establish rather strong doubts concerning both the statistical method and the theoretical usefulness of the concept.

We will address ourselves accordingly in the following discussion primarily to three fundamental questions: (1) What is the theoretical foundation of the elasticity of substitution? (2) Can we devise a method of analyzing the data which is likely to disclose the numerical value of the theoretical concept? (3) Assuming we knew the true value of the elasticity, how would it be interpreted and used? In most of what follows, we shall have time series in mind. While cross-section estimation will be treated explicitly later in the chapter, it should be emphasized that many of the points to be made concerning time series apply equally to cross sections.

THEORETICAL FOUNDATION

The elasticity of substitution is rigorously defined with respect to movement along a single indifference curve. Such a situation is depicted in Figure 3.1, with II as the indifference curve, AB as the original price line, and A'B' as the final price line.

In this case the elasticity may be estimated by

\[ e = \frac{\Delta q_1}{q_1} \times \frac{\Delta (p_1/p_2)}{p_1/p_2} = q_1q_2^{-1} - q_1q_2^{-1} \times \frac{A'/B' - A/B}{A/B} \]

In general, the value of \( e \) will depend on the particular indifference curve that is selected, as well as on the value of \( p_1/p_2 \). Such a situation is cumbersome from a theoretical point of view and, as will be shown, can be crippling to any empirical work. It will behoove us, therefore, to determine the conditions under which the value of the elasticity of substitution depends on the value of \( p_1/p_2 \) alone. The proper requirement suggests itself immediately: the slope of the indifference curve must depend on \( q_1/q_2 \) and not on any scale factor. Another way of saying this is that the income elasticities of the two goods are identical.

For the sake of realism, we should take other goods besides \( q_1 \) and \( q_2 \) into account. The additional requirements affecting \( e \) in such an event are

4 See Morrisett [17] for a historical sketch and careful theoretical discussion of the concept. Another source of insight is Morgan and Corlett [16].

5 See Morrisett [17] for a proof. Linear homogeneous functions have this property. Monotone transformations of linear homogeneous functions do as well.
THEORETICAL FOUNDATION

stancy to result not from explicit assumption but rather from ignoring the problems surrounding the choice of functional form on the basis of the general relationship

\[ \frac{q_1}{q_2} = f\left(\frac{p_1}{p_2}\right) \]  \hspace{1cm} (3.2)

Besides the fact that Equation (3.1) is open to criticism for ignoring the problem of functional form, an important additional criticism concerns the implicit assumption that \( q_1/q_2 \) is dependent only on \( p_1/p_2 \), which requires the rather strong symmetry assumptions discussed above.

Thus suppose for the moment that the symmetry assumptions do not hold, yet we persist in running the regression suggested in Equation (3.1). As evident from Figure 3.2, the resulting estimate need bear no relationship to the theoretical concept. That is, a fall in the price of \( q_1 \), indicated by the shift from the price line \( AB \) to \( AB' \), has resulted in a fall in \( q_1/q_2 \) to \( q_1'/q_2' \). The measured elasticity of substitution would thus turn out to be positive rather than negative in this instance and would provide no insight into the true elasticity of substitution defined along the indifference curve.\(^4\)

\(^3\) To put this in another way, one can think of the indifference curves in Figure 3.1 drawn with the other variables held constant. The condition for \( e \) to depend on \( p_1/p_2 \) alone is then that the indifference map appears the same for all choices of the other variables.

\(^4\) See Stern and Zupnick (22, pp. 484-86) for a similar demonstration using partial equilibrium demand and supply schedules.
To this point our discussion has implicitly assumed that the importing country has a well-behaved indifference map. In view, however, of the well-known conceptual difficulties involved in such community indifference maps, it may be more fruitful to examine the elasticity of substitution in the framework of conventional demand analysis. Let us write the following export-demand functions

\[ q_1 = f(p_1, p_2, y, p_a) \quad \text{and} \quad q_2 = g(p_1, p_2, y, p_a) \]  
(3.3)

