Algebra of the Heckscher-Ohlin Model

Production Function for Sector j with n inputs

$$Q_j = f_j(v_{1j}, v_{2j}, v_{3j}, \ldots, v_{nj}) = v_{nj} f_j(v_{1j}/v_{nj}, v_{2j}/v_{nj}, v_{3j}/v_{nj}, \ldots, 1) = v_{nj}\, f_j(\boldsymbol{v}_j / \boldsymbol{v}_{nj})$$

Unit-Isoquant

$$1 = f_j(v_{1j}, v_{2j}, v_{3j}, \ldots, v_{nj})$$

Cost Minimization to find input intensities A

$$\min_{v \text{ such that } 1 = f_i(v)} \; w'v \qquad = w'A_i$$

Input intensity matrix with m products (each column is a vector of input intensities sector i.

$$A = \begin{bmatrix} A_1 & A_2 & \ldots & A_m \end{bmatrix}$$

Factor market equilibrium **AQ=v**

Zero Profits **A' w = p**

Stolper-Samuelson Derivatives $(d\,\mathbf{w}) = (\mathbf{A}')^{-1}(d\mathbf{p})$

$\mathbf{A}'(d\,\mathbf{w}) + (d\,\mathbf{A}')\mathbf{w} = d\mathbf{p}$

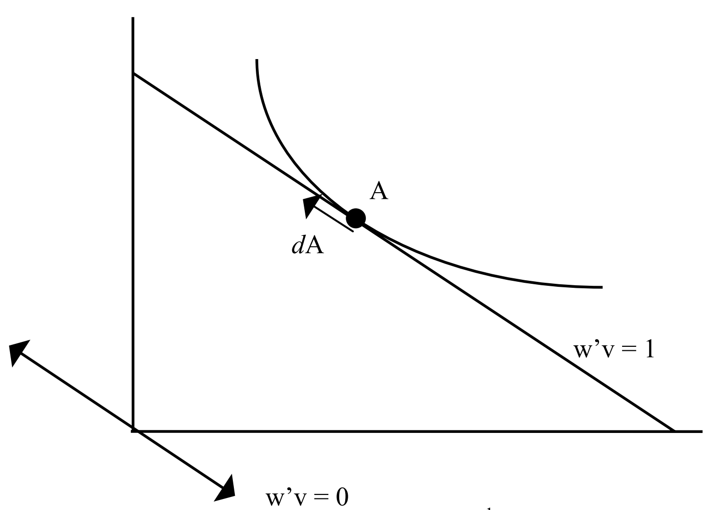
because cost minimization implies $(d\,\mathbf{A}')\mathbf{w} = 0$ for the reasons below.

Differentiating the unit isoquant $1 = f(\mathbf{v})$ implies $0 = \mathbf{f}'\, d\mathbf{v}$

Minimizing costs subject to the unit isoquant constraint implies $0 = \mathbf{w} + \lambda f'(\mathbf{v})$

Premultiply by $d\mathbf{v}'$: $0 = d\mathbf{v}'\mathbf{w} + \lambda\, d\mathbf{v}'\, f'(\mathbf{v}) = d\mathbf{v}'\mathbf{w}$

Figure:



Rybczynski Derivatives $(d\,\mathbf{q}) = (\mathbf{A})^{-1}(d\mathbf{v})$

Samuelson Reciprocity Relations $(d\,\mathbf{q}) / (d\mathbf{v}) = [d\mathbf{w} / d\mathbf{p}]'$

Heckscher-Ohlin-Vanek Theorem (Even model)

Homothetic Tastes: $\mathbf{C}_i = s_i\, \mathbf{Q}_{world}$

Trade: $\mathbf{T}_i = \mathbf{Q}_i - \mathbf{C}_i = \mathbf{A}^{-1}\, \mathbf{v}_i - s_i\, \mathbf{Q}_{world} = \mathbf{A}^{-1}\, (\mathbf{v}_i - s_i\, \mathbf{v}_{world})$

INTERPRET : $\mathbf{A}\, \mathbf{T}_i = \mathbf{v}_i - s_i\, \mathbf{v}_{world}$

Trade Balance $\mathbf{p}'\, \mathbf{T}_i = 0,$

Thus $0 = \mathbf{p}'\mathbf{A}^{-1}\, \mathbf{v}_i - s_i\, \mathbf{p}'\mathbf{A}^{-1}\, \mathbf{v}_{world} = GNP_i - s_i\, GNP_{world}$

$$s_i = \frac{p' A^{-1} v_i}{p' A^{-1} v_{world}} = \frac{w' v_i}{w' v_{world}} = \frac{GNP_i}{GNP_{world}}$$

$$A^{-1} = \begin{bmatrix} A_{K1} & A_{K2} \\ A_{L1} & A_{L2} \end{bmatrix}^{-1} = \begin{bmatrix} A_{L2} & -A_{K2} \\ -A_{L1} & A_{K1} \end{bmatrix} / \,(A_{K1}A_{L2} - A_{K2}A_{L1})$$

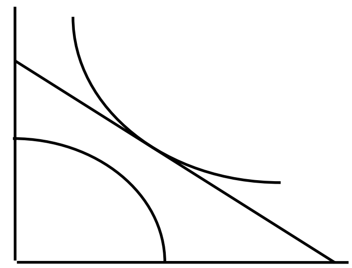
$$= \begin{bmatrix} A_{L2} & -A_{K2} \\ -A_{L1} & A_{K1} \end{bmatrix} / \, A_{L1}A_{L2}\left(\frac{A_{K1}}{A_{L1}} - \frac{A_{K2}}{A_{L2}}\right)$$

Thus if sector 1 is capital intensive, then the inverse of A has the sign pattern $\begin{bmatrix} + & - \\ - & + \end{bmatrix}$

The HOV theorem is:

$$\begin{bmatrix} + & - \\ - & + \end{bmatrix}\begin{bmatrix} + \\ - \end{bmatrix} = \begin{bmatrix} + \\ - \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} + & - \\ - & + \end{bmatrix}\begin{bmatrix} - \\ + \end{bmatrix} = \begin{bmatrix} - \\ + \end{bmatrix}$$

What economic phenomenon is depicted in this diagram:
(Exercise: Label the diagram)

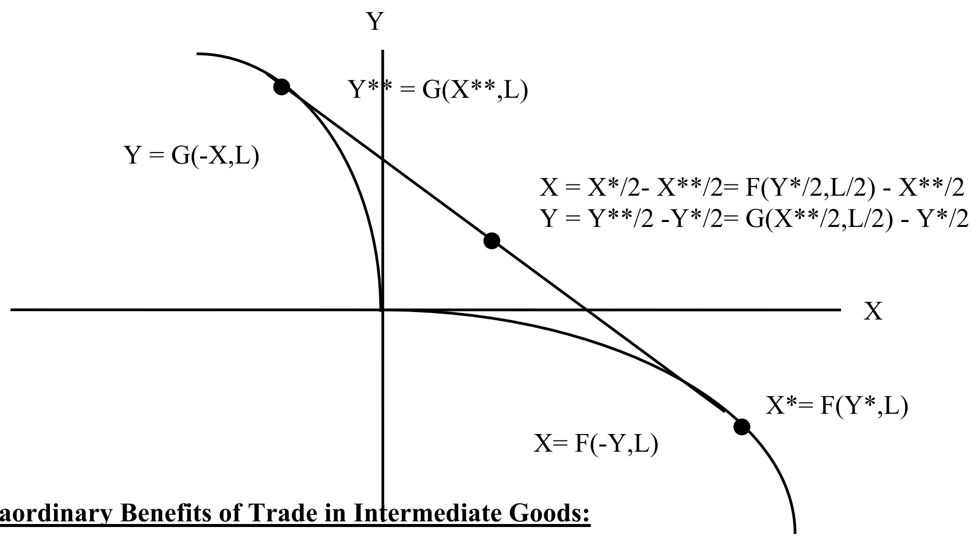


Intermediate Inputs (one direct input, labor)

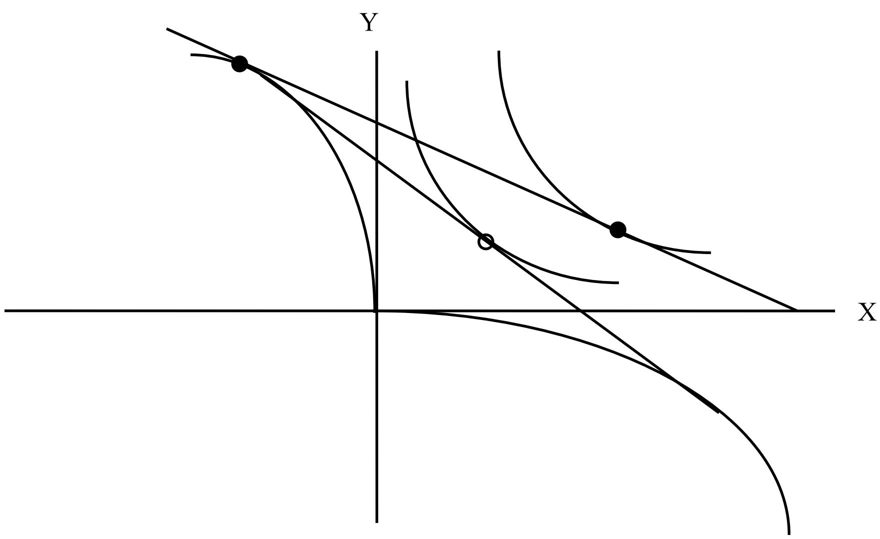
$$X = F(Y_x, L_x) - X_y = L_x f(Y_x/L_x) \quad - X_y$$

$$Y = G(X_y, L_y) - Y_x = L_y g(X_y/L_y) \quad - Y_x$$

X, Y feasible final output combinations:



Extraordinary Benefits of Trade in Intermediate Goods:



Exercise: Suppose that there are consumer goods and producer durables. Consumer goods are made with labor and producer durables. Producer durables are made with labor and producer durables. What does the picture look like? How does it depend on whether you have a comparative advantage in consumer goods or producer durables?

Pathologies / Fragility Issues

	Complete Specialization	Factor Intensity Reversals	Many, but equal number of goods and factors	More factors than goods	More goods than factors
Factor Price Equalization	Fails	Fails	OK		
Stolper Samuelson	OK	OK?	Fail - C		
Rybczynski	OK	OK?	Fail - C		
Reciprocity	OK	OK?	OK		
Heckscher_Ohlin	OK	Fails	Fail - C		
HOV	??	Fails	OK		

Fail - M means the mathematics doesn't go through
Fail - C means that the "message" of the model changes.