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A META-MODEL FOR A REGIONAL BLOOD BANKING SYSTEM*

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by

HARRY YEN* AND WILLIAM P. PIERSKALLA**

Abstract

A computer simulation model for a regional blood banking system was constructed. The inter-related storage, transhipment and ordering functions were built in the model. Provisions were made to allow for the return of unused demands, the disposal of outdated stock, the transhipment of inventory among local blood banks to reduce shortage and outdate, and the limited supply to the central blood bank.

Due to the numerous possible combinations of the input variables, a full factorial or even partial factorial sensitity analysis would be very expensive. For this reason a meta-model which approximates the simulation model was constructed. The independent variables of the meta-model are: the number and size of hospital blood banks in the system, the inventory levels at all locations, the probability of return and the time lapse before return. The dependent variables of the meta-model are: The shortage quantity and outdate quantity of the system. The work is in progress to test the validity of the meta-model in terms of the input and output variables. In some early analysis the predictions of the two dependent variables from the meta-model have been found to be consistent with the values from the simulation model.

One of the objectives of this continuing research is to provide a shortage-outdate input model to a full regional blood banking model which allows for organizational change, facilities location and allocation, and transportation and quality control. The work has also begun on this latter phase of the problem.

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I Introduction and Literature Review:

This paper shall investigate some problems related with the operation and design of a regional blood banking system. The operational problems of interest are: the ordering policies, the sensitivity of shortage and outdate quantities by varying some factors such as the probability of transfusion of a crossmatched unit and the return of unused units. The design problem considered is the construction of an overall system cost function which can be used for organizational changes, facilities location and allocation, and transportation and quality control. The work has also begun on this latter phase of the operation.

A simulation model is constructed to represent the operation of a regional blood banking system. But due to the numerous possible combinations of the input variables, a full factorial or even partial factorial sensitivity analysis would be very expensive. For this reason a meta-model which approximates the simulation model was constructed. The independent variables of the meta-model are: the number and size of hospital blood banks in the system, the inventory levels at all locations, the probability of return of crossmatched but not transfused units and the time lapse before return. The dependent variables of the meta-model are: the shortage quantity and outdate quantity of the system. In some early analysis the predictions of the two dependent variables from the meta-model have been found consistent with the values from the simulation model.

There is much literature in studying blood inventory management from a simulation approach. Only the more recent will be mentioned here. These referenced works describe the earlier studies on simulation and management of blood inventory systems.

The optimal inventory level is found via a Fibonacci search in Pinson (1973) for a single hospital blood bank. In her model, units not transfused are returned to the inventory after a time lapse λ periods, and there is a random donor provision to act as exogenous supply source. In Cohen and Pierskalla (1973), the impact on units outdated by varying λ are studied. In Rabinowitz (1970), three different crossmatching policies were studied: namely, to double crossmatch a unit; to crossmatch older units to patients who have a higher transfusion probability; and to crossmatch Rh negative blood to an Rh positive patient when there is an excessive inventory of that blood type. All of these policies were found to reduce outdates. In Jennings (1970) (1972), a simulation model for a regional blood banking system is constructed. The regional blood banking system is formulated by grouping a number of identical hospitals together. However, there is no central blood bank in the model. Transhipment policies to reduce either shortage or outdate are allowed in the model. The performance of these transhipment policies is evaluated by varying the number of hospitals in the system, the underlying reasoning being that the larger the system the more transhipments, and hence the more economical the system will be due to reduced shortage and outdates. However, there are transportation and information costs which increase as the number of transhipments increases. Jennings found that the marginal benefit in reducing shortage or outdate via transhipments by adding a new hospital into the system was not significant after five hospitals were already in the system. It should be noted that this regional model without a central bank will tend to exaggerate the amount transhipped in a real situation. This exaggeration occurs because a central blood bank having a buffer inventory will allocate more of its inventory to a hospital likelier to experience shortage, and allocate more of a younger inventory to a hospital likelier to experience outdate.

In the next section of this paper, an overview of the simulation is presented. In section III the results of the simulation are used in the construction of a set of empirical equations which will be representative of the behavior of the system. These equations form a "metamodel" in the sense of Blanning (1974) and Kleijnen (1975). Specifically, the two key measures of the system, the outdate and the shortage quantities, are expressed in mathematical equations which consist of only the input parameters such as demand distribution, inventory levels at all facilities, etc. The validity of these mathematical equations are demonstrated by the consistency of the two key measures from the equations and from the simulation.

In section IV further comparison between the empirical equations and the simulation are done to draw conclusions regarding the inputs on system costs from the supply policy and the transhipment policy. In addition, optimal inventory levels and the sensitivity of the system cost versus the number and size of hospitals in the system are presented through an analysis on the empirical equations. In the last section, a summary is given.

