Compte Rendu
de la Conférence
Internationale
sur la Recherche
en Matière
de Transport

Proceedings
of the
International
Conference on
Transportation
Research

PREMIÈRE CONFÉRENCE

FIRST CONFERENCE

Dearborn / June '74

Bruges, Belgium
Juin, 1973

Bruges, Belgium
June, 1973

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THIS PAPER describes a set of techniques developed to assist in planning the operations of a commuter railroad and similar types of public transport. The underlying hypothesis of this research was that a carefully developed method for service and operations planning could, when applied, result in either reductions in operating and capital cost while maintaining current levels of service, or improve the level of service while holding expenditures constant, or a combination of both.

Because of the current wide spread investment in new equipment for commuter railroads, proposals for and actual extensions of such services, the transition of managerial responsibility for such services from essentially freight railroads to urban transit authorities, and the efforts of the Urban Mass Transportation Administration to develop improved planning and managerial methods for public transportation it was felt that the climate is reasonably receptive to new methods for operations planning.

General Approach

The construction of a complete service plan for a commuter railroad is obviously a large and complex problem with a large number of decision variables such as whether or not to run express as well as local trains, the number of cars per train, etc., and with a great deal of interaction among the variables in terms of achievement of system objectives. In the context of urban railroad commuter operations, two primary objectives seem appropriate—one relating to the quality of service to the user and the other relating to the cost of operation to the operator and owner. User criteria primarily relate to (1) travel time and (2) waiting or schedule delay (having to shift your arrival or departure time to suit the train schedule). Cost criteria, which are of importance to both the user and the operator and owner, relate to the operating cost, and perhaps somewhat separately to the capital cost—especially for new equipment—since often capital expenditures are borne partly by general government sources rather than by the operator or users. Fares are not treated in the analysis although the cost information clearly would be useful in the evaluation and selection of fares.

Commuter rail operations are heavily peaked; typically in the U.S.A. approximately 80% of the total traffic occurs in the four peak hours of each week day. Almost all costs are determined by the peak hour operations. The number of cars and locomotives owned and maintained is determined by peak loads, there being more than sufficient cars for the off peak once peak loads can be accommodated. Labor costs are somewhat similar, because once an operating employee is called to duty, he must be paid for eight hours work (or its equivalent in train mileage), and time or mileage available after peak period tasks have been performed is usually more than sufficient to staff all off peak trains. Maintenance and energy costs are typically small relative to the two other major cost categories. Because costs are thus largely determined by peak period operations, and because there is usually plenty of equipment and staff to accommodate off peak operations, it is appropriate to focus upon peak period schedule planning.

The approach taken is essentially to attempt to divide the problem into a number of components which can be treated relatively independently of one another while nevertheless leading toward a good or "best" solution. The four major components of the method developed are presented in Figure 1.

The first, entitled schedule planning, is to explore the most general questions related to the problem, such as whether or not to operate express service, the approximate number and size of trains, etc. The result of this analysis is information on the tradeoff between user costs (travel time and its components), operating costs, and capital costs (cars). With this information on tradeoffs, the analyst can select a trial schedule plan (implying decisions regarding these very general variables) for further, more detailed analysis. The more detailed analysis, timetable construction, is concerned with development of the detailed timetable for train operations and crew assignments, and it will yield precise information on user costs, operating costs, and capital costs. If this refined analysis results in a timetable which is satisfactory from the standpoint of these various groups, then the problem has been solved. If not, the analyst must cycle back to the tradeoff information on schedule plans and select another trial schedule plan. Through such iterations the final schedule plan and detailed timetable can be selected.

Schedule Planning

There are very strong interrelationships between the primary choice variables and the various criteria. Figure 2 presents information on the travel time of trains and the utilization of the seat capacity of those trains as the general pattern of the schedule is varied between an all-stop schedule, a skip-stop schedule (alternate trains stopping at every other station), and zone schedules (trains stopping at a few adjacent stations and

Schedule Planning and Timetable Construction for Suburban Railways†

by

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then operating non-stop between those stations and the downtown area).[1] Train speeds tend to be lowest for the all-stop schedule and highest for the zone schedules. Also, equipment utilization (in terms of seats being occupied) tends to be highest for the zone schedules and least for the all-stop schedule. Thus both the number of crews required and the number of cars required is likely to be decreased as service is shifted from an all-stop schedule to a skip-stop schedule and finally to a zone type of schedule. However, in order for these efficiencies to be achieved, the frequency of service from any one station necessarily is reduced, but depending upon the tradeoff between running time and headway, the travelers' total travel time may nevertheless be reduced.

A method has been developed to consider these interrelationships and to generate information on the tradeoff between user costs, operating costs (in terms of crew requirements and total car miles) and capital costs (in terms of cars required). The operating and capital cost can of course be reduced to a common unit of measurement such as dollars per day,[2] and user costs could also be so reduced if one could accept a value of travel time, but it is useful to retain these as distinct measures because different groups may be responsible for these different components of cost.

Before proceeding to more details of the methods developed, it should be

pointed out that they were originally developed for and applied to an electrified suburban railway with self-propelled cars (and hence no locomotives). Also, since the morning peak was the heaviest, the analysis focused upon that one. In the presentation of methods, the discussion will be in terms of the inbound peak, but with minor modification the methods can be applied to the other peak period.

In order to make the rudiments of the approach understandable, it is necessary to define a few terms which will be used frequently in the following discussion. These are:

Peak period: a period of the day during which the peak direction hourly ridership levels are higher than the average hourly ridership levels. There is a morning and an evening peak period.

