

REGIONAL BLOOD INVENTORY CONTROL AND DISTRIBUTION

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We shall present the results of a number of projects which have been carried out to analyze specific operational decisions at the hospital blood bank, transfusion service and central blood bank levels. We will discuss and develop the following areas of operational decisionmaking: forecasting of demand for blood products, inventory control at the transfusion service level, inventory control in a central blood bank system and finally transshipment allocation policy for a central blood bank system.

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Each of the following sections is a relatively self-contained report of the research project concerned with these areas of operational decisionmaking. Specific decisions, resources and performance measures for each area are introduced and where appropriate, analytic results on optimal policies and procedures are presented. The key findings of the various research projects are summarized in the final section.

FORECASTING

Demand for the variety of blood products carried in inventory is a major source of uncertainty in the management of blood banks. Accurate forecasts of the quantity and timing of future demands are key inputs to inventory control and donor recruiting decisionmaking. In particular, decisions relating to the quantities of blood products to be carried in stock, the scheduling of drawings from donor lists or mobile drawings and ordering from other blood banks should all be made with such forecasts in mind.

It is clear that demand for blood products is inherently stochastic (random) due to the complexity and unpredictability of the phenomena which generate these demands. In our research we have noted that even very large transfusion services have wide fluctuations. Ultimately this demand for blood products can be traced to the occurrence of those medical conditions and outcomes for which blood or component transfusions are indicated. Such conditions include emergency treatment for trauma, scheduled and unscheduled surgeries and a large number of disease conditions. Since it is quite difficult to consider the prediction of these causal factors, which in combination lead to blood demand, it is not realistic to consider the development of a multivariate model to forecast demand.

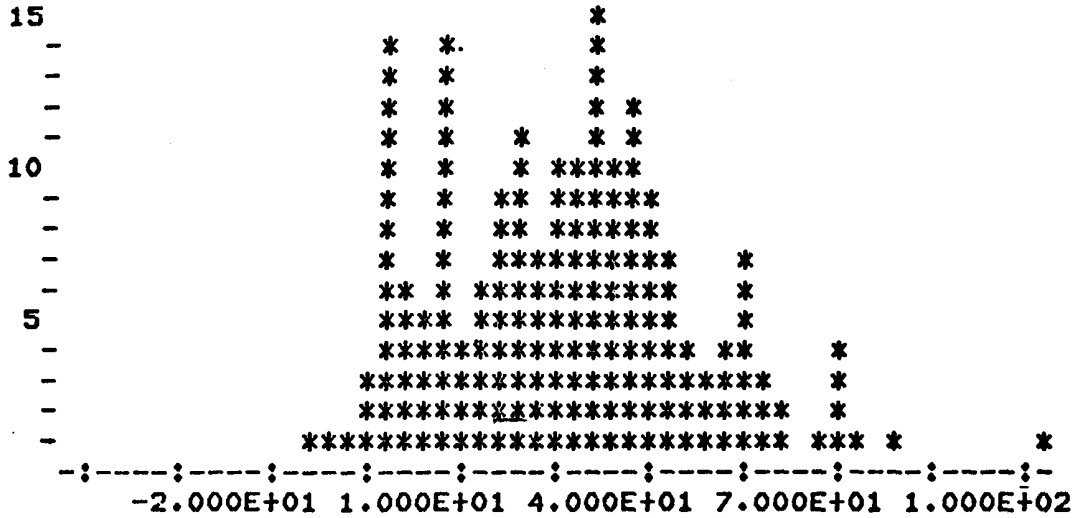
The approach considered in this section forecasts demand by the analysis of the past history, called the time series, of past blood demands (cross-match requests, cross-match quantities or transfusion quantities). Time series analysis is based on the premise that observed patterns of variation contain sufficient information for the prediction of future patterns, hence this method is much easier to implement than multivariate approaches which require data on both past demand and past causal factors. We will develop models which can be used to forecast the quantity (number of units) of blood requested for cross-match for each blood type. We will consider both daily and monthly time series.

The first issue to evaluate is the nature of the probability distribution of cross-match quantities for a particular blood product on any given day. This issue is of particular importance since blood inventory stocking decisions will be directly affected by the shape of this demand distribution. For example, a highly skewed, highly dispersed distribution of demand could indicate the need for more inventory to avoid shortages than would be required for the case where demand is characterized by a symmetric, low variance distribution. Figure 1 is an example of a frequency histogram for daily cross-match quantities for O Rh positive (O+) units taken from 1 year of data from Rush Presbyterian St. Luke's Hospital (RPSL) in Chicago. There is some skewness in this distribution and considerable variance.

In the course of earlier research, Yen¹ successfully analyzed the underlying process generating demand for blood in a hospital blood bank by decomposing the demand random variables which were first introduced by Elston and Pickrel.^{2,3}

HISTOGRAM

ABS. FREQ.



O+X

MEAN = 4.1715E+01
STD. DEV = 2.0909E+01
SAMPLE SIZE = 186

Figure 1. Frequency Histogram of Units Cross-matched (O+) from RPSL

Demand for blood products can be computed by observing the number of those patients in a hospital who require transfusion services on any given day (cross-match requests) and the number of units requested for cross-match for each patient. Mean or average demand (cross-match quantity) on any given day is simply the product of the mean number of requests times the mean number of units per request.

In order to specify the probability distribution for the number of units of a specific category or type, Yen demonstrated that it is sufficient to estimate two parameters, the mean number of patients per day requiring transfusion (d_N) and the mean number of units requested for each patient (d_R). Moreover, the Neyman A distribution characterized by these two parameter values gave an adequate representation of the demand distribution obtained from data collected from a particular hospital. Subsequent analysis with regard to target blood bank inventory decisions (see the second section of this paper and Cohen and Pierskalla^{4,5}) indicated that it was not necessary to keep track of these two components separately since effective system performance can be obtained by basing blood inventory decisions on mean demand ($d_N d_R$). Thus, in order to effectively control the blood inventory, forecasts of mean daily demand must be generated.

Figure 2 is a graph of 4 weeks of observed daily crossmatch quantities for O+ taken from data obtained from RPSL (indicated by the symbol *). It is clear from the graph that the data display considerable variation and a marked weekly (7 day) cycle. Examination of monthly demand data also indicated high variance and the possibility of seasonal variation.

In order to forecast these blood demands it is necessary to apply time series analysis methods to demand (cross-match quantity) data for each blood type and component. In the remainder

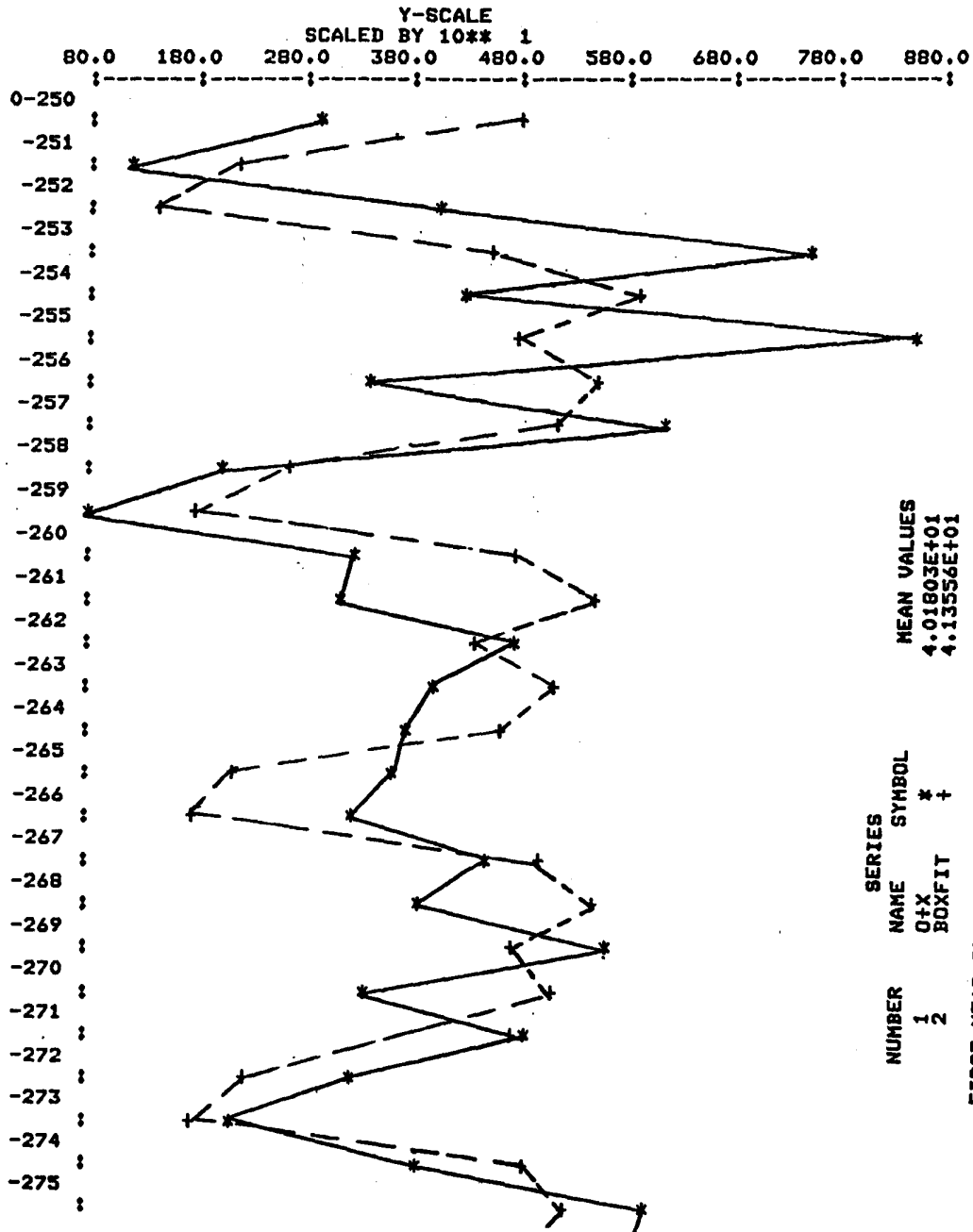


Figure 2. Time Series of Units Cross-Matched (O+) from RPSL

of this section we will illustrate the application of time series analysis to blood demand forecasting by presenting the results of an analysis for a 365-day sample of blood demand for each blood type and a 12-year sample of monthly observations for total (sum over all blood types) cross-match quantities of whole blood.

Again, for illustration purposes, we will only consider the O+ cross-match time series in this section. Results for all eight blood types are summarized by Cohen, Pierskalla, Sasseti and Walkky.⁶

Time Series Analysis of Demand. Figure 3 is a graph of the autocorrelation function of the daily O+ cross-match quantity time series. This function indicates the extent to which current observations are correlated with lagged values of past observations. Extensive analysis of the relationship between the shape of the autocorrelation function and the nature of the time series functional form has been carried out by Box and Jenkins.⁷ It is possible to discern a peak at lag values of 7, 14, 21 and 28 days which indicates that a cycle of length 7 days is present in the time series.

Extensive testing of alternative Box-Jenkins formulations yielded an optimal forecasting model of the form:

$$\begin{aligned}
 (1) \quad D(t) &= C_1 Z(t-1) - C_2 A(t-1) \\
 &+ (1+C_3) Z(t-7) - C_4 A(t-7) \\
 &- C_1 (1+C_3) Z(t-8) + C_2 C_4 A(t-8) \\
 &- C_3 Z(t-14) - C_1 C_3 Z(t-15)
 \end{aligned}$$

where C_1 = Nonseasonal autoregressive parameter

C_2 = Nonseasonal moving average parameter

C_3 = Seasonal autoregressive parameter

C_4 = Seasonal moving average parameter

$D(t)$ = Forecast Daily Demand period t

$Z(t)$ = Actual Daily Demand period t

$A(t)$ = Forecasting Error period t ; $(Z(t)-D(t))$.

This model structure is referred to as ARMA(1,1) x ARIMA(1,0,1) multiplicative seasonal model in the Box-Jenkins terminology.

Optimal values of the parameters C_1 to C_4 were computed by a search algorithm using data for the first half of the 365-day series. The following values were obtained for the time series 0+: $C_1 = 0.9288$, $C_2 = 0.8701$, $C_3 = -0.192$ and $C_4 = 0.9154$. Figure 2 is a plot of actual and predicted values. This graph indicates that it is difficult to accurately predict demand on a daily basis. This is due to the very high variance associated with daily cross-match requests. Consequently the relative magnitude of the unpredictable "noise" component of the time series is high relative to the magnitude of the predictable components. It is interesting to note that the forecasting model does provide a reasonable prediction of the weekly cycle.

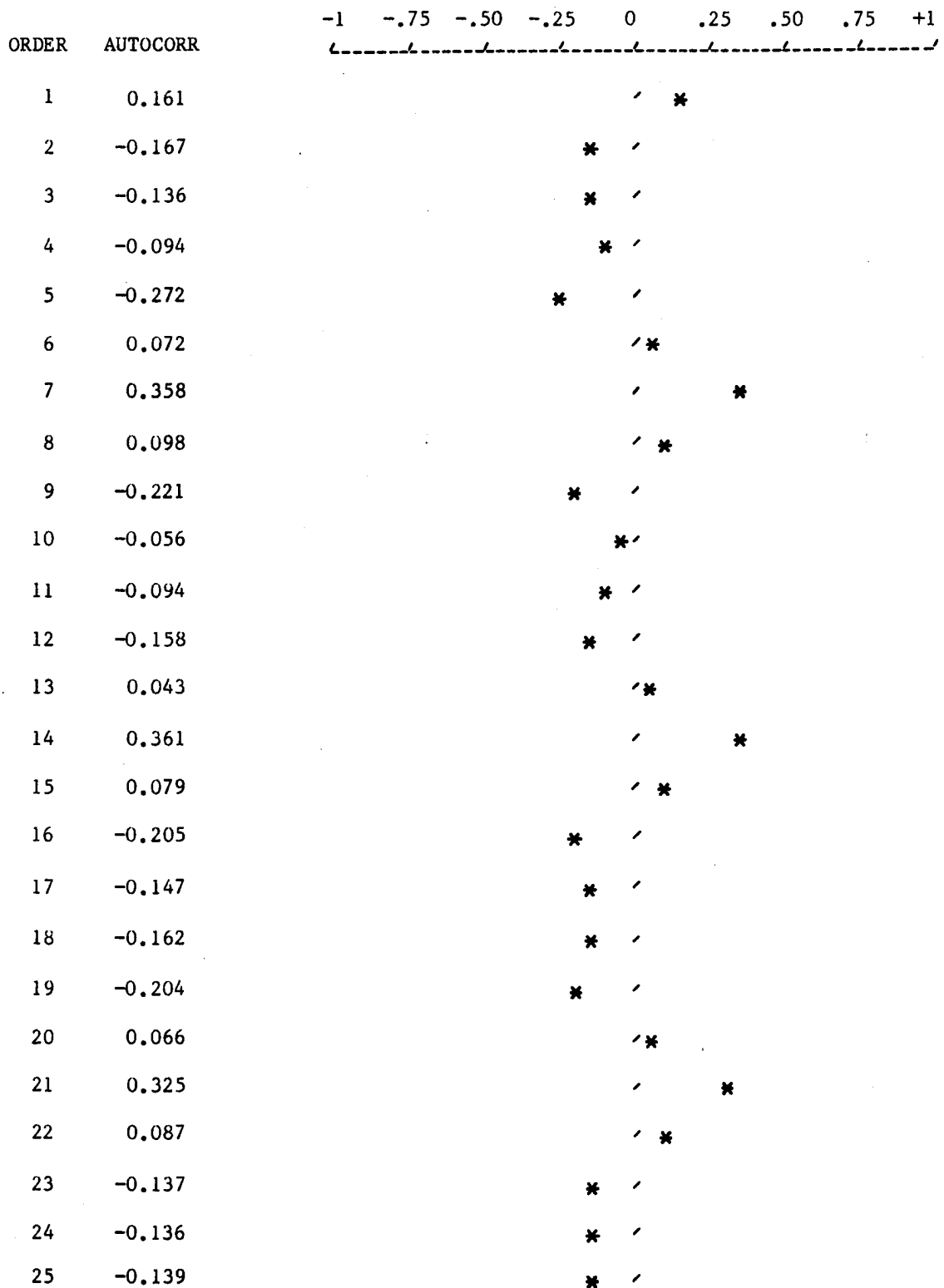


Figure 3. Autocorrelation Function of Cross-match quantity time series (O+)

Stability of the model was estimated by comparing forecast errors of models optimized using data for the first half of the year, and models optimized using data for the second half of the year. The following results were obtained for O+.

