Perishable Inventory Theory and Its Application to Blood Bank Management*

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Abstract

The existence of optimal ordering policies for the perishable product has been demonstrated by Fries and Nahmias and Pierskalla. Unfortunately, computation of the optimal order function is difficult due to the large state spaces needed to describe the process. Consequently, the existing theory has limited utility because it is difficult to understand and virtually impossible to implement, especially in the context of a blood bank inventory. This paper examines a class of simple decision rule ordering policies which eliminate the objections noted above. Application of this theory and further results on issuing and inventory return policies is then made to the blood bank situation. Stability of the optimal policies under certain parametric changes is examined and a decision rule for the computation of the optimal order quantity is estimated. In addition, information and data processing requirements are discussed.

I. Introduction

The analysis of inventory systems with a perishable product has been underway for the past few years. Over a longer period of time, blood bank administrators and management scientists have been looking at the management of a blood inventory. The legal requirement that whole blood be transfused within 21 days of being drawn, the mechanics of issuing blood to meet demands and the organizational structure of regional blood banks lead to a complex inventory system. This paper will consider the management of a regional blood bank in terms of an inventory model and will analyze various components of optimal operating policy.

The inventory system associated with a blood bank is characterized by a perishable product with a fixed lifetime of 21 days. The system is subject to both stochastic supply and demand. Blood supply can be of any age between zero and 21 days where fresh units are usually donated and older units enter the system through shipments from other blood banks. Procurement (supply generation) policy can be defined in terms of call-up sequences from lists of previous donors, investment in donor recruitment campaigns to attract new donors and orders from other blood banks. Ordering policy is concerned with the amount to order under the assumption that the order will be filled.[1] A further area of policy control which may be identified relates to issuing policy in terms of the age sequence of units issued to meet demands. The most common examples are FIFO (first-in-first-out) and LIFO (last-in-first-out) which are equivalent to issuing the oldest and youngest available units, respectively, when all supply is assumed to be fresh.

[1] The results of this paper will be applicable to the single hospital blood bank and those regional blood banks where the central bank must meet all orders placed by the local bank.

The blood inventory is structured into assigned and unassigned subinventories due to the phenomenon of cross-matching. Demands for blood are associated with particular patients. Units issued to meet these demands must be cross-matched (assigned) to the patients before actual transfusion takes place. This procedure involves testing a sample of the issued unit with a sample of the patient's blood and it insures that transfused units will not have an adverse physiological reaction with the patient's blood. Accordingly, uncross-matched units make up the unassigned inventory and cross-matched units make up the assigned inventory.

In response to the costs associated with delays due to cross-matching blood in the face of an emergency, many doctors overorder when requesting blood. Consequently there are a large number of returns of stock from the assigned to the unassigned inventory. The amount returned is random since the blood bank has no *a priori* information on how many issued units will ultimately be returned. The delay, in days, between initial issue and subsequent return for nontransfused units can be viewed as a further control parameter, λ, and is referred to as the cross-match (return) policy. We note that supply is received and that outdated (spoiled) units are retired from the system at the unassigned inventory.

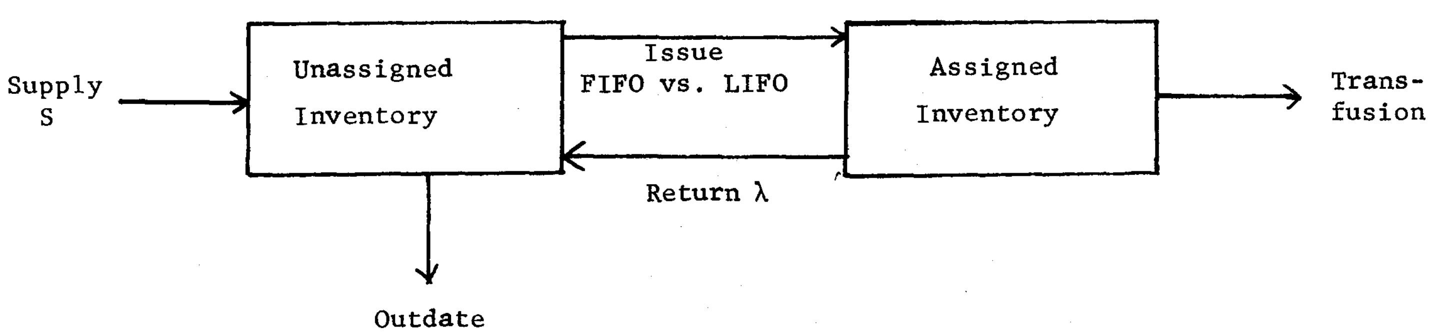

Figure 1

The two stage structure of the blood inventory (see Figure 1) leads to a breakdown of demand into two categories. Permanent demands are those demands which lead to transfusion for the issued unit. Temporary demands lead to return of the issued unit to the unassigned inventory after a delay of λ days. Finally, in general, units are stored at all locations of the regional bank and control of the inventory is usually shared between the central bank and the local hospital banks.