Peak period load: the number of passengers travelling in the peak direction past the maximum flow point (also called peak load point). For a commuter railroad operation the peak load point will generally be the stretch between the last station and the downtown station. If there is more than one downtown station, it precedes the first downtown station in the inbound direction.

Critical demand: the highest peak-direction load during an interval of the peak period equal in length to the round trip time. A particular scheduling policy has its particular critical demand.

Critical demand period: the time period for which the critical demand applies. The length of the critical demand period is equal to the round trip time, while the period itself is defined by a starting point and an end point.

The problem becomes one of deriving the critical demand for a specific stop-schedule, because the number of cars and crews is determined by this demand.

The basic relationship for the car requirement can be written as:

Number of cars required = maximum number of cars required to accommodate passenger flows on trains arriving at CBD in an interval equal to the round-trip time. (1)

Demands
    Origin-destination
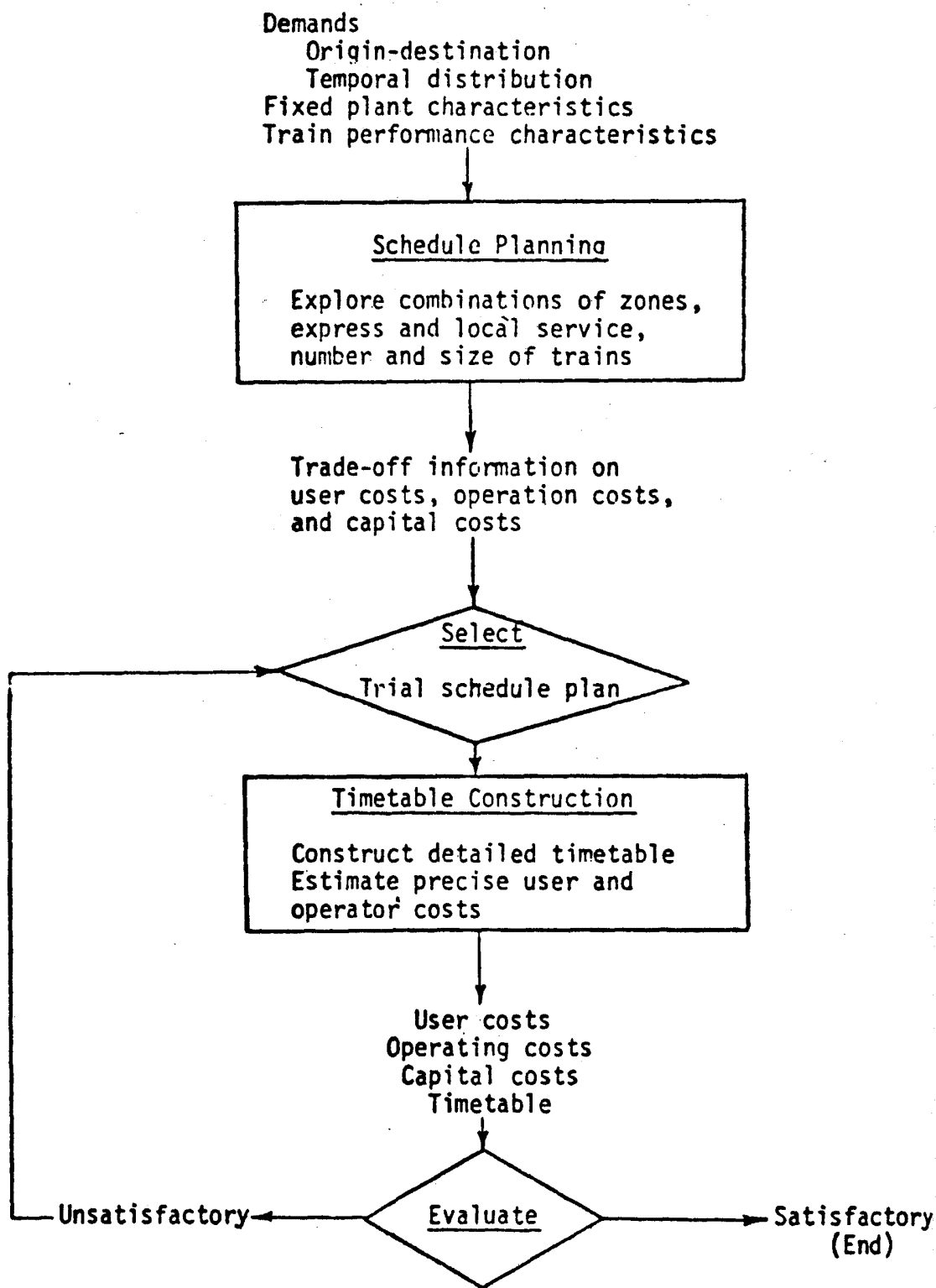   Temporal distribution
Fixed plant characteristics
Train performance characteristics

```
┌─────────────────────────────────┐
│      Schedule Planning          │
│                                 │
│  Explore combinations of zones, │
│  express and local service,     │
│  number and size of trains      │
└─────────────────────────────────┘
```

Trade-off information on
user costs, operation costs,
and capital costs

```
            ◇ Select ◇
        Trial schedule plan
```

```
┌─────────────────────────────────┐
│     Timetable Construction      │
│                                 │
│  Construct detailed timetable   │
│  Estimate precise user and      │
│  operator costs                 │
└─────────────────────────────────┘
```

User costs
Operating costs
Capital costs
Timetable

Unsatisfactory◄────── ◇ Evaluate ◇ ──────► Satisfactory
                                                (End)