II Overview of the Model:

The centralized blood banking system considered in the simulation is structured as a wheel with a central blood bank supplying a number of lower echelon hospital blood banks. Each facility carries the common product with a maximum usable age of twenty-one days. Disposal of all units older than the maximum usable age is required. The flow chart for a central blood banking system (CBB) and the flow chart for a hospital blood bank (HBB) are illustrated in Figures 1 and 2 on the following pages respectively. Explanations for some of the boxes are given after the flow charts.

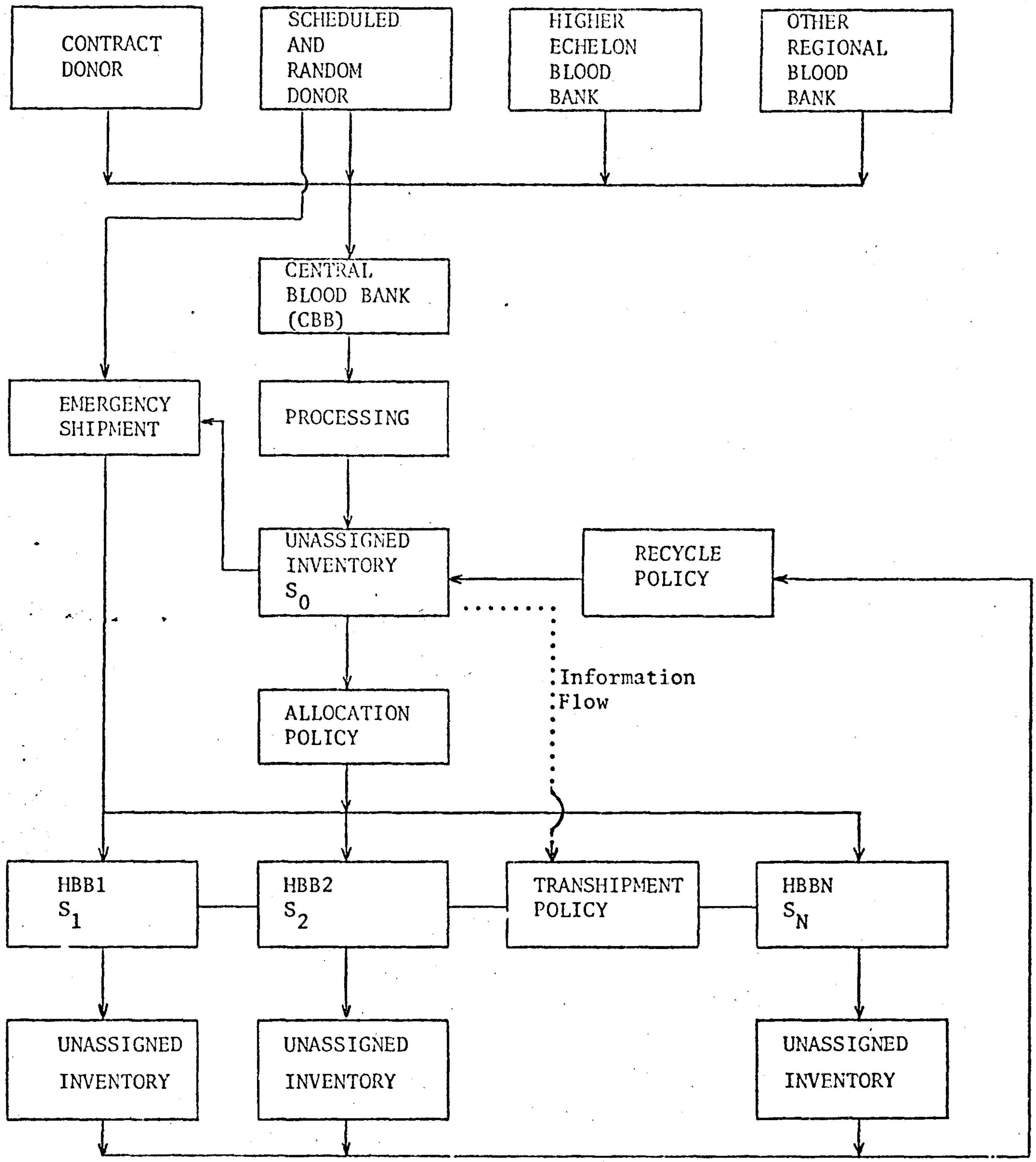

Fig. 1 Flow Chart for Central Blood Banking System

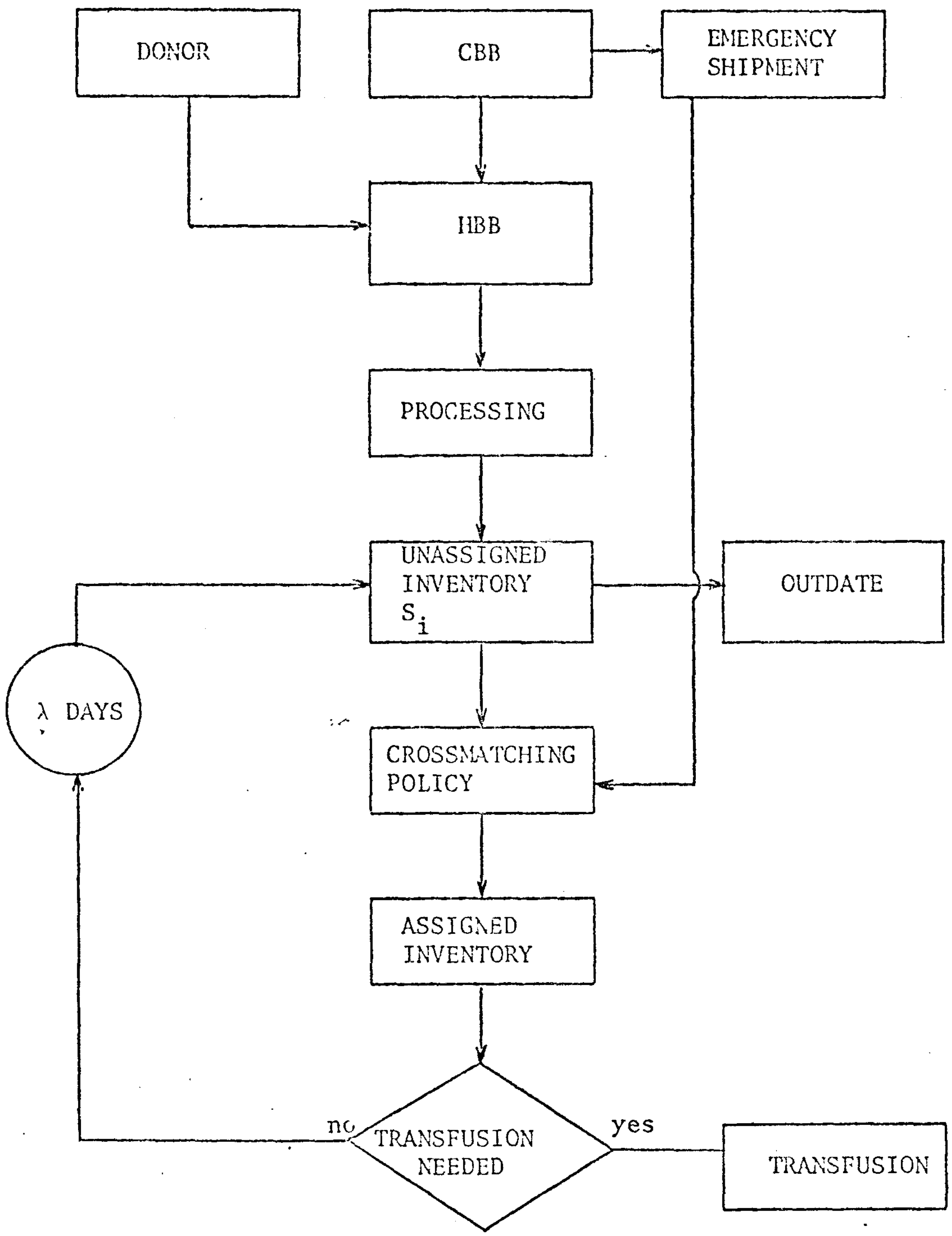

Fig. 2 Flow Chart of a Blood Unit in a Hospital Blood Bank

When an HBB receives a request for blood with specified type and quantity from physicians, called a patient demand, units which are compatible with the type specified in the request are selected via a crossmatch procedure and put in reserve for later transfusion use for that particular patient.

The inventory in the HBB which can be drawn from for crossmatching is referred to as the unassigned inventory while the inventory of crossmatched units is referred to as the assigned inventory or the reserved inventory. Since the probability of an incompatible crossmatch of a unit of a certain type and Rh factor to a particular patient demand of the same type and Rh factor is about 0.1, and since the probability of a particular unit being incompatibel to all the patient demands of that type and Rh factor is 0.1 raised to the power of the number of patient demands on that day, this incompatibility is rather small and may be ignored. Hence, we assume the oldest unit is crossmatched first, i.e., a FIFO issuing policy for crossmatching is followed.

When the sum of all the patient demands exceeds the total unassigned inventory in a HBB, the HBB may backlog the excess demand by postponing surgical procedures or may fulfill the excess demand by placing an emergency order to the central blood bank. In this model the emergency order is always placed. If the emergency order cannot be met by the stock in the CBB, it will be met by the stock from other HBB's. If all other HBB's in the system cannot meet the emergency order, then it will be met by an exogenous source (such as another CBB, on-call conors, hospital walking inventory, or frozen packed red cells). There are different costs involved depending upon which source fills the excess demand.

Once a unit is in the assigned inventory, it is either transfused or after λ periods it is returned to the unassigned inventory. A physician may order several units for a given patient. When one of these units is needed for transfusion, there are various possibilities for deciding which unit to transfuse.