Analysis of a product which undergoes deterioration throughout the planning horizon and which has a maximum useable lifetime began with the work of Van Zyl [15] who considered the two period lifetime case. Fries [3] and Nahmias and Pierskalla [8] and Nahmias [7] extended this work and were able to solve for the optimal order policy for an inventory system with constant utility of the product throughout its useable lifetime, no return of issued stock, fresh supply, arbitrary lifetime of m periods, FIFO issuing and an independent (identically) distributed demand process. The cost structure included shortage, holding, ordering and outdate (wastage) costs. Order policies to minimize discounted total expected costs were examined and the resulting optimal order function turned out to be nonstationary and of a noncritical number form in that it could not be expressed in terms of a single order-up-to quantity. Indeed, the amount ordered in any given period was a function of the age distribution of the inventory on hand at the beginning of that period. Moreover, the optimal policy was to be computed by solving a dynamic program with state variable in R^m.

To illustrate, consider the following:

Let $\underset{\sim}{x} = (x_{m-1}, x_2, \ldots, x_1)$ be the vector of initial inventory at the beginning of the review period and before ordering.

x_i = amount of stock with i periods of useable lifetime remaining.

Let $B_n(\underset{\sim}{x}, y)$ = expected n period discounted total cost function, given an initial inventory of $\underset{\sim}{x}$ and an order of y in period n.

We seek an order function $y_n(x)$ such that

$$C_n(\underset{\sim}{x}) = \inf_{y \geqq 0} \{B_n(\underset{\sim}{x}, y)\} = B_n\left(\underset{\sim}{x}, y_n(\underset{\sim}{x})\right).$$

From the results described in Nahmias and Pierskalla [8], we note the following:

$\forall \ \underset{\sim}{x} \in R^{m-1}$:

1. $B_n(\underset{\sim}{x}, y)$ is convex in y.

2. $$\lim_{y \to 0^+} \frac{\partial B_n(\underset{\sim}{x}, y)}{\partial y} < 0 .$$

3. $$\lim_{y \to \infty} \frac{\partial B_n(\underset{\sim}{x}, y)}{\partial y} > 0 .$$

4. $\exists$ unique $y_n(\underset{\sim}{x})$ solving

$$\left. \frac{\partial B_n(\underset{\sim}{x}, y)}{\partial y} \right|_{y = y_n(\underset{\sim}{x})} = 0$$

and $y_n(\underset{\sim}{x}) \in (0, \infty)$.

5. $$-1 \leq \frac{\partial y_n(\underset{\sim}{x})}{\partial x_1} \leq \ldots \leq \frac{\partial y_n(\underset{\sim}{x})}{\partial x_{m-1}} < 0 .$$

The inequalities in (5) order the contribution of an incremental unit of initial inventory to the amount ordered in period n.

It should be pointed out that a nonstationary, noncritical number order policy would be difficult to implement in a blood bank situation. Recently attempts have been made to examine the optimal stationary critical number (S type) order policy for the simplified system we have been discussing. Results for the m = 2, two period lifetime case can be found in Cohen [2] and Nahmias [9].

In [2], Cohen demonstrated that the limiting expected average cost function (shortages plus outdates) for the m = 2 case can be expressed in terms of the order quantity S and the demand distribution F as follows:

$$C(S) \equiv \lim_{n \to \infty} C_n(S)$$

$$= s \int_S^{\infty} (u - S)dF(u) \quad + \quad \theta \int_0^S \frac{F(u)F(S-u) - F^2(u)F(S-u)du}{1 - F(u)F(S-u)}$$

where

$C_n(S)$ = expected average cost for n periods when an order-up-to quantity of S is in force

s = unit shortage cost

θ = unit outdate cost.

Moreover, C(S) is convex and

i) $\lim_{S \to 0^+} \frac{\partial C(S)}{\partial S} < 0$ if $F(0) = 0$ or $s > \theta F^2(0) \Big/ \left(1 - F^2(0)\right)$

ii) $\lim_{S \to \infty} \frac{\partial C(S)}{\partial S} \geqq 0$.

Thus, C(S) has at least one minimum point, say S^*, which solves:

$$\left. \frac{\partial C(S)}{\partial S} \right|_{S = S^*} = 0 .$$

The extension of the analysis to the case of an arbitrary lifetime of m periods is clearly needed before the theory can be applied to the blood inventory model.