Period Optimized: forecast	Percent of Variance Explained by Model
1:1	27.81
2:2	36.29
1:2	37.57

Thus, for example, a model developed from first period data and used to forecast demand for the first period explained 27.81 percent of the variance. The same model used to forecast over the second half of the year explained 37.57 percent of the variance.

The Box-Jenkins model specified in equation (1) also was evaluated to be relatively insensitive to errors in parameter estimation and robust in its applicability across blood types.

The inherent instability of blood demand as illustrated in figure 2 is, in fact, the primary motivation for holding blood inventories as insurance against the risk of shortage. Moreover, demand variability at the local blood bank level also provides an incentive for regionalization since central and regional blood banks have the effect of pooling demand over many transfusion centers. This aggregation of demand will serve to reduce the relative variance of demand at the central or regional banks.

Figure 4 is a plot of the autocorrelation of the forecast errors (residuals). The relatively small values and random scatter indicate that the forecast model as specified in equation (1) has removed all systematic variation in the data. Consequently, the residuals represent a purely random error term (white noise) and the forecast equation in (1) cannot be significantly improved upon by the addition of further terms or by changing coefficient values.

Time Series Analysis of Monthly Demand. Most blood inventory decisions are not reevaluated on a daily basis. In particular, target inventory levels set by using a procedure like that of Cohen and Pierskalla⁵ (discussed in the third section of this paper) probably would be updated on a monthly or quarterly basis. Figure 13 on page 62 illustrates such a target level, computed on a quarterly basis, on a graph which also indicates daily demand variation. We note the relatively low incidence of shortages using these target levels.

In order to forecast monthly demand and to identify seasonal cycles, it is necessary to collect data for a period in excess of 1 year. Unfortunately, only aggregate data (summed over all blood types) were available for such an extended period. For many planning purposes such aggregate forecasts are sufficient. In those cases where forecasts specific by blood type are needed, a reasonable approach would be to forecast demand levels on the basis of the aggregate data and to use estimates of the distribution of demand over blood types as a means of disaggregating these estimates into blood type specific forecasts. We tested the validity of this approach by examining the standard deviation of blood type fractions over 1 year of observations (see table 1). These standard deviations were observed to be relatively small when compared to the mean for the more common blood types. Moreover, the demand fraction for these

ORDER	AUTOCORR	-1	-.75	-.50	-.25	0	.25	.50	.75	+1
1	0.013					*				
2	-0.018					*				
3	-0.020					*				
4	0.057					'*				
5	-0.165					*'				
6	-0.075					*'				
7	-0.035					*'				
8	-0.021					*				
9	-0.064					*'				
10	0.124					'*				
11	0.055					'*				
12	0.029					'*				
13	-0.106					*'				
14	-0.050					*'				
15	-0.027					*'				
16	-0.018					*				
17	-0.020					*				
18	-0.068					*'				
19	-0.055					*'				
20	-0.055					*'				
21	-0.077					*'				
22	-0.018					*				
23	0.105					'*				
24	-0.010					*				
25	-0.060					*'				

FIGURE 4. AUTOCORRELATION FUNCTION OF FORECAST ERRORS (O+)

<u>BLOOD TYPE</u>	<u>MEAN FRACTION</u>	<u>STANDARD DEVIATION</u>
A+	0.3438	0.1104
A-	0.0489	0.0517
O+	0.3804	0.1081
O-	0.0517	0.0447
B+	0.1220	0.0738
B-	0.0132	0.0226
AB+	0.0374	0.0397
AB-	0.0026	0.0076

Table 1: Transfusion Requests as Fraction of
Total Demand (by blood type)

blood types also was observed to be symmetrically distributed about its mean value. The rarer blood type fractions exhibited significant variation relative to their mean and their distribution tended to be skewed. Since the rarer types do not influence the aggregate blood demand significantly, we may conclude that forecasting of aggregate demand and subsequent disaggregation is a reasonable approach to generating longer term blood type specific forecasts for the common blood types. Further analysis, however, is needed for the rarer blood types. (Cohen, Pierskalla, Sasseti and Walky⁶ show that equation (1) can be used with reasonable accuracy for all blood types.)

The best fit for the monthly demand series (12 years of monthly data from RPSL) using Box-Jenkins methodology is as follows:

$$(2) \quad D(t) = Z(t-1) - 0.812 A(t-1) + 0.259 A(t-12) - 0.211 A(t-13)$$

where $Z(t)$ = the monthly transfusion request level

$A(t)$ = the forecast error

$(Z(t)-D(t))$ and $D(t)$ = the computed average for month t .

This forecast equation indicates that the moving average (error) term has cyclical components with periods of 1, 12 and 13 months. Figure 5 illustrates actual and forecast values using this equation. We observe that the forecast equation generates a curve which is considerably smoother than the actual demand series and that there was an observable trend. Moreover, even for aggregate monthly figures there is significant variance, hence it is difficult to predict monthly aggregate cross-match request quantities at the single hospital blood bank level.

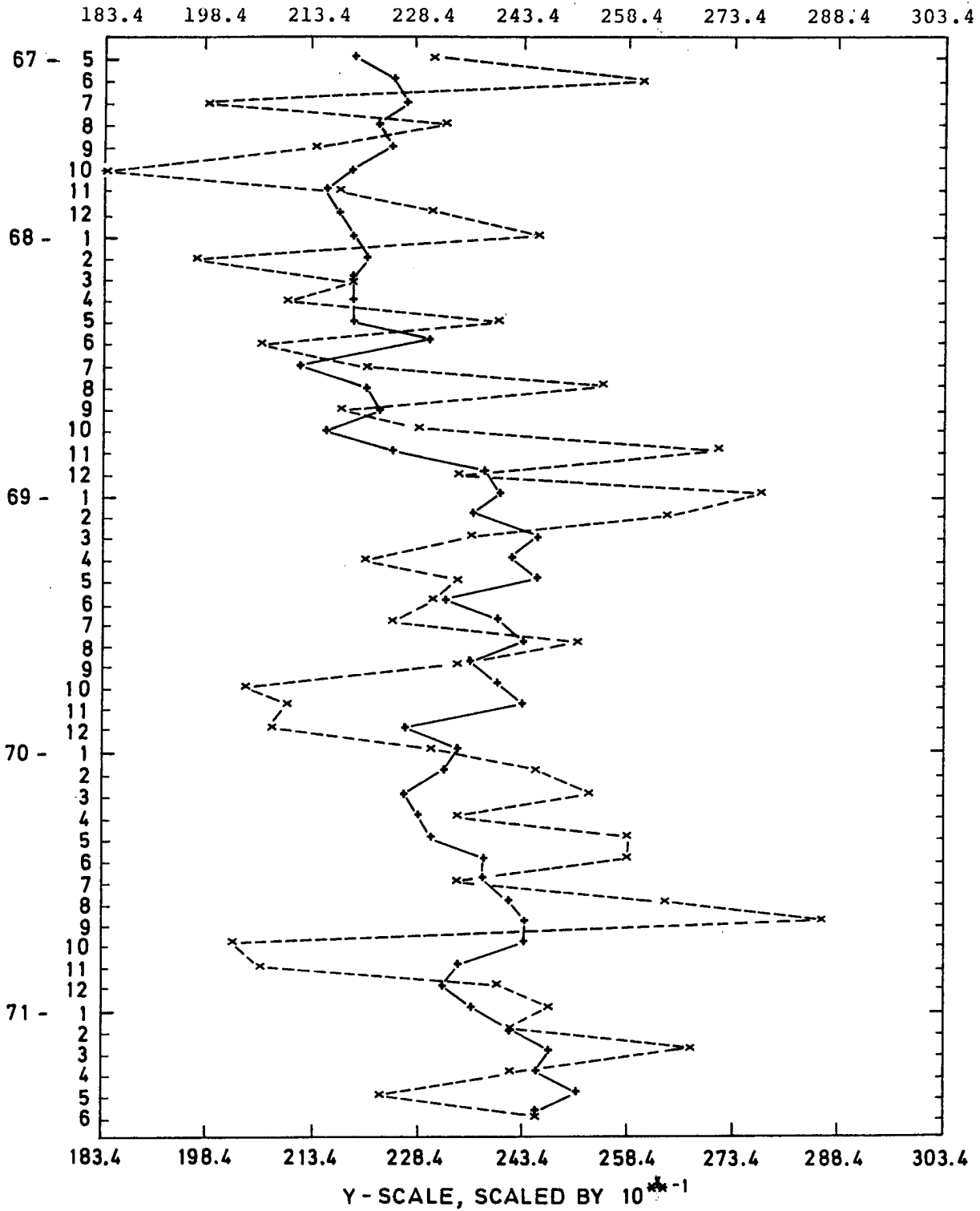


Figure 5. Monthly Time Series of Units Cross-matched (O+):
 *observed, +forecast

This section has illustrated the application of statistical forecasting methods to blood demand data. Such data easily could be obtained from transfusion and cross-match reports generated by the blood bank information system. Forecast values of expected daily cross-match requests form a primary input to inventory control formulae, which are developed in the following sections.

TARGET INVENTORY LEVELS FOR A TOTAL BLOOD BANK OR A DECENTRALIZED REGIONAL BLOOD BANKING SYSTEM

As noted previously, the major responsibility of a hospital blood bank is to ensure that all blood-related demands are met in a manner which minimizes wastage through outdates and spoilage, maintains high quality standards and reduces shortages that require either emergency shipments from other blood banks, emergency demands on donors, appeals to the hospital staff for donations or the delay of nonemergency and elective medical procedures. In order to achieve these goals, it is important for the hospitals, the central blood banks (CBB's) and the regional blood bank (RBB) or regional blood center (RBC) to set inventory levels which trade off shortage versus outdate rates and minimize total operating costs.*

*Throughout this section we use the definitions: Demand: The number of blood units of any one type that are set aside for possible transfusion (i.e., cross-matched) on a given day. Shortage: A situation when the demand exceeds the number of units of blood in inventory. Shortage rate: The long-term fraction (or percentage) of days on which a shortage occurs. Usage: The number of blood units of any one type transfused on a given day. Outdate: A blood unit discarded because its age exceeds the maximum of 21 days. Outdate rate: The ratio of mean number of blood units outdated to mean number of blood units transfused plus those outdated.

This section establishes a simple decision rule which yields the optimal inventory level for each blood type for whole blood and red cells with a 21 day expiration as a function of various factors in the blood bank environment. In using this rule, it is not necessary for the blood bank administrator to choose a shortage rate for system operation since the inventory level recommended by the rule reflects the optimal trade-off between shortages and outdates.

For any given inventory level it is possible to compute the anticipated shortage rate when operating at that level. When using the inventory level obtained from the optimal decision rule, the "expected" shortage rate is near 0.001 for large volume blood types and 0.01 for rarer or small volume types. On the other hand, the outdate rate is shown, in general, to be quite sensitive to certain specific factors in the blood bank management and environment such as age of supply, transfusion to cross-match ratio and the cross-match release period. Depending on these factors, the rule yields outdate rates which vary from 0.001 to 0.07. The rule is simple to use because it can be completely described by the mean or forecast value of the daily demand, the length of the cross-match release period and the ratio of total units transfused to total units cross-matched. The unused, cross-matched units eventually are released from the assigned to the unassigned inventory after a delay of a number of days. This delay time, which we call the "cross-match release period," will be denoted by the variable D . For a hospital blood bank, D is usually one or two days.

As noted in the previous section, mean demand was broken down into the two factors used by Elston and Pickrel^{2,3}: 1) the mean daily number of patients for whom blood is cross-matched, and 2) the mean number of blood units cross-matched per patient. We also considered other factors such as the order (age sequence) of issuing units and the ages of units coming from external suppliers. It was found that the optimal decision rule defined in terms of these two demand factors was not significantly better than the decision rule defined in terms of the mean daily demand. Moreover, the extra cost and effort of estimating these two factors--as opposed to estimating mean daily demand--further mitigated against their use. The other factors mentioned above were also found not to be significant in the determination of the optimal inventory levels.

Based on Yen's work, we also have conducted further research to obtain the optimal target inventory levels for a community blood center which maintains control over the units at the member location. It appears that for such systems the target levels are somewhat lower than for decentralized systems because it is easier to transship units if necessary. These and other results related to the operation of such community blood systems are presented in a subsequent section of this paper.

Methods. Data from Rush Presbyterian St. Luke's Hospital, Evanston Hospital and the North Suburban Blood Center in the metropolitan Chicago area were used in the analysis. The methods of analysis involved various statistical estimation techniques, economic modeling, simulation and inventory operations-analysis. As mentioned in the work by Cohen and Pierskalla,⁴ changes in operating policy clearly will have an impact on the performance of the blood bank. Because the target inventory level is affected by many environmental factors, it is necessary to construct a model of the blood bank in order to test the com-

plex interactions and effects of these factors. An expanded version of the basic model structure of an earlier paper, which was extensively validated in Pinson,⁸ is used in this section. The model requires specification of input factors relating to system environment and control policy. The factors considered in this analysis included parameters to specify the daily demand process, parameters to specify the age process (of units arriving at the bank) the transfusion to cross-match ratio p , target inventory levels, issuing policy, cross-match release time D , shortage cost and outdate cost.

