DEVELOPMENT AND APPLICATION OF A
HIGHWAY NETWORK DESIGN MODEL

Volume 1 of 2
Main Report

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July 1973

FINAL REPORT
Prepared for
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Environmental Planning Branch
Washington, D.C. 20590
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The contents of this report reflect the views of the Department of Civil Engineering of Northwestern University, which is responsible for the facts and accuracy of the data presented herein. The contents do not necessarily reflect the official views or policy of the Department of Transportation. This report does not constitute a standard, specification or regulation.
DEVELOPMENT AND APPLICATION OF A HIGHWAY NETWORK DESIGN MODEL


Department of Civil Engineering
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Environmental Planning Branch
Federal Highway Administration
U.S. Dept. of Transportation
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Two volume report: Volume 1. Main Report
Volume 2. Appendices

The Highway Network Design Model was developed to assist transportation planners in the initial stages of development of system plans. It considers a wide variety of improvements to the system and their economic, social and environmental impacts. By treating the network at a fairly aggregate level, it is possible to explore many alternatives quickly and with a minimum of resources. Only data which is readily available in urban transportation studies today is required.

The model is quite promising as a sketch planning tool. Furthermore, it is well-suited for use in community interaction, because of its efficiency in answering questions posed and because of the inclusion of relatively simple, easily understood, descriptors of system impacts and system characteristics. A program of testing and development in the context of an actual transportation planning effort is recommended. This could equally well be in urban, state-wide, regional, or national planning, for the model is readily adapted to any of these applications.

Urban Transportation Planning
Network Design
Sketch Planning
Mathematical Programming

Unclassified

Unclassified
This research on the development of a Highway Network Design Model represents the efforts of persons from many different fields to develop a sketch planning tool which is responsive to current needs for consideration of a wide variety of impacts associated with highway improvements as well as consideration of the widest possible range of transportation system alternatives. As in the case of most research in a university, this project did not exist in isolation, and we have drawn freely from more exploratory research (on network design and equilibrium models and evaluation methodology) conducted under a National Science Foundation Grant to the Urban Systems Engineering Center of Northwestern University. The diversity of backgrounds and talents available, and the ability to draw results from concurrent related research efforts, are two of the major advantages of conducting research at a university.

We were especially appreciative of the opportunity to perform this research for an agency which is a potential user of the model and which also has considerable influence over the methods adopted and used in transportation planning by states and metropolitan areas. This afforded us the opportunity to develop and demonstrate through example applications the features of this type of model and to convey these findings to an agency which has the resources and power to make the model part of the repertoire of models used in transportation planning.

Of course, this research could not have been performed without the financial support of the Federal Highway Administration, through contract number DOT-FH-11-7862, entitled Network Design. Of equal importance was the climate in which the research was conducted, and we are indebted to Mr. Samuel Zimmerman, Project Manager, and to his predecessor, Mr. Daniel Cohen, for that climate and for understanding the necessarily trial and error nature of much of the research. The suggestions of Mr. Zimmerman regarding fruitful directions of research, his very helpful comments on the final report, and most of all his enthusiasm for the project, were of immense value to us.

Finally, we wish to thank the typists, Ms. Janet Schumer and Pamela Frye, and the draftsman, Mr. Thomas Wenzel, for their excellent work.

Edward K. Morlok
Joseph L. Schofer
Co-Principal Investigators

July 31, 1973
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AUTHORSHIP

In any project of this size and scope, it is necessary to divide the various tasks and assign responsibility. This was done in terms of authorship of chapters and appendices as indicated below. However, it is equally true that much interchange of ideas occurred during both the research and writing phases, so that in a sense almost all participants were associated to some extent with every aspect of the work.

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CHAPTER 1
EXECUTIVE SUMMARY

Purpose and Scope

The Network Design Model was developed to assist transportation planners in the initial stages of development of network plans, in which they wish to explore a wide range of alternatives in order to identify those which appear most promising. Alternatives to be considered usually include a variety of network structures, transportation modes or technologies, variations in the extent of investment and the location of particular facilities, as well as various mixes of capital intensive versus operational changes. These initial explorations are usually conducted at a fairly abstract or aggregate level in order to conserve resources which might be applied at later stages to more detailed analyses and refinement of specific plans. The Network Design Model offers an efficient, and relatively objective, analytic approach for both identifying and assessing many alternative network plans in a short period of time. In this sense it might be viewed as a computer-supported sketch planning tool.

The macroscopic evaluation of these alternatives should focus upon the trade-offs between gains and costs - measures of objective attainment - affecting various segments of the community. Particular community groups may be affected by transportation investments in many ways, some positive and some negative. For example, those living adjacent to a new freeway may experience reduced travel times and increased accessibility to opportunities due to its construction, while at the same time their neighborhood may be adversely affected by the taking of land and by increased noise and air pollution usually associated with such facilities.
It is essential that information on impact trade-offs be developed, early in the transportation planning process, so that the evaluation of generalized alternatives can be performed with reasonable knowledge of the nature and extent of the most important expected impacts. This information should provide effective support for decisions regarding the feasibility and desirability of alternative macro-plans. Otherwise, both planners and decision-makers stand the risk of premature commitment to network plans without knowing the most significant impact implications of those alternatives. The Network Design Model has been developed to serve two, interlinked purposes: to identify efficiently many good network plans at a gross scale, and to provide an early evaluation of the impacts of such plans. The plan identification or search process is explicitly guided by the nature of, and the preferences for, the impacts of alternative plans. While this search process is supported by the analytic model, it is an open loop strategy which provides significant opportunities for decision-maker involvement. Organized approaches to these issues have not been available in the past.

The Network Design Model can provide easily understood information on trade-offs among transportation impacts or levels of objective achievement offered by alternative network plans. An example of this type of information is shown in Figure 1-1 which shows the trade-offs between two objectives, minimizing user travel time and minimizing houses taken for right-of-way. Various network alternatives achieve these objectives to differing degrees; in this example, as greater investment in new highway facilities is made, more and more houses are taken but at the same time, traffic flows more freely, resulting in a reduction in user travel time. Note that each point on this curve represents an alternative network design; a feature of the Network Design Model is its ability to identify and evaluate--in this example,
in two dimensions—the large number of networks necessary to realize this trade-off relationship in a very short time and at low cost.

![Graph showing trade-off between User Travel Time and Houses Taken for Right-of-Way.](image)

Figure 1-1. Example of Information on Trade-Offs in Objective Achievement Generated by the Highway Network Design Model

On the basis of this type of information, the gains and costs accruing to different groups resulting from increasing levels of investment in transportation, as reflected in alternative network structures, can be identified and the desirability of alternative network plans assessed in a variety of dimensions. Using this information, then, it is possible to make rational decisions regarding which generalized network plans should be subjected to more detailed feasibility and design studies. Typically, of course, many more than two dimensions are required to adequately assess transportation plans. The Network Design Model has the capability to accommodate many more evaluative dimensions, so long as each impact to be considered can be measured in a quantitative manner.

**Characteristics of the Model**

The Network Design Model has been developed to be used in an interactive
decision-making environment, providing rapid response to question and ideas posed by its users. The model accepts information on the desired direction of system improvements and synthesizes a set of improvements which, in fact, move in that direction. More specifically, the model requires information on objectives, measured in terms of the impacts of concern, expected travel demand, and existing network structure, and generates alternatives which are most conformal to the stated objectives. The model has the capability to use information on the relative importance of each objective, as provided by the decision-maker or to aid the decision-maker in establishing his own priorities. Thus the model in effect works "backward" from a statement of objectives to a system design most conformal to those objectives, internally designing the network. This is in contrast to the usual planning model, in which the impacts are merely predicted for each externally provided alternative, requiring the user to generate alternatives which conform to his objectives. Thus the Network Design Model focuses the planner's attention upon impacts and evaluation, as well as providing him with the usual information on the network alternative, enabling him to more effectively devote his attention to the necessarily human task of selection of the best alternative.

The model is formulated and solved as a mathematical program rather than using the simulation approach more common to contemporary urban transportation planning. Its users do not, however, need any knowledge of mathematical programming. The Network Design Model will not usually replace more traditional planning models, but will offer an efficient way to develop more and better alternative plans to be tested in more detail using existing methods and models. The nature of the inputs required for the Network Design Model is compatible with the input data required by the existing approaches to urban
transportation planning.

The model has already been used to design simple highway networks according to a wide variety of quantifiable objectives, including:

- minimize user travel time;
- minimize construction costs;
- minimize the number of dwelling units required for rights-of-way;
- meet Federal highway noise standards;
- minimize vehicle miles of travel (an indicator of pollution emissions);
- remain within a pre-specified budget constraint.

A feature of the mathematical programming formulation is that it permits very rapid and inexpensive evaluation of the sensitivity of objective attainment to variations in assumptions about costs of construction, budget levels, the locations of major activity centers, and demand estimates.

Various means have been devised to accept information into the model on the user's preferences and priorities among various objectives to guide in the search for the best alternative. He may express some objectives as minimal standards which must be met, such as might be used for noise intrusion. He might provide information on the relative importance of objectives, such as the value of time provides in the case of travel time relative to monetary expenditures. Or, he may ask the model to identify the "undominated" solutions representing the best possible combinations of the attainment of several objectives. Undominated solutions are the alternative networks which represent the best possible levels of attainment of several objectives; if the attainment of one objective is to be increased beyond the value offered in the undominated solution, the attainment of another objective must be decreased. Using any combination of these approaches, the model helps the user to explore the set of most desirable, feasible solutions. It cannot tell him
what his own value trade-offs are, but it can show him what physical trade-offs are possible. Experiments have shown that the subsequent choice of desirable network plans is made considerably easier and less costly in time and money.

The Network Design Model performs two tasks simultaneously: it chooses the optimal set of links to be improved or added to the existing network, based on the measures of objective attainment specified; and it assigns the (given) travel demands to that new network. It provides, as its standard products, the specifications of the proposed network, including the location and magnitude of link improvements, the flows and levels-of-service on the various links, and the levels of attainment for each objective.

As it is normally used, the model provided a "good" network or a set of networks, each time it is run. The sets of undominated networks, including perhaps one-hundred or more different networks, comprise a trade-off relationship which is given to local planners or policy-makers. Their preferences regarding alternative networks, expressed in terms of levels of objective attainment, are then returned to the model for further analysis. In a short time, the search process generally leads to one or more alternatives suitable for more detailed analysis.

At this point, the user can be sure that he has explored a sufficiently large number of good alternatives to assure that he has greatly increased the likelihood that he will find one of the best, if not the best, network design. He can also be confident that he has given appropriate consideration to a variety of impact measures even at the level of macroscopic, network planning.

**Requirements for Use of the Model**

Who can use the Network Design Model? Any planning agency which is
committed to the rapid and efficient identification and evaluation of aggregate network plans. The agency should, for example, be willing to assess networks which can be generally described using less than seventy-five nodes and perhaps 150 links. The model will identify promising alternatives which can then be evaluated in considerably more detail with traditional techniques. Agencies which are facing significant decisions regarding trade-offs between major objectives will find the model especially helpful.

What is required to use the model? While network design models have been in the literature for a decade or more, operational versions are just coming into use. None is yet in a pre-packaged form suitable for use in the manner of the FHWA or UTPS planning software packages. Some outside assistance, provided by the developers of the model, will be required. The planning agency itself, however, needs no special expertise in mathematical programming. It is necessary to have a fairly large computer available, on the order of a CDC 6400 or an IBM 360-65, with 150K (octal) or 64K (decimal) core storage. A well-stated set of planning objectives is also needed, along with gross, design year estimates of inter-area travel demands by mode. An aggregate version of the existing and committed transportation network will have to be coded, but this is usually a simple task. All links which are candidates for new construction or improvement must be identified.

Also required is the willingness of decision-makers or senior transportation planners to interpret and evaluate intermediate outputs of the model to provide local policy guidance for the network search process. The man-hour and calendar time requirements will vary with the application, but meaningful results should be available within about thirty days of initiation.
CHAPTER 2

INTRODUCTION AND STRUCTURE OF THE REPORT

Decisions regarding the structure of transportation networks, and the general levels-of-service to be offered on them, are among the most important choices to be made in the transportation planning process. These issues are typically treated at a macroscopic, or sketch planning level, at which point detailed characteristics of the network alternatives and their impacts are not available. The important ramifications which such choices have for later stages of the planning process, and for the performance of the implemented transportation system, suggest the need for identifying the best possible candidate networks as early as possible.

Previous approaches to generalized network design have been relatively informal. Rough comparisons have been made between the capacity of the existing and committed transportation network and the expected, design year, travel demands. Where deficiencies have been noted, proposals have been developed for the addition of new links, or for the expansion of existing facilities. The problem with this approach is that it does not insure that the best possible networks will be identified in the sketch planning stage.

There has been interest in recent years in the application of advanced analytic techniques to the search for good alternative transportation networks. Network design models based on mathematical programming have been formulated and solved for small problems. Yet these approaches, too, have suffered from some serious deficiencies.

Firstly, they have tended to be limited to the consideration of measures of network effectiveness which focus primarily on impacts which can be easily measured in monetary units. This is a critical limitation considering the
increasing importance of social and environmental factors in transportation planning. Secondly, such models have been formulated in ways such that their generalizability has been restricted; applications to other networks, in other locations, would be difficult, requiring considerable expertise in mathematical programming, and specialized computer software. Thirdly, such design models have been structured for solution by relatively inefficient techniques, using slow computers. The resulting limitation in the network size which could be treated with a reasonable amount of computer time has been severe. Yet recent advances in computer technology, and in solution algorithms, may permit significant increases in model efficiency.

The purpose of this report is to describe an investigation into the feasibility of creating more responsive, and operational, approaches to optimal transportation network design. Mathematical programming models for network design have been formulated and tested to overcome some of the deficiencies listed above. In particular, models which are simple, sensitive to a broader set of transportation impacts, and more efficient in solution, have been developed and tested.

These models focus on highway networks, although their extension to transit and multimodal networks was given brief consideration. While the examples treated involve only small networks, the extension of the models to large networks was examined and found to be promising. In fact, much attention was given to the question of the feasibility of constructing more efficient methods for solving large scale network design problems, since networks larger than the ones dealt with in this research are of interest. Preliminary results, drawing from work performed under this project and in other related research, suggest that significant increases in solution efficiency may be achievable through additional effort devoted to algorithm and computer code development.

The report itself is organized into a main text and a set of appendices.
The latter provide additional details on the structure of the model, the results of component research efforts which form the foundation of this project, and mathematical discussions which may require some additional understanding of mathematical programming than required to grasp the main text.

The main text comprises nine chapters. The first is the executive summary of the report; the second is this overview chapter. Chapter 3 provides a more detailed description of the functional characteristics of the network design model, and the role it is expected to play in transportation planning.

Chapter 4 presents the "core" model, the basic formulation of a continuous variable network design model. It also provides the solution to an example problem and demonstrates some of the advantageous features of the mathematical programming formulation.

Chapter 5 describes a series of approaches for introducing greater sensitivity to impacts into the network design model. Four strategies for accomplishing this are illustrated through the development of four specific impact measures in forms compatible with the model discussed in Chapter 4. One of these measures, related to noise impacts, is exercised through the use of a numerical example.

Chapter 6 presents a strategy for using the network design model in multiple-objective planning contexts. Methods for generating trade-off curves describing objective attainment, and ways in which to use those trade-offs for interaction with decision makers, are discussed.

Chapter 7 reviews some of the alternative methods applied to solving the model, along with the computational experience with these methods accumulated in the course of this research. Recommendations for further improvements in computational efficiency are presented.

Chapter 8 discusses some of the more promising contexts in which the
network design model might be used. It also describes some alternative strategies for its use in a typical transportation planning environment.

Chapter 9 contains recommendations for further development, implementation, and extension of the model.

More details on specific aspects of this research are contained in the appendices, which are referred to at appropriate points in the main text.
CHAPTER 3

GENERAL DESCRIPTION OF THE HIGHWAY NETWORK DESIGN MODEL

Purpose

The Highway Network Design Model was developed to assist urban transportation planners in the initial stages of development of plans in which they wish to explore a wide range of alternatives and their impacts, in order to identify those which appear most promising. Alternatives to be considered usually include a variety of transportation modes, network structures, and substantial variations in the extent of investment and the location of particular new facilities. These initial explorations are usually conducted at a fairly abstract or aggregate level in order to conserve resources which might be applied at later stages to more detailed analyses and refinement of specific types of plans. Thus, the model may be viewed as a tool for sketch planning.

The evaluation of gross network alternatives should focus upon the distribution of gains and costs among the various affected groups within an urban area (Thomas and Schofer, 1970). In addition to variations among groups, any particular group may be affected in many ways, some positive and some negative. For example, a group living in a neighborhood traversed by a new freeway may experience reduced travel time and increased accessibility to opportunities due to its construction, while at the same time their neighborhood may be adversely affected by the taking of land for this facility and the noise and air pollution usually associated with its use. It is essential that information on the broad range of impacts be developed at an early stage in the planning process, so that the evaluation of general network alternatives can be comprehensive. Otherwise, a general plan might be selected for
further analysis and refinement which will later be found to be unacceptable because of some of its impacts.

It should be equally clear that in the initial stages of sketch planning the entire range of possible alternatives should be considered, so that the best among these has a chance for consideration and adoption. In order to treat a broad spectrum of possible types of alternatives, such as mixes of freeways and arterial improvements or public transport, it is necessary to limit the detail with which each alternative is examined. By using this model to eliminate the poorer alternatives and to identify those which appear more promising, the total resources used for planning at this initial stage are kept to a minimum. This allows resources to be devoted to other tasks, such as refinement of selected sketch plans, selection of a particular detailed plan, the programming thereof, etc. To accomplish this, the Network Design Model focuses upon options at the corridor level, rather than detailed individual link level.

Perhaps the most advantageous use of the Network Design Model will be in interaction with the various community groups and individuals concerned with transportation decisions. It is clear that the direct involvement in planning of those who are affected by transport plans is essential for their views to be adequately represented and for the achievement of a plan which in their view is equitable in the distribution of its benefits and costs. Furthermore, the more direct involvement of political leaders and other decision-makers in the plan evaluation process will increase the likelihood of plan acceptance and hence significantly increase the effectiveness of the planning process itself. This involvement is not satisfactory if it is only after a few plans have been selected by planners for their consideration, but rather must be involved more directly in the process. By such involve-
ment, the goals adopted, the impacts considered, and the alternatives reviewed, will be better matched to the information needs and positions of those who must ultimately decide upon or approve a particular plan. For such interaction to be effective, it is necessary to respond rapidly and efficiently to questions posed. In addition, the methods or models used must be capable of exploring a wide range of alternatives and predicting the impacts in a manner understandable to non-technical persons. The Network Design Model provides this type of capability, by dealing at an aggregate level, yet being able to explore a wide spectrum of alternatives and the associated impacts on specific groups.

**Goal-Directed Planning Models**

The development of the network design model has been guided by two major premises. The first is that it is essential to explore a wide range of alternatives, so that the "best" plan is not inadvertently overlooked. Unfortunately, this is a very formidable task, because the number of options in any realistic network problem is very large; for example, in a highway network containing only 100 links, each of which can be improved in only one way, there are over one million possible combinations. Even if many (or most) of these could be eliminated on a priori grounds, the number is still formidable.

The second premise is that the user(s) of the model must be integrally involved in the process of evaluating alternatives, because preferences for achievement of various objectives may be difficult if not possible to determine at the outset. These are more likely to be revealed as the analysis proceeds and the range and magnitude of impacts become known. As preferences are revealed and operationally described, the added information which
they provide to the planner can be used in even more ambitious analyses.

The implications of these premises for an effective model system are substantial (Morlok, 1969). They require that the model be very robust with respect to the kind of evaluative information it can accept and utilize. Such information may be unstructured, and subject to change as the analysis proceeds. To evaluate a representative sample of the large number of network options, the model should include some means for internally eliminating those alternatives which are clearly unsatisfactory, given whatever is known about preferences. This allows the user of the model to focus his attention on those alternatives which are most promising. The model should have a capability to generate internally the combinatorial alternatives which characterize this problem, so that this need not be done at great expense outside the model. Furthermore, the kind of information the user wants is likely to vary as the analysis proceeds, so flexibility in this regard should be provided. Finally, the model must be structured so that the decision maker can interact with it effectively and efficiently.

A type of model which meets these requirements is the goal-directed model (Morlok, 1970). Such a model would accept information from the decision maker on the goals and impacts of importance, and the relative emphasis to be placed on each. The model then generates various alternatives, and selects the one (or perhaps a few) which best matches the objective achievement requirements. Information on the expected levels of achievement of these objectives is then given to the user, along with information describing the physical characteristics of the best alternative. This information can then be used by the decision maker to further refine his statements of preferences, which can then be used as the basis for further analysis in the model. This interaction is depicted in Figure 3-1.

3-4
Figure 3-1. Strategy for Using the Goal-Directed Model.
This goal-directed, or backward-seeking, model is distinguished from
the more typical type of planning model, in which a subjectively determined
design alternative is tested using models which yield information on the
performance and impacts and hence on the levels of achievement of various
objectives. Such a model has the disadvantage of requiring the user to
specify externally each alternative to be explored, and usually has no pro-
vision for internally rejecting poor alternatives. The goal-directed model
begins with information on the preferences for the attainment of objectives
and performs the mechanical task of testing alternative designs and identi-
fying that one (or set) which most nearly corresponds to the specified pre-
ferences.

In the process of using the model, information is developed describing
the phenomenological trade-offs between impacts and the achievement of var-ious objectives. In some applications, it may be desirable to use the model
to produce such trade-off information in advance of interactions with the
decision maker. This would show him what is technologically possible, at
the outset, in order to assist the decision maker and it would therefore
provide him with guidance in the selection of his preferences. However,
just as there are a large number of alternatives, there are many objectives
and trade-offs, so only a sample of these, rather than all, are likely to be
generated.

This type of trade-off information is exemplified by Figure 3-2, which
shows how alternative plans meet two different objectives, minimizing user
tavel time and minimizing houses taken for rights-of-way. Various alter-
natives achieve these objectives to differing degrees, and the expected
relationship will be that as greater investment in new highway facilities is
made, more and more houses are taken, but at the same time traffic flows
Figure 3-2. Example of Trade-Off Information Generated by the Highway Network Design Model.
more and more freely, resulting in a reduction in user travel time. On the basis of this type of information, the gains and costs accruing to different groups resulting from increasing levels of investment in road capacity can be identified more easily and the desirability of that additional investment assessed from various viewpoints. This information should be of considerable value in making the selection of good alternatives to be explored in more detail in the next phase of the planning process.

**Strategy for Model Use**

The network design model opens up new approaches to the planning of transport systems, and it requires development of an effective strategy for use.

