u)\), as many functional evaluations are usually required.
Blankenship (1973) discusses four different methods exploiting relaxation method for use in combination with BOXSTEP.

Though our computational experience with the network design problem and various solution approaches is quite limited, some general observations can still be made. It appears that the Dantzig-Wolfe decomposition algorithm is not suitable for the network design problem because of the number of coupling constraints. There are as many coupling constraints as the number of links in the network. The results of Dantzig-Wolfe decomposition have been very discouraging. The success of the BOXSTEP approach is also severely limited by the number of coupling constraints. Therefore it appears that the best approach is a dedicated LP code which capitalizes heavily on the structure of the matrix which is extremely sparse and those non-zero entries are almost all +1 or -1. The only non +1 or -1 entries are in the matrix B of Figure F-1.
References


This appendix is a reprint of the paper "A Transportation Network Design Model for Regional Development Planning," by Surendra K. Agarwal and Joseph L. Schofer, which was presented at the Twentieth International Meeting of The Institute of Management Sciences, June 24-29, 1973, in Tel Aviv, Israel.
ABSTRACT

A multimodal transportation network design model is proposed for evaluating alternative regional development policies in developing countries. Given travel demand, quantifiable objectives, and impact constraints, this model specifies an optimal network design. The model may include both passenger and commodity flows. Efficient techniques for the design of large-scale transportation networks are proposed. The results of example applications, including sensitivity analyses, are presented. The use of multiple objective formulations for the purpose of simplifying objective-achievement trade-offs is discussed, and the use of the model in an interactive decision-making environment is proposed. The difficulties involved in estimating accurate demand data are discussed, and some ways of overcoming these are presented.

Introduction - Transport Planning in Developing Regions

The role of transportation investments in fostering social and economic development has been much debated in the literature. There seems to be general agreement at this time that transportation, alone, cannot create increments in development; a variety of other ingredients, which represent an effective potential for such growth, must also be present. It is clear, however, that efficient and effective transportation is an essential concomitant to the regional development process.
In this context, decisions regarding transportation improvements in developing countries are of central importance, if not primary, in the process of attaining development goals. In such situations, where the connectivity of the existing transportation network is incomplete, the transportation planning process must be responsive to the system-wide effects of proposed projects, rather than simply the potential economic benefits and costs associated with individual link additions. This is because project level benefit-cost analysis is clearly insufficient where each project is likely to have broadly distributed effects on facilitating improved network flows throughout an entire region or nation.

Adopting a regional or national perspective in the planning of transportation investments requires comprehensive consideration of social and economic development objectives, along with objectives related only to transportation. Particularly in this case, transportation cannot be planned in a vacuum, but must be related to its potential contribution to the attainment of higher level goals. This suggests that traditional approaches to transportation planning, which have evolved largely in the urban context, are not likely to meet the policy development needs in developing regions.

Such urban-oriented approaches are primarily oriented toward the forecasting of travel demand. Land use plans or forecasts are used as the basis for producing relatively precise estimates. Such a planning strategy is reasonable in situations where the land use pattern, which is the fundamental determinate of demand, is relatively fixed, as it would be in an existing city. In a developing region, however, the spatial arrangement of activities is normally changing at a significant rate. Furthermore, the land use pattern
itself represents a fundamental decision variable, one which is perhaps more important, in terms of development objectives, than decisions about transportation.

A more rational planning strategy for developing regions would be one which is designed to test the efficacy of alternative regional development plans, which represent the major policy options. This could be accomplished by focusing on the implications, in terms of costs and benefits, of such alternative plans. Of course, a principal concomitant of any such plan is its associated transportation requirements. Explicit exploration of the transport implications of development plans represents a fundamentally different approach to transportation planning than that normally utilized in the urban context.

Such an approach requires an efficient strategy for determining, or designing, the most appropriate transportation plan associated with each development option. That transportation plan would be the one which "best" serves the travel demand created by the development plan, within whatever budgetary and impact constraints exist. It is the set of travel demands, in fact, which links the development plan to its transportation requirements.

Creation of that "best" transportation plan, of course, is a major challenge. In the case of developing countries or regions, many transport projects, in the form of links to be added or improved, may be candidates for implementation. An efficient strategy for searching through this set of candidates is clearly required. This transportation search process might then be structured within the development plan search process, creating an hierarchical search process for overall policy evaluation.
Network Design Models

Optimal network design models, generally based on mathematical programming formulations, have been suggested previously for accomplishing the formidable process of searching through all feasible combinations of transportation improvements to find a set which is in some way best. Such models, proposed for use in developed countries and urban areas, select a set of link additions which are optimal under predefined goal set, and within specified constraints, given the description of the existing network, all possible network improvements, improvement cost data, and the expected travel demands. Their operational application has been limited in the past because of the relatively high costs associated with solving large scale problems with them, due to their very significant requirements for computer capacity and computation time. A further limitation has been the difficulty in accommodating all but the simplest objectives and constraints in the formulation of such models.

While little consideration has been given to such applications in the past, the use of such optimizing models for transportation and development planning studies in emerging regions seems especially promising at this time. Among the factors increasing the significance of this opportunity are the following:

1. The sparseness of existing and foreseeable transportation networks in developing countries serves to limit the scale of the network analysis and design problem, particularly in terms of the number of nodes and links which must be considered. This limitation may serve to bring the requirements for computer core storage and computation time down to levels which are reasonable in the context of the information which may be derived from such models;

2. Improvements in computer capacity, speed, and availability should facilitate the use of network design models in this context;
(3) Recent developments in improving the efficiency of computer codes for solving such special classes of mathematical programming problems also contribute to reductions in the cost of application.

Beyond these more pragmatic problems, the information needs for planning in developing countries make the use of optimizing network design strategies in this environment particularly attractive. For the network design approach makes it both feasible and efficient to conduct the two stage, hierarchical search process described above. That is, given transportation demands associated with a development alternative, the design model could produce an "optimal" plan, or a series of optima, which themselves can be evaluated; information produced about the costs of providing effective transportation services for the development plan could then be used as a basis for evaluating alternative development strategies. Thus, an efficient network design model can provide a mechanism for exploring the development-alternative-impacts space in a relatively short time period, supporting the policy decision process at both the level of transport choices and development decisions.

Finally, the use of a mathematical programming formulation also facilitates the conduct of sensitivity analyses which serve to support further, more detailed exploration of the solution space, and, as illustrated in this paper provides a sound basis for trade-offs studies.

The remainder of this paper discusses a network design model formulated specifically for applications in developing regions. The structure of the model is presented, and strategies for its use are illustrated through an hypothetical example. The model proposed is oriented toward commodity flows, which often represent the fundamental mobility need in such cases. While it is presented as a commodity flow model, its extension to the consideration of passenger flows is simple. The example is developed and solved for a single travel mode and single commodity in the text; its extension to the
multimodal case is explained in the appendix.

Structure of the Proposed Network Design Model

The structure of the network design problem is as follows: given a set of possible link development projects, select that combination of projects which produces the "best" traffic network, evaluated in terms of specified, quantitative criteria while maintaining expenditures within the given budget. The problem is purely one of synthesis.

In this section, a single-mode, single time period model is presented where there are no upper limits on the amounts of improvement on each link. Only the shipment of commodities will be considered, although it is quite simple to include the passenger flows in the analysis, treating passengers as a separate commodity using a special mode (e.g., buses instead of trucks). This is realistic in a developing region where the amount of passenger travel by private cars is negligible. If this is not the case, private-auto transportation should be treated as an additional commodity. Conceptually, these extensions do not create any difficulties.

Extension of this simple model to include multiple modes is discussed in the appendix.

The given network has a set of nodes N (production points, consumption points, cities and intersections) and a set of links denoted by A. Each link in A connects two nodes i and j and could therefore be denoted as (i, j) or (j, i).

We make the following definitions:

\[ C_{ij}^k = \text{the cost (travel time) of shipment, per truck, of commodity } k \text{ on link } ij. \]
\( k_{ij} \) = the flow of commodity \( k \) (in tons) from \( i \) to \( j \) with destination \( s \).

\( k_{ij}^s \) = the amount of commodity \( k \) that originates at node \( j \) with node \( s \) as its destination.

\( k_{ij}^s \) = the amount of commodity \( k \) (in tons) per truck (load carrying capacity of a truck).

\( e^k \) = equivalent passenger car units (P.C.U.) per truck for commodity \( k \) (required because link capacity is generally stated in passenger car units).

\( K_{ij} \) = capacity of link \( ij \), P.C.U. per day.

\( Z_{ij} \) = improvement decision variable for link \( ij \).

\( K_{ij} \) = increase in capacity of link \( ij \) per unit of improvement \((Z_{ij})\).

\( B_{ij} \) = improvement cost of link \( ij \) per unit of \( Z_{ij} \).

\( B \) = Total budget available for improvement of the network.

\( K \) = the total number of commodity types.

\( S \) = the total number of destination nodes.

The network design model for the design of road network can formally be stated as follows:

Minimize total transportation costs.

\[
\sum_{i,j \in A} \sum_{k=1}^{K} \frac{k_{ij}^s}{L_{ij}} \sum_{s=1}^{S} \left( k_{ij}^s + k_{ji}^s \right)
\]

Subject to the following constraints:

Traffic flow conservation constraints:

\[
\sum_{j \in A} k_{j1}^s - \sum_{j \in B} k_{1j}^s = D_j^s \quad \text{for} \quad j = 1, \ldots, N; \quad s = 1, \ldots, S; \text{ and } \quad k = 1, \ldots, K.
\]
These are simply conservation of flow equations, requiring that all traffic flowing into a node, including traffic generated there, for each destination node, must leave the node.

(3) Capacity constraints:

$$\sum_{k=1}^{K} \sum_{i=1}^{k} \sum_{s=1}^{S} (x_{ij}^k + k z_{ij}) \leq K_{ij} + k_j z_{ij} \quad \text{for all } ij \in A.$$ 

(4) Budget constraint:

$$\sum_{ij \in A} B_{ij} Z_{ij} \leq B$$

which states that cost of capacity additions must fall within a budgeted amount, B, allocated for network construction and improvement.

and

(5) Non-negativity constraints:

$$x_{ij}, z_{ij} \geq 0 \quad \text{for all } ij \in A; \quad k = 1, \ldots, K; \text{ and } s = 1, \ldots, S.$$ 

This is a linear programming model which can therefore be solved by any of the existing linear programming codes.

This model assumes that the travel time is virtually independent of the traffic volume. In reality, travel time on a link is a function of the volume of traffic on that link, which makes the objective function of the model nonlinear. The nonlinear travel time function can be linearized by piece-wise approximation, and therefore the model can again be solved as a linear programming problem \([11,12]\).

Note that there are no upper limits on the amounts of improvements permitted on each link. If improvements on some links are to be limited in size because of technical, economical, social or other considerations,
then the following constraints should be included.

\[ z_{ij} \leq z_{ij}^{\text{max}} \]

This model considers both improvements to existing links and the construction of entirely new links. In case of new links, there is no existing capacity and hence \( k_{ij} = 0 \).

Bottlenecks in transportation capacity can be present because of limited physical capacity of the transport network or limited supply of rolling stock (as trucks, wagons, locomotives, etc.). Therefore, one alternative for improving the performance of transport system which should be considered is the addition of new rolling stock. To introduce the possibility of expanding the rolling stock new constraints should be added into the model as follows:

Define \( N_{\text{o}} \) = existing fleet size of trucks.
\( B^0 \) = capital cost of buying a new truck.
\( N \) = number of new trucks to be purchased (decision variable) and
\( U \) = utilization rate of trucks per day.

\[ \sum_{k=1}^{K} \frac{1}{f^k} \sum_{i \in A} \sum_{s=1}^{S} \left( x_{ij}^s + k_s x_{ji}^s \right) \leq U(N_{\text{o}} + N) \]

which states that the total number of trucks required to ship the traffic can not exceed the total number of trucks available (existing fleet purchased). And the budget constraint (4) should be modified to

\[ \sum_{i \in A} B_{ij} z_{ij} + B^0 N \leq B. \]

This formulation of the model has only continuous variables, requiring the assumption that capacity can be added to any link in any increment.

If certain links must either be expanded directly to a specified capacity level or not expanded at all, then \( z_{ij} \) have to be constrained to take only the values of 0 or 1. This changes the model into a mixed integer programming
problem which can be solved using well known techniques of integer pro-
gramming [14]. While the mixed integer model may be more realistic in
some applications, the penalties in computation time favor the use of a
continuous version [12]. This may result in the necessity to round the
results produced by the model where integer solutions are required.

Treatment of Capital Costs: An Improved Formulation

In the model described above, the objective is to design that trans-
port system which will minimize the total travel time of commodity shipment
subject to the constraint that the total expenditures for improvements
(capital costs) fall within the budgeted amount. An alternative criterion
commonly used in transport planning is to design the transport system so
as to minimize the total cost (shipment travel cost (in money units) plus
capital cost) subject to budget constraint. The use of this combined
objective function is desirable in that it allows the optimizing model
to trade-off capital cost for operating costs internally in order to achieve
the least total cost network. This formulation becomes difficult to apply,
however, because capital and operating costs (time) are incommensurate.
And therefore the combination of shipment time and construction cost within
the objective function would require the derivation of a money value of
time. While much research has been devoted to the isolation of such a
value, there is still considerable disagreement in the field regarding
the precise value to be used in a given case [16]. Furthermore, deter-
mining the appropriate value to use in a particular developing country
would require additional research. Therefore, this combined, or total
cost, objective function is not recommended.

Use of the simple, total shipment time minimization model does present
some problems. Since this model would choose to improve all links to inde-
finately high levels in the absence of a budget constraint, the latter
plays a very critical role in determining the optimal solution. Yet in most cases there is no a priori limitation on the available budget. While policy makers might logically prefer to keep capital costs as low as possible, there is no obvious strategy, or even a rule of thumb, for fixing the regional transportation budget. In fact, a most promising method for choosing a budget level would be to determine precisely what user cost savings (time), and hence what economic development benefits, would result from various budget levels, and then to select the best attractive combination of both.

A simple modification of the network design model serves to facilitate such trade-off analysis. This involves the application of Bi-Criteria Analysis [3] to explore the trade-offs between total commodity shipment time and capital costs.