Figure I.   The Components of the Service Planning Process
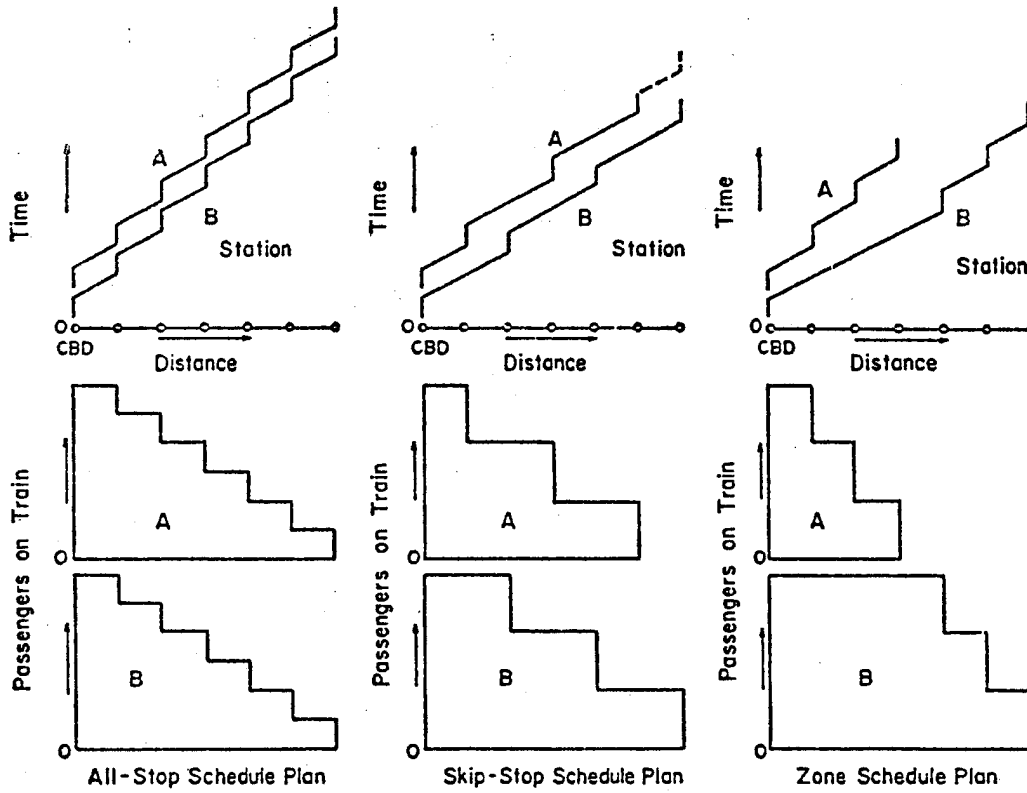
Figure 2. Characteristics of Alternative Schedule Plan

Demands

Origin-destination
Temporal distribution

System

Tracks, station locations
Train performance characteristics

Schedule Planning

Explore combinations of:
zones and skip-stops
train length
number of trains

Alternative
Service Levels

Frequency

Trade-off Information:

User costs
Frequency
Running time
Operating costs
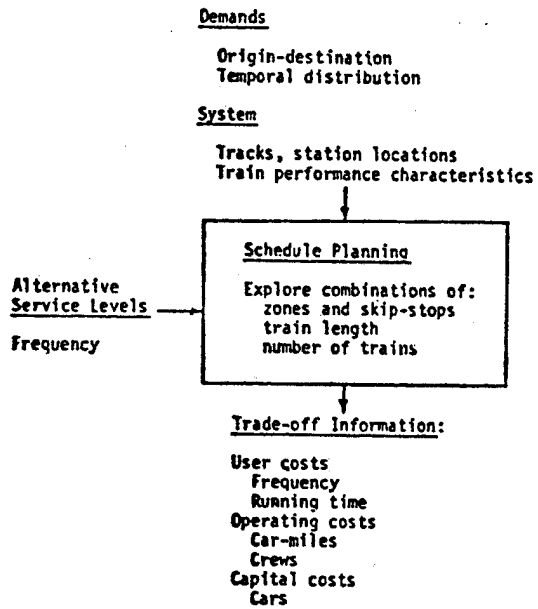Car-miles
Crews
Capital costs
Cars

Figure 3. The Schedule Planning Process

In this formulation[3] the round trip time includes the train travel time in the peak direction and in the reverse direction, as well as a layover time at the two end points. The rationale behind relationship (1) is simple: after a car passes a point on a rail line, in a given direction, it must make a round trip before it can pass that point again in the same direction. Therefore, during this interval (the round trip time) the number of seats required has to equal or exceed the number of passengers arriving at the CBD stations. This formulation implies that all the passengers have a seat (no standees). Once the operator has determined a target peak period load factor (e.g. 90%), the car requirement can be calculated from the above seat requirement.

It should be obvious that different scheduling policies result in different round trip times, which in turn are a critical factor in the determination of the fleet size. The same type of relationship holds for crew requirements.

The critical demand for a particular stop-schedule can be obtained from (2).[4]

$$cd = dem \cdot \max_{a \leq x \leq b-rtt} \left(\int_{x}^{x+rtt} f(t)dt \right) \tag{2}$$

where:

cd = critical demand

dem = total peak period demand originating at the stations which belong to the stop-schedule

a,b = begin and end of the peak period

rtt = round trip time

x = start of the critical demand period

x + rtt = end of the critical demand period

f(t) = desired arrival time distribution for the peak direction morning peak rides

The critical demand for a particular schedule configuration can then be computed using (2) for each stop-schedule (zone). The information required for this computation includes: a) the round trip time for each zone, b) the total demand generated in each zone during the peak period, c) a desired arrival time distribution, which might vary among stations.

