

QUANTITATIVE PROCEDURES FOR NURSE
STAFFING MANAGEMENT -- A SURVEY
AND RECOMMENDED PROCEDURES
FOR APPLICATION

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ABSTRACT

During the last two decades, a wide variety of quantitative procedures have been developed to support nurse staffing management. In this article these procedures are critically reviewed in the context of a conceptual framework that includes activity analysis and workload prediction, tactical staffing decisions, performance monitoring, strategic planning, and coordination with other hospital activities. Strengths and deficiencies of suggested procedures are pointed out, and important aspects that have been overlooked are identified and discussed. Those procedures which have undergone recent successful implementation are presented in some detail. The concluding section suggests areas where more research and development seem desirable.

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A great deal of attention has been given by operations researchers and industrial engineers during the past several decades to the nurse staffing problem -- how to assign nursing staff to provide a desired level of patient care. This has been a natural area for research, because of the size and high cost of nursing activities and the large range of seemingly applicable operations research techniques available.

This area will become even more important in the future. Hospital costs will be subject to increasingly tighter public regulation. It is expected that new reimbursement methods, which provide incentives to halt escalation of costs, will be instituted, and the output and process performance rather than input resources will increasingly be used as a basis for both reimbursement and inter-hospital comparison. Clearly, hospitals will continue to be under increasing pressure to bring about improved cost control. We believe that the development of innovative approaches to the organization and management of nursing resources holds great promise for cost savings in the delivery of hospital services.

In this article we discuss quantitative procedures that have been developed to support nurse staffing activities. The strengths and deficiencies of these techniques are analyzed, and extensions of certain techniques are suggested. A major difference between this review and those of Stimson and Stimson [58] and Aydelotte [7]

is that the suggested procedures are viewed in the context of a conceptual framework of nurse staffing management. Successfully applied procedures and methodologies are presented for those who wish to undertake further applications as well as those who wish to extend current knowledge.

This framework includes five areas of management activity: (a) activity analysis and workload prediction; (b) tactical nurse staffing decisions -- corrective allocations, shift scheduling, and manpower planning; (c) performance monitoring; (d) strategic planning; and (e) coordination with other hospital activities. This framework provides a basis for pinpointing those activities and aspects that have been largely overlooked by operations researchers and industrial engineers, but which are potentially important for improving overall nurse staff utilization and performance.

After the framework is presented, we review studies which have analyzed nursing activities and how the workload varies with the numbers and types of patients to be served. Methods developed to provide forecasts of demand and use rates are also discussed. The next section reviews quantitative procedures that have been developed to assist in making tactical nurse staffing decisions, and describes how these procedures can be integrated with performance monitoring, strategic planning, and other hospital activities as well. The final section summarizes our major conclusions and suggests areas where more research and development seem desirable.

We found that the great bulk of nurse staffing research has concentrated on activity analysis and workload prediction, and on two of the three levels of tactical nurse staffing decisions --

corrective allocations and shift scheduling. There has been little examination of potential improvements in the third tactical decision level -- manpower planning -- or of the interrelationships among the three levels. Finally, minimal attention has been given to improving performance monitoring, strategic planning, or coordination with other hospital activities such as admission and treatment scheduling.

It is our hope that this article will serve several purposes.

(a) It should guide future nurse staffing research toward more comprehensive and fruitful problem-solving approaches; (b) it should give hospital systems analysts a good appreciation for the state-of-the-art and help them select procedures which might be applicable in their own setting; (c) it should help hospital administrators identify areas where, although no ready-made solutions are currently available, they might want to support specific studies in their own hospitals; and (d) because it focuses on a major manpower administration aspect of health care, it should provide a basis for developing comprehensive plans of care for patients and for determining the most effective facilities in which to provide patient care.

The Nurse Staffing Process

The nurse staffing process can be conceptualized as a hierarchy of three tactical decision levels which operate over different time horizons and with different precision. These three decision levels will be called corrective allocations, shift scheduling, and manpower planning.

Within a shift, the staff capacities among units may be adjusted to unpredicted demand fluctuations and absenteeism by using float, part-time, relief, pulling, overtime, and voluntary absenteeism. ("float" refers to a pool of cross-trained nurses who are floated among units to smooth demand

fluctuations; "pulling" refers to temporary reallocation of a nurse to a unit other than where she normally works.) These "corrective allocations" should be based upon the individual's preferences and capabilities, and they are restricted by shift schedules and the capabilities of the employees.

The next decision level is "shift scheduling" to bring about uniform and smooth matching between expected workload and staff capacity among units on a week-to-week and day-to-day basis. For each individual employee days on and off as well as shift rotation and time for classes may be determined. The individual's preferences should be considered to bring about high personnel satisfaction and ensure that personal capabilities are made use of in the best way.

These two "scheduling" levels concern the utilization of personnel already existing within the organization; they have a known mix of specialization and experience. However, the long-term balance of size and capability of nursing personnel among units is obtained by hiring, training, transferring between jobs, and discharging. We call this decision level "manpower planning." Because of the time lags involved, manpower planning actions should be taken early to meet anticipated long-term fluctuations in demand and supply. As we shall see, very few studies have addressed this decision level.

The vital interdependence of the three levels must be recognized in order to bring about systematic nurse staffing improvements. Each level is constrained by available resources, by previous commitments made at higher levels, and by the degree of flexibility for later correction at lower levels. Therefore, each decision level is strongly dependent on the other two; one level should not simply be considered in isolation.

In general, the later a decision is made the more reliable the available information is, but the alternatives that remain are fewer in number and generally more expensive. Hence, there is a tradeoff between early and inexpensive actions based upon unreliable information (e.g., transferring), versus late and expensive corrections made when more reliable information is available (e.g., using overtime). This tradeoff can be likened to the balance between fire-fighting capability and fire prevention.

Furthermore, decisions at each level should be coordinated not only with decisions at the other two levels, but also with future and past events within the level itself. Coordination with the future should be accomplished through the planning stages: forecasting, tentative planning, action planning, and execution. All of the uncontrollable or partially controllable variables that have a major influence on the staffing process have to be forecasted. Examples of these variables are workloads for each skill category, hiring prospects, turnover, and absenteeism. However, the planning process at each level should be dependent on the other two. The plans can be said to be "gliding" or "rolling." That is, an action plan for one level should be the basis for a tentative plan for the level below, and they should be updated and made firmer as execution is approached and more information becomes available.

These three decision levels are depicted in Figure 1. It can be seen from this figure that there are important interactions between these tactical nurse staffing decision levels and other management activities. As shown in the box at the left of Figure 1, coordination with the past should be accomplished through a monitoring system, which should (a) take

an inventory--number and capabilities--of employed personnel; (b) measure, control, evaluate, and correct staffing performance; and (c) gather statistics to be used as a basis for forecasting.