The network design model can be written as a Bi-Criteria problem as follows:

\[
\begin{bmatrix}
    y_1 = \sum_{ij \in A} \sum_{k=1}^K \frac{k c_{ij}}{f^k} \sum_{s=1}^S (k x_{ij}^s + k x_{ji}^s) \\
    y_2 = \sum_{ij \in A} B_{ij} Z_{ij}
\end{bmatrix}
\]

(8) Minimize \( y = y_1 \)

Subject to (2), (3), (4), and (5);

where \( y_1 \) represents the user or shipment travel times and \( y_2 \) represents the construction costs.

This revised model can be solved once for either objective \( y_1 \) or \( y_2 \) and then parametric analysis may be applied, to generate the efficient frontier which describes the surface of non-inferior (undominated or Pareto-Optimal) solutions. A solution which is not inferior is one for which the value of a given criterion function can only be improved at the expense of at least one
other criterion. In the case of only two objectives, as illustrated above, the display of the trade-offs between criteria is quite simply accomplished through the use of the two dimensional graph of the surface (line) of non-inferior solutions [10].

**Multi-dimensional Trade-off Analysis**

In the case of the proposed network design problem, any point on the trade-off curve will give a non-inferior network design alternative, represented by a value of total shipment (user) travel time and amount of budget (capital costs) spent. With this type of information, the decision maker need not justify his choice of selecting a network design alternative in terms of an explicit, a priori, relative weighting of the two objectives, shipment time and construction costs, but only in terms of the localized trade-off between specific alternatives on the two dimensional efficient frontier. This approach has the further advantage of showing the decision maker, at a glance, the nature of the phenomenological trade-offs open to him. He may, if he desires, easily compute the quantitative value of these trade-offs between any two solutions on the efficient frontier.

It should be noted that the extension of this approach to additional dimensions presents no conceptual problems. Thus objectives such as minimization of vehicle-miles of travel (related to air pollution emissions), minimization of land required for rights-of-way, etc., could easily be introduced into this framework.

A problem does arise, however, in presenting such multi-dimensional information to decision makers. While with three objectives the efficient frontiers could be displayed using physical models, four or more objectives require the consideration of hyper-surfaces. Research is currently
underway to evaluate the efficacy of aiding the decision maker's search through such hyper-spaces through the use of multiple two-dimensional graphs displayed using interactive computer graphics.

**Solution of an Example Problem**

To illustrate the use of the network design model, a simple network problem was solved. It was assumed that there is only one mode of travel, and that there is only one commodity. Moreover, it was also assumed that only single time period is to be considered. The network, shown in Figure 1, consists of 9 nodes and 13 links. Two of 13 links, represent yet-unbuilt facilities which are candidates for construction. All of the links were open for improvements. This simple example serves to better illustrate the usefulness of the proposed model-formulation and trade-off analysis strategy.

The input data are summarized in Table 1 and 2, and will not be discussed further. The model was solved on the CDC 6400 computer and FTN3 (FORTRAN Extended, Version 3.0) compiler using a linear programming code "SEXOP" [8] at Northwestern University.\(^1\) The total computer central processor time required to solve the linear program (86 constraints and 247 variables) once and to perform the parametric analysis to generate entire efficient frontier was 48 seconds.

There were sixty nondominated solutions (network designs), which were obtained from parametric analysis. These are illustrated in the form of a two-dimensional efficient frontier in Figure 2. The phenomenological trade-offs between shipment time and capital costs for network improvement are clearly displayed and should be of direct value to the decision-maker.

---

\(^1\) The block angular structure of the network design model can be used to develop some efficient solution techniques [9].
Figure 1: The Example Network
<table>
<thead>
<tr>
<th>Nodes</th>
<th>Length (miles)</th>
<th>Existing Speed (MPH)</th>
<th>Existing Travel Times (Hours)</th>
<th>Existing Capacity (1000s p.c.u./Day)</th>
<th>Discounted Improvement Cost (1000s of Rupees/Mile/Lane)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 2</td>
<td>100.0</td>
<td>25.0</td>
<td>4.0</td>
<td>7.5</td>
<td>50.0</td>
</tr>
<tr>
<td>2 5</td>
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<td>20.0</td>
<td>6.5</td>
<td>3.0</td>
<td>30.0</td>
</tr>
<tr>
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<td>25.0</td>
<td>2.8</td>
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<td>50.0</td>
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<tr>
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<td>3.0</td>
<td>30.0</td>
</tr>
<tr>
<td>4 5</td>
<td>120.0</td>
<td>20.0</td>
<td>6.0</td>
<td>3.0</td>
<td>30.0</td>
</tr>
<tr>
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</tr>
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<td>20.0</td>
<td>2.5</td>
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<td>60.0</td>
</tr>
<tr>
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<td>20.0</td>
<td>4.0</td>
<td>3.0</td>
<td>30.0</td>
</tr>
<tr>
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<td>20.0</td>
<td>4.0</td>
<td>0.0</td>
<td>60.0</td>
</tr>
<tr>
<td>8 9</td>
<td>120.0</td>
<td>20.0</td>
<td>6.0</td>
<td>3.0</td>
<td>30.0</td>
</tr>
</tbody>
</table>
Table 2: TRIP INPUT DATA

(Trucks/Day)

Factory Located at Node 1.

<table>
<thead>
<tr>
<th>From:</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
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<td>Node</td>
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<td>2200</td>
<td>575</td>
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<td>0</td>
<td>800</td>
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<td>300</td>
</tr>
<tr>
<td></td>
<td>250</td>
<td>200</td>
<td>300</td>
<td>0</td>
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<td>450</td>
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<td>470</td>
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<td>200</td>
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<td>300</td>
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<td>100</td>
<td>200</td>
</tr>
<tr>
<td></td>
<td>550</td>
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<td>400</td>
<td>0</td>
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<td>0</td>
<td>50</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>540</td>
<td>0</td>
<td>0</td>
<td>300</td>
<td>200</td>
<td>0</td>
<td>0</td>
<td>100</td>
<td>0</td>
</tr>
</tbody>
</table>
Figure 2: Two-Dimensional Efficient Frontier for the Example Problem

Region I
Region II
Region III

Discounted Value of Construction Costs (1000's of rupees)
For example, it is relatively evident that alternatives in region I in Figure 2 are undesirable, for (relatively) small increases in construction cost will bring about much larger reductions in shipment times if the decision maker moves to region II. Furthermore, region III in Figure 2 is certainly not attractive because spending more in construction costs to move from II to III results in only small reductions in shipment times. The most reasonable solutions lie in Region II. The final choice must be based on the relative importance of shipment travel times and construction costs. The use of Figure 2 allows the decision maker to see the nature of the trade-offs he must make in choosing between alternatives networks in region II.

More detailed information on the solutions contained in the efficient frontier of Figure 2 is shown in the structure of selected, optional networks illustrated in Figure 3. Scanning these selected networks, along with reference to Figure 2, suggests a reasonable priority ordering for construction of link improvements. For example, it is evident that links 2-5 and 8-9 should have the highest priority for implementation, followed by 6-9. At the end of the priority list are links 2-3 and 1-4. This represents a reasonable set of priorities because in this way limited construction resources will first go into those alternatives which, from a regional network perspective, produce the greatest benefits in shipment times reduction.

This concept can also be illustrated by Table 3, which identifies the links included for improvement in various solutions on the efficient frontier. Because the solutions appear in sequence starting at the upper left portion of the efficient frontier, those links listed in the left portion of Table 3 should have higher priority. From another perspective, the high priority link improvements are those included in the largest number of efficient solutions.
T.T. = Total shipment time in 1000's vehicle hours/day
C.C. = Total Discounted cost of improvement in 1000's rupees

The number on links represent the amounts of optimal improvements
(in number of lanes).

Figure 3: Selected Nondominated Network Design Alternatives.
T.T. = Total shipment time in 1000's vehicle hours/day
C.C. = Total Discounted cost of improvement in 1000's rupees

The number on links represent the amounts of optimal improvements (in number of lanes).

Figure 3: Selected Nondominated Network Design Alternatives.
<table>
<thead>
<tr>
<th>Alternative</th>
<th>Link</th>
<th>2-5</th>
<th>8-9</th>
<th>6-9</th>
<th>3-6</th>
<th>3-5</th>
<th>5-6</th>
<th>5-8</th>
<th>2-3</th>
<th>1-4</th>
<th>4-5</th>
<th>4-7</th>
<th>1-2</th>
<th>7-8</th>
</tr>
</thead>
<tbody>
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Evaluation of Alternative Development Patterns

The transport demands used as inputs to this model reflect the spatial arrangement and level of production and consumption activities, as specified in the regional development plan. The network design model can be efficiently utilized to test alternative development patterns by solving it with different demand sets. For example, the model might be used to test alternative locations for a production plant in this manner.

Consider a factory which, after all other factors have been evaluated (including location of factors of production and markets), can be located either at node 1 or node 9 in Figure 1. The network design model can be used to test the transport implications of these locational alternatives.

In the previous example, it was assumed that this factory was located at node 1. Then a further computer run was made for the demand set modified to reflect the location of the factory at node 9. The trip input data for this case is given in Table 4. The efficient frontiers from both of these solutions are shown in Figure 4. It is obvious from this figure that a plant location at node 1 enjoys the benefits of both lower shipment times and lower construction costs in all solutions on the efficient frontiers. Therefore, if all other locational factors are equal, the site at node 1 is to be preferred.

A Note on the Problem of Demand Estimation

Because data on transportation demands serve as a primary input to the network design model, having a determining effect on the nature of the optimal set of network improvements and because these data serve as the fundamental link between the development plan and the transportation plan, some discussion of demand data and estimation is necessary. This is particularly important in the case of developing countries, where data describing transport
Table 4: Trip Input Data

(Trucks/Day)

Factory Located at Node 9.

To: Node

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Figure 4: Efficient Frontiers for Factories Located at Nodes 1 and 9.
demand and other such phenomena are typically very poor. This is often true not only for future forecasts of flows, but also for data on existing flows, travel costs, and facility improvement costs.

It would be unreasonable to suggest that, to apply the proposed network design model, it is necessary to collect highly precise demand data, and to use those data as a basis for forecasting future flows. Furthermore, traditional demand forecasting techniques, which are quite sensitive to transport costs, would be difficult to use since the ultimate cost of shipment is determined inside the network design model. Of course, the use of an iterative modeling strategy would be logical in this case [6], but such an approach is necessarily complex and expensive.

A simpler strategy would be to use a more aggregate method to derive general flow patterns and volumes directly from development plans. This could be accomplished, for example by using a simple input-output model [5], or even a linear programming model. In the latter case gross assumptions would have to be made regarding node-to-node transport costs. This should be performed with caution, however, because, when the demand estimates are subsequently used in the network design model, the cost assumptions will indirectly influence the optimal solutions.

It should be noted that the proposed model is especially well suited for testing the sensitivity of optimal network structures to various levels of demand. This could be accomplished by solving for the efficient frontiers associated with each alternative demand estimate, and comparing the results as shown in Figure 2. Even more information could be derived by plotting one set of optimal networks as shown in Figure 3. The latter form of presentation will help to define those links which should receive the highest improvement priority in the face of uncertain demand.
Conclusion

The proposed network design model, particularly in the multi-criteria formulation, can produce information of special value in supporting regional development planning. The use of efficient frontiers offers a simple strategy for assisting decision makers in making trade-offs between alternative levels of network investment. The model also offers a simplified approach to selecting improvement priorities. Used within a two stage, hierarchical search process, the model can provide a stronger quantitative basis for the evaluation and selection of regional development plans.
APPENDIX - A Multi-Mode Model

A multiple-mode network design problem can be described as follows:
Given the characteristics of each mode (costs, capacities etc.), traffic
demands and improvement projects available for each mode; determine the
best improvement to be made in each mode and the best allocation of traffic
demands among different modes.

Consider that there are M modes denoted by m (m=1,...,M). Using the
same notation as of previous network design model and adding superscript
m to each variable to identify the modal-characteristics, the multi-modal
network design model can formally be stated as follows.

Minimize total shipment costs

\[ (a) \quad \sum_{m=1}^{M} \sum_{k=1}^{K} \sum_{i \in A} \sum_{j \in A} \sum_{k=1}^{K} \sum_{s=1}^{S} c_{ij}^{mk} s \left( x_{ij}^{mk} s + x_{ji}^{mk} s \right) \]

Subject to the following constraints:

(b) Traffic flow conservation constraints for each mode:

\[ \sum_{m=1}^{M} x_{jl}^{mk} s - \sum_{m=1}^{M} x_{lj}^{mk} s = \phi_j^{mk} s \quad \text{for} \quad j = 1, \ldots, N; \]
\[ s = 1, \ldots, S; \]
\[ k = 1, \ldots, K; \quad \text{and} \]
\[ m = 1, \ldots, M-1. \]

and

\[ \sum_{m=1}^{M} x_{jl}^{mk} s - \sum_{m=1}^{M} x_{lj}^{mk} s = d_j^{mk} s - \sum_{m=1}^{M-1} \phi_j^{mk} s \quad \text{for} \quad j = 1, \ldots, N; \]
\[ s = 1, \ldots, S; \]
\[ k = 1, \ldots, K. \]

Here \( \phi_j^{mk} s \) is a decision variable and represents the amount of commodity
type k, which is generated at node j for destination node s and is using
mode m.

(c) Capacity constraints for each mode m:

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\[ \sum_{k=1}^{K} \sum_{m,k}^{e} \sum_{s=1}^{S} \left( m_k x_{i,j}^s + m_k x_{j,i}^s \right) \leq k_{ij}^m + k_{ij}^m z_{ij}^m \text{ for all } ij \in A^m; \]
and \( m=1, \ldots, M. \)

(d) Budget constraint:

\[ \sum_{m=1}^{M} \sum_{ij \in A^m} B_{ij}^m z_{ij}^m \leq B \]

which simply states that the cost of capacity additions to all modes must fall within the budgeted amount.

and

(e) Non negativity constraints:

\[ m_k^s x_{i,j}^s, m_k^s x_{j,i}^s, z_{ij}^m, \emptyset_j^s \geq 0 \text{ for all } ij \in A^m; \]
\[ k = 1, \ldots, K; \]
\[ m = 1, \ldots, M; \text{ and } \]
\[ s = 1, \ldots, S. \]

It should be noted that this is a very simple model, which ignores a variety of real problems, including particularly one possibility of transferring commodities between modes (modal shift), etc.
REFERENCES


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APPENDIX H

A DISCRETE LINK ADDITION NETWORK DESIGN MODEL

Basic Core Model

The core model in Chapter 4 was also formulated as a discrete link addition problem using mixed integer programming. In the discrete problem, we are given a set of distinct projects, each corresponding to adding a specified link to the network. The objective is to choose that set of projects which results in the smallest total user cost, and whose combined construction cost does not exceed the budget available for network improvement. The mixed integer programming model closely resembles the core model in Chapter 4. The notation in the two models is identical except that the integer variables $\delta_{ij}$ are defined to be 1 if link $(i,j)$ is added to the network and 0 otherwise. The model for the discrete link addition problem is as given in Figure H-1.