Once the critical demand is obtained, both car and train requirements can be derived, and the costs for a specific schedule configuration can be computed. Typical relationships between the various measures of costs and the number of zones are presented in Figure 4.

Car requirements and total car miles drop markedly as the number of zones is increased, reflecting the higher average train speeds and better utilization of equipment. Crew requirements are very sensitive to the level of service require-
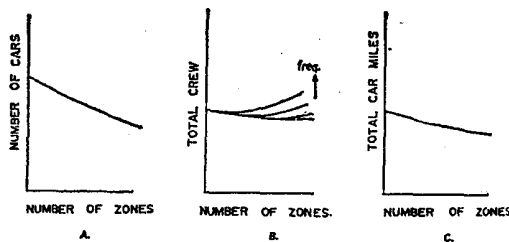
Trade-offs Between User and Owner/Operator Criteria

FIGURE 4

ments (expressd in terms of frequency) imposed from the viewpoint of the users.

Timetable Construction.

The general outline of the timetable construction model for a single zone is given in Figure 5. The purpose of this model is to construct a detailed timetable as well as a car assignment for the trains that will be operated in that zone. This involves determining the departure times for all trains from the origin station of the zone, and the number of cars each train consist of. Given the departure times of the trains from the zone origins, and the train running times, it is easy to derive the departure times at the remaining stations of the zone. The timetable must take into account the specification of the schedule plan arrived at earlier: the number of trains to be operated and the total number of cars to be used for that zone.

Because operating costs (crew size, car miles) and capital costs (fleet size) are essentially fixed, the only variable criteria are those related to the user. Appropriate user criteria are travel time, waiting time and seat availability. Because the travel times are fixed under zone operation, only the last two criteria will be incorporated in the model. The problem then is essentially a mathematical programming problem, in which the objective function is to minimize total traveler waiting time or schedule delay, subject to 1) trains being operated so as to give every passenger a seat, 2) cars assigned to the trains being equal to the number calculated in the preceding analysis, and 3) the number of trains operated being equal to the number computed in the previous analysis.

In the case of the morning peak period problem, the schedule delay consists of the difference between the actual arrival time and the desired arrival time at the downtown station. A similar analysis could be applied to the evening peak period. In that case the schedule delay would consist of a waiting time at the downtown station before boarding a train.

Trial Schedule Plan
zone specification (endpoint, number
of stations)
number of trains to be dispatched
total number of cars
service level (frequency)
↓
Timetable Construction
minimum user schedule delay
(waiting or early arrival)
Subject to:
all passengers seated
maximum number of trains operated
maximum number of cars used
↓
Timetable
Precise user costs
Travel time
Schedule delay
Precise operating costs
Total car miles
Total crew size
Precise capital costs

SINGLE ZONE TIMETABLE CONSTRUCTION MODEL FIGURE 5

The model is formulated as a dynamic programming model. It can be seen as an extension of the model developed by Bisbee [1, 2], to schedule vehicle departures considering waiting time and a given maximum number of departures.

The recurrence relation for the single state dynamic programming formulation is given in (3).[5]

$$G_k(m) = \min_{d} (H_k(d) + G_{k-1}(m'))$$

$$\qquad\qquad\qquad\qquad\qquad (3)$$

where
m = number of passengers incurring a schedule delay (arriving before the desired arrival time)
d = decision at a stage
$d = 0$ no train arrival is scheduled
$d = 1$ a train arrival is scheduled
$d_k^*(m)$ = optimal decision at time t_k if m passengers incur a schedule delay
m' = number of passengers incurring a schedule delay at the preceding stage. Given m passengers at time t_k then:

$$m' = m(1 - d_k^*(m)) +$$
$$(F(t_k) - F(t_{k-1})) \qquad (4)$$

$H_k(1)$ = the dispatching cost associated with a train arrival at time t_k

$H_k(0)$ = the cost of additional schedule delay imposed on the passengers by not having an arrival at time t_k

$G_k(m)$ = minimum cost (schedule delay cost and dispatch cost) at time t_k for all time periods preceding time t_k, given m passengers incurring a schedule delay at time t_k

In the formulation, the stage variable corresponds to time t_k with $k = 1, \ldots N$.

Because we are analyzing downtown train arrivals, the time frame coincides with the critical demand period for the zone under consideration, and we have:
t_1 = a the start of the critical demand period for the zone
t_N = b the end of the critical demand period for the zone

The state variable corresponds to passengers incurring a schedule delay. Those are passengers that will arrive at the downtown station before their desired downtown arrival. If we were constructing a timetable for outbound evening peak trains the state variable would represent passengers waiting to board a train.

A number of refinements can be incorporated in the general formulation. A first refinement relates to load factors. We would like to provide a seat for all passengers (a load factor of 100% or less). Therefore:

$$m \leq c \qquad (5)$$

where
c = capacity of a train.

It should become clear that (5) is not necessary. In this model the number of dispatches (arrivals), and the total dispatching costs, is fixed. A dispatch when $m > c$ would merely result in additional schedule delay costs. The algorithm, therefore, will automatically dispatch a train if and when the train capacity is reached.

