How Should Firms Hedge Market Risk?

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November 2013

Abstract

We consider optimal hedging decisions for a firm whose stock returns are affected by market returns and an idiosyncratic factor that is orthogonal to the market return. We show that the level of the firm’s cash flows depend on the level of the market and the level of the idiosyncratic factor multiplicatively because of compounding. Minimizing the variance of cash flows requires a substantial offsetting position in the market index. However, minimizing the costs of financial distress associated with low cash flow realizations below a threshold requires only a modest hedge against the market factor when firm debt levels are also moderate. This holds even in continuous time and with dynamic hedging policies. We clarify that using return regressions to measure economic exposure to generate optimal hedging deltas may be erroneous and we provide a simple rule of thumb for hedging market risk.

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1UCLA Anderson School. Email all correspondence to bhagwan@anderson.ucla.edu. We thank Jeremy Stein, René Stulz and Ivo Welch for many insightful conversations. We also thank seminar participants at UCLA Anderson brown-bag seminar series and at the UCLA-Lugano Finance Conference. We thank anonymous referees for many useful comments, and are deeply grateful to one who provided the analytical proof for the main result in the paper.
1. Introduction

There is an extensive literature that shows that firms can, under some circumstances, increase shareholder wealth by reducing the volatility of their cash flows. In particular, firms that face significant costs of financial distress if they experience abnormally low cash flows can decrease the present value of financial distress through hedging. In a seminal paper, Froot, Scharfstein and Stein (1993) show that firms that have to finance their investments out of their cash flows are forced to give up positive net present value projects if they experience poor cash flows. Such firms benefit from hedging because it enables them to take advantage of investment opportunities they would have to forsake or give up otherwise. A number of other reasons for why firms can benefit from decreasing total cash flow volatility have been discussed in the literature. Total cash flow volatility is a function of firm idiosyncratic volatility and of volatility induced by systematic risk. Consequently, although this point is rarely discussed, the literature implies that firms can create shareholder wealth by reducing their exposure to systematic risk. However, we do not observe firms hedging their exposure to the market either by shorting a market index or by using financial derivatives on the market index. Nor do we hear many academics or practitioners recommending that firms do that. Fischer Black pointed out this embarrassing fact many years ago.

We show that the simple intuition which would suggest that a firm with positive exposure to market movements (i.e., a positive beta) hedge by taking an offsetting position in the overall market requires careful consideration when more than one source of uncertainty affects the variability of firm’s cash flow. This is because the effects of different sources of uncertainty on a firm’s level of cash flows at a distant date in the future are multiplicative even when in stock returns over short horizons, these effects appear to be separable.

The intuition for the main insight in the paper is illustrated by the following simple example. Suppose a firm’s cash flow, say 5 years from now, is $100 on average. Suppose that the realized cash flow is determined by a factor that is idiosyncratic to the firm and also by overall market conditions. Assume that the idiosyncratic factor alone can make the realized cash flow go up or down by 60% with equal probability, and that the market factor alone can make the realized cash flow go up or down by 50% with equal probability. When both factors are present and are uncorrelated, the realized cash flows can take four different values:

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2 For instance, Smith and Stulz (1983) show that lower cash flow volatility can reduce the present value of taxes; Stulz (1984) makes the case that high cash flow volatility can make the firm’s insiders more risk-averse; using different mechanisms, Breeden and Viswanathan (1998), and Duffie and DeMarzo (1995) show that lower cash flow volatility can help outsiders in assessing the performance of firms.

3 See Bolton, Chen and Wang (2011) for an exception.

4 We learnt this in our discussions with René Stulz.
Suppose now that we short $100 of the market to hedge market risk, so that when the market goes up we lose $50, but when the market goes down we gain $50. The hedged cash flows are the following:

<table>
<thead>
<tr>
<th></th>
<th>Market up by 50%</th>
<th>Market down by 50%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Idiosyncratic factor up by 60%</td>
<td>$100x1.60x1.50=$240</td>
<td>$100x1.60x0.50=$80</td>
</tr>
<tr>
<td>Idiosyncratic factor down by 60%</td>
<td>$100x0.40x1.50=$60</td>
<td>$100x0.40x0.50=$20</td>
</tr>
</tbody>
</table>

As can be seen from the Table, hedging reduces the range of cash flows from $80-$240 to $130-$190 when the idiosyncratic factor is up. However, the worst cash flow realization deteriorates from $20 to $10 when the idiosyncratic factor is down. If the firm had debt of $15, it would be bankrupt with hedging but not if the firm did not hedge the market risk. Our paper will show that the optimal market hedge is a lot more conservative (closer to $15) than hedging $100 of market risk.

