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AN INTERACTIVE APPROACH FOR MULTI-CRITERION OPTIMIZATION, WITH AN APPLICATION TO THE OPERATION OF AN ACADEMIC DEPARTMENT*†

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An interactive mathematical programming approach to multi-criterion optimization is developed, and then illustrated by an application to the aggregated operating problem of an academic department.

1. Introduction

One of the most common difficulties obstructing the successful application of mathematical programming techniques to real problems is the presence of multiple criteria. When there are only two criteria, a commonly accepted approach is to compute numerically the relevant portion of the tradeoff curve (efficient set, Pareto-optimal set) in criterion space and let the decision-maker select the point which he most prefers. However, when the computation of the tradeoff curve is onerous or when there are more than two criteria, there is no general agreement regarding how to proceed. Numerous prescriptions have been offered, nearly all of them ad hoc. See, for instance, the extensive surveys of Johnsen [8] and Roy [9].

One of the present authors recently proposed [5] a man-machine interactive mathematical programming approach to multi-criterion optimization. It assumes that a large-step gradient ascent algorithm would be applicable if the decision-maker were somehow able to specify an overall "preference function" to resolve the conflicts inherent in the given multiple criteria, but never actually requires this preference function to be identified explicitly. Instead, the algorithm calls only for such local information about the preference function as is actually needed to carry out the optimizing calculations. In other words, the viewpoint taken is this: adopt a mathematical programming technique of known efficiency, but implement it so as to require only the necessary information from the decision-maker concerning his preferences over the criteria.

It is surprising that such an obvious approach does not appear to have been studied previously. We are able to cite only the very recent independent work by Boyd [3] as being in a similar vein. There are, of course, other approaches that might be termed "interactive mathematical programming"—such as the Progressive Orientation Procedure [2] for multi-criterion linear programming—but in these other approaches the interaction is usually superimposed in an ad hoc manner rather than being dictated by the mathematical programming algorithm itself.

The interactive approach is described in the following section in the context of a specific mathematical programming algorithm (the Frank-Wolfe method). §3 then details a numerical application to the operation of an academic department. This example is taken from actual experience with a pilot implementation of the approach for the Graduate School of Management at UCLA, where we are attempting to install

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it as a permanent decision-making tool. A synopsis of the underlying mathematical model is presented in the Appendix.

2. An Interactive Mathematical Programming Approach to Multi-Criterion Optimization

We shall describe the proposed approach in the specific context of the well-known Frank-Wolfe algorithm. Many other particular algorithms could also be rendered interactive in a similar way, but we have selected Frank-Wolfe because of its simplicity, its robust convergence properties [7], [10], and because it is embodied in the computer program used to obtain the numerical results presented in §3.

The multi-criterion problem will be considered in the following form:

$$\text{(1)} \qquad \text{Maximize } U[f_1(x), f_2(x), \cdots, f_r(x)], \qquad \text{subject to } x \in X,$$

where $f_1, \cdots, f_r$ are r distinct criterion functions of the decision vector x, X is the constrained set of feasible decisions, and U is the decision-maker's overall preference function defined on the values of the criteria. The functions f_i and the set X are assumed to be explicitly known, but to retain a genuine multi-criterion flavor we *cannot* assume that U is explicitly known.

In order that (1) be amenable to solution by mathematical programming algorithms in general and the Frank-Wolfe method in particular, we shall assume henceforth that $X \subseteq R^n$ is a compact, convex set and that the objective function of (1) is differentiable and concave on X. Two useful sufficient conditions for the assumption of concavity are: (i) U is concave and each f_i is linear; (ii) U is concave increasing and each f_i is concave.

It is appropriate to review the mechanics of the Frank-Wolfe method applied to (1), ignoring for the moment the difficulty caused by the fact that U is only implicitly known:

Step 0. Choose an initial point $x_1 \in X$. Put $k = 1$.

Step 1. Determine an optimal solution y_k of the direction-finding problem

$$\text{(2)} \qquad \text{Maximize}_{y \in X} \; \nabla_x U[f_1(x_k), \cdots, f_r(x_k)] \cdot y.$$

Put $d_k = y_k - x_k$.