Another refinement concerns the size of the trains that can be dispatched. In general it is possible to couple rail cars to form trains. The decision at a stage then involves not only a dispatch decision but also a decision on the size of the train dispatched.

if $m \leq c_i$ d relates to a train with i cars

where
c_i = capacity of a train with i cars
d = decision at a stage
In this case, the dispatching cost associated with a train arrival at time t_k, namely $H_k(1)$, may vary with the train size.

An additional refinement concerns the imposition of a maximum headway. The algorithm may automatically dispatch a train if and when the maximum headway is reached, even if this train has not achieved a desirable load factor.

Finally, it is possible to consider nonlinear costs for schedule delay. Under a linear cost assumption the dynamic programming model will minimize the average schedule delay incurred by passengers. In this case the disutility of being 20 minutes early is considered double the disutility of being 10 minutes

early. Non-linear cost assumptions will be instrumental in eliminating large deviations in schedule delay. This will also result in greater variance of load per dispatch than is the case in linear schedule delay costs.[6]

To execute the dynamic program, boundary conditions have to be established. They are:

$$G_0(m) = 0 \quad \forall\, m \tag{6}$$

To be consistent with the critical demand period concept the first train arrival from the zone is scheduled at time t_1, the start of the critical demand period. This same train, after having completed a round trip, will be scheduled to arrive downtown subsequently at time t_N, the end of the critical demand period for the zone.

In the recurrence relation (3) of the dynamic program, two types of costs are considered at each stage: a user oriented cost reflecting the cost associated to additional schedule delay if no arrival is scheduled, namely $H_k(0)$, and an operator oriented cost measuring the cost of a dispatch associated to a train arrival, $H_k(1)$. The latter cost may vary with the number of cars being dispatched per dispatch.

The total number of railcars required to accommodate the passengers riding during the critical demand period is determined prior to this stage. The number of dispatches and therefore the number of train arrivals, is also determined for a given service level.

To obtain a solution specifying the exact number of trains operated, and the correct rail car fleet, we will parametrically control the cost variable associated to the schedule delay. The correct solution, which corresponds to the minimum schedule delay with a prespecified number of departures, has been obtained in a few iterations of the total model.

Timetable Evaluation

Before a timetable can be suggested for implementation, the service level it provides should be at least as attractive as that of the timetable it may replace. In this section we develop a simple method that will enable us to compare and evaluate timetables in terms of passenger convenience.

For each station in the zone we construct a graph (Figure 6) which depicts desired arrival time and a step function for station boarding times for each of the two timetables under study. Consider a passenger desiring to arrive downtown at time at_i. Under timetable one, he would have to board a train at time bt_{i_1} in order to arrive no later than time at_i, while under timetable two, he could board a train at a later time, that is at time bt_{i_2}. Assuming that a passenger will prefer that timetable which allows him to board later, timetable two is preferred.[7]

The case described above can be generalized to cover all passengers boarding at a particular station and desiring to
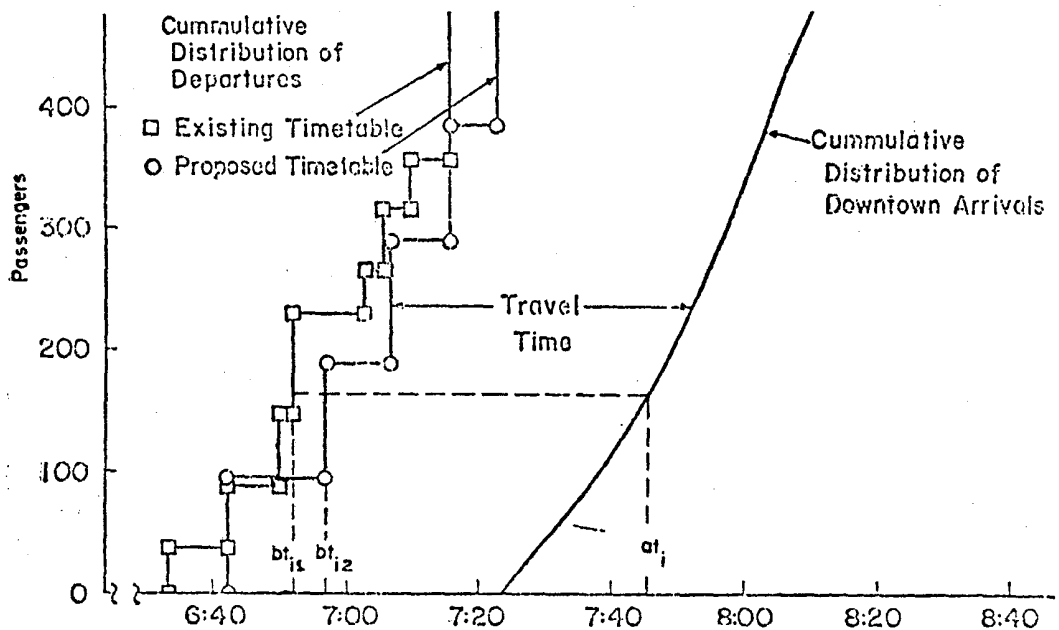

Figure 6   Example of Travel Time Differences Between Schedules

arrive downtown in the period under study. Two propositions will guide us in comparing timetables:

Proposition 1

For a given station a timetable will be preferred to another one if it is true that the first timetable allows some passengers to board a train at a later time and requires no passengers to board at an earlier time. (They may board at the same time.)