The intuition is as follows. Suppose the firm takes a short position in the market assuming the average realization of cash flow. Then, if the realized cash flow turns out to be low, the short market hedge would be excessive and in fact may lead to significant losses if the market goes up. This could be devastating. On the other hand, if the realized cash flow turns out to be high, the market hedge based on the average cash flow would be inadequate, but that is not so critical. This would induce the firm to be conservative in hedging market risk.

If the firm’s objective were to minimize the total variance of the cash flow, then the appropriate market hedge would be large because the firm wants to reduce variance both when cash flow realizations are large as well as when they are small. However, because bankruptcy is relevant only when the cash flow realizations are low, it is more important that firms reduce variance when cash flows are likely to be low, even if it increases variance when cash flows tend to be high.

That there is a tradeoff between financing and risk management is identified in Holmstrom and Tirole (2000), Mello and Parsons (2000), and Rampini and Viswanathan (2010, 2013). A dynamic model of such tradeoffs is also developed in Rampini, Sufi and Viswanathan (2013). Our results also rely on the fact that low realizations of cash flow caused by idiosyncratic shocks can make consequences of over-hedging precarious. However, we believe that the implications of the vulnerability caused by low cash flow realizations for firms’ decision not to hedge market risk aggressively using financial instruments has not been appreciated adequately. We provide a simple model to illustrate this robust intuition. In the discussion section at the end of the paper.
we provide several examples of how this has led many researchers to conclude erroneously that firm behavior and the practice of risk management seem inconsistent with commonly understood theories of risk management.

2. The Model

Consider a firm and a market index whose short-run return dynamics can be expressed as:

\[ r_t = \alpha + \beta r_t^M + \epsilon_t \]

\[ r_t^M = \alpha^M + \epsilon_t^M \]

where \( r_t \) represents stock return, \( r_t^M \) represents contemporaneous return on a market index, \( \beta \) represents exposure to systematic risk, and \( \epsilon_t \) represents the idiosyncratic component of firm’s stock return (which has a mean of 0). Suppose that \( \beta \) is positive and significant and the variance of \( \epsilon_t \) is also significant. This is a canonical example where exposure to \( r_t^M \) is significant and therefore an offsetting market hedge appears intuitively correct. Textbooks often suggest return regressions to estimate the coefficient \( \beta \), which is then argued to be the optimal hedge ratio because it minimizes the variance of returns. We will show that this is incorrect if the goal is to avoid financial distress.

To simplify, let us assume that \( \beta = 1 \). The firm’s incentive to hedge in our model will arise from its desire to avoid financial distress, and not from risk aversion. Thus all agents are risk-neutral so that the return dynamics over a finite period of, say, one year, are:

\[ e^{r_t^M} = (1 + R_t^M) = (1 + R^f)(1 + \epsilon_t^M) \]

and

\[ e^{r_t} = (1 + R_t) = (1 + R^f)(1 + \epsilon_t^M)(1 + \epsilon_t) \]

with \( E(\epsilon_t) = E(\epsilon_t^M) = 0 \), and

\[ E(R_t) = E(R_t^M) = R^f \]

where \( R^f \) is the rate for a risk-free investment. If \( r_t, r_t^M, \epsilon_t \) and \( \epsilon_t^M \) are Normally distributed, \( R_t, R_t^M, \epsilon_t \) and \( \epsilon_t^M \) (which are not the same as the instantaneous variables \( \epsilon_t \) and \( \epsilon_t^M \)) are Log-normally distributed.

Now, consider a two date model in which a firm generates one cash flow at date 1, \( C_1 \). Because we are assuming risk neutrality, the value of the firm at date 0 is:

\[ S_0 = \frac{E_0(C_1)}{1 + R^f} \]
Since \( 1 + R_1 = \frac{c_1}{s_0} \),
\[
C_1 = S_0(1 + R^f)(1 + \varepsilon_1^M) \left(1 + \varepsilon_1\right).
\]
The cash flow depends on the market risk and the idiosyncratic risk multiplicatively.