The objective function (1), the sum of the linearized user cost functions, and constraints (2), (3), (5) are identical to those in the continuous version of the problem. Constraint (4) prohibits flow on any proposed arc $(i,j) \in P$ unless $\delta_{ij} = 1$, e.g., unless the arc is constructed. Constraint (6) is the budget constraint; if $\delta_{ij} = 1$, then the construction cost $B_{ij}$ is incurred. Constraints (7) and (8) are the non-negativity and integrality requirements, respectively.

The model above can be solved with a standard mixed integer linear programming computer code. Unfortunately, the size of the problem is such that the computational requirements of standard codes become prohibitive for networks with fifty nodes or more. A special purpose implicit enumeration code which was considerably more efficient than
Minimize total user costs.

\[
\text{Min. } \sum_{i,j \in A} \sum_{m=1}^{M} C_{ij}^m x_{ij}^m + \sum_{i,j \in A} \sum_{s=1}^{S} 0 \cdot s x_{ij}^s + \sum_{i,j \in P} 0 \cdot 0 \cdot \delta_{ij} \tag{H-1}
\]

Subject to the constraints:

Traffic flow is conserved at each node.

\[
\sum_{k \in A} s_{x_{jk}} - \sum_{i \in B} s_{x_{ij}} = s_{d_j} - s_{d_j}, \forall j, s \tag{H-2}
\]

Flow for all destinations on an arc must be assigned to the capacity of that arc.

\[
\sum_{m=1}^{M} x_{ij}^m - \sum_{s=1}^{S} s_{x_{ij}}^s = 0, \forall i,j \in A \tag{H-3}
\]

Flow must not exceed capacity on each arc.

\[
x_{ij}^m \leq k_{ij}^m \delta_{ij}, \forall m, i,j \in P \tag{H-4}
\]

\[
x_{ij}^m \leq k_{ij}^m, \forall i,j \in E, m = 1, 2, \ldots, M \tag{H-5}
\]

The capital budget can not be exceeded.

\[
\sum_{i,j \in P} B_{ij} \delta_{ij} \leq B \tag{H-6}
\]

Non-negativity.

\[
\delta_{ij} = 0 \text{ or } 1 \tag{H-7}
\]

\[
s_{x_{ij}}, x_{ij}^m \geq 0 \tag{H-8}
\]

Figure H-1. The Integer Core Network Design Model.
where:

$A$ = set of all arcs in network ($A = P + E$)

$A_j$ = set of all nodes departing from node j (after node j)

$B$ = total capital budget available

$B_j$ = set of all nodes leading into node j (before node j)

$B_{ij}$ = cost per unit of maximum capacity added on arc $ij$

$C_{ij}$ = user cost of $m$th increment of capacity on arc $ij$

$s_{ij}$ = total flow destined for node $s$ terminating at node $j$

(by definition = 0, $j \neq s$)

$E$ = set of all existing arcs in the network

$F_{ij}^m$ = fraction of maximum capacity added on arc $ij$ assigned to user cost increment $m$

$K_{ij}$ = capacity of segment $m$ of arc $ij$

$M$ = number of user cost increments on each arc

$s_j$ = total flow originating in node $j$ and destined for node $s$

$P$ = set of all proposed arcs in network

$x_{ij}^m$ = total flow on user cost increment $m$ of link $ij$.

$s_{ij}$ = total flow on link $ij$ destined to node $s$

$e$ = designation of a destination, $2 = 1, \ldots, S.$

$\delta_{ij}$ = 0 or 1 choice variable for arc construction

$S$ = set of all nodes in the network

Figure H-1. The Integer Core Network Design Model, continued.
general codes was written for solving the model above. Although the special solution technique is promising for networks with a moderate number of proposed link additions, the continuous version is more attractive for truly large scale problems.

Possible Modifications

There are many possible modifications to the core integer model which might be desired to reflect various situations. This brief discussion will include means for considering (1) improvements to existing links, and (2) network characteristic constraints, and (3) modelling of peak period traffic only.

Improvements to Existing Links

In the construction of entirely new links, there is no necessary corresponding reduction in capacity of existing links. However, if an existing link is upgraded in any of various ways, the original link ceases to exist. It is not difficult to incorporate this into the model. A new set of links must be defined to augment E and P:

U = set of existing links on which upgrading is proposed.

Thus, on each such link there are two possible user cost functions—one corresponding to the existing links, and the other corresponding to the upgraded link:

\[ C_{ije}^m \equiv \text{user cost parameters of existing arc} \]

\[ C_{iju}^m \equiv \text{user cost parameters of upgraded arc} \]

\[ K_{ije}^m \equiv \text{volume break points for existing arc} \]

\[ K_{iju}^m \equiv \text{volume break points for upgraded arc} \]

\[ B_{ij} \equiv \text{capital cost of proposed arc} \]
The objective function remains identical to the original one, except of course the summation of the user costs must be over arcs in \( E, P \) and \( U \), and the links in set \( U \) are now designated by three subscripts instead of two. Also, flow can occur only on the existing or proposed version of a link, depending upon which is in existence. These changes yield a model as follows:

\[
\begin{align*}
\text{minimize} & \quad \sum_{ij \in P} 0 \cdot \delta_{ij} + \sum_{ij \in U} 0 \cdot \delta_{ij} + \sum_{ij \in (E+P)} C_{ij}^m x_{ij}^m + \\
& \quad \sum_{ij \in (E+P+U)} \left( C_{ij}^m x_{ije}^m + C_{ij}^m x_{iju}^m \right) + \sum_{ij \in (E+P+U)} M \cdot s_{ij}^m
\end{align*} \tag{H-9}
\]

subject to:

\[
\begin{align*}
\sum_{m=1}^{M} x_{ij}^m &= \sum_{s=1}^{S} x_{ij}^s, \forall ij \in (E+P) \tag{H-10} \\
\sum_{m=1}^{M} \left( x_{ije}^m + x_{iju}^m \right) &= \sum_{s=1}^{S} x_{ij}^s, \forall ij \in U \tag{H-11} \\
\sum_{i \in B} \sum_{j \in A} x_{ij}^m - \sum_{j \in B} \sum_{i \in A} x_{jk}^m &= s_{D_j} - s_{i_j}, \forall j, s \tag{H-12} \\
x_{ij}^m &\leq K_{ij}^m, \forall m, ij \in E \tag{H-13} \\
x_{ij}^m &\leq K_{ij}^m \cdot \delta_{ij}, \forall m, ij \in P \tag{H-14} \\
x_{iju}^m &\leq K_{iju}^m \cdot \delta_{ij}, \forall m, ij \in U \tag{H-15} \\
x_{ije}^m &\leq K_{ije}^m - K_{ije}^m \cdot \delta_{ij}, \forall m, ij \in U \tag{H-16} \\
\sum_{ij \in P} B_{ij} \cdot \delta_{ij} + \sum_{ij \in U} B_{ij} \cdot \delta_{ij} &\leq B \tag{H-17}
\end{align*}
\]
\[ x_{ij}^m, x_{ije}^m, x_{iju}^m \geq 0 \]  
(H-18)

\[ s x_{ij} \geq 0 \]  
(H-19)

\[ \delta_{ij} = (0,1) \]  
(H-20)

Relationships (H-15) and (H-16) are the only really new ones, and these merely restrict flow to only the version of the potentially upgraded link which is actually open.

**Multiple Improvements**

It is likely that there will be many improvements considered on some links, such as varying degrees of widening or signalization of a street or varying the number of lanes of a freeway. This can be considered a number of \((0,1)\) choice variables equal to the number of possible investments: \(\delta_{ij1}, \delta_{ij2}, \delta_{ij3}, \text{ etc.}\) Each would appear in the objective function.

The treatment of flow would be similar to that used above, with

\[ x_{ij1}^m, x_{ij2}^m, \text{ etc.} \]

\[ x_{ijp}^m \text{ flow on } m^{th} \text{ segment of arc } ij \text{ with improvement type } p. \]

\[ p_{ij} = \text{ the number of improvement types on link } ij, p=1,2,\ldots, p_{ij}. \]

The modifications to the original model require simply the inclusion of all possible types of investment on the link and a constraint limiting flow to the one link constructed. If only a few links have possible multiple improvements, then this can be treated as a special case. For each such link, the usual terms in the objective function

\[ 0 \cdot \delta_{ij} + \sum_{m=1}^{M} c_{ij}^m \cdot x_{ij}^m + \sum_{s=1}^{S} 0 \cdot s x_{ij} \]
are replaced by

\[
P_{ij} \sum_{p=1}^{P} \delta_{ijp} + \sum_{p=1}^{P} \left( \sum_{m=1}^{M} C_{ijp}^m \cdot x_{ijp}^m \right) + \sum_{s=1}^{S} 0 \cdot s_{ij} = 0
\]

The arc flow constraint is modified for each such link, as follows

\[
P_{ij} \sum_{p=1}^{P} \left( \sum_{m=1}^{M} x_{ijp}^m \right) = \sum_{s=1}^{S} s_{ij} x_{ij}
\]

while H-15 remains unchanged. The capacity constraint for these links is modified to be

\[x_{ijp}^m \leq k_{ij}^m \cdot \delta_{ijp} \quad \forall p\]

A constraint stating that these are mutually exclusive improvements is required if they are for an entirely new link

\[
P_{ij} \sum_{p=1}^{P} \delta_{ijp} \leq 1
\]

and one stating that they are mutually exclusive and exhaustive is required if they correspond to an existing link (and it will not be abandoned):

\[
P_{ij} \sum_{p=1}^{P} \delta_{ijp} = 1
\]

The term in the budget constraint for link \(ij\) must be modified to include all improvements.

\[
P_{ij} \sum_{p=1}^{P} b_{ijp} \delta_{ijp}
\]

The usual non-negativity and integer requirements are retained.

**Network Characteristic Constraints**

It may be desirable to restrict the choices of improvements to links in a variety of ways which reflect policies regarding such investments.

One such policy may be that a sequence (or chain) of links must be either
all upgraded or constructed or none may be. This would be especially applicable to a series of freeway links. If only one type of improvement or new construction is contemplated, then the choice variable for all such links can be treated as a single choice variable $\hat{\delta}_{ijklm}$, where the series of subscripts would indicate the series of nodes on the links.

If many different improvements are possible (e.g., different widths of freeway links), and if these are for entirely new links, then the desired condition is that if any one link is constructed, then all must be constructed, presumably to any one of the specified types. This condition is imposed by adding the constraint given below for each adjacent pair of links:

$$\sum_{p=1}^{P} \delta_{ijp} = \sum_{p=1}^{P} \delta_{jkp}. $$

**Peak Period Model**

It might be desirable to apply the model to different periods of a day, such as peak periods, rather than to the entire day, in some applications. This can easily be done, using the model as presented above. This form of the model makes no assumptions regarding the symmetry of travel demand or of the network.

The number of constraints (other than upper bounds) in the original model is:

$$E + P(1 + M) + S^2 + 1.$$  

where $E$, $M$, $P$, and $S$ are the numbers of elements in their respective sets. The number of choice variables is

$$\hat{P} + (E + P)M + (E + P)S.$$  

where a $\hat{}$ indicates a link rather than arc.
For a network with $P = 10$, $E = 20$, $S = 20$, and $M = 2$, the original version of the model without simplifying assumptions has 501 constraints and 1,330 choice variables.
APPENDIX I

TRANSIT AND MULTI-MODAL EXTENSIONS

Introduction

The increasing inability to expand highway capacity within urban areas, particularly using freeway technology, and the increasing concern for the quality of man's environment, along with many other factors, have resulted in increased interest in and consideration of public transport as a means of accommodating more urban travel in the future. Although this research was primarily devoted to the development of a highway network design model, it is imperative that consideration be given to possible extensions of the model to include public transportation. Therefore, considerable effort was devoted to the formulation of versions of the model which could consider public transportation along with automobile travel, and this appendix includes various versions of such models. Although no example applications of a multi-modal model were constructed and tested, this appendix should present sufficient information on the formulation of the model and the estimation of the parameters that it would not be difficult to construct and test (with an application) the suggested models.

In order to facilitate understanding of the model, it is presented in successively more complex and complete versions. First, a purely transit network design model is presented, to serve as a means of introducing the new variables and parameters required to represent that mode. Then, attention is devoted to merging this model with the core highway network design model in order to build a multiple mode model. Also included is a discussion of the types of output which might be expected
from the models, along with a discussion of the inputs required and a means for obtaining or developing these inputs.

**The Transit Model**

**Problem**

The problem addressed in this initial version of the model is that of designing a bus transportation system for an urban area, in the sense of specifying the links on which buses are to be operated (and by implication those on which transit service would not be provided), the level of service provided on each link as described by the frequency of bus trips, and the capacity to be provided on each link. This initial version of the model does not take into account internally any interaction with other modes, so that a fixed demand for travel which is to be accommodated on transit is assumed. The model must select the network in such a manner that all of the demands can be routed through the network given the choices as to the operating characteristics of the link. The choices regarding the network and the accommodation of traffic therein are made considering four types of objectives:

1. Minimize users' travel time on board the vehicles.
2. Meet level of service constraints which might require the operation of service on certain links and possibly require at least a certain frequency of bus trips to be operated on certain links.
3. Minimize capital expenditures for buses.
4. Minimize operating costs.

Since these four objectives are incommensurate, one is treated in the objective function and the others are treated as constraints. In order to be compatible with the core highway network design model, the minimization
of the users' travel time was selected as the objective to be included in the objective function in the initial version, and the others are dealt with as constraints which might be varied.

The Network

The network consists of links on which buses might be operated and nodes which represent bus stops. Clearly all possible bus stops can not be included in the model because of the very large number of them, and therefore a single node might be used to represent all bus stops within a traffic generation zone. The primary choice variable for system design is the frequency (such as bus trips per day or bus trips per hour) with which buses are to be operated on each link, a zero indicating that no service is to be provided on that link. Given a specification of the frequency with which buses are operated on each link, it is possible to calculate the capacity provided on each link, given the capacity of the bus type used. Given the origin to destination travel demand pattern, it is possible to assign this traffic to the various bus links operated to ascertain the feasibility of accommodating that traffic with that route structure, making appropriate modifications as necessary.