Proposition 2

For a given station a timetable will be preferred to another one if the total time saved under this timetable outweighs the total time saved under the other, provided any increases in total traveling time experienced by travelers is small in comparison to the savings for others. Clearly the second case is more likely to occur than the first. Also, when considering timetables from an entire zone such a mixture of gains and losses is likely to occur for any given timetable over a number of stations. A selection must be made by judgment. The graphical method may provide immediate visual evidence of the relative superiority of one timetable over the other, as is the case in Figure 6 where timetable two is clearly preferable. However when this is not the case, it is necessary to evaluate the two timetables according to Proposition 2.

Application

In this section the theory and approach developed in this research will be described in terms of the application which was used to test methods. This application was to a commuter railroad in the Chicago metropolitan area. The line is approximately 30 miles in length; on it approximately 250 trains are operated each weekday, carrying approximately 80,000 passengers. 130 new cars are being purchased at a cost of approximately $306,000 each, to replace most of the very old existing fleet, the new cars seating 156 persons and the old 84 persons per car. The inner half of the line consists of four tracks, the outer half two tracks.

In a first phase, the schedule planning process, the measures of costs to users, owners and operators are computed as a function of the number of zones, one zone corresponding to an all-stop schedule. For this particular railroad, a skip-stop service was found to wholly be dominated (no better with respect to any criterion and worse with respect to at least one criterion) by the two-zone schedule, and hence it is not presented.

As would be expected, car requirements and total car miles drop markedly as the number of zones is increased, reflecting the higher average train speeds

and better utilization of equipment. Crew requirements are very sensitive to the level of service requirements imposed from the viewpoint of the users. In Figure 7 these requirements, or tradeoffs, are indicated for four different levels of service, these being minimum train frequencies: level 0, no minimum; level 1, 3 trains per hour; level 2, 4 trains per hour; level 3, 5 trains per hour. As the level of service requirement is increased, the increasing train frequencies tend to negate the efficiencies gained from zone schedules, resulting in greater crew requirements. The development of this tradeoff information completes the analysis of the schedule planning phase.

The first step in the timetable construction process consists of the selection of a trial schedule plan, based upon the output of the previous phase. The trial schedule plan selected for this analysis was one consisting of three zones, with level of service 2, which is slightly inferior, in terms of frequency, to the existing service. Three zones appears reasonable, because with more zones and level of service 2 the crew requirements increase rapidly, while with fewer zones the costs in all categories increase. Furthermore, the number of cars required by this plan is 13 fewer than that required by the current plans for schedules with the new equipment, and the number of crews required is two less than with that schedule plan. This level of service was selected with the hope that the reduced train running time due to the zone scheduling would more or less compensate for the slightly reduced frequency.

The detailed timetable is constructed separately for each of the three zones (Figure 8). The only interaction between these timetables is that trains from different zones may have to use the same tracks, but because trains can be operated on very close headways (without stops, approximately one minute apart) it was felt that the modification to schedules required to accommodate the trains on the same track would be minimal.

The results of this analysis are presented in a manner which makes the evaluation of the resulting schedule relatively simple. In addition to the usual tabular specification of the timetable and car assignments, etc., detailed user travel information is prepared. For all the users from each station, a graph is prepared which shows their desired cumulative arrival time downtown and the times at which they must depart from their home station in order to arrive at or before that time. Also prepared for comparison purposes is the time at which each commuter has to leave with the
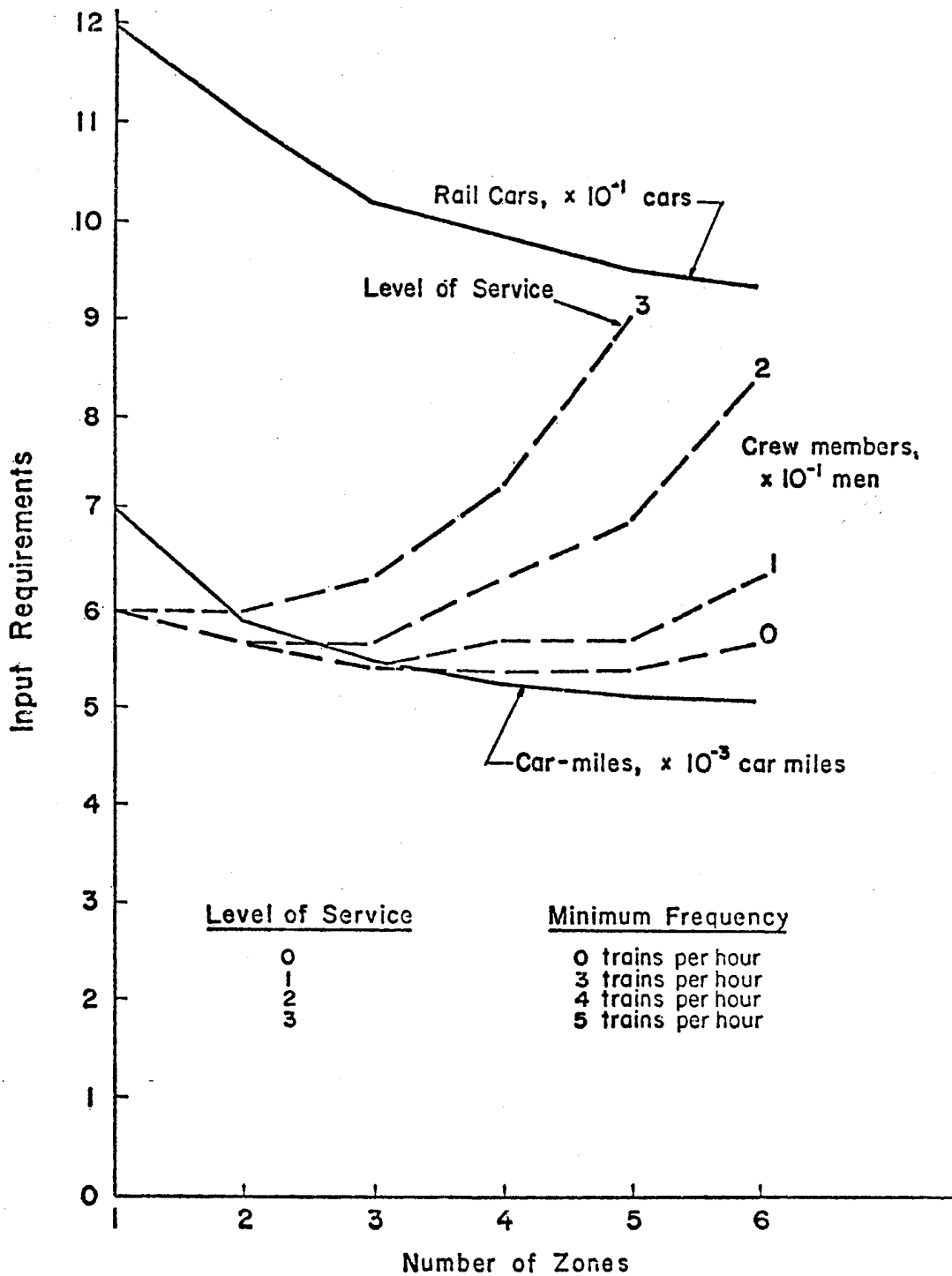
Figure 7 Example Schedule Planning Trade-off Information