Now consider hedging the firm’s cash flow by shorting a forward contract on a market index. Consider a market index with current value \( Z_0 \). Its value at date 1
\[
Z_1 = Z_0(1 + R_1^M) = Z_0(1 + R^f)(1 + \varepsilon_1^M).
\]
The forward price at date 0 of the market index is
\[
F = Z_0(1 + R^f).
\]
If we go short one forward contract on the market index (or equivalently short the market index and invest the proceeds to earn risk-free return on the proceeds), the cash flow on date 1 will be
\[
H_1 = F - Z_1 = -Z_0(1 + R^f)\varepsilon_1^M;
\]
the expected value of the market hedge is zero, with \( H_1 \) positive if market falls and negative if market rises.

Define \( y_0 \equiv \frac{s_0}{Z_0} h_0 \) to be the number of market hedges (short the forward contracts) and \( h_0 \) the fraction of the marked risk hedged. Then the hedged cash flow for the firm is
\[
C_1 + y_0 H_1 = S_0(1 + R^f)[(1 + \varepsilon_1^M)(1 + \varepsilon_1) - h_0 \varepsilon_1^M] = E_0(C_1)[1 + \varepsilon_1 + \varepsilon_1 \varepsilon_1^M + (1 - h_0) \varepsilon_1^M].
\]
If \( B_1 \) denotes the contractual obligations to firm’s creditors or bondholders, then the firm will be bankrupt if its hedged cash flow at date 1
\[
C_1 + y_0 H_1 < B_1.
\]
The bankruptcy condition above can be rewritten as
\[
\varepsilon_1 + \varepsilon_1 \varepsilon_1^M + (1 - h_0) \varepsilon_1^M < -(1 - \varphi), \quad (\text{BC})
\]
where
\[
\varphi \equiv \frac{B_1}{E_0(C_1)} < 1
\]
represents the contractual obligations of the firm as a fraction of its expected cash flow at date 1.
The variance of the hedged cash flows depends on the variance of $\varepsilon_1 + \varepsilon_1^M + (1 - h_0)\varepsilon_1^M$. A hedge that offsets 100% of the variability in cash flows resulting from market movements is optimal when the goal is to minimize the variance of the hedged cash flows. Lemma 1 shows that a 100% hedge does indeed minimize the variance of the hedged cash flow.

**Lemma 1:** The hedge $h_0 = 1$ minimizes the variance of 
$$\varepsilon_1 + \varepsilon_1^M + (1 - h_0)\varepsilon_1^M.$$ 

**Proof:** $\varepsilon_1$ and $\varepsilon_1^M$ have means equal to zero and are independent of each other. Therefore it follows that:

$$\text{Var}(\varepsilon_1 + \varepsilon_1^M + (1 - h_0)\varepsilon_1^M) = \text{Var}(\varepsilon_1) + \text{Var}(\varepsilon_1^M) + (1 - h_0)^2\text{Var}(\varepsilon_1^M).$$

Clearly setting $h_0 = 1$ minimizes the RHS of the equation above.

However, the firm would want to minimize the variance of its cash flows only if minimizing the variance also minimizes the likelihood that its hedged cash flow will fall below a certain threshold. But this is rarely the case.

**Lemma 2:** If the firm has no idiosyncratic risk ($\varepsilon_1 = 0$) then the hedge $h_0$ where $\varphi \leq h_0 \leq 1$ can eliminate financial distress.

**Proof:** Zero idiosyncratic risk implies that $\varepsilon_1 = 0$. Setting $\varepsilon_1 = 0$ in the bankruptcy condition (BC) and then noting that:

$$(1 - h_0)\varepsilon_1^M > (1 - h_0)(-1) = -(1 - h_0) \geq -(1 - \varphi)$$

when $\varphi \leq h_0 \leq 1$ the firm will avoid bankruptcy in all states of the world.

The intuition is that the firm can minimize the likelihood of financial distress by taking an offsetting short position in the market. However, this result does not generalize when variance of idiosyncratic risk is positive and significant. In fact, choosing a conservative market hedge, i.e., $h_0 < 1$, increases the conditional variance when $\varepsilon_1$ is positive and reduces the variance when $\varepsilon_1$ is negative such that the overall variance increases yet the likelihood that the firm will face financial distress goes down.