In its current form, the model accepts various forms of information on the decision maker's preferences and priorities among objectives. For example, objectives may be expressed as minimum standards which must be met, such as might be used for noise intrusion. Information on the relative importance of objectives, such as the value of travel time relative to monetary expenditures, may also be utilized. The decision maker may also use the model to identify the "undominated" solutions representing the best possible combinations of the attainment of several objectives; if the attainment of one objective is to be increased beyond the value offered in the undominated solution, the attainment of another objective must be decreased.

The results of an application of the model may also be used in a "post-processor" (analysis performed outside the network design model) to determine the nature of additional impacts of the alternative networks which cannot be predicted internally. The results of the post-processor analysis may then be used to modify objectives and constraints for further analysis.
in the model itself. Finally characteristics of the best networks, as determined in the model, may be modified or perturbed to perform sensitivity studies of these solutions.

Through combinations of these approaches, the model helps the user to explore the set of most desirable, feasible solutions. It cannot tell him what his own value trade-offs are, but it can show him what physical trade-offs are possible. Experiments have shown that the subsequent choice of desirable network plans is made considerably easier and less costly in time and money.

As it is normally used, the model provides a "good" network or a set of networks, each time it is run. The sets of undominated networks, including perhaps one-hundred or more different networks, comprise trade-off relationships which can be given to local planners or policy-makers. Their preferences regarding alternative networks, expressed in terms of levels of objective attainment, are then returned to the model for further analysis. The search process leads quickly to one or more alternatives suitable for more detailed analysis.

At this point, the user can be sure that a sufficiently large number of good alternatives have been explored to assure that the likelihood of finding one of the best, if not the best, network design, is greatly increased. The decision makers can also be confident that appropriate consideration has been given to a variety of objectives and impacts even at the level of macroscopic, network planning.

The model has been designed to fit into the existing urban transportation planning process as a sketch planning tool. It has been developed and tested for the planning of highway networks in urban areas. The inputs to the model are few in number and generally available within most studies:
- a specified set of objectives and associated measures of achievement;
- a design year interzonal trip table;
- existing or committed networks (including speed-volume relationships on the links);
- possible improvements to that network;
- relationships for predicting impacts of interest;
- externally imposed constraints, such as capital expenditure limits.

The model is formulated and solved as a mathematical program rather than using the simulation approach more common to contemporary urban transportation planning. Its users do not, however, need any knowledge of mathematical programming. The network design model will not usually replace more traditional planning models, but will offer an efficient way to develop more and better alternative plans to be tested in more detail using existing methods and models.

The outputs of the model are manifold. One of course is a single, or perhaps a set of, preferred sketch plans. Each would include a specification of the links to be improved, the extent of such improvement, and the resulting traffic volumes. Corresponding to each alternative is information on the impacts or levels of objective achievement of each. Also, very readily obtained is information on the sensitivity of various results to variations in inputs, such as demand levels, costs of construction, etc.

Conclusion

The network design model is basically designed to be used at the initial stages of transport planning, commonly referred to as sketch planning. It is not designed to replace the existing urban transportation planning methods, but rather to supplement these in an area which is particularly
weak at the present time. As will be described in the following chapters, the model in its current form concentrates primarily upon road improvement decisions within metropolitan areas, but extensions to include public transit and to other contexts are discussed. While the model has been subject to testing and refinement in the present study, it is essential that it be subjected to more rigorous tests in an actual planning environment so as to properly guide further development.

References


CHAPTER 4

THE CORE HIGHWAY NETWORK DESIGN MODEL

**General Description**

The currently formulated versions of the network design model are structured as linear programming problems. The basic structure of a mathematical programming problem is one in which a particular mathematical function is to be maximized or minimized; this is referred to as the objective or criterion function. This optimization process is performed subject to meeting a number of constraints.

The objective function and constraints operate on a set of choice variables, the values of which are determined by the program such that all of the constraints are satisfied and the objective function is maximized or minimized. Mathematical programming provides two ways in which to incorporate the various impacts and related objectives of the transportation planning process: as elements of the objective function, and as constraints which might represent minimal standards or levels of achievement of objectives which are considered acceptable by the user.

Since there can be only one objective function to be extremized, usually only a single objective is included at any time; alternatively, several objectives can be weighted and summed in commensurate units, and the latter function may then be optimized. Such weighting is commonly used in transportation planning when travel time is converted into dollar costs by use of a monetary value of time so that time costs and other monetary costs can be combined into a single measure of total transport cost. To accomplish this, of course, it is necessary to develop the appropriate set of weights (e.g., value of travel time) to permit this aggregation.
In any particular problem, there usually are many options as to which objectives are treated as constraints and which are included in the objective function. Once such structural choices have been made, information on trade-offs between attainment of objectives can be developed quite easily using mathematical programming methods, through varying the required levels of achievement of the objectives which are included as constraints. Another means of generating this information is by varying the weighting values used to incorporate two or more objectives in the criterion function.

The basic structure of the model is as shown in Figure 4-1. The primary choice variables are choices of investments in improvements or construction of roads or public transport, with the possible addition of variables representing the various operating policies of these modes, such as regulations on road traffic flow and the schedules and prices of public transport. To make such choices with respect to these primary decision variables in the model, estimates of those impacts and relationships which must be considered must be prepared initially outside the model. These would include such items as the extent to which the system must accommodate estimated traffic demands, the relation between volumes and travel times on the links, the predicated modal split, the costs of constructing roadway facilities in monetary and other terms, travel time and other costs to users, and measures of noise intrusion into neighborhoods.

For example, if it is required that the existing system plus any new facilities which are added must accommodate estimated future traffic, then one constraint in the model would state quantitatively that the capacity of the system must be sufficient to accept assignment of that predicted traffic. Previous efforts toward the development of network design models focused almost entirely on the inclusion of such operational constraints and relation-
Variables (columns)

Traffic flows on arcs (assignment)  
Link investment (new capacity or shifts in speed-volume function)

Relationships (rows)

Minimize

Total user cost  
(zero)

Subject to:

Demands

Demands must be accommodated by the assignment  
(zero)

Capacity

Traffic can occur only on those links in existence or constructed, and must not exceed capacity provided (existing and new).

Budget

Link investment can not exceed the available budget.

Note: Relationships among the choice variables and the constraints (rows) are indicated in the appropriate rows and columns. A (zero) indicates no direct relationship.

Figure 4-1. Overall Structure of the Highway Network Design Model.
ships (e.g., all demands must be satisfied). The optimization process itself is then performed on easily-measured, user or budget-oriented objectives, such as the minimization of user costs, capital costs, or the summation of these. (For a review see Schwarz, 1968). Only one network design model (Morlok et. al., 1970), which treats intercity rather than urban problems, has been structured so as to deal with a broad range of objectives, but it can not be directly applied to urban transportation without substantial structural changes. Many of the concepts and methods of these prior models are carried forth in this work.

A major feature of the network design models described in this report is that they have been structured so as to accommodate objectives of other affected groups, the attainment of which may not always be subjected to monetary measurement. Thus, if it were required that noise levels in a neighborhood must not exceed a specified level, then traffic flow on links through that neighborhood must be appropriately constrained to keep noise at or below the target value. To introduce this increased sensitivity to impacts into the network design model, the mathematical program must include not only the relationships to predict various impacts, but also the capability to compare impacts to target levels of achievement. If these targets are not met, then the solution is considered infeasible, and is revised until feasibility is achieved.

Important questions relate to the strategies by which the target levels may be set so that the network design process leads toward the identification of a feasible, "best" plan, since in this context "best" is not defined in any rigorous mathematical optimization sense, but rather in the sense that decision-makers select a plan with which is associated particular achievement levels for the various objectives.
The following sections of this chapter describe the mathematical programming formulation for a simple network design model. This initial model focuses on selecting a candidate network based on easily measured physical requirements for facilitating traffic flows, the satisfaction of travel demands, and budget constraints. This model forms the core for further development of an impact-sensitive model, which is described in detail in Chapter 5.

Core Model Components

The core network design model comprises four primary sets of relationships: user cost, capital cost, assignment and demand, and budget. These can be most readily explained as distinct elements, with the notation and definitions being introduced as necessary. These mathematical relationships can then be combined to form the basic model.

User Costs

Travel time on a road is an increasing function of traffic flow. The Federal Highway Administration (1972, III-15) uses the following function to describe the effect of capacity constraints on average travel time:

\[ t = t_0 \left[ 1 + 0.15 \left( \frac{v}{v_p} \right)^4 \right] \]  \quad (4-1)

where

- \( t_0 \) = free speed travel time
- \( t \) = travel time when assigned volume on road is \( V \)
- \( v_p \) = practical capacity of the road

This function is illustrated in Figure 4-2.

The total travel time on the road is then:
Figure 4-2. Highway Travel Time Function.
This non-linear function for total travel time can, for use in a linear programming model, be approximated by a piecewise linear function as shown in Figure 4-3. Although only two linear "pieces" are shown, as many pieces can be used as needed to achieve the desired accuracy. The pieces, or segments, are used to reflect the variation in level-of-service being offered on the roadway at different volumes.

New variables must be defined to describe the traffic volumes in the model. If the volume is less than or equal to $K_1$, then it is fully described by the value of $x_1$, which cannot exceed $K_1$. If it is greater than $K_1$, then the true value is $x_1$ (which equals $K_1$) plus $x_2$. $x_2$ cannot exceed $K_2$, and thus total traffic is limited to $K_1 + K_2$ or the maximum capacity of the roadway. $c_1$ and $c_2$ are the slopes of the two segments, respectively, and these represent the average travel time at the two levels of service—or, if the values are chosen correctly, the average travel cost. Thus, using the two-piece linear approximation, the total travel cost on a road at the volume $x_1 + x_2$ becomes:

\[
T = t_0 \left[ 1 + 0.15 \left( \frac{v}{v_p} \right)^4 \right] \cdot v
\]

(4-2)

Total travel time = $c_1 x_1 + c_2 x_2$

\[
0 \leq x_1 \leq K_1
\]
\[
0 \leq x_2 \leq K_2
\]

(4-3)

The average travel time is:

\[
\text{Average travel time} = \frac{c_1 x_1 + c_2 x_2}{x_1 + x_2}
\]

(4-4)

The connector carrying traffic from node i to node j is referred to as the arc ij. Each arc is capable of accommodating assigned traffic at a number of levels of service, defined by M number of segments, in the linear ap-
Figure 4-3. Piece-wise Linear Approximation to Total Travel Time on a Link.
proximation. In the case shown in Figure 4-3, \( M \) is equal to two. Each arc can thus be described as two segments, each having a different average travel time, \( C_{ij} \). The directional flow on that segment is referred to as \( x_{ij}^m \).

**Capital Costs and Link Improvements**

The total increase in maximum capacity per unit time period made to any arc \( ij \) in the model is \( k_{ij} \). The cost associated with the increase in maximum capacity is calculated as a direct, linear function of that increase in capacity per unit time, with the coefficient expressed in dollars per additional unit of daily capacity. The value of this coefficient, \( B_{ij} \), would be different for each link, reflecting its length and other physical characteristics.

More complex capital cost patterns can be modelled, as required by the particular application. For example, cost per unit of capacity might increase as capacity is increased beyond a certain point, perhaps reflecting the necessity of purchasing more expensive land for right of way as the size of the facility increases. This sort of cost can easily be included by a piece-wise linear approximation, similar to that used for user costs. Only decreasing costs could not be accommodated, because of the restrictions imposed upon the mathematical forms of linear programs.

In effect, arc improvements are modeled as shifts in the average travel time-volume relationships such that \( k^1 \), the "break point," and \( k^1 + k^2 \), the maximum capacity, are increased.

This can be understood by reference to Figure 4-4. Curve A might represent the existing arc flow relationship. The arc might be improved with an expenditure of capital funds so that the travel time-volume relationship is as given by curve B. A further increase in expenditure could shift the

4-9
Figure 4-4. Alternative Improvement to a Link Described as a Shift in the Travel Time-Volume Curve.
curve to C. Such changes can be described by the maximum capacity of each roadway, denoted by \( K_A, K_B \) and \( K_C \) in the figure. All potential improvements on the arc are approximated in the model by the continuous variable, \( k_{ij} \), which is the amount of increase in maximum capacity.

It can be argued that capacity cannot be added in any amount, but only in discrete units. However, it was felt that the continuous solution is preferable to dealing with only discrete capacity additions for three reasons. First, at the general sketch-planning level to which this model is addressed, an arc can represent many individual roads, so only an approximation to added capacity is needed. Secondly, the discrete modeling approach requires all possible improvements to be identified separately, and definition of all the associated variables would increase the size of the model beyond that capable of solution with existing computer codes for all but the simplest networks. Thirdly, through the use of signing, signals, parking restrictions, and channelization, it is possible to achieve small increases in capacity at proportionally lower costs. Thus, the continuous model must be viewed as an efficient approximation for sketch planning which yields the required capacity changes for optimal flow rather than capacity increases resulting from specific roadway designs.

A set of constraints must be included to limit the volume of traffic assigned to each arc and segment thereof to the capacity originally assumed in the problem, plus the additional capacity provided for in the solution.

The constraints are written:

\[
x_{ij}^m - F_{ij}^m k_{ij} \leq k_{ij}^m
\]  
\[(4-5)\]

where: \( k_{ij}^m \) = the initial maximum capacity of the mth segment.
\( F_{ij}^m \) = the fraction of increased link capacity assigned to segment m.
When no improvements are to be permitted on an arc, the variable $k_{ij}$ is not defined in the solution. If the arc does not presently exist, but is proposed by the user, then $k_{ij}^m$ is zero.

A piecewise linear approximation to the travel time-volume curve was devised, with $M = 2$ segments, where $M$ is the designation of the number of segments used. Utilizing a computer program devised to minimize the variation between the actual and the approximate curves, the optimal break-points for the two-arc curve were found to be for $F_{ij}^1 = .75$ and $F_{ij}^2 = .25$ respectively. The deviation of the piecewise linear curve from the actual curve was, in all cases, less than 10 percent, over the range from zero volume to the maximum allowed in the approximation ($K^2_{ij}$). The details of this program are given in Appendix A. More accurate linear approximation, having more than two parts, would require estimation of new values of the $F_{ij}^m$, although this would present no difficulties.

It should be reiterated at this point that different types of improvements and different types of facilities will probably have different cost functions for added capacity. Wherever two different facilities or improvements, with different cost functions, are being considered in the same corridor connecting the same nodes, more than one link which can be constructed or improved can be included in the model. Each of these links connecting the same nodes would be distinguished by perhaps a third subscript, e.g., $K_{ijl}$, $K_{ij2}$, ... for capacity $x_{ij1}$, $x_{ij1}^2$, $x_{ij2}$, $x_{ij2}^2$, ... for flow, etc. Each of these links would be included in the node conservation of flow relationships for the two nodes it connects. Also, a link capacity constraint set would be written for each, and the user costs would be included in the user cost function and the capital costs in the budget constraint. In short, such parallel links would be treated exactly as other links.
Assignment and Demand

The most complex set of constraints in the formulation are the demand and assignment-related constraints. The node-flow constraints require the definition of a new set of choice variables to be described as $s_{xij}$, which refer to the volume on arc $ij$ destined for node $s$. These constraints state that the traffic for a particular destination entering a node (on arcs), plus that generated for that destination at that node (if any), must equal the traffic leaving the node for that destination. Such a relationship must hold for each node except the node corresponding to the destination, since the latter relationship is redundant. This may, then, be stated mathematically as follows:

$$\sum_{k \in B_i} s_{xki} = s_{Di} + \sum_{j \in A_i} s_{xij} \quad (4-6)$$

where:

- $s_{xij}$ = flow on each arc $ij$ going from $i$ to destination $s$.
- $s_{xki}$ = flow on each arc $ki$ going into $i$ and destined for $s$.
- $s_{Di}$ = travel demand from origin $i$ to destination $s$.
- $A_i$ = the set of nodes connected directly by an arc from node $i$ (termed the set of nodes after node $i$)
- $B_i$ = the set of nodes connected directly by an arc to node $i$ (termed the set of nodes before node $i$)

A five node, twelve arc problem would then require definition of sixty $s_{xij}$-type variables and the inclusion of twenty constraints.

Another set of constraints insures that the flow on a link to all destinations must equal the flow on the link for all levels of service. This may be stated mathematically:
\[ \sum_{m=1}^{M} x_{ij}^m - \sum_{s=1}^{S} s_{ij} = 0 \]  \hspace{1cm} (4-7)

where:

\( M \) = number of segments

\( S \) = number of destinations

One constraint of this type is required for each link, the total number of which is small relative to the number of node-flow constraints. However, each contains a large number of variables which serve to link the equations in the constraint set.

**Budget Constraint**

The final applicable constraint limits the total expenditure for arc improvements to a specified value.

This is stated as follows:

\[ \sum_{i,j} B_{ij} k_{ij} \leq B \]  \hspace{1cm} (4-8)

where:

\( B \) = the total capital budget available

\( B_{ij} \) = a calculated coefficient that expresses the dollar cost for each additional unit of maximum capacity for each arc \( ij \).

**Non-negativity**

The linear programming definitional constraint of non-negativity applies to all choice variables \( s_{ij}, x_{ij}^m \), and \( k_{ij} \).

**Objective Function**

The previously defined variables allow the objective function to be
stated in several ways:

(1) minimization of user travel-time: \[ \sum_{ij \in A} \sum_{m=1}^{M} C_{ij}^m x_{ij}^m \]  

(4-9)

(2) minimization of capital investment: \[ \sum_{ij \in E} B_{ij} k_{ij} \]  

(4-10)

(3) minimization of total costs: \[ \sum_{ij \in A} \sum_{m=1}^{V \cdot C_{ij}^m x_{ij}^m} + \sum_{ij \in E} CRF \cdot B_{ij} k_{ij} \]  

(4-11)

where:

\[ A \] = the set of all arcs in the network

\[ C_{ij}^m \] = average travel-time on the mth segment of arc ij

\[ CRF \] = capital recovery factor used to make recurring and capital costs comparable

\[ E \] = the set of all arcs that can be improved

\[ x_{ij}^m \] = volume of traffic on the mth segment of arc ij

\[ M \] = number of segments in the linear approximation

\[ B_{ij} \] = cost per unit of maximum capacity added on arc ij

\[ k_{ij} \] = total maximum capacity added on arc ij

\[ V \] = monetary value of time

**Modeling Special Situations**

It is easy to consider special features of the network under analysis or of the planning context in the basic core model. For example, one-way streets can be simulated by eliminating a potential set of arc flows, either \[ \sum_{m=1}^{M} x_{ij}^m \) or \[ \sum_{m=1}^{M} x_{ji}^m \), to prevent traffic from being assigned to that portion of the road. This can be done by simply eliminating the variables from the formulation or by adding constraints which force their values to zero. At the same time, the segment capacities can be modified to reflect one-way flow. Although these are not usually included in sketch planning, applica-
tions of the model to small areas, as discussed in Chapter 9, might require inclusion of one-way streets.

Peak hour evaluation of the network requires changing only the demand matrix and the capacities of the segments to reflect that period of time.

Applications to Sketch Planning

The model presented above is quite general in its applicability. Some reductions in the size of the model can be accomplished by taking advantage of characteristics of applications to sketch or preliminary planning--the context for which the model was designed.

One of these characteristics is that traffic for an entire day is almost always considered. Also, the capacity of roads is usually considered as a two directional capacity so as to avoid the need to be concerned with the details of design, reversible lanes, peaking of traffic, etc. Therefore, we have formulated the model in a manner which deals with total flows on roads (combining both directions) rather than the directional flows used above. This formulation, which has been used in all the example problems, is presented in Figure 4-5, and will be explained below. Since arcs (which have direction) are not longer appropriate for expressing capacity, the arcs representing a road are now replaced by links, which refer to the capacity and flow in both directions.

The reformulation is quite straightforward. The travel time-volume relationships now used refers to volume in both directions, and hence the new capacity for any segment \( m(k^m_{ij}) \) equals the sum of the two arc capacities for that segment. The user costs \( (C^m_{ij}) \) remain the same. The flows on each road are represented by one flow variable for each segment, not two. These changes are presented in equations 4-12 and 4-16.
Minimize total user transportation costs

\[ \text{Min. } \sum_{i,j \in L} \sum_{m=1}^{M_{ij}} c_{ij}^m x_{ij}^m + \sum_{i,j \in E} \text{CRF} \cdot B_{ij} \cdot k_{ij} \]  

(4-12)

Subject to the constraints:

Traffic flow is conserved at each node.

\[ \sum_{k \in A_j} s_{x_{jk}} - \sum_{l \in B_j} s_{x_{lj}} = s_{D_j}, \forall j, s, j \neq s \]  

(4-13)

Flow to all destinations in both directions on a link must be equal to the total flow on that link as defined by the segment flows.

\[ \sum_{m=1}^{M_{ij}} x_{ij}^m - \sum_{s=1}^{s_{ij}} s_{x_{ij}} = 0, \forall i,j \in L \]  

(4-14)

Flow must not exceed capacity on each link.

\[ x_{ij}^m \leq K_{ij}^m + F_{ij}^m k_{ij}, \forall i,j \in E, m \]  

(4-15)

\[ x_{ij}^m \leq K_{ij}^m, \forall i,j \in N, m \]  

(4-16)

The capital budget cannot be exceeded.

\[ \sum_{i,j \in E} B_{ij} k_{ij} \leq B \]  

(4-17)

Non-negativity.

All \( s_{x_{ij}}, x_{ij}^m, k_{ij} \geq 0 \)

Figure 4-5. The Core Highway Network Design Model.
where:

\[ A = \text{set of all arcs in network} \]

\[ A_j = \text{set of all nodes directly reached by arcs departing} \]

\[ \text{from node } j \]

\[ B = \text{total capital budget available} \]

\[ B_j = \text{set of all nodes directly connected by arcs into node } j \]

\[ B_{ij} = \text{total cost per unit of maximum capacity added on link } ij \]

\[ CRF = \text{capital recovery factor for converting total capital} \]

\[ \text{costs to appropriate daily or annual amount} \]

\[ C^m_{ij} = \text{user cost of } m\text{th increment of capacity on link } ij \]

\[ S_{ij} = \text{total flow originating in node } j \text{ and destined for node } s \]

\[ E = \text{set of all links which can be improved} \]

\[ P_{ij}^m = \text{fraction of maximum capacity added on arc } ij \text{ assigned to} \]

\[ \text{user cost increment } m \]

\[ k_{ij} = \text{total maximum capacity added on link } ij \]

\[ k^m_{ij} = \text{initial capacity on segment } m \text{ of link } ij \]

\[ M_{ij} = \text{number of user cost increments on link } ij \]

\[ N = \text{set of all links in network not improvable} \]

\[ x^m_{ij} = \text{total flow on user cost increment } m \text{ of link } ij \]

\[ s_{x_{ij}} = \text{total flow on arc } ij \text{ destined to node } s \]

\[ L = \text{set of links in the network } (L = E + N) \]

---

Figure 4-5. The Core Highway Network Design Model, continued.
The relationships used to describe the demand for travel and the assignment of that demand, that is, the node flow conservation relationships (4-13), are unchanged. The definitional relationship that the total flow on an arc by destination equals the total flow on the various segments of that arc must be altered. The change is simply to a relationship that states total flow by destination in both directions must equal the total flow on the link segments, as shown in equation (4-14).