Step 2. Determine an optimal solution t_k of the step-size problem

$$\text{(3)} \qquad \text{Maximize}_{0 \leq t \leq 1} \; U[f_1(x_k + td_k), \cdots, f_r(x_k + td_k)].$$

Put $x_{k+1} = x_k + t_k d_k$, $k = k + 1$, and return to Step 1.

The usual interpretation of Step 1 is that it determines the "best" direction d_k (based on a local linear approximation to the decision-maker's utility function) in which to move away from x_k. At Step 2, the decision-maker determines the amount (t) of movement in this direction which maximizes his utility for this one-dimensional restriction of the overall problem. This scheme yields a sequence of improving feasible solutions to (1) which must converge to an optimal solution [7], [10].

Since U is not explicitly known, however, neither Step 1 nor Step 2 can be performed entirely by computer. They both require certain information about U. Fortunately the amount of required information is not great. Its acquisition from the decision-maker as needed gives rise to the man-machine interaction in our approach, as we now explain in more detail.

Step 1

The chain rule yields

$$\nabla_x U[f_1(x_k), \cdots, f_r(x_k)] = \sum_{i=1}^{r} \left(\frac{\partial U}{\partial f_i}\right)^k \nabla_x f_i(x_k), \tag{4}$$

where $(\partial U/\partial f_i)^k$ is the ith partial derivative of U evaluated at the point $[f_1(x_k), \cdots, f_r(x_k)]$, and $\nabla_x f_i(x_k)$ is the gradient of f_i evaluated at x_k. Thus the (linear) objective function of (2) is incompletely known because the $(\partial U/\partial f_i)^k$'s are not known. Notice, however, that the optimal solution y_k of (2) is not affected by positive scaling of the objective function. Hence one may divide the objective function by any positive coefficient $(\partial U/\partial f_i)^k$. The criterion which plays this distinguished role will be called the *reference criterion.*[1] We may assume without loss of generality that the reference criterion is the first one, so that (2) is equivalent to

$$\text{Maximize}_{y \in X} \sum_{i=1}^{r} w_i^k \nabla_x f_i(x_k) \cdot y, \tag{2'}$$

where we define

$$w_i^k \triangleq (\partial U/\partial f_i)^k / (\partial U/\partial f_1)^k, \qquad i = 1, \cdots, r. \tag{5}$$

The "weights" w_i^k reflect the decision-maker's *tradeoff* between criterion i and criterion 1 at the current point, and must be obtained before (2′) can be solved. There are two principal ways of approximating them. The first is based on the equation of the tangent hyperplane to the indifference (level) surface of U at the current point. When divided by $(\partial U/\partial f_1)^k$, this equation is

$$1(f_1 - f_1(x_k)) + w_2^k(f_2 - f_2(x_k)) + \cdots + w_r^k(f_r - f_r(x_k)) = 0.$$

Thus if the decision-maker is indifferent to a change in the values of criteria 1 and i in in the respective amounts Δ_1 and Δ_i, while all other criteria stay at their current values, then we have approximately

$$w_i^k = -\Delta_1/\Delta_i. \tag{6}$$

The approximation becomes arbitrarily exact as the amounts of change approach 0. Hence one way to obtain w_i^k is to determine what (infinitesimal) change Δ_1 in the first criterion "exactly compensates" for a change Δ_i in the ith criterion, with all other criteria remaining at their current values. This is generally termed the "indifference tradeoff" or "marginal rate of substitution" between the two criteria.

The other way to approximate w_i^k is based on the property of the gradient of U as the direction of steepest ascent. When all criteria except the first and ith are held constant, we have

$$w_i^k = \delta_i/\delta_1, \tag{7}$$

where $\delta_i:\delta_1$ is the "ideal" marginal proportion of change for these two criteria; that is, U increases most rapidly in an incremental sense if criterion i changes by δ_i units for each δ_1 change in criterion 1.

The relationship between formulae (6) and (7) is just the negative reciprocal relationship between the slopes of two perpendicular lines. This is pictured in Figure 1,

[1] Since the coefficients $(\partial U/\partial f_i)^k$ will usually be positive in practice, this is the case considered here. It is also possible to use a criterion with a negative coefficient as a reference criterion, in which case one would obviously divide through by the *negative* of this coefficient.