Model outputs include a detailed record of all inventory transactions and the trajectory of the age distributions of both assigned (cross-matched) and unassigned inventories. For the purposes of decisionmaking we seek to minimize mean daily shortage plus outdate costs. Thus a cumulative record of total outdates and shortages is kept. Upon multiplication by the appropriate unit cost and division by the number of days in the run, the desired average cost is obtained. The generation of average outdate plus shortage costs for a fixed set of inputs over a range of different values for target inventory level (S) yields an average cost curve. Figure 6 illustrates a typical set of such curves for a range of different values of D , the cross-match return parameter (the transfusion to cross-match ratio is fixed at 0.5). These curves indicate stability on the part of the optimal target inventory level with regard to D as was previously noted by Cohen and Pierskalla.⁴ That is, even as D increases from 2 to 5 to 7 days the optimal inventory level (S^*), which minimizes the costs, remains between 30 and 40 units. Furthermore, as noted in figure 6, the cost function is relatively flat over a wide range of inventory levels: essentially from $S = 30$ to $S = 50$, the costs do not vary greatly especially for the more reasonable value of $D = 2$. This flatness is caused by the fact that as S increases beyond 30 the

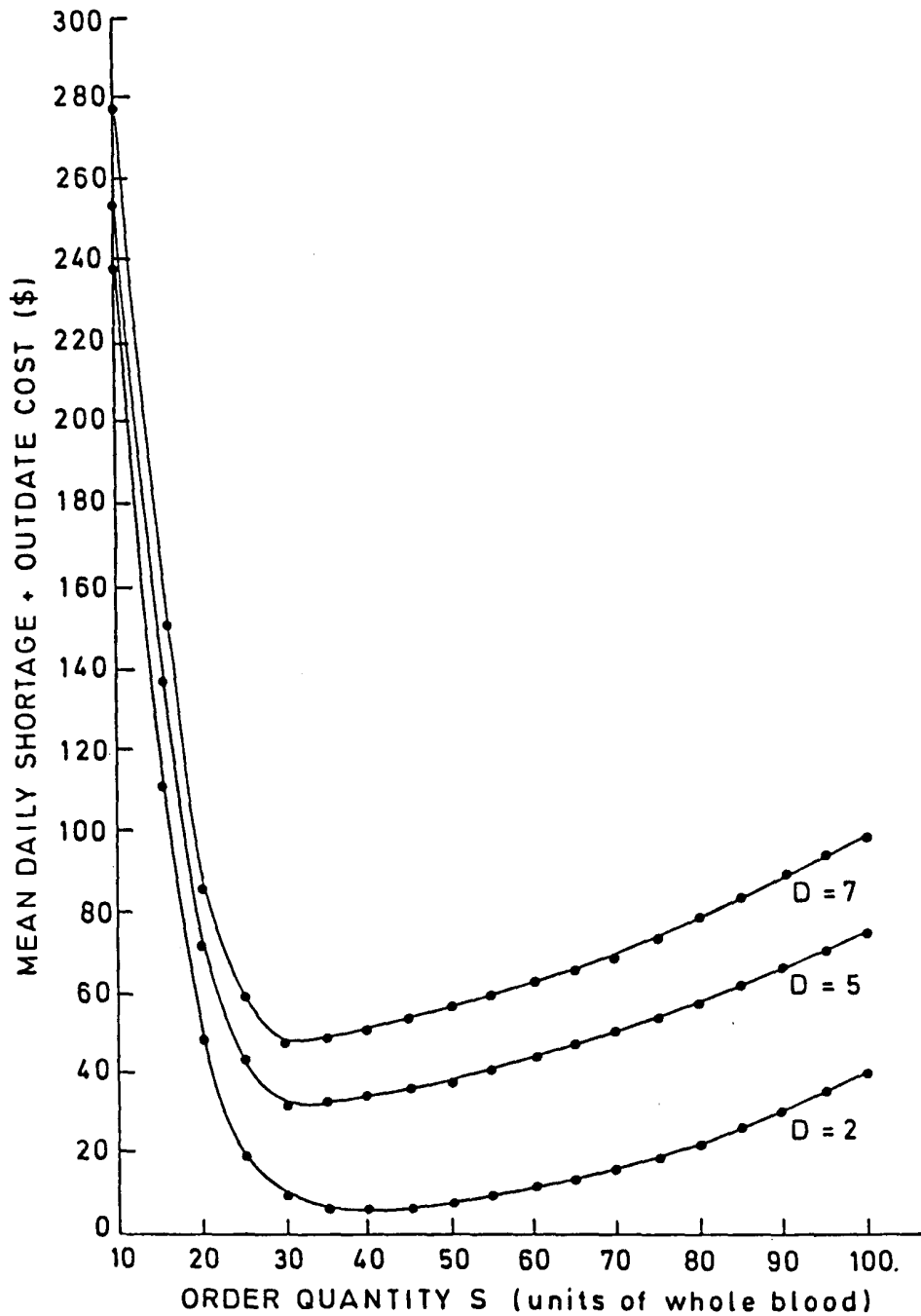


Figure 6. Average shortage plus outdate costs and order quantity (FIFO issuing, \$55 unit shortage cost, \$25 unit outdate cost, a 300-day simulation period, $d_m = 12$, $p = 0.5$ and $A = 6$.)

number of shortages drops to zero and although the number of outdates increases as S increases, it increases very slowly until S passes 60. This slow increase in outdates is caused by the fact that there are 21 days in which to transfuse a unit and for S below 60 it is possible to transfuse most units before they outdate. As the transfusion to cross-match ratio decreases to 0.25 or as the cross-match release time (D) increases to 5 or 7, the response of outdates to an increase in the value of S is more rapid. S is the target daily inventory level, not the amount ordered each day. The amount ordered is only the amount of transfusion plus outdates of the previous day which then will bring the inventory back up to S .

Figure 7 gives the number of shortages and the number of outdates for the case in figure 6 when $D = 5$. Figure 8 plots the cost curves for shortages and for outdates and their sum which is the total cost. The implications of these observations are as follows: 1) The effect on shortages and outdates of the ordering policy is minimal for S 's in the neighborhood of S^* . The insensitivity of the S 's in the neighborhood of S^* is important because a blood bank cannot always achieve S^* each day. Indeed, large drawings of blood through donor plans often can disrupt a policy of achieving S^* on a daily basis. The hospital blood bank administrator must seek an average S^* over time, and 2) the optimal inventory level is relatively insensitive to the value of D . This means that the blood bank administrator can set S^* and then concentrate inventory management control on reducing D knowing full well that S^* will not change significantly.

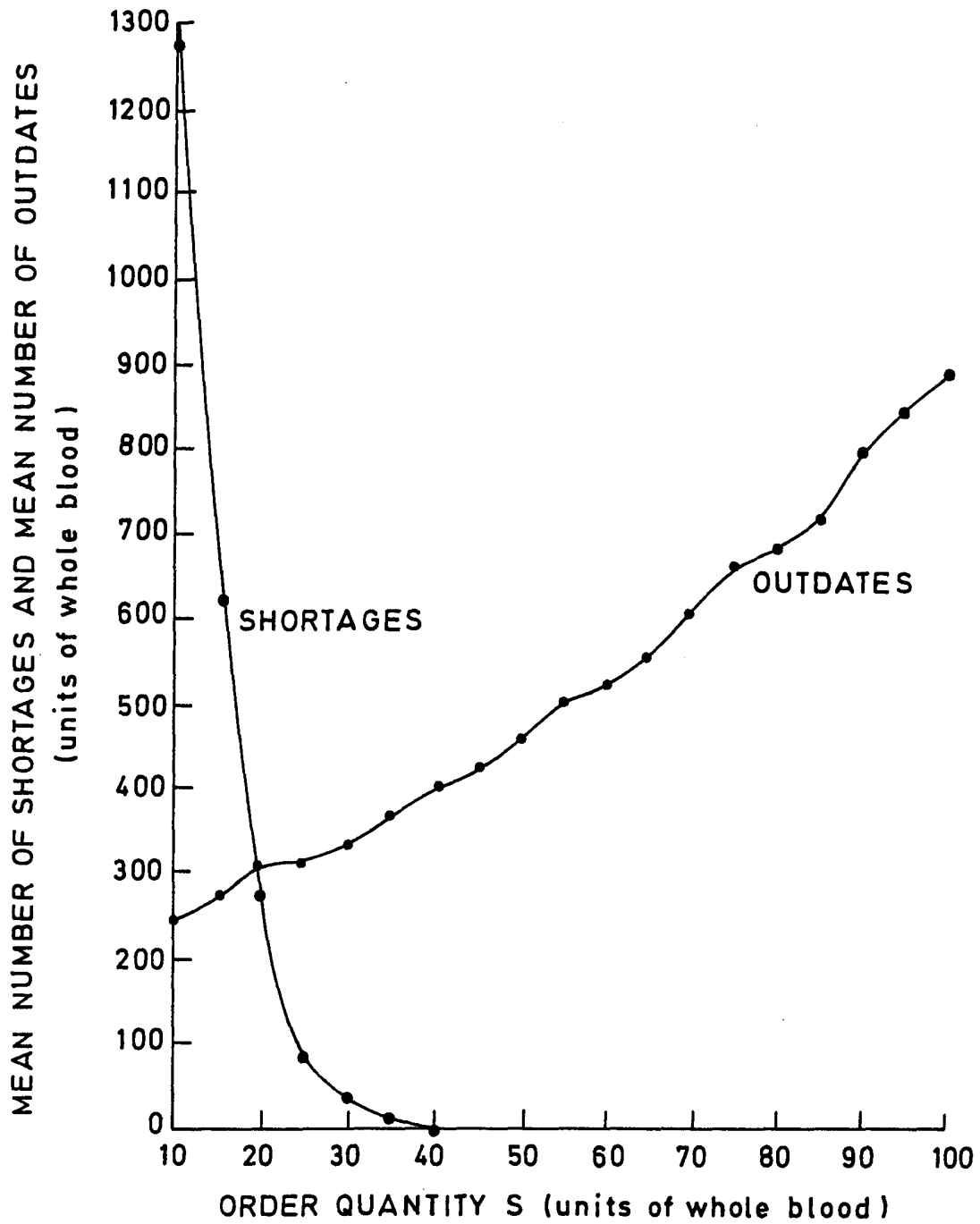


Figure 7. The average shortages and outdates used to produce the cost curves of Figure 1A for the case $D = 5$.

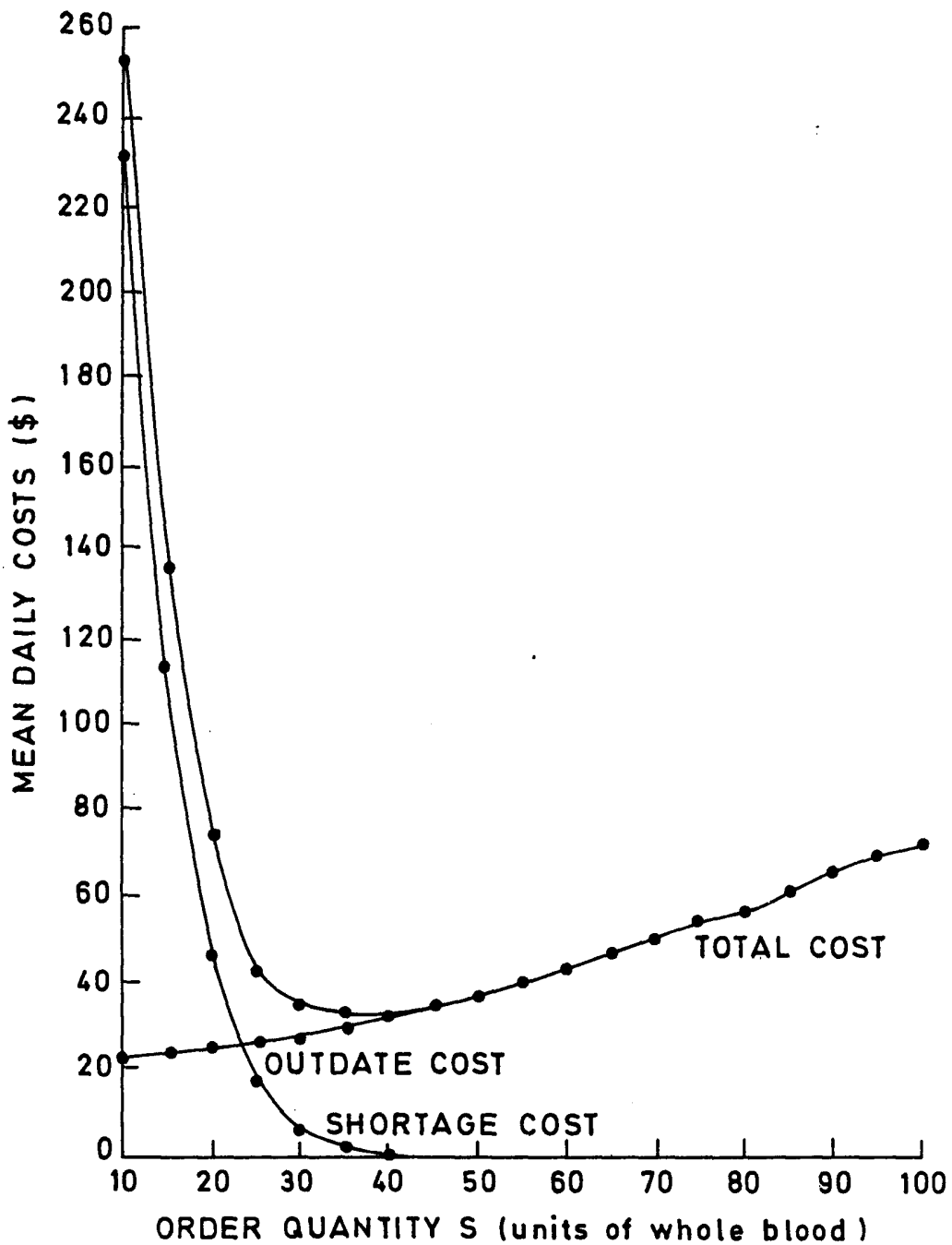


Figure 8. The average total daily costs decomposed into its two components: outdate cost and shortage cost when $D = 5$.

Optimal Decision Rule. There are many exogenous and policy control factors associated with any blood banking system. The exogenous factors identified for the purposes of this discussion include the parameters specifying the mean daily demand, the parameters specifying the age of units supplied for distribution, the per unit shortage and outdate costs and the fraction of total daily demand which is transfused. The control factors for the system include the issuing policy, the cross-match release period (D) and the inventory level (S).