Capacity is now added to each link, not each arc. The budget constraint is appropriately modified, as is the capacity constraint set for each link which might be improved. These changes are given in equations (4-15) and (4-17).

**Characteristics of the Solution**

The previous section has described the basic continuous Network Design Model Formulation and the required form of the input data. The inputs required to use this model are essentially the same as those needed to use the existing Urban Transportation Planning Models package, although they are expressed in different form. Using the notation of the previous section, the coefficients \( C_{ij}^m \) are the travel times on the network at the mth level of service, the constants \( k_{ij}^m \) are the link capacities at the mth level of service, the coefficients \( F_{ij}^m \) describe the shape of the total travel time-volume curve, \( B \) is the maximum expendable capital budget, and \( B_{ij} \) describes the cost of link improvements. These inputs represent descriptions of the physical characteristics of the links which make up the network and, with the exception of the cost of improvement and budget amounts, are similar to information required for the traditional network distribution and assignment process.
The demand matrix elements, represented by the previous notation $s_{Dj}$, can be obtained from a trip distribution matrix generated by existing UTP procedures or may represent existing demand.

The solution of the Network Design Model provides a set of values for the choice variables in the formulation in addition to the slack variables generated by the program.* These values provide the user with the following information:

1. The value of the objective function for the optimal solution, e.g., total travel time on the network, total capital investment, or total transportation cost.

2. The system-optimal link improvements possible within the given budget and their total cost.

3. The system-optimal flow pattern on the resulting network, expressed both as link flows and network "trees".

4. The unused but constructed capacity on every link.

5. The unspent budget.

The methods used to obtain this information will be explained in an example problem to follow.

The set of values giving this information comprises the optimal solution for the given input information. Concomitant products of the model, at the optimal solution, indicate the sensitivity of the objective function to incremental changes in the input data. This provides valuable information to the transportation planner, enabling him to determine the effect of changes in some, or all, inter-nodal demands, the benefits resulting from increasing the permitted maximum capacity addition to any of the links, and the benefits resulting from increasing the allowable budget. This information is provided by the values of the "dual variables" associated with each constraint. Very

*If the reader wishes a more complete description of linear programming, he is referred to a standard text on the subject, such as Gass (1964).
simply, there is a dual variable associated with each constraint equation, defining the change in the value of the objective function resulting from a unit change in the constant on the right hand side. This provides a very efficient and direct form of sensitivity analysis. However, these values for the rate of change of the value of the objective function with respect to each of the constraints only apply as long as the set of variables comprising the optimal solution does not change. The values of these variables may change, but they must remain in solution, if the values of the dual variables are to be valid.

It may be important for the planner to determine the degree to which a demand element, capacity, or budget amount may be changed without altering these relationships. For example, this would enable the determination of the range of errors in demand estimates for which the given solution remains optimal. There are post-processing options to perform this task, greatly enhancing the potential value of the Network Design Model as an aid in the transportation planning process.

One such post-processing feature is the so-called "ranging" option, where any number of constraints or coefficients are increased or decreased until they cause a change in the variables present in the optimal solution. That is, they are perturbed to the point where they bring about a change in the network flow pattern or set of links to be improved. At this point, the planner knows that he can increase or decrease demands between certain node pairs to a particular level without altering the optimal flow pattern or the specific links to be improved, although more improvement may be called for or flow may be increased.

The same technique applied to the budget figure would tell the planner the degree to which investment would have to change before another basic
plan would become optimal. If ranging were applied to the coefficients in the budget constraint, increases (or decreases) in the construction costs could be evaluated with respect to their effects in the optimality of a given plan.

Another similar tool is termed parameterization. By using this approach, constraints and coefficients may be changed by different relative amounts as determined by the user, and the effects may be observed. In this manner, for example, the elements in the demand matrix may be assigned weights based upon the reliability of the demand forecast, with zero reflecting high expected accuracy (no change considered), and other values (less than or greater than zero) reflecting relatively lower levels of reliability. Each element of the demand matrix is changed by a proportional amount. Each element in the demand matrix is thus altered until the previously optimal plan is no longer optimal (and a new optimal solution must be found), or until an infeasible situation occurs. This tells the planner the degree to which the demands can change without affecting the optimality of the plan. This information is of particular value when there is high uncertainty associated with demand projections.

Parameterization can also be applied to travel time coefficients to study the effects of changes in technology, and also to the capital cost coefficients to assess the effects of changes in projected costs of types of facilities. Future changes in relative costs may change the optimal solution significantly.

The user of the model may apply these techniques to a solution obtained from a particular set of inputs, and based on the results, may then resolve the model for input conditions beyond the range for which the previous "point" solution was optimal. He is then capable of generating alternative
sets of flow patterns and link improvements (plans) which are optimal, in
terms of his stated objective function, and according to the values defining
network characteristics, demand, and cost. These subsets of alternatives re-
represent the "efficient frontier", the collection of networks which are op-
timal under various conditions. In addition to providing the planner with a
basis for selecting good networks, the efficient frontiers provide guidelines
for the choice of policies, goals, and constraints. These will be discussed
in more detail in Chapter 6.

Example Problem

To illustrate some of the capabilities of the core network design model
discussed in the previous section, the solution of a sample problem is dis-
cussed below. The network considered in the problem has been structured to
represent the central portion of larger, urban networks and comprises five
nodes and six links. All links are assumed to accommodate travel in both
directions. The example network is shown in Figure 4-6.

The inter-nodal demands, summarized in Table 4-1, were chosen arbi-
trarily, but were made large enough to overload the initial network. A two-
segment approximation was made to the total travel time-volume relationship,
as discussed previously. The arc travel times and the maximum two-way arc
capacities are shown in Table 4-2. The capital cost coefficients were de-
developed, assuming that improvement costs are a linear function of the ca-
pacity added on the links. These are also given on Table 4-2, both as total
capital costs for use in the budget constraint, and as daily costs, for use
in the objective function, calculated using a capital recovery factor for
10% interest and a 20-year life.

Figure 4-7 shows the equations used to represent this problem using
Figure 4-6. Schematic Representation of Sample Problem.
Table 4-1. Origin-Destination Matrix for the Example Problem.

<table>
<thead>
<tr>
<th>From Node (Origin Zone)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15000</td>
<td>4516</td>
<td>5222</td>
<td>7000</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>7685</td>
<td>6218</td>
<td>10414</td>
<td>1122</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>5517</td>
<td>3229</td>
<td></td>
<td>711</td>
<td>2217</td>
</tr>
<tr>
<td>4</td>
<td>6224</td>
<td>3429</td>
<td>1704</td>
<td></td>
<td>858</td>
</tr>
<tr>
<td>5</td>
<td>1333</td>
<td>2129</td>
<td>3209</td>
<td>5817</td>
<td></td>
</tr>
</tbody>
</table>
Table 4-2. Link Characteristics for the Example Problem.

<table>
<thead>
<tr>
<th>Link (segment)</th>
<th>Average Daily Travel Time (Hours)</th>
<th>Average Daily Capacity (Veh./Day)</th>
<th>Cost of Added Capacity Total ($/1000 veh./day)</th>
<th>Daily ($/1000 veh./day) CRF*B&lt;sub&gt;ij&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>i j(m)</td>
<td>C&lt;sub&gt;ij&lt;/sub&gt;</td>
<td>K&lt;sub&gt;ij&lt;/sub&gt;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12(1)</td>
<td>.12143</td>
<td>15,000</td>
<td>600,400</td>
<td>200.13</td>
</tr>
<tr>
<td>12(2)</td>
<td>.80580</td>
<td>5,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14(1)</td>
<td>.04000</td>
<td>40,500</td>
<td>252,800</td>
<td>84.26</td>
</tr>
<tr>
<td>14(2)</td>
<td>.24673</td>
<td>12,500</td>
<td></td>
<td></td>
</tr>
<tr>
<td>23(1)</td>
<td>.06429</td>
<td>15,000</td>
<td>376,200</td>
<td>125.40</td>
</tr>
<tr>
<td>23(2)</td>
<td>.39627</td>
<td>5,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>34(1)</td>
<td>.05714</td>
<td>15,000</td>
<td>347,600</td>
<td>115.86</td>
</tr>
<tr>
<td>34(2)</td>
<td>.37667</td>
<td>5,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>45(1)</td>
<td>.10000</td>
<td>15,000</td>
<td>126,400</td>
<td>42.13</td>
</tr>
<tr>
<td>45(2)</td>
<td>.67787</td>
<td>5,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>35(1)</td>
<td>.02143</td>
<td>15,000</td>
<td>505,600</td>
<td>168.53</td>
</tr>
<tr>
<td>35(2)</td>
<td>.13927</td>
<td>5,000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
the standard notation discussed earlier in this chapter. There are 78 variables and 40 relationships. The objective function consists of twelve user costs terms (two for each link), each comprising a coefficient expressing average travel time and the variable defining the traffic volume assigned to each arc. Also in the objective function are six other terms, representing construction costs for improvements to each link. All other variables enter the objective function with a zero coefficient, since the objective being used is that of minimizing total transportation costs (as given in equation 4-12 of Figure 4-5).

The next set of equations insure that the flow on each link to all destinations equals the flow used in the non-linear travel time calculation. These follow the general form of equation 4-14 in Figure 4-5. The first of these equations includes a verbal description of the variables as an aid in understanding it. Since there are six links, there are six such equations.

The following set is the node conservation of flow and demand constraints, which are patterned after equation 4-13 in Figure 4-5. For each destination, a separate equation is written at each node except the node which is the destination. The first equation is for destination node 1, at node 2 (the equation for node 1 being omitted since that node is the destination node). Other equations for destination 1 (in order) are for node 3, node 4, and node 5. Then destination 2 is considered, with equations for nodes 1, 3, 4, and 5, and so on. Since there are five destinations, and five nodes, there are a total of 20 such relationships—four for each destination.

Presented next are the capacity constraints, the general form is given by equations 4-15 and 4-16 in Figure 4-5. Since in this example problem it is assumed that each link can be improved, equation 4-15 is the prototype
Objective function: to minimize total user cost

Minimize: $0.29143 \times_{12}^{1} + 1.93392 \times_{12}^{2} + 0.09600 \times_{14}^{1} + 0.59431 \times_{14}^{2}$

$+ 0.15429 \times_{23}^{1} + 0.95104 \times_{23}^{2} + 0.13713 \times_{34}^{1} + 0.90400 \times_{34}^{2} + 0.2400 \times_{45}^{1}$

$+ 1.62682 \times_{45}^{2} + 0.05143 \times_{35}^{1} + 0.33425 \times_{35}^{2}$

(user cost in dollars)  (flow in thousands)

(per vehicle for link)  (of vehicles per day)

(35, segment 2)  (on 2nd segment of link 35).

$+ 200.13 k_{12} + 84.26 k_{14} + 125.40 k_{23} + 115.86 k_{34}$

$+ 42.13 k_{45} + 168.53 k_{35}$

(Daily cost in dollars)  (capacity increase)

(per thousand vehicles)  (in thousands of vehicles per day)

Subject to:

Traffic Assignment Constraints

$\sum_{\text{arc } ij} x_{ij} = k_i$, for each node $i$.

$\sum_{\text{arc } ij} x_{ij} = \sum_{\text{route } \text{ including } ij} x_{ij}$, for each route.

$\sum_{\text{arc } ij} x_{ij} = 0$, for each empty arc.

flow on arcs 21 and 12 destined for node 1

flow on segment 1 and 2 of link 12

$1 \times_{21}^{1} + \times_{12}^{2} + \times_{21}^{2} + \times_{12}^{3} + \times_{12}^{4} + \times_{21}^{5} + \times_{12}^{6} - \times_{12}^{1} - \times_{12}^{2} = 0$

$1 \times_{14}^{1} + \times_{14}^{2} + \times_{14}^{3} + \times_{14}^{4} + \times_{14}^{5} - \times_{14}^{1} - \times_{14}^{2} = 0$

$1 \times_{23}^{1} + \times_{23}^{2} + \times_{23}^{3} + \times_{23}^{4} + \times_{23}^{5} - \times_{23}^{1} - \times_{23}^{2} = 0$

$1 \times_{34}^{1} + \times_{34}^{2} + \times_{34}^{3} + \times_{34}^{4} + \times_{34}^{5} - \times_{34}^{1} - \times_{34}^{2} = 0$

$(\text{continued on next page})$

Figure 4-7. Equations of Example Problem.
\[
\begin{align*}
\frac{1}{x_{35}} + \frac{1}{x_{33}} + \frac{2}{x_{35}} + \frac{2}{x_{33}} + 3\frac{1}{x_{35}} + 3\frac{1}{x_{33}} + 4\frac{1}{x_{35}} + 4\frac{1}{x_{33}} + 5\frac{1}{x_{35}} + 5\frac{1}{x_{33}} - \frac{1}{x_{35}} - \frac{1}{x_{33}} &= 0
\end{align*}
\]

Demand-Node Flow Constraints

\[
\frac{1}{x_{21}} + \frac{1}{x_{23}} - \frac{1}{x_{12}} - \frac{1}{x_{32}} = 7.685
\]

flow out of flow into demand generated
node 2 destined for node 2 destined for node 1. (in
node 1 node 1 thousands)

\[
\begin{align*}
-\frac{1}{x_{23}} - \frac{1}{x_{43}} + \frac{1}{x_{35}} + \frac{1}{x_{32}} + \frac{1}{x_{34}} - \frac{1}{x_{53}} &= 5.517 \\
-\frac{1}{x_{14}} + \frac{1}{x_{43}} + \frac{1}{x_{45}} + \frac{1}{x_{41}} - \frac{1}{x_{34}} - \frac{1}{x_{54}} &= 6.224 \\
-\frac{1}{x_{45}} - \frac{1}{x_{35}} + \frac{1}{x_{54}} + \frac{1}{x_{53}} &= 1.333 \\
-\frac{2}{x_{21}} + \frac{2}{x_{14}} + \frac{2}{x_{12}} - \frac{2}{x_{41}} &= 15.0 \\
-\frac{2}{x_{23}} - \frac{2}{x_{43}} + \frac{2}{x_{35}} + \frac{2}{x_{32}} + \frac{2}{x_{34}} - \frac{2}{x_{53}} &= 3.229 \\
\frac{2}{x_{43}} + \frac{2}{x_{41}} + \frac{2}{x_{45}} - \frac{2}{x_{14}} - \frac{2}{x_{34}} - \frac{2}{x_{54}} &= 3.429 \\
\frac{2}{x_{54}} + \frac{2}{x_{53}} - \frac{2}{x_{35}} - \frac{2}{x_{45}} &= 2.129 \\
\frac{3}{x_{12}} + \frac{3}{x_{14}} - \frac{3}{x_{21}} - \frac{3}{x_{41}} &= 4.516 \\
\frac{3}{x_{21}} + \frac{3}{x_{23}} - \frac{3}{x_{32}} - \frac{3}{x_{12}} &= 6.218 \\
\frac{3}{x_{41}} + \frac{3}{x_{43}} + \frac{3}{x_{45}} - \frac{3}{x_{14}} - \frac{3}{x_{34}} - \frac{3}{x_{54}} &= 1.704 \\
\frac{3}{x_{53}} + \frac{3}{x_{54}} - \frac{3}{x_{35}} - \frac{3}{x_{45}} &= 3.209
\end{align*}
\]

(continued on next page)

Figure 4-7. Equations of Example Problem, continued.
\[ + 4x_{14} + 4x_{12} - 4x_{21} - 4x_{21} = 5.222 \]
\[ + 4x_{21} + 4x_{23} - 4x_{12} - 4x_{32} = 10.414 \]
\[ + 4x_{34} + 4x_{32} + 4x_{35} - 4x_{53} - 4x_{43} - 4x_{23} = 0.711 \]
\[ + 4x_{53} + 4x_{54} - 4x_{35} - 4x_{45} = 5.817 \]
\[ + 5x_{12} + 5x_{14} - 5x_{21} - 5x_{41} = 7.000 \]
\[ + 5x_{21} + 5x_{23} - 5x_{32} - 5x_{12} = 1.122 \]
\[ + 5x_{32} + 5x_{34} + 5x_{35} - 5x_{53} - 5x_{43} = 2.217 \]
\[ + 5x_{41} + 5x_{43} + 5x_{45} - 5x_{54} - 5x_{34} - 5x_{14} = 0.858 \]

Capacity Constraints

\[
\begin{align*}
\quad & x_{12} - 0.75 k_{12} \leq 15.0 \\
\text{assigned} & \text{ capacity} & \text{existing capacity in} \\
\text{flow to} & \text{ added to} & \text{thousands of vehicles} \\
\text{1st/seg-} & \text{1st/seg-} & \text{per day on link 12,} \\
\text{ment of} & \text{ment of} & \text{segment 1} \\
\text{link 12} & \text{link 12} & \\
\end{align*}
\]

\[ x_{12}^2 - 0.25 k_{12} \leq 5.0 \]
\[ x_{14}^1 - 0.75 k_{14} \leq 40.5 \]
\[ x_{14}^2 - 0.25 k_{14} \leq 12.5 \]
\[ x_{23}^1 - 0.75 k_{23} \leq 15.0 \]
\[ x_{23}^2 - 0.25 k_{23} \leq 5.0 \]

(continued on next page)

Figure 4-7. Equations of Example Problem, continued.
\begin{align*}
x_{34} & - 0.75 \ k_{34} \leq 15.0 \\
x_{34} & - 0.25 \ k_{34} \leq 5.0 \\
x_{45} & - 0.75 \ k_{45} \leq 15.0 \\
x_{45} & - 0.25 \ k_{45} \leq 5.0 \\
x_{35} & - 0.75 \ k_{35} \leq 15.0 \\
x_{35} & - 0.25 \ k_{35} \leq 5.0 \\
\end{align*}

Budget Constraint

\[6.004 \ k_{12} + 2.528 \ k_{14} + 3.762 \ k_{23} + 3.476 \ k_{34} + 1.264 \ k_{45} + 5.056 \ k_{35} \leq 50.0\]

\begin{tabular}{lll}
\text{cost in } 10^5 \text{ dollars} & \text{capacity} & \text{budget in } 10^5 \\
\text{per 1000 vehicle increase in capacity} & \text{increase} & \text{dollars} \\
on link 35 & on link 35 & \\
\end{tabular}

Figure 4-7. Equations of Example Problem, continued.
for each link in this problem. One such relationship is written for each segment of each link for a total of twelve.

The final constraint is simply the budget constraint, which follows the form of equation 4-17 of Figure 4-5.

The only other equations are standard ones for all linear programming problems, namely that all variables must be greater than or equal to zero. These are not presented in Figure 4-7.

Once a problem is specified as in Figure 4-7, then all information needed to input the problem to a standard linear program solving computer code is available. All that need be done is to translate the problem into the format required by the particular code and computer being used. This process is discussed in detail in Appendix A.

**Results**

The problem described above was solved using the standard linear programming code OPTIMA on Northwestern University's CDC 6400 computer. In order to facilitate understanding of the problem, both total user costs and total transportation costs, the sum of daily travel time and daily capital costs (based on 300 days per year) were calculated.

Initially the example problem was solved using the budget figure of five million dollars to obtain a starting point for the analysis. At this value, the optimal pattern of capacity additions is as shown in Table 4-3, with the entire budget being spent. The total transportation cost is $44,082 per day. The dual value associated with the budget constraint was $514.52. This means that for each extra unit ($100,000) spent in capital improvements (added capacity), there would be a saving of $514.52 in total daily transportation costs (which it should be noted include repayment of the
Table 4-3. Tabulated Solution Results.

<table>
<thead>
<tr>
<th>Solution Range ($ Millions)</th>
<th>Budget ($ Millions)</th>
<th>Dual(^a) Value ($/$)</th>
<th>Capacity Additions (Vehicles) (^b)</th>
<th>Exchange Values (Dollars) (^c)</th>
<th>Value (^d) of Obj. Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unfeasible</td>
<td>3.974</td>
<td></td>
<td>Link 1 2 3 4 5 6 Link 1 2 3 4 5 6</td>
<td>Link 1 2 3 4 5 6 Link 1 2 3 4 5 6</td>
<td></td>
</tr>
<tr>
<td>3.974-4.234</td>
<td>4.20</td>
<td>849.67 2306 -- 6919 -- 1677 -- -- 2237 -- 1442 -- 3838</td>
<td></td>
<td>48,312</td>
<td></td>
</tr>
<tr>
<td>4.234-5.354</td>
<td>5.00</td>
<td>514.52 6070 -- 3155 -- 1331 -- -- 1390 -- 829 -- 2239</td>
<td></td>
<td>44,082</td>
<td></td>
</tr>
<tr>
<td>5.354-10.88</td>
<td>5.40</td>
<td>178.24 7728 -- 1573 -- 1331 -- -- 540 -- 225 -- 786</td>
<td></td>
<td>42,176</td>
<td></td>
</tr>
<tr>
<td>10.88-11.78</td>
<td>11.00</td>
<td>139.31 16870 -- 1573 -- 2207 -- -- 442 -- 214 -- 676</td>
<td></td>
<td>32,237</td>
<td></td>
</tr>
<tr>
<td>11.78-14.49</td>
<td>14.00</td>
<td>129.57 16870 -- 7450 -- 8441 -- -- 417 -- 211 -- 645</td>
<td></td>
<td>28,273</td>
<td></td>
</tr>
</tbody>
</table>

(a) The decrease in calculated daily total transportation costs per $100,000 invested.
(b) In vehicles per day.
(c) Change (decrease) in objective function for each thousand units of capacity added to the corresponding link which is forced into the solution.
(d) Assumed value of time was $2.50 per hour, construction cost allocated to user based on 30 year life and 10% interest rate.
principal and interest on capital expenditures).

The dual is useful in many ways. Firstly, if the analysis is being performed to ascertain a desirable level of capital expenditure, then the dual clearly suggests that more can be spent so as to reduce total transportation costs. This suggests trying higher budget levels. Secondly, even if the budget has been set, it gives a measure of the value of changing the budget. If the gains from increasing the allowed expenditure are very great as in this example—then this information provides very strong support for an increase in the budget. Regardless of the specific context of the analysis such information should be of substantial value to the planner.