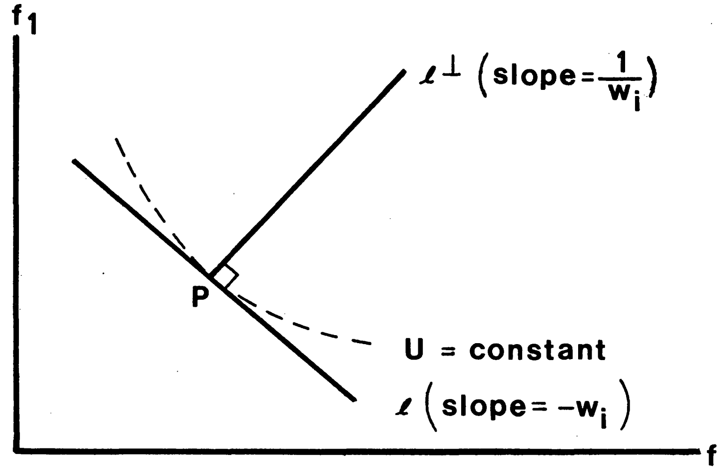

FIGURE 1

where line l represents the tangent to the (dotted) iso-preference curve through the current point $P = [f_1(x_k), \cdots, f_r(x_k)]$, and $l^{\perp}$ is its normal.

It is perhaps worth emphasizing that it is unnecessary in principle to consider all possible pairwise tradeoffs between criteria, although it might be advisable as a consistency check to obtain from the decision-maker more than the minimal r-1 tradeoffs necessary.

This completes our general discussion of how Step 1 can be carried out. In applications, of course, one would want to give very careful thought to the experimental procedure by which the tradeoff weights are elicited from the decision-maker [10].

Step 2

Since U is not explicitly available, (3) must be done directly by the decision-maker. This is not as difficult as it sounds because there is only *one variable*, t, and plots can be made of the values of all r criteria $f_i(x_k + td_k)$ as a function of t between 0 and 1. The decision-maker may superimpose these plots, if he wishes, to better visualize the problem of selecting the best value of t (this is done in the example presented in the next section). Alternatively, the computer can tabulate the values of the criteria at selected values of t.

We emphasize that what is required of the decision-maker is much easier than the comprehension of an arbitrary set of choices in r-space, which would indeed be a hopeless task. He need only comprehend a singly-parameterized curve in r-space, an object which he can see directly in component-wise parametric form. In the important case where all criteria are linear, the curve and its component-wise representation are simply line segments. Of course, this is not to deny that the task of carrying out Step 2 becomes more difficult as criteria become more numerous. But available evidence suggests that one can probably cope with at least 6–8 criteria without undue difficulty.

The Interactive Frank-Wolfe Algorithm

Thus we see that the Frank-Wolfe algorithm for (1) can be executed interactively with the decision-maker as follows:

Step 0. The decision-maker chooses an initial point $x_1 \in X$. Put $k = 1$.

Step 1. a. The decision-maker assesses his tradeoff weights w_i^k by subjective analysis of the current trial point via the relations (6) or (7).

b. Compute an optimal solution y_k of (2′). Put $d_k = y_k - x_k$.

Step 2. Plot the functions $f_i(x_k + td_k)$ over the unit interval, and have the decision-maker subjectively determine an optimal solution t_k to problem (3) by inspection. Put $x_{k+1} = x_k + t_k d_k$, $k = k + 1$, and return to Step 1.

Except perhaps for Step 0, the entire procedure can be viewed by the decision-maker as taking place in *criterion space* rather than in decision-variable space (a space that is usually of much higher dimension). This assumes, of course, that the decision-maker relegates Step 1b and the mechanical portions of Step 2 to a technical assistant or a computer program. Often the initialization, Step 0, can also be done from the same viewpoint by numerically solving for x_1 corresponding to an attainable point in criterion space provided by the decision-maker. The advantage of functioning entirely in criterion space is that it allows the decision-maker to concentrate on making tradeoff judgments with no extraneous details to distract him.