The most common choices are transfuse the oldest unit, transfuse the youngest unit, or randomly pick the unit with an equal probability for each unit. The first policy is called the FIFO transfusion policy, the second is the LIFO transfusion policy, and the last is the random transfusion policy. Now there is a fairly likely possibility that some patients may use all their crossmatched units while others may use some or none of the crossmatched units because a large amount of units crossmatched are for precautionary reasons. Hence, even if a FIFO transfusion policy is followed for each patient, the returning units for all patients combined resembles more of a random selection from the combined crossmatched (assigned) inventory. To handle this fact, in the simulation, the assigned inventory consists of the units crossmatched from the unassigned inventory on a FIFO basis without specifying which units are for which particular patient. Then when transfusions occur the units in the assigned inventory are randomly selected. Therefore, a random transfusion policy, a FIFO crossmatching policy and a lumped assigned inventory for all patients is adopted to simulate the reality which has a FIFO transfusion policy, a FIFO crossmatching policy and a separate assigned inventory for each patient. The random transfusion policy and an aggregate assigned inventory was used in the simulation to reduce the large amount of record keeping needed for each patient and also because it was a good representation of reality.

When a unit is not transfused, then the earlier the unit is returned to unassigned inventory in each HBB, the more chance it will again be crossmatched and possibly transfused. The time period between the crossmatch and the transfusion, and between the transfusion and the return are called the hold and the release parameters respectively. If the hold parameter is too small, there may not be sufficient time to have all requested units processed before transfusion time. If the hold parameter is too large, the units returned have a shorter usable life. The hold parameter is primarily determined by the laboratory workload and processing times. To shorten the hold parameter it may be necessary to add more personnel and equipment. It is assumed that the hold parameter is constant in the simulation.

The release parameter is needed for medical and management reasons. The units cannot be released if the future medical development of the patient is uncertain or if there are transportation or communication problems within the blood bank management. This parameter plays an important role in affecting the outdate amount and in determining the optimal inventory level in HBB's (see Cohen (1974) and Cohen and Pierskalla (1975)). It is one or two days in a well managed hospital or clinical system and three or more days in a poorer system.

For practical purposes, these two parameters are combined into the single parameter λ which specifies the number of periods (hold plus release time) a unit is allowed to remain in the assigned inventory. After a CBB receives all the orders of a specified type and Rh factor from the HBB's, the orders are filled by drawing from the inventory in the CBB using a FIFO issuing policy. For purposes of simplification as well as good medical practice, each type and Rh factor is considered independent of the other types and Rh factors. When the sum of all HBB demands exceeds the total inventory in the CBB, the CBB may backlog the excess demand or may fill all demands by calling in donors, by contacting other CBB's, by using forzen packed red cells, or by requesting an emergency shipment from still higher echelon blood banks. In this simulation the CBB uses different treatment for the excess demand depending upon whether the orders are routine or emergency. Routine orders are placed by the HBB's in the beginning of each period to build up their inventory to a specified level. Emergency orders are placed during the period while the inventory of the HBB's cannot meet their respective users' demands. For routine orders, the CBB will fill the orders as far as its inventory lasts and disregard the excess demands, if any. Consequently, the HBB's may not receive the full amount they ordered. For emergency orders, the CBB still fills the orders as far as its inventory lasts. However, if there are excess demands, the CBB will attempt to fill them from the inventory of other HBB's within the system. Furthermore, if there is insufficient stock in the whole system to fill the excess demands, then the CBB will fill them by contacting exogenous sources. The rationale of the different treatments for excess demands between routine and emergency orders is that the routine orders are used to build up the buffer inventory in the HBB's which may not be needed in the period, and therefore if the excess demands are held unsatisfied a shortage will not necessarily occur. On the other hand, the emergency orders if held unsatisfied will surely create a shortage since the buffer inventory in the HBB has to be completely depleted before the HBB will place an emergency order.

Since each HBB may not receive all that they have ordered, a systematic process is needed to allocate the available stock in the CBB to HBB's. This allocation process will be called the allocation policy and the simulation model contains the following three options:

1. The CBB picks a random hospital and fills its demand by the FIFO issuing policy then goes on to fill the next randomly picked hospital until all the stock runs out or all demands from HBB's are filled. This type of allocation process resembles the first come first served practice which exists in many blood banking systems.

2. The CBB ships an amount to each HBB such that each HBB has the same ratio of the amount received to the amount ordered. Furthermore, all HBB's have the same ratio of the amount of different ages received to the amount ordered. This type of allocation process resembles the rationing of scarce resources intended to be fair to all users and was the policy which was contained in the mathematical model of Chapter II.

3. The CBB ships each unit to the hospital where the shortage probability is the highest in the system. In other words, the delivery of each unit is intended to adjust the system stock configuration such that the system shortage probabilities may be improved.

After all HBB's receive their orders it may be desirable to tranship units among them. Basically there are two reasons for such transhipments: 1) the shortage anticipating transhipment; and 2) the outdate anticipating transhipment. If one location anticipates a shortage while another location does not, then a transhipment from the latter to the former may be beneficial to the system in reducing the shortage cost. Similarly, if one location has an excessive amount of old units while another location does not, an outdate anticipating transhipment can be initiated for the benefit of the system. But before a transhipment is made the exact stock configurations of the locations as well as the demand distributions of the locations must be known in order to evaluate the benefit of the transhipment. When such information is available, the CBB is in the best position to direct the transhipments in the system. Obviously, for these types of actions a sophisticated information gathering and transmitting system is needed.