where \( y \) is money income in the importing country and \( p_a \) is the general price level in this country of commodities other than 1 and 2, including perhaps competing imports. For purposes of simplification we will assume constant-elasticity approximations to (3.3)

\[ q_1 = a p_1^\alpha p_2^\beta y^\gamma p_a^{\alpha_0} \quad \text{and} \quad q_2 = b p_1^{\beta_1} p_2^{\beta_2} y^{\delta_1} p_a^{\beta_0} \]  
(3.4)

where the \( \alpha \)'s and \( \beta \)'s refer to the elasticities of the respective variables. We can then write

\[ q_1 = \frac{a}{b} p_1^{\alpha - \beta} p_2^{\beta - \beta} y^{\gamma - \beta} p_a^{\alpha_0 - \beta_0} \]  
(3.5)

The elasticity of substitution may now be conveniently defined holding money income \( y \) and other prices \( p_a \) constant.

It should be evident from Equation (3.5) that \( q_1/q_2 \) will be functionally related to \( p_1/p_2 \) only if the exponents of the price variables are equal

\[ e = \alpha_1 - \beta_1 = \beta_2 - \alpha_2 \]

or

\[ \alpha_1 + \alpha_2 = \beta_1 + \beta_2 \]  
(3.6)

Equation (3.6) asserts that the sum of the direct and cross elasticities of demand be the same for each commodity. This is quite similar to the symmetry conditions discussed earlier in connection with the utility analysis, and the same conclusion holds. Commodities \( q_1 \) and \( q_2 \) must be quite similar but not too similar.

It will be further evident from Equation (3.5) that there are two variables, \( y \) and \( p_a \), that do not appear in the regression equation (3.1). This is justifiable only when \( \alpha_1 = \beta_1 \) and \( \alpha_2 = \beta_2 \), that is, when the income elasticities of each commodity are comparable and when the cross-price elasticities with respect to other goods are also comparable.

The points just made are essentially empirical problems, and it would seem advisable to test their validity in a regression of the form

\[ \log \left( \frac{q_1}{q_2} \right) = a + b_1 \log p_1 + b_2 \log p_2 + c \log y + d \log p_a \]  
(3.7)

The hypothesis represented by Equation (3.6) would then be examined by testing whether \( b_1 = -b_2 \). Similarly, \( \alpha_2 = \beta_2 \) and \( \alpha_1 = \beta_1 \) could be examined by testing whether \( c = 0 \) and \( d = 0 \).

The question arises again as to what happens if we insist on regressions of the form (3.1) when the condition (3.6) is unwarranted. The answer is that the concept of the elasticity of substitution as a demand phenomenon degenerates since the observed value depends on the particular paths taken by the individual prices. Suppose, for example, that \( \alpha_1 - \beta_1 = -1.0, \beta_1 - \alpha_2 = -0.1, \Delta p_1/p_1 = 0.9, \) and \( \Delta p_2/p_2 = 1.0 \). Since \( p_1 \) has risen by a smaller percentage than \( p_2 \), the ratio \( p_1/p_2 \) has fallen, and we expect \( q_1/q_2 \) to rise. However, from Equation (3.5) we see that

\[ \frac{\Delta (q_1/q_2)}{q_1/q_2} = \frac{(\alpha_1 - \beta_1) \Delta p_1}{p_1} - (\beta_2 - \alpha_2) \frac{\Delta p_2}{p_2} \]

\[ = 1.0(0.9) - (-0.1)(1.0) \]

\[ = -0.9 + 0.1 = -0.8 \]

Contrary to what is expected, a fall in \( p_1/p_2 \) has resulted in a fall in \( q_1/q_2 \) and the observed elasticity of substitution is positive. The observed elasticity of

\[ \frac{\Delta (q_1/q_2)}{q_1/q_2} = \frac{(\alpha_1 - \beta_1) \Delta p_1}{p_1} - (\beta_2 - \alpha_2) \frac{\Delta p_2}{p_2} \]

\[ = 1.0(0.9) - (-0.1)(1.0) \]

\[ = -0.9 + 0.1 = -0.8 \]