In practice, the decision-maker will be able to furnish only approximations to the w_i^k's and t_k. Fortunately the convergence of this procedure is quite robust in the face of such approximation errors. Hogan [7] has recently shown that, loosely speaking, infinite convergence still holds provided the degree of approximation to the w_i^k's becomes ever more exact as k increases, and provided also that an "adequate" fraction of the possible gain is achieved at Step 2.

Of course, it is not the infinite convergence of the procedure that really matters in practical applications, since only a modest number of interactive iterations can actually be carried out. Rather it is the *initial rate* of convergence which is of interest. Very little is known about the initial rate of convergence of mathematical programming algorithms in general, but for the Frank-Wolfe method the following intriguing result has been demonstrated by Wolfe [10] (see also Amor [1]): if the objective function of (1) is boundedly concave (i.e., is concave and has continuous second derivatives on X, and there is a uniform lower bound on all eigenvalues of the Hessian) and X is a bounded convex polytope, then for the first K iterations (K is unknown)

$$\left(\frac{V - U[f_1(x_k), \cdots, f_r(x_k)]}{V - U[f_1(x_{k-1}), \cdots, f_r(x_{k-1})]}\right) \leqq \frac{1}{2}, \qquad k \leqq K,$$

where V is the optimal value of (1). In other words, the error in objective function value is at least halved at each of the first K iterations.

3. An Application

We now describe an application of the foregoing approach to the operation of a single academic department on a large university campus. This numerical example is based on actual data from the 1970–1971 operations of the Graduate School of Management at UCLA. To emphasize that the procedure can be carried out entirely in criterion space, at least so far as the decision-maker is concerned, we suppress all reference to decision variables and defer discussion of the detailed mathematical model of departmental operations to the Appendix. Only a brief scenario of the general setting of the problem and definitions of the individual criteria are presented in this section.

3.1 *Purpose of the Model and Criteria Definitions*

The faculty of an academic department is viewed as engaging in three principal activities: formal teaching, departmental service duties (e.g., administration and curriculum development), and other tasks such as research and student counseling. Formal teaching takes place at the graduate, lower division undergraduate, and upper division undergraduate levels. We shall be concerned primarily with the allocation of faculty effort among these activities, given an exogenous budget for personnel in terms of Full Time Equivalent (FTE) positions. This allocation may be expressed in terms of course sections or "equivalent course sections," one unit of which is defined as the time and effort equal to the actual teaching of one course section. Expressing all activities in related or equivalent units is not required by our procedure, but it does facilitate the assessment of the tradeoff weights. The use of nonacademic personnel or other resources, such as supplies and expenses, is not considered in this model, nor do we address the more "micro" operating problems related to course scheduling or to the assignment of faculty and students to particular course sections and classrooms.

The first three criteria of the model are the number of course sections offered by the department at the graduate, lower division undergraduate, and upper division undergraduate levels, respectively. Criterion four is the amount of teaching assistant time used for the support of classroom instruction by the faculty. The fifth criterion is the regular faculty effort devoted to major departmental service duties (here the equivalent course sections correspond to reduced teaching loads). Finally, criterion six is the regular faculty effort devoted to additional activities such as research, student counseling, and minor administrative tasks, again measured in equivalent course sections.

This model can be used by the department to develop aggregate annual operating plans. A numerical example of this use will now be presented.

3.2 *Initialization* (*Step* 0)

An initial operating point was determined by modifying the actual operating point of the department for the previous academic year. The result, provided in terms of the criteria (f_i, $i = 1, \cdots, 6$), is presented in Table 1.

3.3 *Estimation of Tradeoffs* (*Step* 1)

The next step requires the decision-maker to provide the tradeoff weights associated with each of the criteria. The numerical tradeoff weights and step sizes presented in this example were indeed determined by an administrator, but for obvious reasons the discussion below regarding their rationale must be considered as purely illustrative. We select f_1, the number of graduate sections offered, as the reference criterion. By convention, its tradeoff weight (w_1) is unity.

