

Target Inventory Levels for a Hospital Blood Bank or a Decentralized Regional Blood Banking System

M. A. COHEN AND W. P. PIERSKALLA

From the University of Pennsylvania, Philadelphia, Pennsylvania

For any blood type, there is a complex interaction among the optimal inventory level, daily demand level, the transfusion to crossmatch ratio, the crossmatch release period and the age of arriving units that determine the shortage and outdate rate. The blood bank administrator should establish optimal target inventory levels based on a simple equation (decision rule) relating these factors. Evaluation of this rule indicates that its implementation can lead to a very low shortage rate and a reasonable low outdate rate if the blood bank administrator makes efforts to control the crossmatch release period and the average transfusion to crossmatch ratio.

THE MAJOR responsibility of a hospital blood bank is to administer the collection, processing, storage and distribution of whole blood and blood products throughout the hospital in a manner that ensures all blood-related demands are met. In addition to this primary goal, the hospital blood bank is also concerned with the minimization of wastage through outdates and spoilage, the maintenance of high quality standards and the reduction of shortages that require either emergency shipments from other blood banks, emergency demands on donors, appeals to the hospital staff for donations or the delay of nonemergency and elective medical procedures. These same responsibilities and objectives apply to a regional blood bank as well. In order to achieve these goals, it is important for both the hospital and the regional blood banks to set inventory levels which trade off shortage versus

outdate rates and minimize total operating costs.

Brodheim, Hirsch and Prastacos², developed curves relating recommended whole blood/red blood cell inventory levels and mean daily demand for various specified shortage rates.*

This paper extends the analysis to establish a simple decision rule which yields the optimal inventory level for each blood type as a function of various factors in the blood bank environment. In using this rule, it is not necessary for the blood bank administrator to choose a shortage rate for system operation since the inventory level recommended by the rule reflects the optimal trade-off between shortages and outdates. The elimination of the requirement to pick and justify a shortage rate exclusive of other system outcome measures greatly simplifies the administrator's decision problem.

For any given inventory level it is possible to compute the anticipated shortage rate when operating at that level. When using the inventory level obtained from the optimal decision rule, the "implicit" shortage

* Throughout this paper we use the definitions: Demand: The number of blood units of any one type that are set aside for possible transfusion (*i.e.*, cross-matched) on a given day. Shortage: A situation when the demand exceeds the number of units of blood in inventory. Shortage rate: The long-term fraction (or percentage) of days on which a shortage occurs. Usage: The number of blood units of any one type transfused on a given day. Outdate: A blood unit discarded because of exceeding the maximum age of 21 days. Outdate rate: The ratio of mean number of blood units outdated to mean number of blood units transfused plus those outdated.

Received for publication March 27, 1978; accepted July 18, 1978.

Supported in part by PHS-HS00786 from the National Center for Health Services Research and Development and by NSF-GK40034.

rate is near .001 for large volume blood types and .01 for rarer or small volume types. On the other hand, the outdate rate is shown, in general, to be quite sensitive to certain specific factors in the blood bank management and environment (age of supply, transfusion to crossmatch ratio, and the crossmatch release period). Depending on these factors, the rule yields outdate rates which vary from .001 to .07. The rule is simple to use because it can be completely described by the mean value of the daily demand, the length of the crossmatch release period and the ratio of total units transfused to total crossmatched. The unused, crossmatched units are eventually released from the assigned to the unassigned inventory after a delay of a number of days. This delay time, which we call the "crossmatch release period," will be denoted by the variable D . For a hospital blood bank, D is usually one or two days.

We also considered the breakdown of the mean demand into the two factors used by Elston and Pickrel^{6,7}: 1) the mean daily number of patients for whom blood is crossmatched, and 2) the mean number of blood units crossmatched per patient, as well as other factors such as the order (age sequence) of issuing the units and the ages of units coming from external suppliers. It was found that the optimal decision rule defined in terms of these two demand factors was not significantly better than the decision rule defined in terms of the mean daily demand. Moreover, the extra cost and effort of estimating these two factors (as opposed to estimating mean daily demand) further mitigated against their use. The other factors mentioned above were also found to be not significant in the determination of the optimal inventory levels.

The primary use of the optimal decision rule will be to establish minimum cost inventory levels for whole blood and red blood cell inventories for a hospital blood bank or a transfusion service. However, the rule

will also be applicable to those regional blood banks where the central bank must meet all orders placed by the local member banks and control of the units passes to the member banks. For a hospital blood bank, the administrator would merely compute the mean daily demand for each blood type, the length of the crossmatch release period and the average transfusion to crossmatch ratio, and then apply the optimal decision rule to compute the optimal target inventory level. For a regional blood bank that relinquishes control of the units to its member banks, the administrator would compute the same numbers, *i.e.*, the mean daily demand by type, the crossmatch reserve period and the average transfusion to crossmatch ratio, for each member in the regional system and then use the rule to compute the target inventory levels for his member transfusion locations. The regional administrator could then adjust his donor scheduling to achieve these target levels.