The level of achievement of the various objectives can be estimated given the above information on the network design and the pattern of flows through the network. As will be discussed below, the number of vehicles required to operate the system can be estimated, knowing the links on which service is provided, the travel times of those links, and the frequency of the service. This yields an estimate of capital costs. Operating costs are similarly related to the number of vehicles being operated and the vehicle miles and/or vehicle hours of operation required.
Users' travel time through the network can of course be estimated knowing the paths of travelers through the system and the travel time on the various links which they are using.

A few remarks regarding this model are in order. First, although the model will specify the links to be operated and the frequency to be operated on each link, the model does not specify the precise route structure of the system. It remains for the user of the model to combine links into appropriate chains, each of which would comprise an entire route. Second, the model does not explicitly consider the waiting times of users in estimating their travel time, although a more complex version of the model to be presented later includes this. Third, the model is most naturally applied to an entire day, although it can be applied to an individual peak hour or other period. The precise formulation presented below is for an entire day, and it is assumed that the service operated on one direction of a link is identical to the service in the opposite direction. However, the application to a day does require certain assumptions regarding the peaking of traffic and of service to be provided.

With these introductory comments, then, it is appropriate to turn to a detailed description of the various components of the model. The notation used will be introduced as required.

User Travel Time

In this presentation of the model, user travel time is included in the objective function, to be minimized, while other objectives are treated in the constraints. We assume that the travel time on each link is constant, and therefore all that is necessary in order to predict total user travel time on each link is the total number of users. The flow of passengers in this transit model is described
in a manner identical to that used in the highway model, in which a separate variable describes the flow of passengers on each arc for each destination in the network. The total flow of passengers on any arc is then the sum of all flows on that arc over all possible destinations. Thus the objective function may be written:

\[
\text{Minimize } \sum_{ij \in L_b} T_{ij} \left( \sum_{s=1}^{S} S_{bij} + \sum_{s=1}^{S} S_{bji} \right) \tag{I-1}
\]

where:

\[S_{bij} = \text{flow of passengers on arc } ij \text{ with destination } s\] (\[S_{bji}\] is the flow in the opposite direction, from \(j\) to \(i\))

\[T_{ij} = \text{bus travel time on link } ij\]

\[L_b = \text{links in the bus system}\]

**Capacity Constraint**

It is necessary to insure that the flow of passengers on any arc in the system does not exceed the capacity to move passengers provided by the buses flowing on that arc. One of the choice variables is necessarily the frequency of buses on each arc. Knowing the capacity of each bus, and the frequency with which buses are operated on an arc, it is possible to calculate the capacity provided on that arc and to insure that it is at least as great as the flow on that arc, as follows:

\[
Q \cdot f_{ij} \geq \sum_{s=1}^{S} S_{bij}, \forall ij \in A_b \tag{I-2}
\]

where:

\[Q = \text{capacity of each bus}\]

\[f_{ij} = \text{frequency of buses on link } ij\]

\[A_b = \text{the set of all arcs in the bus network}\]
Note that there is only one frequency variable for a link, not two (one for each direction), since we assume service is identical in both directions.

Node Conservation of Flow Constraints

The flows of passengers on each arc must be related to the travel demand from origin to destination, in order to insure that all travellers are in fact assigned in a feasible way to the network. This is done in a manner precisely identical to that used in the highway model. At each node in the network, a relationship concerning the flows through that node for each destination is written. Part of this relationship includes any traffic generated for that destination at the node, or alternatively traffic terminated at that node. The general form of this relationship is:

\[ \sum_{i \in B_j} s_{bij} + s_{bj} = \sum_{k \in A_j} s_{bjk}, \quad j=1, \ldots, N, \quad s=1, \ldots, S, \quad j \neq s \]  

(I-3)

where:  
- \( s_{bij} \) = bus passengers originating in node \( j \) destined for node \( s \).  
- \( A_j \) = set of nodes after node \( j \)  
- \( B_j \) = set of nodes before node \( j \)

As in the case of the highway network model, such a relationship must be written for each destination at each node in the network, except the relationship for one node for each destination may be omitted, and the node selected is usually that of the destination.

Operating Costs

The operating costs of bus transit systems are usually linearly related to vehicle miles and/or vehicle hours. Given the fixed length
of each link, and the fixed travel time on that link, the total vehicle miles on an arc is simply a linear function of the frequency, as is the total vehicle hours. Thus the total operating cost for a system can be estimated using an expression of the form shown in the left-hand side of the relationship below. Given the fixed over-all travel demand, it is possible to estimate total revenues under any specified fare scheme. To this might be added any operating subsidy for purposes of constraining total operating costs to be less than operating income. Such a relationship is shown below:

$$\sum_{ij \in L_b} F_{ij} f_{ij} \leq R + S$$  \hspace{1cm} (I-4)

where:

- $F_{ij}$ = cost per bus trip on link $ij$
- $R$ = operating revenue from passengers
- $S$ = subsidy for operating expenses

**Capital Cost**

The capital costs of operating a bus service are largely the costs incurred in purchasing the vehicles. Given the assumption of fixed travel time on each link, and certain other assumptions regarding the temporal distribution of traffic, the total number of buses required can be related to total vehicle hours, which in turn is related to the frequency of service on each arc. Thus total capital costs can be expressed as a linear function of frequency. The constraint on total capital expenditures can be expressed as shown below. First, total vehicle requirements are calculated, and then the cost of new vehicles is
restricted to be below the allowable expenditure.

\[ \sum_{ij \in L_b} \frac{2}{U} T_{ij} f_{ij} \leq N + n \]  \hspace{1cm} (I-5)

\[ Y \cdot n \leq B_b \]  \hspace{1cm} (I-6)

where:

- \( U \) = utilization rate of buses, vehicle-hours per day per bus
- \( N \) = number of existing buses
- \( n \) = number of new buses required
- \( Y \) = cost of each new bus
- \( B_b \) = maximum allowable expenditure on new buses.

Level of Service Constraint

The requirement that certain links be retained in any solution to the model and also that the links be operated with a certain minimum frequency of service can be easily incorporated into the model. The source of such constraints is likely to be regulatory requirements. Alternatively, a certain minimal level of transit service might be desired in order to meet other objectives, such as deliberately providing a high level of accessibility to certain zones. Regardless of the source of these requirements, they can be easily incorporated in the model as a lower bound on the selection of the frequency to be operated on appropriate arcs, as shown below:

\[ f_{ij} \geq M_{ij} \]  \hspace{1cm} (I-7)

where:

- \( M_{ij} \) = minimum acceptable frequency on arc \( ij \)
Summary

For convenience, the entire model in this form is presented on Figure I-2. Also shown are the definitions of the various parameters and the variables. Figure I-1 reviews the flow variables and network representation.

Expected Output

Just as the highway models, this model has been constructed in order to present useful information on the possible trade-offs in the achievement of the various objectives to be considered. Although this preliminary version of the model includes only criteria related to user costs and system costs, nevertheless meaningful trade-off information can be provided. For example, the model might be used to explore the effect of different levels of capital grants or operating subsidies to the system. Treating just the capital grant constraint for the moment, it is expected that the results in any application would be as shown in Figure I-3, with total user travel time decreasing, at a decreasing rate, as the magnitude of the capital grant program were increased. This decrease would continue until a point is reached at which additional capital funds would provide buses which cannot be effectively used by the system and hence no longer result in any reduction in users' travel time. Information such as this would be helpful in ascertaining what level of capital grant is most appropriate for a particular system, and should be useful in attempting to develop capital grant applications.

A similar situation would probably exist with respect to the relationship between users travel time, and operating subsidies. As the subsidy were increased, presumably more routes and more frequent service could be operated, resulting in a reduction in users' travel time, as shown in Figure I-4.
Major route intersection and/or zone of traffic generation.

\[ f_{ij} = \text{frequency of buses per day in one direction.} \]

\[ b_{ij} = \text{flows of travelers from } i \text{ to } j \text{ destined to node } s. \]

Figure I-1. Bus Network Conventions
Objective function:

\[
\text{Minimize } \sum_{ij \in L_b, \ell} T_{ij} \left( \sum_{s=1}^{S} S_{b_{ij}} + \sum_{s=1}^{S} S_{b_{ji}} \right) \tag{I-8}
\]

subject to:

Capacity constraints:

\[
Q \cdot f_{ij} \geq \sum_{s=1}^{S} S_{b_{ij}}, \forall ij \in A_b \tag{I-9}
\]

Node conservation of flow constraints:

\[
\sum_{i \notin B_j} S_{b_{ij}} + S_{D_j} = \sum_{k \notin A_j} S_{b_{jk}} \quad j = 1, \ldots, N, \quad s = 1, \ldots, S, \quad j \neq s \tag{I-10}
\]

Operating Cost constraint:

\[
\sum_{ij \in L_b} F_{ij} \cdot f_{ij} \leq R + S \tag{I-11}
\]

Vehicle Availability constraint:

\[
\sum_{ij \in L_b} \frac{2}{U} T_{ij} \cdot f_{ij} = N + n \tag{I-12}
\]

Capital Budget constraint:

\[
Y \cdot n \leq B_b \tag{I-13}
\]

Level of Service constraints:

\[
f_{ij} \geq M_{ij} \tag{I-14}
\]

where:

- \( F_{ij} \) = cost of operating a vehicle between node i and node j, including cost in both directions.

- \( f_{ij} \) = frequency, or number of vehicles operating from node i to node j per unit time period (assumed same in other direction).

Figure I-2. Transit Network Design Model.
\( M_{ij} \) = minimum frequency acceptable on link \( ij \).
\( s_{ij} \) = flow on transit arc \( ij \) destined for node \( s \).
\( S_{Dj} \) = transit demand from node \( j \) destined for node \( s \).
\( Q \) = capacity of a vehicle.
\( T_{ij} \) = travel time by transit from node \( i \) to node \( j \).
\( N \) = number of existing vehicles.
\( n \) = number of new vehicles required.
\( B_b \) = capital budget available.
\( Y \) = cost of new vehicle and supporting equipment.
\( S \) = operating subsidy.
\( R \) = revenues from system.
\( U \) = average vehicle utilization, vehicle-hours per vehicle owned.
\( A_b \) = arcs of the bus system.
\( L \) = links of the bus system.

Figure I-2. Transit Network Design Model, continued.
Figure I-3. Hypothetical Trade-off between Capital Expenditures and Travel Time.

Figure I-4. Hypothetical Trade-off between Operating Subsidy and Travel Time.
Closely related to this trade-off is the question of what an appropriate fare level of the system would be. Assuming that revenues obtained from fares would be used in addition to the subsidy to offset operating costs, and that these operating costs would have to be less than or equal to the subsidy plus revenues from fares, the relationship is expected to be as shown. As fares increase, the total amount of service provided could be increased resulting in reduced total travel time. The important question is to select an appropriate mix of users' travel time, fares to be charged, and operating subsidy. A portrayal of the trade-offs available such as that shown in Figure I-4 should help greatly in making this decision.

A more detailed discussion of the types of information available from this model, possible means of using this in evaluation and selecting system configurations, etc., would be inappropriate at this point, because the real focus of this section is preliminary to the development of a multiple mode model. The presentation of the multiple-mode model follows a brief discussion of means of estimating parameters required for the transit model, which follows. This discussion may be skipped by anyone interested in only the general characteristics of the models.

Estimation of Input Parameters

Although this transit model and the following multiple mode model were not applied to an example problem, considerable attention was given to the methods for estimating all of the required input parameters. It is appropriate that these be presented in order that anyone interested in developing and testing these models could benefit from the consideration already given to this problem.
Perhaps the most troublesome aspect of the purely transit model is the assumption of a fixed demand. The demand matrix might be obtained from the results of a previously completed urban transportation planning study. However, in order to estimate that demand presumably some assumptions were made regarding the level of service on the transit system, and at least the assumptions relevant to major transit flows should be included as constraints on the level of service in the transit network design model. An alternative and more satisfactory approach would of course be to iterate between the estimation of transit demand and the design of the transit system, in much the same manner as was suggested for the highway network design model in Chapter 8. This would be relatively expensive and time consuming but probably would be the most satisfactory approach. The estimation of demand for this transit model will not be considered in more detail, because presumably the model of most interest is the multiple mode model, to be presented in the next section, and in this model the total travel demand is internally split among the various modes.

The estimation of transit system operating and capital costs is not difficult. Many studies have concluded that bus transit system capital costs are essentially determined by the number of buses required and that operating costs are closely related to vehicle miles or vehicle hours (e.g., Miller and Rea (1973) and Meyer et al. (1965)).

The number of buses required is of course determined by the number of buses required during the period in which the largest number of vehicles are in use—presumably in the morning and/or the evening weekday peak period. Considering first a single link in the transit network
during a peak period, the number of buses which would be required to operate service at a frequency of (buses per hour in one direction) is:

\[ \text{Buses required} = 2T \alpha f \]

\( \alpha \) is the fraction of the total daily frequency which is operated in the peak hour.

This formula assumes that a bus operates in one direction (presumably the peak flow direction), turns around immediately and returns to the other terminal, where it is again available for operation in the peak direction. The formula holds provided the duration of time in which service is provided at a frequency \( f \) is at least as great as the round trip time of the bus—a condition likely to occur on urban transit routes. The formula also ignores the obvious condition that the number of buses must be an integer, and if the formula yields a non-integer value, then the buses required is the smallest integer containing the values so calculated.

The formula can readily be extended to consider many links simply by summing the buses required on each of the links, as given below.

\[ \text{Buses required on system} = \sum_{ij \in L_b} 2T_{ij} \alpha_{ij} f \]

The total cost of purchasing all of these buses is simply the price per bus multiplied by the number of buses required. If it is desired that these capital costs be expressed in terms of a daily or yearly equivalent, then an appropriate capital recovery factor is used as shown below:

\[ \text{Total cost of buses} = 2 \cdot Y \cdot \text{CRF} \sum_{ij \in L_b} T_{ij} \alpha_{ij} f \]

If the costs of the maintenance facilities for buses is to be included
in these capital costs, then an appropriate approximation would probably be an additional amount be added to the cost per bus. Similarly, the requirement of reserve vehicles in the fleet could also be accommodated by multiplying the required number of buses by an appropriate factor. Bus vehicle costs are presented in Morlok et al. (1971).