As an example of another type of useful analysis, the right-hand side of the budget constraint (the allowable expenditure) was then ranged to determine the range of budget values over which that particular "plan" remained optimal. Over this range, the solution remains the same in the sense that the same variables remain in the basis, but the values of these variables may change, e.g., the capacity added to a road might increase, but no roads to which capacity was not added in the original solution now have capacity added or the volumes may differ but no altered routing can occur. Solutions were then obtained for budgets slightly greater and lesser in value than the range values and the right-hand of the budget constraint was again ranged. This process was repeated until the end-points for the problem were reached. It can be seen from Figure 4-8 that an investment of $3,974,000 is required simply to meet demands, and an investment of $14,493,000 would allow all travel to occur at the highest level of service. As shown in Figure 4-8 and tabulated in Table 4-3, there are five distinct alternative sets of improvements and flows for the given network that could be considered the best for the different budget ranges. Although the same set of links
Figure 4-8. Value of Objective Function Versus Budget.
are improved for each alternative, the relative share of improvement on each link varies considerably as does the optimal flow pattern.

Since the demand matrix is not symmetric, the flows cannot be expected to be equal in both directions. Initial examination of the flows indicates a higher imbalance at the lower budget levels, with a tendency toward greater equalization as greater amounts of budget are spent. These results may indicate the need for the provision of unbalanced capacity, which is what the model is providing given the non-symmetric demands.

Use of the model as a planning tool requires an understanding of the output for complete interpretation. Solutions subject to a particular budget constraint provided a starting point for analysis. It should be remembered that it is not strictly a "point" solution, since any of the values may "range" without altering the set of linear relationships that hold for a solution, although the objective function value would change. At the endpoints of each range, the basis (or solution set), is altered by a variable entering the basis and one leaving the basis. A new set of linear relationships is then defined. Information defining these relationships between the variables is perhaps more valuable than the actual "point" solution. The dual values define these relationships in the solution output.

This can best be demonstrated in our sample problem by examination of Figure 4-8. If the problem were solved subject to a budget constraint of $5,000,000 and an assumed values of time of $2.50/hour, the dual value associated with the budget constraint would be $514.52 which can be interpreted as the rate of change in the objective function per additional unit ($100,000) of budget. If this value were to hold for all budget levels, the solution curve would look like the dashed line on Figure 4-8.

It is obvious that at some point, additional investment will begin to
yield decreasing returns. This point can be determined easily through the use of the network design model, and this should be quite evident from Figure 4-8. Information of this type, even if developed using an aggregated version of the real network, should be of considerable value to planners and decision makers. The trade-off information of the type presented in Figure 4-8 is especially useful in determining a proper balance of achievement of two conflicting objectives. The minimization of total transportation costs and the minimization of capital expenditures provide an example of this, in the sense that (up to a point) increasing capital costs will decrease total transport costs. As shown in Figure 4-8, solutions with high capital costs and low transport costs can be chosen, as well as those with low capital costs and high transport costs. Total transport costs can be approximately halved (from approximately $48,300 to $28,200) be a four-fold increase of capital improvement expenditures (from approximately $4.0 million to $14.5 million). Figures such as 4-8 present the range of feasible solutions, and the level of achievement of the various objectives, on the basis of which the judgemental decision as to which solution is best can be selected. This use of the output will be discussed more fully in Chapter 6.

Another useful item of output is the so-called exchange values, also shown in Table 4-3. These values exist for all the variables of the problem not in the basis of the optimal solution. These variables have a zero value in that solution. The exchange value is the rate of change in the objective function value if one unit of the variable is forced into the solution. From Table 4-3 it can be seen that link 2 has no capacity additions in the optimal solution. However, the exchange value for that link's capacity added variable indicates that if one unit of capacity were added on that link (one unit being 1,000 vehicles per day), then the total transportation cost would in-
crease by $2,237 per day. Similar information is presented for the other links to which capacity was not added. This information would be useful if there were pressure to construct a particular improvement which did not appear in the optimal solution.

The travel demands can also be examined usefully in light of the dual variables, ranging, parameterization, etc. For example, there is a dual associated with each node flow conservation constraint. The dual associated with each constraint (which is for a node and a destination) is the marginal cost of accommodating the traffic from that node to its destination. In the solution to the example problem, the dual for traffic at node 3 destined for node 2 is $2.438. In other words, the cost of accommodating an extra unit of traffic (one vehicle per day) from 3 to 2 is $2.438. Other values are given in Table 4-4. If road pricing were to be considered, these values would be indispensable.

More related to the near future is the fact that these are the savings that would occur if that traffic were to be accommodated by some other means, such as by a transit system from node 3 to node 2. Each unit of traffic so diverted would save $2.43 per day in road system costs—a savings that must be weighed against the costs of building and operating the transit system. If it were desired to examine these costs in terms of cost per vehicle-mile, this could be done by simply dividing the costs per vehicle trip by the distance between the nodes. Thus the duals can provide information useful in the identification of zone pairs or markets where transit could have a significant social benefit. As pointed out above, the dual values hold only so long as the basis remains unchanged. When the basis changes, the duals may (and generally do) change. The dual discussed above holds for a range in budget from $4.24 to $5.35 million.
<table>
<thead>
<tr>
<th>Node Origin</th>
<th>Node Destination</th>
<th>Dual Value ($/Vehicle)</th>
<th>Budget Range</th>
<th>Budget Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>$3.03</td>
<td>$2.01</td>
<td>$3.03</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1.79</td>
<td>1.04</td>
<td>1.79</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>0.92</td>
<td>0.49</td>
<td>0.92</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>4.03</td>
<td>2.01</td>
<td>4.03</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>2.43</td>
<td>1.17</td>
<td>2.43</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>4.13</td>
<td>2.11</td>
<td>4.13</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>3.31</td>
<td>1.71</td>
<td>3.31</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>1.79</td>
<td>1.04</td>
<td>1.79</td>
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<td>2.43</td>
<td>1.17</td>
<td>2.43</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>1.69</td>
<td>0.94</td>
<td>1.69</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>0.87</td>
<td>0.54</td>
<td>0.87</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>4.13</td>
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<tr>
<td>3</td>
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<tr>
<td>5</td>
<td>4</td>
<td>0.82</td>
<td>0.39</td>
<td>0.82</td>
</tr>
<tr>
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<td>5</td>
<td>0.92</td>
<td>0.49</td>
<td>0.92</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>3.31</td>
<td>1.71</td>
<td>3.31</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>0.87</td>
<td>0.54</td>
<td>0.87</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>0.82</td>
<td>0.39</td>
<td>0.82</td>
</tr>
</tbody>
</table>
Demands can also be subject to ranging and parameterization. This is particularly important if they are subject to uncertainty. Continuing with the same example of node 3 to node 2, the demand was originally estimated to be 3,229 vehicles per day. Ranging indicated that the solution to the problem would remain unchanged—in the sense that there is no change in the links to be improved or in the routing of traffic—if this demand were as low as 2,285 vehicles per day or as high as 5,246 vehicles per day. The dual value of $2.43 can be used to calculate the effect of such variations on total transportation costs. All internodal demands could be examined in the same manner.

To illustrate the value of parameterization, this example problem with a $5 million budget was subject to a parametric analysis of the demand estimates. The original demand matrix is shown in Table 4-1. It is well-known that the methods of demand estimation in transportation planning are subject to uncertainty, and hence an examination of the effect of different levels of demand (and origin-destination pattern, etc.) on the choice of the best system is very desirable.

In this example it is assumed that the total origins and destinations at node 2 are considered predicted with reasonable certainty, while others are not. Node 2 could represent an area of stable land use patterns, while other zones might be expected to experience considerable growth. Demand between these other zones was assumed to vary in the same proportion, i.e., a 10% increase from 1 to 3 would occur with a 10% increase from 3 to 4, etc.

Through parameterization, the effect of successively greater (and if desired smaller) levels of demand were identified. The linear programming model just yields information on the extent to which demand can increase without a change in the basis—a change in the links improved and the assign-
ment pattern. This is given in Table 4-5, an 8.18% increase in demand being possible. Along with this result is the change in the objective function and the change in all the variables.

Then the model considers a larger change. This changes the basis, and the new solution variables and values are given. This new basis holds to a 14.99% increase. And the process continues until either told to stop, or, as in this case, the problem becomes infeasible. This means that there is no way capacity can be added to accommodate any further increase in demand, this occurring at a 24.56% increase for a $5 million budget.

Table 4-6 shows the optimum capacity additions at the original and maximum demand levels. It is interesting to note that the same links are improved in both. This suggests that these link improvements are very flexible in their ability to accommodate a wide range of future traffic. Although the exact amounts of capacity addition differ, it suggests that improvements to those three links are best, and that they should be designed such that more capacity can be easily added if the original design is to be followed.

A final very useful type of output is the travel time on various links and the routing of traffic from origin to destination. The travel time is easily calculated, for in the solution is the volume of traffic on each link segment (given by the $x_{ij}^m$). The user cost coefficients are known. Hence total user cost on the link can be computed using equation 4-3 and average user cost or travel time using equation 4-4. This is shown in Figure 4-9 for the example problem with a $5 million budget.

The routing of traffic is also easily obtained from the output. If an $x_{ij}$ variable is zero, it means that no traffic at node i destined for node s used link ij to move toward its destination. If an $s x_{ij}$ variable is non-zero, its numerical value is the vehicles per day which travel over link ij

4-41
<table>
<thead>
<tr>
<th>Percentage Increase in Demand</th>
<th>Total Transportation Cost (Dollars per Day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>$44,169</td>
</tr>
<tr>
<td>8.18</td>
<td>47,461</td>
</tr>
<tr>
<td>14.99</td>
<td>50,198</td>
</tr>
<tr>
<td>16.57</td>
<td>50,940</td>
</tr>
<tr>
<td>21.98</td>
<td>53,487</td>
</tr>
<tr>
<td>24.56 (Max.)</td>
<td>54,877</td>
</tr>
</tbody>
</table>
Table 4-6. Comparison of Optimum Capacity Additions for Original and Maximum Possible Demands.

<table>
<thead>
<tr>
<th>Link Number</th>
<th>Increase in Capacity Original Demand</th>
<th>(1000 veh. per day) Maximum Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6,070</td>
<td>1,773</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>3,155</td>
<td>7,452</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>1,331</td>
<td>8,452</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Total Transportation Cost ($ per day) $44,169 $54,877
Figure 4-9. Average Link Travel Times in Hours for Optimum Solution at Five Million Dollar Budget Level.
to reach s. By examining the variables for each destination separately, the routing pattern for each destination from each other node can be obtained. An example of these results, for the example problem and a budget of $5 million, is given in Figure 4-10.

It is appropriate to mention at this point, in discussing the assignment results obtained by use of the model, that the assignment criterion used is slightly different than that generally used to model driver's behavior. Specifically, in this model the assignment is made so as to minimize total user travel time, while it is generally accepted that each individual driver selects a route so as to minimize his own individual travel time. These can lead to different assignments, although intuitively it seems as though they would generally be quite similar. Since it is necessary to use the minimization of total user time in this model if the objective function is to be meaningful, it was important to determine how different the assignments are likely to be. This was done in two ways. First, precise mathematical descriptions of the two different methods were prepared and compared, and these further reinforced the intuitive feeling that the assignments would not differ greatly. These are described in Appendix B. Second, a precise equilibrium assignment using the individual user time minimization criterion was compared to the assignment using the total time criterion on a 24 node, 76 arc representation of an actual network. The results were remarkably similar, with virtually identical vehicle-hours and vehicle-miles of travel, and flows on most links were very similar. This also is discussed in Appendix B. Thus the assignment procedure used in the core model appears quite satisfactory. This will be discussed again in the context of inclusion of other impacts (in addition to travel time) in the objective function, where other issues arise.
Note: All flows in 1000 vehicles per day.

Figure 4-10. Destination Specific Network Flow at Five Million Dollar Budget Level.
Conclusion

Thus, even this highly simplified version of the network design model promises to make important contributions to the design of better, more responsive transportation networks. In the next chapter, strategies for extending this model to consider a broader set of goals and impacts are discussed.

References


CHAPTER 5
THE IMPACT SENSITIVE NETWORK DESIGN MODEL

Introduction

The previous chapter has proposed a simple version of a basic transportation network design model, and has demonstrated the potential usefulness of such models in the planning process. This Chapter focuses on extensions to that core model which will make it sensitive to a broader range of impacts and goals.

The contributions which network design models can be expected to make toward improving transportation planning are very much related to the breadth of issues which those models can take into account. It is evident from recent conflicts and controversies associated with transportation proposals, particularly in urban areas, that narrow, economic measures of user benefits and direct costs cannot remain the only factors considered in transportation investment decisions. Major transportation facilities bring about a broad variety of impacts in the communities through which they pass. While such facilities are constructed in order to produce benefits to those communities, the negative consequences cannot be ignored. Furthermore, those benefits which are not easily measured in strict, monetary terms must also receive careful consideration.

For example, it is characteristic of transportation facilities that they produce widely distributed, and individually small, benefits to many people, while imposing relatively high personal costs on a few people. A network design capability which facilitates the rapid and efficient search through many alternative systems can have its most significant effect on transportation planning if it can include methods for considering such impacts and
trade-offs. That is, to the extent that such models can take into account the widest possible range of consequences, they can assist the planner and decision maker in identifying those networks which will not only be attractive from a simple, economic feasibility perspective, but which will also be generally acceptable to the community at large.

To introduce this feature in an analytic network design capability such as that proposed in the previous chapter, it will be essential to find ways to measure the most important impacts at a high level of quantification. Those consequences which are not subject to such measurement cannot be included in an optimal network seeking process, and must be handled, much as they are now, in a subjective, ad hoc manner through political negotiation. The political process, of course, has evolved to meet precisely these kinds of issues, and has become relatively effective at doing so. However, when more significant impacts can be treated in quantitative ways, it will be possible to increase the objectivity with which decisions are made. At the same time, increased quantification should relieve the political process of some of the burdens, of inferring in place of information missing because it cannot be quantified, thus allowing the political process to function more effectively in treating the remaining, qualitative issues.

In recent years, the ability to quantify impacts of public investments such as transportation facilities has increased markedly. For example, noise impacts, which were often totally ignored in transportation planning ten years ago, can be quantified at a high level and can be predicted with considerable accuracy. The same may be said for air pollution resulting from motor vehicle operations. The consequence of taking valued public facilities, such as schools, churches, businesses, historical features, and parks, still cannot be measured with complete objectivity. In few cases is it possible to
convert such impacts to a meaningful scale of dollar values.

The features of the core network design model permit the inclusion of most readily quantified impacts in the process of searching for the best network. Furthermore, some impacts which cannot be measured objectively may also be given a reasonable consideration because of the characteristics of the model. The remainder of this chapter will provide some examples of ways in which the network design model can be made increasingly sensitive to such impacts.

**Methods for Introducing Impact Sensitivity**

There are two broad strategies for introducing impact sensitivity into the core network design model. The first, and preferred, approach is to modify directly the structure or solution method for the model, and the second is to analyse further the outputs of the model. In the first case, impact measures are permitted to have a direct, internal effect on the optimal solution produced by the model. This takes better advantage of the features of the model, placing more burden on the optimization process to find a network solution which is "best" in terms of several impact dimensions. In the second case, impacts treated externally can only affect the solution provided by the model after some form of "post-processing" analysis, which is an analysis performed outside of the main model using outputs from it. This is then followed by an assessment by the planner, who must then modify the inputs to a subsequent run of the model. In the second, or "open loop" case, then, greater reliance is placed on the planner to interpret and utilize the initial results of the network design model for directing later stages of the search process.

Clearly, the first approach is preferred, since it capitalizes on the
features of the analytic model to identify networks which are optimal in a broader sense. Unfortunately, because of the nature of the mathematical programming framework, only impacts which are measurable in certain ways may be included directly in the model. Direct inclusion requires that it be possible to develop quantitative relationships compatible with the model which forecast impacts based on properties of solutions to the network design problem. These predicted impacts are then allowed to influence the search for, and selection of, the best solution.

The inclusion of these other impacts directly in the model will, however, require more care in interpreting the model output. The reason is that consideration of these impacts will influence the assignment of traffic to the network, along with the influence of travel time. The result is that both the link improvements selected and the assignment will be based upon considerations of the objectives or impacts included in the objective function and constraints. The traffic assignment will therefore generally differ from that which would occur if users were to select their own routes based upon travel time and other factors usually considered.

In this context, the assignment or flow pattern must be considered a socially desirable one, rather than one which would naturally occur given the link improvements specified in the same solution. In order to achieve the desired flow pattern, various traffic control or restraint procedures would have to be considered. These might take the form of reducing speed limits, ramp metering on freeways, closing of links to through traffic, inducing certain turn movements and not others, priority schemes, etc. Such specific schemes can not be explicitly considered in the model because of the level of aggregation necessary for broad sketch planning, any more than the precise location of new links is included. However, the basis for the
design of specific control strategies is included in the output of the model in the form of directional link flows, travel times, and traffic routing patterns, as described in the previous chapter.

Yet direct inclusion is only possible when the forecasting equations are linear, since linear programming is being used to solve the model. For example, consideration of accessibility measured in the traditional sense,

$$A_i = \sum_{j=1}^{n} E_j t_{ij}^{-b}$$  \hspace{1cm} (5-1)

where:

$$A_i = \text{accessibility of place } i$$
$$E_j = \text{attractiveness of place } j$$
$$t_{ij} = \text{travel time or cost from } i \text{ to } j$$
$$b = \text{empirically derived constant}$$
$$n = \text{number of destination zones}$$

cannot be included in the model because it is a nonlinear form. Thus, while it would be of considerable interest to develop a model which would design accessibility-optimal networks in this sense, this is impossible under the proposed formulation. However, it is possible to apply a post-processing technique for computing accessibility in this way to facilitate the evaluation of otherwise-optimal solutions outside of the model.

Similarly, the effect of transportation on ambient air quality cannot be included directly in the model because the forecasting relationships are decidedly nonlinear. Furthermore, the predictive models required to relate alternative networks and their flows to air pollution are quite complex, and including them within the design model would greatly increase the computational time required. Thus, air pollution impacts must also be considered in a post-
processor mode.

Some nonlinear impacts, such as noise pollution, can be included directly in the model because standards for highway noise emissions have been established at the federal level. Prior to running the model, the link between these standards, as they would apply to each affected neighborhood, and network characteristics, can be defined quantitatively and included in the model structure.

Those impacts considered within the model may be treated in one of three ways. Some may be included directly in the objective function, so that their values are optimized; others may be included in the constraint set, so that they are limited to pre-specified, acceptable values; others may be treated through trade-off analyses which take advantage of the properties of mathematical programming models.

To accommodate an impact measure within the objective function, it is necessary to find a set of weights which permit the conversion of that measure into units which are commensurate with the other measures in the objective function. Since the structure of the core model requires that some measure of travel cost or time be minimized in the objective function so that the model may assign traffic flows to links, the impact measure considered for use in the objective function must be converted to time or cost units. This is easy if an acceptable monetary value for the impact measure can be found, but this is not often the case. More advanced methods for treating multiple objective problems are described in detail in Chapter 6.

To accommodate impact measures in the constraint set, it must be possible to define the minimum or maximum acceptable level for such impacts in advance of the use of the model. Noise pollution, for which standards have been set, is an example of an impact which can be treated in this way. In addition, of
course, it must also be possible to relate the constrained impact to some other elements of the model structure, such as network flows or links to be added, in order for the constraint to have an influence on determining the optimal solution to the problem. The sensitivity analysis features of linear programming models can, of course, be used to assess the significance of the effect of the constraining value of the standard. In this way, the network design model could be useful in the evaluation and selection of standards themselves.

Impacts associated with specific links, such as the taking of valued community facilities for rights-of-way, can be examined through the use of trade-off analysis. This does not require the ability to measure and value the impacts in convenient ways; instead, it simply allows the planner to assess, with a high degree of efficiency, the cost to the objective function of removing a threatening link from the optimal solution. By identifying the cost implications associated with eliminating a potentially unattractive component of the network, the model may help the planner and decision maker to establish their own assessment of the social worth of a threatened facility or institution.

The following sections discuss specific methods for treating four different impacts of transportation systems in the network design model.

**Accessibility**

The major reason for providing transportation facilities in an area is to make places accessible to people. Yet, in most transportation planning efforts, the goal of providing accessibility has been treated only implicitly. Recently, interest has grown in finding more direct ways to examine accessibility, particularly in terms of the ways in which it is differentially dis-
tributed among different population groups.

From this standpoint, then, it would be highly desirable to modify the core network design model so that it could identify networks which are optimal from the perspective of accessibility. Alternatively, the model could be used to find networks which were optimal in other important dimensions, but which met certain pre-established accessibility standards. These alternative approaches reflect using accessibility in the objective function, or the constraint set, respectively, of the mathematical programming formulation.

A review of the literature on accessibility measures was conducted to identify alternative ways of measurement, in order to find methods which were compatible with the network design model. This literature review is reported in Appendix E.

The previously defined measure,

$$A_i = \sum_{j=1}^{n} E_{ij} t_{ij}^{-b}$$

has become the most common form of accessibility measurement. However, because this measure is non-linear in terms of travel time, it cannot be used directly in the design model. It does offer a useful way to characterize accessibility, and thus it may be desirable to develop this measure for "optimal" networks in a post-processor module. That is, otherwise optimal networks could be identified within the model, and their accessibility properties, measured in this form, could then be evaluated using a separate software package. The results of both the network design model and the post-processor analysis could then be provided to the decision maker. The essential disadvantage of this post-processor mode, of course, is that; if the decision maker wishes to shift the accessibility pattern resulting from the model, the planner must rely on his judgment to discover ways in which to
accomplish such shifts.

Another common measure of accessibility is simply an enumeration of the number of trip end opportunities within a certain travel time of each trip origin (Olsen, 1972). For example, how many job opportunities are within 30 minutes by auto of a specified origin? This measure has the advantage of being easily understood by both the planner and the decision maker. Again, however, it is not compatible with the network design model because it has no regular mathematical form which would allow it to be computed efficiently. It would, of course, be possible to treat this measure in a post-processor mode as well.

In fact, all the information necessary to determine the route and compute travel time between any pair of nodes is available directly from the core model. Hence any measure of accessibility based on travel time could be generated in a post-processor, provided the other information needed is available.

As discussed in detail in Appendix E, a less commonly utilized measure of accessibility is the simple summation of travel times for all trip makers on a network. This has the advantage of being easy to understand, and it is weighted by the number of trips to each destination, because it represents the sum of all travel times. It fails to reflect both intuitive feelings and empirical findings related to the traveler's preference for short trips, a concept which is introduced in the first measure discussed above by the exponential weight, b. This measure, however, is easily incorporated into the model because of its linear form. In fact, the version of the model proposed in the previous chapter used as its objective function the minimization of total travel time (or cost). Thus, if one is willing to accept this measure of accessibility, the proposed model already includes it explicitly as
the primary objective.

While the model, then, can be viewed as a method for optimizing accessibility defined in this manner, this approach has some notable deficiencies. In particular, accessibility in this sense is only maximized in terms of the pre-specified trip table. Therefore, no consideration is given to the existence of latent demand; that is, people who travel very little now because of poor accessibility would experience little accessibility improvement in the networks identified by the design model. In addition, if a policy were established to provide improved accessibility to a part of the region where development was to be encouraged, the model would not respond to this need because trips to such an area would not appear in the given demand table.

It is possible to get around these limitations by arbitrarily factoring up existing trips in the demand matrix, or by adding new, fictitious trips. The model would then attempt to minimize travel times for those new trips, and would thereby provide those areas with better transportation services.

For example, if a particular population groups were poorly served at present, and if the planner wished to improve the accessibility of that group, he might multiply all trips made by that group in the trip table by 2.0. This weighting would force the model to place the accessibility needs of that group at a higher priority level than the needs of other travelers. In situations where no trips at all existed in the trip table, arbitrary numbers of trips could be added prior to application of the network design model. Determination of the trip weighting factors and numbers of fictitious trips to be introduced would have to be accomplished through an experimental, trial-and-error process.

A related strategy for introducing preferential accessibility to speci-
fic groups would be to weight the travel times or costs for those groups. That is, wherever those travelers appeared on the network, their costs would be computed separately, using a unique cost schedule. To accomplish this, however, it would be necessary to increase the number of variables in the model in order to keep track of the special traveler groups. The required change in the model is described in more detail in Appendix E.

**Noise Emissions**

Noise is a major concomitant of motor vehicle traffic; it has been a frequent source of complaint at the community level. However, noise impacts are rarely considered in macroscopic network planning, since precise locational and design characteristics of facilities are not yet established at that stage. Furthermore, it is not possible to aggregate noise emissions over an entire network, as is possible for air pollution. Noise levels are closely related to the specific characteristics of a transportation facility and its traffic, and relatively detailed studies are required to predict them. In addition, noise impacts are related not only to emission levels, but also to the nature of the activities (land uses) occurring in the area.

It is especially desirable, however, to develop a capability to consider noise impacts in network sketch planning, since in this way they may be taken into account prior to the point at which plans are finalized. Otherwise, it is likely that the regional or area-wide network plan will be selected without regard to such effects. When the latter occurs, localized noise impacts are typically ignored in the planning of specific facilities in order to facilitate achievement of the regional plan, prepared without consideration of noise. Alternatively, the design engineers may be forced to accept the difficult task of reducing noise emissions through the modification of
detailed design features, or through the construction of noise barriers. Unfortunately, the chances of achieving significant noise reductions in these ways are quite limited (Harmelink, 1972). Therefore, a special effort was made, as a part of this project, to introduce consideration of noise impacts into the network design model.

As documented in Appendix C, which reports in detail on the investigation of highway noise conducted for this project, a number of noise emission forecasting models have been developed. Typically, these models relate noise levels to traffic volume, speed, characteristics of the roadway and its immediate environment, and distance from the road to the point of observation. The model selected for use within the network design model was developed by Bolt, Beranek and Newmann, (1969), and is stated below:

\[ L = 20 - 10 \log_{10} D_E + 20 \log_{10} S + 10 \log_{10} V, \]  

(5-2)

where:

- \( L \) = mean noise level in decibels measured on the A-weighted scale (dBA);
- \( D_E \) = distance in feet from the center of the single lane equivalent to the observer;
- \( S \) = average vehicle speed in miles per hour;
- \( V \) = volume of traffic, aggregated over all lanes, in vehicles per hour.

The assumptions underlying this model are discussed in Appendix C. Basically, it is assumed that the traffic stream can be treated as a line source of noise, and it does not take into account noise related to stopping and starting; therefore, it relates only to freeway type operations.

The noise-sensitive version of the network design model does not deal with the situation of parallel links which are so close to one another that noise generated on both contributes to noise levels in the environment of in-
terest. The theory for "adding" the noise from both sources exists, of course. However, it was felt that at the sketch planning level for which the model is designed it is likely that only one link (which might represent many nearby parallel roads) would be considered. Therefore two or more distinct facilities in the same corridor were not considered in the noise model; thus modifications to the model would be necessary to include this situation.

This model may be adjusted to take into account truck noise by addition of the term $L_t$, which is a tabular function of the percentage of trucks in the traffic stream. As used in this report, the model does not consider grades, curves, or noise barriers, since it was felt that, at this aggregate level of analysis, such detailed information would not be available. It does, however, account for median widths.

To introduce such a model into the network design process, there are, again, two options: it could be built into the objective function of the constraint set. Using this formulation in the objective function, however, would be incompatible with the requirement that the model must also minimize travel times or costs in order to account for traffic assignment. Furthermore, since highway noise emission standards have recently been promulgated at the federal level (PPM90-2), it is possible to establish maximum permissible noise levels to be used to define constraints in the model. Finally, since the noise model includes traffic volume as an independent variable, such a constraint could be easily linked to the other components of the design model. That is, given maximum acceptable noise levels, the network design model could adjust volumes and link improvements to remain within these limits.

A difficulty arises in the attempt to translate the noise model for use
as a constraint in the network design process, in that the noise forecasting relationship ignores the well-known relationship between speed and volume on a highway. Instead, the noise model has been calibrated using data from many types of highways, so that a wide variety of speed-volume combinations are possible. Yet it is known that, for a given facility, with a fixed number of lanes, speed and volume are quite closely related: as volume increases, speed decreases and, as congestion sets in, both speed and volume decrease (see Appendix C, Figure 2). Since, in the network design model, the noise relationship would be applied to specific facility types, this link between speed and volume had to be taken into account. This was accomplished by substituting two of the better known speed-volume models, given a reasonable value for mean free (zero volume) speed, into the noise equation, resulting in two alternative noise models which could be written in terms of volume, distance from the roadway, and percent trucks.

This model can be operationalized for use in the network design context by solving it for the traffic volume in terms of the remaining variables. Then, pre-specifying the critical distance from the roadway, at which the noise standard would be applied, the total number of lanes, the noise standard level, and the mean free (zero volume) speed on the facility, as well as the typical percent trucks, the noise constraint may be computed in terms of limiting values of traffic volumes alone.

The resulting, simplified model is shown schematically in Figure 5-1, repeated from Appendix C. In this figure, for a given facility type, percent trucks, and critical distance from the roadway, noise is shown to be a non-linear function of traffic volume per lane: as volume increases, noise increases for a while; then, as congestion sets in, and speeds decrease, noise levels begin to decline. Note that the effect of the number of lanes is ac-
Figure 5-1. Traffic Volumes Exceeding Given Noise Standards.
counted for by dealing with only average volume per lane.

Figure 5-1 also shows that, given a maximum noise level standard, (shown as a horizontal line), the intersection of that standard with the noise-volume relationship specifies limiting volume levels required to meet that standard. Of course, because of the shape of the noise-volume relationship, any noise standard level may result in none, one, or two volume limitation levels. Where the two relationships do not intersect, no possible volume on the facility would lead to a violation of the noise standard. Where the standard line touches the noise-volume relationship at one point (the maximum noise level to be expected from that facility), the corresponding volume represents the maximum permissible flow if noise standards are to be met. Finally, if the noise standard is low enough, or the noise emissions high enough, it is possible for the two relationships to meet at two points, a high volume and a low volume level. The constraining value of volume, of course, is always the lower level. For the relationship shows that, above a certain volume, the noise standard would be met, but it would be impossible to require a facility to operate continuously at a high volume level. Instead, it is logical to keep the volume level below the first limiting value, shown in the figure as $V_1$.

To use this relationship in the network design model, it is first necessary to identify those links on which noise constraints will be applied. Assuming a reasonable percentage of trucks, and a typical configuration of the facility type, and specifying an appropriate measurement distance and noise standard based on adjacent land uses, the noise model in Appendix C is then applied to establish upper bound constraints on per-lane traffic volumes. These are then introduced into the network design model; they pose no additional difficulties in solving the model. A computer program for computing
volume constraints directly is also included in Appendix C.

The model itself will assign traffic to the noise-critical links only until the constraints are met; then, it will assign any remaining traffic to alternative routes which are not noise-constrained. The model may also add capacity improvements to the noise-critical links to keep per lane volumes within limiting values; more typically, it will add capacity to links on other paths which are not noise-constrained, and which, thus, are forced to carry heavier volumes.

To study the effect of such noise constraints on the solutions produced by the network design model, a series of tests were performed using the 8 node, 11 link network shown in Figure 5-2. All other data related to this example are shown in Tables 5-1 and 5-2. The critical distance from the roadway was set at 100 feet, the median width at 20 feet, and it was assumed that 5% of the vehicles were heavy trucks, resulting in a truck correction increment of +2 dBA.

Using a 70 dBA standard level for initial explorations, and comparing the constraining traffic volumes with a previous assignment of this network, it became clear that the resulting volume constraints would have no effect on the network design model solution. Furthermore, for the relatively low mean free speeds assumed initially, estimates of the noise emissions for the links indicated that the use of a 65 dBA standard would also have little constraining influence.

Therefore, the example was modified so that links 6 and 8 were assumed to be freeways having mean free speeds in the range of 45 to 60 miles per hours; further, these facilities were defined as having 4 lanes, two in each direction, a twenty foot median width, and a practical capacity of 1800 vehicles per lane per hour.
Figure 5-2. Noise Impact Example Network.
Table 5-1. Demand Matrix for Noise Impact Example Problem.

(in 1000's p.c.u./day)

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
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<td></td>
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<td>2.5</td>
<td>2.0</td>
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<td>5.0</td>
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<td></td>
<td></td>
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<td>2.6</td>
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<td></td>
<td>1.5</td>
<td>1.0</td>
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<tr>
<td>7</td>
<td>2.6</td>
<td>2.0</td>
<td>3.0</td>
<td>2.0</td>
<td>1.0</td>
<td>2.0</td>
<td></td>
<td>1.0</td>
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<tr>
<td>8</td>
<td>2.6</td>
<td>2.0</td>
<td>5.0</td>
<td>2.0</td>
<td>2.5</td>
<td>2.0</td>
<td>1.5</td>
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</tbody>
</table>
Table 5-2. Description of Links of Noise Impact Example.

<table>
<thead>
<tr>
<th>Link Number</th>
<th>Practical capacity (vph/lane)</th>
<th>Mean free speed (mph)</th>
<th>Number of lanes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1000</td>
<td>25</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>1350</td>
<td>20</td>
<td>4</td>
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<td>1000</td>
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<td>2</td>
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<td>1000</td>
<td>25</td>
<td>2</td>
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<tr>
<td>5</td>
<td>1000</td>
<td>25</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>1800</td>
<td>45-60*</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>2000</td>
<td>35</td>
<td>2</td>
</tr>
<tr>
<td>8</td>
<td>1800</td>
<td>45-60*</td>
<td>4</td>
</tr>
<tr>
<td>9</td>
<td>1000</td>
<td>35</td>
<td>2</td>
</tr>
<tr>
<td>10</td>
<td>1000</td>
<td>30</td>
<td>2</td>
</tr>
<tr>
<td>11</td>
<td>500</td>
<td>25</td>
<td>2</td>
</tr>
</tbody>
</table>

*in 5 mph increments
Under this modification, it was immediately observed that the volume constraints resulting from the use of both 60 and 65 dBA noise standard levels at 100 feet from the roadway would very likely result in an infeasible solution in the network design model. That is, there would be no way to accommodate the zone-to-zone travel demands and meet the noise standards at the same time. This suggests some interesting potential uses for the noise-constrained network design model. It could be used to evaluate the feasibility of meeting noise standards, given a fixed network and travel demands. It might also be used to evaluate the effects upon transportation network/capacity requirements of changing land use, causing an associated shift of noise standards for various links.

Given the above findings, the network design model was solved for the example problem using the 70 dBA standard level. Since the noise model requires peak period volumes, while the design model operates on average daily traffic, it was assumed that 12% of the ADT traffic was carried in the peak hour. Initially, the model was run without the noise constraints to establish a baseline for comparison purposes. The results, assuming a budget level of $30 million and a mean free speed on links 6 and 8 of 65 miles per hour, indicated a total travel time on the network on 11,216 hours, with total vehicle miles at the level of 302,800. The volume/capacity ratios for all links were at or below 0.75. Link 6 was improved considerably by the model, with the addition of almost two additional lanes.

Next, the noise constraints for links 6 and 8 were introduced, and the model was solved again. Limiting volumes were 10,400 vehicles per day on each constrained link. The result was a very significant diversion of traffic from the high speed—and thus, high noise—links, 6 and 8, which was necessary to achieve the noise constraints. Links 3, 4 and 9 carried capacity flows,
and a three lane improvement was added to link 7. Daily vehicle mileage increased to 334,600, and the total travel time went up to 16,673 hours per day. The freeway links, as a result, carried a very small daily volume (10,400) at a very high level of service.

In addition to constraining traffic volumes, perhaps through ramp metering, alternative approaches to meeting the noise standards include reducing the percentage of trucks (e.g., restricting trucks from using the noise-sensitive facilities), establishing more stringent standards on the noise emissions of individual vehicles, increasing the value of the noise standard itself, or decreasing the mean free speed (reducing speed limits).

To further explore the effect of speeds on the solution to the network design model, the model was solved using mean free speeds for links 6 and 8 of 40, 45, 50, 55, and 60 miles per hour, as well as the previously tested 65 miles per hour. The volume constraints for these links at various speeds are shown in Table 5-3. The results of these experiments are shown in Table 5-4 and Figure 5-3. It can be seen that, as freeway mean free speed is lowered, both total daily travel time and vehicle miles of travel decrease, since more vehicles may utilize the freeway links, taking advantage of more direct routings while still meeting the noise constraints. Figure 5-3 also shows the results of baseline runs of the model for various mean free speeds without the noise constraints. In these cases, as speeds decrease on links 6 and 8, total network travel time goes up, as would be expected, since the average level of service for travelers is going down. With the noise constraints, decreasing the mean free speeds allows increasing numbers of vehicles on the freeway links, and thus serves to increase average level of service. Travel time savings would continue to accrue as speeds are lowered on the noise-constrained network, until the point is reached at which the
Table 5-3. Volume Constraints on Links 6 and 8 for Different Mean Free Speeds

<table>
<thead>
<tr>
<th>Mean Free Speed (mph)</th>
<th>Volume Constraint (ADT)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>28000</td>
</tr>
<tr>
<td>45</td>
<td>21900</td>
</tr>
<tr>
<td>50</td>
<td>17700</td>
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<tr>
<td>55</td>
<td>14600</td>
</tr>
<tr>
<td>60</td>
<td>12200</td>
</tr>
<tr>
<td>65</td>
<td>10400</td>
</tr>
</tbody>
</table>
Table 5-4. Travel Costs on the Noise Constrained Network for Various Mean Free Speeds

<table>
<thead>
<tr>
<th>Freeway Mean Free Speed (mph)</th>
<th>Daily Network Travel Time (hours)</th>
<th>Daily Network Veh. - Miles</th>
</tr>
</thead>
<tbody>
<tr>
<td>65 (unconstrained)</td>
<td>11,216</td>
<td>302.8</td>
</tr>
<tr>
<td>65</td>
<td>16,673</td>
<td>334.6</td>
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<td>310.5</td>
</tr>
<tr>
<td>40</td>
<td>13,023</td>
<td>306.3</td>
</tr>
</tbody>
</table>
Travel Time VS. Mean Free Speed on Freeway for 8 Node Network for Budget of 30 Million.

Figure 5-3. Effect of Mean Free Speed on Links 6 and 8 on Total Travel Time on the Example Network.
speeds on links 6 and 8 go below the speeds on complementary, non-freeway paths. Subsequently, travel times would begin to increase again.

The network design model also provides useful information on the capital cost implications of alternative ways to meet the noise constraints. For example, at a mean free speed on the freeway links of 65 miles per hour, $22 million in capital improvements were required to meet the travel demands on the network. For a speed of 40 miles per hour, only about $14 million would be needed. Again, this is because, at the higher freeway speeds, volumes on links 6 and 8 were severely constrained, necessitating the improvement of complementary links to carry the traffic.

The results of these experiments suggest that the inclusion of noise constraints in the network design model represents a potentially valuable improvement. Their use is easy, they do not effect the efficiency of solution of the model significantly, and with them, the model provides a unique way to include such environmental impacts in aggregate network planning.

Noise is by nature a very localized phenomenon, and to consider it in the network design model requires specific information about individual links. This presents a minor drawback, since the design model itself is most appropriate for macroscopic studies, or sketch planning. In such an application, it is likely that a number of smaller links will be aggregated into a single, large pseudo-link. Such aggregation reduces the size of the design problem and thus increases the efficiency of solution of the model. It is expected that the planner would encounter some difficulty in attempting to specify the characteristics of such an aggregated link in a manner appropriate for use in calculating the noise constraints.

Even at the sketch planning level, however, it is likely that those links which are to be noise-contrained will be few in number, and these may
be restricted to the proposed new freeway facilities only. If this is the
case, then treating all noise-constrained links individually, without aggre-
gation, should be possible.

Air Pollution

Air pollution emissions produced by motor vehicles are becoming of in-
creasing concern, particularly in larger urban areas. While federal and
state governments are attempting to increase the strictness of emission stan-
dards for individual vehicles, both logic and public interest make it impos-
sible to forego the consideration of air pollution in transportation network
planning. In particular, it would be highly desirable to develop an effi-
cient capability to assess the air quality implications of alternative high-
way network plans.

Direct consideration of air quality impacts in the network design model
is not feasible at this time. This is because air quality can only be as-
sessed by predicting not only the basic vehicular emissions, but also the
dispersion of those emissions through the atmosphere. Current dispersion
models are both preliminary in nature and computationally complex. Computer
capacity and time requirements for reasonable large dispersion models would
probably exceed the relatively large requirements imposed by the network de-
sign model itself. Furthermore, realistic consideration of air quality im-
plications would require treatment of non-vehicular pollution sources as
well, a task which would be impossible in transportation sketch planning.

A number of models are available for estimating air pollution emissions
alone, however, and these might be candidates for inclusion in the network
design model. Introducing them in the objective function, again, would not
be possible because of the need for that function to include the minimiza-
tion of travel time, and since emissions are generally non-linearly related to speed, volume, and vehicle type.

Simplistic use of emission models to establish volume constraints for the design model, as in the case of noise impacts, would be feasible. This is particularly true in the case of one important pollutant, carbon monoxide, because its concentration tends to decrease rapidly with distance from the source, making emissions closely correlated with concentration on air quality in the region of highest concentration where air quality is of concern. However, this is not the case for the two other most important pollutants, hydrocarbons and oxides of nitrogen. No standards currently exists, nor are they likely to be promulgated, for total emissions of various pollutants from a given transportation facility. In part, this is because the primary issue is air quality, which is determined not only by vehicular emissions, but also by emissions from other sources and by micro-atmospheric conditions such as winds, temperature, topography, and the characteristics of man-made structures.

As a result, it seems necessary to treat air pollution in a post-processor mode in conjunction with the network design model. It is relatively easy to add a vehicular emissions post-processor to the design model, since the outputs of the latter provide most of the inputs needed to compute emissions with sufficient accuracy. The design model could then be used to explore the air pollution implications of alternative networks, using the planner and decision maker in an open-loop process, reviewing the impacts of a given optimal network and restructuring the inputs for another optimal seeking effort.

To develop a model post-processor, a review of the literature on vehicular pollution emissions was conducted, and is reported in Appendix D. It
was found that the pollutants of primary concern are carbon monoxide, hydrocarbons, and oxides of nitrogen. Furthermore, a number of estimating relationships have been developed to predict such emission, in terms of grams produced per mile of operation. The fundamental independent variables of these models are traffic volumes, travel speeds, automobile-truck mix, and vehicle age mix. The latter is of importance because vehicles produced in different years were required to meet different emission standards. The network design model estimates volumes and, since mean free speed is a parameter of the volume-travel time functions it uses, it is possible to use one of the well-established speed volume relationships to compute operating speeds on the links.

The data and models used to develop the post-processor described in Appendix D were developed by the U. S. Environmental Protection Agency and by Argonne National Laboratory. It would be a simple matter to revise these relationships as new information becomes available. Carbon monoxide and hydrocarbon emissions are calculated based on vehicle age, volumes, and the auto-truck mix; an adjustment is made for operating speed, since, at higher speeds, internal combustion engines are somewhat more efficient, producing less pollutants. The model parameters also account for the "cold start" effect, which causes vehicles to emit greater quantities of pollutants during their warm-up periods. Prediction of oxides of nitrogen emissions is accomplished similarly.

The proposed post-processor requires the user to select a forecasting year, and the computer program developed (Appendix D) automatically determines the vehicle age mix; however, data are stored only for the years 1970-75 in the current model. The emissions model itself operates on a peak period basis only. Since the network design model works with average daily
traffic, it is necessary to convert these figures to a peak period volume estimate. This is accomplished using methods specified in the Highway Capacity Manual for estimating 30th highest hour volumes.

Link speeds are calculated using the speed-volume function used in many models for capacity restrained traffic assignment (see equation 4-1); an alternative model has also been developed using a more realistic parabolic speed-volume curve. In either case, the model requires mean free speed and practical capacity for each link; these establish the speed-volume curve, from which, given volumes from the network design model, operating speeds may be calculated.

In its current form, the emissions model estimates the production of carbon monoxide, hydrocarbons, and oxides of nitrogen, in pounds per (peak) hour, produced by each link. These are reported in both absolute numbers and percentages. This permits the planner to assess the absolute and relative contribution to area-wide air pollution made by each link. Link-specific information should be of particular value to the planner in determining what changes should be made to improve emission characteristics of networks produced in subsequent runs of the design model. The typical form of the output of the emissions model is shown in Figure 5-4. The data shown are from an application of the network design model to a twenty-four node, thirty-eight link problem based on an aggregated version of the highway network for Sioux Falls, South Dakota.

In addition, the model provides network-wide summaries of these three emissions in terms of total pounds produced. These quantities should be of value in comparing alternative networks developed by the design model.

It is interesting to note that two of the three primary air pollutants are produced in smaller quantities as link speeds increase, while, as dis-
<table>
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<tr>
<th>LINK (MPH)</th>
<th>TRAFFIC CHARACTERISTICS</th>
<th>1970 EXHAUST EMISSION TABLE</th>
<th>HYDROCARBONS</th>
<th>OXIDES OF NITROGEN</th>
</tr>
</thead>
<tbody>
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<td></td>
<td>(VPH) (LBP) (LANES) (LENGTH)</td>
<td>(CARBON MONOXIDE (LR/HR), (CO/0 TOT) LR/HR)</td>
<td>(LR/HR), (CO/0 TOT) LR/HR)</td>
<td>(LR/HR), (CO/0 TOT) LR/HR)</td>
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<td>84.2 4.5</td>
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<td>324 7.0</td>
<td>58.2 2.7</td>
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<td>127.3 6.0</td>
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**Figure 5-4. Pollution Emission Estimates Produced By the Emissions Post Processor.**

**TOTAL CARBON MONOXIDE EMISSION BY NETWORK** 235.1 (LB PER 1000 VEHICLE MI)

**TOTAL HYDROCARBON EMISSION BY NETWORK** 2736.2 (LB/HR)

**TOTAL OXIDES OF NITROGEN EMISSION BY NETWORK** 1879.5 (LB/HR)

**TOTAL CARBON MONOXIDE EMISSION BY NETWORK** 235.1 (LB PER 1000 VEHICLE MI)

**TOTAL HYDROCARBON EMISSION BY NETWORK** 27.3 (LB PER 1000 VEHICLE MI)

**TOTAL OXIDES OF NITROGEN EMISSION BY NETWORK** 49.8 (LB PER 1000 VEHICLE MI)
cussed in the previous section, noise impacts tend to **increase** as speeds increase. This suggests that an environmentally-important trade-off must be made between these impacts in the transportation network planning process. It seems clear that the network design model, because of its capability to develop efficiently large numbers of aggregated networks, offers a useful mechanism for exploring this trade-off relationship in the early stages of the planning process.

**Assessing the Cost of Preserving Valued Facilities**

Whenever improvements or additions to an existing transportation system are considered, controversy inevitably arises concerning the desirability of saving threatened facilities in the path of those improvements, such as schools, churches, historic sites, scenic areas, recreation areas, wildlife refuges, special communities and businesses which could not survive relocation. Obviously, certain elements of the community value facilities in different ways. For example, construction of a new roadway through a wildlife refuge may provoke acrimonious debate from conservation-minded people, while other members of the community may not object at all.

Because it is so difficult in most cases to assess the monetary value of a facility, a method is proposed whereby one may determine the cost of preserving it. Existing methods, e.g., performing a traffic assignment with and without the threatening link, may be used to determine the cost, but even for a few facilities, the costs of utilizing contemporary transportation planning models are prohibitively high. Furthermore, it is rare to see such approaches applied in the early stages of sketch planning, where many critical choices about network structure are made. What is sought, therefore, is a relatively inexpensive and efficient method which enables one to determine
how much it would cost the users of the transportation system (in time or money) to avoid destroying one or more valued facilities in question. This is the primary trade-off of interest, since typically network planning involves decisions involving the sacrifice of local values to produce user savings which are distributed regionally. Once this "avoidance" cost is determined, the information is available to make a decision concerning whether a facility is worth the total social avoidance cost.

The mathematical properties of the network design model are such that the determination of this avoidance cost is very easy and efficient. The recommended approach is simply to modify the model so that the improvement is not made, yielding the user costs, optimal pattern of link investments, etc. for the best plan without taking the valued land. The impacts and levels of objective achievement for this "without taking valued land" solution can be compared with the "with taking the valued land" solution to assess the costs of saving the valued land in terms of other objectives. This is the desired information.

The approach described above requires two separate runs of the model. It is possible to reduce the computer time and hence cost of the "with" and "without" computations by taking advantage of the mathematical structure of the model. This alternative approach is described in Appendix K.

Simply stated, this method applies an arbitrarily large, dummy user cost to each link which threatens an important community facility. Once an optimal solution to the initial ("with taking valued land") network design problem has been found, and its associated user cost determined, these dummy costs are introduced, and the solution process is re-initiated, beginning at the previously established optimal point. The solution of this modified problem, arrived at in a small number of iterations because it was begun at

5-33
the point of the previous optimal, defines the user cost of the modified net-
work. The difference in user costs represents the cost associated with saving
the valued facility by not constructing the link in question. This process
would then be repeated for each threatening link under consideration, as well
as to a series of links to determine the costs of saving collections of valued
facilities.

It should be noted that the use of the values of the dual variables as-
associated with the upperbound capacity-added constraints to interpret the ef-
flect on the objective function of restricting the addition of capacity to
save valued facilities is not possible. This is because the interpretation
of dual variables in sensitivity analysis applies only as long as the basis
of the mathematical program does not change. If a link must be deleted from
the optimal solution to preserve a valued facility, then by definition the
optimal basis changes.

Implementation of this approach requires the planner to alter the user
cost coefficients in the objective function associated with the threatening
links. There are, at present, few computer codes which permit the analyst
to change these coefficients conveniently in the midst of a solution run.
Typically, such codes have no re-start capability, and thus the objective
function and constraint matrix associated with the previously optimal solu-
tion will not be retained in core storage. Without this re-start feature,
the new starting point for solution (i.e., based on the modified cost coef-
ficients) must be prepared by the analyst in the form of data inputs for a
completely new run of the model. This approach will be more costly, in com-
puter time, than would be the case if a re-start capability existed.
Conclusion

This chapter has discussed methods of treating four important impacts of transportation investments. These are offered as examples of strategies for including broader impact considerations in the application of the network design model to transportation sketch planning. The four impacts discussed were each treated using distinctly different techniques, again, to demonstrate the flexibility of the design model. These techniques were (1) inclusion in the objective function—accessibility; (2) inclusion in the constraint set—noise impacts; (3) treatment in a post processor mode—air pollution emissions; and (4) perturbation of the solution of the model—valued community facilities.

Clearly, a variety of additional goals and impacts can be treated in the network design model using variations of these techniques. The model offers the flexibility and efficiency to permit reasonable consideration of such impacts in the earlier stages of the transportation planning process. Effective use of this capability should facilitate the introduction of impact sensitivity into sketch planning, so that general commitments can be made to network plans with fuller recognition of such impacts. This should increase the likelihood that such generalized plans will be feasible not only in a technological sense, but also in terms of their social, economic, and environmental consequences.

References


CHAPTER 6
THE MULTI-OBJECTIVE MODEL

The Problem

Transportation systems have many and widespread impacts. Decisions made with respect to them have consequences that affect many aspects of the relevant environment, extending far beyond the transport system itself. Transportation can both enhance and damage the quality of the environment; it can act as a stimulus or as a deterrent to urban growth and development. Thus, the ability to make enlightened transportation decisions may, to a large extent, determine the society's success in achieving wider policy objectives.

Objectives differ significantly depending upon the point of view from which the transportation system is being considered. Therefore, different groups will have different objectives which often may be conflicting with each other. For the purpose of transportation planning, the primary interest groups might be classified as follows:

1. The community
2. The non-users
3. The users
4. The system owners and/or operators

In the planning and decision making process, the perspectives of all interest groups must be considered.

Until recently, the most commonly used criterion for investment decisions was the incremental benefit-cost ratio which states that the changes in benefits, to whomsoever they may accrue, must be in excess of the change in costs, to whomever they accrue. In the definition of costs, undesirable consequences such as smog, accidents, noise, etc., are often not considered.
Moreover if considered, the usual practice is to attempt to convert these into money costs, a difficult and imprecise task.

Ignoring the incidence or distribution of gains and costs has often led to the selection of plans which possess substantial net benefits for some (especially intensive freeway users), and substantial negative net benefits for others (usually lower income groups in the path of proposed facilities). Those who feel quite negatively affected have often developed the power to block some highway improvement projects. This is of considerable concern to all urban residents and groups, for it suggests that there has been a serious deficiency in the transport planning process. Therefore, there is a need for developing analysis methods that will generate alternatives which will be acceptable to all interest groups affected by transportation planning in meeting their objectives.

Contemporary methods for designing and evaluating alternative transportation plans have been extended to consider the impacts upon the various affected groups, at a rather disaggregated level such that the distribution of gains and costs can be determined; and improvement in these methods is continuing. An example is the Plan Information Matrix developed by the Federal Highway Administration. However, these methods have been developed primarily with a view toward evaluating rather detailed alternatives, using information typically generated in one of the standard urban transportation network models (such as the so-called BPR package) as input. This limits these methods to essentially a role of evaluation after many design decisions have been made, since neither they nor the detailed network models are suited to exploring the full range of alternatives.

It is essential that the basic network designs to be explored and refined using such detailed models be based upon considerations of their full
impacts. In this chapter, an organized search process, using the capabilities of the highway network design model, to generate and evaluate good network alternatives in a multi-dimensional context is presented. This approach allows the plan identification or search process to be guided explicitly by the nature of, and the preferences for, the multi-dimensional impacts of alternative plans. As will be discussed in Chapter 8, this process is such that interaction with appropriate decision-makers and community groups is facilitated, with the model providing an information generating and processing capability.

A Network Design Model with Two Objectives

In Chapter 4, a core network design model was developed. The objective of that model was to design that transportation system which will minimize the total travel time subject to the constraint that the total expenditures for improvements (capital costs) fall within the budgeted amount. An alternative criterion commonly used in transport planning is to design the transport system so as to minimize the total cost (total travel time, evaluated in money units, plus capital cost) subject to a budget constraint. The use of this combined objective function is desirable in that it allows the optimizing model to trade-off capital cost for travel costs internally in order to achieve the least total cost network. This formulation becomes difficult to apply, however, because combining travel time and construction cost within the objective function requires the derivation of a money value of time. While much research has been devoted to the isolation of such a value, there is still considerable disagreement in the field regarding the precise value to be used in a given case. Therefore, this combined, or total cost, objective function is not recommended.
Use of the simple, total travel time minimization model does present some problems. Since this model would choose to improve all links to indefinitely high levels in the absence of a budget constraint, the latter plays a very critical role in determining the optimal solution. Yet in most cases there is no a priori limitation on the available budget. While policy makers might logically prefer to keep capital costs as low as possible, there is no obvious strategy, or even a rule of thumb, for fixing the transportation budget. In fact, a most promising method for choosing a budget level would be to determined precisely what user cost savings (time) would result from various budget levels, and then to select the most attractive combination of both.

Thus what is really desired in the context of multiple objectives is information on the possible levels of achievement of the various objectives. Considering first just two objectives, for each value or level of achievement of one, it is desirable to know the maximum possible level of achievement of the other objective. Any alternative which is the same in terms of the first objective and poorer than another alternative in terms of the second presumably would be of no interest, since an alternative equal in the first objective and better in the second is known. The poorer alternative is clearly inferior to the other, and is said to be "dominated" by the other. Only undominated alternatives are of interest, because for any dominated alternative, there exists an alternative which is better in terms of one objective and at least as good in terms of the other objective.

This can be restated with an example. Assume that the two objectives are (1) minimization of travel time and (2) minimization of capital expenditures for road improvements. Alternative A has a cost of $10 million and a travel time of 100,000 person-hours per day. Alternative B has a cost of
$10 million also, but a travel time of 120,000 person-hours per day. Assuming for the moment these are the only two objectives, it can be stated that alternative A is clearly preferred to B--without knowing anything about the money value of time. Similarly, an alternative C, with a cost of $11 million and a travel time of 110,000 person-hours per day is dominated by alternative A, and of no interest. On the other hand, alternative D, with a cost of $12 million and a travel time of 90,000 person-hours per day is an undominated, or so-called "efficient" alternative. The word efficient is used because it means that the achievement of one objective cannot be improved without degrading the achievement of another.

These alternatives are shown in Figure 6-1. Also shown there is the efficient frontier, defined as the entire set of efficient or undominated alternatives. These would be the only alternatives of interest in this two objective context, because any other alternatives can be improved upon in terms of objective attainment for one objective without degrading performance in terms of the other objective. A means of generating these alternatives--or the efficient frontier--is necessary.

**Efficient Frontier Generation**

The mathematical form of the network design model makes it very easy to generate the desired trade-off information in terms of only efficient alternatives. It is of course very desirable to explore only efficient alternatives, since others can be rejected on a priori grounds. The method used is drawn from a general method presented by Geoffrion (1967), which is termed the bi-criteria method. This is best explained through an example.

Considering the same objectives as above, minimizing travel time and minimizing capital costs, these can be written for the core model as follows.
Figure 6-1. Example of Trade-Off Information and Efficient Frontier.
(using the notation developed in Chapter 4):

Minimize travel time

\[ f_1 = \sum_{ij \in L} (c_{ij}^1 x_{ij}^1 + c_{ij}^2 x_{ij}^2) \quad (6-1) \]

Minimize capital cost

\[ f_2 = \sum_{ij \in E} B_{ij} k_{ij} \quad (6-2) \]

The recommended means for generating the desired trade-off curve is to use an objective function in the model which combines the two objectives of interest. This takes the form shown below, in which weights, designated \( \alpha \) and \((1 - \alpha)\), are used to weight the two objectives into a composite objective:

\[ \text{Minimize} \ (1 - \alpha) f_1 + \alpha f_2 \quad (6-3) \]

or

\[ \text{Minimize} \ (1 - \alpha) \sum_{ij \in L} (c_{ij}^1 x_{ij}^1 + c_{ij}^2 x_{ij}^2) + \alpha \sum_{ij \in E} B_{ij} k_{ij} \quad (6-4) \]

\[ 0 \leq \alpha \leq 1 \quad (6-5) \]

This combined objective is of course minimized subject to all the constraints in the original core model, as given in Chapter 4 in Figure 4-5, except the budget constraint (since we wish to explore several budget levels).

Note that when \( \alpha = 0 \), the model searches for the minimum travel time solution, ignoring capital costs. In this way the lowest time and highest capital cost alternative is identified. When \( \alpha = 1 \), it ignores time and attempts to minimize capital expenditures, this defining the other extreme possible (or feasible) solution. As \( \alpha \) is varied between 0 and 1, other so-
olutions, with intermediate values of travel time and capital expenditures, are generated. In fact, a particular characteristic of linear programs can be taken advantage of, so that only a limited number of solutions (the expenditure points) need be generated, from which all other solutions can be generated by simple interpolation. The details of the method are presented in Geoffrion (1967).

Example of Bi-criteria Trade-Off Analysis

An efficient frontier was generated for the example network of 8 nodes and 11 links for the two objectives (1) minimize the total travel time, and (2) minimize the total construction cost. The network is the same as used in Chapter 5 for the noise constraint example. The efficient frontier is shown in Figure 6-2. It should be noted that each point on this trade-off curve represents a unique network design composed of links to be added and improvements to existing links, and that each design can be obtained from the computer solution to the problem.

The usefulness of this trade-off relationship can be illustrated by considering the reasoning which a decision-maker might go through in studying it. For example, it is relatively evident that the alternatives in region I are undesirable, for (relatively) small increases in construction cost will bring about much larger reductions in travel times if the decision maker is willing to move into region II. Furthermore, region III is certainly not attractive, because spending more on construction costs to move from II to III results in only small reductions in travel times. The most reasonable solutions seem to lie in region II. The use of Figure 6-2 allows the decision maker to see the nature of the trade-offs he must make in choosing between alternative networks in region II.
Figure 6-2. Efficient Frontier of Vehicle Hours Versus Construction Cost.
This efficient frontier also provides the immediate and explicit results of sensitivity analysis with respect to amount of budget available for improvement of the transport system. If the budget were \( B_1 \), the best transport system, which can be designed by spending all resources, would have \( T_1 \) as the total travel time. Similarly for budget level \( B_2 \), the best transport system will have total travel time \( T_2 \). This information can be of direct value in assisting in the choice of an overall budget level, which would otherwise be a most difficult problem.

Any point on the efficient frontier represents the combination of construction cost (dollars) and travel time (vehicle hours) which is valid for any value of travel time. This is a significant aid to the transport planner, as it helps him to evaluate transport project without any value judgements. For example, by moving from efficient point D to E, the decision maker knows the cost \( \Delta f_2 = f_2^E - f_2^D = B_2 - B_1 \) for reducing the total travel time \( \Delta f_1 = f_1^D - f_1^E = T_1 - T_2 \). That is, the cost expended to reduce travel time by one unit in moving from network D to network E is:

\[
\frac{(B_2 - B_1)}{(T_1 - T_2)}.
\]

Given this information, the decision maker need only decide whether or not he is willing to accept this level of cost per unit of time reduction. This also permits the easy assessment of the sensitivity of benefit-cost ratios for alternative plans with respect to the assumed value of travel time.

Such trade-off information is also useful in assessing priorities for various system improvements.

If each efficient point is related to the network design, it can be seen what links are in the solution. Scanning the selected networks (for example, of efficient points A, B, C, D, E, F) to identify links which are com-
mon to all of these solutions may suggest a reasonable priority ordering for construction of link improvements. A link or links which are common to all or almost all and in the solution for the lower budget levels are good candidates for implementation first. Then those links in the higher budget solution but not already implemented would be considered, and so on. Some deviation from optimality in each period is to be expected, but both judgement and additional runs of the models can be used to help select the proper priority.

**Multiple (More Than Two) Objectives**

The extension of the approach presented above to additional dimensions presents no conceptual problems. Thus, objectives such as minimization of vehicle-miles of travel (related to air pollution emissions), minimization of land required for rights-of-way, etc., could easily be introduced into this framework.

However, the displaying of the trade-offs becomes much more complicated when more than two objectives are involved. In such cases, two different approaches may be taken. The first is that trade-off curves (efficient frontiers) could be developed for each pair of objectives, without considering other objectives. This information (i.e., two dimensional trade-off curves) should assist the decision maker in understanding some of the feasible alternatives open to him and their impacts on different objectives. The other is the generation of trade-off curves between two objectives, holding the level of achievement of the other objectives fixed through use of the constraints. Both of these merit further discussion.
Independent Trade-off Curves

Trade-off curves have been developed for the 8 node, 11 link problem described above, considering four objectives. These are: (1) minimize total travel time, (2) minimize total construction cost, (3) minimize total vehicle-miles (an indicator of air pollutants emissions), and (4) minimize total number of dwelling units taken (an indicator of social disruption). (The details of this example are presented in Agarwal (1973)).

Thus there are six possible trade-off curves, each of two dimensions. These are generated considering only the two objectives to be portrayed. The results are shown in Figures 6-2 through 6-7. Note that, in general, the efficient points of one trade-off curve do not correspond to the same solutions as on other trade-off curves.

These curves present the feasible set of solutions and should be quite valuable in guiding the search for the best alternatives. Once a region of desired or attractive solutions has been identified, that is, a region of combinations of achievement of the various objectives, then that information can be used to guide the selection of better weights for the objective function. Also, if certain levels of achievement of some objectives are considered essential, then these could be included in the constraint set. This process would continue until sufficient information on the options and impacts of interest has been generated.

Tri-Objective Trade-Off Curves

The alternative approach is one which enables the presentation of three objectives graphically. An example is shown in Figure 6-8. Here the trade-off between travel time and capital cost is assessed for various fixed levels of achievement of a noise objective: meet various levels of noise
Figure 6-3. Efficient Frontier of Vehicle Hours Versus Houses Taken.
Figure 6-4. Efficient Frontier Vehicle Miles Versus Construction Cost.
Figure 6-5. Efficient Frontier Vehicle Miles Versus Houses Taken.
Figure 6-6. Efficient Frontier Vehicle Miles Versus Vehicle Hours.
Figure 6-7. Efficient Frontier Construction Cost Versus Houses Taken.
Figure 6-8. Hypothetical Example of Three Objective Trade-off Curves.
standards. As would be expected, more stringent noise standards can be met only with greater capital cost or greater travel time, or a combination of both. This information would be used in essentially the same manner as discussed above for other types of trade-off information.

**Conclusion**

It should be recognized that the number of efficient point solutions can become quite large as the number of variables in the problem becomes large. Thus, constructing the set of all efficient point solutions can become costly unless an extremely efficient algorithm is used. Instead, it is desirable to employ a structured procedure which will assist the decision-maker(s) in proceeding rapidly toward the best alternative. Of course, this is not an easy task, for the decision-maker in general will not know his relative weights for (or importance of) different objectives, nor will he in general have any minimum acceptable level of achievement of his objectives. Some recently developed methods for interacting with decision-makers are discussed below.

**Interactive Methods**

To overcome difficulties of two dimensional trade-off curves, some interactive, multi-objective programming techniques can be used. These approaches do not assume a global objective, but rather require the decision maker to provide his local weights, or relative values of different objectives, in the neighborhood of a feasible alternative. These relative values are used in a local objective function for a mathematical programming algorithm to generate an optimal solution for that objective. The decision-maker then has an opportunity to provide new values, which again serve as in-
puts to the algorithm. This process continues until the decision maker no longer wishes to revise his weights, and so an optimal solution is reached.

By nature, these approaches are iterative and always progress toward the best solution, which is undefined initially but is discovered as the process progresses. There are two steps in every iteration: a calculation phase and a decision making phase. There are various methods for getting the information from the decision maker and for ordering the steps. This leads to many different approaches to interactive, mathematical programming for solving multi-objective programming problems; a good summary of these may be found in Roy (1971) and MacCrimmon (1968).

Two of the most recent approaches within the mathematical programming framework are (1) Geoffrion's interactive approach to a multi-objective decision problem (Geoffrion, et. al., 1972), and (2) the STEM method developed by Benayoun, et. al. (1971). Though both approaches involve an interactive procedure to reach the most satisfactory solution, they are quite different in their details and assumptions about knowledge of the decision-maker(s) utility function.

Geoffrion's Interactive Approach to Multi-objective Decision Making

Geoffrion considers the case of a linear programming problem such as ours, with many objective functions. The preference function of the decision maker is only implicitly known, and therefore it is assumed that the decision maker can only provide specific kinds of information about it: marginal substitution rates (or "trade-offs" or weights) between the ith criterion (objective) and a fixed reference criterion, say the list, for all objectives, at any solution (feasible or allowable). This is equivalent to assuming that, for a given solution to the network design problem, having a
total travel time of A hours, and a capital cost of B dollars, the decision
maker is able to state that he would be willing to spend r additional dollars
to save s additional hours of travel time. From this information the direc-
tion of change in which his utility increases most rapidly can be determined.
Then a mathematical programming method is used to determine a direction of
change which is allowable in terms of the model constraints. The model is
solved to give the decision maker a range of solutions which move in his de-
sired direction, and the decision maker his most preferred solution. He is
then again asked for his marginal substitution rate or trade-off values at
the new solution. The process continues with the definition of a new direc-
tion of change, and so on. The process is terminated either when the de-
cision maker is satisfied with the solution achieved at any stage or when
the solution remains unchanged with the new trade-off values.

The STEM Approach of Benyoun, et. al.

In the STEM method, the procedure begins first by finding extreme so-
lutions for each objective function considered independently from the others,
from which a pay-off table is constructed. These are ideal solutions from
the standpoint of each objective taken independently.

The process is iterative.