Now posit that the firm minimizes a cost associated with financial distress\(^5\) that is proportional to the difference between its contractual obligation and its hedged cash flow in states in which it is bankrupt. Formally the firm minimizes

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\(^5\) We do not posit that the firm minimizes the probability of bankruptcy for two reasons. First, minimizing the probability of bankruptcy introduces a discontinuity when the firm is just at the boundary of bankruptcy. Second, in some states of the world when the firm’s unhedged cash
where
\[ x \equiv C_1 + y_0 H_1, \]

\( f(x) \) is the density function of hedged cash flow \( x \) and \( K \) is a scaling constant that measures the cost of financial distress.\(^6\) Normalizing all cash flow numbers by \( E_0(C_1) \), and setting \( K = 1 \), the firms’ objective function can be rewritten as:

\[
\Gamma \equiv \min_{h_0} \int_{-1}^{\infty} \int_{-1}^{\infty} \max \left[ -(1 - \varphi) - \{ \varepsilon_1 + (\varepsilon_1 + 1 - h_0) \varepsilon_1^M \} , 0 \right] f(\varepsilon_1^M) f(\varepsilon_1) d\varepsilon_1^M d\varepsilon_1.
\]

Notice that the Max in the integrand ensures that when the hedged cash flow is higher than the threshold, the bankruptcy cost is zero.

**Main Theorem:** The firm minimizes the expected cost of bankruptcy for \( h_0 = \varphi \equiv \frac{B_1}{E_0(C_1)} \).

**Proof:** See Appendix.\(^7\)

### 3. Numerical Simulations

We have shown that the optimal market hedge (when the firm’s return sensitivity to market factor is one-to-one) is equal to the level of firm’s contractual obligations as a fraction of expected cash flow. We now plot firm’s hedged cash flow for various reasonable parameter values. We assume Log-normal distributions for idiosyncratic shock \( \varepsilon_1 \) and for the systematic shock \( \varepsilon_1^M \) with means equal to zero and vary the standard deviations as well as the bankruptcy threshold \( \varphi \).

First consider \( \sigma(\varepsilon_1) = \sigma(\varepsilon_1^M) = 40\% \) - which is a reasonable estimate for a 4 year horizon - and \( \varphi = 0.3 \) which implies that the firm’s contractual obligations are about 30% of its expected cash flow \( C_1 < B_1 \), the firm may have a perverse incentive to have a *speculative* short position on the market index.

\(^6\) Notice that because we allow the hedged cash flow to become negative, we are in effect assuming that the firm has unlimited liability and thus it will honor its obligations on the short market hedge. Thus the pricing of the forward contract that assumed no default is appropriate. Assuming limited liability complicates the analysis – the derivative short position must be priced to account for default and an additional perverse incentive to hold a speculative position. This additional complexity does not lead to any additional insights that have not already been analyzed in the related papers mentioned in the introduction. Therefore we stay with the simpler formulation of unlimited liability.
flows. Figure 1 confirms that the optimal hedge that minimizes the expected cost of bankruptcy $h_0$ is equal to 30%. The chart also shows the probability of bankruptcy at various hedge levels.

Figure 2 shows the distribution of the hedged cash flow with a 100% hedge and the optimal hedge of 30%. Notice that the firm is bankrupt for values less than $-(1 - \varphi) = -0.70$. The optimal hedge creates more values in the upper tail and reduces values in the lower tail. The standard deviation, however, is higher with the optimal hedge (54%) than with a 100% hedge (43%). This confirms our result that the hedge that minimizes the bankruptcy costs or the probability of bankruptcy is not the hedge that minimizes the variance.

If we instead choose a lower $\varphi = 0.2$, the optimal hedge also reduces to 20% as seen in Figure 3. Here no hedging is not very different from the optimal hedge whereas 100% hedge is far from optimal.

In general, minimizing the probability of bankruptcy leads to a higher optimal hedge than the hedge that minimizes the cost of bankruptcy. This is because minimizing the probability of bankruptcy, in some states of the world when the firm’s unhedged cash flow $C_1 < B_1$, may provide a perverse incentive to have a speculative short position on the market index. This is seen more clearly in Figure 4 in which bankruptcy threshold is increased to $\varphi = 0.5$.