As seen in the above presentation, there are important strategic policy and design decisions in a hospital that restrict the number of alternatives available at each of the three tactical decision levels. This box is depicted at the top of Figure 1. Examples might include policies about the use of float personnel, the control of admissions, the skill mix of the nursing personnel, or the number of nurses to be in the training pool at any point in time. These policies should be part of any investigation into new methods to improve productivity. The impact upon cost, quality and satisfaction of the policies in use should be recognized by the administrator; only then can the long-run value of maintaining them be intelligently assessed. Hence, the strategic planning level forms a decision level of its own which (a) dictates the range of options available in each of the three tactical decision levels, (b) sets the standards to be used for performance evaluation, and (c) makes decisions on the designs and the policies to be used in all hospital functions.

We have stressed the importance of recognizing the interdependence among the three tactical decision levels as well as among strategic, tactical, forecasting, and monitoring activities. However, it is equally important to consider the interdependence between nurse staffing the other hospital activities, shown at the right of Figure 1. Each nurse staffing level should cooperate with levels in the demand and facility control systems that operate with the same horizon and precision. That is, corrective

allocation should be coordinated with task assignment; shift scheduling with scheduling of admissions, operation rooms, treatment, and supportive services; and manpower planning with budgeting, recruitment, training, and facility planning. Gross imbalances among these activities are common due to a lack of organizational and motivational incentives for coordination. This reflects ambiguous and sometimes conflicting objectives based on traditional and strong value systems. Unfortunately, most studies have omitted these crucial horizontal interdependencies.

In summary, the nurse staffing process can be conceptualized as a hierarchy of three highly interdependent tactical decision levels that are restricted by strategic decisions. Staffing decisions should be coordinated with other hospital activities and the performance of the staffing process should be measured and evaluated frequently in order to obtain overall staffing effectiveness. (For greater elaboration of this framework, see Wandel and Hershey, [65]).

Activity Analysis and Workload Prediction

Before examining the quantitative procedures that have been proposed to assist nurse staffing, we will review studies which have analyzed the activities nurses perform and how nursing load varies with the numbers and types of patients. In particular, we will review how these studies have been used to develop methods for workload prediction -- a necessary input into any planning or scheduling procedure. (See Aydelotte, [7], for an extensive discussion of this literature.)

For each tactical decision level -- corrective allocation, shift scheduling and manpower planning-- the activity analysis and workload prediction needs vary with respect to the amount of detail needed and the

corresponding accuracy of the workload measures given by skill level on each nursing unit. On a shift basis the actual patient mix and illness severity along many dimensions which affect staffing needs must be accurately predicted at the beginning of the shift to allocate available personnel appropriately. On a daily and weekly basis, the average workload and minimum staffing needs on each shift must be predicted with cyclic and/or seasonal patterns in order to build 2, 4, 6, or 8 week work schedules for the personnel on each unit. Finally, on a monthly, quarterly, semi-annual and annual basis, the average workload and minimum staff by skill levels must be predicted based on trends, seasonal patterns, and other hospital activities in order to follow appropriate hiring, training and staffing policies.

Staffing Based upon Average Patient Requirements

For many years, it was assumed that nursing units should make staffing decisions for each specialty unit (e.g., medicine, surgery, and obstetrics) by multiplying the average number of nursing hours required per patient by the number of patients (see Stimson & Stimson, [58].) As early as 1940, Pfefferknorn and Rovetta used this approach [50] and similar procedures were developed by the American Hospital Association [5] and George and Kuehn [24].

In 1947, the National League for Nursing developed a patient classification scheme for the care of pediatric patients. An attempt was made to relate the amount of nursing time required to the classifications but the times were not sufficiently refined to provide a sensitive tool for staffing needs.

Binhammer et al. [10], Bernstein et al. [9], and Claussen [13] assigned patients to categories according to their individual needs for direct nursing care, and the time requirements for direct nursing care per patient in each category were more effectively established. Then the average mix of patients was determined for each unit, and average staffing requirements were calculated. These requirements were assumed to remain constant over time when used as a basis for staffing.

In the 1950's, work sampling and time study were introduced to (a) clarify the various functions that nurses of different categories perform, (b) determine the skill levels required to perform these tasks devoted to tasks above or below the category's skill level. If there were striking deviations from what was expected, changes could be made in the organization and operation of the nursing units. [29, 4, 6, 63, and 19].

Activity Studies Based on Direct Care Indices

In the late 1950's and early 1960's, a group at Johns Hopkins University [14, 15, 16 and 17]; and Flagle et al., [20 and 21] revealed wide swings in demand for nursing care within each unit from day to day, even when the total number of patients remained the same, because of variations in the distribution of patients in various "need" categories. Three need categories -- total care, partial care, and self-care -- were defined through checklists of observable patient characteristics. Work sampling studies were then used to determine the means and standard deviations of minutes per day spent on direct nursing

care for patients in each category. It was also shown that the distribution of direct care times could be well approximated with normal distributions (Connor, [14]). By multiplying the number of patients in each care category by the average direct care times for that category, and then summing over all categories, a "direct care index" was obtained. This index could be used to estimate the number of hours of direct nursing care required in each unit and each shift to meet existing standards of care for a given patient load.

Some of the advantages of this approach are its simplicity, objectivity and reliability. As to simplicity, it is easy to train personnel (such as a charge nurse) to fill out the data collection form quickly and consistently. On a thirty bed unit it can be done in about five minutes. The approach is objective and reliable since it consists of evaluating the status of patients with regard to physical and hygienic needs so that evaluation bias is kept to a minimum.

Some criticisms of this approach is the strong emphasis on physical and hygienic needs and the exclusion of instructional, observational and emotional support needs of patients. For some patients the intensity of these latter needs correlate highly with physical and hygienic needs but for others they do not.

Now the direct care load based on their definition of direct care composed only about 30% of the total work load. Therefore, Connor [14 and 15] made further work sampling studies to determine the relationships between the time nursing personnel spend in productive

activities (direct patient care, indirect patient care, paper work, communications, etc.) and the following three independent variables: (a) the amount of direct care to be given (direct care index), (b) the census, and (c) personnel hours available. Using a step-wise linear regression analysis, he concluded that the time spent in productive activities varied significantly with both the index and the personnel hours available, but not with the census.

Connor also noticed that when hours available increased beyond a level required to give adequate care, the extra time was not used for increasing the level of direct care administered. Instead, it was used mostly for personal time, with a small percentage for communication, paperwork, and indirect care. One important implication is that if labor-saving equipment is introduced, the number of nursing hours should be reduced simultaneously. Otherwise, the personal time saved will not go into direct care but into increased idle time.