In case such information is not available, then the value of transhipping is uncertain and no transhipment would be made directly from one HBB to another. However, each HBB still knows its own stock configuration. Hence, the excessively old units can be returned to the CBB and recycled to another hospital from there. This particular type of outdate anticipating transhipment will be called the recycle policy. The names "outdate anticipating transhipment" and "shortage anticipating transhipment" will hereafter refer exclusively to transhipments between HBB's directly.

III Meta-Model:

The overall design of this simulation experiment is aimed to establish a metamodel, i.e., a collection of empirical equations such that by given a set of input variables, the output of the system can be predicted by the collection of empirical equations. The primary output of the system, mainly the number of outdated units and the number of shortage units, are measures of the system's performance for a set of input variables. The empirical equations are constructed through our reasoning of the behavior of the system. The simulation provides a mechanism to justify our reasoning by comparing the output from the simulation and from the empirical equations.

In order to express the empirical equations in a compact form, the following notation are used:

D_0 = the demand at the central blood bank.

D_j = the demand at the j^{th} hospital blood bank.
It is assumed that D_j is a Neyman Type A distributed random

variable with two parameters, α the mean number of patients per day and β the mean units requested per patient.

λ = the return parameter, the time lapse before a unit is returned to the unassigned inventory if not transfused (in periods).

S_j = the inventory level ordered up to at location j (in units).

N = the number of HBB's in the system.

p_j = the probability a unit corssmatched will be transfused at location j.

v_j = the shortage at location j (in units).

q_j = the outdate at location j (in units).

n = the number of times a unit is crossmatched in its lifetime.

a = the age of a unit when it is crossmatched for the first time.

Now we shall derive the empirical equations. On any day at location j, there are D_j units issued. Of them D_jp_j will be transfused and $D_j(1-p_j)$ returned after λ periods. In addition, there are q_j units outdated and promptly removed from the inventory. So the gross depletion of the unassigned inventory is the sum of the demand and the outdate. But there are units returned from the assigned inventory, namely, the units issued λ days earlier and not transfused. Assuming we ignore the differences between the number of units to be returned after λ periods and the number of units returned on the day, the actual number of units depleted becomed $D_jp_j+q_j=D_j+q_j-D_j(1-p_j)$. Under a fairly wide range of order levels S_j, the q_j are small and negligible relative to D_jp_j, thus approximately only D_jp_j units will be ordered in the following day. Therefore, the daily demand at the CBB, D_0, is essentially the sum of D_jp_j for

$j = 1,\ldots, N$. Consequently, the distribution of D_0 is the N-fold convolution of $D_j p_j$ for $j = 1,\ldots, N$. The expected shortage at the CBB can thus be calculated by the following equation.

$$(1) \qquad Ev_0 = \sum_{k=1}^{\infty} kP(D_0=S_0+k) = \sum_{k=1}^{\infty} P(D_0 \geq S_0+k).$$

Once there is a shortage in the CBB, it implies that not all the HBB's may get what they ordered. The amount recieved by each HBB depends on how the available inventory in the CBB is allocated.

Since the HBB orders up to a predetermined inventory level, it will order more if there is less stock on hand. But if there is less stock on hand then there is a higher shortage probability. So the shortage probability is directly related to the amount ordered. Due to this fact options 2 and 3 of the allocation policy in the simulation, namely to allocate proportionally the available stock to HBB's by the ratio of the order from each HBB and the sum of all orders from HBB's and to allocate the available stock one by one to the HBB with the highest shortage probability, are essentially the same. The simulation runs reported in this chapter adopted option 2.

Assume all the demands at the HBB's are independent and identically distributed. Based on Corollary 2.19 in the previous chapter, all the ordered up to inventory levels will be the same. Consequently, the shortage probabilities are identical for all locations. Hence the CBB will allocate its shortage evenly among the HBB's. That is, only $S_j - \frac{Ev_0}{N}$ units are on hand at location j in the beginning of each period. Consequently, the expected shortage at location j can be calculated in the following equation.

$$(3.2) \qquad Ev_j = \sum_{k=1}^{\infty} kP(D_j=S_j-\frac{Ev_0}{N} + k) = \sum_{k=1}^{\infty} P(D_j \geq S_j-\frac{Ev_0}{N} + k).$$

Combining (1) and (2), one of the important measures of the performance of a centralized blood banking system, namely the shortage units at the CBB and the HBB's, are expressed in terms of input parameters which are the demands and inventory levels at all locations.