An alternate but essentially equivalent method for estimating vehicle requirements makes use of information on the average utilization rate of buses, as measured by vehicle-hours of service provided by a bus in a day. Let $U$ be that average utilization rate. The total daily vehicle hours operated is $\sum_{ij \in L_b} 2 T_{ij} f_{ij}$, the $2$ being required because both $T_{ij}$ and $f_{ij}$ refer to one-way flows and service is provided in both directions. The number of buses required is given by

$$\text{Buses required on system} = \frac{1}{U} \sum_{ij \in L_b} 2 T_{ij} f_{ij} \quad \text{(I-15)}$$

This is a linear function of frequency, just as is the prior equation. Either can be used with equal ease.

The operating costs of buses can be treated in a similar fashion. The major operating costs are associated with the maintenance and provision of fuel and other supplies for the operation of vehicles, and also the payment of drivers to operate those vehicles. Maintenance costs are generally estimated on the basis of vehicle miles. The total vehicle miles operated in the transit system is given by the formula below:

$$\text{Total vehicle-miles operated} = \frac{1}{U} \sum_{ij \in L_b} L_{ij} f_{ij} \quad \text{(I-16)}$$

where:

$L_{ij} =$ length of link $ij$. 

I-17
From transit operator records the maintenance cost per vehicle mile can usually be estimated without difficulty, and is multiplied by the total vehicle miles operated to yield total maintenance costs. For examples of such computations, see Morlok et al. (1971).

The costs associated with the drivers of the bus system are usually estimated on the basis of total vehicle hours or total vehicle miles driven. The standard transit system accounts permit easy estimation of average costs per vehicle hour or per vehicle mile for drivers, and these can be used as an estimate of the unit costs. The formula for vehicle miles was given above, and if that is the basis for operator cost estimation, then the appropriate cost formula is given below, with \( o_m \) as the estimate of operator costs on the basis of miles:

\[
2 \cdot o_m \sum_{i,j \in L_b} L_{ij} f_{ij} \quad (I-17)
\]

If the basis is vehicle hours, then the total vehicle hours operated multiplied by the cost per vehicle hour \( (o_h) \) is given by the following formula:

\[
2 \cdot o_h \sum_{i,j \in L_b} T_{ij} f_{ij} \quad (I-18)
\]

Thus there is no difficulty incorporating either cost estimation procedure into the model.

Other expenses, mainly insurance and administrative costs, are usually estimated as a constant fraction of all other costs, and hence can be reflected in the model by simply multiplying all cost factors by an appropriate fractional amount.

Thus total capital and total operating costs are of the general form:

\[
\sum_{i,j \in L_b} F_{ij} f_{ij} + \sum_{i,j \in L_b} Y \cdot CRF \frac{2}{U} T_{ij} f_{ij} \quad (I-19)
\]
The estimation of other parameters required for the model must be
done on the basis of policies which are either given or to be explored in
the use of the model. For example, level of service constraints would
presumably reflect either regulatory requirements imposed upon a particular
transit system or be used to reflect different policies regarding the
quality of service to be provided. Similarly, the inclusion of relationship
dealing with operating costs, capital expenditures, and subsidies
would be based upon characteristics of the particular situation being
analyzed. In fact, in most situations it probably would be desirable to
explore different capital budgets, different levels of subsidies, etc.,
in order to assess their relationships with other measures of performance
of the system.
Multiple-Mode Model

The construction of a preliminary multiple mode urban transportation model is not difficult given the existing core highway network design model and the transit network design model described above. The merging of these into a multiple mode model requires that two conditions be met. The first is that a set of objectives be incorporated in the model which is suitable for consideration of both modes simultaneously. Given the focus of the two models on general objectives such as users' travel time, operating costs, capital expenditures, and various impacts on the environment of the system, this requirement poses no difficulty. The second requirement is that a suitable means for estimating the split of traffic between the two modes be included within the model. This is the most troublesome requirement and will be discussed in detail below prior to presentation of the suggested model.

Alternative Approaches

There are basically two approaches to the inclusion of mode choice within the model. The first is to allow the model to select the split of traffic among the modes in such a manner that is most appropriate to achieving the objectives being considered (incorporated into the model). This is consistent with the approach used in the highway network design model for assignment, in which the routing of traffic through the network was determined by considerations of achieving whatever objectives were

I-20
included in the criterion function and meeting whatever objectives were incorporated as requirements in the constraints. Thus the assignment could yield traffic flows which are quite different from those which would be realized with motorists choosing their routes on the basis of their own criterion (presumably minimization of travel time and similar perceived costs). Thus the model requires that appropriate inducements be devised to yield traffic flow patterns similar to those resulting from the model. In a similar manner, this approach to mode split would require the use of inducements to travellers to select modes in the proportions which the model specified as being optimal, given the set of objectives considered. These inducements would probably take the form of pricing schemes, traffic restraints, rationing of parking spaces, etc. Despite the difficulties in implementing the model results, such results are attractive in that they specify conditions which are optimal with respect to the overall objectives considered.

The alternate approach is to attempt to incorporate into the model the usual mode choice relationships used in urban transportation planning. Such relationships usually estimate mode choice on the basis of the relative differences in travel time and cost for the two modes. Such an approach is not possible in this type of model if linear relationships are to be retained, and the retention of linear relationships is essential if standard computer codes are to be used in solving relatively large problems. The reason it is not possible to incorporate such relationships into the model is that the calculation of travel time by a highway, for example, requires the use of a non-linear expression. For example, on an individual link, the travel time of an individual motorist is given by the following relationship:
link travel time = \frac{C^m x_1 + C^m x_2}{x_1 + x_2} \quad (I-20)

where, as may be recalled from Chapter 4, \( C^m \) is the user cost on the \( m \)th piecewise linear segment and \( x^m \) is the choice variable flow on that segment.

An additional difficulty is that the routing of traffic through the network is not explicitly generated in a usable form within the model, so that some means for generating this information would have to be incorporated into the model before the travel times on the various links on a path from an origin to destination could be summed to yield total travel time. Even though the travel time from any node to any destination node can be ascertained from the dual values associated with the node conservation of flow constraints in the core version of the model with the minimization of travel time objective, this information is not available for use in the primal problem in the manner that would be required for incorporation of the mode choice relationships. Furthermore, when other impacts are considered in the objective function these dual values no longer represent travel time only, and hence do not provide the desired information.

Perhaps it should be noted at this point that a multiple mode urban transportation model has been formulated which does explicitly deal with travel time and does incorporate the mode choice relationships has been developed, although in its initial form it dealt only with user, operating, and capital costs within a single urban corridor. This is reported in Hay, Morlok and Charnes (1965). However, that model is now being extended to consider other impacts, although it might not be possible to extend this type of formulation to a network because of the large number of
constraint relationships that would be required.

Thus the approach which will be taken in this multiple mode model is that of allowing the modal split to be determined in a manner most appropriate to the achievement of the objective function considered in the model.

**The Model**

The construction and presentation of the model is rather straightforward, the model being essentially a combination of the transit model presented above and the highway core model presented earlier. The combining of the models can be accomplished by constructing a network which consists of both highway links for automobiles and bus transit links, as defined above. The nodes would continue to be nodes at which traffic originates or terminates and at which transit lines and/or highways intersect one another. The same variables as defined previously are used to describe the choices on each of the modes. The bus system is described by choices regarding the frequency of service, which in turn determines which links exist in the network and which do not and what the capacity provided on those links is. The highway system is described by choices with respect to the expansion of capacity of existing links or the construction of new links. Flows on both of the systems were described by directional flows on an arc with respect to each destination, and this convention is retained in the combined model. The only major change in the structural relationships of the model is that at each node a flow conservation relationship is written for each destination which includes the flows into and out of the node via both modes.

The resulting model is presented in Figure I-5. The objective
Objective Function

\[
\text{Minimize } \sum_{ij \in L_B} T_{ij} \left( \sum_{s=1}^S S_{sij} + \sum_{s=1}^S S_{sbij} \right) + \sum_{ij \in L_B} F_{ij} f_{ij} + \sum_{ij \in L_B} \text{CRF} \cdot \frac{1}{U} T_{ij} f_{ij} \\
\text{Transit: User time} \quad \text{Operating costs} \quad \text{Capital costs}
\]

\[+ \sum_{ij \in L_A} \sum_{m=1}^M C_{ij}^m X_{ij}^m + \sum_{ij \in E_A} \text{CRF} \cdot B_{ij}^k \cdot k_{ij} \quad \text{(I-21)}
\]

\[
\text{Auto: User time} \quad \text{Capital costs}
\]

Subject to:

Node conservation of flow-demand constraints

\[
\sum_{i \in B_j} S_{bij} + \sum_{i \in B_j} S_{x_{ij}} + S_{D_j} = \sum_{k \in A_j} S_{bij} + \sum_{k \in A_j} S_{x_{ij}}, \forall j, s, s \neq j
\]

\[
\text{Transit} \quad \text{Auto} \quad \text{Traffic generated} \quad \text{Transit} \quad \text{Auto} \\
\text{Traffic into node at node j} \quad \text{Traffic leaving node j} \\
\text{on arcs} \quad \text{for s} \quad \text{on arcs} \quad \text{(I-22)}
\]

Transit capacity

\[
Q \cdot f_{ij} \geq \sum_{s=1}^S S_{sbij}, \forall i \in A_b \quad \text{(I-23)}
\]

Transit vehicle availability

\[
\sum_{ij \in L_B} \frac{2}{U} T_{ij} f_{ij} = N + n 
\]

Transit level of service

\[
f_{ij} \geq M_{ij} \quad \text{(I-25)}
\]

Transit Operating subsidy

\[
\sum_{ij \in L_B} F_{ij} f_{ij} \leq R + S 
\]

Figure I-5. The Multiple-Mode Model Equations.
Auto assignment

\[ \sum_{m=1}^{M} x_{ij}^m = \sum_{s=1}^{S} s_{ij}^s + \sum_{s=1}^{S} s_{ij}^s \gamma_{ij}^e L_a \]  

(I-27)

Road capacity

\[ x_{ij}^m \leq K_{ij}^m + F_{ij}^m k_{ij}^m, \gamma_{ij}^e E_a \]  

(I-28)

\[ x_{ij}^m \leq K_{ij}^m, \gamma_{ij}^e N_a \]

Capital expenditures

\[ Y \cdot n + \sum_{ij}^{E_a} B_{ij} k_{ij} \leq B \]  

(I-29)

\[ \text{cost of new buses} \quad \text{cost of new road construction} \quad \text{Total capital budget} \]

where:

All variables and parameters are as defined previously.

A subscript a or b is used to denote sets of arcs and links for auto (road) and bus, respectively.

Figure I-5. The Multiple-Mode Model Equations, continued.
function used in this version of the model is one including a number of objectives which are weighted: user cost, operating cost, and capital cost. The reason for the use of this composite objective function is that such an objective function is likely to be the only one appropriate for this multiple mode model. If only a single objective, such as minimization of users travel time, were to be used, then it is likely that a very strange flow pattern would emerge, such as in this particular case all the traffic moving via highway, which in general would consume less time on any link than bus transit does. However, any desired combination of objectives could be used; the one presented is simply used as an example.

As mentioned, the node flow conservation relationships differ in this model from those of the single mode models in that both modes flows must be combined. This new type of relationship, equation I-22, states that the total flow into a node for a particular destination on the arcs of both the highway and the transit system, plus the traffic generated at that node for that destination (if any) must equal the total flow out of that node on the arcs of the two modes for that destination. Such a relationship must be written at each node for each destination, except that one node's relationship for each destination might be omitted, and the easiest one to omit is that of the node corresponding to the destination.

The other relationships, equations I-23 through I-28, apply to an individual mode and are identical to the ones for the single mode model. These require no further explanation.

The constraints which reflect other impacts might be combined for the
two modes. For example, the metropolitan area may have a single budget for expenditures on transportation which cannot be exceeded. In this case, the capital expenditures for the transit system as well as those for the local share of highway improvements would be summed, with the requirement that the total expenditures be less than a certain amount. This type of relationship is shown in equation I-29. On the other hand, funds for each of the two modes capital expenditures might be separate, in which case the individual relationships used previously in the single mode versions would be incorporated.

It should be noted at this point that the use of the model requires the estimation of only parameters which have been discussed previously in the text, so that no further difficulties in estimating the various inputs should be encountered.

Expected Results

The types of results which might be expected from this model are similar in general form to the outputs to be received from the single mode models. However, the results differ, in that the substitutability of one mode for another in terms of accommodating future traffic and also in assisting in the achievement of various objectives is revealed by this model's output. A typical form of the results might be as shown in Figure I-6. Here total user costs are related to total capital expenditures for transportation in the area as well as to the subsidy made available for the support of public transport systems. The expected relationship is probably as shown, namely that as the subsidy increases the total user cost for any given level of capital expenditures decreases. Using trade-off information such as this, intelligent choices might be
Figure I-6. Hypothetical Expected Results of Multiple Mode Model Application.
made regarding the proper combination of capital expenditures on the various modes and subsidies. Of course, underlying each point on such a trade-off curve set is a specific set of improvements to both the transit system and the highway network, and a particular operating plan for the transit system.

Possible Extensions

Since this model was not actually implemented on an example problem, experience cannot be relied upon as a basis for suggesting extensions and improvements to the model. However, a number of variations of the model were considered, and the more promising of these are discussed below.

It is likely that many travellers within an urban area are essentially captives of the public transit system. These can be easily distinguished in the model by requiring a certain minimal level of traffic flow on the transit system—that traffic being the trips of transit captives. These could be represented in the model by replacing each existing transit flow variable by two variables—one representing captives and the other choice riders. Separate node flow conservation equations would be written for choice and captive riders—the captives being restricted to use the transit system only.

A problem which might occur in this formulation of the model is that a traveller from a particular origin to a particular destination might switch frequently between transit and highway in an unrealistic manner. While it is possible that a traveller might drive from his home to a transit station and then take transit to his destination, it is virtually impossible to switch back and forth more times between automobile
and public transport. If such a problem should arise, then substitution of the following types of relationships for the node conservation of flow constraints would preclude such switching among modes:

1. Write separate node flow relationships for each mode. Include in these a variable for traffic generated at, say, node j for destination s.

2. Set the transit and auto traffic generated variables equal to total trips generated at each node for each destination.

A major omission of the model in terms of user costs and quality of service is the waiting time of travellers by transit. If it is desired to include waiting time in the user cost information, then appropriate piecewise linear approximations to the relationship of waiting time to frequency of transit service can be included. Once these relationships are included, then the waiting time can be included in the objective function.