The functional relationship between S^* and the various control and exogenous factors will be called the "optimal decision rule." A statistical experiment was carried out in which the assorted factors were varied throughout their range and the simulation model used to compute the S^* value associated with each factor value configuration. The results of this experiment then were used in a curve fitting analysis to identify the desired functional relationship, i.e., the appropriate optimal response surface. It is important to note that the average cost surface is not being identified, but rather the surface of cost minimizing values of S.

The function we seek can in general be written as:

$$(3) \quad S^* = f(d_1, d_2, \dots, A_1, A_2, \dots, P, D, C_S, C_O, I)$$

where

d_i = parameters describing the demand process;

A_i = parameters describing the age of supply process;

p = fraction of total daily demand transfused

(i.e., the transfusion to crossmatch ratio);

D = crossmatch release period;

I = issuing policy indicator.

Due to the previously observed domination of first-in first-out (FIFO) as the best issuing policy for $D=5$ and $p=0.25$, the experimental design was restricted to the following factors.

1. Mean Daily Demands, d_M

d_M takes the values 2, 3, 12, 16, 18, 24, 32, 48

2. Mean Age of Supply, A

A takes the values 1, 6.03

3. Crossmatch Release Period, D

D takes the values 1, 2, 4

4. Shortage Cost, C_S

C_S takes the values \$35, \$55

5. Transfusion to Crossmatch Ratio, p

p takes the values 0.25, 0.5

The lower values--2 and 3--of mean daily demand correspond to rarer types of blood or to transfusion locations with low daily demands for common types. The large values--32 and 48--correspond to daily demands at large hospitals or a community blood center for more common types of blood. The age composition of arriving supply was drawn from two classes of distributions. An empirical distribution estimated from hospital data (with a mean age of supply of 6.03 days) and a degenerate uniform distribution yielding fresh supply were considered. Other distributions had been tested in previous work and were found to provide no additional information or policy changes.

Outdating cost was fixed at \$25, issuing policy was set as FIFO and the transfusion fraction was set at 0.25 and 0.5. The actual costs of \$25 for outdates and \$35 or \$55 for shortages are not important in absolute value for determining S^* . What is important is their ratio or relative value. The absolute quantities merely reflect the estimated average cost of outdate and shortages. The target inventory level S^* is not sensitive to reasonable ranges of the ratio of outdate to shortage costs.

A full factorial design for this experiment would involve 192 separate simulation-optimization runs. A $1/2$ factorial design was chosen and thus 96 separate observations of S^* were generated. The actual design and the results of the experiment are indicated in Cohen and Pierskalla.⁵ These results were analyzed by standard regression techniques to determine which input parameters were significant in the computation of S^* and what the functional form of the decision function should be. Both linear and log-linear functional forms were investigated. The log-linear function gave significantly better results; the results of the regression run are presented in table 2.

TABLE 2

Blood Decision Rule All Variables (Log-Linear)

<u>Variable</u>	<u>Coefficient</u>	<u>Statistical Error</u>	<u>T-Value</u>
ln (d_M)	0.7604	0.00972	78.24
ln (p)	0.1216	0.02925	4.16
ln (D)	-0.0677	0.01791	-3.78
ln (A)	-0.0138	0.01128	-1.22
ln (C_S)	0.0520	0.04485	1.16

Intercept 1.61248

Regression

Error

Degrees of Freedom 5
 Sum of Squares 60.72035
 Mean Square 12.14407

Degrees of Freedom 90
 Sum of Squares 0.88765
 Mean Square 0.00986

S.E. of Estimate 0.09931

F-Value 1231.3

Multiple R-Squared 98.56

The most significant explanatory variable is the mean daily demand. However, the transfusion/cross-match ratio (p) and the cross-match release period (D) are also significant, but their coefficients of 0.1216 and -0.0677, respectively, in the regression equation indicate that their influence on the optimal S^* is not nearly as large in magnitude as the d_M . The other two independent variables, A and C_S , are not significant and their coefficients also are small. Consequently, these last two variables were dropped and the regression was run again using only the significant variables. In the second regression the intercept term changed but all other coefficients and statistics coincided to at least three significant digits. Using the results of the second regression, the optimal decision rule can be written by equation:

$$(4) \quad \ln S^* = 1.7967 + 0.7604 \ln(d_M) \\ + 0.1216 \ln(p) - 0.0677 \ln(D)$$

or equivalently by equation

$$(5) \quad S^* = 6.03 (d_M)^{0.7604} (p)^{0.1216} (D)^{0.0677}.$$

In figure 9, this optimal decision rule is graphed for the case when $p = 0.25$ and $p = 0.5$ and $D = 1$ day. Figure 10 presents the optimal decision rule for $p = 0.25$ and $p = 0.5$ when $D = 2$ and 4 . For fixed p and D it is interesting to note from figure 9 that a positive coefficient of 0.7604 for mean daily demand in the optimal decision rule indicates a concave shape to the rule. For a blood bank this would mean, for example, that as the mean daily demand is increased, a less than proportional increase in the order quantity would be optimal. Another way of viewing the rule is that a blood bank which doubles its size (in terms of mean daily demand) should increase its optimal inven-

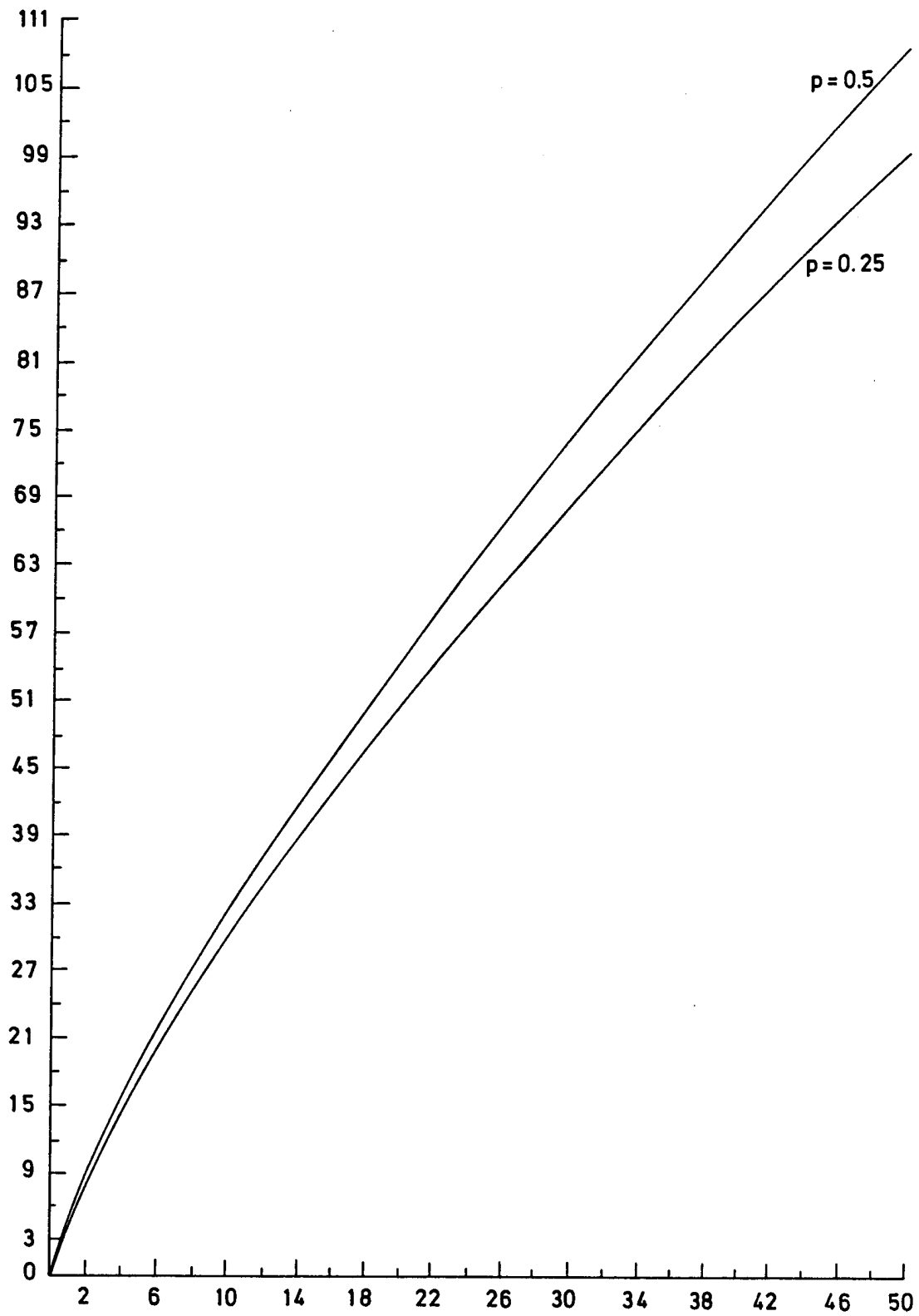


Figure 9. Optimal target inventory levels for different mean daily demand levels when $D = 1$ and $p = 0.25$ and 0.5 .

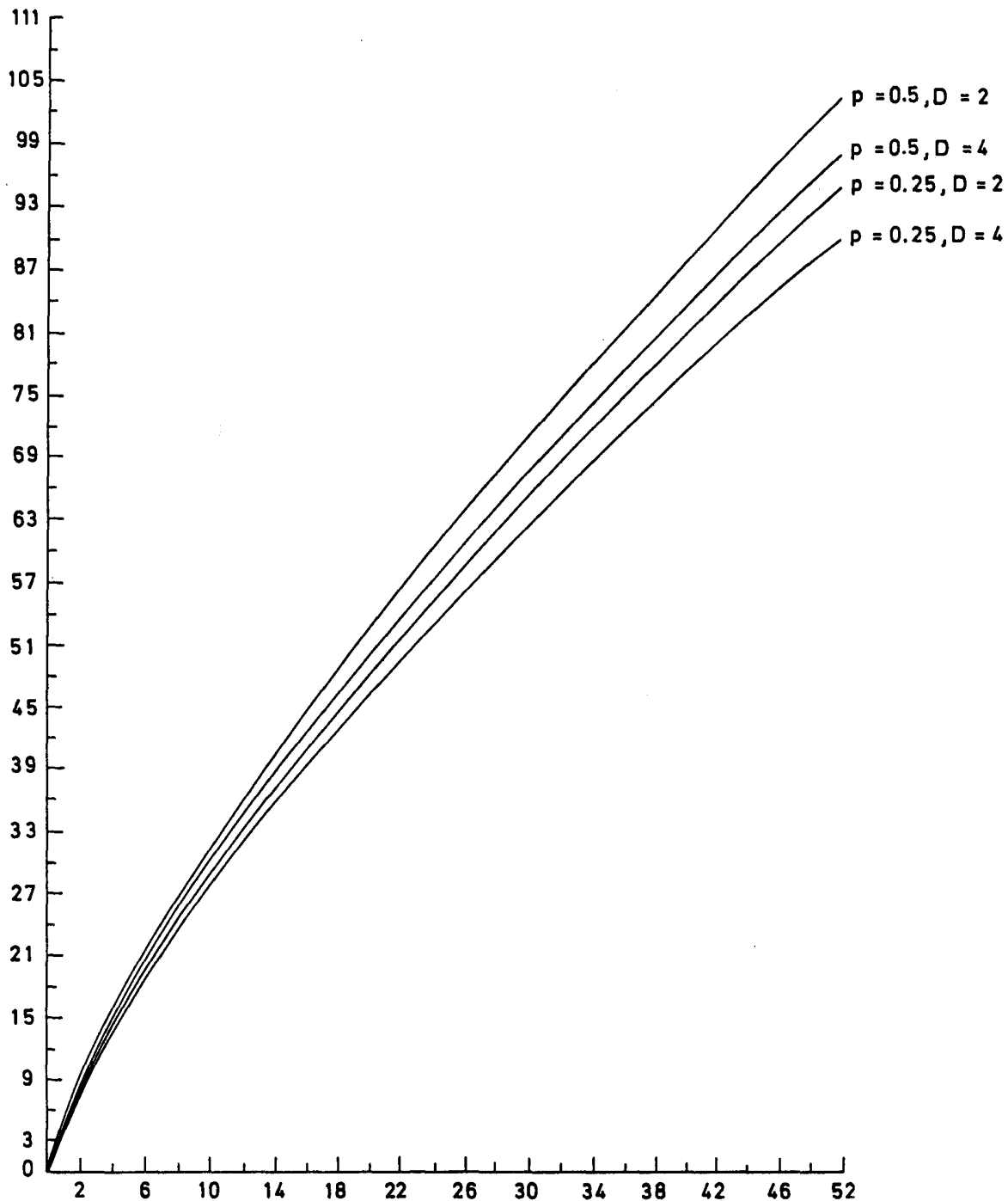


Figure 10. Optimal target inventory levels for different mean daily demand levels when $D = 2$ and 4 and $p = 0.25$ and 0.5 .

tory level by less than 70 percent (provided the p's and D's remain the same). Using equation (4) or (5), the blood bank administrator can compute the optimal target inventory level for each blood type merely by inserting the mean daily demand for each blood type, the average transfusion to cross-match ratio p and the cross-match release period D.

Evaluation of the Decision Rule. The mean daily amount of blood transfused (usage) is determined by the transfusion to cross-match ratio times the mean daily demand. A range of about 1 to 25 units transfused per day was considered in the experimental design. This corresponds to an annual volume of between 300 to 10,000 transfusions. Because almost all blood banks have type specific mean demand volumes which fall into these ranges, it was felt that testing the extrapolation of the decision rule for even larger volumes was not necessary.

A more important evaluation involves a comparison of the results obtained from using the decision rule with the data from the Chicago area and from other published sources (Brodheim, Hirsch, and Prastacos⁹). We already have shown in figures 5 and 6 that use of the optimal target inventory level for O+ blood at Evanston Hospital results in virtually zero shortages and a very low outdate rate. The actual shortage rate is less than 0.004 for this particular blood type and the outdate rate is less than 0.02.

Tables 3 to 6 show the target inventory levels and their corresponding shortage and outdate rates for a large range of mean daily demands for various p and A values. When the mean daily demand for cross-matching a particular blood type is two units ($p = 0.5$; $A = 1$; $D = 1$) then the optimal target inventory to maintain on hand each day is eight units. By maintaining this level the blood bank will experience a shortage rate of

about 0.022 and an outdate rate of about 0.013. As the volume of activity increases, i.e., mean daily demand is larger, the statistical law of large numbers comes into play. When the mean daily demand is 16 units and the optimal target inventory level of 38 units is maintained, the shortage rate and outdate rate virtually drop to zero (both are less than 0.006) when $D = 1$ or 2 .