The literature on setting target inventory levels for blood banks is extensive, but only in the past decade or so has inventory theory been used in these determinations. Brodheim *et al.*² have reviewed the work of Rockwell *et al.*,¹⁴ Jennings,⁹ Yahnke *et al.*,¹⁵ and Hirsch and Schorr⁸ regarding the relationship between mean demand and inventory levels. The work of Elston and Pickrel^{6,7} was also described there in some detail.² We briefly note some further work regarding inventory levels. Yen,¹⁶ extending the work of Jennings,¹⁰ analyzed the effects of various allocation policies, transshipment policies and inventory levels on shortages and outdates in a centralized blood bank or community blood center. He derived several equations for computing the expected shortages and outdates at the central bank and the shortages at the member banks. Based on Yen's work, we are currently conducting further research to obtain the optimal target inventory levels for a community blood center which maintains

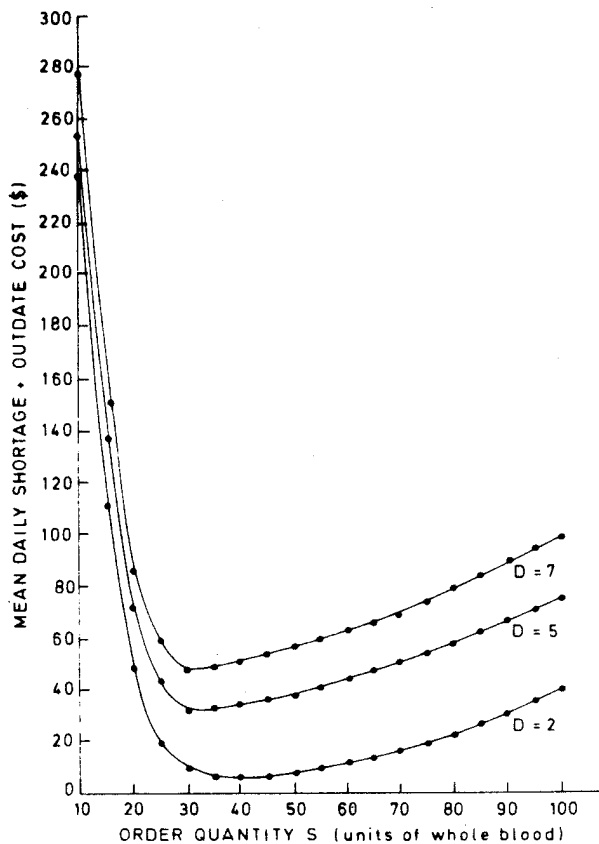


FIG. 1. Average shortage plus outdate costs and order quantity (FIFO issuing, a 300 day simulation period, $d_M = 12$, $p = 0.5$ and $A = 6$).

control over the units at the member locations. It appears that for such systems the target levels are somewhat lower than for decentralized systems because it is easier to transship units if necessary. These results will be presented in a later paper.

Methods

Data from Rush-Presbyterian-St. Luke Medical Center, Evanston Hospital and the North Suburban Blood Center in the metropolitan Chicago area were used in the analysis. In addition, some of the data published by Brodheim, Hirsch and Prastacos² were used as a basis for comparing the shortage rates, inventory levels, mean daily demand and the demand distribution in this study.

The methods of analyses involved various statistical estimation techniques, economic modeling, simulation and inventory operations-analysis. As mentioned in our earlier paper,³ changes in operating policy will clearly have an impact on the performance of the blood bank. Because

the target inventory level is affected by many environmental factors, it was necessary to construct a model of the blood bank in order to test the complex interactions and effects of these factors. An expanded version of the basic model structure of our earlier paper (which was extensively validated in Pinson¹²) is used in this paper. The model requires specification of input factors relating to system environment and control policy. The factors considered in this analysis included: parameters to specify the daily demand process, parameters to specify the age (of units arriving at the bank) process, the transfusion to crossmatch ratio p , target inventory levels, issuing policy, crossmatch release time D , shortage cost and outdate cost.

Model outputs include a detailed record of all inventory transactions and the trajectory of the age distributions of both assigned (crossmatched) and unassigned inventories. For the purposes of decision making we seek to minimize mean daily shortage plus outdate costs. Thus a cumulative record of total outdates and shortages is kept. Upon multiplication by the appropriate unit cost and division by the number of days in the run, the desired average cost is obtained. The generation of average outdate plus shortage costs for a fixed set of inputs over a range of different values for target inventory levels (S) yields an average cost curve. Figure 1 illustrates a typical set of such curves for a range of different values of D , the crossmatch return parameter (the transfusion to crossmatch ratio is fixed at .5). These curves indicate the stability on the part of the optimal target inventory level with regard to D as was previously noted.³ That is, even as D increases from 2 to 5 to 7 days the optimal inventory level (S^*) which minimizes the costs remains between 30 and 40 units. Furthermore, as noted in Figure 1, the cost function is relatively flat over a wide range of inventory levels; essentially from $S = 30$ to $S = 50$ the costs do not vary greatly especially for the more reasonable value of $D = 2$. This flatness is caused by the fact that as S increases beyond 30 the number of shortages drops to zero and although the number of outdates increases as S increases they increase very slowly until S passes 60. This slow increase in outdates is caused by the fact that there are 21 days in which to transfuse a unit and for S below 60 it is possible to transfuse most units before they outdate. As either the transfusion to crossmatch ratio decreases to .25 or as the crossmatch release time (D) increases to 5 or 7, the response of outdates to an increase in the value of S is more rapid. S is the target daily inventory level, not the amount ordered each day. The amount

ordered is only the amount of transfusions plus outdates of the previous day which will then bring the inventory back up to S.