These functions have the desirable property that if all prices and money income are multiplied by the same factor no change in demand occurs. Equation (3.5) then becomes

\[ q_1 = \alpha_1 + \beta_1 (p_1/p_2) + \gamma_1 (y/p_2) + \delta_1 (p_a/p_2) \]

\[ q_2 = \alpha_2 + \beta_2 (p_1/p_2) + \gamma_2 (y/p_2) + \delta_2 (p_a/p_2) \]

With use of this function, the elasticity of substitution depends not only on the levels of all the variables but also on the paths taken by \( p_1 \) and \( p_2 \), which will depend on supply conditions. In other words, \( q_1/q_2 \) is not functionally dependent on \( p_1/p_2 \), and the concept of the elasticity of substitution degenerates.

If the variables \( y \) and \( p_a \) are excluded, the resulting estimate of the elasticity of substitution will necessarily be inefficient, statistically speaking. It will be unbiased only if \( y \) and \( p_a \) are uncorrelated with \( p_1/p_2 \). See Morgan and Corlett [16] for some not-too-encouraging experiments with an income term.
substitution will depend in general on the particular choice of $\Delta p_1/p_1$ and $\Delta p_2/p_2$, that is, on the paths taken by the individual prices. An elasticity of substitution estimated during one period will hold for another period only if the paths of these variables are retraced.

The particular paths followed by the explanatory variables will depend, in general, on the interaction of all the other economic variables. But it may be noted that if it were possible to specify other economic relationships that completely determine the paths of the explanatory variables, the effect will be to make the elasticity of substitution unique. Supply functions come to mind immediately. For example, it may be that monetary inflations in Countries 1 and 2 behave similarly and in consequence $p_1/p_2$ follows the same path. Thus, Country 1 may undergo approximately a 6 percent rate of inflation while Country 2 suffers only a 3 percent rate consistently over time. In this case, $p_1/p_2$ would grow consistently at a 3 percent rate, and the observed elasticity of substitution would be roughly constant over the period. Such an estimate would only be useful of course if we could be confident of a continuance of the inflation rates in the two countries. But even though it is quite possible that the interaction of demand and supply will create a close relationship between $q_1/q_2$ and $p_1/p_2$, such a relationship is only a description of the time series and not an analysis useful in understanding the underlying economic forces or in predicting future events. What can be concluded from our discussion of the theoretical foundation of the elasticity of substitution is that a regression of the form

$$\log \frac{q_1}{q_2} = a + b \log \frac{p_1}{p_2}$$ (3.8)

requires the following assumptions:

(i) The algebraic sum of cross and direct elasticities of demand for the two commodities must be equal.

(ii) The income and any other price elasticities of demand for the two commodities must be equal. This implies roughly that the two commodities be alike in all economic respects except that they are not perfect substitutes. If they are perfect substitutes then $b$ becomes $-\infty$ and $p_1/p_2 = 1$ as long as some of both commodities is being sold. In this case, $\log p_1/p_2$ is a constant (as is $a$) and Equation (3.8) cannot be estimated.

The foregoing objections may be met by a regression of the form

$$\log \frac{q_1}{q_2} = a + b_1 \log p_1 + b_2 \log p_2 + c \log y + d \log p_s$$ (3.9)

which is what we suggest be used, with form (3.8) being avoided. However, form (3.9) has the disadvantage that data must be collected on income $y$ and other prices $p_s$. Inasmuch as the coefficients $c$ and $d$ are likely to be small, we may on grounds of economy drop these two terms and fit

$$\log \frac{q_1}{q_2} = a + b_1 \log p_1 + b_2 \log p_2$$ (3.10)

In effect, our preference for Equations (3.9) and (3.10) represents a rejection of the elasticity of substitution on theoretical grounds. The elasticity of substitution requirement that $b_1 = -b_2$ in these relations imposes assumptions we do not regard as suitable for a priori imposition upon the data. However, empirical tests of relationships (3.9) and (3.10) may prove that the elasticity of substitution is a useful approximation in certain contexts.

Let us now turn to the question of measurement of the elasticity of substitution. Assuming that (i) and (ii) just mentioned hold, we must inquire whether a least squares regression of the form (3.8) yields a good estimate of the true elasticity of substitution: $e = \alpha_1 - \beta_1 = \beta_2 - \alpha_2$. In the previous chapter we observed that the existence of a supply relationship biases toward zero any least squares estimate of a price elasticity of demand. This is due to the fact that the error term in the demand relationship has a positive correlation with the price term. Unfortunately, the identical simultaneity problem exists with regard to a regression of the form (3.8).

That is, a disturbance to (3.8) such as a temporary shift in demand in favor of $q_1$ will be associated with an accommodating movement of $p_1/p_2$ as

This makes the demand functions homogeneous of degree zero so that doubling all prices and money income will not change the quantities demanded. If we also assume as in (ii) in the text that

$$\alpha_y - \beta_y = \beta_s - \alpha_s = 0$$ (b)

we can subtract the two equations in (a) to get

$$\alpha_1 + \alpha_2 = \beta_1 + \beta_2$$ (c)

which is the same as Equation (3.6). Therefore (i) and (ii) in the text can be replaced by (a), the absence of money illusion, and (b), identical elasticities with respect to the other variables.
suppliers of $q_1$ raise prices to ration the available quantity, and suppliers of $q_2$ lower prices to eliminate accumulatings stocks. Just as in the case of price elasticities, the estimate of the elasticity of substitution will be biased toward zero unless supply elasticities are infinite. The size of the bias will be large accordingly when the disturbances to (3.8) are large relative to the disturbances to the supply functions. There is a presumption, however, that the elasticity of substitution relation will be more stable than the corresponding demand relation. Disturbances to one of the demand functions in (3.3) are likely to have their counterparts in disturbances to the other demand function. Accordingly, when we divide these demand functions, the one disturbance will tend to cancel out the other and the elasticity-of-substitution relation may be quite stable on the demand side. On the supply side, on the other hand, the individual disturbances reflect events in two different countries and are therefore less likely to cancel each other out. The increased stability on the demand side in the absence of the same on the supply side may therefore reduce the bias in the estimate associated with the simultaneous interaction of demand and supply.

It appears from our discussion that there are many reservations, both theoretical and statistical, concerning the concept of the elasticity of substitution. It is only natural to ask then why so much effort has been devoted to estimating it. The most obvious answer that suggests itself is that the estimated elasticities of substitution are generally more negative and more significant statistically than the estimated demand elasticities. Such results are often considered to provide better evidence of the workings of the international price mechanism. This conclusion may not be warranted, however. That is, in contrast to the typical demand elasticity that is approximated by $\alpha_1$, or $\beta_1$ (both negative), the elasticity of substitution approximates $\alpha_1 - \beta_1$, or $\beta_1 - \alpha_2$. These latter approximations are clearly more negative than $\alpha_1$ or $\beta_1$. Furthermore, when the assumption $\alpha_1 - \beta_1 = \beta_2 - \alpha_2$ is erroneous, large negative estimates may result simply due to the paths followed by $p_1$ and $p_2$.

INTERPRETATION AND USE OF RESULTS

Let us now consider the question of interpreting and using the measured values of the elasticity of substitution. Since the theory of international monetary relations and the balance-of-payments adjustment process is traditionally

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The transition from Equation (3.12) to (3.13) is not wholly acceptable. The parameters \( \alpha_1, \beta_1, \) and \( \gamma_1 \) are total elasticities describing the effect of a price change inclusive of the income effect. In contrast to this, the elasticities in (3.12) describe only the substitution effect and require a compensating variation of money income \( y \) to constrain choice to the initial indifference level. This can be made clearer by referring to the indifference map in Figure 3.3.

\[ \frac{\partial u}{\partial q_1} \text{ and } \frac{\partial u}{\partial q_2} \]

Solving (d) and (e) with (c) yields
\[ dq_1 = Zp_1u_s dp_1 \]
\[ dq_2 = Zp_2u_s dp_1 \]

where
\[ Z = (\rho u_{11} - 2p_1p_2u_{12} + p_2^2u_{22})^{-1} \]

Therefore
\[ (p_1q_1)\alpha_1 + (p_2q_2)\beta_1 = (p_1q_1)\frac{\partial u}{\partial q_1} + (p_2q_2)\frac{\partial u}{\partial q_2} \]
\[ = p_1^2Zp_1u_{22} - p_2^2Zp_2u_{11} \]
\[ = 0 \]

\[ \alpha_1 = \frac{q_2p_2}{q_1p_1 + q_2p_2} \gamma_1 = \frac{q_1p_1 + q_2p_2}{q_1} \gamma_1 \]  \hspace{1cm} (3.15)

If commodity \( q_2 \) is a substitute for \( q_1 \), the value of \( \gamma_1 \) will be positive and
\[ \tilde{\alpha}_1 = \frac{-q_2p_2}{q_1p_1 + q_2p_2} \gamma_1 \]  \hspace{1cm} (3.16)

will be less negative than \( \alpha_1 \). Thus \( \tilde{\alpha}_1 \) will underestimate \( \alpha_1 \) in absolute value.

The foregoing approach to estimating the price elasticity of demand tends to impinge upon one's tolerance of statistical tricks. This is especially the case in view of the highly questionable nature of some of the steps involved. It may also be that the utility analysis itself is of dubious validity in this context. We could not doubt introduce other equations for the purpose of refining or altering the estimates of the direct elasticities. But the question is, why estimate these elasticities in such an indirect manner? If one is interested in the

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price elasticity of demand, would it not be better to use the more direct techniques discussed in the preceding chapter? We have already indicated at some length the numerous difficulties that arise in estimating the elasticity of substitution. This, combined with the further assumptions necessary to calculate the price elasticity from it, certainly puts a great strain on the analysis and the interpretation of results.

In view of what has just been said, what is the case, if any, that can be made for estimating the elasticity of substitution in international trade? On purely theoretical and statistical grounds, we have argued in favor of computing price elasticities directly if this is the object of the analysis. It may be, however, that the measurement of direct elasticities yields poor results in comparison with the measurement of substitution elasticities due to the reduced simultaneity bias as discussed before. It is also true that with use of this analysis the need for data other than relative quantities and prices may be obviated, with concomitant economy of operation. But the validity of these points is primarily a question of fact, which ought to be investigated by estimating both types of elasticities in a given situation and comparing the results.

Nonetheless, there are situations in which it may be useful to pose hypotheses in terms of the elasticity of substitution, in particular when interest may not center entirely on the elasticity but as well on other influences that affect export sales. One such situation might be when we wanted to explain existing trade patterns of particular exporting countries vis-à-vis another one. Such knowledge could be useful in establishing an industry’s export-price policy and in formulating government policies affecting exports. Ginsburg’s work [5], which is based upon the pooling of time-series and cross-section data, is especially interesting in this regard and therefore worth discussing briefly. It should be noted, however, that his work is subject to our criticisms noted earlier concerning the choice of explanatory variables and the separation of the price variables.

where \( q_i \) and \( q_s \) are imports into a third country from Countries 1 and 2 and \( q_{so} \) is total imports. The elasticity of \( q_i \) with respect to \( p_s \) can therefore be expressed as the sum of three terms: the elasticities of \( q_i / (q_i + q_s) \) of \( q_i + q_s / q_{so} \) and of \( q_{so} \) with respect to \( p_s \). All three can be expected to be negative and therefore the first will be less negative than the number we are after, \( \alpha_i \), the elasticity of \( q_i \) with respect to \( p_s \). Thus another lower limit estimate of \( \alpha_i \) is

\[
\alpha_i = \frac{d(q_i / (q_i + q_s))}{dp_s} \cdot \frac{p_s}{q_i / (q_i + q_s)} = \frac{d((q_i / q_s) / (q_i / q_s + 1))}{dp_s} \cdot \frac{p_s}{((q_i / q_s) / (q_i / q_s + 1))}
\]

Substituting in Equation (3.2), \( f = q_i / q_s \) yields

\[
\alpha_i = \frac{df}{df + 1} \cdot \frac{p_s}{f / (f + 1)} = (f + 1)^{-1}e = \frac{q_i}{q_i + q_s}e
\]

where \( e \) is the elasticity of substitution. This estimate closely resembles the first one in Equation (3.16).

POOLED TIME-SERIES AND CROSS-SECTION ESTIMATION

We have been proceeding as if time-series data on relative quantities and relative prices were used to estimate the elasticity of substitution. Alternatively one could use cross-section data relating to some specified period of time. The difference in the regressions can be seen as follows

\[
\log \left( \frac{q_i}{q_{so}} \right) = \alpha_i + b_i \log \left( \frac{p_i}{p_{so}} \right) \quad \text{(fixed)} \quad (3.17)
\]

\[
\log \left( \frac{q_i}{q_{so}} \right) = \alpha_i + b_i \log \left( \frac{p_i}{p_{si}} \right) \quad \text{(fixed)} \quad (3.18)
\]

where \((i = 1, \ldots, N)\) and \((t = 1, \ldots, M)\). The subscripts 1 and 2 in the quantity and price ratios are now to be interpreted as referring to Country 1 and Country 2. Equation (3.17) thus represents the time-series approach across all years for a given commodity, \((q_i / q_s)\). Equation (3.18) is the cross-section approach across all commodities for a given year. In the case of Equation (3.17), there will be \(N\) separate regressions yielding an elasticity of substitution \(b_i\) for each of the commodities. For Equation (3.18) there will be \(M\) separate regressions yielding an elasticity of substitution \(b_i\) for all of the commodities in a given year. Implicit in the cross-section approach is the assumption that the behavior of the various commodities included in the analysis is commensurable, so that \(b_i\) can be interpreted as a kind of average elasticity of substitution for all the commodities. Since the cross section should comprise only reasonably close substitutes, it is evident that we need to make the same assumptions concerning elasticities that were mentioned earlier in connection with Equation (3.10).

These two separate approaches can be logically combined. This can be done by pooling data cross sections for different time periods into a single regression equation as follows

\[
\log \left( \frac{q_i}{q_{so}} \right) = \alpha_{1i} + b_{1i} \log \left( \frac{p_i}{p_{so}} \right)
\]

\[
= (a + \alpha_i + \beta_i) + b_i \log \left( \frac{p_i}{p_{si}} \right) \quad (3.19)
\]

The elasticity \(b_i\) is assumed to vary between commodities but to be constant over time, and the level of the function \((a + \alpha_i + \beta_i)\) is assumed to vary among commodities and over time in such a way that \(\beta_i\), the change from year to year, influences all commodities identically.\(^{16}\)

\(^{16}\) This formulation is due to Ginsburg and Stern [6]. It was suggested by an analysis of covariance technique developed originally by D. B. Suits.
When the regression indicated by Equation (3.19) is actually performed, the commodity and time characteristics, \( \alpha_t \) and \( \beta_t \), are represented by dummy variables. Thus in the regression a value of one or zero is given to the commodity dummy corresponding to \( \alpha_t \), depending on whether the particular price and quantity observation comes from the \( i \)th commodity or not, and the same is true for a particular year \( t \). There will consequently be separate regression coefficients for each commodity dummy variable \( \alpha_i \), and each year \( \beta_t \). Now if \( p_{1i}/p_{2i} = 1 \), which means that the relative prices of commodity \( i \) are equal, it follows from Equation (3.19) that

\[
\log \left( \frac{q_1}{q_2} \right)_{it} = a + \alpha_i + \beta_t
\]

(3.20)

A value of \((a + \alpha_i + \beta_t)\) greater than zero signifies the extent of the "nonprice" preference of the importing country (i.e., the rest of the world) for Country 1 goods. A negative value of this sum measures the nonprice preference for Country 2 exports.

The constant term \( a \) measures the average preference of the importing country for all the commodities in all the time periods covered by the sample. Its value does not depend on a particular commodity or year. The value of the commodity variable \( a_i \) determines whether the preference for a particular commodity differs from the average preference \( a \) for all commodities. The \( \alpha_i \)’s will vary for particular commodities, depending on such factors as transport costs, quality differences, and the demand characteristics of particular import markets. The \( \beta_t \)’s measure how relative preferences vary with the time periods due to such factors as changes in world or regional incomes or changes in commercial policy.

The \( b_t \)’s in Equation (3.19) measure the price elasticities of substitution. These elasticities are constrained by the form of this equation to be constant for each commodity, but the elasticities are permitted to vary among commodities. Thus, if the \( b_t \)’s are negative, as we would hypothesize, Country 1 will experience, when its price is lower, a greater export demand than Country 2.

It should be evident that the pooling of the cross-section and time-series approaches provides a much richer analysis than either approach individually. Combining the two approaches also makes possible the assessment of the importance of qualitative variables. It is noteworthy that further experimentation with Equation (3.19) by Ginsburg [5] has shown that the results can be improved upon considerably by disaggregating the quantity and price ratios according to individual importing regions and by segmenting the price ratio into intervals in order to allow for complex curvilinearity in the relationship. It would thus appear that a great deal of interesting work can be done using a combination of cross-section and time-series data in estimating the variety of factors, including prices, which determine export ratios and market shares.

**CONCLUSION**

Our intention in this chapter has been to provide arguments for viewing with skepticism the often-measured elasticity of substitution in international trade. It was shown in particular that the commonly used estimation procedure was valid in the case of two commodities only when there was equality of the algebraic sum of cross and direct elasticities of demand and equality of the income and any other price elasticities of demand. A suggested way in principle to take these conditions into account was to regress relative quantities on separate price variables for the goods in question, income, and the prices of other goods. In case the coefficients on the latter two variables were believed to be small, a regression of relative quantities on the separate price variables (rather than on the price ratio) might be acceptable.

\[\left( \frac{q_1}{q_1 + q_2} \right)_{it} = (a + \alpha_i + \beta_i + \gamma_i) + [(b + \delta_i + \zeta_i + \theta_i) \log \left( \frac{p_1}{p_1 + \Sigma j(\lambda_j)} \right)]\]

This formulation expresses \((q_1/(q_1 + q_2))_{it} = \), Country 1’s market share of combined 1 and 2 exports to region \( r \), of commodity \( i \), in year \( t \) as a regression on \( \log (p_1/p_1 + \Sigma j(\lambda_j)) \), the relative price of the commodity in that region and year. It will be noted that the dependent variable here is expressed in terms of a market share. Observations were included only when both countries exported to a particular region, thus ruling out zero or 100 percent market share. Ginsburg is now investigating methods for taking these extreme observations into account.

Both the intercept \((a + \alpha_i + \beta_i + \gamma_i)\) and the slope \((b + \delta_i + \zeta_i + \theta_i)\) vary among commodities, years, and regions. Since the slope determines the elasticity of market shares due to the factors just mentioned, this formulation permits detailed analysis of the determinants of elasticities. The intercept measures the influence of nonprice preferences on Country 1’s market share. Since the log \((p_1/p_1 + \Sigma j(\lambda_j)) \) is zero when both prices are identical, nonprice preferences are measured by the sum of the intercept coefficients. If this sum exceeds 0.5, then Country 1 goods are favored.

The remaining price coefficients \( \lambda_1 \) measure nonlinearities in the relation between market shares and relative prices. Each \( \lambda_1 \) determines how an importer’s reactions occurring in a specified price range differ from the average within a central price range, as measured by \( \delta \). Separate elasticities can thus be calculated for particular price intervals.

\[\left( \frac{q_1}{q_1 + q_2} \right)_{it} = (a + \alpha_i + \beta_i + \gamma_i) + [(b + \delta_i + \zeta_i + \theta_i) \log \left( \frac{p_1}{p_1 + \Sigma j(\lambda_j)} \right)]\]

1 Ginsburg’s expanded version of Equation (3.19) is as follows

17 A full explanation of the use and interpretation of the dummy variables is given in Ginsburg [5].
We sought next to question the basis for preferring the measurement of the elasticity of substitution over the measurement of direct demand elasticities as evidence of the international price mechanism. It was shown that the elasticity of substitution was bound to be more negative than the direct elasticity, and that such more negative estimates might result simply from the paths followed by the prices in question. There was some reason to believe, however, that the elasticity-of-substitution relation might be more stable than the separate demand functions.

We then investigated the manner in which the direct elasticity could be derived from the measured substitution elasticities. The question here, however, was why such an indirect procedure was necessary. It may be, as stated, that the indirect approach reduces simultaneity bias and is also more economical in terms of data requirements. These are factual considerations, however, which should be investigated in their own right.

It was suggested finally that there might be particular empirical circumstances involving the export behavior of individual countries when the elasticity of substitution could be measured in conjunction with other explanatory variables. Ginsburg's work on the pooling of time series and cross sections of export behavior was shown in this regard to be potentially very fruitful.

APPENDIX TO CHAPTER 3

In order to investigate the elasticity of substitution when Equation (3.6) is not satisfied, let us differentiate Equation (3.5) to yield

\[
\frac{d \log q_1/q_2}{d \log p_1/p_2} = (\alpha_1 - \beta_1) \left( 1 - \frac{d \log p_2}{d \log p_1} \right)^{-1} + (\beta_2 - \alpha_2) \left( 1 - \frac{d \log p_1}{d \log p_2} \right)^{-1} \\
+ (\alpha_3 - \beta_3) \left( \frac{d \log p_1}{d \log y} - \frac{d \log p_2}{d \log y} \right)^{-1} \\
+ (\alpha_4 - \beta_4) \left( \frac{d \log p_1}{d \log p_n} - \frac{d \log p_2}{d \log p_n} \right)^{-1}
\]

The last two terms should not affect our estimate of the elasticity of substitution since they reflect changes in \(q_1/q_2\) due to causes other than price movements. If the \(y\) and \(p_n\) terms are included in the regression equation, the effects of changes in \(y\) and \(p_n\) on \(q_1/q_2\) will be removed via the multiple correlation technique. If they are not included, the estimated elasticity will be biased unless

\[
\alpha_y = \beta_y
\]

(a)

or

\[
\frac{d \log y}{(d \log p_1 - d \log p_2)} = \frac{d \log y}{d \log (p_1/p_2)} = 0
\]

(b)

(that is, \(y\) does not respond to \(p_1/p_2\)). A similar statement applies to the last term.

\[
|\beta_4 - \alpha_4| < |\alpha_1 - \beta_1|
\]

\[
|\beta_2 - \alpha_2| > |\alpha_1 - \beta_1|
\]

FIGURE 3.A.1

Range of Values of Elasticity of Substitution

Turning now to the first two terms, we see that the elasticity depends on \(d \log p_1/d \log p_2\), the particular path followed by the price relative. However, the elasticity will be \(\alpha_1 - \beta_1\) when \(\beta_2 - \alpha_3 = \alpha_1 - \beta_1\).
Letting
\[ Z = \frac{d \log p_1}{d \log p_2} = \frac{dp_1/p_1}{dp_2/p_2} \]
we can solve for \( e \) as
\[ e = \frac{(\alpha_1 - \beta_1)Z + (\alpha_2 - \beta_2)}{Z - 1} \]

We have, then, the elasticity of substitution as a sort of weighted average of the direct and cross elasticities of demand, with the weights depending on \( Z \), the path of the price relative. Two cases can occur and are graphed in Figure 3.A.1.

The value of \( e \) will approximate \( \alpha_1 - \beta_1 \) at the extreme values of \( Z \), that is, when the variation of \( p_1 \) dominates the variation of \( p_2 \). When \( p_1 \) varies little, \( Z \) will be near zero and \( e \) will approximate \( \beta_2 - \alpha_2 \). The reader will note that there are very distinct regions where \( e \) is positive. These regions are characterized by movements of \( p_1 \) and \( p_2 \) that are similar in magnitude and identical in sign.

REFERENCES