Even if the blood bank administrator operates at the optimal target inventory level S^* it is still necessary to control the cross-match release period D , the average transfusion to cross-match ratio p , and the average age of arriving units A , in order to keep shortages and outdates down. This control is especially important for transfusion locations where the mean daily demand for a blood type is small. The worst case shown is in table 6 where for $d_M = 2$, $D=2$, $p=0.25$ and $A=6$, the optimal S^* is 7. However, at this S^* the shortage rate is 3.7 percent. On the other hand, from table 3 holding $d_M=2$ but reducing D to 1, A to 1 and increasing p to 0.5, the optimal $S^* = 8$ but more importantly the shortage rate is 2.2 percent and the outdate rate is 1.3 percent. This drop in the shortage rate occurs because the units are available in unassigned inventory more frequently since $D = 1$ versus $D = 2$ and $A = 1$ versus $A = 6$ and the probability of transfusion at each cross-match p is higher: $p = 0.5$ versus $p = 0.25$.

Indeed, by looking at tables 3 to 6 it is possible to isolate the effects of any single parameter holding the other parameters constant. One of the more important control parameters for the administrator is the cross-match release period D . It is also one of the most significant parameters in reducing outdating. Less easily controlled parameters are p and A . However, if the administrator can reduce A and/or increase p , then the outdate rate also will drop significantly.

Table 3

Shortage and Outdate Rates Using the Optimal
Decision Rule for Given $p = 0.5$ and $A = 1$

d_M	S^*		Shortage Rate		Outdate Rate	
	<u>D = 1</u>	<u>D = 2</u>	<u>D = 1</u>	<u>D = 2</u>	<u>D = 1</u>	<u>D = 2</u>
2	8	8	0.022	0.022	0.013	0.064
16	38	36	0.003	0.006	0.001	0.004
32	64	61	0.002	0.004	0.0001	0.002
48	88	84	0.001	0.003	0.0001	0.001

Table 4

Shortage and Outdate Rates Using the Optimal
Decision Rule for Given $p = 0.5$ and $A = 6$

d_M	S^*		Shortage Rate		Outdate Rate	
	<u>D = 1</u>	<u>D = 2</u>	<u>D = 1</u>	<u>D = 2</u>	<u>D = 1</u>	<u>D = 2</u>
2	8	8	0.022	0.022	0.068	0.155
16	38	36	0.003	0.006	0.005	0.027
32	64	61	0.002	0.004	0.005	0.019
48	88	84	0.001	0.003	0.004	0.019

Table 5

Shortage and Outdate Rates Using the Optimal Decision Rule for Given $p = 0.25$ and $A = 1$

d_M	S^*		Shortage Rate		Outdate Rate	
	<u>D = 1</u>	<u>D = 2</u>	<u>D = 1</u>	<u>D = 2</u>	<u>D = 1</u>	<u>D = 2</u>
2	7	7	0.037	0.037	0.220	0.357
16	35	33	0.008	0.014	0.020	0.120
32	59	56	0.006	0.012	0.009	0.082
48	81	77	0.006	0.012	0.009	0.081

Table 6

Shortage and Outdate Rates Using the Optimal Decision Rule for Given $p = 0.25$ and $A = 6$

d_M	S^*		Shortage Rate		Outdate Rate	
	<u>D = 1</u>	<u>D = 2</u>	<u>D = 1</u>	<u>D = 2</u>	<u>D = 1</u>	<u>D = 2</u>
2	7	7	0.037	0.037	0.340	0.447
16	35	33	0.008	0.014	0.091	0.204
32	59	56	0.006	0.012	0.049	0.171
48	80	77	0.006	0.012	0.049	0.164

The parameters D , p and A have less impact on the shortage rate. In fact, the optimal target inventory level S^* plays the major role with regard to reducing the shortage rate. As was mentioned earlier, the S^* tends to be the inventory level at which the shortages are virtually zero especially when there is good control over D , p and A .

It should be iterated here that if the administrator chooses to operate above the optimal S^* , as shown in figures 7, 8 and 9, then the outdates and costs will rise with little additional impact on reducing shortages. Conversely, operating below S^* means that outdates will drop but shortages and costs will rise. Fortunately, when D is small, e.g., $D = 1$ and p is large, e.g., $p = 0.5$ the cost curves are relatively flat near S^* so there is a considerable amount of flexibility in achieving S^* on a day-to-day basis.

A final important fact concerns the parameter A , the average age of arriving units at the transfusion location. Tables 3 to 6 indicate that the larger A becomes the larger the resulting outdate rate becomes. When a transfusion location depends upon a central bank or supplementary sources for its supply it may not be able to control the ages of arriving units. For large volumes of demand this lack of control does not greatly impact on outdating if the administrator maintains $D = 1$ and $p = 0.5$. However, for small daily demand levels such as $d_M = 2$, especially with large D and/or small p , the control of A is very important. Often, small blood banks have such small daily demand levels. At such locations the supplied blood should be as fresh as possible. For optimal efficiency the blood inventory at these small banks should be replaced periodically with all fresh units and the unassigned old units should be returned to the regional supplier.

Brodheim, Hirsch and Prastacos⁹ give a series of inventory level curves based on shortage rates of 0.2, 0.1, 0.05, 0.02 and 0.01. They suggest that the blood bank administrator should choose the shortage rate at which to operate. The administrator can read off the inventory level from the appropriate curve using the mean daily demand for each blood type. Our results indicate that it is not necessary for the blood bank administrator to choose a shortage rate. In fact a shortage rate much above 0.01 or 0.001 (depending on the mean daily demand p and D) will result in an extremely high cost nonoptimal level. It is true that at a shortage rate of 0.2 the outdate rate will be lower because outdates are essentially caused by two factors, one of which is only slightly affected by the shortage rate. The first factor is that the supply of arriving stocks of blood at the bank is variable. On days of mobile drawings or of donor recruiting campaigns the supply of blood jumps up and on other days it is significantly lower. The second factor is random cross-match demands and actual transfusions resulting therefrom. There is a significant daily fluctuation in demands and transfusions. In those periods in which the supply is higher on average and demand is lower on average, outdating will occur regardless of fairly large differences in the average shortage rate chosen by a blood bank administrator. Consequently, choosing a high shortage rate will not reduce outdating much since outdating tends to occur in batches. It will, though, significantly increase shortages at other times when supply is low and demand is average or above average (see figure 7).

The preceding analysis demonstrates that the choice of a target inventory level is determined by trading off the cost of shortages with the cost of outdates. We also have seen that it is possible to associate an expected shortage rate with each possible value of the target inventory. The range of costs considered in the estimation of the decision function results in shortage rates at the optimal target inventory level of about 0.01 or less when D , p and A are at reasonable levels.

It is interesting to consider the conditions under which target inventory levels yielding a shortage rate of 0.05 or 0.1 would be optimal. In particular, one may wonder about the relative value placed on outdates and shortages by a manager who, in setting a target inventory level, chooses a higher shortage rate. It is important to note that each possible value of target inventory, and its corresponding shortage rate, has an associated "imputed" cost of shortage which would make that inventory level optimal.

Figure 11 illustrates the range of such imputed costs for a large blood bank. In a system where the ratio has a value of 500 an outdated unit would be 500 times more costly than a shortage unit. Similarly, a value of 0.5 means that an outdated unit costs one-half the cost of a shortage unit. In our earlier studies it was observed that in 1973 and 1974 an outdated unit cost approximately \$25 (primarily lost processing cost) and a shortage unit cost about \$35 to \$55 (primarily telephone calls and emergency transportation or the cost of the freezing and thawing process for a frozen unit). Consequently, realistic ratios of outdate to shortage costs would be in the range $\$25/\$35 = 0.714$ to $\$25/\$55 = 0.455$. Ratios in excess of 1.0 make very little realistic sense since such ratios mean outdates cost more than shortages. However, as can be seen from the average cost in figure 11, if the blood bank administrator were to choose to operate at shortage rates of 0.2, 0.1, 0.05 or 0.01, the implication is that the administrator views outdates 367, 207, 110 and 31 times as important, respectively, than shortages. In fact, at the optimal target inventory level S^* for this volume and common blood type, the shortage rate is 0.001 when the outdate cost to shortage cost ratio is 0.5, i.e., an outdated unit costs about one-half that of a shortage unit in time and/or money.

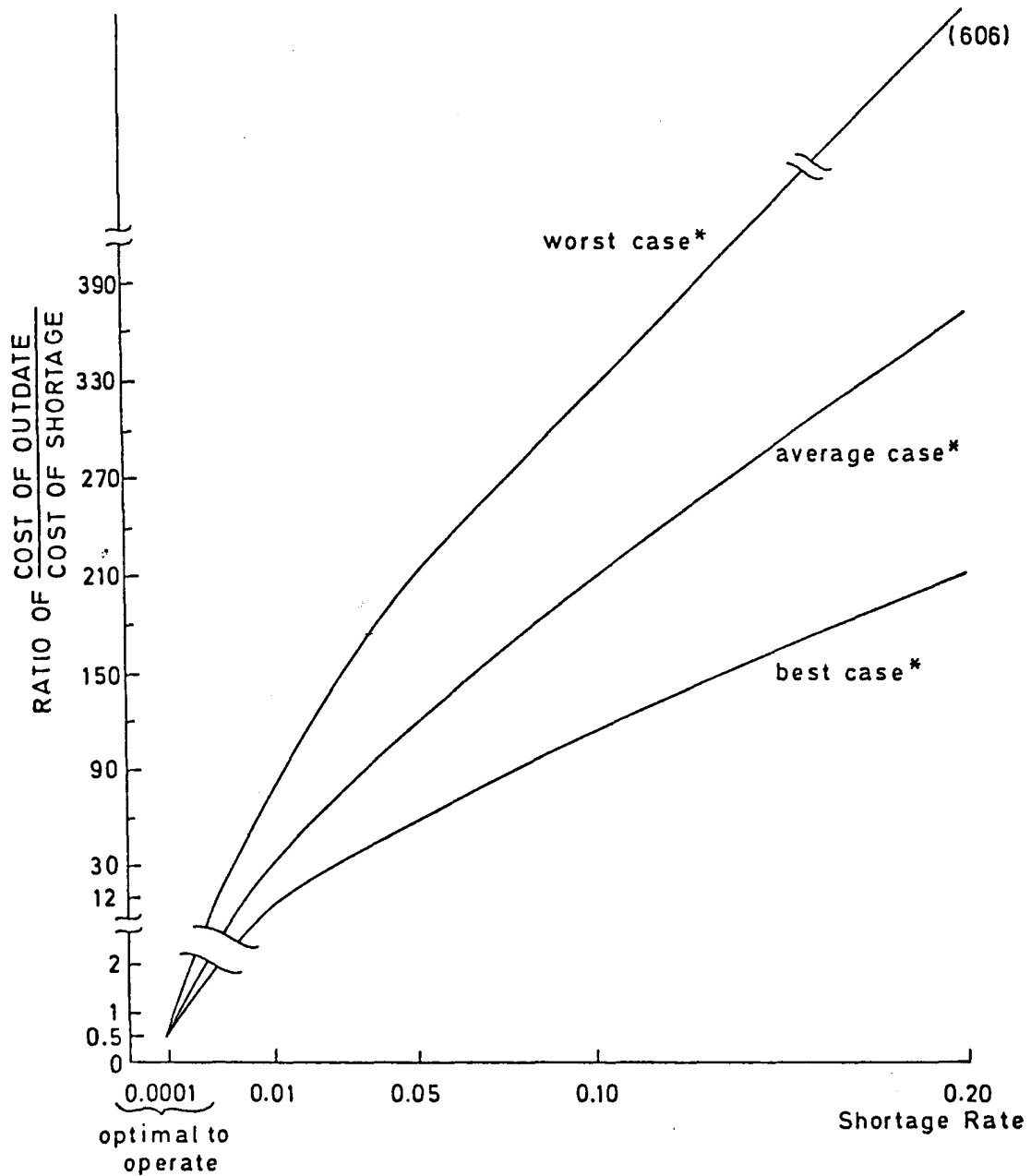


Figure 11: The Imputed Cost Ratio of Outdate Cost/Shortage Cost when Operating a Large Hospital Blood Bank ($d_M = 48$) at Different Shortages Rates for Blood

*The worst, average and best cases were computed from the regression results in Table 1 using the standard error of estimate and the standard errors of the regression coefficients.

The imputed cost ratios of operating at any shortage rate for any level of mean daily demand can be computed to generate curves similar to those in figure 11. We have performed this calculation for a large number of cases. In each, we observed that although the imputed ratios for shortage rates at 0.2, 0.1, 0.05 and 0.01 differ for each blood type and mean daily demand level, the ratios all were far in excess of 1.0. This means that a hospital administrator choosing a target inventory with these large shortage rates is saying implicitly that an outdated unit is far more important than a shortage unit. Our exposure to the Chicago area blood banking system indicates that, in fact, the converse is true, namely, a unit short is far more important and more costly than a unit outdated.

This analysis indicates the inconsistency of choosing to operate a blood bank at what appear to be reasonable shortage rates of 0.1 or 0.05. By selecting a target inventory level according to equation (4) the blood bank manager can achieve far better performance for all reasonable values of the outdate shortage cost ratio.

A final issue considered was the relationship between the decision rule results and the ordering rules which have been observed in practice. Many blood bank administrators keep sufficient blood on hand to meet anticipated needs for 6-8 days. It is possible to compute the number of days of transfusion supply on hand from the order-up-to quantity as follows:

$$\text{Number of days of transfusion supply} = S^*/(p d_M).$$

Figure 12 is a graph of the number of days of transfusion supply on hand versus mean daily usage $p d_M$. As the system scale increases, the optimal number of days of transfusion supply on hand decreases. This is another example of scale econo-

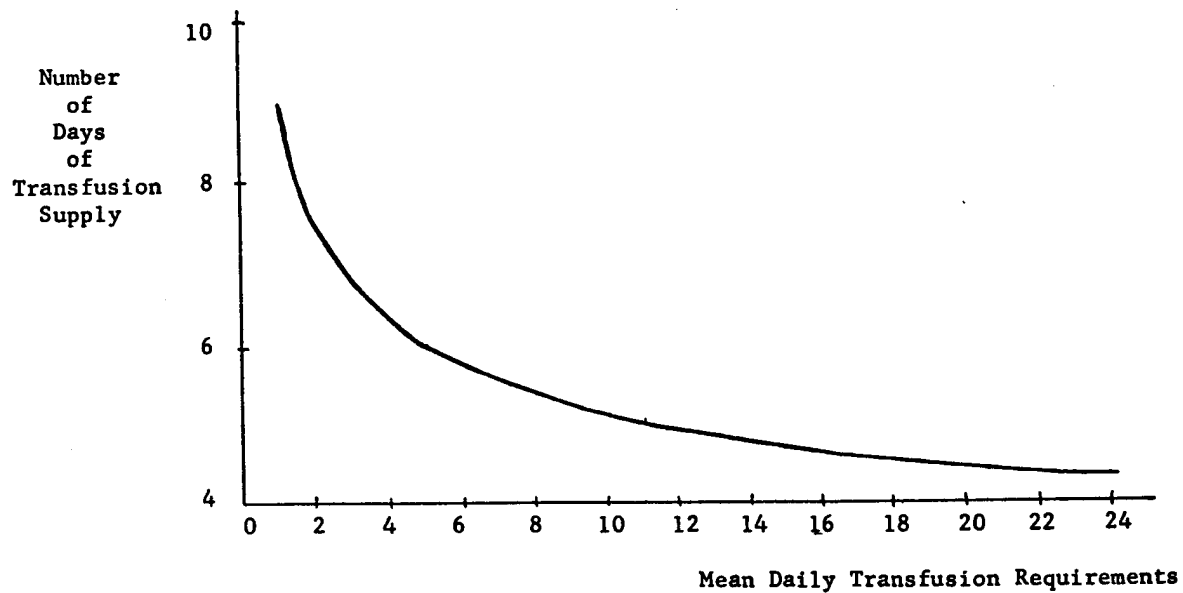


Figure 12: Number of Days Transfusion Supply versus the Mean Daily Transfusion Requirements when $p = 0.5$ and $D = 1$

mies. Moreover, the curve is convex, and is relatively flat at a value of approximately 6 days of transfusion supply for a broad range of system scales.

Figure 13 illustrates the application of the target inventory level determined by equation (5) to the data collected in Chicago from RPSL. The mean level of demand was computed from a quarterly moving average. It is interesting to note that the incidence of shortage which would have been experienced with this level of inventory is well within the range predicted by our model.

TARGET INVENTORY LEVELS FOR A COMMUNITY BLOOD CENTER SYSTEM OR A CENTRALIZED REGIONAL BLOOD BANKING SYSTEM

A community blood center inventory system is characterized by a network of blood banks serving a specific geographical region. Managerial control of the network is exercised or coordinated through a central facility. The degree of control depends upon which activities are performed at the community blood center or at its satellite transfusion services (TS's) and on whether the central facility maintains "authority" over the units in the unassigned inventories at the TS's.

In a central blood bank or regional blood center the management of inventories of whole blood and components involves a complex and often interrelated set of decisions concerning collection, processing, record keeping, storage, issuing and transportation of units. In this section, some management decision problems are analyzed to determine easily implemented rules which yield the "best," or at least "very good" operating re-

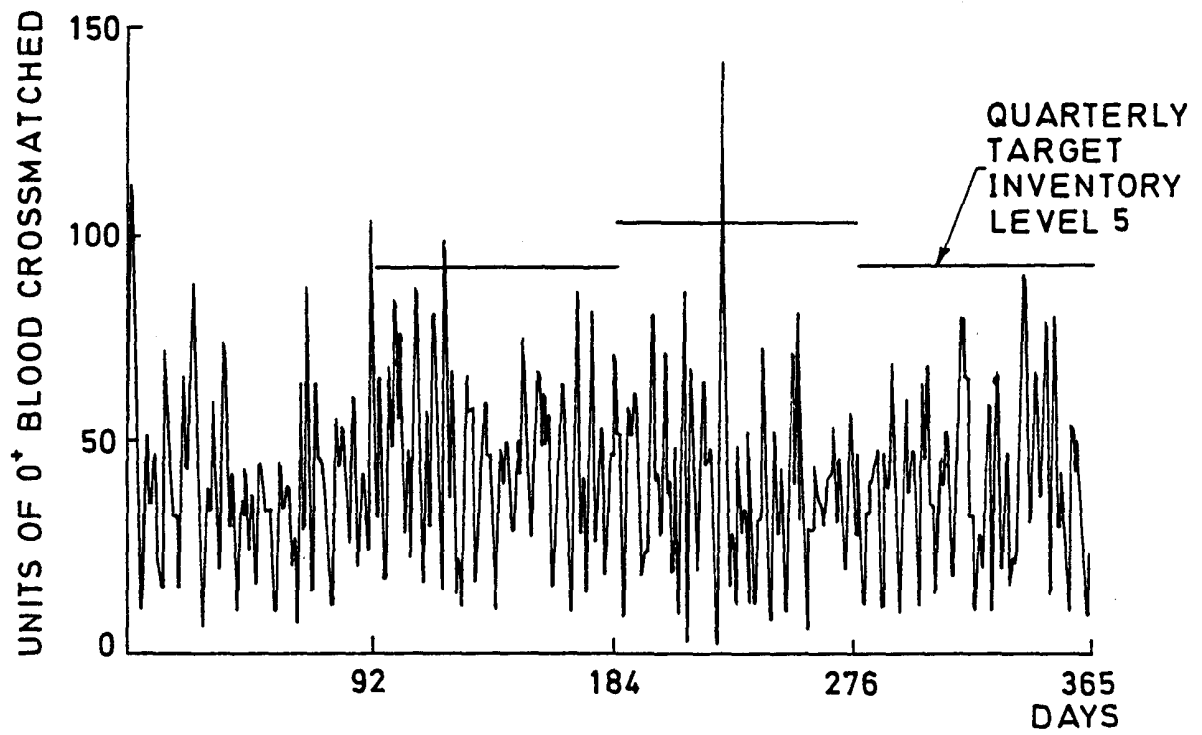


Figure 13. Daily units of O+ blood cross-matched at a metropolitan medical center (with target inventory levels indicated)

sults at the CBB, or RBC and its satellite transfusion locations.¹⁰

It has been recognized that benefits can be derived by pooling resources in the form of a community central blood bank. The most apparent benefit is that the hospital blood bank staff is relieved of the responsibility of donor recruitment, blood procurement and blood processing. This permits the transfusion service to channel its energies and efforts toward the resolution of patient related transfusion problems. Another advantage to the hospital is the opportunity to pool widely fluctuating, largely unpredictable demands with those of other hospitals in the system. Within the system the variations cancel each other and produce a smoother, more predictable aggregate demand. This will enable member blood banks to maintain lower inventories without degrading their outdate and shortage performance.

Since the demand to which the central blood bank must respond is generated outside its control, its decisionmaking processes must focus primarily on inventory management. While management decisions regarding donor recruitment, phlebotomy and processing are essential, they can be handled effectively only after efficient optimal inventory control policies have been implemented.

Various areas of blood handling require careful managerial decisions for the most cost-effective operations of the blood bank. Inventory control at the central blood bank must set inventory levels to maintain the optimal trade-off between excess inventory with consequent outdating, and excessive numbers of shortages. Inventory levels at hospital blood banks and transfusion locations must be set at a level which will minimize the need for emergency shipments from the central blood bank while minimizing the amount of outdating or return of excessively aged

blood. Issuing procedures to govern the distribution of blood from the central blood bank to the various transfusion locations and decision policies governing the transshipment of blood from one transfusion location to another must be developed and modified. Recycle control within the system as well as within a hospital must be exercised with monitoring of the recycle time. Policies must be set regarding the return of aging blood to the central blood bank for reallocation to transfusion locations where the probability of transfusion is high. These concerns result in a management decision problem relating to the routing, frequency and volume of deliveries to the transfusion locations.

Managerial decisions relating to component production levels are gaining steadily increased importance as blood transfusion therapy increases in complexity and quantity. These decisions involve the determination of both the levels and site (in-house or in the field) of routine component preparation. Another factor is the use of blood component separation devices which permit the preparation of large quantities of components such as granulocytes, platelets and plasma, but which also require a higher level of technology.

Inventory levels can be set at both the central and member banks by using an outdating/shortage cost-minimizing procedure similar to that which was described in the previous section for the single hospital blood bank. The optimal inventory level for the central blood bank for each blood type is a complex function of the number of transfusion locations, their mean daily transfusions and the transfusion fraction for all locations served by the central blood bank. Associated with the optimal inventory function for the central blood bank is an optimal inventory level for each member hospital bank which is a function of the demand and transfusion fraction for that location. For those

hospital blood banks which belong to a centralized system it is to be expected that their optimal inventory levels will differ from the values indicated by the decision rule of the previous section which is appropriate for hospitals in a decentralized system. Figure 14 illustrates the CBB structure and the decisions needed to answer the questions posed above.

In order to study some of the benefits and shortcomings from a centralized blood banking system, a simulation model was constructed. The simulation model was quite complex and is described in detail in Yen.¹ Among the issues to be discussed in this section are the optimal inventory levels (S_i), the impact on total system cost of high cross-match to transfusion ratios, the allocation of units from CBB to the TS's, the transshipment policy among TS's and the effect of a limited and somewhat random supply to the central blood bank. In addition, the sensitivity of system cost to changes in the number and size (number of units demanded) of TS's in the system were considered. All conclusions presented in this section were based on the analysis of the results obtained in Yen's model.

As one might expect, the results indicate that the size of optimal inventory levels in the individual hospital blood banks levels off with incorporation of each additional blood bank in the centralized system. Also, after a certain system scale is reached the marginal benefits received from lower shortages and lower outdates can be expected to approach zero as more hospital blood banks are added to the system. Finally, as more hospital blood banks are included in a centralized blood banking system, the total average distances between the CBB and the TS's, as well as their information needs, increase and thus the corresponding transportation and information costs also increase. So,

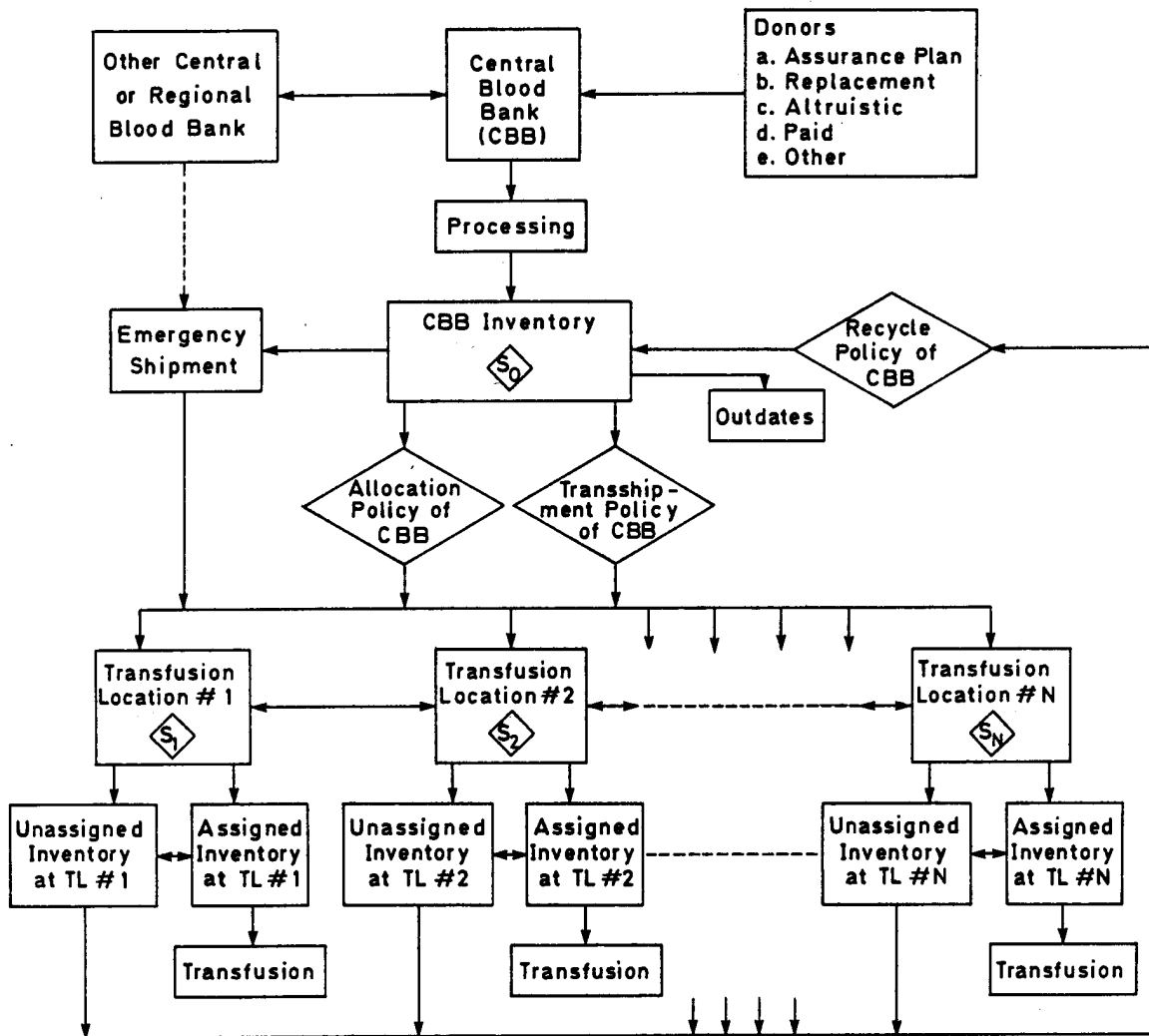


FIGURE 14. FLOW CHART FOR A CENTRAL BLOOD BANKING SYSTEM SHOWING LOCATIONS AND CBB DECISION POLICIES

after a saturation number of hospital blood banks in the system is reached, further inclusion of local banks is not likely to reduce the system cost per unit and indeed as has been shown appears to lead to diseconomies of scale.

Before proceeding to detailed discussions of the results for a CBB, it should be noted that all of the optimal decision rules concerning inventory amounts, cross-match release policies, issuing policies, transshipment policies, vehicle routes and other factors interact with one another. That is, if one changes the policy in one area, it will affect the policies being followed in the other areas. These interactions will become more apparent as we proceed through this section and more will be said about them in subsequent discussions. Since it is not possible to present all of these policies simultaneously, each will be presented separately and the reader should keep in mind that they all interact.

The Optimal Daily Inventories at the CBB. The daily amount of whole blood and components which must be maintained at the CBB depends upon the amounts maintained at each TS in the system. If the total inventories at the TS's are large, then the amount at the CBB may be small and vice versa. However, large inventories at the TS's could result in more outdates and/or outdate-anticipating transshipments. Similarly, small inventories might incur more emergency shipments and/or shortage-anticipating transshipments. Because of these possibilities, there must be a balance between the inventory at the CBB and the inventories at the TS's.

Equations for determining optimal inventories of whole blood and packed cells at the TS's were given in the previous section. Using a similar simulation-optimization-regression approach and making the same reasonable assumptions concerning the system costs of shortages and outdates, the optimal inventory level at the CBB was established.

