Multi-market Competition in Packaged Goods: Sustaining Large Local Market Advantages with Little Product Differentiation

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Abstract

Local shares for weakly differentiated brands of packaged goods tend to be very different in U.S. markets, despite direct competition between them. For instance, it is not uncommon that two brands of fast-moving consumer goods sustain market shares of say 40% and 10% respectively in the same market despite the fact that the product they sell is physically very similar. Curiously, the same two brands may sustain market shares of 10% and 40% respectively in another market. It is shown that the stability of this surprising asymmetry can be explained from two realities of competing in packaged goods: multi-market contact, or high positioning cost (e.g., advertising costs or retailer slotting fees). Common to these explanations is the role of horizontal product differentiation. Specifically, when two products are undifferentiated, it is more likely that their observed local outputs will be very different. Surprisingly, with spatially concentrated outputs, equilibrium profits in undifferentiated categories are higher than in moderately differentiated categories. Indeed, it is shown that undifferentiated packaged-goods industries may be more profitable than industries with differentiated goods.

It is further found that manufacturers may be more profitable with than without costly retailers. This happens when positioning or distribution costs –such as slotting fees– deter competitors from seeking a “fair” share in markets where another (physically similar) brand leads. Distribution costs are more effective in this deterring role when products are undifferentiated. Finally, the results of the analysis suggest that firms selling undifferentiated goods should focus on defending their strong markets and stay away from attacking in markets where a competitor leads.

JEL Classification: L11, L15, L22, L66, M30, R12
1 Introduction

Consumer goods in the United States often lack meaningful product differentiation on attributes other than brand labels (Carpenter, Glazer, and Nakamoto 1994; Trout and Rivkin 2000). If two products are physically identical, except perhaps for brand labels, utility maximizing consumers should be relatively indifferent between them. All else equal, therefore, demand for such brands should be similar—or at least not systematically different—within and across markets.

However, the same national brand of repeat purchase goods often has very different market shares across different domestic markets, even after controlling for the influence of regional or local brands. Consider Figure 1, which shows market shares for the two largest manufacturers of brands of Mexican salsa, Campbell and Frito-Lay. These manufacturers market the Pace and Tostitos brands, respectively. Both brands originate in Texas and offer very similar products. Within and across markets, the two firms have very different shares and seem to divide the domestic U.S. market in two territories, one for each firm.\textsuperscript{1} Tostitos dominates along the East Coast, whereas Pace leads west of the Mississippi. While market-shares are clearly not constant across markets, they are in fact constant across time.\textsuperscript{2} Given any one market, and given the similarity of the two brands, the question is: How is it possible that in direct competition, these firms sustain such diverse yet persistent market divisions? Put differently, why can large local market advantages persist in the face of little product differentiation?

I present two explanations for this puzzle. First, reciprocal local market advantages, e.g., where all competitors have some strong—and accept some weak—markets, can be sustained as the outcome of multi-market competition. Second, when it is costly to position as the market leader in communication—or distribution channels, it is still possible for two physically identical products to end up in an asymmetric equilibrium, even in a single market (i.e., without reciprocity).

With both arguments, the key contingency in this paper is that with less product differentiation, asymmetries in competitive equilibria occur more often and—if they rely on orientation toward future profits—are more easily sustained. Thus, sustenance of geographic concentration of market shares should be especially expected in mature and undifferentiated product categories.

\textsuperscript{1}The pattern in Figure 1 is not exceptional. Similar patterns are observed for categories such as ground coffee, margarines, and mayonnaise.

\textsuperscript{2}This fact is illustrated by the fact that Figure 1 represents the annual averages of market shares for 1996, suggesting that the differences in share are not simply due to temporary local marketing programs.
Traditionally, geographic concentration of outputs and prices has been attributed to geographic cost differences and spatial competition (see e.g., Greenhut 1981). In this tradition, the prices and outputs of firms are affected by the location of firms through transportation cost (Anderson and de Palma 1992; Fujita, Krugman, and Venables 1999) and a central argument in this literature is that locating oneself closer to consumers creates a cost advantage. Thus, it is the transportation cost of firms that drives the spatial distribution of prices and outputs.

However, the initial location of the manufacturers whose outputs are represented in Figure 1 was similar and therefore transportation cost of firms does not seem to explain the observed data well. Rather than focusing on the transportation cost of firms, I focus on (1) the transportation cost of consumers and (2) that firms compete in multiple geographic markets.

Geographic markets are defined as areas without consumers overlap and without consumer arbitrage. In other words, I use as a defining characteristic of a geographic market that consumers do not travel from one market to the next to benefit from price differences across markets. This definition is particularly applicable in the domestic US with its discrete population centers (metropolitan areas) separated by sparsely populated space. Consumer transportation cost across such markets is often too high compared to the potential gains from traveling, especially in the context of packaged goods. Although almost entirely omitted from theoretical analysis, the “no consumer arbitrage” property of local markets is important to understanding multi-market conduct of firms. At a minimum, the opposite assumption, i.e., that firms would not seek to benefit from the de facto immobility of
consumers across markets, seems lacking as a theoretical point of departure.

Allowing firms to set different prices in different markets, I show that firms have an incentive to maintain advantages that may have grown historically in some markets and accept historical disadvantages in other markets. This incentive increases as the differentiation between products diminishes and may lead to coordination by firms across markets. Therefore, even without transportation cost arguments and even with the same types of consumers across markets, large market advantages can be sustained, especially in the face of little product differentiation.

This paper aims to contribute to a growing literature in economics and marketing about the role of geography and space. In this context, the “New Economic Geography” (Fujita, Venables, and Krugman 1999) focuses on providing answers to two fundamental questions about economic activity. These are (1) when does spatial symmetry of economic activity break, and (2) why do spatial asymmetries in economic activity persist. Because of the empirical observation of spatial concentration in packaged goods categories, I focus here on the second question. The origins and nature of spatial concentration (the first question) are pursued elsewhere.

The paper is organized as follows. The next section reviews research that suggest consumers take non-product attributes such as advertising and distribution as perceptual cues for product quality. Section 3 discusses a logit demand model. Section 4 establishes the basic relation between profits, perceived quality and prices in a single market framework. Section 5 analyses when asymmetries can be sustained in a multimarket economy even when it is costless to locally reposition from low to high perceived quality. Section 6 shows how the asymmetries can be sustained when there are significant costs to locally positioning as a high quality firm. It also focuses on the role of retailers in sustaining spatial concentration of outputs. Section 7 discusses and interprets the main results in the context of packaged goods. Section 8 concludes.

2 Local determinants of consumer quality perceptions, mind— and shelf-space.

Consumers form brand perceptions from environmental cues other than the product itself. As Keller (1993) puts it “although the judicious choice of brand identities can contribute significantly to customer-based brand equity, the primary input comes from [...] the various product, price, advertising, promotion, and distribution decisions.”
Obviously, an important impetus to quality perceptions remains the physical product itself. However, as the quote above seems to imply, perceptual advantages for packaged goods also originate in differences across brands in awareness among consumers and support in the distribution channel. Corstjens and Corstjens (1995) note that brand awareness and distribution support are frequently zero-sum “assets” to firms because of limits on consumer information processing and on retailer shelf-space. The premise of this paper is that brand awareness and distribution support such as shelf-space are used by consumers as quality cues.

For instance, Kirmani and Wright (1989) find a positive relation between advertising and expectations about product quality. It is therefore not surprising that brand awareness is often a determinant of choice, especially for low involvement decisions (Corstjens and Corstjens 1995; Bettman and Park 1980; Hoyer and Brown 1990; Park and Lessig 1981).

Simonson (1993) concludes that consumers construct preferences at the point of purchase. For packaged goods this means that preferences for different brands are often formed at the supermarket shelf. Shelf space allocations then affect choices in at least two ways. First, consumers may take large shelf space allocations of packaged goods as cues that those brands are popular in a given local market. Thus, if consumers do not acquire brand information themselves (Dickson and Sawyer 1990, Hoyer 1984), they may rely on (what they believe are) the preferences of others. Second, the spatial arrangement of products including shelf-space allocations raise brand awareness at point of purchase (Fazio, Powell, and Williams, 1989).

In sum, while consumers from different markets may face the same physical product, perceptions about the quality of these products are co-determined by local advertising and distribution strategies of firms. It is exactly the point of this paper that even if such influences on quality perceptions are small they can be of substantial consequence and profitable.

I consider two types of perceived quality. In section 4 and 5, I use a concept of perceived quality, called “historical perceived quality,” that is an endowment from the past. Its cost is sunk. An example is order-of-entry effects on top-of-mind awareness for brands or on favorable treatment by retailers (Bowman and Gatignon 1990; Robinson and Fornell 1985). In section 6 and 7, I use a source of perceived quality that can be “bought” in the form of advertising or shelf-space allocations, its defining characteristic being that it is costly. This is termed “managed perceived quality.”
3 A duopoly model of demand

Utility I use an address model of consumer demand. In this model, consumers $h$ are characterized by a position $z_h$ in a $K$-dimensional attribute space in $\mathbb{R}^K$. Whereas the consumer’s ideal point $z_h$ is unobserved, its distribution across $h$ is known. Products $i = 1, 2$ are defined by a known address $z_i \in \mathbb{R}^K$ in the attribute space. Consumers $h$ have a quadratic disutility for distance between ideal points $z_h$ and the location of products $z_i$ (d’Aspremont, Gabszewicz, and Thisse 1979). Utility for brand $i$ by household $h = 1, \ldots, N_m$ in market $m$ is given by

$$V_i(h, m) = Y_h + a_{im} - p_{im} - \frac{\mu}{2} \sum_{k=1}^{K} (z_{kh} - z_{ki})^2,$$

(1)

where $Y_h$ is income of household $h$, and $a_{im}$ is the perceived quality of a firm in given market. The attribute $a_{im}$ is common to all households in market $m$. Further, $p_{im}$ is the price of the product in market $m$. The scalar $\mu$ measures the consumer’s disutility of products being far away from his ideal point. The utility model (1) thus acknowledges the presence of household, market, and brand specific components.

Quality perceptions The local attribute $a_{im}$ represents local quality perceptions. As discussed previously, quality perceptions can either reflect historical advantages, such as order of entry effects, selective consumer learning, etc., or can be influenced by shelf-space allocations by retailers in local markets or local advertising of the brand. Alternatively, the quality perceptions $a_{im}$ can capture versions of the same product. For instance, services in the airline industry are spatially versioned, with individual firms offering much more travel flexibility in some regions than in others (see e.g., Karnani and Wernerfelt 1985). However, we focus on the first two interpretations, i.e., market reach or historical advantages.

Product positions in physical attribute space I assume that there is one physical attribute $z_i^k (K = 1)$, in addition to the quality perceptions $a_{im}$. This attribute is common to all consumers and markets. To rule out a consumer focused explanation of asymmetries, I initially assume that the location of products and consumers is symmetric around zero. Owing to the presence of the multiplier $\mu$, it can be assumed without loss in generality that the position of product 1 is given by $-\frac{1}{2}$ and of product 2 by $+\frac{1}{2}$. The difference in positions of the two products introduces horizontal differentiation in the model.

Location of consumers in physical attribute space The consumer ideal points $z_h \in \mathbb{R}$ represent
the idiosyncratic component of utility. I assume the logistic density for the location of consumers

\[ g(z) = \frac{\exp -z}{(1 + \exp -z)^2}, z \in \mathbb{R}. \]  

Expected demand Consumers choose that alternative that maximizes their utility. Expected demand of product \( i \) for \( N_m \) consumers in market \( m \) is thus obtained by integrating of the utility equation (1) over the support of product \( i \) using the consumer density of equation (2). Given the formulation of the utility function the components \( Y_h \) and \( z_h^2 \), do not affect choice (they do not affect utility maximization because they are common to both alternatives). Given the symmetric positions, the utility component \( z_i^2 \) \((i = 1, 2)\) also drops out of the utility comparisons. What remains is the interaction \( z_h z_i \) of the location of consumers and products. Thus the location of the consumers enters the utility comparison as a linear term and demand is given by a logit model.

\[ s_{im} = N_m \Pr (V_i(h, m) \geq V_j(h, m)) \]
\[ = N_m \Pr \left( z_h \leq \frac{(a_{im} - p_{im}) - (a_{jm} - p_{jm})}{\mu} \right) \]
\[ = N_m \exp \left[ \frac{(a_{im} - p_{im})}{\mu} \right] \sum_{j \neq i} \exp \left[ \frac{(a_{jm} - p_{jm})}{\mu} \right], i, j = 1, 2 \]

In this formulation, the effective degree of horizontal differentiation manifests itself as the disutility for quadratic distance between product \( i \) and the consumer’s ideal point scale parameter of the logit, \( \mu \). For convenience and because its role turns out to be largely passive, I arbitrarily scale \( N_m = 1 \).

The logit demand formulation has broad appeal in both theoretical (e.g., Anderson, de Palma, and Thisse 1992), as well as empirical work (e.g., Berry, Levinsohn, and Pakes 1995). It is noted that with a uniform distribution for \( g(z) \), the Hotelling model obtains. This does not change basic results in this paper.

Because I initially wish to separate multi-market contact effects from demand expansion, the standard model used here does not account for an outside good. This maybe justified by realizing that for mature categories such as coffee, Mexican salsas, and alike, demand expansion is small (Nijs et al 2002). Nonetheless, it is desirable to explore the robustness of our main results to the introduction of an outside good. Hence, after establishing several results with the standard model, these results will be shown to generalize to the case of demand with an outside good.
4 Perceived quality, prices and profits

4.1 A basic relation

Of initial interest is how perceived quality $a_{im}$ affects prices $p_{im}$, and profits $\pi_{im}$. Subsequently, these findings are applied to whether cross-market variation in perceptual positions $a_{im}$ is beneficial to firms, i.e., whether firm $i$’s profits $\sum_m \pi_{im}$ are positively impacted by “spatial concentration.” The latter term is defined as having markets with high and markets with low demand compared to the “fair” demand of the product. I next investigate whether (1) such concentration can be sustained as a multimarket asymmetric equilibrium, and (2) the same intuition persists when additional phenomena are included in the analytic framework.

In the competitive interaction between firms, the timing of events is as follows. Firms first position their products by simultaneously setting $a_{im}$. Conditional on positions, firms next simultaneously set prices. Marginal cost $c_{im}$ and fixed cost $K_{im}$ are initially quality-independent and fixed. Thus, for now, firms can increase perceived quality at no additional cost. This assumption gives a strong result. If firms do not wish to increase perceived quality to the highest attainable level in each market (and break spatial concentration) even when it is costless to do so, they will not do so when attacking is expensive.

In a two-product case, demands are given by

$$s_{1m} = \frac{\exp \left( \frac{(a_{1m} - p_{1m})}{\mu} \right)}{\exp \left( \frac{(a_{1m} - p_{1m})}{\mu} \right) + \exp \left( \frac{(a_{2m} - p_{2m})}{\mu} \right)},$$

(4)

with $s_{2m} = 1 - s_{1m}$ in the absence of an outside good. The profit function for firm $i$ is $\pi_{im} = (p_{im} - c_{im}) \cdot s_{im} - K_{im}$.

Given the sequence of decisions, prices are solved first. Caplin and Nalebuff (1991) have shown that a unique Bertrand-Nash equilibrium in prices exists in a demand system such as equation (4). Rearranging the f.o.c. for firm $i$

$$\frac{d\pi_{im}}{dp_{im}} = (p_{im} - c_{im}) \cdot s'_{im} + s_{im} = 0,$$

(5)

an implicit equation for the prices of interest is obtained

$$p^*_{im} - c_{im} = \frac{\mu}{1 - s_{im}}, \quad i = 1, 2.$$

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3Currently these prices are to be interpreted as market prices. Later when the case of retailers is considered, the manufacturers will set whole sale prices.

4I later consider cases where costs depend on $a_{im}$. 
These price equations are implicit because the right-hand side of the expression for the markup contains all prices, and positioning information (through $s_{im}$). Using the last equation to solve for $s_{im}$ and substituting in the profit function gives that at optimal prices

$$\pi^*_i = p^*_i - c_{im} - \mu - K_{im}. \quad (7)$$

Define the local positioning difference as $a_m \equiv a_{1m} - a_{2m}$. Two useful dependencies of local prices (and profits, given the last equation) on the positioning differential are given in the first proposition.

**Proposition 1 (Optimal Prices)**

1. The price of product 1 is increasing in $a_m$, and the price of product 2 is decreasing in $a_m$.
2. The price increase (decrease) of product 1 (product 2) is never larger than the increase in the positioning differential, i.e.,

$$0 < \frac{dp^*_1}{da_m} < 1, \text{ and } -1 < \frac{dp^*_2}{da_m} < 0.$$

**Proof:** see appendix A

This result states that prices for product 1 increase as its positioning advantage over product 2 widens. However, if one product improves its positioning, its equilibrium price does not rise in equal measure. Consumers get at least part of the utility stemming from positioning improvement. For a related result, see Anderson, de Palma and Thisse (1992), and Anderson and de Palma (2001).

**Proposition 2 (Convexity)** The optimal prices of both products, $p^*_{1m}$ and $p^*_{2m}$ are convex in their positioning difference, $a_m$.

**Proof:** see appendix A

This result states that with a widening positioning gap between the two products, the marginal effect of $a_m = a_{1m} - a_{2m}$, $m = 1, ..., M$, on prices increases. Given equation (7) this result also applies to profits.

What does this result imply for multimarket profits? First, firms can set $a_{im}$ in each market. The convexity result then implies that both firms, competing on $M$ markets, would prefer to have a distribution of uneven market-specific positions $a_m$ over an even set of product positions of $\bar{a} = \frac{1}{M} \sum a_m$ in each market.

In practical terms, the result therefore suggests the possibility that each firm is better off having some strong markets and some weak markets rather than having many “average” markets. Put a different way, both firms have an incentive to sustain uneven market-division as long as each firm
has a sufficient number of “strong” markets: a condition to be explored more fully in the coming sections. This result is a necessary—albeit insufficient—condition to sustain asymmetric equilibria in a multi-market economy.\(^5\) I now present a benchmark result in a single market.

4.2 The case of a single market

Firms can choose among two levels of perceived quality \(a_h\) and \(a_\ell\), with \(a_h > a_\ell\). Firms first set \(a_{im}\) simultaneously and subsequently choose prices. The following proposition conveys that, in a single market, both firms will end up positioning at \(a_h\).

**Proposition 3 (Single Market)** In the single market equilibrium both firms position at \(a_h\) and charge a price of \(c + 2\mu\). Profits are equal to \(\mu - K\).

**Proof:** see appendix A

That is to say, given proposition 1 both firms want to set perceived quality high. This will give both players equal shares in the market and give the optimal prices in the market. As expected, profits and prices rise in the degree of horizontal differentiation.\(^6\) In other words, if there is no effective horizontal differentiation, price competition will drive profits to zero.

I now consider how the above result can be avoided as a function of a key reality of firm competition in packaged goods, namely that firms meet in multiple markets that are characterized by absence of consumer arbitrage.

5 Sustaining historical asymmetries through multi-market contact

5.1 Spatial concentration and horizontal product differentiation

The base-scenario analyzed in this section is of two firms \(i\), two markets \(m\), and two levels of perceived quality: high \((a_h)\) and low \((a_\ell)\). Perceived quality can be set by firms in both markets at no cost. Firms each maximize multi-market profits by choosing positioning \(a_{im}\) first, and setting prices \(p_{im}\) next. In addition, in a multimarket context, it is unnecessarily restrictive to limit the analysis to one-period games. Therefore, firms are allowed to interact repeatedly in an infinite horizon game.\(^7\)

\(^5\) The sufficient conditions are the focus of the next sections.

\(^6\) Soberman (2002) shows however that in a single market, if consumers differ with respect to their awareness of products, the monotonicity of profits in differentiation may fail to hold.

\(^7\) Comparisons to the single period single market game in the previous section are thus not immediate. However, unless consumers accept the idea of each firm taking periodic turns at being the “high-quality” player, a single market repeated game will result in the same equilibrium as the single period game.
Consistent with empirical observation, e.g., Figure 1, it is assumed that there is an initial degree of spatial concentration. In the modeling framework, this translates in each firm being endowed with one market in which it is the leader in perceived quality. In the other market, the firm is perceived to be of lower quality. This initial condition is exogenous to the analysis and reflects a history that may include order of entry effects, selective consumer learning, brand awareness, shelf space allocations, etc. Arbitrarily, firm 1 is positioned at $a_h$ in market 1 while firm 2 is positioned at $a_L$. In market 2 the opposite happens. This section studies whether this initial condition is sustainable as an equilibrium.

Denote the ratio of output of product 1 to that of product 2 at optimal prices in market 1 by $\Phi \equiv s^*_1/s^*_2$. Further, denote the equilibrium profits of firm $i$ by $\pi^*_i \equiv \sum_m \pi^*_im = \pi^*_i1 + \pi^*_i2$. Given equal cost, the prices of products mirror each other across markets, i.e. $p^*_11 = p^*_22$, and $p^*_12 = p^*_21$. From the definition of the ratio of outputs, it is therefore obvious that in market 2, $s^*_12/s^*_22 = \Phi^{-1}$.

By proposition 1, $\Phi > 1$, i.e., in market 1, firm 1 is the product with the higher perceived quality, prices, and demand. Equations (6) and (7) give that

$$\pi^*_im = \frac{s^*_im}{1 - s^*_im} \cdot \mu - K \quad (8)$$

Therefore, with asymmetric positioning, multi-market profits are

$$\pi^*_i = \sum_m \pi^*_im = (\Phi + \Phi^{-1}) \mu - 2K, \quad i = 1, 2. \quad (9)$$

If repositioning is free, firm 1 is tempted in the short run to reposition from $a_L$ to $a_h$ in market 2. Because both products are then positioned at $a_h$ in market 2, the ratio of their outputs is 1. Thus, the one-time pay off of deviation for firm 1 is $\pi^d_1 = (\Phi + 1) \mu - 2K$. Given this deviation, it is easy to show that firm 2 now optimally repositions in market 1 from $a_L$ to $a_h$. Subsequently there is no profitable deviation for either firm, and hence the payoff for both firms is equal to $\pi^0_i = 2\mu - 2K$ for ever. As an implication of proposition 2, $\pi^*_i > \pi^0_i$. Thus, in order for firm 1 to resist repositioning at no cost in market 2, its valuation of future profits needs to be high enough. Specifically, firm 1 does not reposition for all discount rates $\delta$ that satisfy

$$\pi^*_i(1 + \delta + \delta^2 + \cdots) > \pi^d_1 + \pi^0_1(\delta + \delta^2 + \delta^3 + \cdots). \quad (10)$$

From this expression, $\delta$ is obtained as

$$\delta > \delta^* \equiv \frac{\pi^d_1 - \pi^*_i}{\pi^d_1 - \pi^0_1} = \frac{1}{\Phi} \quad (11)$$
Given that $\Phi > 1$ by construction, $0 < \delta^* < 1$. A lower $\delta^*$ means that even when future profits are worth little to a manager, she prefers keeping the balanced asymmetry, over taking the immediate gain of breaking it. The following proposition is proven in the appendix.

**Proposition 4 (base result)**

1. There is always a mutual profit incentive to sustain spatial concentration, i.e., $\pi^*_i > \pi^0_i$, with $\pi^*_i$ and $\pi^0_i$ the profits from asymmetric and symmetric positioning respectively.

2. The minimum discount factor that sustains spatial concentration is equal to the ratio of each firm’s smaller and larger market share, i.e., $\delta^* = \Phi^{-1}$.

3. The motivation to sustain spatial concentration decreases monotonically with the degree of product differentiation, i.e., $\partial \delta^* / \partial \mu > 0$.

**Proof:** see appendix A

The first part of the proposition states that there is always a profit incentive to sustain spatial concentration.\(^8\) This result is implied by proposition 2. Indeed, with asymmetric positioning the positioning difference in market 1 is $a_1 = a_h - a_l$, whereas in market 2 it is $-a_1$. The positioning difference when both competitors position at $a_h$ is equal to 0 in both markets. It follows from the proposition that $\pi(a_1) + \pi(-a_1) > 2\pi(0)$.

The second part of the proposition implies that existing spatial concentration is sustainable—even when breaking it is free— as long as firms value future profits sufficiently. As long as a unit of tomorrow’s profit is at least worth $\Phi^{-1} < 1$ times a unit of today’s profit, asymmetry is sustainable. The more asymmetric the outputs of the firms, the lower $\Phi^{-1}$, and the more easily spatial concentration is sustained.

The third part of the proposition states that minimal valuation of future profits needed to sustain spatial concentration, decreases as products become more similar. As the horizontal differentiation of the products diminishes, the value of $\delta^*$ tends to zero, i.e., $\lim_{\mu \to 0} \delta^* = 0$. This result basically implies that as markets are less and less differentiated, managers should be more and more interested in sustaining spatial concentration even when improving their business in weak markets is costless.

What is the role of horizontal differentiation on profits in the case of spatially concentrated equilibria? As was shown in the previous section, for a single market economy profits rise monotonically as the degree of effective horizontal differentiation increases, i.e., as $\mu$ increases. However, surprisingly

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\(^8\)In anticipation of results to come, it is therefore interesting to speculate that manufacturers might prefer the costly presence of a third party (e.g., retailers) that diminishes the short term temptation to position high in all markets and shield them from competing head on.
perhaps at first glance, this is not true in spatially concentrated equilibria of the type considered here. The following proposition states that competing in undifferentiated product categories, i.e., when \( \mu \) tends to 0, is more profitable than competing in differentiated product categories, when \( 0 < \mu < \mu_\ast \).

**Proposition 5 (differentiation)**

In a spatially concentrated equilibrium, multi-market profits are higher when products are undifferentiated than when products are moderately differentiated. Technically, \( \frac{\partial \pi^*}{\partial \mu} < 0 \) for small enough \( \mu \).

The result does not require a specific functional form of the demand specification. Without any horizontal differentiation, i.e., when \( \mu \downarrow 0 \), the prices charged by a firm are equal to \( p^* = c + a_h - a_\ell \) in the market where it leads, and \( p^* = c \) in the market where its competitor leads. The firm with the best positioning will take all demand \( (N_m = 1) \). Therefore, each firm, having one market in which it leads and one in which it lags, makes a profit of \( (p^* - c) \cdot 1 + (p^* - c) \cdot 0 = a_h - a_\ell \). With a slightly higher \( \mu \),

\[ p^* = c + a_h - a_\ell \]  

each firm has a bit less demand in its favorable market, where the margins are close to \( a_h - a_\ell \), but has a bit more demand (in fact the exact amount that it lost in the profitable segment) in its unfavorable market where the margins are close to 0. Hence total demand remains the same, but the effective margin shrinks. In sum, profits for undifferentiated categories are larger than profits in categories with “intermediate” levels of differentiation (see also next section for an example in the presence of an outside good). Profits ultimately will increase in \( \mu \) but only for \( \mu \) sufficiently large.

The intuition of the negative impact of \( \mu \) on profits is that differentiation on perceived quality is more effective when there is no actual horizontal differentiation between the products. The demand effect of positioning \( a_{im} \) is amplified (dampened) by absence (presence) of horizontal differentiation. Oddly, it is the loss of monopoly power in the strong market due to horizontal differentiation that initially causes profits to drop when \( \mu \) increases. Klemperer (1992) shows a related result, namely that firms may be less competitive when their products are the same than when they offer different products. The driving force behind his result is consumer travel cost, the role of which is not unlike that of \( \mu \) in my model.

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Footnote:

9 For the moment, I am interpreting \( \mu \) as a characteristic of consumers, that is not under control of firms. This interpretation can be made more general because \( \mu \) can alternatively be interpreted as a multiplication of the distance of the brands in the attribute space and the disutility of distance between a given brand and the consumer’s ideal point.
The result in proposition 5 suggests that the casual observation that many consumer packaged goods lack physical product differentiation may not harm profitability. Indeed, as long as there is a spatially concentrated equilibrium, intermediately differentiated product-markets are less profitable than undifferentiated product-markets.

5.2 Duopoly with outside good

The previous analysis generates a simple and unconditional result because it isolates one specific aspect of multimarket competition, namely, that spatial concentration creates local market power. However, it is likely that breaking the spatial concentration by positioning both products at $a_h$ raises category-demand. To investigate which of these effects (market power vs demand expansion) dominates, I investigate positioning and pricing decisions in the presence of an outside good. Demand is given by

$$s_{im} = N_m \cdot \frac{\exp ((a_{im} - p_{im})/\mu)}{\exp ((a_{1m} - p_{1m})/\mu) + \exp ((a_{2m} - p_{2m})/\mu) + \exp [V_0/\mu]}, \quad i = 1, 2, \quad m = 1, 2,$$

where $V_0$ is the value of the outside good. As before, set $N_m = 1$ in both markets.\(^\text{10}\)

Equation (6) still describes optimal prices given positioning. To facilitate discussion of the results in the presence of an outside good, some additional notation is helpful. Let $T$ stand for the combined share of the inside goods (at optimal prices) when both are positioned at $a_h$. $S$ is the combined share of the inside goods (again at optimal prices) when one is positioned at $a_h$ and the other at $a_\ell$. Finally, among the inside goods, $R$ is the share of the product positioned at $a_h$ if the other product is positioned at $a_\ell$, i.e., $R = s_1/(s_1 + s_2)$, if product 1 is positioned at $a_h$ and product 2 at $a_\ell$. Applying these definitions, $s_1 = SR$ if product 1 is positioned at $a_h$ and product 2 at $a_\ell$, whereas $s_1 = 0.5T$ if both products are positioned at $a_h$. The following result is shown in the appendix.

**Proposition 6 (Outside good)**

1. There is a mutual profit incentive to sustain spatial concentration as long as

$$T - S < \frac{S^2(1 - 2R)^2}{(2 - S)},$$

\(^\text{10}\)A more general result can be obtained using a nested logit model with separate nests for the inside and outside goods and different scale parameters for the choice among nests and the choice among the inside goods. The results for this model are analytically very cumbersome without adding much insight. These results are available upon request.
2. If the above inequality holds, the minimum discount factor that sustains spatial concentration equals,

\[ \delta^* = \frac{1}{\Phi} \frac{T\Phi - S(2 - T)}{\Phi S(2 - T)\Phi - T} \]

where \( \Phi = \frac{R}{(1 - R)} \)

**Proof:** see appendix A

The interpretation of the first part of the proposition is as follows. The right hand side of the inequality is always positive. Hence as long as \( T - S \leq 0 \), the presence of a pay-off incentive to sustain any existing asymmetries is guaranteed. However, the more interesting case is \( T - S > 0 \) (demand expansion from both positioning at \( a_h \)). The proposition states that as long as demand expansion from both products positioning at \( a_h \) is not too large, there is a profit incentive to maintain a reciprocal form of asymmetric positioning. The long-term category demand effects of marketing mix investments were found to be absent or small in the overwhelming number of mature categories of packaged goods (Nijs et al. 2001). Sustenance of spatial concentration in packaged goods seems therefore only modestly impacted by demand expansion effects.

Figure 3 gives a visual interpretation of the profitability of spatial concentration with an outside good. Using a numerical solver, this figure depicts the difference between the profits at optimal price when positioning is asymmetric (in one market at \( a_h \), in the other market at \( a_l \)) versus when positioning is symmetric at \( a_h \) in each market. The parameters used in this graph are \( V_0 = -1 \) and \( a_l = 0 \). Focusing on the line that applies to \( \mu = 0.75 \), it can be seen that for small values of \( a_h \) (in the figure, smaller than 2.4) profits are higher if both firms position at \( a_h \) in each market. In this case, the expansion effect outweighs the benefit of having differential market power in one market. However, for values of \( a_h \) that are larger than 2.4, it is beneficial to sustain asymmetric positions. Thus in such cases, the incentive to sustain the asymmetries is present. Note that positioning asymmetrically when symmetric positions are optimal, presents less of an opportunity loss than vice versa.

Focusing on the cases where spatial concentration is more profitable than symmetric competition leads to two speculations. First, the profit difference between asymmetric and symmetric positioning is not very large when the firms are selling horizontally differentiated goods. For instance, the profit differential that applies to the case \( \mu = 1.25 \) is close to 0 for all \( a_h \). However, when the products are not horizontally differentiated, the difference in profits becomes very substantial. For instance, the

\(^{11}\text{The choice that } V_0 = -1 \text{ is done to reflect our earlier point that for many mature categories the long term impact of marketing variables on category volume was found to be non-existent in 98% of categories (Nijs et al 2001).}\)
profit differences at $\mu = 0.25$ are relatively sensitive to $a_h$.

Second, the profit incentive is not present for values of $a_h$ close to $a_\ell$ (here chosen to be 0). Thus, initial conditions in a market that are characterized by “small” asymmetries in perceived quality, are likely to be quickly competed away. This implies that one would tend to see a survival bias towards spatial concentration being either strong or absent but not weak.

It is informative to compare profits with symmetric and asymmetric positioning as a function of $\mu$. Figure 3 shows the optimal profits for the same example as above, when $a_h = 1$, $a_\ell = 0$, and $V_0 = -1$ as a function of $\mu$. Two points are noteworthy. As claimed in proposition 5, profits decrease in differentiation $\mu$ for $\mu$ small enough. Spatially concentrated industries with “intermediate” levels of product differentiation are less profitable than industries without product differentiation. Point $A$ in Figure 3 shows that profits from spatial concentration are 1 when $\mu = 0$. Not until $\mu = 1.47$ is there a multimarket policy with equal profits (point $B$).

Second, in the presence of an outside good, the two profit curves intersect. Hence, beyond a certain value for $\mu$, firms are more profitable when they both position at $a_h$ and compete for “fair”
Figure 3: Multimarket profits with symmetric and with asymmetric positioning

share. For instance, if $\mu = 1$, it is more profitable to compete symmetrically.

Before investigating how this contingency is impacted by non-zero positioning cost, I first establish how robust the multi-market argument is against deviations from complete reciprocity.

5.3 The case of $M$ markets

For spatial concentration to be sustained, it was so far assumed that the firms are “balanced” across markets, i.e., that firms are globally equally well off. This section investigates how imbalance across markets still leaves ample possibility for local asymmetries to persist, even when the smaller firm could obtain substantial demand expansion at the expense of the larger firm at no cost. I analyze $M$ markets, two products, without an outside good. In all markets, initial positioning on perceived quality is asymmetric. For historical reasons, firm 1 leads in $L < M$ markets. Firm 2 leads in $M - L$ markets. Again, without much consequence, $N_m = 1$ for all markets. Once more, $a_{im}$ can assume two values: $a_h$ and $a_l$. To simplify the analysis $a_{1m} = a_h$, $a_{2m} = a_l$ for markets $1, \ldots, L$ and vice versa for markets $L + 1, \ldots, M$. Denote the high share that is associated with positioning at $a_h$ when
the competitor positions at \( a_\ell \) by \( R \) (\( R > 0.5 \)).

The following proposition is proven in the appendix.

**Proposition 7 (\( M \) markets)**

1. There is a mutual profit incentive to sustain spatial concentration as long as

\[
\frac{M - L}{M} < R
\]

2. This boundary condition becomes more easily met as the degree of horizontal differentiation diminishes. Formally,

\[
\frac{dR}{d\mu} < 0
\]

3. The critical discount factor that sustains spatial concentration is equal to

\[
\delta^* = \frac{1}{\Phi} \frac{M - L}{L}
\]

When the profit incentive above holds, \( \delta^* < 1 \).

**Proof:** see appendix A

The interpretation of the first part of the proposition is that when for each firm the within-market share in their strong markets, \( R \), is higher than the across-market share of weak markets \((M - L)/M\), both firms are better off with local asymmetries than with local symmetries. This condition is guaranteed for the product that leads in most markets, but it poses a boundary condition on the presence of a profit incentive for the other product. The first result in the proposition also implies that \( R/(1 - R) \equiv \Phi > (M - L)/L \).

The firm with the smaller number of “strong” markets therefore has a profit incentive only if the asymmetries are strong enough. Hence, whereas the previous section focused on the role of the outside good in suggesting that observations of weak asymmetries may be rarer than observations of strong asymmetries, this section finds another reason why weak asymmetries are hard to sustain. Namely, small asymmetries (those with \( R \) close to 0.5) will not survive if competing firms do not share an equal number of “strong” and “weak” markets.

However, many different combinations of \( R > 0.5 \) and \( L = 1, \ldots, M - 1 \) do provide a more profitable division of the total economy than with \( R = 0.5 \) in all markets. In the extreme, even products with only 1 strong market and \( M - 1 \) markets where it is small, may be motivated to maintain such a situation if it is a really large product in its “home market.”
The second part of the proposition states that, in the case of $M$ markets, as was the case for a $2$ market duopoly, and as was the case for a $2$ market duopoly with an outside good, the profit incentive to sustain asymmetries is more generally present when firms are less horizontally differentiated. I again obtain the result that especially when differentiation among products is low, firms—each with historical advantages in selected markets—should be strategically motivated to abstain from attacking in markets where it has a low product development index (BDI). The latter point is in contrast to popular marketing strategy recommendations (for a discussion see Kotler 2003).

The third part of the proposition focuses on the critical discount factor that sustains the scenario of this section as an equilibrium in which one firm has $L$ strong markets and one firm has $M - L$ strong markets. Mathematically, the discount factor is equal to the relative within-market share $(1 - R)/R$ in weak markets (at optimal prices) times the relative across-market share of weak markets $(M - L)/L$. Given that this result only holds if there is a principle reason to sustain asymmetric positions, this discount rate is guaranteed to be less than 1. The result in proposition 4 that $d\Phi/d\mu < 0$, implies that—as before— even relatively impatient managers will resist the short term temptation to attack and increase their demand in “weak” markets when product differentiation diminishes.

6 The role of retailers

6.1 Representation of retailers

In addition to multi-market competition, a further characteristic of competition in packaged goods is that local product positioning, i.e., acquiring shelf and mind space, is costly. To study whether such costs promote the sustenance of spatial concentration, this section focuses on manufacturer competition subject to slotting— and other distribution fees as a manifestation of positioning cost.

To model the necessary aspects of retailer-behavior in the context of multi-market competition among firms, two simple modifications are made to the model. To the manufacturer, slotting fees, promotion allowances, and other retailer support are a (periodic) fixed cost, $K$, that does not depend on quantity sold. In other words, to the manufacturer, retailer support for existing products is akin to paying rent for shelf-space.\footnote{Slotting fees are meant here to capture “pay-to-stay” fees. Such fees are charged by retailers to manufacturers in return for special treatment at the supermarket shelf. See e.g., Federal Trade Commission (2001).} Retailer support for firm $i$ in market $m$ enhances the quality
perceptions of consumers $a_{im}$. The fixed cost $K(a)$ increases in $a$ ($K'(a) > 0$) and is assumed to be low enough for two manufacturing firms to enter in any given market. The latter accords with the empirical fact that all local markets are entered by multiple manufacturers.

The second modification is the introduction of a retailer mark-up. Prices are modeled as $p_{im} = w_{im} + u_{im}$, where $u_{im}$ is the markup set by all retailers in market $m$ and $w_{im}$ is the wholesale price that is set by the firm. The local retailer mark-ups reflect intra-market competitive phenomena (e.g., competition between retailers) that are not likely to give rise to spatial concentration at the national level. Hence, I use a constant mark-up $u_{im} = u$ (for a similar simplification, see Vilcassim, Kadiyali, and Chintagunta 1999).

Exogenous slotting fees and markups are a useful approximation of the reality that retailers face many different product categories and therefore often use a heuristic approach to setting margins and slotting fees. More to the point, even with passive retailers, the model is informative about how the presence of costly retailers might sustain spatial concentration.

### 6.2 Single market competition

Consider a single market (drop the subscript $m$), logit demand, no outside good, two manufacturing firms, and the presence of retailers with slotting fees $K(a_i)$, and markup $u$. Manufacturers first set positioning simultaneously, and then simultaneously decide on prices. To get tractable results, I again assume that there are two possible levels of quality perceptions $a_h$ and $a_l$. Demand for good $i$ is equal to

$$s_i = \frac{\exp((a_1 - p)/\mu)}{\exp((a_1 - p_1)/\mu) + \exp((a_2 - p_2)/\mu)}$$

with $p_i = w_i + u$. In a single market context, profit for each of the manufacturing firms is equal to

$$\pi_i = s_i (w_i - c) - K (a_i)$$

From the first-order conditions wholesale prices are equal to

$$w_i = c + \frac{\mu}{1 - s_i}.$$ 

Note that these prices are not the same as before. That is to say, the retailer mark-up is represented in the shelf price, which in turn impacts $s_i$. Profits at these wholesale prices are equal to

$$\pi_i^* = \frac{\mu s_i}{1 - s_i} - K (a_i).$$

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Of initial interest is whether an asymmetric equilibrium in which one firm positions at $a_h$ and the other at $a_\ell$ can emerge because of slotting fees $K(a)$.

**Asymmetric positioning**  
Consider first the case where product 1 is positioned at $a_h$ while product 2 is positioned at $a_\ell$. Profits for firm 1, which is positioned “high” are equal to $\pi_1^* = \mu \Phi - K(a_h)$, with $\Phi = s_1 / (1 - s_1) > 1$. Suppose firm 1 considers repositioning to $a_\ell$. If so, it splits the market evenly with firm 2 (which is also positioned at $a_\ell$). Its profits would equal $\mu - K(a_\ell)$. Thus, firm 1 will not reposition to $a_\ell$ as long as $\mu \Phi - K(a_h) > \mu - K(a_\ell)$ because $a_h > a_\ell$.

Firm 2, positioned “low,” will not reposition if the payoff of sustaining $a_\ell$ is larger than imitating firm 1 and repositioning to $a_h$. This implies that $\mu \Phi - K(a_h) > \mu - K(a_\ell)$. By combining these results, neither firm has an incentive to deviate from asymmetric positioning as long as

$$\mu (1 - \Phi^{-1}) < \Delta K < \mu (\Phi - 1),$$

with $\Delta K \equiv K(a_h) - K(a_\ell)$.

What can be said about this condition? First, there always exist slotting fees that make asymmetric positioning an equilibrium. This is true because $\mu (1 - \Phi^{-1}) < \mu (\Phi - 1)$ iff $\Phi > 1$, i.e. as long as the product with the highest perceived quality obtains the highest market share which is true given our assumptions. Second, the latitude of $\Delta K$ that supports the asymmetric equilibrium is, after some algebraic manipulation equal to $\mu (\Phi - 1) (1 - \Phi^{-1})$. More about the dependence of this range on $\mu$ will be stated momentarily.

**Symmetric positioning**  
The conditions under which a symmetric equilibrium exists are described next. Under symmetric positioning at $a_\ell$, both firms have the following profits.

$$\pi_i^* = \mu - K(a_\ell)$$

If either firm repositioning to $a_h$, profits of that firm will be $\pi_1^* = \mu \Phi - K(a_h)$. Thus, if $\Delta K > \mu (\Phi - 1)$, then repositioning will not occur and a symmetric equilibrium with both firms positioned at $a_\ell$ holds.

Following similar logic, a symmetric equilibrium at $a_h$ is obtained when it is not profitable for either firm to reposition to $a_\ell$. This happens when $\Delta K < \mu (1 - \Phi^{-1})$. In words, if it is cheap enough to position at $a_h$, all firms will do so.

The following proposition summarizes these results (proofs are in the appendix)
Proposition 8 (Retailer – single market)

1. Whole sale prices are \( w_i^* = c + \frac{\mu}{(1 - s_i)}, i = 1, 2 \)

2. In the presence of a retailer, there are three unique equilibria whose support is given by the difference in slotting fees \( \Delta K \) and by the degree of horizontal differentiation \( \mu \) as follows

   (a) If \( \Delta K < \mu (1 - \Phi^{-1}) \) then both products position symmetrically at \( a_h \)

   (b) If \( \mu (1 - \Phi^{-1}) \leq \Delta K \leq \mu (\Phi - 1) \) then one product positions at \( a_h \) and one product positions at \( a_\ell \) (or vice versa)

   (c) If \( \Delta K > \mu (\Phi - 1) \) then both products position symmetrically at \( a_\ell \)

3. The cost differential \( \Delta K \) over which asymmetric positioning is the only equilibrium has the following characteristics

   (a) \( \lim_{\mu \downarrow 0} \{ \mu (1 - \Phi^{-1}), \mu (\Phi - 1) \} = \{0, a_h - a_\ell\} \)

   (b) \( \lim_{\mu \to \infty} \{ \mu (1 - \Phi^{-1}), \mu (\Phi - 1) \} = \{(a_h - a_\ell)/3, (a_h - a_\ell)/3\} \)

**Proof:** see appendix A

The first two parts of this proposition were discussed above.

The third part of the proposition merits further discussion and interpretation. When product differentiation is low, i.e., \( \mu \downarrow 0 \), the difference in gross profits (before positioning cost) between the two firms in the unit-sized market tends to \( a_h - a_\ell \). This statement echoes proposition 1 which showed that price (and profit) increases are never larger than the increases in positioning. Therefore, the upper limit of the difference in fixed cost \( \Delta K \) that supports an asymmetric equilibrium is equal to \( a_h - a_\ell \).

Conversely, when the category becomes more differentiated –as \( \mu \) increases– positioning has less influence on profitability. In the limiting case when products are no longer substitutes, the condition states that \( (a_h - a_\ell)/3 \leq \Delta K \leq (a_h - a_\ell)/3 \). In other words as the products are more horizontally differentiated, no slotting fees (except in the limit \( K(a) = K_0 + \frac{1}{3} a \)) will support an asymmetric equilibrium.

The width of the interval \( \{ \mu (1 - (\Phi)_1^{-1}), \mu (\Phi_1 - 1) \} \) can be loosely interpreted as the generality with which an arbitrary cost function \( K(a), a > 0 \), obeys \( \mu (1 - \Phi^{-1}) \leq \Delta K \leq \mu (\Phi - 1) \). If the interval is wide, any cost differential will support an asymmetric market outcome as the only equilibrium outcome. Conversely, if the interval is very narrow, only very small cost-differences will support an asymmetric market outcome. Hence, again, asymmetric outcomes happen under more general conditions (in this case: cost-functions) when goods are undifferentiated.
In sum, the slotting fees $K(a)$ can sustain an asymmetric equilibrium between firms in a single market. Therefore this outcome can not be due to multimarket contact. Rather, in this case, the asymmetry is due to the inherent non-linearity in demand.\textsuperscript{13} When goods are undifferentiated, there is only “room” for one high quality player in the market. If two products try to both be high quality players, neither of them will make enough profits to make up for the increased costs in positioning relative to positioning lower. Because the profitability gap between the two firms increases, when there is little horizontal differentiation in a category, the occurrence of asymmetric equilibria will occur even with high $\Delta K$.

6.3 Multimarket competition in the presence of retailers

I now consider the case of two markets instead of one. Retailers are again passive players, with a fixed mark-up and slotting fees, $K(a)$, that depend on the level of support, $a$, given to the manufacturer’s products. As before, I use a repeated interaction framework with infinite horizon to explore the multi-market nature of competition. In each period, firms position their products first (either at $a_h$ or $a_\ell$) and then set prices.

Four possible positioning cases need to be considered. These cases are listed below.

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<thead>
<tr>
<th>CASE 1</th>
<th>firm 1</th>
<th>firm 2</th>
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<tbody>
<tr>
<td>market 1</td>
<td>$a_\ell$</td>
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<tr>
<td>market 2</td>
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<thead>
<tr>
<th>CASE 2</th>
<th>firm 1</th>
<th>firm 2</th>
</tr>
</thead>
<tbody>
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<td>market 2</td>
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<tr>
<th>CASE 3</th>
<th>firm 1</th>
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<td>market 2</td>
<td>$a_\ell$</td>
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<tr>
<th>CASE 4</th>
<th>firm 1</th>
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<tbody>
<tr>
<td>market 1</td>
<td>$a_h$</td>
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<tr>
<td>market 2</td>
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</table>

All other possible combinations merely involve label switching of firms or markets and are therefore redundant. Case 1 (positioning low by all firms in all markets) falls under the previous proposition. If it is too expensive to position at $a_h$ in one market, it is also too expensive to position at $a_\ell$ in multiple markets.

Case 2 can not be a multimarket equilibrium because it can not be the case (given $\mu$ and $K(a)$) that both firms position at $a_h$ in one market and at $a_\ell$ in the other. If positioning is cheap enough

\textsuperscript{13}Yarrow (1989) considers the specific case that $K(a) = \exp(a)$. Not unlike this paper, he finds that asymmetric equilibria are possible in a single market. For another instance of the latter result see also Moorthy (1988), who makes the additional argument that the asymmetric equilibrium may be interpreted as a possible advantage of the first entrant. Both papers focus on asymmetric equilibria in a single market.
for both firms to select $a_h$ in market 1, there is a profitable deviation from both playing $a_\ell$ in market 2. Conversely, if advertising is expensive enough for both firms to choose $a_\ell$ in market 1, there is a profitable deviation from both selecting $a_h$ in market 2.

This leaves cases 3 and 4. For logical reasons, the paper focuses on case 3 first. Product 1 leads product 2 in market 1 and vice versa in market 2. At optimal prices the ratio of shares of the larger over the smaller product is denoted by $\Phi$, as before. If case 3 is an equilibrium, both firms have a profit of

$$\pi_i^* = (\Phi + \Phi^{-1}) \mu - (K(a_h) + K(a_\ell)) . \quad (18)$$

A possible deviation for a given firm is to reposition to $a_\ell$ in the market where it was positioned at $a_h$. This deviation is attractive when positioning cost is high enough. However, if it is optimal for one firm to reposition from $a_h$ to $a_\ell$ it is optimal for the other firm to do the same in the other market. The optimality of this repositioning is therefore considered in case 1 and is covered by proposition 8.

Another deviation is to reposition to $a_h$ in the market in which it was positioned at $a_\ell$. If a firm does this it ends up with one market in which it positions at $a_h$ against $a_\ell$ by its competitor, and one market where both firms position at $a_h$. This repositioning will give to the firm that repositions the following profits

$$\pi_i^d = (\Phi + 1) \mu - 2K(a_h) . \quad (19)$$

This deviation is attractive in the short run to the firm that repositions if the cost of positioning at $a_h$ is small enough. Specifically, comparison of profits gives that if $\Delta K < \mu \left(1 - \Phi^{-1}\right)$, then $\pi_i^* < \pi_i^d$ and firm $i$ has a short term incentive to deviate. If this condition is met for one firm in one market, it logically also meets for the other firm in the other market. Assume that other firm will then also position at $a_h$ (I now obtain case 4). Upon this retaliation from the second firm, the profits for either firm will forever equal

$$\pi_i^0 = 2\mu - 2K(a_h) , \quad (20)$$

which is less than $\pi_i^*$. Using the same arguments as those preceding proposition 4, case 4 will not occur if firms value future profits enough. The minimum discount rate that keeps firms in asymmetric equilibrium and out of case 4 is equal to

$$\delta^* = \frac{\pi_i^d - \pi_i^*}{\pi_i^d - \pi_i^0} = \frac{1}{\Phi} - \frac{\Delta K}{\mu (\Phi - 1)} . \quad (21)$$
Note that in proposition 4, the minimal discount rate equals $\Phi^{-1}$ which is larger than the discount rate above. Thus compared to the results in that proposition, the presence of retailers who charge for retail support makes the asymmetric equilibrium more easily sustained even with myopic firms. These results are formally stated in the following proposition

**Proposition 9 (Retailer – two markets)**

1. Wholesale prices are $w_{im}^* = \mu/(1-s_{im}) + c$, $i = 1, 2$

2. When firms meet in multiple markets the following equilibria exist in the presence of a retailer

   (a) If $\Delta K > \mu (\Phi - 1)$ then both products position symmetrically at $a_\ell$ in all markets.

   (b) If $\mu (1 - \Phi^{-1}) \leq \Delta K \leq \mu (\Phi - 1)$ then one product positions at $a_h$ and one product positions at $a_\ell$ in each market.

   (c) If $\Delta K < \mu (1 - \Phi^{-1})$ then

      i. if the value of future profits exceeds $\delta^*$ times the value of today’s profits both firms will sustain a reciprocal asymmetric positioning.

      ii. if the firms are myopic, both products position symmetrically at $a_h$ in each market.

3. The minimum current value of future profits that will admit an asymmetric multi-market equilibrium is equal to

   $$\delta^* = \frac{1}{\Phi} - \frac{\Delta K}{\mu (\Phi - 1)}$$

   **Proof:** see appendix A

Figure 4 helps to interpret the proposition. It outlines the equilibria that exist for an arbitrary cost function $K(a)$ (subject to the constraint that the profits for both firms needs to be non-negative) and degree of horizontal differentiation $\mu$. This figure was generated, using a numerical solver, for the scenario in which $a_h = 1$, and $a_\ell = 0$. There are 4 zones in this figure. The top zone outlines the cases where the cost difference between positioning high and low is so large that both products position low in all markets.

The second, shaded, zone represents the cases where a single-market asymmetric equilibrium exists. This zone widens as horizontal differentiation diminishes ($\mu \downarrow 0$). As was pointed out in proposition 8 point 3, the cost differential $\Delta K$ for which an asymmetric equilibrium is obtained widens as the horizontal product differentiation diminishes. In a this zone, a product may lead in both markets, in one, or in none. In none of these cases is there an incentive to deviate in any single market. Consequently, deviations in multiple markets are also unprofitable.

The third zone, with only the diagonal pattern, identifies when asymmetric equilibria are sustainable in two markets but not in a single market. In other words, it identifies the cases where the
cost difference between positioning high or low is so small that in a single market case, all products would position at $a_h$. However, in a two-market context, the spatially concentrated equilibrium holds in which firms prefer one market in which one firm is positioned at $a_h$ while its competitor is positioned at $a_l$. That is, this equilibrium holds if the firms value the future enough. Figure 4 was created with $\delta = 0.75$. Thus even if firms only value next period’s profits at 75% of current profits, the area over which spatial concentration is sustainable increases from the shaded to the area with the diagonal pattern.

Finally, the fourth zone (the bottom zone), contains all cases where firms position at $a_h$ in all markets. As the firm’s discount rate $\delta$ increases, the fourth zone will diminish.

The third part of the proposition focuses on the future orientation needed for spatial concentration to occur more generally than in the cases outlined in the shaded area. Even myopic firms can more easily sustain asymmetric positioning if retailers charge them for such positioning because it makes attacking in low demand markets less profitable. Indeed, comparing proposition 4 with proposition
9, \( \delta^* \) (today's minimum valuation for a unit of tomorrow's profit) is smaller in the presence of positioning cost than without.

### 6.4 Do slotting fees harm firms' profits?

It has been suggested that slotting or pay-to-stay fees are anticompetitive and harmful to firms' profits (see Kelly 2001 for a discussion rather than an endorsement of this point of view). The combination of spatial concentration and slotting fees however can be more profitable than a world without slotting allowances. To see this, consider firms who compete in multiple markets and whose discount rates obey

\[
\frac{1}{\Phi} - \frac{\Delta K}{\mu (\Phi - 1)} < \delta < \frac{1}{\Phi}. \tag{22}
\]

For these firms, the presence of costly retailers can be an unambiguous blessing. Namely, without costly retailers, these firms would not resist the short term temptation to attack in the market where they are small. However, with retailers such firms are sufficiently discouraged from attacking in their competitors most profitable markets.

In fact, incumbent firms can be more profitable with costly retailers than without. Specifically, for the cases identified by equation (22), firms do better with costly retailers than without, if the profit from the combination of spatial concentration and the presence retailers is higher than the profit from the combination of symmetric positioning and free or no retailers, i.e., if

\[
(\Phi + \Phi^{-1}) \mu - (K(a_h) + K(a_l)) > 2\mu - 0, \tag{23}
\]

which alternatively can be rewritten as

\[
K(a_h) + K(a_l) < (\Phi - 1) (1 - \Phi^{-1}) \mu \tag{24}
\]

It is clear that there exist \( \delta \) and slotting fees \( K(a) \) that obey the conditions (22) and (24). For instance, to obey (22) slotting fees should discriminate across different levels of \( a \). Indeed, slotting fees will not deter local laggards from attacking, if attacking is not made expensive by the retailer. Further, to meet (24) slotting fees should not be too high, i.e., firms' profitability will be ultimately harmed by ever increasing slotting allowances. The following proposition formalizes the conditions under which firms may benefit from the presence of retailers, and illustrates the dependence of these conditions on horizontal differentiation.
Proposition 10 (retailers and incumbents)

1. Firms which are neither too myopic nor sufficiently forward looking, i.e., for which \( \frac{1}{\Phi} - \frac{\Delta K}{\mu (\Phi - 1)} < \delta < \frac{1}{\Phi} \), will be able to sustain spatial concentration with costly retailers but not without.

2. For such firms, if the slotting fees are not too high, i.e., if \( K (a_h) + K (a_\ell) < (\Phi - 1) (1 - \Phi^{-1}) \mu \), it is more profitable to have a spatially concentrated equilibrium sustained by costly retailers than to serve consumers directly through no or free retailers.

3. The slotting fees under which firms are still better off are higher with low degrees of differentiation than with high degrees of differentiation. That is,

\[
\lim_{\mu \to 0} (\Phi - 1) (1 - \Phi^{-1}) \mu = a_h - a_\ell,
\]

whereas

\[
\lim_{\mu \to \infty} (\Phi - 1) (1 - \Phi^{-1}) \mu = 0.
\]

Proof: see appendix A

The first two parts of the proposition 10 were addressed above. The last part states that as the horizontal differentiation between products increases, the firms are better off with retailers only when these retailers are free. Yet when products are very close substitutes, it is more profitable to have costly retailers –who sustain spatial concentration– as long as they are not too expensive, i.e., as long as \( K (a_h) + K (a_\ell) < (a_h - a_\ell) \). An interesting link emerges to work by McGuire and Staelin (1983) who found that as products become closer substitutes, firms prefer to shield themselves from competition by selling through a retailer. In contrast to their single-market framework, the result here relies on the fact that firms meet in multiple markets.

6.5 An alternative interpretation for advertising

The previous section analyzed the role of positioning cost in the context of shelf space allocations or other retailer-support. Keller (1993) and Kirmani and Wright (1989) have argued that inferences about product quality are alternatively affected by advertising investments. Therefore, a discussion and an interpretation of the previous results in the context of advertising is appropriate.

In an advertising interpretation, positioning at \( a_h (a_\ell) \) translates into advertising at a high (low) level. The cost difference \( \Delta K \) is the marginal cost of advertising. The previous section suggests there are three advertising cases to consider.

First, if advertising costs are sufficiently high, nobody will advertise at \( a_h \). This is equivalent to the case in proposition 8 where \( \Delta K \) is large. Second, for intermediate values of \( \Delta K \) an asymmetric
advertising equilibrium exists. In such an equilibrium, there is only “room” for one player to advertise at a high level. The other player will realize that it is impossible to mimic the success of the first player given that this player occupies the \( a_h \) position. It is therefore natural to think in this context of a defensible first mover advantage (see also Moorthy 1988). Finally, for really low values of \( \Delta K \) both firms will advertise. As products are less and less differentiated, a unique single-market asymmetric equilibrium becomes more and more likely, because the range of values for \( \Delta K \) – for which such an equilibrium is unique – expands.

In a multimarket setting, the results imply that spatial concentration is an equilibrium under general conditions, even if the cost of advertising is low, i.e., even if it is tempting to advertise heavily in markets where another product leads.

### 6.6 The presence of an outside good

Turning now to a quick comparison with the case where an outside good is present, I use numerical methods to show that the presence of an outside good does not fundamentally change the previous analysis. The use of numerical methods is required because analytical results are difficult to obtain. Demand in a single market is given by

\[
s_i = \frac{\exp[(a_i - p_i^*)/\mu]}{\exp[(a_1 - p_1^*)/\mu] + \exp[(a_2 - p_2^*)/\mu] + \exp[V_0/\mu]},
\]

(25)

Wholesale prices given positioning are equal to

\[
w_1^* - c = \frac{\mu}{1 - s_1^*} \quad \text{and} \quad w_2^* - c = \frac{\mu}{1 - s_2^*}.
\]

(26)

Define the following abbreviations. As before, the share of the inside goods at optimal prices is denoted by \( S \) when one product is positioned at \( a_h \) and product 2 is positioned at \( a_{\ell} \). I further define \( T_h \) (\( T_{\ell} \)) as the share of the inside goods when both products are positioned at \( a_h \) (\( a_{\ell} \)). Finally, \( R \) denotes the share of product 1 among the inside goods. Suppose that product 1 is positioned at \( a_h \) and product 2 is positioned at \( a_{\ell} \). Thus for firm 1 it is not profitable to reposition to \( a_{\ell} \) if

\[
\frac{RS\mu}{1 - RS} - K(a_h) > \frac{0.5T_{\ell}\mu}{1 - 0.5T_{\ell}} - K(a_{\ell}),
\]

(27)

whereas for firm 2 there is no profitable deviation if

\[
\frac{(1 - R)S\mu}{1 - (1 - R)S} - K(a_{\ell}) > \frac{0.5T_h\mu}{1 - 0.5T_h} - K(a_h),
\]

(28)
Summarizing the results for both firms, there is no profitable deviation if

$$\mu \left( \frac{0.5T_h}{1 - 0.5T_h} - \frac{(1 - R) S}{1 - (1 - R) S} \right) < \Delta K < \mu \left( \frac{RS}{1 - RS} - \frac{0.5T_\ell}{1 - 0.5T_\ell} \right).$$

Using this equation and a numerical solver the results from the previous section can be compared with the current case. Figure 5 shows the possible equilibria in the presence of an outside good. As in the previous section, the scenario considered here is $a_h = 1$, and $a_\ell = 0$. I now also set $V_0 = -1$.

The equilibria obtained are very similar to those obtained without the presence of an outside good. Again, when the “inside goods” are not well differentiated, a wide range of positioning costs generate a single-market asymmetric equilibrium. In contrast, as the goods are more and more differentiated, a symmetric equilibrium is obtained for more and more values of $\Delta K$. Second, when firms compete in multiple markets, spatial concentration is sustainable under more general conditions than single-market asymmetry. Depending on the discount rate $\delta$ symmetric equilibria of “high” positioning exist for low $\Delta K$. For instance, Figure 5 was generated with a discount rate of $\delta = 0.75$. With
increasing $\delta$ these symmetric equilibria at $a_h$ become less and less prevalent.

However, there is also a notable difference with the previous case. Given that positioning now has primary demand effects (in addition to selective demand effects), there is a greater tendency to position both at $a_h$. This is obvious from Figure 5 by noting that the area of symmetric positioning at $a_h$ is larger than that of Figure 4.

7 Discussion

The main idea of this paper is that spatial concentration may persist for products of undifferentiated consumer goods. This idea was motivated by illustrating the surprising degree of spatial concentration of weakly differentiated categories such as Mexican salsas. It was noted that the same degree of spatial concentration holds for products such as ground coffee and mayonnaise. However, a second result was that opportunities and/or incentives to sustain spatial concentration are less strong when products are more clearly differentiated.

If the argument about differentiation is empirically important, local differences should not persist to the same extent in categories with differentiated goods. An example such a category is breakfast cereals, i.e., consumers will in general have little difficulty distinguishing between say Kellogg’s Corn Flakes and General Mills Cheerios. More broadly, the product portfolios of the top manufacturers contain few if any products that are physically indistinguishable. Indeed, as Nevo (2001) observes, the top manufacturers of breakfast cereals do not imitate each other’s products. Therefore, it is not unreasonable to claim that breakfast cereals are more horizontally differentiated than brands of Mexican salsas.

Figure 6 shows the 1992 local shares of Kellogg and General Mills in the breakfast cereal market. Compared to Figure 1, the most obvious feature of the cereal data is the absence of spatial concentration. That is, whereas there are differences in market shares for each product across markets they are only modest compared to the example of Salsas.

This anecdote suggests that (lack of) product differentiation may be an important factor in the spatial concentration of an industry. It is not claimed that differentiation is the only important factor. Specifically, cost efficiencies of spatial concentration or the presence of actors whose decisions have spatial footprints (such as retailers or distributors) could well contribute to spatial concentration. However, if the striking difference between Figure 6 and 1 is any indication, the degree of product
differentiation does seem to play a role in the sustainance of spatial concentration.

8 Conclusion

There are many reasons why competing firms of undifferentiated goods face different initial conditions in a given markets, e.g., order-of-entry effects, pre-emption of mind-space (e.g., selective learning by consumers) and shelf-space (e.g., selective availability of facings), etc. These phenomena can lead to initial differences in market shares and profitability. This paper has argued that even after the original reasons for the asymmetries that arise from such initial conditions vanish, spatial concentration of prices and outputs can be sustained despite immediate competition between goods. Two different explanations for this fact were presented.

The first explanation is that multi-market contact provides a mechanism to sustain geographically concentrated advantages. If such advantages is distributed equitably, competitors are better off compared to uniformly competing head-on with each other in all local markets.

The second explanation focuses on the possibility that firms can create a local form of vertical differentiation by through costly positioning at differential qualities. For a variety of positioning cost, there is “room for only one high quality player in a market.” Thus spatial concentration in multiple markets can be sustained if local positioning is costly. It is also shown that costly intermediaries such as retailers, may help to sustain asymmetries by making it expensive for lagging firms to compete for “fair” market share.
The contingency that spans both explanations is that sustenance of spatial concentration should be expected especially when goods are physically similar. This leads to the surprising statement that if goods are the same, initial market conditions may cast very long shadows, whereas if products are differentiated, these initial market conditions will not be sustainable. Indeed, in the latter case, all competitors tend to compete for a “fair share” in all local markets. Thus in undifferentiated categories, launch strategies are very important and may initiate a market division that will be very resistant to change later on.

Provided that spatial concentration leaves both firms with at least some strong markets, this paper further suggested that profitability of packaged-goods categories does not need to rely exclusively on horizontal product differentiation. Local asymmetries in product positioning on perceived quality may suffice as a source of differentiation. Also, local differentiation on perceived quality is more effective when horizontal product differentiation is absent than when it is moderate.

Finally, situations wherein category demand is high but market share is low, are oft seen as a business opportunity (see e.g., Kotler 2003; Schultz, Martin and Brown, 1984). The results in this paper are cautionary with respect to attacking in such markets. Specifically, in cases of spatial concentration a firm has to consider what will happen in one’s own high-share markets as a consequence. The results in this paper suggest that spatial concentration may dissolve and all firms will be worse off ever after. This is especially true in mature categories with a low degree of product differentiation, i.e., for many packaged goods categories.

There are several limitations to this paper. First, I have analyzed duopolies in markets of equal size. Whereas the consideration of oligopolies or markets of varying size will have some impact on the results, such impact is small and perhaps of limited theoretical interest. Second, I have focused mainly on sustenance of existing asymmetries. In future research, it is desirable to address the emergence of concentrations in market shares. Indeed, given the opportunity of sustenance of asymmetries especially when there is little product differentiation, the question of what causes these asymmetries takes on some urgency. Third, empirically—and Figure 1 nicely illustrates this—the patterns of share concentration are highly spatial. Given the regularity of the patterns, and the degree of spatial variability of market shares, it seems important to study the origins of this phenomenon.
9 References


Schultz, Don E., Dennis Martin, and William P. Brown (1984), Strategic Advertising Campaigns, Crain Books, Chicago, IL.


A Proofs

Proof of proposition 1  For convenience, drop all subscripts \( m \). Recall that
\[
s_1 = \frac{\exp[(a - p_1)/\mu]}{\exp[(a - p_1)/\mu] + \exp[(-p_2)/\mu]}, a = a_1 - a_2 \tag{A.1}
\]
Some useful relations are \( \frac{ds_1}{dp_1} = -\frac{1}{\mu} s_1 (1 - s_1), \frac{ds_1}{dp_2} = \frac{1}{\mu} s_1 s_2, \frac{ds_1}{da} = \frac{1}{\mu} s_1 (1 - s_1). \) Taking the first order condition for firm 1 gives,
\[
F(p_1, p_2, a) \equiv p_1 - c_1 - \frac{\mu}{1 - s_1} = 0 \tag{A.2}
\]
The total differential of this function is
\[
F_{p_1} dp_1 + F_{p_2} dp_2 + F_a da = 0
\]
It can further be shown that \( F_{p_2} \) and \( F_a \) are both equal to \(-\Phi\). Substitution in the total differential for \( F \) gives
\[
(1 + \Phi) dp_1 - \Phi dp_2 - \Phi da = 0 \tag{A.4}
\]
Now, totally differentiate the first order condition for firm 2.
\[
G(p_1, p_2, a) \equiv p_2 - c_2 - \frac{\mu}{s_1} = 0 \tag{A.5}
\]
The total differential of this function is \( G_{p_1} dp_1 + G_{p_2} dp_2 + G_a da = 0. \) Once more it is easy to show that
\[
G_{p_1} = -\frac{1}{\Phi}, \quad G_{p_2} = 1 + \frac{1}{\Phi}, \quad G_a = \frac{1}{\Phi} \tag{A.6}
\]
Substitution in the total differential of \( G \) gives
\[
-\frac{1}{\Phi} dp_1 + \left(1 + \frac{1}{\Phi}\right) dp_2 + \frac{1}{\Phi} da = 0 \tag{A.7}
\]
Finally, combining (A.4) and (A.7), gives that
\[
\frac{dp_1^*}{da} = \frac{\Phi^2}{1 + \Phi + \Phi^2} > 0, \quad \frac{dp_2^*}{da} = \frac{-1}{1 + \Phi + \Phi^2} < 0. \tag{A.8}
\]
This proofs proposition 1. The result states further that changes in \( a \) are never priced by the firm to the market completely. Indeed, it may be noted from the definition of \( \Phi \) that the sensitivity of \( p_1 \) to changes in \( a \) is always between 0 and 1.

Proof of proposition 2  Once again, the subscript \( m \) is dropped from the notation. It needs to be shown that the profits of both firms are convex in \( a \). Thus, the second order derivative of profits with respect to \( a \) needs to be evaluated at the equilibrium prices. It is sufficient that
\[
\frac{d}{da} \left( \frac{ds_i^*}{aq} \right) = \frac{d^2 \pi_i^*}{da^2} = \frac{d^2 p_i^*}{da^2} > 0, \quad i = 1, 2 \tag{A.9}
\]
To simplify the derivation, I can use the expressions in (A.8) and take the derivative of both expressions with respect to $a$. For both firms it is possible to write

$$\frac{d^2 p_i}{da^2} = \frac{df_i(\Phi)}{d\Phi} \frac{d\Phi}{da}$$

(A.10)

with $f_i(\Phi)$ given by equation (A.8). It can be shown that

$$\frac{df_1(\Phi)}{d\Phi} = \frac{(2 + \Phi)\Phi}{(1 + \Phi + \Phi^2)^2} > 0 \quad \text{and} \quad \frac{df_2(\Phi)}{d\Phi} = \frac{(2 + \Phi)}{(1 + \Phi + \Phi^2)^2} > 0$$

(A.11)

Recalling that $\Phi = \exp\left[(-p_1 + p_2 + a) / \mu\right]$, the derivative $d\Phi/da$ of the ratio of outputs with respect to $a$ is

$$\frac{d\Phi}{da} = \frac{d}{da} \left(\exp\left[(-p_1^* + p_2^* + a) / \mu\right]\right)$$

$$= \exp\left[(-p_1^* + p_2^* + a) / \mu\right] \cdot \frac{d\left((-(p_1^* + p_2^* + a) / \mu\right)}{da}$$

$$= \Phi \cdot \frac{1}{\mu} \left(\frac{-dp_1^*}{da} + \frac{dp_2^*}{da} + 1\right)$$

$$= \frac{1}{\mu} \frac{\Phi^2}{1 + \Phi + \Phi^2} > 0.$$  

(A.12)

Substitution of (A.11) and (A.12) into (A.10) proves that the profits of both firms are convex in $a$. □

**Proof of proposition 3**  
By proposition 1 both firms positioning with low perceived quality is not an equilibrium. Prices and profits increase in the positioning gap $a_m$ and therefore unilaterally setting a high level of perceived quality is a profitable deviation from mutually positioning at low perceived quality.

If one firm has high perceived quality and the other firm has low perceived quality, it is optimal for the low perceived quality firm to increase its perceived quality. This is implied by the finding in proposition 1 that

$$\frac{dp_2}{da} = \frac{-1}{1 + \Phi + \Phi^2}.$$  

(A.13)

If both firms have high perceived quality they both set prices of $c + 2\mu$. These prices stem from $p_i^* = (c_i + \mu) / (1 - s_i)$, and from the obvious result that if both have the same positioning $s_i = 1/2$. It is easily verified that there are no unilateral deviations from this proposed equilibrium. □

**Proof of proposition 4**

1. The presence of the profit incentive is easily derived from the comparison of total profit across the two markets under asymmetric market positions vs. symmetric positioning.

$$\mu \Phi + \mu \Phi^{-1} - 2K \geq 2\mu - 2K.$$  

(A.14)

The LHS is minimized for $\Phi = 1$, which is the case of symmetry. Hence, the above inequality always holds.
2. Proved in the text.

3. I need to show that \( \frac{d\Phi}{d\mu} > 0 \) or equivalently that \( \frac{d\Phi}{d\mu} < 0 \) as long as \( \Phi > 1 \). I can rearrange the definition of \( \Phi \) at optimal prices to obtain the implicit equation that

\[
\Phi = \exp\left(\frac{(a - p_1^* + p_2^*)}{\mu}\right) \tag{A.15}
\]

with \( p_1^* = \mu/(1 - s_1) = \mu(1 + \Phi) \), \( p_2^* = \mu/(1 - s_2) = \mu(1 + \Phi^{-1}) \). Thus, at optimal prices the following relation exists,

\[
\Phi = \exp\left(\frac{a}{\mu} - \Phi + \frac{1}{\Phi}\right) \tag{A.16}
\]

which is larger than 1 if \( a > 0 \) (see proposition 1). From this equation, take the derivative to obtain that

\[
\frac{d\Phi}{d\mu} = -\exp\left(\frac{a}{\mu} - \Phi + \frac{1}{\Phi}\right)\left(\frac{a}{\mu^2} + \frac{d\Phi}{d\mu} + \frac{1}{\Phi^2} \frac{d\Phi}{d\mu}\right) \tag{A.17}
\]

Rearranging gives that.

\[
\frac{d\Phi}{d\mu} = -\frac{a}{\mu^2} \frac{\Phi^2}{(1 + \Phi + \Phi^2)} \tag{A.18}
\]

which is strictly negative as long as \( a > 0 \) (which is always true).  

Proof of proposition 5

1. See the example in the text.  

Proof of proposition 6:

1. I use the symbols \( S, T, \) and \( R \) as they are defined in the text. At optimal prices, the profit that a firm will get from asymmetric positioning is equal to

\[
\pi^* = \frac{SR\mu}{1 - SR} + \frac{S(1 - R)\mu}{1 - S(1 - R)} - 2K, \tag{A.19}
\]

whereas the profit (at optimal prices) a firm will get if all firms position at \( a_h \) in all markets is equal to

\[
\pi^0 = \frac{T\mu}{1 - 0.5T} - 2K. \tag{A.20}
\]

Manipulating the inequality \( \pi^* - \pi^0 > 0 \) gives the result that

\[
(T - S) < \frac{S^2 (1 - 2R)^2}{(2 - S)}. \tag{A.21}
\]
2. Suppose the incentive condition holds and that both firms are positioning asymmetrically. Arbitrarily taking $R > 0.5$, the profit a player would enjoy if it deviated from the asymmetric positioning is

$$\pi^d = \frac{RS}{1-RS} + \frac{0.5T\mu}{1-0.5T} - 2K,$$

(A.22)

which is always more than $\pi^*$. Using that

$$\delta^* = \frac{\pi^d - \pi^*}{\pi^d - \pi^0},$$

(A.23)

gives the result $\blacksquare$

**Proof of proposition 7:**

1. Say $R > 0.5$ in markets 1 through $L$. Then the profit incentive condition follows from the following comparison. If the firm stays in asymmetric equilibria it gets high profits in market 1 through $L$ and lower profits in all other markets. If the firm is in symmetric equilibrium, then it gets the symmetric duopoly profits in each market. Formally, with spatial concentration profits are equal to

$$\pi_1 = L \left( \frac{\mu R}{1-R} - K \right) + (M-L) \left( \frac{\mu (1-R)}{R} - K \right),$$

(A.24)

whereas in the symmetric case ($R = 0.5$), they are equal to

$$\pi_1 = M (\mu - K).$$

(A.25)

Solving for the superiority of the payoffs under spatial concentration, I get for firm 1 the following condition

$$L \frac{R}{1-R} + (M-L) \frac{(1-R)}{R} - M > 0$$

(A.26)

which solves to

$$\frac{M-L}{M} < R$$

(A.27)

This condition is guaranteed for the firms that leads in the majority of markets. For the other firm there is a boundary condition.

2. The derivative of $R$ with respect to $\mu$. At optimal prices,

$$R = \frac{\exp((a-p_1^*+p_2^*)/\mu)}{\exp((a-p_1^*+p_2^*)/\mu) + 1}$$

(A.28)

from which it follows that

$$R = \frac{\exp(a/\mu - 1/(1-R) + 1/R)}{\exp(a/\mu - 1/(1-R) + 1/R) + 1} = \frac{f(\mu)}{f(\mu) + 1},$$

(A.29)

with $f(\mu) > 0$. Taking derivatives implies that

$$R' = \frac{f'(\mu)}{f(\mu) + 1} - \frac{f'(\mu)f(\mu)}{(f(\mu) + 1)^2} = \frac{f'(\mu)}{(f(\mu) + 1)^2}$$

(A.30)
and in order to get an explicit expression one can develop
\[ f'(\mu) = f(\mu) \left( \frac{-a}{\mu^2} - \frac{R'}{(1-R)^2} - \frac{R'}{R^2} \right). \]  \hfill (A.31)
Combination of the last two equations gives the following result.
\[ R' = -\frac{a}{\mu^2} \left( \frac{(1-R)^2 R^3}{(1-R)^2 + R^3} \right) \]  \hfill (A.32)
thus if \( a > 0 \) then \( R' < 0 \). This means that if firm 1 is positioned advantageously, its share will become larger as horizontal differentiation is further diminished (as \( \mu \) goes down).

3. What holds for the minimal discount rate \( \delta^* \) in this case? A necessary condition is that the profit incentive to sustain the local asymmetries must hold. Therefore,
\[ \frac{M - L}{M} < R \text{ and } 1 - R < \frac{L}{M} \]  \hfill (A.33)
so that when there is a profit incentive to sustain the asymmetries, the following inequality is guaranteed.
\[ \frac{R}{1 - R} > \frac{M - L}{L} \]  \hfill (A.34)
The profit from a one time deviation from mutual asymmetric positioning is equal to \( \pi^d = \mu \left( LR / (1 - R) + (M - L) \right) - MK \). Thus,
\[ \delta^* = \frac{\pi^d - \pi^0}{\pi^d} = \frac{1}{\Phi} \frac{(M - L)}{L} \]  \hfill (A.35)
The critical discount factor that sustains the asymmetric positioning is smaller than 1. \hfill  

**Proof of proposition 8:**

1. Proof is the same as that given in the text preceding proposition 1

2. These statements are directly proven in the text

   (a) It is obvious that \( \lim_{\mu \to 0} \mu \left( 1 - \Phi^{-1} \right) = 0 \). Applying l’Hospital’s rule to \( \lim_{\mu \to 0} \mu \left( \Phi - 1 \right) \), I get
   \[ \lim_{\mu \to 0} \mu \left( \Phi - 1 \right) = \lim_{\mu \to 0} \frac{\Phi'}{1/\mu^2} = \lim_{\mu \to 0} \frac{(a_h - a_{\ell}) \Phi^2}{\Phi + \Phi^2} = (a_h - a_{\ell}) \]  \hfill (A.36)

   (b) Again, applying l’Hospital’s rule,
   \[ \lim_{\mu \to \infty} \mu \left( \Phi - 1 \right) = \lim_{\mu \to \infty} \frac{\Phi'}{1/\mu^2} = \lim_{\mu \to \infty} \frac{(a_h - a_{\ell}) \Phi^2}{\Phi + \Phi^2} = \frac{(a_h - a_{\ell})}{3}. \]  \hfill (A.37)

   Further,
   \[ \lim_{\mu \to \infty} \mu \left( 1 - \Phi^{-1} \right) = \lim_{\mu \to \infty} \frac{\Phi'}{\Phi^2/\mu^2} = \lim_{\mu \to \infty} \frac{(a_h - a_{\ell})}{\Phi + \Phi^2} = \frac{(a_h - a_{\ell})}{3}. \]  \hfill (A.38)

   This completes the proof. \hfill ■
Proof of proposition 9

1. Proof is the same as that given in the text preceding proposition 1

2. These statements are directly proven in the text. The only comment worth making is that if

\[ \mu (1 - \Phi^{-1}) \leq \Delta K \leq \mu (\Phi - 1) \]

one firm will position at \( a_h \) while the other firm will position at \( a_\ell \). Because this result holds in a single market, it will necessarily also hold for two markets given the lack of arbitrage across markets. In simple terms, consumer demand is independent across markets, and therefore if there is no profitable deviation for the leader or the lagger in a single market, there is no profitable deviation in \( M \) markets, regardless of who leads and who lags or where. \( \blacksquare \)

Proof of proposition 10

1. & 2. These results are derived in the text.

3. Applying l'Hôpital’s rule,

\[
\lim_{\mu \to 0} \mu (\Phi - 1) (1 - \Phi^{-1}) = \lim_{\mu \to 0} \mu \frac{\Phi + \Phi^{-1} - 2}{1/\mu} = \lim_{\mu \to 0} \mu (a_h - a_\ell) \frac{\Phi^2 (1 - 1/\Phi^2)}{(1 + \Phi + \Phi^2)} = (a_h - a_\ell)
\]

Following the same steps it is easy to show that \( \lim_{\mu \to \infty} \mu (\Phi - 1) (1 - \Phi^{-1}) = 0. \) \( \blacksquare \)

B Computing a relation between \( \Delta K \), \( \mu \), and \( \delta^* \).

Suppose that there are two markets and that there is an incentive to reposition from \( a_\ell \) to \( a_h \). The profit with asymmetric positioning is

\[
\pi_i^* = \mu \left( \frac{RS}{1 - RS} + \frac{(1 - R) S}{1 - (1 - R) S} \right) - (K(a_h) + K(a_\ell)) \quad \text{(B.1)}
\]

The profit during deviation is equal to

\[
\pi_i^d = \mu \left( \frac{RS}{1 - RS} + \frac{0.5T_h}{1 - 0.5T_h} \right) - 2K(a_h) \quad \text{(B.2)}
\]

after which profits will be equal to

\[
\pi_i^0 = \frac{T_h}{1 - 0.5T_h} \mu - 2K(a_h) \quad \text{(B.3)}
\]

forever. Thus knowing that

\[
\delta^* = \frac{\pi_i^d - \pi_i^*}{\pi_i^d - \pi_i^0}, \quad \text{(B.4)}
\]

an implicit relation between \( \Delta K \), and \( \mu \) exists for each value of \( \delta^* \).