We now wish to obtain a tradeoff weight for f_2, the number of lower division sections offered, versus f_1. This weight will be estimated by considering the "ideal" proportional change. The ratio of f_2 to f_1 at the current operating point is $16/314 = 0.051$. In other words, approximately 1 lower division section is offered for every 20 graduate sections. Suppose the administrator feels that the undergraduate offerings of the department should be given increased emphasis relative to the graduate offerings. Then he may feel that an ideal proportional change from the current values for these two criteria is more like 1 lower division undergraduate section for every 10 graduate sections, or

$$w_2 = \delta_2/\delta_1 = 1/10 = 0.1.$$

Similar reasoning relating f_3 to f_1 led to $w_3 = 0.25$.

TABLE 1

Criterion	Initial Value in (Equivalent) Sections
sections offered—graduate (f_1)	314.0
sections offered—lower division (f_2)	16.0
sections offered–upper division (f_3)	50.0
teaching assistant time used for support (f_4)	53.0
releases for departmental service duties (f_5)	50.0
additional activities of the regular faculty (f_6)	293.5

To obtain w_5, the administrator must state his preference for changes in the number of graduate sections versus changes in the number of departmental service releases. He realizes, of course, that the granting of one release reduces the total number of sections offered by one. However, he may feel that the loss of 2 releases for departmental service from the current number of 50 would be just offset, *in terms of his preferences*, by a gain of 3 graduate sections. In other words, an increase of 3 additional units of f_1 would exactly compensate for a loss of 2 units of f_5, so that

$$w_5 = -(\Delta_1/\Delta_5) = -(3/-2) = 1.50.$$

These examples have illustrated both approaches to the estimation of tradeoff weights. The "ideal proportional change" approach (see (7)) was illustrated by the determination of w_2, and the "indifference" approach was illustrated by the estimation of w_5 (see (6)). To obtain w_4, we shall illustrate a tactic useful when it is difficult to relate a criterion to the reference criterion, but relatively easy to relate it to some other criterion. In this case, f_4 is related to f_6, which in turn is related to f_1.

Suppose the administrator has difficulty in trading off teaching assistant support time (f_4) against the number of graduate sections (f_1). However, he judges that teaching assistant support for 8 sections taught by regular faculty would be worth 1 equivalent course section of a regular faculty member's time devoted to "additional activities." It follows that $-(\Delta_6/\Delta_4) = 1/8$ estimates $(\partial U/\partial f_4)/(\partial U/\partial f_6)$ (it would serve as the weight for f_4 *if* f_6 were the reference criterion). However, since $w_6 = (\partial U/\partial f_6)/(\partial U/\partial f_1)$, w_4 can be obtained from the relation

$$w_4 = \frac{(\partial U/\partial f_4)}{(\partial U/\partial f_6)} \cdot \frac{(\partial U/\partial f_6)}{(\partial U/\partial f_1)} = -\left(\frac{\Delta_6}{\Delta_4}\right) w_6 = \frac{w_6}{8}.$$

Only w_6 remains to be estimated.

Suppose the administrator feels that time for additional duties (research, counseling, etc.) by the faculty is particularly important. He may feel that the ideal proportional change is an increase of 2 section equivalents in f_6 for every additional graduate section. Then, $w_6 = \delta_6/\delta_1 = 2/1 = 2.0$, and consequently $w_4 = (0.125)(2.0) = 0.25$.

3.4 *Step-Size Determination* (*Step* 2)

These tradeoff weights are used to compute a new feasible operating point by solving the optimization problem (2′). As described in §2, we can now plot the values of all r criteria as a function of t. Since all of our criteria are measured in similar units, it is natural to superimpose their plots on the same graph as shown in Figure 2. The values of the criteria at $t = 0$ are the initial operating point of Table 1, and the corresponding values for $t = 1$ are the solutions to (2′). Since all of our criteria happen to be linear (see the Appendix), the plots of their values are line segments.