Figure 2 gives the number of shortages and the number of outdates for the case in Figure 1 when $D = 5$. Figure 3 plots the cost curves for shortages and for outdates and their sum which is the total cost. The implications of these observations are as follows: 1) The effect on shortages and outdates of the ordering policy is minimal for S's in the neighborhood of the optimal S^* . The insensitivity of the S's in the neighborhood of S^* is important because a blood bank cannot always achieve S^* each day. Indeed, large drawings of blood through donor plans can often disrupt a policy of achieving S^* on a daily basis and the hospital blood bank administrator must seek an average S^* over time, and 2) The optimal inventory level is relatively insensitive to the value of D. This means that the blood bank administrator can set S^* and then concentrate inventory management control on reducing D knowing full well that S^* will not change significantly.

Optimal Decision Rule

There are many exogenous and policy control factors associated with any blood banking sys-

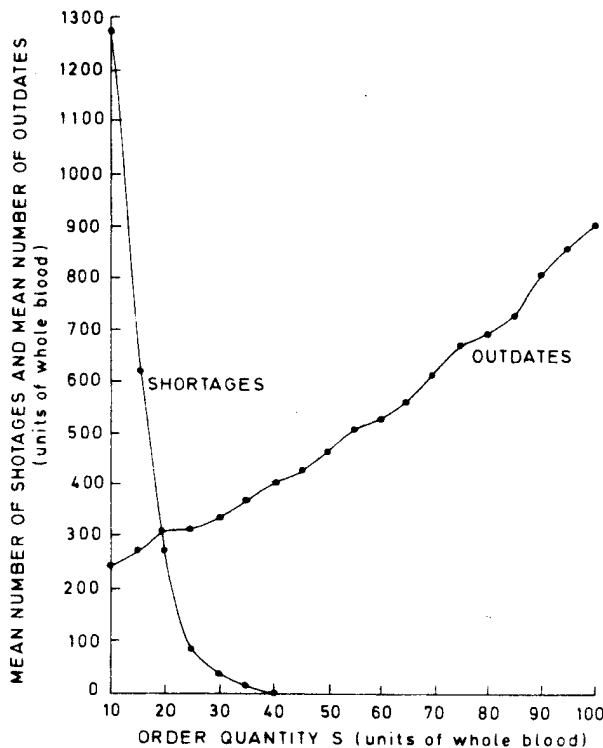


FIG. 2. Average shortages and outdates used to produce the cost curves of Figure 1 for the case $D = 5$.

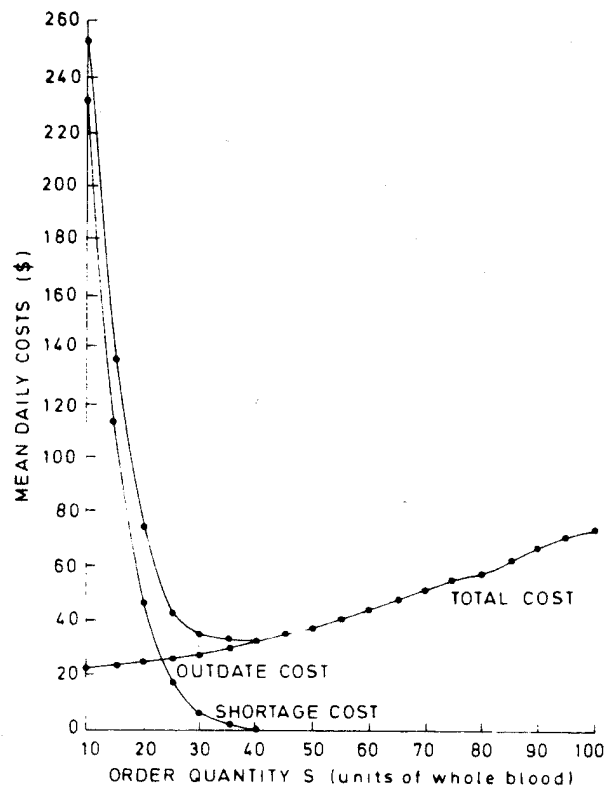


FIG. 3. Average total daily costs decomposed into its two components: outdate cost and shortage cost when $D = 5$.

tem. The exogenous factors identified for the purposes of this discussion include the parameter specifying the mean daily demand, the parameters specifying the age of units supplied for distribution, the per unit shortage and outdate costs and the fraction of total daily demand which is transfused. The control factors for the system include the issuing policy, the crossmatch release period (D) and the inventory level (S).

The functional relationship between S^* and the various control and exogenous factors will be called the "optimal decision rule." A statistical experiment was carried out. The assorted factors were varied throughout their range and the simulation model was used to compute the S^* value associated with each factor value configuration. The results of this experiment were then used in a curve fitting analysis to identify the desired functional relationship (*i.e.*, the appropriate optimal response surface). It is important to note that the average cost surface is not being identified, but rather the surface of cost minimizing values of S.

The functional we seek can in general be written as equation:

$$S^* = f(d_1, d_2, \dots, A_1, A_2, \dots, p, C_S, C_0, D, I) \quad [1]$$

where

- d_i = parameters describing the demand process;
- A_i = parameters describing the age of supply process;
- p = fraction of total daily demand transfused (*i.e.*, the transfusion to crossmatch ratio);
- C_s = unit shortage cost;
- C_o = unit outdate cost;
- D = crossmatch release period;
- I = issuing policy indicator.

Due to the previously observed³ domination of first-in first-out (FIFO) as the best issuing policy for $D \leq 5$ and $p \geq .25$, the experimental design was restricted to the following factors.

1. Mean Daily Demands, d_M
 d_M takes the values {2,3,12,16,18,24,32,48}
2. Mean Age of Supply, A
 A takes the values {1,6.03}
3. Crossmatch Release Period, D
 D takes the values {1,2,4}
4. Shortage Cost, C_s
 C_s takes the values {\$35,\$55}
5. Transfusion to Crossmatch Ratio, p
 p takes the values {.25,.5}.

The lower values 2 and 3 of mean daily demand correspond to rarer types of blood or to transfusion locations with low daily demands for common types. The large values 32 and 48 correspond to daily demands at large hospitals or a community blood center for more common types of blood. The age composition of arriving supply was drawn from two classes of distributions. An empirical distribution estimated from hospital data (with a mean age of supply of 6.03 days) and a degenerate uniform distribution yielding fresh supply were considered. Other distributions had been tested in previous work^{3,12} and were found to provide no additional information or policy changes.

Outdating cost was fixed at \$25, issuing policy was set to be first-in first-out (FIFO) and the transfusion fraction was set at .25 and .5. The actual costs of \$25 for outdates and \$35 or \$55 for shortages are not important in absolute value for determining S^* . What is important is their ratio or relative value.^{12,16} The absolute quantities merely reflect the estimated average cost of outdates and shortages. The target inventory level S^* is not sensitive to reasonable ranges of the ratio of outdate to shortage costs.

A full factorial design for this experiment would involve 192 separate simulation-optimization runs. A $\frac{1}{2}$ factorial design⁵ was chosen and

thus 96 separate observations of S^* were generated. The actual design and the results of the experiment are indicated in Cohen and Pierskalla.⁴ These results were analyzed by standard regression techniques to determine which input parameters were significant in the computation of S^* and what the functional form of the decision function should be. Both linear and log-linear functional forms were investigated. The log-linear function gave significantly better results and the results of the regression run are presented in Table 1.

The most significant explanatory variable is the mean daily demand, (d_M). However, the transfusion/crossmatch ratio (p) and the crossmatch release period (D) are also significant, but their coefficients of 0.12159 and -0.06769 , respectively, in the regression equation indicate that their influence on the optimal S^* is not nearly as large in magnitude as that of d_M . The other two independent variables, A and C_s , are not significant and also their coefficients are small. Consequently, these last two variables were dropped and the regression was run again using only the significant variables. In the second regression the intercept term changed but all other coefficients and statistics coincided to at least three significant digits. Using the results of the second regression, the optimal decision rule can be written by equation:

$$\ln S^* = 1.7967 + 0.7604 \ln (d_M) + 0.1216 \ln (p) - 0.0677 \ln (D) \quad [2]$$

or equivalently by equation:

$$S^* = 6.03(d_M)^{.7604}(p)^{.1216}(D)^{-.0677} \quad [3]$$

where

- d_M is the mean daily demand for a blood type;
- p is the average transfusion to crossmatch ratio, and
- D is the crossmatch release period.

In Figure 4, this optimal decision rule is graphed for the case when $p = .25$ and $p = .5$ and $D = 1$ day. Figure 5 presents the optimal decision rule for $p = .25$ and $p = .5$ when $D = 2$ and 4. For fixed p and D it is interesting to note from Figure 4 that a positive coefficient of 0.7604 for mean daily demand in the optimal decision rule indicates a concave shape to the rule. For a blood bank this would mean that if, for example, as the mean daily demand is increased, then a less than proportional increase in the order quantity would be optimal. Or another way of viewing the rule is that a blood bank which doubles its size (in terms of mean daily demand) should in-

Table 1. Blood Decision Rule All Variables (Log-Linear)*

Variable	Coefficient	Statistical Error	t Value
ln (d _M)	0.7604	0.00972	78.24
ln (p)	0.1216	0.02925	4.16
ln (D)	-0.0677	0.01791	-3.78
ln (A)	-0.0138	0.01128	-1.22
ln (C _S)	0.0520	0.04485	1.16
Intercept	1.61248		
Regression		Error	
Degrees of Freedom	5	Degrees of Freedom	90
Sum of Squares	60.72035	Sum of Squares	0.88765
Mean Square	12.14407	Mean Square	0.00986
SE of Estimate	0.09931		
F-Value	1231.3		
Multiple R-Squared	98.56		

* Logarithms are natural or Napierian.

crease its optimal inventory level by less than 70 per cent (provided the p's and D's remain the same). Using equation (2) or (3), the blood bank administrator can compute the optimal target inventory level for each blood type merely by inserting the mean daily demand for each blood type, the average transfusion/crossmatch ratio, p and the crossmatch release period, D.

Evaluation of the Decision Rule

The mean daily amount of blood transfused (usage) is determined by the transfusion to crossmatch ratio times the mean daily demand. A range of about 1 to 25 units transfused per day were considered in the experimental design. This corresponds to an annual volume of between 300 to 10,000 transfusions. Because almost all blood banks have type specific mean demand volumes which fall into these ranges, it was felt that testing the extrapolation of the decision rule for even larger volumes was not necessary.

A more important evaluation is how do the results of using the decision rule compared with the data from the Chicago area and by Brodheim, Hirsch and Prastacos.² We have already shown in Figures 2 and 3 that use of the optimal target inventory level at Evanston Hospital for O⁺ blood results in virtually zero shortages and a very low outdate rate. The actual shortage rate is less than .004 for this particular blood type and the outdate rate is less than .02.

Tables 2 to 5 show the target inventory levels and their corresponding shortage and outdate rates for a large range of mean daily demands for varying p and A (the mean age of arriving units). When the mean daily demand for cross-

matching a particular blood type is two units (p = .5, A = 1 and D = 1) then the optimal target inventory to maintain on hand each day is eight units. By maintaining this level the blood bank

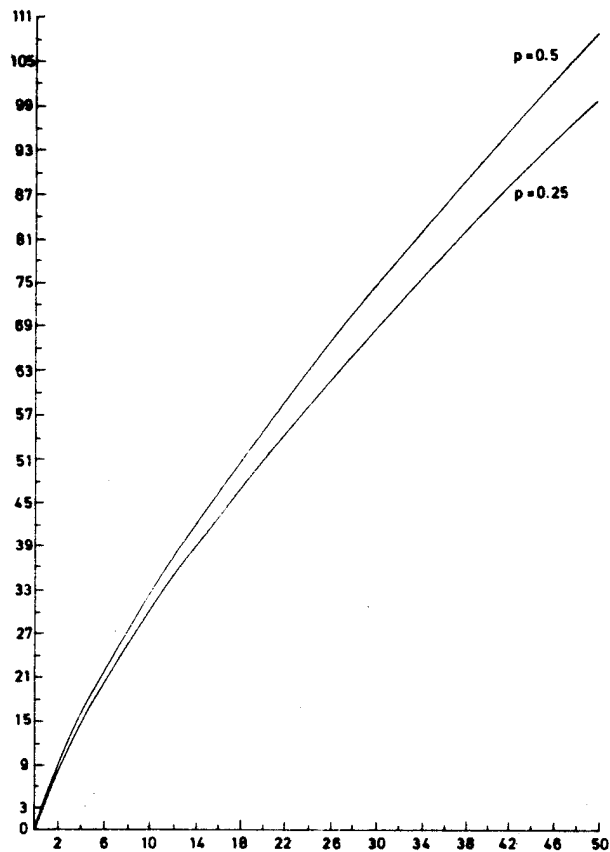


FIG. 4. Optimal target inventory levels for different mean daily demand levels when D = 1 and p = 0.25 and 0.5.

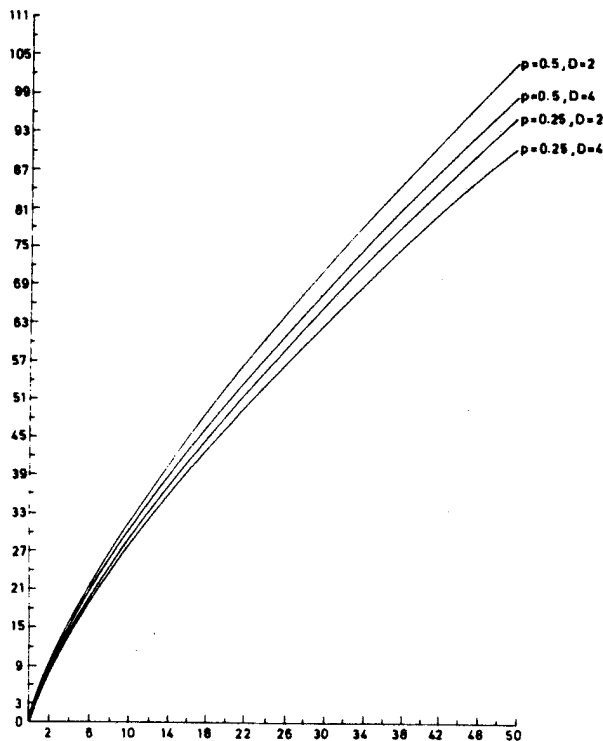


FIG. 5. Optimal target inventory levels for different mean daily demand levels when $D = 2$ and 4 , and $p = 0.25$ and 0.5 .

will experience a shortage rate of about .022 and an outdate rate of about .013. As the volume of activity increases, *i.e.*, mean daily demand is larger, the statistical law of large numbers comes into play. When the mean daily demand is 16 units and the optimal target inventory level of 38 units is maintained, the shortage rate and outdate rate virtually drop to zero (both are less than .006) when $D = 1$ or 2 .