existing (or any other) timetable. An example of this output is presented in Figure 9, for a particular station. The curved line on the right is the cumulative desired arrival time distribution of travelers at the central business district in the morning. The departure times at the station where these commuters originate are shown by vertical lines connecting the circles. Commuters are assigned to the latest train which will enable them to reach their destination at or before their desired arrival times, and those commuters assigned to each train
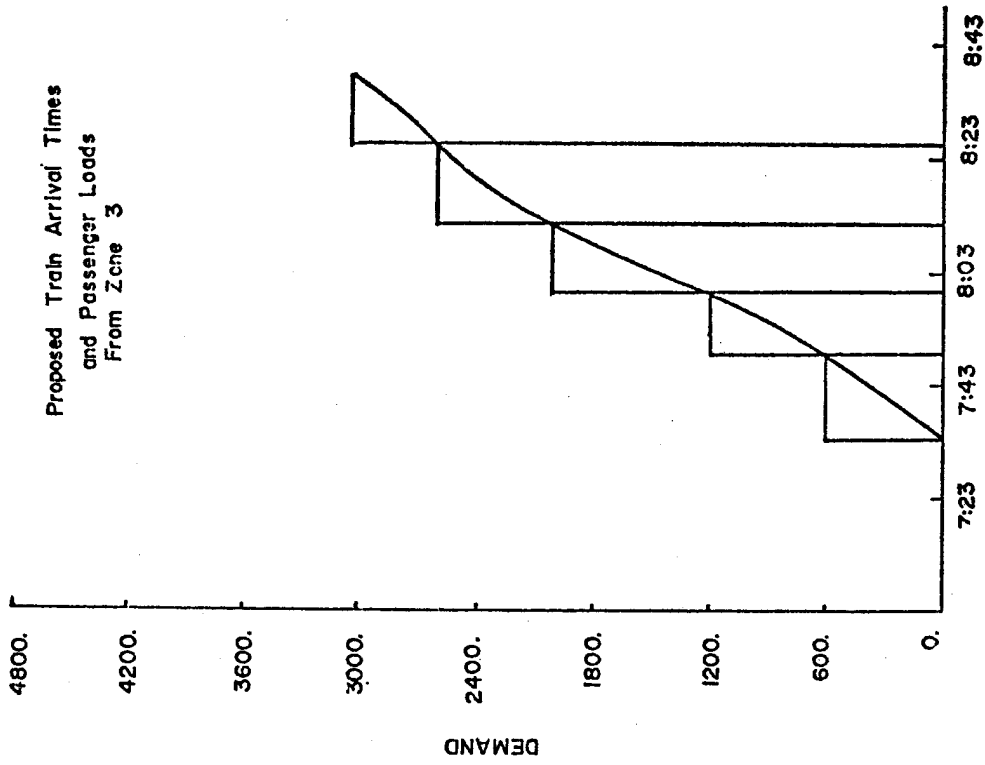
Proposed Train Arrival Times
and Passenger Loads
From Zone 3

DEMAND

Figure 8. Example Zone Schedule

Cummulative
Distribution of
Downtown Arrivals

Travel
Time

Cummulative
Distribution of
Departures
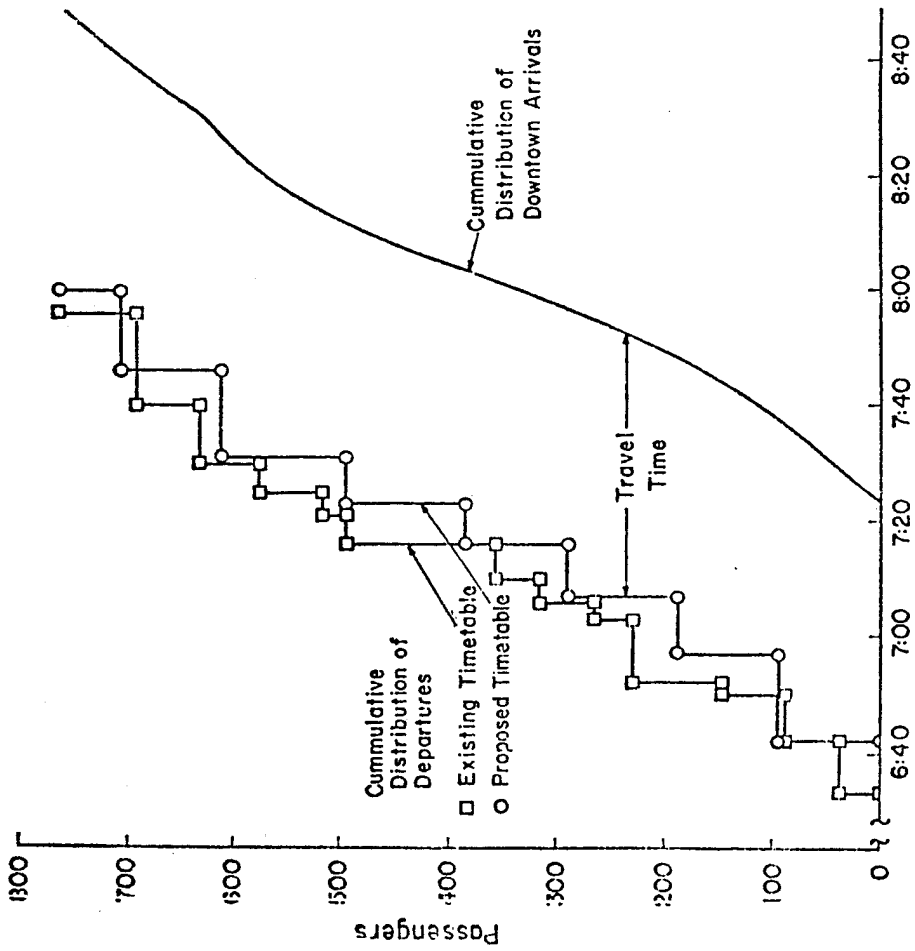
□ Existing Timetable
○ Proposed Timetable

Passengers

Figure 9 Example of Total Travel Time for Morning
Commuters from One Station

are specified by the location of the vertical line corresponding to each departure. For example, commuters desiring to arrive at 7:30 A.M. must take the 6:43 A.M. departure. Approximately 100 commuters, those desiring to arrive between 7:23 A.M. and 7:35 A.M., find that that train is the best one for them. Thus the total area between the desired arrival time and line connecting the circles is the total travel time plus schedule delay of commuters using the optimal zone schedule.

Also shown on this figure is the time at which the same commuters must depart with the currently planned schedules with new equipment[8] in order to arrive at work on time, and the cumulative departure distribution for them is given by the line connecting the small squares. The interesting fact related to the proposed zone schedule is that virtually all commuters can leave home later in the morning and still arrive at work as desired if the proposed zone schedule were to replace the present schedule. This is true even though the planned schedule includes operation of more trains than the zone schedule. The more careful planning of the zone schedule in terms of departures, and the higher train speeds resulting from express operation, produce a substantial gain for almost all present commuters, while a few are required to leave up to ten minutes earlier.

A similar analysis of travel time from each and every station has been carried out for this three-zone schedule, and the results are essentially identical: almost all commuters are better off as a result of the new schedule, and only a very tiny number are required to depart earlier. Thus it would seem that the three-zone schedule is in fact a quite satisfactory one, so that the analysis might be terminated at this point. This is especially likely, considering the fact that the timetable developed by this process used 13 fewer cars and slightly fewer crews than the currently planned schedule. If, on the other hand, this schedule did not turn out to be satisfactory, then another general schedule plan might be selected from the large number of options, and that schedule plan refined so that it can be evaluated. This process would continue until either a satisfactory schedule plan has been identified or until all alternatives have been explored, requiring that a choice be made from this set.

Conclusions

The general outline of a method for schedule planning and timetable construction on suburban commuter railroads has been presented, along with the results of a test application to an actual railroad.

There is really very little in the methods which limit them to suburban railroads, the methods being equally applicable to any urban transit with essentially fixed schedules and routes and heavily peaked traffic patterns. Thus the methods should be applicable to rail rapid transit, buses operating on public streets as well as on private rights of way, street cars on public streets and private rights of way, as well as new technologies with similar characteristics.

Perhaps the most surprising result of this effort was not that analytical techniques could be developed and reasonably successfully applied to the operations planning of urban public transportation, but that the result of this particular analysis indicated that there is an operations plan for this commuter railroad which is better for both the railroad (in terms of reduced car requirements and car miles) and better for virtually all of its commuters in terms of reduced travel time plus schedule delay. This indicates that urban transit might be very substantially improved, both from the standpoint of its economics and the quality of its service, by the application of analytical techniques to its planning.

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FOOTNOTES

1 For a presentation of stop schedules see Eisele [5].

2 The methods used in developing the cost model are presented in [6].

3 This formulation is also discussed in [3] and [7].

4 We have used integration in equation (2) even though it is technically not correct. The function $f(t)$ is not continuous. Only in theory is it possible to construct a desired arrival time distribution function.

5 The formulation is similar to the one given in [1], [4] and [8].

6 See Ward [8].

7 A passenger may like to depart earlier to "beat the crowd." This will occur especially in those situations where seats are at a premium. In this analysis we plan a timetable that will provide load factors under 100%, thereby guaranteeing a seat to everyone.

8 All new cars have not arrived yet, so these schedules are not in effect.