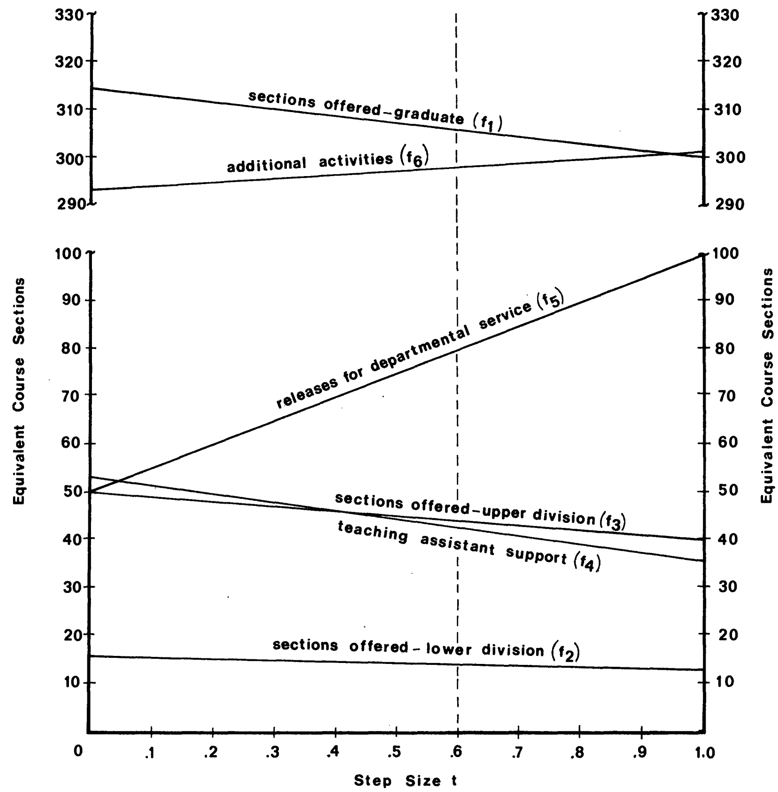

FIGURE 2. Step 2 of Iteration 1.

To complete an iteration of the procedure, the decision-maker must determine a value of t for which the corresponding values of the criteria are most preferred. A vertical line may be visualized as superimposed on Figure 2, intersecting the plots of the criteria and the t axis ; by shifting this line from left to right, the decision-maker may visualize all of the feasible solutions to this restricted one-dimensional optimization problem. In this particular example, the administrator selected $t = 0.6$ (the dashed line in Figure 2). The corresponding criteria values become his revised operating point, and he is ready to perform another iteration.

3.5 *Further Iterations of the Procedure*

After considering the point selected in Step 2 of the first iteration, the administrator revised two of his tradeoff weights. He reduced w_5, the tradeoff weight for releases for departmental service duties, from 1.50 to 0.80, and increased w_4, the tradeoff weight for teaching assistant support, from 0.25 to 0.35. He felt that his relative preferences for the other criteria remained unchanged at this new point.

The revised weights and new operating point were used to compute the results presented in Figure 3. To complete the second iteration, the decision-maker selected $t = 1.0$ as his preferred solution to the one-dimensional optimization problem (Step 2).

For the third iteration, the administrator modified only w_5 from 0.80 to 0.50. The new operating point selected at the conclusion of the second iteration is an extreme point solution to (2′). A post-optimality analysis of the direction-finding linear program indicated that the solution would be unchanged for the new value of w_5. Since no improvement would occur in the third iteration, the procedure was terminated.

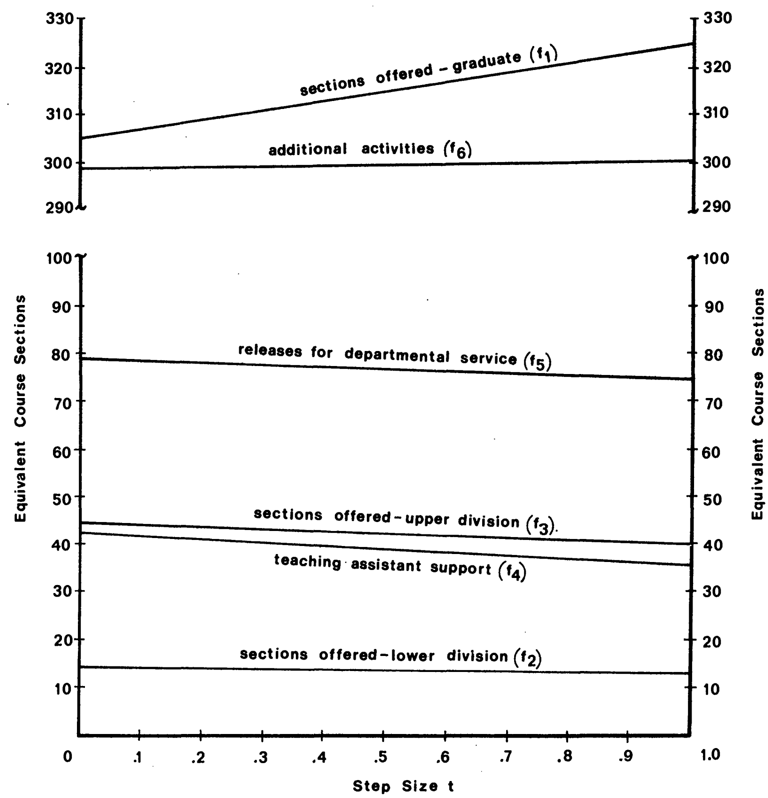

FIGURE 3. Step 2 of Iteration 2.

In closing, a word of explanation may be in order concerning why the solution arrived at required a significant reallocation of faculty effort from teaching to releases for departmental service. Immediately prior to the implementation of our model, the faculty of the Graduate School of Management adopted significant modifications in the existing master's degree program. The successful mounting of this new program required a major short-term commitment of effort in such activities as curriculum design and revision, program evaluation, and computer software development. The tradeoffs provided by the administrator reflect these considerations. Subsequent use of the model to plan for the 1972–1973 academic year generated a solution with releases at a lower level.

4. Conclusion

We have presented an interactive mathematical programming approach to multi-criterion optimization and an example of its application to the operating problem of an academic department. Our initial experience with this procedure has indicated that decision-makers can provide the required information without significant difficulty. Based on the favorable response and suggestions of the administrators of the Graduate School of Management at UCLA, we have further refined the model to include additional detail in terms of course and student levels. This revised model has been used to assist in planning the future operations of the Department. In addition, we have used the model to aid the administration in investigating the impact of proposed changes in policies which would significantly affect the mix and number of required courses.

This work suggests several topics for further investigation. Foremost among those of general interest are the development of reliable experimental procedures for estimating tradeoffs (marginal rates of substitution) between criteria, and the comparative study of initial rates of convergence for various mathematical programming algorithms with potential for interactive implementation. The present literature on both of these topics is surprisingly thin.

Future applications in the area of academic management, and perhaps in other areas as well, would benefit from the ability to treat certain parameters of the model not as exogenously given, but as interacting decisions made at another level. For instance, the resource budgets for an academic department are actually determined by a central campus administration which observes and reacts to the actual or intended operating plans of the departments. This requires a hierarchical model. The interactive procedure described in this paper has been generalized by Geoffrion and Hogan [6] to a two-level organization composed of a coordinator and several semi-autonomous operating units. This enables the departmental operating problem to be treated as part of a larger problem in which the campus administration interactively coordinates the budgets of the departments so as to pursue its objectives for the entire campus.

Appendix

A brief discussion and overview of our model of the departmental operating problem was presented in §3. We now describe the details of this model. Some of the following may represent aspects unique to the University of California system.

Variables Under the Control of the Department

The departmental decision variables included in the model are the following:

x_{1j} = the number of course sections offered at level j. Levels $j = 1, 2, 3$ correspond to graduate, lower division undergraduate, and upper division undergraduate courses, respectively.

x_{2k} = the number of regular FTE faculty of type k hired beyond the department's contractual commitments. In our example, types 1 and 2 represent tenured and non-tenured regular faculty, type 3 represents teaching assistants, and types 4 and 5 represent lecturers and senior lecturers, respectively.

x_{3k} = the number of irregular FTE faculty hired at a salary level equivalent to a regular faculty position of type k.

x_{4k} = the number of equivalent FTE of type k released from teaching for departmental service duties.

All of these variables are required to be nonnegative.

Parameters Determined by the Campus Administration

Some departmental parameters are determined exogenously by the campus administration:

y_{1k} = the academic FTE faculty of type k allocated to the department, and

y_{2j} = student enrollment at level j.

We have assumed that course levels and student levels correspond for $j = 1, 2, 3$, although the most recent version of the model makes a finer distinction.

Constraints

The total number of sections taught in the department is determined by the work conservation equation

$$\sum_{j=1}^{3} x_{1j} = \sum_{k=1}^{5} [t_k(c_k + x_{2k} + x_{3k} - a_k - s_k) - g_k(x_{4k} + r_k)], \tag{8}$$

where

t_k = the customary number of sections taught per academic year by a faculty member of type k,

c_k = the estimated number of faculty FTE of type k to whom the department is contractually committed for the planning period,

a_k = the estimated number (in FTE) of leaves without salary by faculty of type k,

s_k = the estimated number (in FTE) of faculty of type k taking sabbatical leave,

r_k = the estimated faculty time (in FTE) of type k released from teaching due to support from outside the department, and

g_k = the number of equivalent sections corresponding to a full workload for an FTE faculty of type k.

The value of g_k also corresponds to the teaching load of lecturers and irregular faculty, since they perform no duties other than classroom teaching.

The hiring of irregular faculty is limited by both the available FTE and salaries generated by vacancies, leaves, and support from outside the department. The resulting constraints are

(9) $$\sum_{k=3}^{5} x_{3k} \leqq \sum_{k=1}^{5} (y_{1k} - c_k - x_{2k} + a_k + r_k),$$

(10) $$\sum_{k=3}^{5} b_k x_{3k} \leqq \sum_{k=1}^{5} b_k (y_{1k} - c_k - x_{2k} + a_k + r_k),$$

where b_k is the average annual salary for a full-time faculty member of type k.

The constraints

(11) $$x_{2k} \leqq y_{1k} - c_k, \qquad k = 1, 2, 3, 4, 5,$$

place an upper limit on the number of faculty hired ;

(12) $$r_k + x_{4k} \leqq t_k (c_k + x_{2k} - a_k - s_k)/g_k, \qquad k = 1, 2, 3,$$

provide that teaching releases of regular faculty cannot exceed the total teaching obligations ; and

(13) $$t_3 (c_3 + x_{23} + x_{33}) \leqq x_{12}$$

restrict the formal teaching by teaching assistants to lower division undergraduate classes.

In addition, there are constraints which reflect policies or commitments of the department. We require that

(14) $$x_{1j} \geqq m_{1j}, \qquad j = 1, 2, 3,$$

where m_{1j} is the minimum number of sections which must be offered at course level j in order to meet departmental commitments. Similarly,

(15) $$m_{2j}\, x_{1j} \leqq \sum_{j'=1}^{3} \sum_i y_{2j'}^{i}\, p_j^{ij'}/h_j \leqq m_{3j}\, x_{1j}, \qquad j = 1, 2, 3,$$

places limits on the average class size, where

m_{2j} = desired lower bound on average class size at level j,

m_{3j} = desired upper bound on average class size at level j,

h_j = average number of student credit hours per student-section for courses given at level j, and

$p_j^{ij'}$ = the preference by students enrolled in department i at level j' for courses offered by *the* department at level j, measured in student credit hours per student for an academic year (obtainable from registration records).

The summation over i in (15) ranges over all departments in the University.

Departmental Criterion Functions

The first three criterion functions,

(16) $$f_j = x_{1j}, \quad j = 1, 2, 3,$$

represent the number of sections offered by the department at the graduate, lower division undergraduate, and upper division undergraduate levels, respectively. The fourth criterion,

(17) $$f_4 = g_3 x_{43},$$

is the number of equivalent sections of teaching assistant time used for the support of instruction by other faculty. Criterion five,

(18) $$f_5 = \sum_{k=1}^{2} g_k x_{4k},$$

is the number of sections from which the regular faculty is released from teaching for departmental service duties. Finally, the sixth criterion,

(19) $$f_6 = \sum_{k=1}^{2} (c_k + x_{2k} - s_k - a_k)(g_k - t_k),$$

represents the balance of the regular faculty's time (measured in equivalent sections) devoted to areas other than classroom instruction exclusive of releases from formal teaching duties.

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