The next important measure, outdated units in the system, will be covered now. It is assumed in the simulation that the units entering the CBB are of age 1. After a unit enters the system at CBB, it will be stored there for a period of $\frac{S_0}{ED_0}$ days before being issued to one of the HBB's. In other words, suppose there is a three day supply at the CBB. The units issued to the HBB will be four days old. After the HBB receives its order the units will be placed in the unassigned inventory and be crossmatched at the appropriate time. Since each day there are an average of ED_j units crossmatched and $ED_j(1-p_j)$ returned, the units will be stored in the HBB for an average time of $\frac{S_j - ED_j}{ED_j p_j}$ days before being crossmatched for the first time. Hence, the average age of a unit when crossmatched for the first time a is:

$$a = \frac{S_0}{ED_0} + \frac{S_j - ED_j}{ED_j p_j} \quad . \tag{3}$$

Once a unit is crossmatched it will return to the unassigned inventory if not transfused after λ days. Since a FIFO crossmatching policy is followed, those returned units must be at least as old as all the units which have not ever been crossmatched. So they have a high probability of being issued again the same day they are returned. Assuming the maximum useful life of a unit is 21 days including the twenty-first day, the average life span of a unit subject to crossmatching is therefore 22-a days. Therefore, a unit can be crossmatched for about $\frac{22-a}{\lambda}$ times before it reaches an age between $22-\lambda$ and 21 days. But if a unit is returned at ages between $22-\lambda$ and 21 days, the unit can only be crossmatched at most one more time. This is because the next time the unit is returned it already exceeds the allowable age for crossmatching purposes. Assuming there is an equal probability for the units to be returned at ages $22-\lambda$, $22-\lambda+1, \ldots, 21$, then the expected number of times of crossmatching for units between $22-\lambda$ and 21 days is $\frac{1}{\lambda}$ times. If we assume the returned units are always crossmatched again on the day they are returned, the total number of times a unit can be crossmatched is approximately:

$$(4) \qquad n = \frac{22-a}{\lambda} + \frac{1}{\lambda}.$$

Since a unit can be outdated only if every time it is crossmatched it is not transfused, the expected daily outdate at location j is approximately:

$$(5) \qquad Eq_j = ED_j(1-p_j)^n.$$

Outdates can also occur at the CBB if there is more than 21 days supply stored there, which is very unlikely. So an empirical equation for Eq_0 has not been developed.

Expressions (1) to (5) are the empirical equations. If they are representative of the behavior of the system, the values of Ev_j and Eq_j calculated from the empirical equations should be comparable to the corresponding values from the simulation for each set of input variables. Table 3.1 is a summary of comparative simulated and calculated results for outdate and shortage from forty-one sets of input variables. A regression analysis between the simulated and calculated results of the form:

Simulated results = B • (calculated results) + c

has been done. Results of the regression analysis are summarized in Table 3.2.

Two sets of regressions were presented: one with the constant term included in the regression equation, and the other not. Furthermore, each set of regressions consists of two equations: one between the simulated outdate and calculated outdate, and the other between the simulated shortage and calculated shortage. So there are a total of four regression equations. The R^2 value for each of them is at least .93 and the F value is at least 679.69. Thus, we may conclude that the results from the empirical equations are consistent with the results from the simulation.

However, further statistical validation should be performed to test if the empirical equations can indeed represent the behavior of the system. Two methods are proposed below and under current research.

A. First to test if, for any given set of input variables, the simulation results are normally distributed around the values from the empirical equations. If this hypothesis is accepted, then a chi-square or Kolmogorov-Smirnov goodness of fit test should be used to see if the two response surfaces, one from the simulation and the other from the empirical equations, are well fit or not.

B. In the derivation of the empirical equations there were several gross approximations and assumptions. Once these gross approximations and assumptions can be statistically shown valid, then the resulting empirical equations can be concluded as a representation of the system behavior. The following is a list of the hypotheses needed to be tested.

1. The average daily demand at the CBB is the sum of $D_j p_j$ for $j=1,2,\ldots,N$.
2. The average inventory level after ordering is $S_j - \frac{Ev_0}{N}$ for $j=1,2,\ldots,N$.
3. The outdate and shortage anticipate transfer policies has no effect on outdate and shortage.
4. The average age for a unit to be crossmatched for the first time is a.
5. The average number of times a unit can be crossmatched is n.