The outdate costs were assumed to be \$25 per unit at all locations. The shortage costs were assumed to be \$55 per unit at the TS's and \$35 at the central blood bank. These figures were based upon the following reasoning. The outdate cost consists primarily of the average costs per unit of recruiting, processing, storing and transporting one unit. When a unit outdates, these costs are basically lost. Actually a more appropriate cost to charge for outdates would be the marginal per unit costs of these blood bank activities rather than average per unit costs. However, it is not easy to obtain actual marginal costs at present since the cost figures available are not sufficient to define the appropriate marginal relationship. Furthermore, since the average cost includes many variable items such as bag costs, record keeping and hours of work, it is reasonably representative of the marginal cost. The \$25 figure for the outdate cost was based on estimates from blood bank administrators of their average per unit cost in 1974.

The shortage cost of \$35 per unit in 1974 at the CBB was based on a \$25 average per unit cost for processing and handling a unit on an emergency basis and \$10 for recruiting and/or transportation from another source on an emergency basis. Again, marginal costs per unit would be better but they were not available. The shortage cost of \$55 per unit in 1974 at the TS's was based on the average per unit cost of maintaining a buffer stock of frozen blood units either at the TS or the CBB or shipping a unit(s) by emergency shipment from another regional center. Finally, it should be recalled that what is important about these costs is not their absolute levels, but rather their relative magnitudes. Hence, if inflation should cause them to rise in the same relative proportions, the results still hold. Furthermore, the results hold even when the relative magnitudes are varied over reasonable ranges.

Before presenting the optimal stock levels and the other optimal policies and equations for the CBB, it will be useful to list some notations:

- d_0 = Mean demand for whole blood and packed red cells at the central blood bank.
- d_j = Mean demand for whole blood (WB) and packed red cells (RC) at the hospital blood bank. It is assumed that demand at the bank is a Neyman type A distributed random variable characterized by the mean number of patients per day and the mean units requested per patient. Yen¹ demonstrated that the Neyman A fits the data well.
- D = The return parameter, the time lapse before a unit is returned to the unassigned inventory if not transfused (in days).
- S_0 = Target inventory level at the CBB (in units of WB and RC).
- S_j = Target inventory level at location j (in units of WB and RC).
- N = Number of TS's in the system.
- P_j = The probability of a cross-matched unit of WB or RC being transfused at location j .
- v_0 = Shortage at the CBB (in units of WB and RC).
- v_j = Shortage at location j (in units of WB and RC).
- q_j = Outdate at location j (in units of WB and RC).
- n = Number of times a unit of WB or RC is cross-matched in its life time.
- a = Age of a unit of WB or RC when it is cross-matched for the first time.

The variables d_0 , d_j , S_0 , S_j , v_0 , v_j and q_j are computed for each type and Rh factor; rather than have two subscripts, one for location and the other for ABO and Rh, the second subscript has been suppressed for ease of writing the results. However, for low volume rare blood groups when the target levels and the demands are small, say, one or two units, it is better to maintain the stock at the CBB rather than incur excessive transshipping of units.

The optimal level target inventory level at the CBB is:

$$(6) \quad S_0^* = 3.14 (d_0)^{.72} (N)^{.93}$$

$$R^2 = 0.993$$

$$F = 3529$$

Significant at 0.001 for all coefficients.

The corresponding optimal target inventory level for these TS's which belong to the central bank system is:

$$(7) \quad S_j^* = 7.99 (d_j)^{.78}$$

$$R^2 = 0.995$$

$$F = 8675$$

Significant at 0.001 for all coefficients.

These results were estimated by a procedure similar to that used for the independent hospital blood bank and are relevant for the special case where $P_j = 0.5$ and $D = 2$.

The relationship between the level of demand and the optimal inventory level in terms of days of blood usage for both an independent bank and a member of a central system is illustrated in figure 15. These figures were computed from the equations above and from equation (4) for target inventory at an indepen-

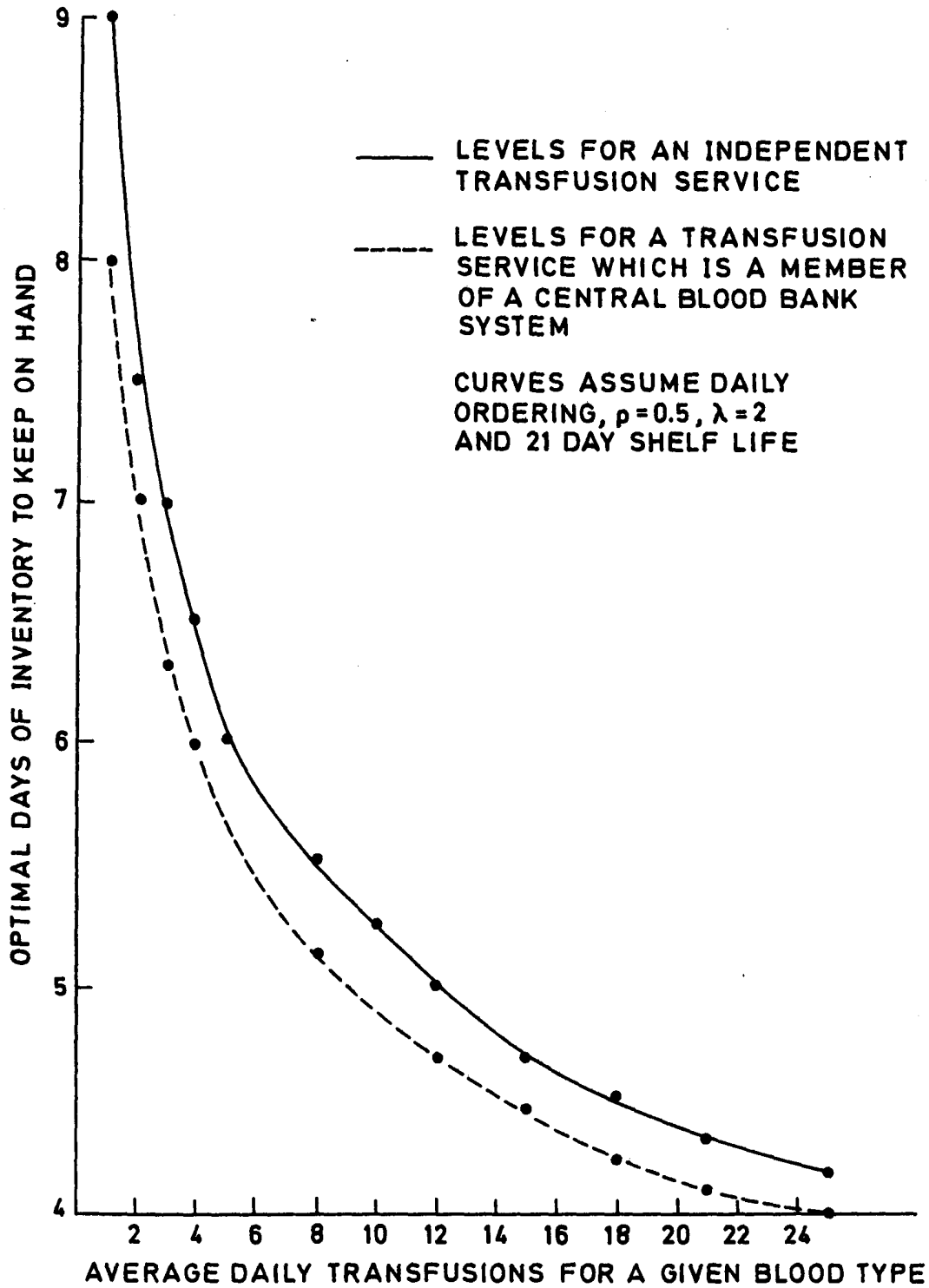


FIGURE 15. OPTIMAL DAYS OF INVENTORY TO KEEP ON HAND TO MEET TRANSFUSIONS FOR A GIVEN BLOOD TYPE

dent bank for the case where the transfusion fraction at each bank, p , is 0.5 and the cross-match reserve time, D , is 2 days. The level of optimum inventory at a transfusion service can be reduced by 5 to 12 percent if the service is a "total services" member of a community blood center.

Centralized Blood Bank Issuing and Allocation Decision.

After the CBB receives all the requests from the TS's, the orders are filled by drawing from the inventory in the CBB using a FIFO issuing policy. For purposes of simplification as well as good medical practice, each type and Rh factor is considered independent of the other types and Rh factors. When the sum of all TS demands exceeds the total inventory in the CBB, the CBB may backlog the excess demand or may fill all demands by calling in donors, by contacting other CBB's, by using frozen packed red cells or by requesting an emergency shipment from still higher echelon (regional) blood banks. In this analysis the CBB uses different approaches to handle the excess demand depending upon whether the orders are routine or emergency. Routine orders are placed by the TS's at the beginning of each day to build up their inventory to a specific level. Emergency orders are placed during the day when the inventory of the TS's cannot meet their respective users' demands. For routine orders, the CBB will fill the orders as long as its inventory lasts and disregard the excess demands, if any. Consequently, the TS's may not receive the full amount they ordered. For emergency orders, the CBB still fills the orders as long as its inventory lasts. However, if there are excess emergency demands, the CBB will attempt to fill them from the inventory of the TS's within the system. Furthermore, if there is insufficient stock in the whole system to fill the excess emergency demands, then the CBB will fill them by contacting exogenous sources. The rationale of the different treatments for the three types of excess demands, i.e., the three types of "shortages" between routine and

emergency orders, is that the routine orders are used to build up the buffer inventory in the TS's. These routine orders may not represent transfusion demands that day. Therefore if the excess of the routine orders over the available inventory at the CBB is not filled, a true shortage will not necessarily occur. On the other hand, the emergency orders, if not filled, will surely create a shortage, since the buffer inventory in the TS has to be essentially depleted before the TS will place an emergency order.

Since each TS may not receive all that it has ordered, a systematic process is needed to allocate the available stock in the CBB to TS's. This allocation process is called the allocation policy. Essentially there are three distinct practical alternatives:

- 1) The CBB picks a TS and fills its demand by the FIFO issuing policy then goes on to fill the next TS until all the stock runs out or all demands from TS's are filled. This type of allocation process resembles the first-come first-serve practice which exists in some blood banking systems.

- 2) The CBB ships an amount to each TS such that the ratio of the amount received to the amount ordered is the same for each TS. Furthermore, all TS's have the same ratio of the amount of different ages received to the amount ordered. This type of allocation process resembles proportional rationing of scarce resources and is intended to be fair to all users with regard to their stated target needs by treating each user equitably. (See Cohen, Pierskalla and Yen¹¹ for a theoretical treatment of this problem.)

3) The CBB ships each unit to the hospital where the shortage probability is the highest in the system. In other words, the delivery of each unit is intended to adjust the system stock configuration such that total system shortage probabilities may be improved. If the target level needs in policy 2) above are based on shortage probabilities, then this alternative policy coincides with policy 2). However, if the target level needs are based on some trade-off between shortages and outdates, then policies 2) and 3) may differ slightly.

After all TS's receive their orders it may be desirable to transship units among them. Basically there are two reasons for, or types of, such transshipments: 1) the shortage anticipating transshipment; and 2) the outdate anticipating transshipment (see Jennings^{12,13} or Yen¹). If one location anticipates a shortage while another location does not, then a transshipment from the latter to the former may be beneficial to the system in reducing the shortage cost. Similarly, if one location has an excessive amount of old units while another location does not, an outdate anticipating transshipment can be initiated for the benefit of the system. Before a transshipment is made the exact stock configurations of the locations, as well as the demand distributions of the locations, must be known in order to evaluate the benefit of the transshipment. When such information is available, the CBB is in the best position to direct the transshipments in the system. Obviously, for these types of actions a sophisticated information processing system is needed (see the second section of this paper).

In the case where such information is not available, the benefits of transshipping are uncertain and no transshipment should be made directly from one TS to another. However, since each TS knows its own stock age configuration it can choose to

return excessively old units to the CBB. In this way old units are recycled to other hospitals in the system. This particular type of outdate anticipating transshipment will be called the recycle policy. The terms "outdate anticipating transshipment" and "shortage anticipating transshipment" will hereafter refer exclusively to transshipment between TS's directly.

Optimal Cross-matched Release and Issuing Policies From the CBB. Cohen and Pierskalla⁴ show that if a unit is cross-matched and not reported transfused within a short time (D equals 1, 2 or 3 days) at a TS, further information should be obtained on the status of the demand for which the unit was issued. If the demand had disappeared, the unit should be made available for possible reassignment either at the same bank or another hospital blood bank. In this manner, the cross-match release time, D , should be kept as low as possible. As long as D can be maintained below 7 days, the FIFO issuing policy should be followed at the CBB for those TS's which receive daily or at least tri-weekly deliveries from the CBB. If D exceeds 7 days, last-in first-out (LIFO) will be somewhat better than FIFO but both policies will then have excessive outdates and shortages.

In another study of issuing policies in a TS by Deuermeyer, Pierskalla and Sasseti,¹⁴ it was shown that for a department which has low usage and low values of P_j , a LIFO issuing policy for that department should be followed. The underlying reason why LIFO should be followed rather than FIFO was to increase the probability of transfusion of the cross-matched unit. This same reasoning applies to certain TS's in a CBB system, namely, those TS's which require infrequent deliveries (at most twice a week) and have low transfusion probabilities. For these TS's the CBB should issue by LIFO and then at the next delivery pick up any nontransfused units, replacing them with younger units. The older units which are then picked up may be made available to TS's with higher volume needs which have higher transfusion probabilities.

Optimal policies include:

- 1) Cross-match Release Period, D , should be 1 or 2 days (the smaller that D is the lower are the shortages, out dates, and costs).
- 2) For TS's which received daily or triweekly deliveries, the units which are shipped to them should be issued on a FIFO basis (unless fresh units are needed for special purposes such as cardiac surgery).
- 3) For TS's with infrequent deliveries (once or twice a week), the units which are shipped to them should be issued on a LIFO basis and unused units from the prior shipment should be picked up and replaced with younger units.

Also in the study by Cohen and Pierskalla,⁴ it was shown that the optimal target inventory level at the central bank, S_0 , was not greatly dependent upon the cross-match release time D , provided that D was not allowed to get too large. This result is the only instance found so far where the choice of one aspect of operational policy, namely, "how large should D be," is not strongly interactive with the ordering policy S_0 at the CBB. If the size of D does get large, such as $D > 4$ days, then its value will influence the target optimal inventory levels at each TS (see equations 4 and 5).

Transshipment Policies Between TS's. Several conditions may trigger a transshipment by the CBB between two TS's. The most important condition is when a TS has an emergency demand and the CBB does not have sufficient stock on hand to meet it.