Even if the blood bank administrator operates at the optimal target inventory level (S^*) it is still necessary to control the crossmatch release period (D) the average transfusion to crossmatch ratio (p) and the average age of arriving units (A) in order to keep shortages and outdates

Table 2. Shortage and Outdate Rates Using the Optimal Decision Rule for Given $p = .5$ and $A = 1$

d_M	S^*		Shortage Rate		Outdate Rate	
	$D = 1$	$D = 2$	$D = 1$	$D = 2$	$D = 1$	$D = 2$
2	8	8	.022	.022	.013	.064
16	38	36	.003	.006	.001	.004
32	64	61	.002	.004	.0001	.002
48	88	84	.001	.003	.0001	.001

Table 3. Shortage and Outdate Rates Using the Optimal Decision Rule for Given $p = .5$ and $A = 6$

d_M	S^*		Shortage Rate		Outdate Rate	
	$D = 1$	$D = 2$	$D = 1$	$D = 2$	$D = 1$	$D = 2$
2	8	8	.022	.022	.068	.155
16	38	36	.003	.006	.005	.027
32	64	61	.002	.004	.005	.019
48	88	84	.001	.003	.004	.019

down. This control is especially important for transfusion locations where the mean daily demand for a blood type is small. The worst case shown is in Table 5. For $d_M = 2$, $D = 2$, $p = .25$ and $A = 6$, the optimal S^* is 7. However, at this S^* the shortage rate is 3.7 per cent and the outdate rate is 44.7 per cent. On the other hand, from Table 2 holding $d_M = 2$ but reducing D to 1, A to 1 and increasing p to .5, the optimal $S^* = 8$ but more importantly the shortage rate is 2.2 per cent and the outdate rate is 1.3 per cent. This drop in the shortage rate occurs because there is one more unit on hand each day, *i.e.*, $S^* = 8$ versus $S^* = 7$. The very significant drop in the outdate rate occurs because the units are available in unassigned inventory more frequently since $D = 1$ versus $D = 2$ and $A = 1$ versus $A = 6$ and the probability of transfusion at each crossmatch p is higher $p = .5$ versus $p = .25$.

Indeed, by looking at Tables 2 to 5 it is possible to isolate the effects of any single parameter holding the other parameters constant. One of the most important control parameters for the administrator is the crossmatch release period (D). It is also one of the most significant parameters in reducing outdated. Less easily controlled parameters are p and A . However, if the administrator can reduce A and/or increase p , then the outdate rate will also drop significantly.

Table 4. Shortage and Outdate Rates Using the Optimal Decision Rule for Given $p = .25$ and $A = 1$

d_M	S^*		Shortage Rate		Outdate Rate	
	$D = 1$	$D = 2$	$D = 1$	$D = 2$	$D = 1$	$D = 2$
2	7	7	.037	.037	.220	.357
16	35	33	.008	.014	.020	.120
32	59	56	.006	.012	.009	.082
48	81	77	.006	.012	.009	.081

The parameters D , p and A have less impact on the shortage rate. In fact, the optimal target inventory level (S^*) plays the major role with regard to reducing the shortage rate. As was mentioned earlier, the S^* tends to be the inventory level at which the shortages are virtually zero especially when D , p and A are in good control.

It should be reiterated here that if the administrator chooses to operate above the optimal S^* , as shown in Figures 1, 2 and 3, the outdates (and costs) will rise with little additional impact on reducing shortages. Conversely operating below S^* means that outdates will drop but shortages (and costs) will rise. Fortunately, when D is small (say, $D = 1$) and p is large (say, $p = .5$), then the cost curves are relatively flat near S^* so there is a considerable amount of flexibility in achieving S^* on a day-to-day basis.

A final important fact to notice concerns the parameter A , the average age of arriving units at the transfusion location. Tables 2-5 indicate the larger that A becomes the significantly larger is the outdate rate. When a transfusion location depends upon a central bank or supplementary sources for its supply, it may not be able to control the ages of arriving units. For large volumes of demand, this lack of control does not greatly impact outdating if the administrator maintains $D = 1$ and $p = .5$. However, for small daily demand levels such as $d_M = 2$ (especially with large D and/or small p), the control of A is very important. Often small blood banks have such small daily demand levels. For such locations the supplied blood should be as fresh as possible. For optimal efficiency these small banks should have their blood inventory periodically replaced with all fresh units and the unassigned old units returned to the regional supplier.

Discussion

Brodheim, Hirsch and Prastacos² give a series of inventory level curves based on shortage rates of .2, .1, .05, .02 and .01. They suggest that the blood bank administrator should choose the shortage rate at which to operate the blood bank. The administrator can read off the inventory level from the appropriate curve using the mean daily demand for each blood type. Our results indicate that it is not necessary for the blood bank administrator to choose a shortage rate. In fact a shortage rate much above .01 or .001 (depending on the mean daily demand p and D) will result in an extremely

Table 5. Shortage and Outdate Rates Using the Optimal Decision Rule for Given $p = .25$ and $A = 6$

d_M	S^*		Shortage Rate		Outdate Rate	
	$D = 1$	$D = 2$	$D = 1$	$D = 2$	$D = 1$	$D = 2$
2	7	7	.037	.037	.340	.447
16	35	33	.008	.014	.091	.204
32	59	56	.006	.012	.049	.171
48	80	77	.006	.012	.049	.164

high cost nonoptimal level. It is true that a shortage rate of .2 the outdate rate will be lower than at a shortage rate of .01. However, it will not be very much lower because outdates are essentially caused by two factors, one of which is only slightly affected by the shortage rate. The first factor is that the supply of arriving stocks of blood at the bank is variable. On days of mobile drawings or a donor recruiting campaign the supply of blood jumps up and on other days it is significantly lower. The second factor is the random crossmatch demands and actual transfusions resulting therefrom. There is a significant daily fluctuation in demands and transfusions. In those periods of time where the supply is higher on the average and demand is lower on the average, outdating will occur regardless of fairly large differences in the average shortage rate chosen by a blood bank administrator. Consequently, choosing a high shortage rate will not reduce outdating by much since outdating tends to occur in batches but it will significantly increase shortages at the other times when supply is low and demand is average or above average (see Figure 2).