Conclusion

A network design model for the consideration of both highway and public transit has been presented. It has most of the desirable features of the highway network design model in terms of efficiency, sensitivity to impacts and potential ease of use and understanding. Only actual application will provide a basis for evaluating the model and its usefulness, yet it appears quite promising as a useful tool based upon the initial formulation.
References


APPENDIX J

RECOMMENDED APPROACH TO DEDICATED ALGORITHM AND COMPUTER CODE

Introduction

In this appendix we develop an efficient method for performing the computations required by the simplex method when solving the linear programming core model in Chapter 4. We will show that the structure of the constraint matrix for the core model is such that the computational requirements for each step in the simplex method is vastly reduced. The computational requirements of the suggested procedure will then be compared with those of the straightforward application of the simplex method. The material in this appendix is primarily from (LeBlanc, 1973).

The notation used here will be slightly different from that used previously. Also, the network design model in this appendix will have limitations on flows due to noise pollution considerations. These constraints are

\[ p_{ij} (x_{ij}^1 + x_{ij}^2) \leq p_{ij} \]

It is shown in (LeBlanc, 1973) that this is equivalent to the constraints

\[ k_{ij} \leq p_{ij}^{-1} (p_{ij}) - 1 k_{ij} - 1 k_{ij} \]

The problem we wish to solve may now be formally stated. We will use the following notation.

- \( m \) the number of arcs in the network.
- \( n \) the number of nodes in the network.
- \( p \) the number of nodes which are destinations.
- \( x_{ij}^1, x_{ij}^2 \) the flow along the first and second linear parts of arc \((i,j)\).
- \( k_{ij}^1, k_{ij}^2 \) the existing capacity of the first and second linear parts of arc \((i,j)\).
- \( k_{ij} \) the capacity we add to arc \((i,j)\).
- \( b_{ij} \) the unit cost of adding capacity to arc \((i,j)\).
- \( \bar{b} \) the budget available for increasing capacity in the network.
- \( x_{ij}^s \) the flow along arc \((i,j)\) with destination \(s\).
The level of emissions from a flow rate of $x_{ij}$

The maximum allowable level of emissions on arc (i,j)

The demand (number of trips) between origin j and destination s

The unit cost (slope) of the first and second linear parts of the total cost function for arc (i,j)

\[
\text{(LP)} \quad \begin{align*}
\text{Min} & \quad \sum_{i,j} \left( 1_{c_{ij}} x_{ij} + 2_{c_{ij}} x_{ij} \right) \\
1_{c_{ij}} x_{ij} & \geq 0 \\
2_{c_{ij}} x_{ij} & \geq 0 \\
x_{ij} & \geq 0 \\
D(j,s) + \sum_{i} x_{ij} & = \sum_{j} x_{jk} \\
1_{x_{ij}} & \leq 1_{K_{ij}} + .75 k_{ij} \\
2_{x_{ij}} & \leq 2_{K_{ij}} + .25 k_{ij} \\
k_{ij} & \leq p_{ij}^{-1} (p_{ij}) - 1_{K_{ij}} - 2_{K_{ij}} \\
\sum_{i,j} b_{ij} k_{ij} & \leq 5
\end{align*}
\]

Constraints (J-2) and (J-4) - (J-6) hold for all arcs (i,j) such that i < j, and thus the number of these constraints is twice the number of arcs in the network. There is one constraint (J-3) for each node j in the network, for each destination node, s, and one constraint (J-7). In (J-1) and (J-7) the summation is over all arcs (i,j) such that i < j.

The objective function (J-1) is the linearized system travel time function.

Constraints (J-2) relate the variables $1_{x_{ij}}$, $2_{x_{ij}}$, and $x_{ij}^s$. Introducing
the variables \( \ell x_{ij} \) and \( \ell x_{ij} \) to linearize the system cost function results in fewer constraints than using \( \ell x_{ij} \) and \( \ell x_{ij} \) to denote the flow along the first and second parts of arc \((i,j)\) with destination \(s\). Constraint (J-3) is conservation of flow; (J-4) and (J-5) are the capacity constraints due to linearization, and (J-6) is the upper bound on \(k_{ij}\) because of pollution considerations. Finally, (J-7) is the budget constraint.

**Solution for the Network Design Problem with Capacities**

Although real world network design problems can be extremely large, there is a great deal of structure to the matrix of constraint coefficients for the preceding problem. The matrix for the 24 node network of described in Appendix A, appears in Figure J-1. We will label the submatrices in Figure (J-1) as follows.

\[
\begin{array}{cccccc}
A_0 & A_1 & A_2 & \cdots & A_p \\
B_1 & & & & \\
& B_2 & & & \\
& & B_p & & \\
S_0 & S_1 & S_2 & \cdots & S_p
\end{array}
\]

There are \(p\) diagonal submatrices \(B_1 = B_2 = \ldots = B_p\), and each \(B_i = [B, -B]\). The \(s^{th}\) submatrix \([B, -B]\) is the matrix of coefficients of the conservation of flow equations (J-3) for flow travelling to destination \(s\), \(s = 1, 2, \ldots, p\). As is well known (Jorgensen, 1963) of conservation of flow equations for each destination can be written with the node-arc incidence matrix as coefficients. In this network, the
Figure J-1. Constraint Matrix for the Network Design Problem
arcs were numbered such that the number of the arc from node i to node j equals the number of the arc from node j to i (mod 38). Thus the incidence matrix for the network takes the form \([B, -B]\). However, for each destination \(s\), the system of conservation of flow equations contains one redundant equation; therefore the \([B, -B]\) in Figure J-1 are all equal to \([B, -B]\) with one row deleted. We can eliminate any one equation from each set of equations; by eliminating the same equation from each set, each of the submatrices \([B, -B]\) in Figure J-1 is identical; this is helpful in reducing storage requirements. We will assume that the last row of the incidence matrix is deleted so that row \(j\) of \([B, -B]\) is the conservation of flow equation for node \(j\). If there are \(n\) nodes and \(m\) arcs in the network, then each \([B, -B]\) is \((n-1) \times m\) and is of full row rank. We see from Figure J-1 that in the linking constraints, \(A_1 = A_2 = \ldots = A_p = \begin{bmatrix} I & I \\ 0 & 0 \end{bmatrix}\). Each of these submatrices is of dimension \(m_0 \times m\). Reference to the model above shows that \(m_0\), the number of the linking constraints (J-2) and (J-4) through (J-7) is approximately twice the number of arcs in the network. We include the objective function in the first row of the constraint matrix by making the transformation

\[ x^0 = \sum_{ij} c_{ij}^1 x_{ij} + \sum_{ij} c_{ij}^2 x_{ij} \]

(J-8)

The objective function is now \(\min x^0\); \(x^0\) will always be the last basic variable. Note that the columns of the constraint matrix have been partitioned into the sets \(S_0, S_1, \ldots, S_p\).
The matrix for the 24 node, 76 arc network contains 552 conservation of flow equations; there are \( p = 24 \) block diagonal submatrices, and each \([D, -B]\) is 23 x 76. There are 38 identification constraints (J-2) and 38 upper bounding constraints on the added capacity (J-6), one for every other arc. There are 76 arc capacity constraints (J-4) and (J-5), two for every other arc, and one budget constraint (J-7). Including the objective function, each \( \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \) is 154 x 76.

When the revised simplex method is used to solve the problem Min \( cx \) s.t. \( Ax \leq b \), \( x \geq 0 \), the quantities \( c, b, A, \) and \( B^{-1} \) must be stored; \( B_k \) is the basis matrix at iteration \( k \). As is well known, the size of the basis and its inverse is determined by the number of constraints.

When dealing with a network with hundreds of nodes, the matrix in Figure J-1 will have more than 10,000 rows. Obviously, serious storage problems will result if the linear programming problem is treated directly. Fortunately, this problem exhibits such a structure that the storage requirements are considerably less than for general linear programming problems of the same size, and a direct approach via the revised simplex method becomes possible.

Since the matrix of coefficients is block diagonal, an obvious approach is that of Dantzig-Wolfe decomposition.

This approach was attempted recently for the model in the preceding section in a study at Northwestern University; however, the Dantzig-Wolfe routine converged so slowly that the procedure was terminated before optimality.
The slow convergence seemed to be due to the fact that the linking constraints (on the order of twice the number of arcs in the network) very tightly couple the problems. Thus the Dantzig-Wolfe technique had difficulty finding the optimal dual prices to send to the subproblems whose solutions in turn would satisfy the linking constraints in the master problem.

Now any basis for our network design problem can be written in the following form by rearranging the columns and partitioning.

<table>
<thead>
<tr>
<th>m₀ rows</th>
<th>A₁₁</th>
<th>A₂₁</th>
<th>Aᵢ₁</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>n-1 rows</td>
<td>B₁₁</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>n-1 rows</td>
<td></td>
<td>B₂₁</td>
<td></td>
<td>C</td>
</tr>
<tr>
<td>n-1 rows</td>
<td></td>
<td></td>
<td>Bᵢ₁</td>
<td></td>
</tr>
<tr>
<td>m₀ columns</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Here the submatrices $A_{i1}$ and $B_{i1}$ each contain exactly $(n-1)$ columns from the corresponding column sets $S_i$. The submatrix $[B]_{C}$ consists of the remaining basic columns from each $S_i$, as well as the basic columns from $S_0$. Each $B_{i1}$ is $(n-1) \times (n-1)$ and non-singular, and each $A_{i1}$ is $m_0$ by $(n-1)$. Observe that if the rank of any of the $B_{i1}$ were less than $(n-1)$, then the columns above could not span the same space as the matrix of coefficients.

Reference to Figure J-1 shows that since each column of the $A_{i1}$ in (J-9) consists of the first $m_0$ rows of a column from the set $S_i$, each column of the $A_{i1}$ is a unit vector, where $i=1, \ldots, p$. Each column of the $B_{i1}$ contains at most one +1 and at most one −1, since these columns are
from the incidence matrices with one row deleted \([B, -B]\). The columns
of \(C\) from the sets \(S_1, S_2, \ldots, S_p\) are also incidence columns with one
row deleted, and the columns of \(C\) from \(S_0\) are zero vectors. The columns
of \(\hat{B}\) consist of unit vectors, incidence vectors with an objective
function coefficient in the first row, and vectors containing a \(-.75,
-.25, b_{ij}\), and a 1. The basis matrix in (J-9) is almost block tri-
angular. It if could be transformed into a block triangular matrix, then
the required simplex calculations would be greatly simplified. It is
shown in Lasdon [1970] that the matrix \(T\) defined by

\[
T = \begin{bmatrix}
I_1 & V \\
0 & I_2
\end{bmatrix}
\]  

(J-10)

where \(I_1\) is a \(p(n-1) \times p(n-1)\) identity matrix, \(I_2\) is an \(m_0 \times m_0\) identi-
ty matrix, and \(V\) is a matrix \(p(n-1) \times m_0\), is a transformation matrix
such that

\[
\beta T =
\]

\[
\begin{array}{c|c}
A_{11} & A_{21} & \cdots & A_{p1} \\
B_{11} & B_{21} & \cdots & B_{p1} \\
\end{array}
\]

\[
= \begin{bmatrix}
\hat{B} \\
0
\end{bmatrix}
\]

Here \(\hat{B}\) is \(m_0 \times m_0\); in (J-10) \(V\) is the matrix

\[
V = \begin{bmatrix}
B_{11}^{-1} \\
B_{21}^{-1} \\
B_{p1}^{-1}
\end{bmatrix}
\]

\[
C = -\hat{B}^T C
\]  

(J-11)
where $C$ is $P(n-1) \times m_0$ and $V$ is $p(n-1) \times m_0$. The matrices $B$, $T$, and hence $BT$ are of dimension $[p(n-1) + m_0] \times [p(n-1) + m_0]$.

To gain insight into the nature of the $B^{-1}_{11}$, consider the system of equations

$$B_{11}x = e^j$$

where $e^j$ is a unit vector with the one in row $j$. This system has the unique solution

$$x = B^{-1}_{11} e^j$$

and hence $x$ equals column $j$ of $B^{-1}_{11}$.

Note (J-12) is a system of conservation of flow equations, and since $B_{11}$ is a basis for $[B, -B]$, the columns of $B_{11}$ define a tree in the network.

Since the RHS of (J-12) has a 1 in row $j$, the conservation of flow equation for node $j$, one unit of flow enters the network at node $j$.

Since $e^j$ contains no -1, this unit of flow leaves the network at node $n$, the node corresponding to the row deleted from the incidence matrix. Therefore the vector $x$ consists of unit flows along the path in the tree defined by $B_{11}$ between nodes $j$ and $n$. Furthermore, the $i$th component of $x$ equals -1 whenever the flow on the $i$th arc is in the direction opposite to the arc's orientation. An example is given at the end of this chapter.

To estimate the density of the $B^{-1}_{11}$, suppose that the network is a 400 node square grid with 20 nodes on each side. Then if the tree is as shown below, a path between two nodes contains an average of about 12 arcs. Thus the $B^{-1}_{11}$ will be only about 3% dense at each iteration of...
the simplex method.

Because of the structure in the constraint matrix of Figure J-1, the matrix multiplication for computing $V$ in (J-11) can be greatly simplified. Observe that the $j^{th}$ column of $V$, $V^j$, is obtained by multiplying $\bar{S}$ by column $j$ of $C$. As is well known, this can be written as

$$V^j = \sum_{k=1}^{p(n-1)} C(k,j)(\bar{B})^k \quad (J-13)$$

where $(\bar{B})^k$ is column $k$ of $\bar{B}$. But column $j$ of $C$ contains at most one +1 and at most one -1; the remaining entries are zero.

To take advantage of this sparsity, we can conveniently store the columns of $B_{ii}^{-1}$ (and thus the columns $(\bar{B})^k$) by using pointers instead of storing each element. Denote by

- $\text{NUMBEP}(j)$ - the number of +1's in column $j$ of $B_{ii}^{-1}$
- $\text{NUMBEM}(j)$ - the number of -1's in column $j$ of $B_{ii}^{-1}$
- $\text{BINVRS}P(j,k)$, $k=1, \ldots$, $\text{NUMBEP}(j)$ - position of +1's in column $j$ of $B_{ii}^{-1}$
- $\text{BINVRS}M(j,k)$, $k=1, \ldots$, $\text{NUMBEM}(j)$ - position of -1's in column $j$ of $B_{ii}^{-1}$
INDEXC(j,1) - the position of the +1 in column j of C
INDEXC(j,2) - the position of the -1 in column j of C

If there is no +1 or -1 in column j of C, then we will let the corresponding index have value 0. A check for this can be easily included in any code. Then $V^j$ is given by

$$V^j = (B^j)^{INDEXC(j,1)} - (B^j)^{INDEXC(j,2)}$$

(J-14)

We can see by (J-14) that all entries of $V^j$ are zero, except for the rows where a non-zero entry appears in columns $(B^j)^{INDEXC(j,1)}$ and $(B^j)^{INDEXC(j,2)}$. Thus we can form $V^j$ by joining together the representations of the two columns in the RHS of (J-14). Since we are adding together 0's, 1's, and -1's, the non-zero entries of $V^j$ will be -2, -1, 1, and 2. Since each $(B^j)^K$ in (J-14) averages 12 non-zero entries, $V^j$ will contain an average of no more than 24 entries. These may also be stored by pointers in a manner similar to that used for the $B^{-1}_{11}$.

To appreciate the savings in computational effort, suppose that each $B^{-1}_{11}$ is 400 x 400 and that $p = 50$, and that we perform the multiplications in (J-11) in a straightforward manner. Then we must multiply the 20,000 rows of $B$ by column j of C to compute $V^j$. Even if we use the obvious fact that $B$ is block diagonal, we must perform 400 multiplications for each of the 20,000 rows of $B$, or 8,000,000 multiplications.

This is to be contrasted with J-14, where we compute an average of 24 pointers for $V^j$. Furthermore, the operations in J-14 are all additive. Thus there is an enormous reduction in the required computations.

J-11
Now we can transform any basis matrix for our problem into a block triangular matrix. We will now show that all of the steps in the revised simplex method can be performed without storing the entire basis for the problem. Instead, we need only store the inverses $B_{i1}^{-1}$, $i = 1, 2, \ldots, p$, and $\vec{B}^{-1}$, and the $B_{i1}^{-1}$ can be stored by means of pointers as mentioned above. We do not need to know the inverses of the $A_{i1}$. This yields a considerable savings in storage requirements as compared to storing the entire inverse in product form. Now suppose we have a basic feasible solution for this problem, and we want to use the revised simplex method to see if this solution is optimal. We know that $\pi = c_B \beta^{-1}$, but we do not want to store $\beta^{-1}$, even in secondary storage. Since $\beta T$ is block triangular and easier to work with, it is natural to try to use this matrix to compute the dual variables. Now

$$c_B = (0, 0, \ldots, 0, 1) \in \mathbb{R}^{(n-1)p + m_0}$$

since only $x_0$ appears in the objective function, and it is considered to be the last basic variable. Therefore

$$c_B T = (0, 0, \ldots, 0, 1)$$

by inspection of $T$. Now

$$\pi \beta = c_B = \pi (\beta T) = c_B T = (0, 0, \ldots, 0, 1)$$

Expanding this equation, we have

$$\begin{pmatrix} \pi_0 & \pi_1 & \ldots & \pi_p \end{pmatrix} \begin{bmatrix} A_{11} & A_{21} & A_{p1} & \vec{B} \\ B_{11} & B_{21} & 0 \\ \vdots & \vdots & \vdots \\ J-12 & \end{bmatrix} = (0, 0, \ldots, 0, \ldots, 1)$$
where \( \pi_0 \in \mathbb{E}^{m_0} \) is the vector of dual variables for the first \( m_0 \) constraints, and \( \pi_i \in \mathbb{E}^{n-1} \) is the vector of dual variables for the \( i^{th} \) set of \( (n-1) \) constraints, \( i = 1, 2, \ldots, p \). Performing the matrix multiplication, we see that

\[
\pi_0 A_{il} + \pi_i B_{il} = 0 \in \mathbb{E}^{n-1} \quad i = 1, \ldots, p
\]  \hfill (J-15)

\[
\pi_0 \tilde{B} = (0, 0, \ldots, 0, 1) \in \mathbb{E}^{m_0}
\]  \hfill (J-16)

Since the right hand side of (J-16) is a unit vector, \( \pi_0 \) is evidently the last (the \( m_0^{th} \)) row of \( \tilde{B}^{-1} \). Given \( \pi_0 \), the remaining \( \pi_i \) are readily computed as

\[
\pi_i = -\pi_0 A_{il} \tilde{B}_{il}^{-1} \quad i = 1, \ldots, p
\]  \hfill (J-17)

Now we noted earlier that the columns of \( A_{il} \) are unit vectors, and that the \( \tilde{B}_{il}^{-1} \) contain only 0's, 1's, and -1's and are very sparse. Thus the matrix multiplication in (J-17) can be greatly simplified. Define

\[ u = \pi_0 A_{il} \] \( u \) is \( 1 \times n-1 \). Denote by

\[ \text{INDEXA}(j) - \text{the position of the 1 in column } j \text{ of } A_{il} \]

Then

\[
u = (\pi_0(\text{INDEXA}(1)), \pi_0(\text{INDEXA}(2)), \ldots, \pi_0(\text{INDEXA}(n-1)))
\]  \hfill (J-18)

Now \( \pi_i = -u \tilde{B}_{il}^{-1} \), and the \( j^{th} \) component of \( \pi_i \) is the product of \( -u \) with column \( j \) of \( \tilde{B}_{il}^{-1} \). This is the negative of

\[ u(\text{BINVRSP}(j,1)) + u(\text{BINVRSP}(j,2)) + \ldots + u(\text{BINVRSP}(j,\text{NUMBERS}(j))) - u(\text{BINVRSM}(j,1)) - u(\text{BINVRSM}(j,2)) - \ldots - u(\text{BINVRSM}(j,\text{NUMBERS}(j))) \]

But since \( u(i) = \pi_0(\text{INDEXA}(i)) \) by (J-18) we get that the \( j^{th} \) component of \( \pi_i \) is

\[ J-13 \]
\[ \pi_0 \text{INDEXA(BINVRSM(j,l))} + \ldots + \pi_0 \text{INDEXA(BINVRSM(j,NUMBERM(j))} \]
\[ \pi_0 \text{INDEXA(BINVRSPP(j,l))} - \ldots - \pi_0 \text{INDEXA(BINVRSPP(j,NUMBERP(j))} \]

Thus we do not need to store the vector \( u \) after all.

Again, we have a reduction in computations by orders of magnitude over the straightforward matrix multiplication in (J-17). Computing the \( j \)th component of \( \pi_0 \) by actually multiplying (J-17) would require multiplying \( A_{i1} \) by \( B^{-1}_{11} \) and then multiplying \( -\pi_0 \) by the resulting \( j \)th column. Let us first examine the number of operations required to multiply \( A_{i1} \) by \( B^{-1}_{11} \). The example 400 node grid network would have 1600 arcs; thus \( A_{i1} \) would be 3200 x 400 and \( B^{-1}_{11} \) would be 400 x 400. We must multiply each of the 3200 rows of \( A_{i1} \) by each of the 400 columns of \( B^{-1}_{11} \). This is 1,280,000 inner products, each involving 400 multiplications, for a total of 512,000,000 multiplications. In addition, we must multiply \( \pi_0 \) by the resulting \( j \)th column. These half a billion multiplications are to be compared to the average of twelve additions and triply indirect addressings in (J-19).

With this method of computing the prices, we can use a structured routine in which a central program sends the last row of \( B^{-1} \) to \( p \) different subsystem programs. Each subsystem, \( i \), has access to \( B^{-1}_{i1} \) and \( A_{i1} \) and can compute its own multipliers. Subsystem \( i \) can then price out its non-basic columns by computing the \( \tilde{c}_j = -\pi P^j \) for all non-basic columns \( P^j \) of the constraint matrix. Observe that the objective function coefficients are all zero except for the coefficient of \( x_0 \), which is always basic. Now reference to Figure (J-1)
shows that

\[ \tilde{c}_j = -\eta_0 A_j^0 \quad \text{if} \ P_j \in S_0 \]

\[ -\eta_0 A_j^i - \pi_i B_j^i \quad \text{if} \ P_j \in S_i, \ i = 1, \ldots, p \]

where \( A_j^0 \) is the \( k_j \) column of \( A_0 \) (the first \( m_0 \) rows of the \( j \)th column of the matrix of coefficients in Figure J-1. Similarly, \( A_j^k \) and \( B_j^k \) are respectively the \( k_j \) columns of \( A_j \) and \( B_j \); these are the non-zero sections from \( P_j \).

Once again, we can greatly simplify the multiplications in (J-20). For \( P_j \in S_i, \ i = 1, \ldots, p \), \( A_j^k \) is a unit vector, and since \( B_j^k \) is an incidence vector (with one row removed), we can denote by

- \( \text{INDEXB}(k,1) \) - the position of the \(+1\) in column \( k \) of \( B_j^i \)
- \( \text{INDEXB}(k,2) \) - the position of the \(-1\) in column \( k \) of \( B_j^i \)
- \( \text{INDEXA}(k,1) \) - the position of the \(+1\) in column \( k_j \) of \( A_j \)

Recall from Figure J-1 that each \( B_j^i \) equals \([B, -B]\) and that \( A_j = A_2 = \ldots = A_P = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \). Thus the above indices are independent of \( i \). Again, if there is no \(+1\) or \(-1\) in column \( k \) of \( B_j^i \), then the corresponding index has value zero. This can be easily checked before proceeding. The closed form for \( \tilde{c}_j \) is then

\[ \tilde{c}_j = -\eta_0 (\text{INDEXA}(k_j)) - (\pi_i (\text{INDEXB}(k_j,1)) - \pi_i \text{INDEXB}(k_j,2)) \]

If \( P_j \in S_0 \), then \( A_j^0 \) is either a unit vector, an incidence vector with an objective function coefficient in row one, or contains a \(-.75, -.25, b_{ij}, \text{and a 1} \). Thus the vector multiplication \(-\eta_0 A_j^0 \) can also be performed extremely efficiently by simple bookkeeping.

J-15
Assume then that \( \min_j c_j = c_s < 0 \) and that \( P^s \) is a column from column set \( S_r \) of the matrix in Figure J-1. To find column \( s \) of the tableau, \( P^s \), we perform the operation
\[
\tilde{P}^s = \beta^{-1} P^s
\]
This is equivalent to solving the system of \( m_0 + p(n-1) \) linear equations in the \( m_0 + p(n-1) \) variables \( P^s \)
\[
\beta P^s = P^s
\]
where the matrix \( \beta \) and the right hand side \( P^s \) are given. Again we observe that this system would be simpler if \( \beta T \) were the matrix of coefficient, so we solve the system of \( m_0 + p(n-1) \) equations in \( Z^s \) \( E^{m_0+p(n-1)} \) instead
\[
(\beta T) Z^s = P^s \tag{J-21}
\]
Obviously, after we obtain the solution to (J-21), we have
\[
\tilde{P}^s = T Z^s \tag{J-22}
\]
Now if we write out the system of equations (J-21) in partitioned form, we can solve the system by inspection.

\[
\begin{bmatrix}
A_{11} & A_{21} & A_{p1} & \tilde{\bar{s}} \\
B_{11} & & & \\
B_{21} & & & \text{0} \\
B_{p1} & & & \\
\end{bmatrix}
\begin{bmatrix}
z_1^s \\
\cdot \\
\cdot \\
z_p^s \\
z_s^s \\
\end{bmatrix}
= \begin{bmatrix}
k_s \\
A_r \\
0 \\
B_r \\
0 \\
\end{bmatrix} \tag{J-23}
\]
In the RHS of (J-23) $A_r^s$ and $B_r^s$ are respectively the $k^s_r$ columns of $A_r$ and $B_r$ (the non-zero sections of column $P_r^s$ from column set $S_r$). To remind the reader, each $A_{ii}$ is $m_0 \times (n-1)$ and each $B_{ii}$ is $(n-1) \times (n-1)$, so each $z_i^s$ is $(n-1) \times 1$; $\bar{B}$ is $m_0 \times m_0$, so $z^s$ is $m_0 \times 1$. This system is equivalent to

$$\sum_{i=1}^{p} A_{ii} z_i^s + \bar{B} z_r^s = A_r^s$$

$$B_{il} z_i^s = 0 \quad i = 1, \ldots, p; \ i \neq r$$

$$B_{rl} z_r^s = B_r^s$$

By inspection, we have the solution

$$z_i^s = (0, 0, \ldots, 0)^t \in \mathbb{E}^{n-1} \quad i = 1, \ldots, p; \ i \neq r \quad \text{(J-24)}$$

$$z_r^s = \bar{B}^{-1} B_r^s \in \mathbb{E}^{n-1} \quad \text{(J-25)}$$

$$z^s = \bar{B}^{-1} (A_r^s - A_{rl} z_r^s) \in \mathbb{E}^{m_0} \quad \text{(J-26)}$$

The procedure for computing $z_r^s$ in (J-25) is identical to that used in computing $V^j$ in (J-13) and (J-14). We again observe that $B_r^s$ is an incidence vector with one row deleted, and as above, we denote by

INDEXB($k_s^1, 1$) - position of $+1$ in column $B_r^s$

INDEXB($k_s^2, 2$) - position of $-1$ in column $B_r^s$

$(B_{rl}^{-1})^j$ - column $j$ of $B_{rl}^{-1}$

Then in (J-25) we have that

$$z_r^s = (B_{rl}^{-1})^j - \text{INDEXB($k_s^1, 1$)} - (B_{rl}^{-1})^j \text{INDEXB($k_s^2, 2$)} \quad \text{(J-27)}$$

J-17
In the example 400 node network above, the columns of the RHS of (J-27) average 12 non-zero entries, and thus $z_r^s$ averages no more than 24 entries.

To compute $z^s$ in (J-26) we have that $A_r^s$ is a unit vector, and thus $B^{-1}A_r^s$ equals column "INDEXA($k_s^r$)" of $B^{-1}$. We now define

$$U = B^{-1}A_{r1}$$

We see that column $j$ of $U$ equals $B^{-1}(A_{r1})^j$, where $(A_{r1})^j$ is column $j$ of $A_{r1}$. We will also write $U^j$ and $(B^{-1})^j$ for column $j$ of $U$ and $B^{-1}$, respectively. But $(A_{r1})^j$ is a unit vector, and so $B^{-1}(A_{r1})^j$ equals column INDEXA($j$) of $B^{-1}$:

$$U^j = B^{-1}(A_{r1})^j = (B^{-1})^j \cdot \text{INDEXA}(j)$$

Thus each column of $B^{-1}A_{r1}$ is available through a trivial bookkeeping device. Now we perform the multiplication $-Uz_r^s$. We have just determined the positions of the non-zero entries in $z_r^s$; letting $z_r^s(k)$ equal the $k$th component of $z_r^s$, we write

$$-Uz_r^s = - \sum_{j=1}^{n-1} z_r^s(k)U^j = - \sum_{j=1}^{n-1} z_r^s(k)(B^{-1})^j \cdot \text{INDEXA}(j) \quad (J-28)$$

Thus we don't need to store the vector $U$ at all. Since $z_r^s$ averages only 24 non-zero entries, most of the terms in the sum in the RHS of (J-28) are zero.