In this case a check of the other TS's should be conducted and a transshipment made provided the TS which furnished the units will not be placed in a precarious shortage situation, that is, provided the probability of shortage at the sending TS does not become too large after depletion of its stock.

Less important transshipments occur due to shortage or out-date anticipating transshipments. For shortage anticipating transshipments, a unit is transshipped from location A to B if the shortage probability in A is greater than that in B and if the difference of the two probabilities is greater than a certain number. The number should be large enough so that the transshipment will be beneficial to the system. It is calculated according to the following formula:

$$(8) \quad [\text{shortage probability in A} - \text{shortage probability in B}] > \text{transportation cost/shortage cost.}$$

If the transportation cost is estimated to be about 5 percent of the shortage cost, then the number used in the determination of whether or not to transship a unit is 0.05 (i.e., initiate a transshipment if shortage probability is reduced by 0.05). Note that the shortage cost is assumed to be the same for all TS's and the transportation cost is independent of the facilities where the transshipment occurred. This simplification is justified because the majority of the transportation costs are often not the direct costs, e.g., gas and time consumed in the shipment, rather the indirect costs related to the handling, labeling, accounting and information exchanged between the two facilities. All these indirect costs, however, depend upon the size of the system. Therefore, the number 0.05 can at best be described as an educated guess.

For outdate anticipating transshipments, a unit is transshipped from A to B if the outdate probability in A is greater than that in B and if the difference of the two probabilities is greater than the transportation cost divided by the outdate cost:

$$(9) \quad \frac{[\text{outdate probability at A} - \text{outdate probability at B}]}{\text{transportation cost/outdate cost.}}$$

Again, 0.05 was used for this ratio (the number 0.05 is based on similar calculations and assumptions as those used above). It should be noted that in both cases the number 0.05 is somewhat arbitrary since actual costs are not known precisely. However, in the range between 0.03 and 0.20 there appears to be no significant difference in the number of units transshipped. Indeed, for this range, virtually no shortage or outdate anticipating transshipments will occur (based on simulation results from Yen's model).

One reason why there are few shortage anticipating transshipments stems from the allocation policy in the CBB. Recall that units are available for transshipment only after each TS has received its delivery. But under allocation policies 2 or 3, the units in the CBB are issued one by one to the location with the highest shortage probability or proportionally to their target needs. So at the end of the allocation process each TS will have an essentially identical shortage probability except when there is insufficient inventory in the CBB to make them equal or when there is a tie in shortage probabilities before the issuance of the last few units. In both of these cases some discrepancies among shortage probabilities will occur, but they are rather negligible under relatively wide ranges of target inventory levels at all locations. Consequently, the conditions to initiate a

shortage transshipment would rarely occur, hence hardly any units are shortage transshipped. For this reason the shortage transshipment policy has virtually no significant effect on the shortages in the system.

The insensitivity of the outdated units to the outdate transshipment policy can be explained as well. By observing that a unit will be outdated only after several passages through the cross-matching process, the quantity of expected daily outdates is fairly small simply because the probability of outdate given by $(1-P_j)^n$ is usually a very small number. Hence, there are very few units which outdate, when optimal inventory, issuing, P_j and D policies are followed, regardless of whether an outdate transshipment policy is in effect or not. Consequently, the outdate transshipment policy can be expected to have virtually no significant effect on the outdates in the system.

It should be mentioned here that the simulation model indicated that while there are some units transshipped, the actual quantities were insignificant even when the inventory levels at different locations were varied over wide ranges. However, if the actual inventory levels used are far larger than the optimal target inventory levels at the TS's, then as one would expect, outdate transshipments to become significant if the allocation policy is changed to the FIFO allocation policy. Both of these decisions are extreme and should not be followed. That is, the CBB should use optimal target inventory levels and should not use allocation policy 1 (FIFO).

Transshipment and Allocation Policies

1. Use allocation policy 2. Allocation policy 3 is also good but requires more computation and time for implementation.

2. Transship units from one TS to another

- a) If there is an emergency need at a TS, if the CBB is out of stock, and if the sending TS does not incur an excessive probability of shortage (say over 10 per cent).
- b) If the probability of shortage at TS_i minus the probability of shortage at TS_j is greater than or equal to the ratio of unit transportation cost to shortage cost.
- c) If the probability of outdate at TS_i minus the probability of outdate at TS_j is greater than or equal to the ratio of unit transportation cost to outdate cost.

Expected Number of Shortages and Outdates When Optimal Ordering, Issuing, Crossmatch Release, Allocation and Transshipment Policies Are Followed. The equations for the expected number of shortages and outdates are derived in Yen¹ and will be summarized briefly here. A shortage occurs whenever the daily demand D_0 , at the CBB exceeds the available supply, S_0 . However, the daily demand D_0 is a random variable so the expected shortage at the CBB is given by:

$$\text{Expected Shortage} = E[v_0] = \sum_{k=1}^{\infty} P\{D_0 \geq S_0 + k\}$$

where $P\{D_0 \geq S_0 + k\}$ is the probability that the daily demand D_0 exceeds the amount $S_0 + k$.

Since a TS orders up to a target inventory level, S_j , it will order more if there is less stock on hand. If there is less stock on hand there is a higher shortage probability. So the shortage probability is directly related to the amount or-

dered. Furthermore, once there is a shortage in the CBB, it implies that not all the TS's may get what they ordered. The amount received by each TS depends on how the available inventory in the CBB is allocated. For the following equations, allocation policy 2 is followed.

Thus, there is a shortage at the j th TS whenever the demand, D_j , exceeds the estimated supply $S_j - E[v_0](E[D_j]/E[D_0])$ (where $d_j = E[D_j]$ and $d_0 = \sum_{k=1}^S P_j E[D_j]$). Consequently,

$$E[D_0] = \sum_{k=1}^{\infty} P_k E[D_k] = d_0$$

Estimate of the expected shortage at $TS_j = E[v_j] = \sum_{k=1}^{\infty} \{P D_j + (E[D_j]/E[D_0])E[v_0] \geq S_j + k\}$.

In order to determine the expected number of outdates in the total CBB system, it is necessary to estimate the number of of times, n , a unit is likely to be cross-matched before it outdates. Once n has been determined, then, at the j th TS, the probability that the unit will outdate is just $(1 - P_j)^n$, i.e., it is just the probability that the unit is not transfused every time it is cross-matched. Now each day the expected number of units demanded is just $E[D_j]$, so the

(12) Expected daily outdates at $TS_j = E[q_j] = (1 - P_j)^n E[D_j]$. And the total system outdates are

(13) Total daily outdates = $\sum_{j=1}^n (1 - P_j)^n E[D_j]$.

In order to compute n , it is necessary to know the average age of a unit when it is first cross-matched, a , and the cross-match release time, $\lambda(j)$ where $\lambda(j)$ is a parameter which is set

by TS_j (recall that $D(j)$ should be only 1 or 2 days). After a unit enters the system at the CBB, it will be stored there for a period of $S_0/E[D_0]$ days before being issued to one of the TS's. After the TS receives its order, the units will be placed in the unassigned inventory and will be cross-matched at the appropriate time. Since each day there is an average of ED_j units cross-matched and $(1 - P_j)E[D_j]$ returned to the unassigned inventory, the units will be stored in the TS unassigned inventory for an average time of $S_j - ED_j/ED_jP_j$ days before being cross-matched for the first time. Under the assumption that a fresh unit was received at the CBB, its average age when cross-matched for the first time, a , is:

$$(14) a = S_0/ED_0 + (S_j - ED_j)/ED_jP_j.$$

Once a unit is cross-matched it will return to the unassigned inventory if not transfused after D days. Since a FIFO cross-matching policy is followed those returned units must be at least as old as all the units which have not ever been cross-matched; they have a high probability of being issued again the same day they are returned. Assuming the maximum useful life of a unit is 21 days including the twenty-first day, the average life span of a unit subject to cross-matching is therefore $22 - a$ days. Therefore, a unit can be cross-matched for about $(22 - a)/D$ times before it reaches an age between $22 - D$ and 21 days. But if a unit is returned at ages between $22 - D$ and 21 days, the unit can only be cross-matched at most one more time. This is because the next time the unit is returned it already exceeds the allowable age for cross-matching purposes. Assuming there is an equal probability for the units to be returned at ages $22 - D, 22 - D + 1, \dots, 21$, then the expected number of times of cross-matching for units between $22 - D$ and 21 days is about $1 - 1/D$ times because if the unit is of age 21 days, then it cannot be cross-matched for the next day's demands. If we assume the return units are always cross-matched again on the day they are

returned, the the total number of times a unit may be cross-matched is approximately:

$$(15) n = (22-a)/D + 1 - 1/D.$$

The quantities n , a , and $E[q_j]$ are easy to calculate either by hand or with a computer. The quantities $E[v_0]$ and $E[v_j]$ are more difficult to calculate and a computer is needed at the present time (tables of these numbers could be developed).

It is apparent from these formulae that the expected shortages and outdates are complex functions of D , P_j 's, S_j 's, S_0 , d_j 's and d_0 . Consequently, when any blood system control policy is changed in a manner which can affect any of these parameters, this change will also affect most of the other decisions.

Delivery Vehicle Routing. As part of a regional blood bank design model, the problem of vehicle routing for blood product deliveries was considered. The basic problem involves selecting a route for each central blood bank subsystem which minimizes overall transportation costs between the CBB and its member TS's. For each configuration of blood banks, a "sweep" algorithm (see Or¹⁵) was used. The algorithm is a heuristic method which is incorporated into the overall regional blood bank location and central bank allocation model. Figure 16 indicates a typical regional design solution for the metropolitan Chicago area which also contains optimal vehicle routes.

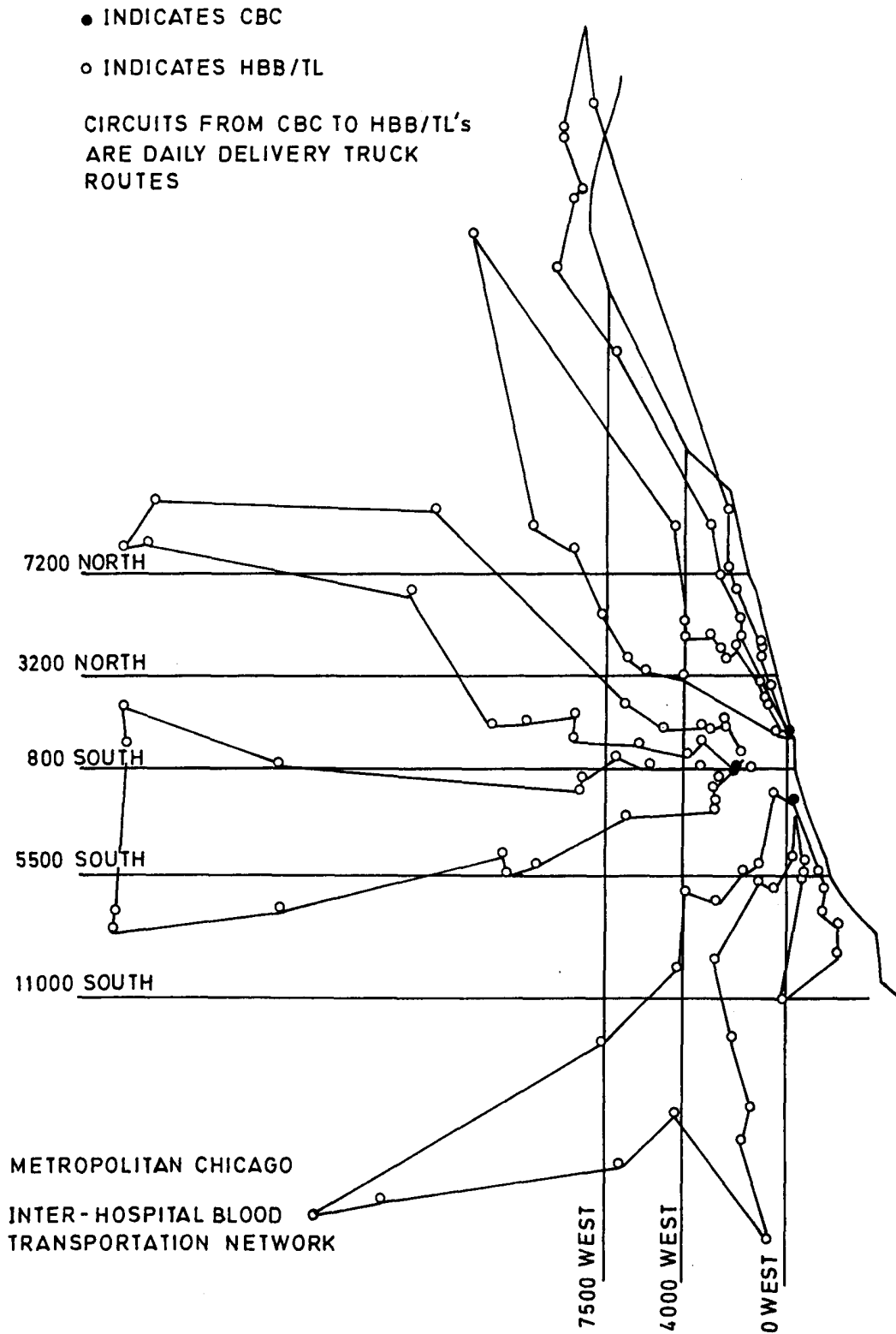


Figure 16. Allocation based on emergency routing system costs

SUMMARY

This paper has considered a number of contributions to the development of operational procedures for blood bank management. The first section illustrated how some of the data maintained in blood bank information systems could be used in blood bank management. In particular, data on cross-matches were analyzed to develop a forecasting model of demand for blood products. The underlying variability of cross-match demand was analyzed. Time series methods were then applied to daily type specific cross-match and monthly total cross-match data. These methods led to models for predicting mean daily demand which is required for inventory control. Conclusions were that on either a daily or monthly basis, blood demand forecasting is extremely difficult and can only be accomplished with the use of relatively complex time series models. Hence, investment in blood inventory, in better management and in demand pooling are indicated as a response to this high level of uncertainty. It was possible, however, to develop a single forecasting equation which was effective for all blood types.

The second section presented the results of applying a simulation model and statistical analysis to the problem of controlling hospital blood bank inventory levels. A comprehensive target inventory level decision function was estimated. This function can be used to set optimal inventory levels in a manner which takes into account the scale and particular environmental features of a hospital blood bank. The mean daily demand, the transfusion to cross-match ratio and the cross-match release period were shown to be significant variables.

The next section considered a variety of operational control issues relevant to a central blood bank system. Decision rules for setting optimal daily inventory levels at the central

blood bank and member transfusion service locations were developed.

In the final section centralized blood bank issuing and allocation decisions rules such as cross-match release guidelines and transshipment guidelines also were analyzed.

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