Recently, attempts have been made to solve for bounds on the expected cost function associated with the S type policy. Nahmias [10] has results for a myopic model approximating the perishable problem and Cohen [2] has generated the following bounds for the expected number of outdates for the general m period lifetime-critical number order policy problem.

Let Z_n = random variable denoting the number of outdates in period n. Then

$$\int_0^S F^{*m}(S) \leq \sum_{k=1}^{m} E[Z_{n+k}] \leq S \quad , \qquad n = 1, 2, \ldots$$

where F^{*m} is the m-fold convolution of the demand distribution. A discussion of the m period lifetime problem is presented in [2].

All of the existing inventory theory (that we are aware of) for a perishable product has thus been primarily concentrated on ordering policy for the simplified case where all issued units are consumed, all supply is fresh and FIFO issuing is in force. This situation clearly does not coincide to that of the previously described regional blood inventory. Consequently, there is some question on the relevance of the policy rules developed to date. Moreover, no attention has been given to return policy and the only useful result for issuing policy is the proof that FIFO minimizes cumulated outdates and shortages in Pierskalla and Roach [11]. This latter result is for a perishable inventory model with no returns and a nonincreasing utility for the product over its lifetime.

Analytic models which capture the complexities of the inventory system and explicitly consider interactions of the various policy components are evidently needed. Such theoretical work is ongoing but the models are very complex and it will take time before they are well understood. An alternative approach, which will be considered in this paper, is that of simulation. The simulation results derived will be associated with a more realistic blood inventory model and will also provide insights into policy interactions which are useful in their own right and are helpful in making simplifying assumptions for further theoretical work. Specifically, we seek a management policy which will be relevant to the blood bank situation and will be both easy to compute and implement.

A number of simulation models for blood bank systems have been constructed (see [5 , 6 , 14]). These models have been primarily concerned with order policy and do not consider interactions of the previously outlined policy components. The empirical results of this paper are based on the simulation models of Cohen and Pierskalla [1] and Pinson [13] which were both specifically designed to test for such policy interaction.

The next section will briefly describe the basic simulation model and its data requirements.

II. Simulation Model and Data Requirements

The purpose of the simulation model is to evaluate the set of alternative blood management policies with respect to shortage and outdate costs. Specifically, we consider the interaction among the three major policies: issuing, ordering and cross-match returns.

The class of order functions considered are given by:

$$y = S - \sum_{j=1}^{21} a_j x_j$$

where

y = amount ordered

S = order-up-to quantity

x_j = amount of inventory of age j

a_j = prespecified order weight

$= 0.5 + (21 - j) * 0.05$.

The weights a_j are a simple linear approximation which satisfy the inequalities listed in (5) previously. Pinson [13] showed that these weights are reasonable but that the optimal cross-match, ordering and issuing policies were not overly sensitive to changes in them. Indeed, weights of $a_j = 1$ are also acceptable. Issuing policy was restricted to FIFO and LIFO and the (cross-match) return policy was specified by a nonnegative integer λ.

Data requirements for the model included the distribution for total daily demand, the breakdown between temporary and permanent demand, the age distribution of ordered units and unit shortage and outdate costs. In much of the analysis, empirical distributions derived from the observed pattern of shipments in local and regional blood banks was used. In addition, for comparative purposes, specified theoretical distributions were used for demand and age distributions.

A simultaneous variation of the three policy components for fixed sets of input data was carried out. The results generated by the simulation model will be briefly summarized in the next section.

III. Results

1. Issuing and Return Policies

LIFO and FIFO issuing policies were considered in conjunction with values of the cross-match release parameter λ of 0 through 7. λ = 0 corresponds to an inventory system where all issued units are consumed and λ = 7 corresponds to a system where untransfused cross-matched units remain in the assigned inventory for a period of one week. In all, 16 issuing-cross-match policy combinations were considered.

Table 1 gives results averaged over a number of runs and Figure 1 is a graph of cumulated outdates vs. λ for both FIFO and LIFO. The following observations can be made.