In each iteration a compromise solution is calculated which is the pos-
sible problem solution with objective function values "nearest" to that of
the ideal values. In this calculation, "weights" are introduced to define
the relative importance of the distances to the ideal solution. The weights
depend upon the relative importance of the objectives known in advance and
are influenced by the values of the objectives in the pay-off table. In the
decision phase, the ideal solution and the compromise solution are shown to
the decision maker. Comparing them, he decides if the compromise solution is satisfactory; if it is, the compromise is the solution required and the procedure terminates. Otherwise he must accept relaxations of those criteria for which he is satisfied to improve the values of others. He then indicates that criterion and the maximum amount of relaxation he can accept. Then the method returns to the calculation phase for the next iteration. With the new information, new weights are determined and a new compomise solution is proposed.

The mathematical details of these approaches are far beyond the scope of this report. Agarwal (1973) has applied these approaches to the design of transportation networks, showing that, through the use of such interactive, multi-objective programming approaches, good network designs can be generated and selected by decision makers with considerable efficiency. It was found to be possible to generate network designs, acceptable to the decision makers, in three to four iterations.

Conclusions

Thus there are many alternative strategies for dealing with multiple objectives in the network design model. Some rather straightforward approaches, using trade-off relationships, were presented and discussed in detail. Even these will make a substantial contribution to improving the objectiveness of transportation planning decisions. Finally, some promising new methods of interaction were discussed.

References


CHAPTER 7
SOLUTION METHODS AND COMPUTATIONAL EXPERIENCE

The potential use of any mathematical model is at least partly determined by the efficiency with which the solution of the model can be obtained. In this chapter, different approaches to solve the continuous form of the network design model are briefly presented and computational experience is given. The mathematical details are given in Appendix F.

The simplest approach to the solution of this model is to treat it as a regular linear program. Other approaches involve exploiting the special structure of the model. In this chapter, then, the solution of the network design model is attempted by three methods: (a) regular linear program using the OPTIMA code, (b) Dantzig-Wolfe decomposition, and (c) a new price-directive algorithm called BOXSTEP method.

Regular Linear Programming Solution

To solve the network design model as a large linear program, the OPTIMA linear programming code was used. This code was developed by Control Data Corporation for use on their computers. The example test problem was an aggregated highway network for Sioux Falls, South Dakota, having 24 nodes and 38 links. (This and the other test networks referred to in this chapter are described in Appendix A.) The linear program for this network had 667 rows and 1,938 variables. The OPTIMA code solved this problem on a CDC 6400 computer in 14 minutes and 16 seconds (central processor time). The reasons that OPTIMA took so long were that OPTIMA was written several years ago and does not have some of the more modern efficient, matrix and GUB (generalized upper bound) constraint approaches. Also, the size of the problem required

7-1
that extended core storage be used. Therefore, a search was initiated for more efficient ways to solve the network design model. An obvious approach was to exploit the block angular structure of the model as shown in Figure 7-1 using the Dantzig-Wolfe decomposition algorithm.

The network design model (see Figure 7-1) is an angular system and decomposes into two subproblems and one master problem. The matrices A (constraints set I) represent the node flow conservation equations of the core model while matrices I and B (constraints set II) are the capacity and budget constraints. Constraint set III of Figure 7-1 is the flow defining constraint set in the core model, and these constraints define the master problem. The two parts I and II of the network design model are independent of each other and are bound together by the linking constraints given by III. Thus, the structure of the network design model is well-suited for using the Dantzig-Wolfe decomposition algorithm.

The price paid for this decomposition is that the master program and subprogram may have to be solved many times. First the master program is solved, and from its solution, objective functions are generated for each of the subprograms. Then these are solved, and from their solution new columns are generated to be added to the master program. The process is then repeated until, after a finite number of cycles, an optimality test is passed (Dantzig, 1963).

The subprogram I of the network design problem is a shortest route problem. Thus solving this amounts to finding the shortest routes for the flow from each node to each destination using the costs given by the master problem. Therefore, subproblem I can be solved by any efficient shortest route algorithm. One of the most efficient algorithms is one due to Hu (1968) which determines the shortest chains between all pairs of nodes in a
Figure 7-1. Graphic Structure of the Core Model.
network. Since the constraint matrix and objective function are the same for each destination s, it is only necessary to store the constraint matrix and objective function once.

Subproblem II will be referred to as the "capacity problem" since it contains all of the capacity constraints. Note that it also contains the budget constraint. Although this problem is a linear program, it can be formulated as a "knapsack" problem with a closed-form solution.

The master problem is also a linear program and therefore can be solved by any conventional linear programming code. The details of this are given in Appendix F.

Computational Experience with the Dantzig-Wolfe Decomposition Algorithm

The test problem used to apply the Dantzig-Wolfe decomposition algorithm is the same 24 node, 38 link network which was solved using the OPTIMA code. The computational experience, while quite limited, has been rather discouraging.

In the first run both subproblems were solved as linear programs. The computer run was stopped after 300 seconds. The total number of iterations performed in 300 seconds was 28. The value of the objective function for each iteration is given in Table 7-1. The best value reached in 300 seconds was 325.71. The optimum is known to be 86.67 from OPTIMA.

In the second run, subproblem I was solved as a shortest route problem using the Hu algorithm (1968). Moreover, to reduce the number of artificial variables in the initial simplex tableau of the master problem subproblem II (the capacity problem) was solved heuristically in the first iteration. Thus a feasible heuristic column was generated from subproblem II which matched
Table 7-1. Dantzig-Wolfe Solution of 24 Node, 38 Link Network

<table>
<thead>
<tr>
<th>Iteration Number</th>
<th>Objective Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7400.000000</td>
</tr>
<tr>
<td>2</td>
<td>1210.326229</td>
</tr>
<tr>
<td>3</td>
<td>1160.016015</td>
</tr>
<tr>
<td>4</td>
<td>1088.057648</td>
</tr>
<tr>
<td>5</td>
<td>943.041778</td>
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<td>6</td>
<td>821.789317</td>
</tr>
<tr>
<td>7</td>
<td>770.651116</td>
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<tr>
<td>8</td>
<td>754.298926</td>
</tr>
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<td>9</td>
<td>737.174855</td>
</tr>
<tr>
<td>10</td>
<td>721.193376</td>
</tr>
<tr>
<td>11</td>
<td>703.274887</td>
</tr>
<tr>
<td>12</td>
<td>677.370424</td>
</tr>
<tr>
<td>13</td>
<td>660.190020</td>
</tr>
<tr>
<td>14</td>
<td>642.673250</td>
</tr>
<tr>
<td>15</td>
<td>636.153311</td>
</tr>
<tr>
<td>16</td>
<td>621.886082</td>
</tr>
<tr>
<td>17</td>
<td>608.391024</td>
</tr>
<tr>
<td>18</td>
<td>582.212026</td>
</tr>
<tr>
<td>19</td>
<td>556.420137</td>
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<td>21</td>
<td>499.384162</td>
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<td>24</td>
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<tr>
<td>25</td>
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<td>27</td>
<td>357.468809</td>
</tr>
<tr>
<td>28</td>
<td>325.710596</td>
</tr>
</tbody>
</table>
the column from the shortest route problem for as many rows as possible. The number of artificial variables was 14 for this case. This time the run was made for 400 seconds. Again the time limit of 400 seconds was selected arbitrarily. The best value of the objective function reached was 307.26 at the 40th iteration. The value of the objective function at each iteration is given in Table 7-2.

The final run was made by modifying the heuristic technique to generate the starting solution of subproblem II. This was achieved by renumbering the links in the ascending order of construction costs. It reduced the number of artificial variables in the master problem to zero. This time the run was made for 150 seconds. But the results were again very discouraging. There was no change in the value of the objective function even after 23 iterations (150 seconds).

All computations were performed using the SEXOP linear programming code (Marsten, 1972) on a CDC 6400 computer.

Sometimes it is much easier to maximize the Lagrangian function, which is the objective function of the master problem of Dantzig-Wolfe with the dualized coupling constraints in it, than to solve Dantzig-Wolfe linear program. This is known as the GLM (Generalized Lagrange Multiplier) approach.

Therefore, the approach of maximizing the Lagrangian L(u) was tried but did not work well. The test problem was again a network design problem but of smaller size. The number of nodes and links were 12 and 18 respectively. A Grinold-type steepest ascent algorithm (Grinold, 1972) "jammed" at about 50.96. With Grinold's primal/dual step-size rule the steps became very short very quickly. The optimal step size rule climbed higher but eventually suffered the same fate—it appeared to be converging to the value 50.96. The maximum was at 56.65 as described later. Therefore, the procedure was ter-
Table 7-2. Dantzig-Wolfe Solution of 24 Node, 38 Link Network

<table>
<thead>
<tr>
<th>Iteration Number</th>
<th>Objective Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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</tr>
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<td>2822.474925</td>
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<td>466.073270</td>
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<td>307.723872</td>
</tr>
<tr>
<td>40</td>
<td>307.264883</td>
</tr>
</tbody>
</table>
minated after several hundred seconds.

Therefore we started looking for other ways to solve the network design problem efficiently. This led to the development of a new strategy called BOXSTEP for large scale mathematical programs (Hogan et al., 1973).

**BOXSTEP Strategy for the Network Design Problem**

It was discovered that a local version of Lagrangian problems could be solved much more quickly than Lagrangian itself. This simple idea suggested the following. First maximize the Lagrangian up to a point before it slows down. Then put a box of chosen size around that point and solve the local box problem. If the solution lies in the interior of the box, then this solution is the globally optimal dual solution. If solution of the local box problem lies on the boundary of the box, then a new box can be placed at that solution and the process repeated. The main drawback of this algorithm is the overlap between successive boxes. This drawback can be reduced by determining an optimal step size in the direction determined by ascent method. Then the box is centered at this new point.

**Computational Experience with BOXSTEP**

The test problem is a network design problem with 12 nodes and 18 links. The first subproblem was solved using a shortest route algorithm. Subproblem II was solved as a continuous knapsack problem in closed form.

A line search (one-dimensional maximization) was performed between successive boxes. This was done with an adaptation of Fisher and Shapiro's efficient method for concave piecewise linear functions (Fisher and Shapiro, 1972). BOXSTEP was implemented using the SEXOP linear programming package (Marsten, 1972).
Table 7-3 summarizes the results of the test problem. The problem was run with several different box sizes. In all cases the problem was run until the optimum was achieved. The optimum value of the objective function was 56.65. Each run was started at the same point—a heuristically determined solution using dual values $u^0 = c^a$ (where the $c^a$ are the average costs of travel on network links). For each box size $\beta$ the columns headed $N(\beta)$ give the average number of constraints generated per box. Notice that this number increases monotonically as the box size increases. For a fixed box size, the number of constraints generated per box did not appear to increase systematically as we approached the global optimum. The column headed time gives the total computation time (cp), in seconds, for a CDC 6400.

The large box ($\beta = 1000$) is equivalent to using the Dantzig-Wolfe algorithm on the problem. The smallest box ($\beta = .1$) produces an ascent that is close to being steepest ascent. A pure steepest ascent algorithm, as proposed by Grinold (1972), was tried on this problem as described earlier. The poor performance of steepest ascent was consistent with our poor results for very small boxes.

The BOXSTEP algorithm was also used to solve the 24 node and 38 link network problem but the results were again rather discouraging. It took about 150 seconds to solve a box of very small size. Therefore, the approach was dropped for this network after solving several boxes.

Note that the computational time of the algorithm BOXSTEP depends on the starting dual values $u$ associated with the coupling constraints $Ax \leq b$. In BOXSTEP, the total computational burden to solve a local problem involves the functional evaluations of the Lagrangian Function and solving the linear programming problem $N$ times, where $N$ is the number of constraints generated within the box. In cases where evaluations of $L(u)$ can be made easily but
Table 7-3. Solution of Test Problem (12 Nodes, 18 Links) by BOXSTEP with Varying Box Sizes

<table>
<thead>
<tr>
<th>( \delta ) (box size)</th>
<th>No. of Boxes Required</th>
<th>( \bar{N}(\delta) )</th>
<th>Time (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>34</td>
<td>12.7</td>
<td>172</td>
</tr>
<tr>
<td>0.5</td>
<td>18</td>
<td>14.2</td>
<td>118</td>
</tr>
<tr>
<td>1.0</td>
<td>13</td>
<td>17.1</td>
<td>104</td>
</tr>
<tr>
<td>2.0</td>
<td>9</td>
<td>17.7</td>
<td>88</td>
</tr>
<tr>
<td>3.0</td>
<td>6</td>
<td>25.0</td>
<td>99</td>
</tr>
<tr>
<td>4.0</td>
<td>4</td>
<td>26.8</td>
<td>76</td>
</tr>
<tr>
<td>5.0</td>
<td>5</td>
<td>33.4</td>
<td>134</td>
</tr>
<tr>
<td>6.0</td>
<td>4</td>
<td>34.3</td>
<td>115</td>
</tr>
<tr>
<td>7.0</td>
<td>3</td>
<td>38.0</td>
<td>119</td>
</tr>
<tr>
<td>20.0</td>
<td>2</td>
<td>67.5</td>
<td>203</td>
</tr>
<tr>
<td>25.0</td>
<td>2</td>
<td>74.0</td>
<td>243</td>
</tr>
<tr>
<td>30.0</td>
<td>1</td>
<td>74.0</td>
<td>128</td>
</tr>
<tr>
<td>1000.0</td>
<td>1</td>
<td>97.0</td>
<td>217</td>
</tr>
</tbody>
</table>
solving the linear program is computationally expensive, an initial good approximation to the optimal dual variables is desired. This will reduce the number of boxes to be solved. The network design problem has this characteristic, namely, evaluations of \( L(u) \) can be made easily since evaluating \( L(u) \) involves solving the shortest route and continuous knapsack problems, whereas the linear program is computationally expensive.

To capitalize on this characteristic, the approach we used was based upon a technique, "the relaxation method" (Motzking and Schoenberg, 1954), for the problem of finding a feasible solution for a system of linear inequalities which was a good approximation to the optimal dual variables. We then solved the network design problem of 12 nodes and 18 links, described earlier, and the results have been encouraging. We were able to reduce the total computational time for this problem by 50%. A box size of 2.0 was used for this run. The results are summarized below.

1. Previous Solution

Start BOXSTEP algorithm at \( u = c^a \).

<table>
<thead>
<tr>
<th>( \beta ) (box size)</th>
<th>no. of boxes required</th>
<th>( \bar{N}(\beta) )</th>
<th>total time (sec.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>9</td>
<td>17.7</td>
<td>88</td>
</tr>
</tbody>
</table>

2. BOXSTEP with initial approximation of the optimal dual variables using the relaxation method.

Starting \( L_B = 5.99 \), \( U_B = 60.00 \), \( = 1.0 \)

Start heuristic method at \( u = c^a \).

\( L(u) \) before entering BOXSTEP = 28.84

<table>
<thead>
<tr>
<th>( \beta ) (box size)</th>
<th>no. of boxes required</th>
<th>( \bar{N}(\beta) )</th>
<th>total time (sec.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>4</td>
<td>17.5</td>
<td>44</td>
</tr>
</tbody>
</table>

Therefore this appears to be useful heuristic to be employed before entering BOXSTEP. For a smaller network design problem of 8 nodes and 11 links,
the computational time was 11.524 seconds while the regular linear program-
ming code (SEXOP) took 26 seconds to solve it.

Conclusions

Though our computational experience with the network design problem and
various solution approaches is quite limited, some general observations can
still be made. It appears that Dantzig-Wolfe decomposition algorithm is not
suitable to the network design problem because of the number of coupling
constraints. There are as many coupling constraints as the number of links
in the network. Similarly, the BOXSTEP algorithm has also not worked well
for large networks on out CDC 6400 computer. Again, a large number of con-
straints have to be generated to solve one box.

Therefore, it appears, at the present time, that the best algorithmic
approach is a dedicated LP code which capitalizes heavily on the structure
of the matrix which is extremely sparse and whose non-zero entries are almost
all +1 or -1. The only non +1 or -1 entries are in the matrix B of Figure
7.1. The dedicated LP code should be developed in a subroutine form so that
it can easily be combined with the standard transportation planning package.
Although we have not developed this dedicated LP code we have generated the
necessary theory for its construction. This theory is given in Appendix J.
A good systems programmer with some knowledge of linear programming could
code this dedicated algorithm and also generate the extensive input-output
subroutines needed for effective implementation.

References

Agarwal, S. K. (1973). Optimization Techniques for the Interactive De-
sign of Transportation Networks Under Multiple Objectives, Ph.D. Dissertation,
Department of Civil Engineering, Northwestern University, Evanston, Illinois.


CHAPTER 8

CONTEXTS AND STRATEGIES FOR APPLICATION OF THE NETWORK DESIGN MODEL

Introduction

The network design model is most appropriate for use in applications where a relatively small number of links and nodes can be used to describe the network. This is because, as the number of links and nodes increases, the time and cost required to solve the model increase rather rapidly. Since the model has been developed to make the transportation planning process more efficient, it is not reasonable to propose applications to the design of very large, dense networks commonly used in traffic assignment today.

This orientation leads to the specification of two general classes of contexts in which the application of the model is most promising. These are the sketch planning, or aggregate planning context, and the treatment of small scale, disaggregate planning problems. The first area has been the focus of most of the discussion in previous chapters and will be elaborated on in the next section of this chapter. Subsequent sections will introduce the small scale application, and will present specific methods for using the network design model in both of these contexts.

An extremely important consideration is the manner in which the network design model would be used in the context of transportation planning, for this will to a large degree determine whether or not the use of the model is advantageous and improves the effectiveness of the planning process. There are two important issues related to strategies for the use of the model. The first is its relationship to the other models and data used in transport planning--matters lying entirely within the planning activity itself. The second issue is concerned with strategies for use of the model...
within a broader planning context—that including the decision-making process and interaction with political leaders and community participation in planning. These will also be discussed.

**Sketch Planning Applications**

Applied to macroscopic network planning, the network design model should be a useful tool for exploring alternative forms of aggregate networks in terms of their implications for user costs, capital costs, and social and environmental impacts. For use of the model in this sketch planning context, it is necessary to aggregate the network from the more detailed form used in other transportation planning models. Also, since the model requires fixed origin-destination demands as input, important questions relate to how these are to be generated.

A more general question is that of how the network design model would be used in the overall context of transportation planning. The model is particularly well-suited for use in a context of interaction between planners, political leaders and decision-makers, and community groups. As discussed in Chapter 3, this is perhaps the most important potential use of the model. This possibility also raises questions regarding strategies for the use of the model.

Strategies for dealing with these questions are presented in the following sections.

**Strategies for Use of the Model**

There are many ways in which the network design model can be used as a sketch planning tool. It might be used as simply a replacement for the current procedures of sketch planning. In most studies, these are largely ad
hoc methods, which suffer from a lack of comprehensiveness, objectivity, and reproducability. A few studies have developed formal methods for sketch planning, but these seems to be limited to consideration of criteria related to only user and capital costs.

One strategy for using the network design model would be simply to replace the existing methods by this model. This would take the general form shown in Figure 8-1. The model would require inputs largely identical to existing sketch planning procedures, in the form of goals, a composite network, and an estimate of future demand (presumably based upon a future land use pattern). The model would be used to generate information on possible networks and their associated impacts. This information could be generated and used by the planners in the manner described in Chapter 6, on the basis of which a set of networks which appear promising would be selected. This selection presumably would be on the basis of a plan having a potentially desirable set of impacts relative to many other options, and on the planner's judgement as to the likelihood of acceptance of the plan.

Once such potentially good or "best" plans are identified, then they would be subject to more detailed testing, refinement and evaluation. This would be accomplished using the more common and detailed urban transportation planning methods (such as the so-called BPR package).

Interaction with the political leaders and decision-makers would occur at the later stages of planning. Some surely is necessary at the network planning level, and of course at the project design level. The same is the case with community participations in planning, which must be extensive at the project level and is increasing at the network or system level.

This strategy for use of the model--as simply a substitute for existing methods--does not take advantage of the model's value in greater community
Figure 8-1. Network Design Model for Sketch Planning with Interaction at Later Stages.
involvement in transportation planning. Unlike most other sketch planning methods, the network design model explicitly considers a broad range of impacts of transportation system changes and the distribution of these impacts. Furthermore, the information is required and generated at an aggregate level, resulting in relatively small amounts of general information upon which discussion can center. The simple nature of this information, and the light it sheds on impacts and trade-offs, should facilitate effective interaction. The model is thus well-suited to serve as one of the tools to be used at the early stages of interaction with political leaders and community groups.

By involving such individuals and groups at early stages in the planning process, it is more likely that the alternatives selected in the sketch planning stage will be ones which will have acceptable impacts when they are subjected to more detailed analysis at later stages. Thus it is more likely that the alternatives considered at each stage will include one which the political decision making process will find attractive and select as the plan. The earlier the decision makers and affected groups are involved, the more quickly the important impacts and critical issues will be identified and the more responsive to these the planning process can become. This should make the process much more effective.

The use of the network design model in such an interactive manner is shown in Figure 8-2. The relationship of the model and its use to other stages of planning is essentially unchanged. The difference is that in this strategy the information generated by the model on various achievable combinations of impacts is used as a basis for preliminary evaluation of sketch plans by political decision makers and communities. Such evaluations are then used as a basis for further exploration of alternatives, of the general characteristics or with the general combination of impacts which seem most
Figure 8-2. Use of the Network Design Model in Interactive Sketch Planning.
preferable, as discussed in Chapter 6. This process would continue until it is concluded that further exploration would yield no more useful alternatives or information.

It is recommended that the network design model--when used as a sketch planning tool--be used in an interactive strategy. While this is likely to best utilize the model's features and result in the largest potential benefit in planning, it also can be used in a non-interactive manner.

Network Aggregation and Disaggregation

In order to use the network design model for sketch planning, it is essential that the network be much more aggregate than that used in the more conventional urban transportation network models. Otherwise the speed and efficiency of the model, which are essential in sketch planning, would be lost. It will be most efficient to aggregate the more detailed description of the existing and committed network so that it can be represented with about 50 to 200 links. It has been found that such aggregation is possible, using the planner's judgmental processes to select collections of smaller, arterial streets to be represented by a single link. The single link would have a capacity approximately equal to the combined capacity of the original links, and the aggregate link travel time-volume curve would be obtained by summing the volumes on the original links for each travel time (or speed). It is well-known that even the detailed networks used today for traffic assignment represent abstractions of the real road network. To use the design model efficiently, it is necessary to carry this abstraction process somewhat further.

It would be desirable, of course, to identify links of a particularly important nature, and to treat them in a disaggregate sense within the
model. For example, proposed freeway additions, and even existing freeways, might best be included directly in the model. Where the number of such links requiring more detailed treatment is large, it may be necessary to eliminate some of the less important, intermediate nodes so that the actual number of links used in the model is kept small.