Another finding [14 and 17], important to nurse staffing, was that the workloads among the units were statistically independent and only some 20% of the total staff needed to be "floaters" in order to smooth the daily fluctuations.

The average time requirements used for calculating the direct care index and the coefficients in the regression equations were based on historical care patterns, not patient needs. To improve the standard of care, the content of patient care should be examined to evaluate the levels of skill and times required before a direct care index and regression equations are used to predict workloads.

Connor's regression analyses were extended to several variables by Jelinek [30, 31 and 32]. He included four categories of nursing personnel (professional nurses, practical nurses, nursing aides, and student nurses); Connor studies only professional nurses. He verified Connor's conclusions, as well as the following important hypotheses: (a) marginal productivity decreased for all personnel except nursing aides; (b) contributions to direct care made by increases in staff were greatest for nursing aides, followed by practical nurses, student nurses, and professional nurses respectively; and (c) staff shortages in a given category were at least partially corrected by substituting nurses for other categories.

The regression analysis approach used by Connor and Jelinek is an important tool, not only to increase the understanding of the existing nursing system, but also to (a) establish ratios and standards for planning and control purposes. (b) obtain models for workload predictions, and (c) forecast effects of changes in the organization of operation of the nursing system.

Workload Predictions Directly Based on Nursing Activities

Besides the patient classification methods of estimating workloads, several methods have been developed that use nursing activities directly. The alleged advantage of this direct method is that the workload for each individual patient is estimated separately, instead of using average workload estimated for patient categories. The most complete method would be to use time standards for all tasks on each

Nursing order to predict direct care time for each patient. The same method could also be used to calculate time requirements for indirect care and non-patient-related activities. Through summation, a total care time could be predicted. The major shortcomings are that (a) nursing orders and care plans are generally not accurate enough, (b) some tasks are mutually dependent, (c) time standards are difficult to estimate, and above all, (d) the method is very cumbersome.

In order to circumvent some of these problems, several methods have been developed that use samples of nursing activities instead of all activities. They use either (a) the most frequent activities of a unit, (b) the most time consuming activities, (c) the activities that require the least skill, (d) the activities that require the most skill, or (e) some combination of the above. A time standard is established for each activity and, by studying the frequency of occurrence for each activity, an index of workload is established for each unit.

One example is the study of White, Quade and White [70]. They used five activity areas -- diet, vital signs, respiratory aids, suction, and cleanliness -- to make estimates of care needs. Clark and Diggs [12] added toileting and turning and/or assisted activity to the list of activity areas and showed that the time required for these activities made up 76% of the total direct care time. The staff utilization and control program developed by CASH [11] also uses direct activity analysis. It gives normative data on how occupancy levels should be translated into number of personnel in each category for each of the three shifts of a unit's operation.

SPRI [57] used step-wise regression analysis to determine the relationship between patient characteristics and patient-dependent workload for each personnel category. Using from 5 to 8 patient characteristics, each with its own scale for measurement, 59-58% of the total workload, both direct and indirect care, for individual patients could be explained.

Markovian Workload Predictors

One major disadvantage with all the methods reviewed above is that they can be used to predict workloads only one or two shifts in advance since the status and number of patients on each unit changes rapidly. Thomas [60 and 61] tried to overcome this by using a Markov model that utilized four phases of patient recovery, each subdivided into three categories. These 12 states, together with entry and exit states, are linked with a matrix of transition probabilities. But since different diagnoses have different length-of-stay distributions, a separate matrix has to be established for each possible diagnosis. This makes the method cumbersome to apply. Singer [56], Balintfy [8], Wendell and Wright [69], Kolesar [36], Warner [66], and Meredith [41] also used Markovian analysis.

Offensend [48] and Smallwood, Sondik and Offensend [56] developed a semi-Markov model. This semi-Markov model was used to calculate the mean and variance of the amount of nursing time required during each hour of the day in each nursing unit for each category of nursing personnel. It should be noted that both the

index and the direct methods could also be extended to predict variance in workload. This would be valuable for calculation of accuracy of workload predictions and as a basis for establishing desired reserve staff capacity.

Forecasting Monthly Workloads

Since the length-of-stay in short term hospitals is normally much less than one month, the movements of patients inside the hospital need not be followed to forecast monthly workloads. Average requirements for nursing hours from each category of personnel in each unit per patient in each admission category during the total length of stay would be sufficient information to translate expected admissions to expected workloads. However, the expected variance of the workload during a month is much more difficult to predict.

Several methods for making forecasts of demand and use rates have been developed [25], but few methods for translating these forecasts into workloads have been reported. Kaplan [35] used historical monthly average data in a linear regression model to determine the relationship between the number of professional nursing hours and patient days for each individual unit. This is an improvement over the commonly used statistic -- average number of nursing hours per patient day -- since a fixed component is added.

The regression model was tested over a seven month period to translate patient days into nursing hours. Most of the variations could be explained by the model and larger deviations were due to inconsistent

staffing or transient effects. This information, which represented an evaluation of the staffing process, was not fed back to the decision-makers for correction and adaptation. Kaplan remarked that providing supervisors with such information might reduce the number of extreme deviations.

Conclusions

The greatest need for accurate workload predictions is to be certain that on each shift the available personnel are efficiently and effectively allocated to the appropriate nursing unit to provide a comprehensive plan of care for each patient. In most hospitals this workload measurement utilized data gathering by the head nurse based on each patient's needs either subjectively through her knowledge of the patient or formally using direct and indirect care activities on patient classification forms. Markovian workload predictions are not widely used if at all.

More and more hospitals are adopting some formal patient classification system. Based on the above readings and experiences a practical methodology to implement a patient classification and nurse workload measurement system is:

- I. Establish a good quality control measure on the nursing units in order to measure the impact of any changes in staffing levels, reassignment, modified organizational structures it will be the "base-line" to measure change. Hopefully this quality of nursing care measure will involve outcome as well as process criteria.

II. Initiate Patient Classification System.

A. Define Objective categories which measure:

1. physical needs of patients
2. hygienic needs of patients
3. instructional needs of patients
4. observational needs of patients
5. emotional support of patients
6. medications and treatments

and which classify patients by age, degree of self-help and other items useful in assessing nursing needs.

These categories will vary by medical services; however, some attempt to achieve parallel categories across services is useful to provide a consistent system for the hospital.

- ### B. It would be desirable to keep the number of items to around twenty (20) - thirty (30) for most units and the acuity categories to four (4) or five (5). The more items and categories, the more accuracy except that there is more resistance from the persons completing the forms since it is more laborious and time consuming. A balance must be struck between the number of items/categories and objective accuracy. (Note: on a twenty-five (25) bed unit the time required to complete the form for all patients should not exceed ten minutes^s by a person with experience on using the form.)