The preceding analysis demonstrates that the choice of a target inventory level is determined by trading off the cost of shortages with the cost of outdates. We have also seen that it is possible to associate an expected shortage rate with each possible value of the target inventory. The range of costs considered in the estimation of the decision function resulted in shortage rates at the

optimal target inventory level of about .01 or less when D , p and A are at reasonable levels.

It is interesting to consider the conditions under which target inventory levels yielding a shortage rate of .05 or .1 would be optimal. In particular, one may wonder at the relative value placed on outdates and shortages by a manager who in setting a target inventory level chooses a higher shortage rate. It is important to note that each possible value of target inventory (and its corresponding shortage rate) has associated with it an "imputed" cost of shortage which would make that inventory level optimal.

Figure 6 illustrates the range of such imputed costs for a large blood bank. In a system where the ratio has a value of 500 an outdated unit would be 500 times more costly than a shortage unit. Similarly, a value of .5 means that an outdated unit costs one-half times the cost of a shortage unit.

In earlier studies^{3,12,16} it was observed that in 1973 and 1974 an outdated unit cost approximately \$25 (primarily lost processing cost) and a shortage unit cost about \$35 to \$55 (primarily telephone calls and emergency transportation or the cost of the freezing and thawing process for a frozen unit). Consequently, realistic ratios of outdate to shortage costs would be in the range $\$25/\$35 = .714$ to $\$25/\$55 = .455$. Ratios in excess of 1 make very little realistic sense since such ratios mean outdates cost more than shortages. However, as can be seen from the average case in Figure 4, if the blood bank administrator were to choose to operate at shortage rates of .2, .1, .05 or .01, he would be implicitly saying that he views outdates 367, 207, 110 and 31 times more important than shortages, respectively. In fact, at the optimal target inventory level S^* for this volume and common blood type, the shortage rate is .001 when the outdate

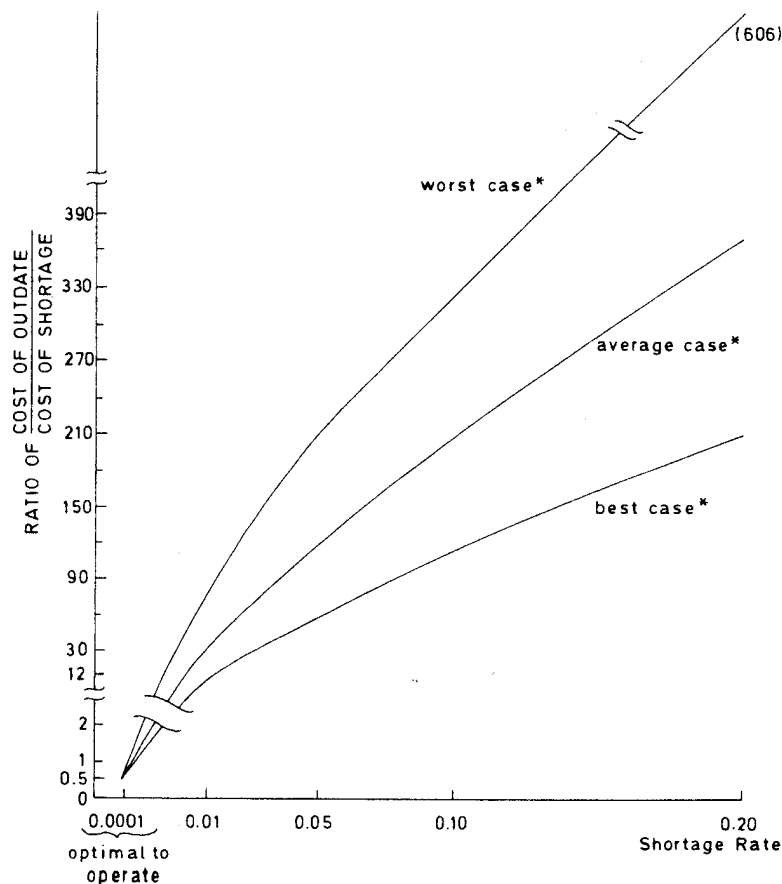


FIG. 6. Imputed cost ratio of outdate cost/shortage cost when operating a large hospital blood bank ($d_M = 48$) at different shortage rates.

cost to shortage cost ratio is .5, *i.e.*, an outdated unit costs about one-half of a shortage unit in time and/or money.