Probably the best strategy to follow in this aggregation process is to first attempt to aggregate the original set of traffic generating nodes to a more manageable number. Again, this would be accomplished by a judgemental process, using criteria related to propinquity and similarity of function. Next, these nodes would be connected by an aggregated network, which would be similar to the "spider networks" typically used in transportation planning today. Care would be taken, of course, to retain the clear identity of those links which are judged to be of special interest in the planning process.

Experience during the course of this project suggests that network aggregation should present no serious problems, although this conclusion must be subject to further tests. The network originally used in planning for Sioux Falls, South Dakota (population 83,684), which contained over 100 nodes and over 700 links, was aggregated to 24 nodes and 38 links for use in the model. The results of trial solutions were reasonable for use in area-wide sketch planning.

The purpose of sketch planning is to assist in the making of macroscopic decisions regarding network structure (link locations) and the general performance characteristics of links (capacity, level-of-service). Precision is neither necessary nor desirable at this level of analysis. The objective is to search efficiently through a large set of networks in order to identify a relatively smaller number of candidate networks for more de-
tailed analysis. Selection of the candidate networks is accomplished by optimizing the objective function subject to the constraint set used in the particular model run.

It must be emphasized that, in this planning context, more detailed analysis of the candidate networks generated by the design model will be required. The model itself will not replace existing approaches to transportation planning, but will merely provide them with much more promising starting points. For example, demand forecasting will be a necessary step prior to use of the network design model. Furthermore, it will be desirable to develop the candidate, aggregate networks prepared by the model at the level of detail used in traffic assignment at the present time. These detailed versions would then be subjected to assignment testing at the very least. When one of the candidates is selected for final analysis, it may also be appropriate to recycle through the demand forecasting process (at least trip distribution and assignment) prior to reaching any firm conclusions regarding its attractiveness.

The process of disaggregating, or detailing, a network produced by the design model would be similar in character to the aggregation process. Engineering judgment would be required to convert the level-of-service specifications produced by the model into feasible, functional design parameters of facilities. The strategy for applying the network design model to sketch planning, then, would both begin and end with disaggregated networks, and it would be the role of the planner to relate these to an aggregate network.

**Estimator of Travel Demands**

In a given application context, one of the most important strategic choices to be made is the selection of a demand matrix for use in the net-
work design model. Demand estimates represent the driving variables in the model, for the linear program attempts to satisfy all demands at minimum costs, subject to some additional constraints such as those related to the budget and impacts.

The difficulty associated with selecting a demand set arises from the fact that, in the model, the structure of the highway network is determined by the demand, while it is well known that demand itself is greatly affected by the characteristics of the highway and the transit network. Thus, in reality there are two simultaneous, interacting relationships; yet, given the complexity of the network design model and the current state-of-the-art in demand (including mode choice) forecasting, these relationships cannot be specified and solved together. For example, no reliable single equation demand model is available for use as a constraint in the network design model, an approach which would solve this problem.

This difficulty is also faced by the contemporary approaches to transportation planning, and here it is met—when it is met—through an iterative solution strategy. That is, demand is estimated, based on some assumptions about the modal network characteristics, and then that network is tested in the context of the demand. Levels-of-service on the network are re-evaluated, and are then used as the basis for making a revised demand and mode choice estimate. This process should continue until it closes, that is, until the series of demand (or level-of-service) estimates begins to stabilize.

This inter-relationship between network structure and demand needs special consideration in the use of the network design model, since the networks developed by the model itself are likely to be even more directly sensitive to demand estimates than those produced by the heuristic process.
currently used in transportation planning. Ideally, this problem should probably be treated through the regular use of an iterative planning strategy, such as that shown in Figure 8-3. Here, the network design model is initiated using current estimates of travel demand, along with a specification of the existing, committed, and proposed highway network. The design model would produce a set of desirable road network improvements; adding these improvements to the network, the process would be recycled, and a new demand estimate (generation, distribution, and mode choice) would be developed. This new demand estimate would be based upon the highway network and a transit network, the latter being a best guess at that stage in the planning process as to the transit network. The fact that demand would change over time due to factors other than those related to the transport system would be taken into account in the land use forecasts and in parameters of the demand models (generation, distribution, mode choice), which would reflect regional growth, etc. Growth estimates would not be introduced once all of the design year growth had been introduced. The criterion for stopping the network design iteration process would be stability in the characteristics of the optimal highway network. Once a desirable network had been determined in this manner, the planning process would proceed to more detailed testing and evaluation.

An obvious problem with this approach is the relatively high costs associated with the iterative process. In particular, re-estimation of demand would be expensive and time consuming if accomplished on a disaggregate basis, and probably unreliable if accomplished at an aggregate level. More seriously, resources spent in the iterative process might be taken from sensitivity and parametric studies using the design model, which would reduce the advantages of the model. On the other hand, this approach increases
Figure 8-3. Incremental Growth Demand Estimation Strategy.
demand incrementally, and so the network would "grow" incrementally, reducing the possibility of the network "blowing up" to an unreasonably large size.

An approach which might increase the rate of closure on a stable solution is shown in Figure 8-4. Here the network design model is given the same network inputs, along with a land use and demand estimate made assuming that all possible highway link improvements were built and assuming a "most likely" transit system. The stopping criterion in this approach is again stability in the optimal network configurations. Closure would probably be rapid because the demand estimate, which would probably be quite high, would likely be best satisfied in the design model by something very close to the maximum network (all improvements built), unless the budget constraint prevented this. The unfortunate result of this approach, then, might be to lead the planner toward unnecessarily large and costly networks and, subsequently to the realization of excessive travel demands.

At the other extreme, a minimal network design could be developed using the strategy proposed in Figure 8-3 without recycling to update the demand estimate. That is, only current demand would be utilized. This approach would be inexpensive, but it would also be unrealistic, and it would very likely lead to unacceptably low levels-of-service in a short time, unless very extensive transit improvements were contemplated.

A more attractive strategy is shown in Figure 8-5. In this case, the initial network design model run uses a demand estimate for the design year based on the existing and committed highway network and the "most likely" transit network. The optimal highway network produced by the model would then serve as the basis for re-estimating design year demand. This is quite similar to the approach used in transportation planning today. Intuitively, it seems likely to over-design the network, but the magnitude of the error
Figure 8-4. Maximum Improvement Demand Estimation Strategy.
Figure 8-5. Recommended Demand Estimation Strategy
would probably be smaller than would be the case with the strategy shown in Figure 8-4. This approach could be made more efficient by developing a software module which would automatically update the initial (existing and committed) network based on the improvements selected by the design model. This would avoid hand processing, and the revised network could be put directly into the travel demand forecasting package. If it were desirable to explore more than one transit network assumption, it would not be difficult to follow this procedure on three or four such networks.

A less costly, but less realistic, alternative to the above strategy would be to use the approach described in Figure 8-5 without recycling through the demand estimation process. This would probably lead to an underestimate of demand, which is characteristic of current planning processes which forego this iteration, but the magnitude of the error may be acceptable. The latter would be true if a strong, continuing transportation planning process were conducted.

Some version of Figure 8-5, then, is the recommended strategy for applying the network design model. The efficacy of this approach, however, and the nature of the biases inherent in it, can only be determined through operational testing of the design model. In particular, a field laboratory approach, in which the model is applied in parallel with a more traditional approach to network planning, would be extremely helpful in both assessing the model and in formulating strategies for its use.

**Extensions to Multi-Modal Planning**

It seems quite important to consider all modes even at the sketch planning level, in order to relieve any modal biases which might otherwise enter transportation planning. Public transportation has played a very important
role in larger cities, and its role there as well as in smaller communities is likely to increase in importance. Therefore extensions of the highway network design model to include public transit were considered and are discussed in detail in Appendix I. This preliminary investigation indicated that a multi-modal model is feasible, and it would be extremely desirable to make operational and test such a model, for it would increase the usefulness of the overall sketch planning process and also avoid some of the difficulties with demand and mode choice discussed above.

**Disaggregate Network Planning**

There are a number of transportation network planning problems for which the complete network, including all significant links and nodes, is small enough to permit full representation within the design model. In such cases, the model can play an even greater role in the planning process.

**Possible Applications**

One of these situations may be the planning of a transportation system for a smaller city, or for a new town. Here, adequate representation of the full network may often be achieved with less than 200 links. Furthermore, because the relationship between network capacity and travel demands may lead to few congestion problems, detailed urban transportation planning, in the contemporary sense, may be unnecessary. Instead, detailed issues may be well-treated through the application of standard traffic operations analysis procedures. In such cases, placing a major portion of the burden in the planning process on the network design model should eventually result in lower planning costs, while at the same time assuring that a wide variety of alternative networks and their impacts are given thorough consideration. It
should be recognized, of course, that, in its current state, the model still requires further operational testing, so that initial applications may require some additional resources for model implementation.

Another opportunity for full scale use of the network design model would be for planning small portions of larger networks. These contexts might include central business district traffic planning, perhaps for the design of traffic restraint schemes. Here, extensive use might be made of the environmental impact measures discussed in Chapter 5. Similar applications might be made to the planning of traffic systems for areas undergoing redevelopment. In each of these cases, it would be necessary, of course, to abstract the connections between the small network under investigation and the larger network which surrounds it; a similar aggregation task would have to be accomplished for traffic flows between the study area and its immediate environment.

An additional application area where the full, relevant network might be included in the design model is transportation planning for a developing region. In this case, the real network may be very sparse if it were to include only primary roadways. In most developing areas, of course, commodity transportation needs may be far more important than passenger flows. It is not difficult, however, to operate the network design model with truck traffic estimates in place of autos. In fact, extended versions of the model could be used for designing railroad, pipeline, and waterway networks as well. Preliminary investigations of urban multimodal design models (Appendix I) suggest that it is also feasible to use the model to establish the optimal mix among modes. An example of the application of the network design model to planning for a developing region is shown in Appendix G.

An alternative application would be state-wide or corridor transporta-
tion planning. Again, the number of nodes and links is likely to be such that the model could be used to model essentially the entire network of interest. For highway planning, the existing model could be used, although it might be desirable to add commodity flows explicitly with a separate demand matrix and explicit treatment of truck movements. A multiple mode model, along the lines of the highway and transit model described in Appendix I, would be appropriate for multiple-mode planning for intercity travel. A more complex model, but one which includes more detailed mode choice and cost sub-models is given in Morlok et. al. (1970).

Models of this sort, applied at the state-wide or corridor level, would be responsive to many of the emerging public issues. These include the need for more intercity freeways, the optimal distribution of freight traffic between rail, truck and water carriers, the optimal mix of air, rail, bus and auto travel in high density intercity corridors, the need for inter-modal coordination to achieve transport efficiency, to name but a few. Included in these are questions of public investment, pricing, subsidies and regulatory constraints.

The General Process

A planning process appropriate for full-scale network applications of the design model is shown in Figure 8-6. There is no need, of course, for network aggregation and disaggregation, although there may be a need to give special consideration to the interfaces with any surrounding network components. The initial network would be based on existing and committed links, as well as any candidate links which the planner wishes to consider. In cases where a small part of a larger network is being planned, it may be appropriate to use only a portion of the average daily traffic demands. For
Figure 8-6. Full-Scale Application of the Network Design Model.
example, in CBD planning, the primary concern is likely to focus on the peak period, and demands occurring during these hours should be utilized in the model. This would require some adjustment of the parameters in the travel time-volume relationships, since heretofore these have been expressed in terms of ADT volumes. Again, the nature of the outputs of the network design model should increase the effectiveness with which the planner interacts with decision makers.

Conclusions

Thus there are many possible applications of the network design model; for both sketch planning and use as a full-scale planning tool. In all the applications discussed, its inclusion of a broad spectrum of impacts, its efficiency in solution, and the wealth of information on alternatives and trade-offs in objective attainment, can be used to advantage. Recommendations for operational testing of the model in any one of these contexts are discussed, along with other recommendations, in the following chapter.

References

CHAPTER 9
RECOMMENDATIONS FOR IMPLEMENTATION AND EXTENSION

Potentials of the Model

The experience gained in the development of the versions of the network design model described in this report suggests that the potential contributions of such models to transportation planning are most promising. At the simplest level, the design model should facilitate the efficient investigation of a large number of alternative network improvement plans in the earliest stages of the planning process. By broadening the preliminary search effort, the network design model can help to assure that at least some very good networks will be considered. The present, ad hoc methods used to generate broad scale network alternatives give no guarantee that such alternatives will be found; in fact, because manual processes discourage the consideration of more than a few networks, except at great costs, using them reduces the likelihood of finding the better networks.

Perhaps more significant than the increased efficiency of the search process brought about by network design models is the potential they offer for considering a variety of impacts of alternative networks at the sketch planning stage. This was discussed in Chapter 5. To the extent that the macroscopic networks identified for further study in initial explorations are selected in terms of their social and environmental impacts, as well as in terms of economic and technological feasibility, it should be possible to reduce the conflicts which occur in the more detailed stages of transportation planning. For example if the design models can help to select networks which are acceptable in terms of their noise and air pollution impacts, these two issues should be less controversial in detailed corridor and functional
planning. The impact dimensions described in Chapter 5 represent only selected examples of the ultimate potential of the network design model.

Finally, the most important impact of the model is expected to come from its facility for supporting interactions between the planner and decision makers. This stems from the fact that the network design model is structured to provide extremely useful, and understandable, information on the impact trade-offs among alternative networks. For example, it is easy and inexpensive to use the model to explore trade-offs between total transportation costs and noise impacts, air pollution, accessibility, and the taking of valued community facilities. While this information could be developed using more traditional transportation planning tools, the network design model can produce such trade-offs as a regular feature of its computational output, with little additional cost in manpower and computer time. In this exploration of trade-offs, the model can be of direct use not only for identifying networks which are promising under a given set of policies, but it can also support the evaluation of the policies themselves. For example, the impact of alternative transportation budget levels can be determined using ranging analysis on the budget constraint. The cost implications of alternative noise standards could be determined through variations in the noise constraints used in the model.

**Strategies for Implementation**

As it stands currently, the network design model is not ready for routine use in transportation planning processes. First, computer solution times using off-the-shelf linear programming codes are relatively long. Furthermore, the use of such codes requires skills not typically available in an operational planning agency. Thus, there is a clear need for the de-
velopment of a simplified, user-oriented software package; this package should be based on the most efficient solution algorithm available.

Once a software package suitable for routine use is available, implementation of the network design model should be relatively easy. However, it is well-known that software development can be a costly undertaking; in addition, the cost of such a program would be increased by the need to create an efficient solution algorithm.

A desirable alternative approach to model implementation is to pursue a careful program of testing and evaluation prior to making a commitment to software development. The testing program would serve to "shake-down" the model and its various features, and to prove the efficacy of its use in transportation planning. There might be considerable value in such advanced proof-testing, for its results would provide stronger support for a decision to invest resources in software development. To test the model in its current form, of course, would require the commitment of sufficient computer time, as well as the availability of persons skilled in the application of large scale linear programming codes. In the context of an operational transportation planning effort, however, the additional resources required to exercise the network design model would be relatively modest.

The process of implementing the model would probably be made more effective, and the resulting model package more responsive to the needs, if such a developmental program were carried out prior to software development. This preferred approach is discussed further in the next section.

Design of a Testing and Evaluation Program

The proposed testing program should be constructed to fulfill the following purposes:
1. Determine the usefulness of the network designs produced by the model, particularly in comparison with the designs produced by contemporary, ad hoc methods.

2. Determine the feasibility of applying all of the features of the network design model, including impact sensitivity components, ranging analysis, and multi-objective trade-off studies, in a single application.

3. Determine the compatibility of the model with a typical transportation planning study, including the relative ease with which professional planners can interact with the model, the problems associated with preparing input data, and difficulties associated with interpreting outputs.

4. Determine the relative importance of model solution times as they influence its potential use in the planning process, as well as further exploring the relationship between solution times and network size.

5. Assessing the overall impact of the network design model on the transportation planning process, including its ability to introduce impact considerations early in that process.

A promising way in which to achieve these purposes is to apply the network design model in the context of an operational transportation planning process. Two or three applications opportunities should be sought, and the initial trial should focus on one of these; as experience is developed, the additional applications should be added to the experimental program in sequence. In this way, learning from the first effort can be used to structure the later applications, so that the overall program of testing and evaluation meets the needs defined above.

It would be desirable to design at least the first application so that the network design model is exercised in parallel with, rather than as a replacement for, traditional sketch planning tools. This fail-safe approach would protect the viability of the operational planning process, as well as offering the opportunity to compare the products of the model with the results of contemporary techniques.

The selected applications environments should represent the better
planning processes underway, rather than situations in which outside help is obviously needed. The planning problems should be challenging, but the resources and the procedures in the target agency should be of such quality as to offer the model a reasonable chance for success, and a high probability of acceptance. The latter point suggests that the network design model should be applied to situations in which the planning staff and decision makers exhibit significant a priori interest in its results.

The specific context should be one in which a network abstraction having on the order of 200 links or less would be reasonable. This suggests an application to a city in the population range of 500,000 or less, or to a study of a problem described in terms of part of a larger scale network. The possibility of an initial application to a statewide or regional development study should also be considered. Statewide transportation planning methods are improving rapidly, and a number of states are involved in the application of advanced techniques in this context. The propensity to consider innovative methods should be sought out as a sign that the agency might have an interest in experiments with the network design model. An advantage of a statewide test is that many state highway networks could be reasonably described with a relatively small number of links. The role that the states are playing in continuing national transportation needs studies offers an attractive opportunity for testing the network design model.

To implement these experiments, target agencies would require some additional resources for applying the model. In particular, the services of professionals skilled in large-scale linear programming applications would be required, although only two to three man-months would be sufficient for an initial test. Furthermore, technical support would be needed to prepare input data for the design model, and to facilitate computer runs. Much of this
support should be available from the technical staffs of the target agencies, with only a small amount of additional aid required from external sources. The final requirement would be computer time, available on a relatively large and efficient machine (e.g., a CDC 6600). It may be appropriate to use government-owned computers, either near the test site, or in the Washington, D.C. area.

Selection of the initial testing sites should begin as soon as possible. Because local agency cooperation is essential to the success of these experiments, the preliminary set of candidate agencies might be constructed from among those local agencies expressing a willingness to participate. To solicit expressions of interests, written descriptions of the model, including this report and other materials provided to the Federal Highway Administration, should be given wide circulation. Oral briefings might also be held in Washington and at other regional centers around the country. The final choice of target agencies should be made from those wishing to be involved, on the basis of the appropriateness of the network design model for use in those candidate planning processes.

Development of a Dedicated Algorithm

The experience gained with solution methods for the network design model, documented in Chapter 7 and Appendix J, suggests that substantial improvements in the efficiency of solution can be achieved by the development of a special, dedicated algorithm for solving the model. Such an algorithm would take advantage of the unique structure of the network design problem (i.e., the structure of the matrix is extremely sparse and the non-zero entries are almost +1 or -1) to permit the solution of large-scale networks with very little computer time and storage. Previous experience with the
development of such algorithms for other problems has been very promising, although it would be unreasonable to speculate on the likely improvement in efficiency for the network design model at the present time. For routine application of the model, however, a dedicated linear programming code is felt to be essential.

It is recommended that the development of such an algorithm be delayed until the testing program has been substantially completed. The results of that program would provide a strong basis for preparing specifications for the algorithm, and this should insure the efficacy of its use in the future. Algorithm development should be accomplished by an organization with proven skills and experience in the area of operations research applications. Care must be taken to insure that the software created to solve the algorithm is compatible with the other elements of the contemporary transportation planning package, as well as with the nature of computer hardware typically available to local planning agencies.

With proper care in algorithm development, it seems feasible to program the network design model as a subroutine which could then be packaged in a suitable way for use in transportation planning, without requiring the user agency to apply any special skills in the area of linear programming.

**Dissemination to User Agencies**

The primary efforts toward dissemination of the results of this overall research and development process should take place after the testing and evaluation program and the software development efforts have been completed. Then, the network design model, with appropriate documentation, could be embedded in the standard transportation planning package and the educational processes which support it.
Prior to that, however, the Federal Highway Administration should also circulate, through its own information channels, special seminars, and the open literature, more general descriptions of the model. This should be done to encourage the early assembly of comments, criticisms, and suggestions which will be useful both in testing and in software preparation. Of course, such preliminary information will also be helpful in identifying target agencies for the testing program.

Extensions of the Network Design Model

A variety of extensions to the model, as it now stands, hold considerable promise for the future. For example, the possibility of adding additional dimensions of impact sensitivity represents an attractive and feasible avenue for improvement of the design model. This could be accomplished during the testing and evaluation program, but it is recommended that the latter effort be restricted to demonstrating the current model, for purposes of efficiency. But, as the model comes into more widespread use, with the support of flexible software, it should be possible to expand its impact sensitivity considerably.

A version of the model applicable to transit network planning was also formulated as a part of this project, as described in Appendix I. The transit model, however, is quite preliminary in its current form, and it is considerably more complex than the highway model. With the availability of additional resources, however, it should be possible to bring the transit model up to the level of the model reported in Chapter 4, from which point a testing and evaluation program might be initiated.

Combining the transit and highway models into a multimodal network design model would then be conceptually feasible. The availability of the
latter type of model should bring about a very substantial improvement in
the state-of-the-art of urban transportation planning. The primary diffi-
culty associated with this extension, however, is the non-availability of a
mode choice model simple enough for inclusion within the network design model
structure. This would require a mode choice formulation which could be
linearized in a single equation. While such models are now in the formative
stage, there is insufficient experience with their application, and not much
is known about their validity. The interest in such simplified approaches
to mode choice forecasting is such that, as the transit and multi-modal
models evolve, the state-of-the-art of mode choice forecasting can be ex-
pected to grow to meet the need.

Similarly, the inclusion of a total travel demand estimation package
within the network design model would be an attractive improvement. As dis-
cussed in Chapter 8, the interdependencies of travel on network level-of-
service, and vice versa, suggest the desirability of marrying these two
modeling processes. Again, the difficulty in achieving this extension lies
in demand forecasting methods. Single equation, "direct" demand models have
been formulated and tested, but the state-of-the-art in this area is not yet
sufficiently strong to support a variable-demand network design model. Such
a development should, however, be possible within the next decade.

Within a much shorter time frame, the adaptation of the versions of
the network design model described in this report for use in an interactive
graphics mode should be considered. This would permit planners and decision
makers to interact with the model more effectively. In particular, a graphics
terminal could be used for "real-time" display of trade-offs between alter-
native networks in several dimensions. Three-dimensional trade-off surfaces
could easily be depicted, and with more than three dimensions of importance,
a series of two-dimensional trade-offs could be displayed. Interactive graphics would also be useful for directly relating the outputs of the design model to the network maps which they represent. This would facilitate understanding the information in more realistic terms than would be possible through the use of numerical data alone. Given the availability of appropriate computer hardware, the development of an interactive graphics capability can be accomplished in a very short time period.