III. Initiate nursing unit work factors study.

- A. Relate the direct patient care to the particular class of patient defined by the P.C.S. "Direct patient care" is loosely defined as "care provided at the patient's bedside in direct support of that patient."
- B. "Indirect or non-direct patient care" must also be measured. This workload is usually expressed as a constant amount of time which depends on the census and not on the P.C.S. Of course, this definition and use of indirect patient care can be changed if so desired.
- C. Before beginning the study be sure to have a good understanding of the designations for direct and indirect care. Much confusion will result if these designations are not clear.
- D. The data on direct and indirect care must be gathered:
 1. over a two to three week time period when the unit is at a reasonably stable capacity
 2. by each shift
 3. by P.C.S. (for direct care)
 4. by skill level (R.N., L.P.N., Aide, other)
 5. by hour of day
- E. The data can be gathered by:
 1. self-recording time forms completed by each person
 2. work sampling by a disinterested third party

- F. The massive amount of data collected must be completely indentified as in "D" above and analyzed. Times for each skill level to do the daily P.C.S. tasks for the categories of patients must be computed (averages are needed and even variances are useful).
- IV. After the workload factors are computed in "III" above they can be incorporated into a computer program.
- V. Generally, the P.C.S. is done on a unit prior to the day shift and nursing workload is computed for the next three shifts. Unusual P.C.S events, happening later in the day, are then factored into the computations as they occur.
- VI. Allocations of personnel can be made based on the workloads computed in "V" above and the nurse supply available prior to and at the start of each shift. Methods for these corrective allocations are described in the next section.

The above workload measurement technique is useful for the daily allocation of available staff. For forecasting daily and weekly unit staffing needs to construct 2,4,6 or 8 week schudules, a practical methodology is to use time series or regression methods which utilize the daily work loads collected over the past 3 - 12 months.

Quantitative Procedures for Nurse Staffing

This section reviews quantitative procedures that have been proposed to assist nurse staffing activities in each of the three tactical decision levels -- corrective allocations, shift scheduling, and manpower planning. In our framework, staffing activities include not only planning and coordination of staffing actions but also monitoring outcomes and analyzing alternative policies and designs. Throughout the discussion, we shall point out those elements and aspects that have been largely neglected, but which are important for overall system effectiveness.

Corrective Allocations

The main purpose of the corrective allocation system is to smooth the workload to staff ratios among the units by using float, pulling, part-time, relief, overtime, voluntary absenteeism, and patient reallocation.

Connor et al. [17] reported the use of the direct care index as a basis for assignment of overtime and float pool. Temporary shifting of personnel among wards has long been a common practice, and many hospitals have implemented some procedure for predicting workloads -- often merely based on census -- to bring about more systematic and equitable personnel allocations.

In order to make the allocations more specific and objective, though often less individualized and flexible, computers have been programmed with heuristic decision-making procedures that evaluate the available information and suggest personnel allocations. For example, Jelinek et al. [34 and 33] have developed a "Personnel Allocation and Scheduling Control System for Patient Care Services" that has the following three interrelated but separate

computer based functions: personnel scheduling, personnel allocation and management reporting. The allocation function is a heuristic procedure based on pre-established priorities that balance the tradeoff between employee dissatisfaction and the workload to staff balance. Another example is Medinte's [40] time-sharing program.

Wolfe [71] and Wolfe and Young [73] used a linear programming model to assess existing nursing personnel of different categories to match given demands from nine groups of tasks. The objective function value for a task from group i assigned to personnel category j is $x_{ij} w_j + v_{ij}$, where x_{ij} is the number of minutes required, w_j the wage per minute, and v_{ij} the intangible costs of having category j perform a task in group i . These intangible costs were assigned by a panel of nurses. Since personnel were assigned only for a full eight hour shift, integer solutions were required and undistributed idle time has to be evaluated. However, a non-integer solution procedure was used. The difficulties of obtaining input data and integer solutions, the fact that the model covers only a single period, and the neglect of preferences and capabilities of individual nurses, indicate that more development is needed before the model can be successfully applied.

Warner [66] and Warner and Prawda [68] formulated the allocation problem as a mixed integer quadratic programming problem. The demand is assumed to be known and the objective function value for personnel category n in ward i during shift t is the quadratic "cost"

$$W_{int} (R_{int} - \sum_m Q_{imnt} U_{imnt})^2,$$

Where R_{int} is the demand, U_{imnt} the number of nurses of category m who are working in category n , Q_{imnt} the associated substitution ratio, and W_{int} the relative seriousness of shortage. The problem was decomposed by a primal re-

source-directive approach into a multiple-choice programming master problem, with quadratic programming subproblems. As with Wolfe's model additional work is required, particularly in estimating the subjective parameters.

Another mathematical programming model was formulated by Liegman [37]. She used a psychometric technique called the Q-sort to measure quantitatively the existing nursing concepts of effective utilization of personnel in an extended care facility. This was the basis for the objective function of a model that generated allocation of personnel given a set of patient requirements and a nursing team configuration. Actual assignments and computer-generated assignments were compared, and the model was found to be valid.

A stochastic allocation model was developed by Miller and Pierskalla [42]. The objective was to minimize the cost of allocating nurses among nursing classes and units subject to constraints on the demand for and the supply of nursing services. Because the number of nurses reporting for work in the various classes and units form a random vector, the allocation model used was a stochastic program with recourse. It was then transformed into a deterministic equivalent and solved.

Finally, we mention Freund and Staats [23] who (a) developed a technique for describing "elements" reflecting care task groups and patient conditions, (b) obtained difficulty values for each element, and (c) developed a heuristic procedure for allocating elements to nurses with difficulty as a measure of assignment "load".

Most of the workload prediction techniques and allocation procedures are not used to monitor and adapt the procedures themselves or to bring about improved long-range staff balance. Quantitative benefit-cost analysis of alternative procedures and policies is generally not considered. However, Offensend [48] used his workload predictor in experiments that were made with various policies for admission control and assignment of patients to nursing teams. He calculated the value of obtaining better information before making decisions, and he performed sensitivity analyses on several parameters.

Tani [59] developed a model of a staffing reserve-resource system that supplies float and relief staff and scheduled voluntary absences and extra shifts. Historical distributions of reserve demand and availability of relief, extra shifts, and voluntary absences were estimated. A cost function was established and various sizes of the float pool were tried in order to find an optimal float pool size. The study exemplifies an approach to benefit-cost analysis that can be used to predict effects of changes in availability of various corrective allocation alternatives or changes in operating policies.

The simulation model developed by Hershey, Abernathy, and Baloff [27] focuses on (1) prediction of savings in manpower requirements of introducing a float pool under a variety of operating and policy parameters, and (2) forecasting manpower requirements for a given allocation procedure and policy. Several criteria which an administrator might adopt for equating levels of patient care under alternative staffing schemes are suggested and studied.

Although the examples indicate savings in manpower requirements of 9-12% from introducing a float pool for the most realistic combinations of parameters, no general conclusions should be drawn because many costs and benefits were not included in the model, and the data were hypothetical.

All of the allocation models require large data bases for their operation. Furthermore these data bases must be accurate for every shift when allocations are performed. To maintain such data bases in the absence of time sharing and/or distributed computer systems is virtually impossible. Consequently none of these mathematical allocation models are operating at the present time. However as more micro and mini-computers are introduced into nursing administration departments simple interactive allocation models will be used to aid in the corrective allocative decisions. Some of this is already underway. It involves the construction of individual nurse data bases which indicate their work schedules, their nursing speciality skills, their previous experience on different hospital units and their preferences for floating or pulling. In addition data is needed on the manpower needs tradeoff of the different skills levels since, one practical nurse does not substitute for one registered nurse (although the reverse holds).

As the shifts change the nursing shortage or excess for each unit and skill level can be computed (based on previously entered patient classification workload calculations) and by interactive heuristic allocation rules nurses can be pulled or floated to appropriate units. Some of these data bases are under construction and others exist in partial form at a few leading medical centers such as Fairview Hospital in Minneapolis, Wn., Presbyterian - St. Luke in Chicago, Ill., and Stanford Hospital in Palo Alto, CA. among others.

With these data bases in place simple and complex allocation models may be tried. More importantly, linkages to other information and decision systems can be introduced. For example, the allocation information system can link to the payroll system to record who worked what shift and unit, to the admissions scheduling system to indicate not only bed availability and patient transfer but personnel availability for under or over staffed units, and to patient billing systems as hospitals begin to allocate manpower as well as supplies costs to particular patients.

Shift Scheduling

The nurse scheduling process may be viewed as one of generating a configuration of nurse schedules that specify the number and identities of the nurses working each day of the scheduling period. By specifying nurse identities, a pattern of scheduled days off and on is created for the individual nurses. These patterns along with the hospital staffing requirements define the nurse scheduling problem: How to generate a configuration of nurse schedules that satisfy the hospital staffing requirements while simultaneously satisfying the individual nurse's preferences for various schedule pattern characteristics.

The most common formalized procedures to schedule shifts on and shifts off for individual nurses are based on the development of a fixed schedule that repeats itself on a cyclic basis, normally every fourth week [28, 22, 46, Price 51]; and Maier-Rothe and Wolfe, [39]. The major shortcomings of such "cyclic schedules" are that they normally cannot take into account fluctuations in work loads and absences, except on a fixed weekday pattern, and they do not consider preferences or capabilities of individual nurses. Such inflexible scheduling procedures require high flexibility in the corrective allocation

and the manpower planning system unless expensive overstaffing is used.

Two noncyclical scheduling papers of note have been by Rothstein[53] and Warner [67]. Rothstein's application was to hospital housekeeping operations. He sought to maximize the number of day off pairs (e.g., Monday- Tuesday) subject to constraints requiring two days of each week and integral assignments. Warner presented a two phase algorithm to solve the nurse scheduling problem. Phase I is involved with finding a feasible solution to various staffing constraints while Phase II seeks to improve the Phase I solution by maximizing individual preferences for various schedule patterns while maintaining the Phase I solution.

Sanders [54] let nurses distribute 1,000 disutility points among seven factors in their schedules, e.f., night duty and split weekend, to measure their attitudes concerning the relative unpleasantness of each factor. Substantial differences in attitudes from nurse to nurse and between wards were found. An evaluation of past schedules' disutility points for each individual showed great variances. He also found that nurses with relatively greater disutility points tended to be absent more often. The problem was then to find a scheduling procedure that minimizes the sum of cumulative disutilities while considering all other constraints. He investigated alternative definitions of equitability, existence of equitable solutions, properties of some scheduling paradigms, and relationships between optimal and equitable solutions. A relatively equitable heuristic procedure seems to be to give the worst schedule to the nurse with the least accumulated disutility points and the best schedule to the one with the most until all schedules have been distributed.

Despite the multitude of quantitative procedures that have been developed to assist in scheduling nursing staff, very few have been implemented and accepted; the vast majority of hospitals schedule nurses on an informal and subjective bases. One of the most successful implementations of quantitative nurse scheduling has been by Miller, Pierskalla and Roth [44]; Miller, Pierce and Pierskalla [43]; Miller Pierce, Pierskalla and Roth [45]; and Jelinek et. al [33 and 34]. This computer-based nurse scheduling system has been successfully implemented in a number of hospitals in the United States and Canada. The theoretical basis is mathematical programming; and the computer basis is the cyclic coordinate descent algorithm. Among the locations where the algorithm has been implemented are:

- (1) Mount Zion Medical Center, San Francisco, California
- (2) Pacific Medical Center, San Francisco, California
- (3) Stanford University Medical Center, Stanford, California
- (4) Kingston General Hospital, Kingston, Ontario
- (5) Rush Presbyterian St. Luke Medical Center, Chicago, Illinois

The mathematical programming model schedules days on and days off for all nurses on a given unit or ward for a given shift for a two, four, six, or eight week scheduling horizon subject to certain hospital policy and employee constraints. Because of the large number of constraints, it is possible that no feasible solutions to the nurse scheduling problem would exist if all the constraints were binding. For this reason the constraints are divided into two classes: Feasibility set constraints, which define the sets of feasible nurse schedules, and nonbinding constraints, whose violation incurs a penalty cost which appears in the objective function. Each hospital has the discretion to define which constraints go into each class.

Constraints: The Feasibility Set

Because of the possibility of special requests by nurses, no constraints are binding in the sense that they hold under all circumstances except those constraints emanating from the special requests. The model however, distinguishes between constraints the hospital would like to hold in the absence of special requests, and those which are always allowed to be violated while incurring a penalty cost.

The former constraints define the feasibility set π_1 , i.e., π_1 = the set of feasible schedule patterns for nurse 1.

In the absence of special requests, this set might include all schedules satisfying:

- (1) A nurse works ten days every pay period (i.e., 14 day scheduling period),
- (2) No work stretches (i.e., stretches of consecutive days on) are allowed in excess of σ days (e.g., $\sigma = 7$)*, and
- (3) No work stretches for τ or fewer days are allowed (e.g., $\tau = 1$)¹

Hence one schedule in a π satisfying these might be (with $\sigma = 7$, $\tau = 1$)

1 1 1 1 1 1 1 0 0 1 1 1 0 0

Now suppose a nurse has special request. For example, suppose the nurse requests the schedule 1 1 1 1 1 1 1 0 1 0 0 0 B, where the B indicates a birthday. In this case all of the above constraints would be violated and π_1 would consist of only the schedule just given. Thus in the general case π_1 is the set of schedules which:

* These are calculated within a scheduling period and also at the interface of a scheduling period with past and future scheduling periods.

- (1) Satisfies a nurse's special requests, and
- (2) Satisfies as many of the constraints the hospital would like to see binding as possible, given the nurse's special request.

The constraints the hospital would like to hold are a function of the hospital in which the model is applied. This, for example, the model could easily specify five out of seven days as ten out of fourteen or specify additional constraints the hospital would like to see satisfied such as one weekend off each pay period.

3.2. Constraints: Nonbinding

Each schedule pattern $\chi^i \in \pi_i$ may violate a number of nonbinding schedule pattern constraints while incurring a penalty cost. Define

N_i = the index set of the nonbinding schedule pattern constraints for nurse i .

For example, if the hospital in which the model was being implemented deemed them as nonbinding, the following constraints might define N_i :

No work stretches longer than S_i days (where $S_i \leq \sigma$);*

No work stretches shorter than T_i days (where $T_i \geq \tau$);*

No day on, day off, day on patterns (1 0 1 pattern);*

No more than κ consecutive 1 0 1 patterns;*

Q_i weekends off every scheduling period (4 to 6 weeks);*

No more than W_i consecutive weekends working each scheduling period;*

No patterns containing four consecutive days off;*

No patterns containing split weekends on (i.e., a Saturday on - Sunday off - pattern, or vice versa).

*These are calculated within a scheduling period and also at the interface of a scheduling period with past and future scheduling periods.

In addition to nonbinding schedule pattern constraints, there are also nonbinding staffing level constraints. Define: d_k = desired staffing level for day k ; and m_k = the minimum staffing level for day k . Then:

- (a) the number of nurses scheduled to work on day k is greater than or equal to m_k and
- (b) the number of nurses scheduled to work on day k is equal to d_k .

3.3. Objective Function

As was mentioned, the objective function is composed of the sum of two classes of penalty costs; penalty costs due to violation of nonbinding staffing level constraints and penalty costs due to violation of nonbinding schedule pattern constraints.

3.4. Staffing Level Costs

Define the group to be scheduled as the set of all the nurses in the unit who are to be scheduled by one application of the solution algorithm. Further define a subgroup as a subset of the group specified by the hospital. For example, the group to be scheduled may be all those nurses assigned to a nursing unit and the subgroups may be Registered Nurses (RNs) and Licensed Practical Nurses (LPNs) and Nursing Aides. Alternatively, the group may be defined as all RNs and a subgroup might be those capable of performing as head nurses.

Then, for each day $k = 1, \dots, 14$ (where there are I nurses), the group staffing level costs are given by:

$$f_k \begin{pmatrix} 1 & i \\ \Sigma & \chi_k \\ i=1 & \end{pmatrix}, \quad \text{where } \chi^i = (x_1^i, \dots, x_{14}^i).$$

For example, this function might appear as in Fig. 2. Now define

B_j = the index set of nursing subgroups j .

J = the index set of all subgroups.

If m_k^j and d_k^j are the minimum and desired number of nurses required on day k for subgroup j , we define the staffing cost for violating those constraints on day k for subgroup j as:

$$h_{jk} \left(\begin{array}{c} \Sigma x_k^i \\ i \in B_j \end{array} \right)$$

where $h_{jk}(\cdot)$ is defined similarly as $f_k(\cdot)$.

Then the total staffing level costs for all 14 days of the pay period

are:

$$\sum_{k=1}^{14} f_k \left(\begin{array}{c} 1 \\ \Sigma x_k^i \\ i=1 \end{array} \right) + \sum_{k=1}^{14} \sum_{j \in J} h_{jk} \left(\begin{array}{c} \Sigma x_k^i \\ i \in B_j \end{array} \right) .$$

3.5. Schedule Pattern Costs

For each nurse $i = 1, \dots, I$ the schedule pattern costs for a particular pattern x^i measure:

- (1) the costs inherent in that pattern in relation to which constraints in N_i are violated;
- (2) How nurse i perceives these costs in the light of nurse i 's schedule preferences;
- (3) How this cost is weighed in the light of nurse i 's schedule history.

For example, for (1) the pattern

1 1 1 1 1 0 0 1 1 1 0 0 1 1

may incur a cost for a nurse whose minimum desired work stretch is 4 days.

This is a cost inherent in the pattern. Considering (2) it is next asked how nurse i perceives violations of the minimum desired stretch constraint, i.e., how severely are violations of this nonbinding constraint viewed vis-a-vis others in N_i . Finally (3) gives some indication of how to weigh this revised schedule pattern cost in the light of the schedules nurse i has received in the past. Intuitively, if nurse i has been receiving poor schedules, the cost of a given schedule should be relatively higher than the costs for schedule of other nurses in order to cause a good schedule to be accepted when the solution algorithm is applied and vice versa. Thus

$g_{in}(x^i)$ = the cost of violating nonbinding constraint $n \in N_i$ of schedule x^i

x_{in} = the "weight" nurse i gives a violation of nonbinding constraint $n \in N_i$ which is called the aversion coefficient.

A_i = the aversion index of nurse i ; i.e., a measure of how good or bad nurse i 's schedule have been historically vis-a-vis nurse i 's preferences.

Then the total schedule pattern cost to nurse i for a schedule pattern x^i is :

$$A_i \sum_{n \in N_i} \alpha_{in} g_{in}(x^i),$$

and the sum of these costs for all nurses $i = 1, \dots, I$ is the total schedule pattern cost.

3.6. Problem Formulation

Let $\lambda \in (0,1)$ be a parameter that staffing level and schedule pattern costs. It is chosen such that the weighted staffing and schedule pattern costs are of approximately equal magnitude. Experience has shown a trial-and-error procedure to be effective in arriving at satisfactory values of λ .

Given λ , the problem is to find x^1, x^2, \dots, x^I which minimize:

$$\lambda \sum_{k=1}^{14} f_k \sum_{i=1}^I x_k^i + \sum_{k=1}^{14} \sum_{j \in J} h_{jk} \sum_{i \in B_j} x_k^i$$

subject to $x^i \in \pi_i, i=1, \dots, I$.

The solution procedure used is a near-optimal algorithm. It starts with an initial configuration of nurse schedules, one for each nurse. Fixing the schedules of all nurses but one, say nurse i , it searches π_i . The lowest present cost and best schedule configuration are updated if, when searching π_i , a schedule is found which results in a lower schedule configuration cost than the lowest cost to date. When all the schedules in π_i have been tested, either 1) a lower cost configuration has been found, or 2) no lower cost configuration has been found. The process cycles among the I nurses and terminates when no lower cost configuration has been found in I consecutive tests.

Each set π_i will always contain at least one feasible schedule, due to the manner in which the feasibility sets are constructed. To arrive at an initial solution one may select one schedule from each π_i in an appropriate manner

(e.g., select the schedule with the lowest dissatisfaction cost).

If the feasibility region is viewed as the cartesian product of the feasibility regions $\pi_1, \pi_2, \dots, \pi_I$, the algorithm is simply a cyclic coordinate descent algorithm along the coordinate directions π_i . Each π_i contains all feasible schedules for nurse i . When 4 days are given off every 14 day period, π_i contains at most $\binom{14}{4} = 1001$ schedules. This number is reduced considerably when previous schedules, special request, and other feasibility set constraints are considered. The convergence of the algorithm is assured since the cartesian product contains a finite number of points, namely, $\prod_{i=1}^I \pi_i$, where π_i is the number of schedule in the set π_i .

The following steps describe the algorithm in detail:

1. Determine the set of feasible schedules for each employee's π_i .
2. Calculate the schedule pattern costs for each schedule $x^i \in \pi_i$, for $i=1, \dots, I$.
3. Choose an initial schedule mix (i.e., a schedule for each nurse $i = 1, \dots, I$) and let BEST = its cost (e.g., choose the lowest cost schedule from π_i).
4. Let $i = 1, K = \pi_i, k = 1$ and CYCLE = 0.
5. Try the k th candidate schedule, x^{ik} , in the schedule mix by temporarily removing the present schedule for nurse i from the current schedule mix and inserting schedule x^{ik} . Let TEST = the cost of this new schedule mix.
6. If TEST < BEST go to Step 8.
7. Let $k = k + 1$. If $k = K + 1$ go to Step 9. Otherwise go to Step 5.
8. Let CYCLE = 0 and BEST = TEST. Insert x^{ik} in place of the current schedule for nurse i in the schedule mix. The Schedule mix now contains the "best schedules found so far". Go to Step 7.

9. If CYCLE = I stop. Otherwise let $i=i+1$ (if $i > 1$, let $i=1$) and let $K = \pi_i$, $k = 1$, and $CYCLE = CYCLE + 1$. Go to Step 5.

The use of this algorithm has resulted in less variation between actual and desired staffing levels, in more weekends off, in less long stretches, in more split days off and in higher personnel satisfaction and lower costs than the previous manual or semi-automated systems. However, the data needs are great and require accurate maintenance. Consequently, in order to implement such a sophisticated system the hospital must have access to systems programmers and a comprehensive mini-or full-sized computer.

As in any implementation which involves the preferences of hundreds (possibly thousands) of individuals, it is important to integrate them into the implementation process. Once the hospital's top administrators have decided to install the computerized algorithm, the implementation process begins. It proceeds through a series of steps over several months. The initial step is to meet with the Director of Nursing to explain the system, gather data on hospital scheduling policies such as the number of weekends off--on maximum and minimum stretches, beginning day and length of pay periods, schedule horizon (usually four or six weeks), rotation, use of part-time and/or float personnel, etc.

Next there is a group meeting with the head nurses to explain the operations of the system, what it can and cannot do for them, the types of reports and schedules they will receive, the time savings to them, the problems which it eliminates for them, and the need for timely data on special request. Emphasis is placed on the importance of cooperation on both sides and that the computerized schedules do not take away any of the authority of the head nurse in approving special requests of changing the schedules to meet unanticipated needs. The computerized algorithm is a tool which removes some onerous tasks so that they may have more time for more important tasks related to health care delivery.

Following the group meeting, individual meetings are scheduled with each head nurse. The purpose of the individual meetings are to answer any system questions and, more importantly, to gather data needed by the algorithm. The data comprise such items as who are the charge nurses, what groups and subgroups must be scheduled together, minimum and desired group and subgroup staffing levels, what specific scheduling problems are on the unit such as parallel people, part-time restrictions, rotation restrictions, fixed patterns, team vs. primary care groups, etc. Another important purpose of this meeting is to explain the limitations of the scheduler. For example, any group on a unit may specify which two days, Friday-Saturday or Saturday-Sunday, constitute a weekend; however, all of the nurses in that group must use the same definition for their weekend.

The next step is an orientation meeting with groups of nurses on the units. The main purpose here is to remove the fear of impersonalization by the computer and emphasize that the head nurse still controls the schedules and that the computer gives them fairer and more individualized schedules that meet their particular requests and preferences.

After the general orientation meetings each individual is interviewed (with more time devoted to nurses' aides, orderlies, and medical technicians) to obtain rankings of her (or his) preferences for weekends, stretches, split days off, etc. and to explain to her the interactions of such preferences on her schedules. They are also informed that all schedule changes or requests must be approved by the head nurse just as in the past.

Following each of the interviews, data are prepared and stored in the Master File of the algorithm. After all of the above data have been stored, trial schedules are run to adjust the various hospital and individual parameters. These trial schedules are reviewed by the respective head nurses to catch any items missed in prior interviews.

The nurse scheduling is now in operating condition and periodic schedules are produced. Even in this production phase, however, the schedules are reviewed every time and adjustments are made for new hires, terminations, changes in workload requirements and/or nurse preferences, etc. On a continuing basis it usually requires the full-time work of one trained high school graduate, who works well with people and enjoys the challenge of producing the best schedule for each head nurse, to operate the algorithm for a 40-unit hospital with 900 full and part-time nursing personnel. If the hospital is one-half this size, then only one-half the work is needed since the effort in running and maintaining the algorithm is essentially linear with respect to the number of units and people being scheduled.

The savings in head nurse time alone has been pointed out in other studies but at a minimum it is one day per month per head nurse and usually two to four days per month. Of course, head nurse's time is not the only advantage of the algorithm. Other advantages are: fairer schedules which meet nurse preferences, more even staffing of units and a flexibility which allows the nursing administration to examine the effects of changes in policies prior to the implementation of such changes.

Although the above model has been implemented in several settings it suffers from the extensive work needed to maintain a large data base, from the need for a large computer (although a mini would do) and from the lack of integration with other systems such as: corrective allocations, payroll, admissions, and other hospital activities. As with the corrective allocation level, little has been done to continuously evaluate and adapt the scheduling systems themselves, and quantitative benefit-cost analysis of alternative design and operating procedures is even more exceptional.