The analytical results in our paper were derived assuming beta was equal to 1. When the beta is different from one, the optimal hedge that minimizes the expected cost of bankruptcy is approximately equal to the product of $\beta$ and $\varphi$. This is seen in Figure 5 which plots firm’s hedged cash flow for beta of 0.5, 1 and 1.5 when $\varphi = 0.5$. The optimal hedge ratios are 25%, 50% and 70% respectively.

To summarize our results from numerical simulations, we find that (a) when beta is equal to one the optimal hedge that minimizes the cost of financial distress is equal to the fraction of contractual debt obligations to expected cash flows which is often smaller than 100%, a variance minimizing hedge, (b) a hedge that minimizes the probability of bankruptcy is typically higher than the hedge that minimizes the cost of bankruptcy, and (c) the optimal hedge when beta is different from one is approximately equal to $\varphi \beta$.


Hedging market risk depends crucially on why the firm wants to hedge. We have argued that if the goal of the firm is to minimize variance of its cash flows, perhaps because owner-manager is risk averse, then fully hedging market risk is appropriate. However, if the goal is to minimize the costs associated with financial distress, a more moderate hedging of market risk is prudent. A key determinant of how much market risk should be hedged is the level of firm’s contractual obligations that may trigger financial distress. For instance a firm with only 25% debt in its capital structure (a typical US manufacturing firm) should hedge market risk roughly half as much as a firm that has 50% debt in its capital structure (for instance an airline company).
Of course, the amount of market risk hedging also depends on the sensitivity of firm’s cash flows to overall market conditions. For example, firms in industries such as automobiles, retail, telecommunications, tend to have higher sensitivity to the market and therefore should hedge more than firms in industries such as food, tobacco, oil and gas, which have lower sensitivity to the market.

We suggest a simple rule of thumb for hedging market risk when the goal is to avoid financial distress. First, estimate the sensitivity of firm’s cash flows to market movements. Second, estimate firm’s fixed obligations as a proportion of expected cash flows. The optimal market hedge as a proportion of expected cash flows is the product of these two estimates.

5. Robustness Checks

Our theoretical analysis shows that because cash flow at date 1,

\[ C_1 = S_0(1 + R^f)(1 + \varepsilon_t^M)(1 + \varepsilon_1) \]

is affected by idiosyncratic shock \( \varepsilon_1 \) and market related shock \( \varepsilon_t^M \) in a multiplicative fashion, a 100% market hedge is not optimal. One might wonder if this result arises because we have imposed a requirement that the hedge is put in place at the beginning of date 0 and have not allowed the hedge to alter at a more frequent interval. We now show that allowing the hedge to change dynamically does not alter the conclusion that a 100% hedge is not optimal.

Suppose we were to subdivide the period from 0 to 1 into \( N \) sub-periods – as \( N \) gets large, we will get the approximation to continuous time. A 100% hedge at the beginning of sub-period \( t \)

\[ y_{t-1} = \frac{S_{t-1}}{Z_{t-1}} \]

would result in a hedge profit at time \( t \)

\[ H_t = -S_{t-1}(1 + r^*) \varepsilon_t^M \]

where \( (1 + r^*) = (1 + R^f)^{\frac{1}{N}} \) and \( \varepsilon_t^M \) represents the (much smaller) shock to the market for one sub-period. If \( H_t \) is invested in the risk free asset until date 1, the future value of it will be

\[
H_t (1 + r^*)^{N-t} = -S_{t-1}(1 + r^*)^{N-t+1} \varepsilon_t^M \\
= -S_0 (1 + r^*)^{t-1} \left[ \prod_{l=1}^{t-1} (1 + \varepsilon_l)(1 + \varepsilon_t^M) \right] (1 + r^*)^{N-t+1} \varepsilon_t^M \\
= -S_0 (1 + R^f) \left[ \prod_{l=1}^{t-1} (1 + \varepsilon_l)(1 + \varepsilon_t^M) \right] \varepsilon_t^M
\]
where \( \epsilon_t \) represents the (much smaller) idiosyncratic shock for one subperiod. The total hedged cash flow at date 1, then is:

\[
C_1 - S_0 (1 + R_f) \sum_{t=1}^{N} \prod_{i=1}^{t-1} (1 + \epsilon_i)(1 + \epsilon_i^M) \epsilon_t^M.
\]

Since

\[
C_1 = S_0 (1 + R_f) \prod_{i=1}^{N} (1 + \epsilon_i)(1 + \epsilon_i^M),
\]

substituting in hedged cash flow above, simplifying, and keeping only terms that have the product of at most two \( \epsilon \) terms, the hedged cash flow is equal to

\[
S_0 (1 + R_f) \sum_{t=1}^{N} \left( \epsilon_t \sum_{i=1}^{t} \epsilon_i^M \right).
\]

So even though each second order product term is small in the above expression, the number of such terms is of the order of \( \frac{N(N+1)}{2} \) and our simulations confirm that standard deviation of the hedged cash flow is of the similar order of magnitude as the yearly standard deviation of return on the market.