Now that we have $Z^s$, we use (J-22) to compute $\tilde{P}^s$, the column of the tableau corresponding to the entering basic column.
\[
\begin{bmatrix}
\tilde{r}_1^S \\
\tilde{r}_2^S \\
\vdots \\
\tilde{r}_p^S \\
\tilde{r}_r^S
\end{bmatrix} = T \begin{bmatrix}
Z^S
\end{bmatrix} = \begin{bmatrix}
I_1 & V_1 \\
& \cdots \\
& & I_p & V_p \\
0 & & & I_2
\end{bmatrix} \begin{bmatrix}
0 \\
\vdots \\
0 \\
Z^S_r
\end{bmatrix} \quad (J-29)
\]

where the matrix \( V \) has been partitioned into the \( n-1 \times m_0 \) submatrices \( V_i \). We now have

\[
\begin{align*}
\tilde{r}_i^S &= v_i z^S & i &= 1, \ldots, p; i \neq r \\
\tilde{r}_r^S &= z^S + v_i z^S & (J-30) \\
\tilde{r}_r^S &= z^S & (J-31)
\end{align*}
\]

When multiplying \( V_i \) and \( V_r \) by \( z^S \), we make use of the fact that each \( V_i \) in (J-29) has extremely low density. We noted earlier that \( V_j \), an entire column of \( V \), averages 24 non-zero entries in the example 400 node network with 50 destinations. Furthermore, each column of the \( V_i \) in (J-29) is a segment of \( V_j \) with only \( 1/50^{th} \) as many rows. The elements of \( V \), and thus the elements of the \( V_i \), consist of elements in \{-2, -1, 0, 1, 2\}. In taking the linear combination of the columns of the \( V_i \) and \( V_r \) in (J-30) and (J-31) we multiply the few entries in the columns of the \( V_i \) and \( V_r \) by the corresponding components of \( z^S \) and join together the representations of these columns.
The column which leaves the basis is chosen by the usual simplex rule. We compute

$$\text{Min} \quad \frac{\tilde{b}_i}{\tilde{p}_i^s} = \frac{\tilde{b}_z}{\tilde{p}_z^s}$$

where $\tilde{b}_i$ is the current value of the $i^{th}$ basic variable, and we choose column $z$ of the basis to become non-basic. Let column $z$ of the basis be column $p_z^j$ of the matrix of coefficients. The new values of the basic variables are computed as usual; $\tilde{p}_z^s$ is the pivot element, and

$$\tilde{b}_z^s = \frac{\tilde{b}_z}{p_z^s} \quad \text{and} \quad \tilde{b}_i^s = \tilde{b}_i - \frac{\tilde{b}_z}{\tilde{p}_z^s} \quad i = 1, 2, \ldots, p; \ i \neq z$$

where $b^*$ is the updated vector of basic variables.

In summary, if we are given any basic feasible solution, we can generate the relative cost factors and the column to enter the basis. The column of the tableau, the updated column, can then be determined so that we can find the column which leaves the basis. All of these operations require only the matrix of coefficients, $B^{-1}$, $A_{il}$, and $B^{-1}$. As we have shown, both the storage and the computational aspects of this procedure can be performed extremely efficiently.

When a new variable enters the basis, we must update the inverse of the new basis. Lasdon [1970] gives the procedure for updating the basis inverse for general linear programming problems which are block diagonal with linking constraints. As might be expected, we can compute the new basis inverse much more efficiently for our problem than for general
linear programming problems.

The column which leaves the basis, \( P^j_z \), is to be replaced by column \( P^s \), when \( P^s \) and \( P^z \) are columns from different column sets, say \( S_q \) and \( S_r \), respectively. Lasdon [1970] shows that we must compute a new \( B^{-1}_{rl} \).

Now we have shown that column \( j \) of \( B^{-1}_{rl} \) is a simply the vector of unit flows between nodes \( j \) and \( n \) along the arcs in the tree defined by \( B_{rl} \). Recall that row \( n \) was the redundant row deleted from the network's node-arc incidence matrix; this resulted in \([B, -B]\) in Figure J-1. Now we can use this information to develop a routine to find \( B^{-1}_{rl} \) without actually performing any pivot operations. The following is a systematic procedure for searching the tree defined by \( B_{rl} \) for the paths (e.g., the arc numbers) between each node and node \( n \). First we will give a verbal description of the procedure, including an example, and then we will state the algorithm formally.

Starting from node \( n \), we move through the tree, coming across each node in turn. We give each node in turn two labels, which help us retrace the path from node \( n \) to the node in question. For each node, \( i \),

\[ \text{LABEL}(i,1) - \text{First intermediate node on path between node } i \text{ and node } n \]
\[ \text{LABEL}(i,2) - \text{First intermediate arc on path between node } i \text{ and node } n \]

Since there are exactly \( n-1 \) arcs and \( n-1 \) nodes (not counting node \( n \)) in the tree, we perform exactly \( n-1 \) iterations of this search procedure.

For any specified row, \( j \), of the incidence matrix of the tree (e.g., for any node \( j \) in the tree), the routine searches across the columns of row \( j \) for a non-zero entry. We then have two possible cases.
In the first case, we locate a non-zero in row j. We call the column containing this non-zero entry column k; then we know that arc k in the tree is adjacent to node j. Next we search the rows of column k of the incidence matrix of the tree for the row containing the other non-zero entry (there are exactly 2 entries in each column); call this row r. Now we have that node r is adjacent to node j via arc k. We give node r the labels

\[
\text{LABEL}(r,1) = j \\
\text{LABEL}(r,2) = k
\]

which indicates that node r was reached from node j via arc k. This will be used in reconstructing the path between node j and node n.

In the second case, we are unsuccessful in our search across the columns of row j for a non-zero entry. This means that there are no more arcs in the tree leaving node j; we have come to a dead end in the tree, and we must backtrack. To do this, we simply use the label of node j: we repeat the procedure by searching the row "LABEL(j,1)".

We now give an example

\[
\begin{array}{cccc}
1 & 2 & 3 & 4 \\
\hline
1 & 0 & 0 & 1 & -1 \\
2 & 0 & -1 & -1 & 0 \\
3 & -1 & 1 & 0 & 0 \\
4 & 1 & 0 & 0 & 1
\end{array}
\quad
\begin{array}{cccc}
1 & 2 & 4 \\
\hline
1 & 0 & 0 & -1 \\
2 & 0 & -1 & 0 \\
3 & -1 & 1 & 0 \\
4 & 1 & 0 & 1
\end{array}
\]

Figure J-2. A Network and Tree and the Corresponding Incidence Matrices
In the network of Figure J-2 the tree consists of the three arcs 1, 2, and 4. Note that the third basic column is the fourth column of the coefficient matrix (incidence matrix) on the left of Figure J-2. Starting with node 4, we search row 4 of the incidence matrix of the tree, on the right in Figure J-2 locating a 1 in column 1. Every time we locate an element in the matrix, we erase it. Now we have that arc 1 is adjacent to node 4. Searching column 1, we locate a - 1 in row 3, which tells us that node 3 is incident to node 4. We have that

\[
\text{LABEL}(3,1) = 4 \\
\text{LABEL}(3,2) = -1
\]

Now we repeat the procedure with row (node) 3. Searching row 3 gives us a 1 in column 2 and then a - 1 in row 2 of column 2. The labels are

\[
\text{LABEL}(2,1) = 3 \\
\text{LABEL}(2,2) = -2
\]

Next we search row 2, but cannot locate a non-zero entry. Therefore, we search row "LABEL(2,1)" , e.g., row 3, but again we are unsuccessful. Thus we search row "LABEL(3,1)" = row 4, finding a 1 in column 4 and a - 1 in row 1 of column 4. The final labels are

\[
\text{LABEL}(1,1) = 4 \\
\text{LABEL}(1,2) = -4
\]

There are 4 nodes in the network, and since we have completed 3 iterations of the procedure, we are finished.

To construct column 1 of the inverse, we recall that column 1 is the
solution to the system of equations

\[
\begin{bmatrix}
0 & 0 & -1 \\
0 & -1 & 0 \\
-1 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_4
\end{bmatrix} =
\begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix}
\]

Now $\text{LABEL}(1,2) = -4$, which tells us that $x_4 = -1$. The first intermediate node on the path between node 1 and node 4 is $\text{LABEL}(1,1) = 4$, and we have completed column 1 of the inverse: column 1 is $(0, 0, -1)^t$. To compute column 2 of the inverse, we see that $\text{LABEL}(2,2) = -2$, so $x_2 = -1$. The first intermediate node in the path between node 2 and node 4 is $\text{LABEL}(2,1) = 3$, and so the second arc in the path is $\text{LABEL}(3,2) = -1$. Therefore $x_1 = -1$. The next node on the path is $\text{LABEL}(1,1) = 4$, and we are done: the column is $(-1, -1, 0)^t$. To compute the third column, we note that $\text{LABEL}(3,2) = -1$, and so $x_3 = -1$. Since $\text{LABEL}(3,1) = 4$, we are finished with the column. It is easily verified that

\[
\begin{bmatrix}
0 & -1 & -1 \\
0 & -1 & 0 \\
-1 & 0 & 0
\end{bmatrix}
\]

is in fact the inverse of the incidence matrix

\[
\begin{bmatrix}
0 & 0 & -1 \\
0 & -1 & 0 \\
-1 & 1 & 0
\end{bmatrix}
\]

We now give the formal statement of the algorithm. We perform $n-1$ iterations of the following procedure.
0. Set NODE = n.

1. Search row "NODE" of the incidence matrix until finding a non-zero entry. Define ARC = the column containing the entry, and erase the entry. Go to 2.

If there is no non-zero entry, set NODE = LABEL(NODE). Go to 1.

2. Search column "ARC" for the non-zero entry, and set SIGN = the algebraic sign of this entry. Set SAVE = NODE and set NODE = the row containing the non-zero entry, and erase the entry. Set LABEL (NODE,1) = SAVE, LABEL(NODE,2) = ARC * SIGN. Go to 1.

References


APPENDIX K

RECOMMENDED MATHEMATICAL APPROACH TO ASSESSING COST OF PRESERVING VALUED FACILITIES

As discussed in Chapter 5, it is possible to take advantage of the mathematical structure of the network design model to reduce the costs of assessing the cost of preserving valued facilities on land which might be taken for right of way. A detailed explanation of this method and its incorporation into the model follows.

Suppose for each pair of adjacent nodes \((i, j)\), either a link in the transportation system already exists or one is proposed. Consider only the proposed links. Suppose further that there is a highly valued facility along the route of proposed link \(i-j\). What is the cost paid by potential system users to avoid taking the facility? It is the cost paid by the using public for not including link \(i-j\) in the "optimum" (in the sense of least cost) network. The procedure for determining this cost is as follows:

Identify all proposed link additions and the valued facilities along the proposed routes. For those links threatening facilities for which the cost of saving is to be determined, place constraints \(\alpha^m_{ij}\) on \(k^m_{ij}\), the added capacity of links \(ij\) at user cost level \(m\). Identify all user costs \(s^m_{ij}\) along links \(i-j\) at level \(m\) from source \(s\). Define for each link \(i-j\) under investigation, source \(s\) and user level \(m\), a multiplier, \(s^m_{ij}\), such that if \(s^m_{ij} \cdot s^m_{ij}\) were the true cost of traversing link \(ij\) from source \(s\) at level \(m\), the model would choose other links for traffic assignment in order to minimize total user cost. That is, establish a multiplier which raises user costs to an unacceptably high level. Now at each iteration of the linear programming code, the problem is to minimize \(c^N_{XX} + c^B_{XX}\).
subject to: \( Ax = Bx_B + Nx_N = b \) where the matrix \( A \) is partitioned into submatrices \( B \) and \( N \) corresponding respectively to the vector \( x_B \) of basic variables and the vector \( x_N \) of non-basic variables. By rewriting this objective function solely in terms of \( x_N \), one finds: minimize \( \begin{bmatrix} c_B B^{-1} b + (c_N - c_B B^{-1} N) x_N \end{bmatrix} \), subject to: \( Bx_B + Nx_N = b \), \( x_B \geq 0 \). Call this the main problem.

Consider a specific link \( i-j \) in the network. Let each cost at level \( m \) from source \( s \) be denoted by \( s_{c_{ij}}^m \). At each iteration, the variables \( s_{x_{ij}}^m \) for \( s = 1, 2, \ldots, S \) and \( k_{ij}^m \) will either all be basic or they will all be non-basic. Then the costs \( s_{c_{ij}}^m, s = 1, \ldots, S, m = 1, \ldots, M \), will be components of either the vector \( c_B \) or \( c_N \). For each iteration of the model in the main problem, calculate the alternate cost function:

\[
z' = \begin{bmatrix} c_B^1 B^{-1} b + (c_N^1 - c_B^1 B^{-1} N) x_N \end{bmatrix}
\]

where the primes denote vectors whose components are the \( s_{\mu_{ij}}^m s_{c_{ij}}^m \) instead of the \( s_{c_{ij}}^m \).

Suppose at the optimum value \( z_0 \) of the objective function of the main problem, the choice variables \( s_{x_{ij}}^m (s = 1, 2, \ldots, S) \) and \( k_{ij}^m \) are all non-zero. This means these variables are basic and take on non-zero values at the optimum. Now it is desirable to force these variables to zero and see how the objective function changes. Therefore, introduce the alternate cost function \( z' \) as the objective function in the main problem and continue the computations until the optimum value, \( z_1 \), of \( z' \) is attained (at which point, because of the high costs \( s_{\mu_{ij}}^m \cdot s_{c_{ij}}^m \) the variables \( s_{x_{ij}}^m, s = 1, \ldots, S \) and \( k_{ij}^m \) will be non-basic). Then the cost of avoiding all valued facilities on link \( i-j \) is \( z_1 - z_0 \). This procedure may then be followed for each link in the original (or secondary) optimal solution(s) which affects valued facilities.