TABLE 1

COMPARISON OF SIMULATION AND EMPIRICAL EQUATIONS

N	α	β	S_0	S_j	λ	q_j SIMULATED	q_j CALCULATED	v_j SIMULATED	v_j CALCULATED
3	3	2	18	16	2	10	9.76	10	8.51
3	3	2	15	16	2	8	8.54	13	9.93
3	3	2	17	16	2	11	9.34	9	8.85
3	3	2	14	16	2	11	8.10	10	10.84
3	3	2	12	17	2	7	8.06	8	7.16
3	3	2	12	18	2	11	8.99	3	3.50
3	3	2	15	17	2	11	9.52	4	4.94
5	3	2	15	17	2	5	6.18	25	21.86
3	3	2	15	18	2	15	10.61	4	2.35
3	3	2	21	18	2	15	13.71	1	1.82
3	3	2	21	20	2	17	17.01	0	0.35
3	3	2	21	21	2	21	19.04	0	0.14
3	3	2	21	21	3	108	88.70	1	0.14
2	3	2	10	16	2	8	8.38	17	13.23
2	3	2	10	17	2	9	9.35	8	6.69
2	3	2	8	18	2	9	8.59	5	5.74
2	3	2	14	18	2	12	13.71	1	2.14
2	3	2	12	18	2	9	12.08	1	2.46
2	3	2	10	18	2	9	10.42	2	3.24
2	3	2	10	19	2	10	11.62	1	1.50
2	3	2	20	20	2	24	23.69	0	0.36
4	3	2	20	19	2	17	12.27	0	0.98
4	3	2	24	19	2	17	13.92	0	0.86
4	3	2	16	17	2	10	8.47	3	6.16
4	3	2	28	21	3	123	89.58	1	0.14
4	3	2	28	21	2	22	19.39	0	0.14
5	3	2	25	18	2	9	10.98	1	2.17
5	3	2	30	18	2	8	12.39	1	1.98
5	3	2	35	21	2	14	19.13	0	0.15
5	3	2	20	17	2	6	8.54	3	5.83
6	3	2	24	17	2	9	10.17	5	5.10
6	3	2	30	17	2	12	14.58	5	3.93
6	3	2	18	17	2	5	7.96	9	12.21
6	3	2	20	17	2	10	8.95	7	7.27
6	3	2	42	21	2	21	22.40	1	0.14
3	5	2	24	27	2	12	16.76	6	1.02
3	8	2	48	32	2	15	21.45	22	13.03
3	8	2	36	40	2	17	25.03	1	0.30
3	5	2	30	20	2	10	12.25	54	42.43
3	5	2	60	30	2	30	44.89	2	0.11

TABLE 2

REGRESSION ANALYSIS ON SIMULATION AND EMPIRICAL EQUATIONS

REGRESSION	B	STD ERROR B	F	R^2	C	F
Outdate	1.26	0.048	679.69	0.95	-4.15	12.13
Shortage	1.24	0.045	754.83	0.95	-0.69	2.71
Outdate	1.14	0.039	853.00	0.93	Regression Forced Through the Origin	
Shortage	1.19	0.037	1030.64	0.95		

III Analysis of the Results:

Since the simulation results and the calculated results from the empirical equations are highly correlated on two of the key performance indicators of the regional blood banking system, namely the outdated units and the shortage units, further analysis may be directed to the behavior of the system from the empirical equations.

The simulation was run with many policies operational which turn out to have no significant impact upon the system's behavior. These policies include the shortage transhipment policy, the outdate transhipment policy, and the limited supply policy. In the simulation model, the shortage and outdate transhipment policies allow units to be transhipped whenever the savings from expected shortage cost or outdate cost exceeds the transportation cost with the latter set to be 5%of either the shortage cost or the outdate cost. The limited supply policy specifies that the amount the CBB received is drawn from a normal distribution with mean the amount ordered and standard deviation 20% of the mean.

There are reasons to support the conclusion that the shortage units are insensitive to these policies. The most important one is from the allocation policy in the CBB. In the simulation the units are warranted to be transhipped only after each HBB has received its delivery. But the units in the CBB are issued one by one to the location with the highest shortage probability. So at the end of the allocation process each HBB will have an identical shortage probability except when there is insufficient inventory in the CBB to make them equal or when there is a tie in shortage probabilities before the issuance of the last few units. In both of these cases some discrepancies among shortage probabilities will occur, but they are rather negligible under relatively wide ranges of ordered up inventory levels of all locations. Consequently, the conditions to initiate a shortage transhipment would rarely occur, and hence hardly any units are shortage transhipped. Naturally, the shortage transhipment policy has no significant effect on the shortage units in the system.

The insensitivity of the outdated units to the outdate transhipment policy can be explained as well. By observing equation (5) it can be seen that a unit will be outdated only after several times of the crossmatching process. So the expected daily outdate is going to be fairly small simply because $(1-p_j)^n$ is a very small number. So there are very few units ever to be outdated regardless of whether an outdate transhipment policy is in effect or not. Consequently, the outdate transhipment policy can be expected to have virtually no significant effect on the outdated quantity.

It should be mentioned here that there are units transhipped but at a very insignificant quantity under a fairly wide range of inventory levels at different locations. The outdate transhipment units do become significant when each location has a very high ordered up to inventory level. On the other hand, the shortage transhipment units become significant when the allocation policy is changed to the FIFO allocation policy. But both of these cases are so extreme, they were run only to test if the computer program of the simulation was valid. Thus, the results of these extreme cases were not reported.

The insignificant effect of the limited supply policy can be explained as follows. Usually within a wide range of inventory levels, there are few shortages in the CBB, so the inventory levels with or without the limited supply policy are not significantly different. Consequently, there is no significant effect on the shortages or outdates.

What are the factors which do affect the two measures? The ordered up to inventory levels S_j for $j=0,1,\ldots,N$ are no doubt the dominant factors in affecting the shortage at their respective locations. But the S_0 and the number of facilities in the system N will also have some effects on shortage at lower echelon facilities. Moreover, because D_0 is an N-fold convolution of $D_j p_j$ for $j=1,2,\ldots,N$, the shortage at the CBB will also depend on the number of facilities in the system as well as the demand distribution and the transfusion probabilities at lower echelon facilities. The single dominant factor in affecting the outdate is the return parameter λ. This observation coincides with the conclusion reached in Cohen and Pierskalla(1974), in which further analyses on λ are presented.

Much about the factors and policies which will or will not affect the shortage units and the outdate units has been discussed. By allowing the allocation policy to ship units where there is the highest shortage probability and by controlling the return parameter, the system cost reduction could be very substantial. However there are two other important policies which can induce further cost reduction: first stocking inventories at an optimal level; and second, regionalizing the hospital blood banks in an optimal manner.

Table 3 on the following page presents the optimal inventory levels for different combinations of parameters (N, α, β) where N is the number of identical hospital blood banks in the system, α is the daily patient arrival rate, and β is the average units requested by patients. It will be demonstrated in the next chapter that a Neyman Type A distribution which has the two parameters (α, β) can satisfactorily represent the demand distribution of a hospital blood bank. Furthermore, the parameter β is found to be two units per patient from our data and from another report by Rabinowitz (1970). He reasoned that whenever the amount of units in need is in doubt, two units are ordered by the physicians. So β is assumed to be two and suppressed from the table. The return parameter λ is set to be two days which is considered the necessary time for a unit to return in a well managed hospital blood bank. The outdate costs are assumed to be \$25 per unit at all locations. The shortage costs are assumed to be \$55 per unit at hospital blood banks and \$35 at the central blood bank. Finally, the transfusion probability P is assumed to be 0.45 based on our own data for two hospitals. All the results presented in the table are derived from the empirical equations.

By inspection, two conclusions regarding the optimal inventory levels and the parameters (N, α, β) are drawn. First, the optimal inventory levels at hospital blood banks are insensitive to the number of hospitals in the system. They depend only on the two parameters α and β. Since α denotes the arrival rate of patients and β is assumed to be two units per request, we shall also refer to α as the size of the hospital because generally the larger the hospital the larger the value of α. By varying α and comparing the optimal inventory levels from the empirical equations with the similar quantities from Cohen and

TABLE 3

COSTS VERSUS NUMBER, SIZES AND INVENTORY LEVELS OF THE FACILITIES IN THE SYSTEM

N	α	S_0	S_j	$\frac{S_0}{N \cdot \alpha \cdot \beta \cdot p}$	COST/HOSPITAL-DAY	COST/UNIT TRANSFUSED
2	2	6	15	1.67	1.27	0.71
3	2	9	15	1.67	1.26	0.70
4	2	12	15	1.67	1.25	0.69
5	2	17	14	1.89	1.24	0.69
6	2	22	14	2.04	1.24	0.69
2	3	9	20	1.67	1.37	0.51
3	3	14	19	1.73	1.36	0.50
4	3	18	19	1.67	1.32	0.49
5	3	22	19	1.63	1.30	0.48
6	3	26	19	1.61	1.29	0.48
2	4	12	24	1.67	1.47	0.41
3	4	17	23	1.57	1.46	0.41
4	4	22	23	1.53	1.42	0.39
5	4	29	23	1.61	1.40	0.39
6	4	34	23	1.57	1.39	0.39
2	5	15	27	1.67	1.59	0.35
3	5	22	27	1.63	1.55	0.34
4	5	28	27	1.56	1.53	0.34
5	5	34	27	1.51	1.51	0.34
6	4	41	27	1.52	1.49	0.33
2	6	17	31	1.57	1.69	0.31
3	6	26	31	1.61	1.66	0.31
4	6	33	31	1.53	1.63	0.30
5	6	41	31	1.52	1.61	0.30
6	6	50	31	1.54	1.59	0.29

Pierskalla (1974) presented in Table 4 below, except that in there the optimal inventory levels are for the hospital blood banks operating independently; it is observed that they are very close. Combining Tables 3 and 4 we may conclude that the optimal inventory level depends only on the sizes of the hospital regardless of the number of hospitals in the system even when the number of hospitals in the system is only one.

TABLE 4

COMPARISON OF INVENTORY LEVELS BETWEEN EMPIRICAL EQUATION AND COHEN AND PIERSKALLA

α	S_j (EMPIRICAL EQUATIONS)	S_j (COHEN AND PIERSKALLA (1974))
2	15	19
3	20	19*
4	24	24
5	27	26*
6	31	28

*These figures are interpolated.

Second, the optimal inventory level at the central blood bank divided by $N \cdot \alpha \cdot \beta \cdot p$ is in the neighborhood of 1.67 from Table 3. