Frugal Materialism and Risk Preferences

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Abstract: Frugal materialism is a tendency of consumers to become more sensitive to durability of products in response to a tightening budget constraint. This paper proposes a model of frugal materialism, and establishes a theoretical link between frugal materialism and the slope of risk aversion: For both their absolute and relative versions, frugal materialism and increasing risk aversion are nearly equivalent to each other.

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Introduction

In a recent award-winning article, Tully, Hershfield, and Meyvis (2015), hereafter “THM,” find that making people feel more financially constrained increases their “concern about the lasting utility of their purchases” (p. 59), and results in higher demand for durable material goods relative to perishable equally-priced versions of the same goods. The consumers that THM studied thus exhibit frugal materialism, whereby a reduction in disposable income (forcing one to become more “frugal”) increases their demand for more durable material goods (“materialism” in the sense of preference for acquiring more durable material possessions instead of buying perishables).

Frugal materialism seems like an intuitive property of consumer preferences: the poorer you are, the more attractive an increase in the durability of the goods you buy should be, ceteris paribus. THM conclude that during an economic downturn, a consumer who exhibits frugal materialism misses out on some of the happiness known to come from experiences. This paper proposes a parsimonious model of frugal materialism, and extends the implications of THM’s finding to the seemingly unrelated domain of risk preferences: It turns out that frugal materialism is nearly equivalent to increasing risk aversion.

Consider a consumer who faces a budget-constrained choice between two goods. One of the goods is a material good in that it can have various degrees of durability, and the other good is perishable (called “experience” throughout the paper to match the THM setting). I analyze a

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THM’s Study 6 provides a clear example of their finding: Subjects were to imagine they are walking around the city when it starts to rain, and they can either stop for a coffee in Starbucks or buy a poncho that is either described as “disposable” or “reusable” between subjects. Price is not an issue in the scenario, because the coffee costs the same as the poncho in all conditions. In a control group, the subjects expressed approximately the same strength of preference for both types of poncho over coffee. However, asking the subjects to “keep in mind their financial constraints” before making the decision dramatically increased their relative preference for the reusable poncho, while decreasing their relative preference for the disposable poncho.

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Note that throughout this paper, “materialism” means an individual-level increase in revealed preference for acquiring material possessions, not a personal value or a belief system as in Richins (2011). In other words, materialism is just a label for an increased consumer sensitivity to durability of material goods.
canonical model of such a consumer’s demand for different amounts of the two goods—an additively separable utility with one utility function $u$ for the material good and a possibly different second utility function $v$ for the experience. Let absolute (relative) frugal materialism be an increase in the absolute (relative) budget allocated to the material good in response to the joint event of (1) increasing the durability of the good and (2) shrinking the overall budget. It is not clear whether THM found only relative or also absolute frugal materialism because their dependent variable is only a single choice between an experience and a material good, so I analyze both versions of the phenomenon. Figure 1 summarizes my findings.

**Figure 1: Summary of results**

The first major finding of this paper is that absolute frugal materialism implies at least one of the two good-specific utility functions exhibits increasing absolute risk aversion (IARA), and both utilities being IARA in turn implies absolute frugal materialism. Therefore, when the two utility functions are affine transformations of each other, absolute frugal materialism is equivalent to IARA. Because most previous research has either found absolute risk aversion to be decreasing or argued a priori that it should be so (e.g. Bernoulli 1738, Pratt 1964, Arrow 1971, Rapoport, 3

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3 Note that researchers studying risk preferences often consider utility functions over different amounts of money, whereas the focus here is on the amount of a good as the argument of each utility function. The math is identical.
a finding of absolute frugal materialism would be surprising on its own. Additionally, a finding of absolute frugal materialism in consumers with CARA or DARA risk preferences in the same product domain and context would indicate an anomaly not captured by the canonical model used here.

The second major finding of this paper is a somewhat restricted analogue of the above relationship that applies to the relative versions of the two constructs: I show that when the two utility functions are affine transformations of each other and the absolute risk aversion is concave, relative frugal materialism is equivalent to increasing relative risk aversion (IRRA). When the two utility functions are distinct, CARA (which are IRRA) preferences imply relative frugal materialism, but CRRA preferences do not. A finding of relative frugal materialism but not absolute frugal materialism in the context of two closely related goods thus zeroes in on non-IARA and IRRA preferences in accordance with Arrow’s (1971) famous hypothesis.

Model

Let there be two goods, one called an experience and one called a material product. Both goods cost the same per unit, and a consumer has a budget $B$ – the total units of both goods he can afford. The material product can be durable in that an expected number $\lambda$ of future consumption opportunities exists during which a unit purchased today will still be available, with $\lambda$ including any potential temporal discounting of future consumption. The material product is neither a substitute nor a complement for the experience, so the utility of consuming $E$ of experience and $M$ of the material product with durability $\lambda$ is additively separable. Moreover, the assumption that the

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4 Both the “experience” and the “product” are just generic goods in this paper; consumer framing of goods as either experiences or products is not modeled. I label the good with variable durability a “material product.”
consumer experiences diminishing marginal utility (concavity of each univariate utility) is standard:

**Assumption:** Utility\( (E, M; \lambda) = (1 + \lambda)u(M) + v(E) \) with \( u \) and \( v \) concave.

To determine his demand, the consumer selects the amount \( M^* \) of product and the amount \( E^* \) of the experience to purchase to maximize his utility such that the budget constraint \( E + M \leq B \) holds:

\[
\{M^*, E^*\} = \arg \max_{M \geq 0, E \geq 0} (1 + \lambda)u(M) + v(E) \text{ subject to } M + E \leq B
\]

(1)

In terms of the above notation, THM find tightening the budget constraint \( B \) increases the difference between demand for a durable version (high \( \lambda \)) and demand for the disposable version (low \( \lambda \)) of the material product. Considering a small change in durability, we can employ the tools of calculus to define local absolute (relative) frugal materialism in terms of the cross partial of (percentage) demand for the material product in budget and durability:

**Definition:** A consumer exhibits absolute frugal materialism when \( \frac{\partial^2 M^*}{\partial \lambda \partial B} < 0 \), and exhibits relative frugal materialism when \( \frac{\partial^2}{\partial \lambda \partial B} \left( \frac{M^*}{B} \right) < 0 \).

The goal of this paper is to explore what the sign of this cross partial teaches us about the shape of \( u \) and \( v \). The first main result of this paper follows (see the Appendix for all proofs in this paper):

**Proposition 1:** In terms of the absolute risk aversions of utilities \( u \) and \( v \) evaluated at the optimal consumption bundle \( A_u = -\frac{u''(M^*)}{u'(M^*)} \) and \( A_v = -\frac{v''(E^*)}{v'(E^*)} \), the demand cross partial driving absolute frugal materialism can be expressed as

\[
\frac{\partial^2 M^*}{\partial \lambda \partial B} = -\frac{A_u' A_u + A_v' A_v}{(1 + \lambda)(A_u + A_v)^3}.
\]
The implications of Proposition 1 are straightforward: Because absolute risk aversions of concave functions are positive by construction, it follows that \( \frac{\partial^2 M^*}{\partial \lambda \partial B} < 0 \iff A_u'A_v + A_u'A_v > 0 \); that is, consumers exhibit absolute frugal materialism iff a weighted average of their absolute risk aversions of \( u \) and \( v \) is increasing. Thus, either \( u \) or \( v \) of absolute frugal materialists must be IARA; the popular CARA and DARA specifications rule out frugal materialism. See Figure 1 for an illustration of these implications. The second main result of this paper follows:

**Proposition 2:** In terms of the absolute risk aversions \( A_u \) and \( A_v \) and the relative risk aversions denoted \( R_u = M^* A_u \) and \( R_v = E^* A_v \), the percentage-demand cross partial driving relative frugal materialism can be expressed as

\[
\frac{\partial^2}{\partial \lambda \partial B} \left( \frac{M^*}{B} \right) = -\frac{(R_u' + R_v')(A_u + A_v) + (A_u' - A_v')(R_v - R_u)}{B^2 (1 + \lambda)(A_u + A_v)^3}.
\]

The implications of Proposition 2 are less stark than those of Proposition 1 because the expression in the numerator is more complicated. Nevertheless, it is immediate that CARA utilities imply \( \frac{\partial^2}{\partial \lambda \partial B} \left( \frac{M^*}{B} \right) < 0 \) because the second term in the numerator is zero and the first term is positive due to CARA implying IRRA.

When \( u = \alpha v \), the sign of the slope of relative risk aversion is tightly connected to relative frugal materialism. It is immediate from Proposition 2 that CRRA implies

\[
\frac{\partial^2}{\partial \lambda \partial B} \left( \frac{M^*}{B} \right) = 0 \quad \text{because both terms are zero (}u = \alpha v \text{ implies } R_v = R_u\text{). It turns out CRRA is precisely the boundary case because of the following corollary:}
\]

**Corollary to Proposition 2:** When \( u = \alpha v \) and ARA is concave, consumers exhibit relative frugal materialism iff the relative risk aversion of \( u \) is increasing.
Concrete Examples of Utility Functions

This section illustrates the two main results on several concrete and popular examples of consumer-preference models. Table 1 at the end of the section collects all the formulae for easy reference.

Quadratic utility (example if IARA)

Consider a quadratic utility

\[ U(M, E; \lambda) = (1 + \lambda) a \left( M - \frac{M^2}{2} \right) + b \left( E - \frac{E^2}{2} \right) \]

of Dixit (1979), where \( a, b > 0 \) are constants that weigh the relative importance of the two goods. Note these preferences involve \( u = \alpha v \) in equation 1, and both \( u \) and \( v \) are IRRA and IARA. The consumer solves

\[
\max_{M \geq 0, E \geq 0} \left( 1 + \lambda \right) a \left( M - \frac{M^2}{2} \right) + b \left( E - \frac{E^2}{2} \right) \text{ subject to } M + E = B
\]

The solution is

\[ M^* = \frac{bB + a(1 + \lambda) - b}{a(1 + \lambda) + b} \]

so consumers exhibit both absolute frugal materialism as predicted by IARA and Proposition 1:

\[ \frac{\partial^2 M^*}{\partial \lambda \partial B} = \frac{-a(1-a)}{(1+a\lambda)^2} < 0 \]

and also relative frugal materialism as predicted by IRRA and the Corollary to Proposition 2:

\[ \frac{\partial^2}{\partial \lambda \partial B} \left( \frac{M^*}{B} \right) = - \frac{2ab}{B^2 \left[ a(1 + \lambda) + b \right]^2} < 0 \]

To gain insight into quadratic preferences, consider the slope of relative demand in the budget:

\[ \frac{\partial}{\partial B} \left( \frac{M^*}{B} \right) = \frac{b - a(1 + \lambda)}{B^2 \left[ a(1 + \lambda) + b \right]} < 0 \iff \lambda > \frac{b}{a} - 1 \]

In words, given sufficient durability to make a unit of the material good preferable to a unit of the experience, an increase in the budget decreases the proportion of the budget spent on the material good. Figure 2 assumes the consumer
values the non-durable versions of the two goods equally, and shows what happens when the budget increases and the product is durable: For small budgets, the consumer buys only the material good, allocating 100% of his budget to it. As his budget increases, he adds some experience into the mix. In this sense, quadratic preferences capture the idea of perishable “experience” as a luxury, and the possible intuition that the THM result is obvious because poorer people should not waste their scarce money on coffee when they can get a durable poncho instead.

**Figure 2: Quadratic preferences with \( a=b=1 \)**

![Figure 2: Quadratic preferences with \( a=b=1 \)](image)

Note to figure: The curves are indifference curves. The thin downward-sloping straight lines are budget constraints. The thick upward-sloping line is the locus of solutions to equation 2.

**Cobb-Douglas utility (example of CRRA, and so DARA)**

Another textbook example of preferences is the Cobb-Douglas utility function

\[
U(M,E; \lambda) = (1 + \lambda) a \log(M) + b \log(E),
\]

where \( a > 0 \) and \( b > 0 \) again represent weights of the individual goods within the total utility. Note that Cobb-Douglas preferences involve \( u = \alpha v \) in equation 1, and both \( u \) and \( v \) are CRRA and DARA. The consumer solves
\[
\max_{M,E} \left(1 + \lambda \right) a \log(M) + b \log(E) \quad \text{subject to} \quad M + E = B
\] (3)

It is well known that the solution to this problem is \( M^* = \frac{a \left( 1 + \lambda \right)}{a \left( 1 + \lambda \right) + b} B \), so the consumer splits his budget according to the effective weight of each good in overall utility. Durability simply increases the effective weight of the product. Clearly, these preferences support the potential intuition that relative frugal materialism is surprising because richer people should just buy more of everything proportionally instead of shifting their relative demand towards one particular good:

Because the percentage demand does not depend on the budget, it is immediate that

\[
\frac{\partial^2}{\partial \lambda \partial B} \left( \frac{M^*}{B} \right) = \frac{\partial}{\partial B} \left( \frac{M^*}{B} \right) = 0,
\]

so Cobb-Douglas preferences rule out relative frugal materialism as predicted by CRRA and the Corollary to Proposition 2. Because

\[
\frac{\partial^2 M^*}{\partial \lambda \partial B} = \frac{a(1-a)}{(1+a\lambda)^2} > 0,
\]

Cobb-Douglas preferences also rule out absolute frugal materialism as predicted by DARA and Proposition 1.

**Figure 3: Cobb-Douglas preferences (example with \( a=b \))**
As an aside, the type of preferences that imply \( \frac{M^*}{B} \) does not depend on \( B \) already have a name—“homothetic.” Formally, a utility function is homothetic when a monotonic transformation of it (i.e., an alternative representation of the same underlying preferences) exists that is homogeneous of degree 1: \( U(cM, cE) = cU(M, E) \). A well-known example of homothetic utility functions is the constant elasticity of substitution (CES) family. Graphically, homothetic preferences have indifference curves whose slopes are constant along rays beginning at the origin. See Figure 3 for an illustration. When \( u = \alpha v \), homotheticity is clearly inconsistent with relative frugal materialism.

**Stone-Geary utility (DRRA, and so DARA):**

So far, we have seen two examples with the crucial cross partials that are either negative or zero. Another example is needed to show the relative-demand cross partial can also be positive, and so its sign is thus not a priori even weakly constrained by standard consumer theory. Consider the following generalization of the Cobb-Douglas preferences, due to Geary (1950) and used in Marketing by Iyengar et al. (2011): \( U(M, E) = a \log(M - m) + b \log(E - e) \), where \( m \geq 0 \) and \( e \geq 0 \) represent minimum amounts of \( M \) and \( E \) that the consumer needs to purchase (Cobb-Douglas is the special case of \( e = m = 0 \)), with the utility only valid for \( M > m \) and \( E > e \). Note that Cobb-Douglas preferences involve \( u = \alpha v \) in equation 1, and both \( u \) and \( v \) are DRRA and DARA.
The solution to the consumer budget-allocation problem is

\[ M^* = m + \frac{a(1 + \lambda)}{a(1 + \lambda) + b}(B - e - m), \]

and the key cross partial for relative frugal materialism is

\[ \frac{\partial^2}{\partial \lambda \partial B} \left( \frac{M^*}{B} \right) = \frac{ab(m + e)}{B^2 \left[a(1 + \lambda) + b\right]^2} > 0. \]

Therefore, Stone-Geary preferences cannot exhibit relative frugal materialism as predicted by the Corollary to Proposition 2.

When I set \( m = 0 < e \), I obtain a model of a consumer for whom increased durability makes him spend a greater part of his discretionary budget \((B-e)\) on ponchos while very financially constrained consumers (i.e., \( B \approx e \)) spend all their money on coffee. Such a consumer’s intuition may be that coffee is a necessity, so a budget reduction shifts their demand to coffee, and the shift is faster with durable ponchos because they obviously represent a bigger chunk of the discretionary budget.

**Hyperbolic absolute risk-aversion utility (a general family)**

A popular utility function in the study of risk preferences is the HARA function

\[ U(x) = \frac{1-\gamma}{\gamma} \left( \frac{ax}{1-\gamma} + b \right)^\gamma, \]

known to allow all three possible combinations of increasing and decreasing absolute and relative risk aversions. For tractability, I consider the following three-parameter \( u=v \) example \((a, b, \text{ and } \gamma \text{ are parameters}):\)

\[ \max_{M,E} \left(1 + \lambda\right) \left( \frac{aM}{1-\gamma} + b \right)^\gamma + \left( \frac{aE}{1-\gamma} + b \right)^\gamma \text{ subject to } M + E = B. \]  \( \text{(4)} \)

It is well known that a HARA function is DARA if \( \gamma<1 \), IARA if \( \gamma>1 \), and CARA as \( \gamma \to \infty \). As Proposition 1 predicts, the sign of the absolute-demand cross partial hinges only on \( \gamma \) because
\[
\frac{\partial^2 M^*}{\partial \lambda \partial B} = \frac{1}{1-\gamma} F(\gamma, L), \text{ where } F(\gamma, L) = \left(1 + \frac{L^{2-\gamma}}{1 + (1 + L)^{\frac{1}{\gamma-1}}}ight)^2 > 0 \text{ for all } \gamma, \text{ and so } \frac{\partial^2 M^*}{\partial \lambda \partial B} < 0 \iff \gamma > 1.
\]

It is also well known that a HARA function is IRRA iff \( b > 0 \). Indeed, the sign of the relative-demand cross partial hinges only on \( b \): \[
\frac{\partial^2}{\partial \lambda \partial B} \left( \frac{M^*}{B} \right) = -\frac{2b}{aB^2} F(\gamma, L) < 0 \iff b > 0.
\]

### Table 1: Summary of concrete examples under the \( u = \alpha v \) assumption

<table>
<thead>
<tr>
<th>Name of utility function</th>
<th>( u(x) )</th>
<th>( \frac{\partial^2}{\partial \lambda \partial B} \left( \frac{M^*}{B} \right) )</th>
<th>( \frac{\partial^2 M^*}{\partial \lambda \partial B} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stone-Geary (DARA, DRRA)</td>
<td>( \log(x - x_{\text{min}}) )</td>
<td>( \frac{\alpha (1-\alpha)(M_{\text{min}} + E_{\text{min}})}{B^2(1+\alpha \lambda)^2} &gt; 0 )</td>
<td>( \frac{\alpha (1-\alpha)}{(1+\alpha \lambda)^2} &gt; 0 )</td>
</tr>
<tr>
<td>Cobb-Douglas (DARA, CRRA 1)</td>
<td>( \log(x) )</td>
<td>0</td>
<td>( \frac{\alpha (1-\alpha)}{(1+\alpha \lambda)^2} &gt; 0 )</td>
</tr>
<tr>
<td>Isoelastic (DARA, CRRA ( r ))</td>
<td>( x^{1-r} - 1 ) ( \frac{1}{1-r} )</td>
<td>0</td>
<td>( \frac{w}{r(1+\lambda)(1+w)} &gt; 0 ) \text{ where } ( w = \sqrt{\frac{\alpha (1+\lambda)}{(1-\alpha)^2}} )</td>
</tr>
<tr>
<td>Exponential (CARA ( a ), IRRA)</td>
<td>( \frac{1-e^{-\alpha x}}{a} )</td>
<td>( \frac{-1}{2B^2a(1+\lambda)} &lt; 0 )</td>
<td>0</td>
</tr>
<tr>
<td>Quadratic (IARA, IRRA)</td>
<td>( x - \frac{x^2}{2} )</td>
<td>( \frac{-2\alpha(1-\alpha)}{B^2(1+\alpha \lambda)^2} &lt; 0 )</td>
<td>( \frac{-\alpha (1-\alpha)}{(1+\alpha \lambda)^2} &lt; 0 )</td>
</tr>
<tr>
<td>Hyperbolic risk aversion (general)</td>
<td>( \left(\frac{ax}{1-\gamma} + b\right)^\gamma )</td>
<td>( -\frac{2b}{aB^2} F(\gamma, L) &lt; 0 \iff b &gt; 0 )</td>
<td>( \frac{1}{1-\gamma} \frac{F(\gamma, L) &gt; 0}{&gt;0} \iff \gamma &gt; 1 )</td>
</tr>
</tbody>
</table>
Discussion

Frugal materialism can be rationalized in a standard microeconomic model with additively separable utility, but it is not a generic property of standard preferences. This paper documents a close relationship between frugal materialism and the seemingly unrelated domain of decision-making under risk in a standard model of consumer demand. I show that under mild assumptions, consumers who exhibit absolute (relative) frugal materialism should exhibit increasing absolute (relative) risk aversion in the same context. This newly discovered relationship both broadens the implications of THM’s results, and suggests further directions for empirical work at the intersection of risk and materialism. I now briefly expand on both of these contributions in turn.

The theoretically appropriate and empirically relevant slope of risk aversion has received much discussion since Pratt’s (1964) definition of the concept. Regarding the slope of absolute risk aversion, most research to date has either found it to be negative (i.e., DARA, e.g., Rapoport, Zwick, and Funk 1988, Levy 1994, and others), or argued a priori that it should be so (Bernoulli 1738, Pratt 1964, Arrow 1971, Gollier and Pratt 1996, and others). Arrow (1971) advanced a DARA-IRRA hypothesis as the most plausible pair of slopes of absolute and relative risk aversion, and recent work by Brocas et al. (2018) finds empirical evidence of Arrow’s hypothesis. The relationship between frugal materialism and the slope of risk aversion thus extends the interpretation of THM as follows: On one hand, a finding of absolute frugal materialism would be quite surprising because subjects who exhibit it should have IARA preferences over at least one of the goods in question. On the other hand, a finding of relative but not absolute frugal materialism would be consistent with Arrow’s hypothesis and the prevailing understanding of risk aversion in the literature. A finding of no frugal materialism would suggest DRRA preferences, found by a relative minority of work to date (for an example of a DRRA finding, see Ogaki and Masao 2001).
The implications for further empirical work are at least threefold: First, we need to find whether and when consumers exhibit both forms of frugal materialism or only the relative version. Second, we need to conduct within-subject measurements of both the intensity of frugal materialism and the slope of risk aversion to empirically test the proposed link. Finally, we need to explore potential relationships between frugal materialism and other important behaviors under risk. For example, frugal materialism may be related to precautionary savings (Kimball 1990), because a durable good may serve as a useful hedge against a future income shock.

Beyond the above implications for risk preferences, frugal materialism of either kind is also incompatible with homothetic preferences commonly used in the empirical literature (e.g., the multinomial logit model of consumer demand). Future modelers need to develop non-homothetic models, especially when attempting to model consumers with varying financial constraints. Such models are rare in the literature; the seminal example is Allenby and Rossi (1991) extended in Allenby, Garratt, and Rossi (2010). Moreover, the sensitivity of THM’s cross partial to the curvature of the utility function suggests specific functional forms of the non-homothetic models matter a lot for matching basic patterns of the data; for example, the popular Stone-Geary model is inconsistent with relative frugal materialism.
References


Appendix: Proofs of Propositions

Proof of Proposition 1: Because both utilities are increasing in quantity consumed, the budget constraint binds and the consumer’s problem in equation 1 is equivalent to
\[
\max_M (1 + \lambda) u(M) + v(B - M),
\]
which has the first-order condition
\[
(1 + \lambda) u'(M^*) = v'(B - M^*). \tag{FOC}
\]
To derive the cross partial of interest, differentiate the FOC twice, starting with the budget, and express in terms of the absolute risk aversions of \(u\) and \(v\) denoted \(A_u = \frac{-u''}{u'}\) and \(A_v = \frac{-v''}{v'}\) respectively:
\[
(1 + \lambda) u'' \frac{\partial M^*}{\partial B} = v'' \left(1 - \frac{\partial M^*}{\partial B}\right) \Rightarrow \frac{\partial M^*}{\partial B} = \frac{v''}{(1 + \lambda) u'' + v''} = \frac{A_v}{A_u + A_v}, \tag{2}
\]
where the arguments of \(u, v, \) and their derivatives have been suppressed for clarity (from this point in, \(u, A_u, \) and their derivatives always have \(M^*\) as the argument, whereas \(v, A_v, \) and their derivatives always have \(E^* = B - M^*\)). The formula is intuitive: When the budget increases, the consumer buys more of the material good when the utility of the experience is diminishing faster (larger \(v''\)) relative to the effective (durability-weighted) utility of the material good. The \((1 + \lambda)\) weight drops out when \(\frac{\partial M^*}{\partial B}\) is expressed in terms of the absolute risk aversions, because the implicit \(M^*\) and \(E^*\) arguments satisfy the FOC.

To finish the derivation of \(\frac{\partial^2 M^*}{\partial B \partial \lambda}\), differentiate equation 2 with respect to \(\lambda\), remembering the argument of \(A_v\) is \(B - M^*\), and hence \(\frac{\partial A_v}{\partial \lambda} = -\frac{\partial M^*}{\partial \lambda} A'_v\).
Finally, differentiate the FOC with respect to $\lambda$, and again express the result in terms of the risk aversions:

$$u' + (1 + \lambda) \frac{\partial M^*}{\partial \lambda} u'' = - \frac{\partial M^*}{\partial \lambda} v'' \Rightarrow \frac{\partial M^*}{\partial \lambda} = - \frac{u'}{(1 + \lambda)u'' + v''} = \frac{1}{(1 + \lambda)(A_u + A_v)}.$$  (4)

Plugging equation 4 into equation 3 completes the proof.

*QED Proposition 1*

**Proof of Proposition 2:** To calculate the cross partial of percentage demand, first differentiate with respect to budget:

$$\frac{\partial}{\partial B} \left( \frac{M^*}{B} \right) = \frac{B \frac{\partial M^*}{\partial B} - M^*}{B^2} = \left( \frac{1}{B^2} \right) \left( B \frac{\partial M^*}{\partial B} - M^* \right).$$

Now differentiate by durability, and plug in the result of Proposition 1:

$$B^2 \frac{\partial^2}{\partial \lambda \partial B} \left( \frac{M^*}{B} \right) = - \frac{\partial M^*}{\partial \lambda} + B \frac{\partial^2 M^*}{\partial \lambda \partial B} = - \frac{(A_u + A_v)^2 + B (A_u A_v + A_v A_u)}{(1 + \lambda)(A_u + A_v)^3}.$$  

When the relative risk aversions are denoted $R_u \equiv M^* A_u, R_v \equiv E^* A_v$, the denominator can be expressed in terms of $R_u' = M^* A_u + A_v, R_v' \equiv E^* A_v + A_v$ as follows to prove the result:

$$A_v (MA_u' + A_u + EA'_v - EA_v' + EA_v' + A_v) + A_u (EA_v' + A_v + MA'_v - MA_v' + MA_v' + A_u)
= A_v (R_u' + R_v' + E (A_u' - A_v')) + A_u (R_u' + R_v' + M (A_v' - A_v'))
= (R_u' + R_v')(A_u + A_v) + (A_u' - A_v')(R_v - R_u).$$

*QED Proposition 2.*
Proof of Corollary to Proposition 2: To show IRRA $\Rightarrow$ relative frugal materialism, it is enough to focus on the DARA case because we already know IARA and CARA are sufficient on their own. IRRA makes the first term in the numerator of $\frac{\partial^2}{\partial \lambda \partial B} \left( \frac{M^*}{B} \right)$ positive, so making the second term also positive, that is, $(A'_u - A'_v)(R_v - R_u) > 0$, is sufficient for relative frugal materialism. There are two cases:

1) When $M>E$, IRRA and $u = \alpha v$ means $R_v < R_u$, so we need $A'_u < A'_v$. From DARA, both slopes of ARA are negative. Because $M>E$, $A'_u < A'_v$ when ARA is steeper at the higher of the two consumption amounts, for which global concavity of ARA is sufficient.

2) When $M<E$, IRRA means $R_v > R_u$, and so we need $A'_u > A'_v$. From DARA, both slopes of ARA are negative. Because $M<E$, $A'_u > A'_v$ is steeper at the higher of the two consumption amounts, for which global concavity of ARA is sufficient.

To show relative frugal materialism $\Rightarrow$ not DRRA, note DRRA makes the first term in the numerator of $\frac{\partial^2}{\partial \lambda \partial B} \left( \frac{M^*}{B} \right)$ negative, so making the second term also negative, that is, $(A'_u - A'_v)(R_v - R_u) < 0$, is sufficient to rule out relative frugal materialism. There are two cases:

1) When $M>E$, DRRA and $u = \alpha v$ means $R_v > R_u$, so we need $A'_u < A'_v$. Because DRRA implies DARA, both slopes are negative, and the same argument as in the above case 1 shows global concavity of ARA is sufficient.

2) When $M<E$, DRRA and $u = \alpha v$ means $R_v < R_u$, so we need $A'_u > A'_v$. Because DRRA implies DARA, both slopes are negative, and the same argument as in the above case 2 shows global concavity of ARA is sufficient. QED Corollary to Proposition 2