Manpower Planning and Performance Monitoring

In contrast to the allocation and scheduling levels, for which many models and systems for decision support have been proposed and some adopted, few quantitative models have been developed to assist planning of task substitution, hiring, training, transfer, discharge, and other personnel allocation decisions. Instead, most studies concerned with this level of nurse staffing have stressed the importance of variable budgeting, that is, procedures for evaluation of outcomes that correct for variations in uncontrollable factors, such as workload, wages and procedural changes.

For example, the purpose of Kaplan's study [35] was to perform monthly evaluations of efficiency and scheduling by comparing actual number of nursing hours with a variable standard expressed as a linear function of the actual number of patient days. The coefficients in the linear function were estimated from historical data.

Olson [49] describes an operating system for bi-weekly monitoring of budget performance. The number of hours worked in each department is compared with standard hours, calculated from a linear function with or without a fixed component. The coefficients are obtained from CASH [11] standards or from historical data. The hours worked to standard hours ratio is plotted together with occupancy rate for each department in order to estimate flexibility in staffing performance and changes in personnel efficiency or utilization.

Davis and Cowie [18] stress measurement and evaluation of both cost and quality by means of variable budgeting and effectiveness indices, but no specific technique is described. Even traditional hospital budgeting and accounting handbooks discuss the tradeoff between cost and quality, and recommend flexible budgets for nursing services (e.g., Hay, 26, pp. 276-286).

The American Hospital Association has a central computer processing service called Hospital Administration Services (HAS). Direct expenses, certain resources measures such as manhours, and some output measures such as patient days, meals, and pounds of laundry are reported monthly to HAS by each participating hospital. In return, each hospital receives statistics comparing its current performance with its historic performance and with the distribution of performance of other hospitals of the same size, geographical location, or function. Very high relative ranges in some performance indicators have been reported (Griffith,[25] p. 299), probably due to great variations in both variable definitions and actual performance. Furthermore, very little information to support nurse staffing can be found in the HAS reports and the data is about five weeks old, which makes it too late to take useful corrective actions. Hospitals must therefore rely on their own information system for staffing monitoring.

A promising budgeting and control research project is the Hospital Management Monitoring System Project (Griffith,[25] pp. 300-302). Costs, manhours, and output measures (patient days for nursing, number of exams for lab, etc.) are accumulated for each responsibility center, a first line supervisor post. The supervisors are then judged by their ability to meet manhours to output standards that have been pre-established through negotiations between the administration and the supervisors. The standards, together with seasonally adjusted output volumes and expected wage rates, are also the basis for the annual budgeting of monthly expenditures. Furthermore, the volume forecast for each responsibility center is updated monthly using regression, indices, and exponential smoothing. Those forecasts, together with the manhours to output standards, help the supervisors to foresee manpower requirements and then use part-time , overtime, transfer, vacation, and attrition of personnel to adjust the work force. Savings of \$150,000 per year were reported due to the introduction

of this variable budgeting and control procedure in the demonstration hospital.

The MEDICUS system (Jelinek et al., 34 and 33) described above also contains a management reporting system. Three categories of data are reported: (1) nursing performance (cost, manpower, and activity levels), (2) nursing personnel status (utilization, over-time, turnover, absenteeism, and personnel satisfaction), and (3) quality level measurements (questions or measurable "process" attributes are randomly selected from a large "question master file" and used to generate questionnaires that are filled out and then used to calculate quality indices).

Abernathy et al. [3] and Wandel [64] divided the staffing process into three decision levels (a) policy decisions, including the operating procedures for nursing units and for the staffing process itself; (b) permanent staff allocation, including hiring, discharge, training and reallocation; and (c) short-term scheduling and allocation of available staff within the constraints determined by the two previous levels. These three levels are used as decomposition stages in the development of a probabilistic programming model of the staffing process. Solution procedures are developed and demonstrated with a hypothetical example application. The example also illustrates the type of information that can be generated by the model and the utility of this information (a) for policy evaluation and decision; (b) as an informational basis for actual staff planning and control; (c) for coordination of staff allocation with patient admission, treatment scheduling, vacation planning and load forecasting; (d) to suggest standards for budgeting purposes; and (e) to monitor nursing performance and staffing effectiveness. However, the adaptability, acceptability, and significance of the model has yet to be demonstrated.

Recommendations and Conclusions

In this article, we have conceptualized the nurse staffing process as a hierarchy of three tactical decision levels. The great bulk of research in nurse staffing has concentrated on the shift scheduling and corrective allocation levels, and on activity analysis and forecasting procedures to support staffing decisions. Exploration of the potential benefits of improved manpower planning procedures has only recently begun. This is an important research area which we believe deserves greater attention in the future.

Another area which should be investigated is the interdependence of the decision levels discussed in this paper. Each decision level is constrained by the resources that have been made available at higher levels, and by the degree of flexibility for later correction at lower levels.

Consider the manpower planning decision level. The week-to-week and day-to-day shift scheduling decisions and shift-by-shift corrective allocations obviously depend upon the permanent staffing levels established through the manpower planning process. On the other hand, the range and flexibility of shift scheduling and corrective allocation options available to management should be known so that their performance can be anticipated and included in the staffing decisions made during the manpower planning process. That is, the overall staff requirements cannot be fully assessed until it has been decided how fluctuations in load are to be accommodated. If overtime, for example, provides the only flexibility in short-term scheduling, the full-time staff requirements might be substantially higher than if part-time help and a float pool were also available.

The interdependencies of the three levels should be recognized to bring about systematic nurse staffing improvements. One general conclusion from our review of the studies surveyed in this article is that greater attention should be given to studying these interdependencies of the decision levels, rather than studying each level in isolation.

The need for coordination between nurse staffing and other hospital activities, such as task assignment, scheduling of admission and operating rooms, budgeting, payroll, and facility planning, should also be studied. Many studies have examined alternative approaches for balancing patient numbers and mix, allocating patients to nursing units, and scheduling patient care activities, but the impact of these admission and treatment scheduling procedures upon nursing utilization and upon the need for flexibility in staffing procedures has not yet been fully explored.

Very few studies have attempted to evaluate systematically alternative designs and methods of implementing nurse staffing systems and operating policies. We think it is crucial that the impact upon cost, quality, and satisfaction of these strategic decisions be carefully investigated. There are many conflicting objectives, legal constraints, and professional value systems in hospitals that place little emphasis on efficiency. More research and demonstration projects have to be undertaken before the adaptability, acceptability and significance of most of the reviewed procedures can be fully understood.

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