λ	0	1	2	3	4	5	6	7
FIFO Issuing								
Transfused	2272.	2272.	2272.	2272.	2272.	2272.	2272.	2242.4
Outdated	0.	16.3	58.7	117.0	173.1	202.1	272.1	330.8
Unassigned Inventory	503.	445.7	364.3	272	161.9	121.9	35.9	0
Assigned Inventory	0	41.0	80.0	114.0	168.0	179.0	195	201.8
Total	2775.	2775.	2775.	2775.	2775.	2775.	2775.	2775.
Shortage	0	0	0	0	0	0	0	72.5
LIFO Issuing								
Transfused	2272.	2272.	2267.8	2266.2	2257.2	2254.0	2241.0	2234.4
Outdated	477.	449.3	428.6	403.6	372.2	366.3	357.5	338.8
Unassigned Inventory	26	12.7	0	0	0	0	0	0
Assigned Inventory	0	41	78.6	105.2	145.6	154.7	176.5	201.8
Total	2775.	2775.	2775.	2775.	2775.	2775.	2775.	2775.
Shortage	0	0	10.9	14.8	47.6	46.3	74.6	90.6

Table 1: BLOOD INVENTORY POLICY EVALUATION RUNS
ISSUING AND CROSS-MATCH POLICIES

1. When $\lambda = 0$, as predicted by the theory of Pierskalla and Roach [11], FIFO is optimal in terms of minimizing the cumulated outdates and shortages.
2. As λ increases under FIFO issuing, system performance deteriorates.
3. As λ increases under LIFO issuing, outdates decrease and shortages increase.
4. For reasonable λ, FIFO dominates LIFO.
5. Although not shown, it is possible to generate examples where outdates under FIFO exceed those under LIFO for λ sufficiently large.
6. There is great sensitivity to changes in λ under both issuing policies.

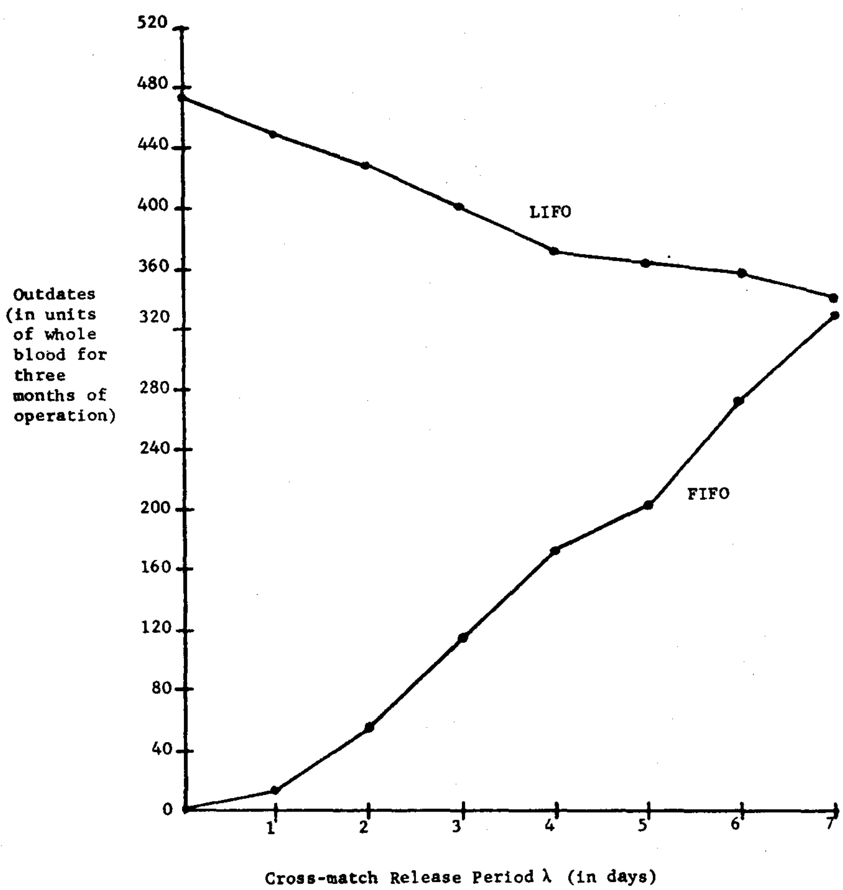

Figure 1: OUTDATES VS. CROSS-MATCH RELEASE PERIOD FOR FIFO AND LIFO ISSUING

2. Centralized vs. Decentralized Cross-Match Control

For the purposes of this study we define centralized control to mean that the cross-match period λ is a constant determined by the central bank administrator. Decentralization is introduced into the model by treating λ as a random variable. Specifically, λ was chosen to have a positive truncated normal distribution. The mean of the distribution represents the broad policy guideline set by the central bank as to the maximum time a unit should remain assigned, and the variance reflects the degree of control exerted by the central bank in enforcing this mean time. Once λ is reached for a particular cross-matched unit, the unit then becomes available to the central bank for redistribution to other hospitals if needed.

The simulation model was thus run with a sample of cross-match return times drawn from the truncated normal distribution with the specified mean and variance. The results are given in Table 2 and Figure 2.

FIFO Issuing							
Variance of λ	0	1	2	3	4	6	8
Transfused	2272	2272	2272	2272	2272	2243	2173
Outdated	165	183	195	223	254	310	350
Unassigned Inventory	170	154	136	91	37	0	0
Assigned Inventory	168	166	172	189	212	222	252
Total	2775	2775	2775	2775	2775	2775	2775
Shortage	0	0	0	0	0	71	211
LIFO Issuing							
Transfused	2257	2255	2258	2245	2230	2198	2153
Outdated	378	378	367	368	366	369	378
Unassigned Inventory	0	0	0	0	0	0	0
Assigned Inventory	140	142	150	162	179	208	244
Total	2775	2775	2775	2775	2775	2775	2775
Shortage	43	49	39	68	100	157	255

Table 2: BLOOD INVENTORY POLICY EVALUATION RUNS FOR CROSS-MATCH RELEASE PARAMETER λ DRAWN FROM A TRUNCATED NORMAL DISTRIBUTION WITH MEAN OF 4 DAYS

The costs of decentralization are clear from these numbers. Under FIFO the number of outdates and shortages increase with increasing variance. Under LIFO shortages and outdates do not change in a monotonic fashion. Of importance here is the observation that the advantages of issuing with FIFO can be lost if the variance for λ is allowed to become excessive. Thus, FIFO and a tighter control on the mean and variance of λ lead to significant reductions in outdating and shortages.

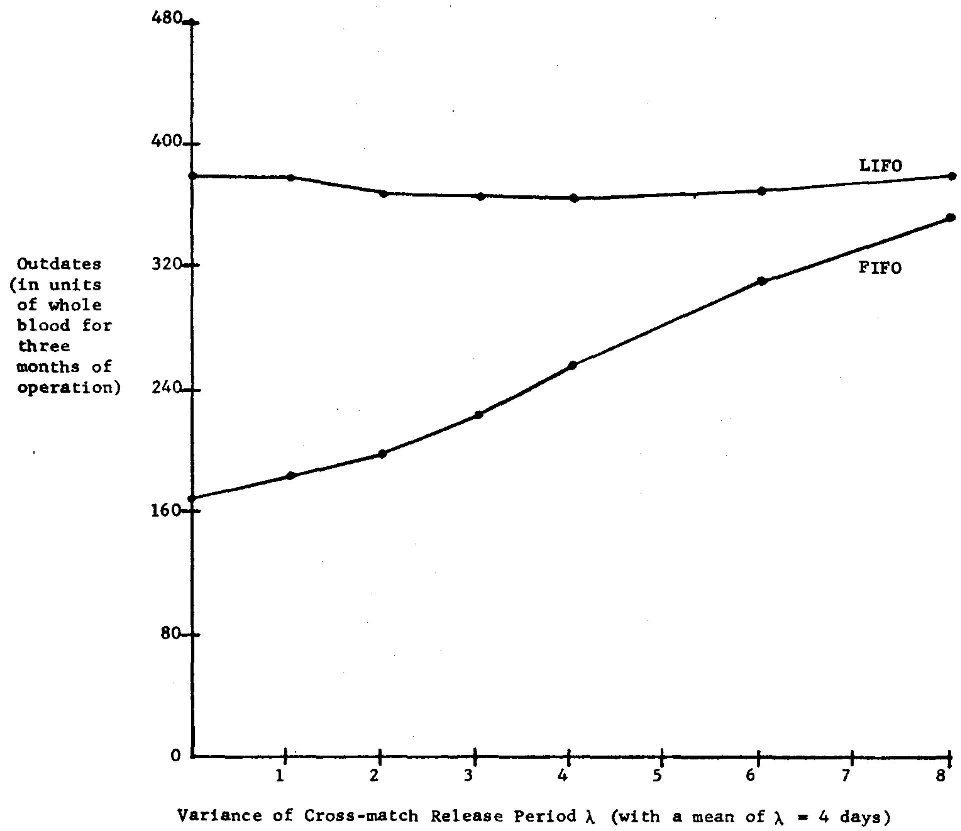

Figure 2: OUTDATES VS. CROSS-MATCH RELEASE PERIOD VARIANCE FOR FIFO AND LIFO ISSUING

3. Optimal Order Quantity

Using the linear ordering function described previously, the variation between S, the order-up-to quantity and average costs (outdates + shortages) was investigated. All runs were made with FIFO issuing and per unit shortage and outdate costs of \$55.00 and \$25.00, respectively. These particular costs were chosen because many studies have estimated the cost of processing one unit of whole blood at about \$25.00 and a unit of emergency blood at \$55.00. The \$55.00 cost could be an emergency drawing and/or shipping cost or the cost of a frozen unit of packed red cells. In the study by Pinson [13], other costs were considered and it was shown that the magnitude and shape of the cost curves and the magnitude of the optimal order quantity S^* were not greatly influenced by reasonable changes in costs. A further verification of this result will be given later on in this paper. The resulting cost curves for λ = 2, 5, 7 are illustrated in Figure 3.

The cost minimizing order quantity, S^*, was computed by using a Fibonacci search technique in conjunction with the simulation model. A simulation run was made with each chosen value for S (all other policy factors and data inputs held constant). By varying S, the cost curve of Figure 3 is built up, point by point. This procedure assumes unimodularity of the cost function; an analytical question under consideration. (As mentioned in Section I, the cost function is convex when m = 2.) The results of the Fibonacci search are in Table 3. We can make the following observations.

Cross-Match Release Period λ	2	5	7
S^*, the Optimal Order-Up-To Quantity	30	32	29
Cost/day at S^*	\$7.25	\$36.55	\$54.55
Outdates at S^*	29	144	216
Shortage at S^*	0	1	1
Transfused at S^*	493	493	493
Average Unassigned Inv. at S^* (before ordering)	23.99	24.87	21.58
Average Assigned Inv. at S^*	18.77	39.42	53.29

Table 3: SYSTEM PERFORMANCE AT OPTIMAL ORDER POINT, S^*

(a) Costs are far more sensitive to cross-match policy λ than they are to the order policy (S). (This is seen by the fact that the mean cost/day increases from $7.25 to $36.55 to $54.55 as λ goes from 2 to 5 to 7 days.)

(b) The cost minimizing <u>value</u>, S^*, is fairly insensitive to the value of λ even though the overall cost <u>level</u> increases significantly with λ. (This is seen by the relatively flat (near S^*) curves of Figure 3.)

(c) The simulated running average cost curve is approximately of the desired convex shape.

(d) Loosely speaking, the optimal S^* is chosen to be the minimum S such that there are no shortages.

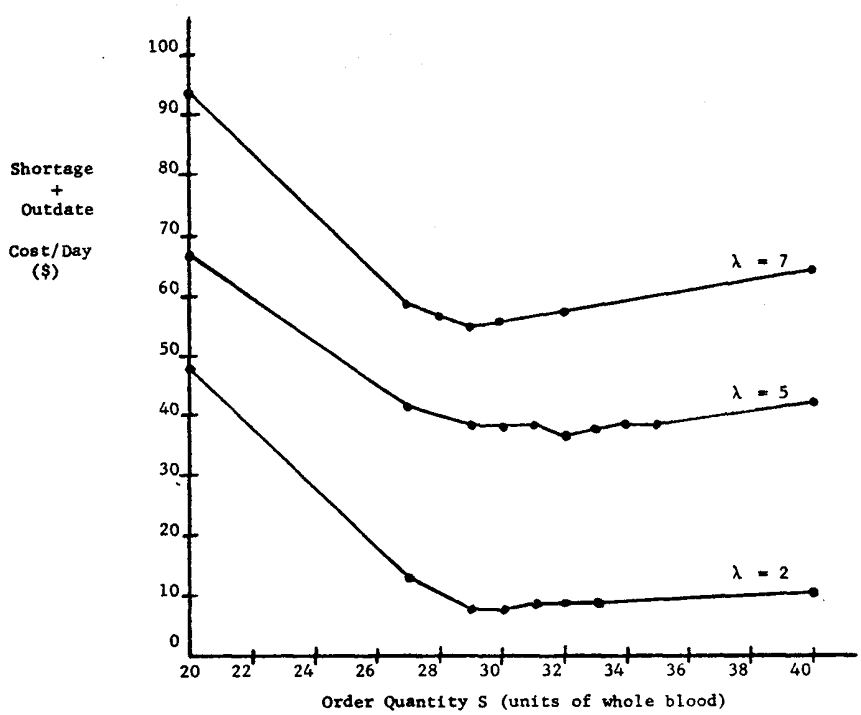

Figure 3: AVERAGE SHORTAGE/OUTDATE COSTS VS. ORDER QUANTITY FOR FIFO ISSUING, $55 UNIT SHORTAGE COST, $25 UNIT OUTDATE COST AND A 100 DAY SIMULATION PERIOD

The implications of the first two observations are as follows:

(i) The effect on shortages and outdates of the ordering policy is minimal for S's in the neighborhood of S^* when compared to either the issuing or cross-match policies. The <u>insensitivity</u> of the S's in the neighborhood of S^* (the optimal S) is important because a central bank cannot always achieve S^* each day. Indeed, large drawings of blood through donor plans often disrupts the central blood bank administrator's policy of achieving S^* on a daily basis and he must seek an average S^* over time, and

(ii) The optimal order quantity is essentially insensitive to the value of λ. This means that the blood bank administrator can set his S^* and then concentrate his inventory management control on reducing λ knowing full well that his S^* will not change significantly.

Thus, from this analysis we may conclude that an optimal strategy for a blood bank would involve the following components.

1. Use FIFO issuing.
2. Work to keep λ as small as possible.
3. Work to keep λ under centralized control.
4. Use the optimal order-up-to quantity, S^*, when computing the daily order.

The results of further investigation indicated a remarkable stability on the part of S^* to changes in age of supply distributions, shortage costs, order weights $\{a_j\}$ and choices of λ. Thus, it would seem that in the absence of an adequate analytic model, a reasonable approach would be to seek out an empirical procedure for computing S^* as a function of those parameters which describe the demand, supply and structure of the particular blood bank under consideration. This approach will be considered in the next section.

IV. Decision Function Analysis

There are, of course, many exogenous and policy control factors associated with any blood bank system. The exogenous factors identified for the purposes of this discussion include the parameters specifying the total demand distribution, the parameters specifying the age of units supplied distribution, the per unit shortage and outdate costs and the fraction of total demand which is transfused. As previously noted, control factors for the system include the issuing policy, the cross-match release period (λ) and the order-up-to quantity (S). The simulation model and Fibonacci search procedure relate S^*, the cost-minimizing order quantity, to these exogenous and control factors by providing an observation for S^* which is also related to demand and age of supply distribution samples and the model structure.

The functional relationship between S^* and the various control and exogenous factors will be useful in making policy recommendations to the blood bank manager. We shall refer to this function as the <u>decision function</u> and we note that it is the usual object of any analytic study of inventory systems. In this section, the simulation model is used to explore the decision function and we will now examine the procedure used to generate the data required for its estimation.

A statistical experiment in which we vary the assorted factors throughout their range and use the simulation model to compute the S^* value associated with each factor configuration was carried out. The results of this experiment were then used in a curve fitting analysis to identify the desired functional relationship.

The function we seek can in general be written as:

$$S^* = f(\delta_1, \delta_2, \ldots, \alpha_1, \alpha_2, \ldots, \gamma, C_s, C_0, \lambda, I)$$

where

δ_i = parameters describing the demand process;

α_i = parameters describing the age of supply process;

γ = % of total demand transfused;

C_s = unit shortage cost;

C_0 = unit outdate cost;

λ = cross-match release period;

I = issuing policy indicator.

Due to the previously noted insensitivity of S^* to changes in cross-match policy and other exogenous factors and due to the costs associated with examining all possible combinations of factor choices, attention was restricted to a subset of the explanatory factors listed above. Specifically, analysis was centered on the relationship between S^* and the parameters specifying the demand distribution. A compound Poisson (specifically Neyman Type A) distribution was postulated for the purposes of the experiment. The advantages of using this distribution are that only two parameters are needed and that each parameter has an unambiguous interpretation in the blood inventory context. Total demand in a given day is generated by the arrival of patients needing blood and the "arrival" of requests of units for each patient. If both arrival processes are Poisson, then specification of the patient/day arrival rate, δ_P, and the request/day arrival rate, δ_R, completely specify the total demand process. The identification of the demand process and other statistical analysis of data from regional blood banks is currently underway by Pierskalla, Sorum and Yen [12].

The desired decision function now has the form:

$$S^* = f(\delta_P, \delta_R).$$

The experiment to generate data to estimate f involves the following steps:

1. vary δ_P, δ_R over a reasonable range of values;
2. generate a sample of demands for each choice of parameters;
3. run the simulation model and search program with each demand sample holding all other data and policy inputs fixed;
4. compute S^* for each pair (δ_P, δ_R);
5. estimate f from the generated data;
6. check the sensitivity of f with respect to changes in other excluded controlling factors.

We note that step 6 is necessary to validate the exclusion of many of the previously noted exogenous and control variables.

Table 4 presents the results from a typical set of runs. All simulations for this table were made with FIFO issuing, $\lambda = 2$, empirical age of supply distribution and unit shortage and outdate costs of \$55.00 and \$25.00, respectively. δ_P had values of 1, 2, 4, and 6, and δ_R had values of 1, 2, and 3, respectively, and thus a total of 12 points were generated. We note that there seems to be a strong relationship between S^* and the mean demand given by $\delta_P \cdot \delta_R$. It is also apparent from Table 4 that there seems to be a monotonic relationship between average costs and changes in arrival rates δ_P and δ_R.

δ_P PATIENTS PER DAY ARRIVAL RATE	δ_R UNITS PER PATIENT ARRIVAL RATE	S^* OPTIMAL CRITICAL NO.	AVERAGE COST/DAY ($) (OUTDATES + SHORTAGES)
1	1	4	5.80
1	2	7	7.10
1	3	13	10.75
2	1	6	6.35
2	2	13	9.05
2	3	17	13.35
4	1	12	6.00
4	2	24	8.50
4	3	30	9.05
6	1	16	7.50
6	2	28	7.80
6	3	42	10.00

$$\ln S^* = \underset{(.0574)}{1.397} + \underset{(.0581)}{.754} \ln \delta_P + \underset{(.0386)}{.933} \ln \delta_R$$

$s^2 = .00833$ $\quad\quad$ $\bar{R}^2 = .98306$

TABLE 4: OPTIMAL ORDER-UP-TO QUANTITY (S^*) AND MINIMAL COST/DAY FOR RETURN PARAMETER (λ) = 2, EMPIRICAL AGE OF SUPPLY DISTRIBUTION, UNIT OUTDATE COST OF $25.00 AND UNIT SHORTAGE COST OF $55.00

After investigating both linear and polynomial functions, a Cobb-Douglas function was chosen. This function has the following form:

$$S^* = B\, \delta_P^{A_1}\, \delta_R^{A_2}$$

and hence is linear in its logarithms, i.e.,

$$\ln S^* = \ln B + A_1 \ln \delta_P + A_2 \ln \delta_R ,$$

where B, A_1, A_2 are the coefficients to be estimated. The Cobb-Douglas formulation has an intuitive appeal since it represents a generalization of a simple linear function relating S^* and mean demand $\delta_P \cdot \delta_R$.

Much has been said about the Cobb-Douglas function in the economic literature in the context of production theory. For example, if the coefficients A_1, A_2 are such that $A_1 + A_2 = 1$, then the firm is said to exhibit constant returns to scale, i.e., homogeneity of degree $A_1 + A_2$. While many of the economic insights do not carry over to our situation, it is interesting to note that positive coefficient values less than one indicate a concave function along either coordinate direction. For a blood bank this would mean that if, for example, the number of units per patient is held fixed and if the number of patients per day is increased, then a less than proportional increase in the order quantity would be optimal.

The results of the regression on the log values is to be found at the bottom of Table 4. The standard error is shown in brackets under each coefficient and the adjusted $\bar{R}^2$ and the adjusted residual variance s^2 are indicated at the bottom of the table. The fit is clearly quite good and the coefficient values are positive fractions close to one. Before turning to the sensitivity of the results to changes in other exogenous and policy factors, we note that some of the difficulties in using linear regression for this class of functions has been analyzed by Goldberger [4]. Our fit was such that the problem of a possible bias in the estimate of the B coefficient is not significant.

Table 5 gives the results of our experiment for $\lambda = 1, 2$, and 4, with all other factors held constant. The numbers indicate that there appears to be little significant response to moderate changes in λ. Further sets of runs for each value of λ were made with a triangular age of supply distribution, with all units supplied fresh and with a per unit shortage cost of $35.00. In all cases, there was almost no change in the observed values for S^* and hence we observe that the estimated decision function is fairly insensitive to these factors.

λ	B	A_1	A_2
1	4.22	.796	.861
2	4.03	.754	.933
4	3.99	.758	.944

TABLE 5: ESTIMATED DECISION FUNCTION COEFFICIENTS FOR VARIOUS VALUES OF CROSS-MATCH RETURN PARAMETER, λ.

We may conclude, therefore, that the Cobb-Douglas relationship presents a family of decision rules which seem to capture the essentials of the true optimal order policy over a fairly broad range of those parameters needed to specify a blood bank inventory system. The insensitivity of the estimated coefficients to changes in λ confirms our earlier observations of section III and is especially useful in a practical situation. By working towards reducing λ, the blood bank administrator does not necessarily need to recompute his order policy. We note also that for the case of a compound Poisson demand process data requirements are minimal since only estimates of the easily observed quantities δ_P and δ_R are needed. The decision function estimated for the appropriate value of λ can then be used to compute S^*, the optimal inventory level.

The results of this experimental investigation of the decision function are illustrative of the type of information which the simulation model can provide. It is, of course, possible to extend the experiment to include a larger set of explanatory variables and other classes of demand distributions. However, due to the relative importance of the issuing and cross-match policy components, it is clear that even approximate ordering rules can be tolerated in a practical situation. For this reason, we may treat the decision function as an acceptable rule of thumb which has the advantages of easy implementation and minimal information and data processing requirements. In conjunction with the previous conclusions on issuing and cross-matching, a management strategy for the blood bank which can effect significant savings in operating costs can then be implemented.

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