The imputed cost ratios of operating at any shortage rate for any level of mean daily demand can be computed to generate curves similar to the ones in Figure 6. We have carried this out for a large number of such cases. In each case, we observed that although the imputed ratios for shortage rates at .2, .1, .05 and .01 differ for each blood type and mean daily demand level, the ratios were all far in excess of 1. This means that a hospital administrator choosing a target inventory with these large shortage rates is implicitly saying that an outdated unit is far more important than a shortage unit. Our exposure to the Chicago area blood banking system indicates that in fact the converse is true, namely, a unit short is far more important and more costly than a unit outdated.

This analysis indicates the inconsistency of choosing to operate a blood bank at what appear to be reasonable shortage rates of .1 or .05. By selecting a target inventory level according to equation (3) the blood bank manager can achieve far better performance for all reasonable values of the outdate shortage cost ratio.

A final issue which was considered was the relationship between the decision rule results and the ordering rules which have been observed in practice. Many blood bank administrators keep sufficient blood on hand to meet anticipated needs for six to eight days. It is possible to compute the number of days of transfusion supply on hand from the order-up-to quantity as follows:

$$\text{Number of days of transfusion supply} = \frac{S^*}{p \cdot d_M}$$

Figure 7 is a graph of the number of days of transfusion supply on hand versus mean daily usage $p \cdot d_M$. As the system scale increases, the optimal number of days of transfusion supply on hand decreases. This

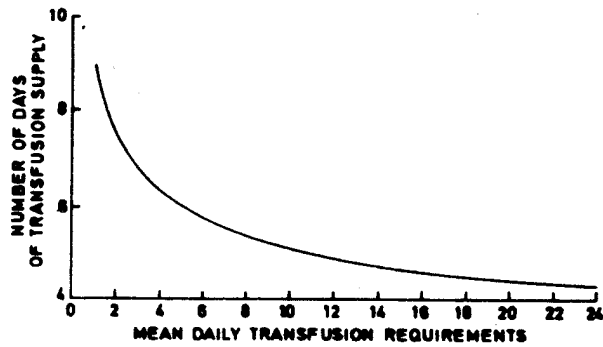


FIG. 7. Number of days transfusion supply versus the mean daily transfusion requirements when $p = .5$ and $D = 1$.

is another example of scale economies.¹ Moreover, the curve is convex, and is relatively flat at a value of approximately six days of transfusion supply for a broad range of system scales.

References

1. Beatzoglou, T., D. Connors, B. Friedman, and W. Pierskalla: Economies of Scale in a Blood Bank. Evanston, Health Services Research Center, Northwestern University, 1976.
2. Brodheim, E., R. Hirsch, and G. Prastacos: Setting inventory levels for hospital blood banks. *Transfusion* 16:63, 1976.
3. Cohen, M. A., and W. P. Pierskalla: Management policies for a regional blood bank. *Transfusion* 15:58, 1975.
4. ———: Target inventory levels for a hospital blood bank or a decentralized regional blood banking system. NHCNC Working Paper No. 8, National Health Care Management Center, University of Pennsylvania, Philadelphia, 1979.
5. Connor, W. S., and S. Young: Fractional Factorial Designs for Experiments with Factors at Two and Three Levels. National Bureau of Standards, Applied Mathematics Series 58, U.S. Department of Commerce, 1961.
6. Elston, R. C., and J. C. Pickrel: A statistical approach to ordering and usage policies for a hospital blood bank. *Transfusion* 3:41, 1963.
7. ———, and J. C. Pickrel: Guides to inventory levels for a hospital blood bank determined by electronic computer simulation. *Transfusion* 5:465, 1965.
8. Hirsch, R. L., and J. B. Schorr: A computerized blood inventory system in the service of the community, the blood donor and the patient recipient. Second Year Report (Section 1), The Community Blood Council of Greater New York, 1970.
9. Jennings, J. B.: An analysis of hospital blood bank

- whole blood inventory control policies. *Transfusion* 8:335, 1968.
10. ———: Blood Bank Inventory Control. *Manage. Sci.* 19:637, 1973.
 11. Neyman, J.: On a new class of 'contagious' distributions applicable in entomology and bacteriology. *Ann. Math. Stat.* 10:35, 1939.
 12. Pinson, S. D.: A study of decision policies in blood bank inventory control: A computer simulation approach, M.S. Thesis, Northwestern University, 1973.
 13. Rabinoqitz, M.: Hospital blood banking: An evaluation of inventory control policies. City University of New York Mount Sinai School of Medicine, 1970.
 14. Rockwell, T. H., G. F. Lindsay and T. E. Hoover: Investigation of commodity blood banking systems: An application of simulation methodology. The Ohio State University Research Foundation Report No. RF1234, 1973.
 15. Yahnke, D. P., A. A. Rimm, C. J. Mundt, R. H. Aster, and T. M. Hurst: Analysis and optimization of a regional blood bank distribution process. *Transfusion* 12:111, 1972.
 16. Yen, H.: Inventory management for a perishable product: Multi-echelon system, Ph.D. Dissertation, Northwestern University, 1975.
-
- Morris A. Cohen, Ph.D., Assistant Professor, Department of Decision Sciences, The Wharton School, University of Pennsylvania, Philadelphia, Pennsylvania 19104.
- William P. Pierskalla, Ph.D., Director, National Health Care Management Center, University of Pennsylvania, Philadelphia, Pennsylvania 19104.