The second robustness check we consider is the situation in which the cash flow to the firm is also generated continuously but what the firm needs to hedge is accumulated cash flow at some date in future. Even in this case, we confirm that even though the market sensitivity of the firm’s cash flow at any given instant could be perfectly hedged by the market hedge, the fact the hedge for both near and distant cash flows must be determined in advance at date 0 precludes the possibility of completely eliminating the sensitivity to market movements.

### 6. Discussion

A long standing puzzle in the risk management literature has been why firms do not hedge their exposure to risks that appear to have a significant influence on their corporate cash flows. The obvious risk to consider is firms’ exposure to market conditions. We have argued that this is because exposure to market risks interacts multiplicatively with other idiosyncratic risks that firms face. Our intuition is that were firms to take a short position in say the market index and if the firms’ realized cash flow were to turn out to be low because of idiosyncratic factors, then precisely in those precarious low cash flow realizations, if the market index is high, firms would face enormous losses on their short hedge positions. We show that this would induce firms to be rather conservative in hedging their exposure to the market.
Our analysis clarifies that a hedge that minimizes the variance of the cash flow is not equivalent to a hedge that minimizes the costs associated with financial distress. Use of stock return regressions on various risk factors is often recommended to quantify exposure to risks and to determine optimal hedge ratios for avoiding unwanted risks. We argue that this approach has several problems. First, hedge ratios that minimize cash flow variance do not minimize the costs of financial distress. Second, stock return dynamics may already anticipate that managers will hedge the firm risk exposures and thus a regression coefficient would tend to underestimate the true exposure. Third, it may be the case that the sensitivity of firms’ stock returns to overall market returns arises not because cash flows are particularly sensitive to market movements but because the discount rates have a large common component. This would make a simple regression of stock returns on market returns show significant sensitivity, but an attempt to hedge cash flows by shorting the market may turn out to be misguided.7

Our arguments also shed some light on discussions in the risk management literature in which it is argued that firms should attempt to hedge its total economic exposure rather than focusing only on transaction exposure. Our analysis suggests that economic exposures are likely to be multiplicative and identification of these exposures using regression methods, as is often advocated, and then determining optimal hedges based on regression coefficients is likely to lead to incorrect results.

Survey evidence presented in Bodnar, Graham, Harvey and Marston (2011) indicates that the most common risks that are managed using financial instruments are interest rate risk, foreign exchange risk, energy price risk, commodity price risk and credit risk. The evidence also suggests that foreign exchange risk that is managed arises largely from transaction exposure caused by contractual commitments. We suspect that interest rate, energy price, commodity and credit risks also arise largely from known transaction exposure whose cash flow value is known in advance and therefore hedging them using financial instruments is straightforward. Although market risk is considered to be the most important concern for firms surveyed, markets risks are rarely hedged using financial instruments.

Finally, our analysis casts serious doubts on proposals that suggest that people hedge too little and should use financial markets to hedge many different types of risks including risks of housing price declines and unemployment risks (see Shiller, 2004). 8 The problem with these recommendations is that they do not appreciate that risks that affect people’s lives arise interactively. Simple financial instruments that hedge each of these risks separately may in fact leave people more vulnerable if a large cash outflow occurs on a particular hedge precisely at a time when the capacity to pay has diminished substantially because of other factors. If one could write contracts that are simultaneously contingent on several risk factors, indeed, significant risk reduction may be possible. The feasibility of complex instruments that fit this description is

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7 We thank René Stulz for discussing this insight with us.
8 We thank Jeremy Stein for discussing these ideas with us.
doubtful. Not only would this require an accurate quantification of risk exposures which are likely to be different for each individual, but also the instruments’ contingencies would have to involve variables that can be easily measured and cannot be manipulated. The possibility of misusing financial instruments to speculate rather than hedge, for personal profit at the risk of putting the organization in peril, and thus the costs of instituting internal controls and systems that can minimize or prevent such abuse, make the case for hedging with financial instruments even more tenuous. We are not surprised that hedging by firms and individuals using financial instruments is not as prevalent as some finance scholars believe it should be.
Appendix

Proof of Main Theorem:

The optimization problem is:

\[
\Gamma \equiv \min_{h_0} \int \int_{-1}^{1} \max_{-1}^{1} \left[ - (1 - \varphi) - \{\varepsilon_1 + (\varepsilon_1 + 1 - h_0)\varepsilon_1^M\}, 0 \right] f(\varepsilon_1^M)f(\varepsilon_1)d\varepsilon_1^Md\varepsilon_1
\]

The maximum function inside the integral is convex and the optimization problem is strictly convex and has a unique minimum, and therefore a local minimum is also the global minimum.

We rewrite the optimization problem as:

\[
\Gamma \equiv \min_{h_0} \int \int_{\Sigma(h_0)} \left[ - (1 - \varphi) - \{\varepsilon_1 + (\varepsilon_1 + 1 - h_0)\varepsilon_1^M\} \right] f(\varepsilon_1^M)f(\varepsilon_1)d\varepsilon_1^Md\varepsilon_1
\]

where \(\Sigma(h_0)\) is the region of integration. This region for \(\varphi \leq h_0 < 1\) is:

\[
\varepsilon_1 \leq -(1 - h_0) \quad \text{and} \quad \varepsilon_1^M \geq \frac{(1-\varphi)+\varepsilon_1}{(\varepsilon_1+1-h_0)}
\]

For \(0 < h_0 < \varphi\):

If \(\varepsilon_1 \geq -(1 - h_0)\) then \(\varepsilon_1^M \geq \frac{(1-\varphi)+\varepsilon_1}{(\varepsilon_1+1-h_0)}\)

and if \(\varepsilon_1 \leq -(1 - h_0)\) then \(\varepsilon_1^M \geq -1\).

The Leibnitz-Reynolds theorem implies that the derivative with respect to \(h_0\) is:

\[
\int \int_{\Sigma(h_0)} \frac{\partial}{\partial h_0} \left[ - (1 - \varphi) - \{\varepsilon_1 + (\varepsilon_1 + 1 - h_0)\varepsilon_1^M\} \right] f(\varepsilon_1^M)f(\varepsilon_1)d\varepsilon_1^Md\varepsilon_1
\]

because \([- (1 - \varphi) - \{\varepsilon_1 + (\varepsilon_1 + 1 - h_0)\varepsilon_1^M\}]\) along the boundary of the region of integration is zero.

For \(\varphi \leq h_0 < 1\) the integral above is:
The first term is zero and the second term is negative because
\[
\frac{\partial \varepsilon_1^M}{\partial \varepsilon_1} = \frac{h_0 - \varphi}{(\varepsilon_1 + 1 - h_0)^2} < 0 \quad \text{for} \quad h_0 < \varphi.
\]
Hence the derivative decreases for \( h_0 < \varphi \) and increases for \( h_0 > \varphi \) and is zero at \( h_0 = \varphi \).
Figure 1: The figure shows the probability of bankruptcy (left axis) and the cost of bankruptcy (right axis) for \( \sigma(e_1) = \sigma(e_1^M) = 40\%, \varphi = 0.3 \)
Figure 2: The figure shows the probability distribution with a 100% hedge and with optimal hedge for $\sigma(\varepsilon_1) = \sigma(\varepsilon_1^M) = 40\%$, $\varphi = 0.3$
Figure 3: The figure shows the probability of bankruptcy (left axis) and the cost of bankruptcy (right axis) for $\sigma(e_1) = \sigma(e_1^M) = 40\%, \varphi = 0.2$
Figure 4: The figure shows the probability of bankruptcy (left axis) and the cost of bankruptcy (right axis) for $\sigma(\epsilon_1) = \sigma(\epsilon_1^M) = 40\%$, $\varphi = 0.5$
**Figure 5:** The figure shows the cost of bankruptcy for $\sigma(\varepsilon_1) = \sigma(\varepsilon_1^H) = 40\%, \varphi = 0.5$, for different betas.
References


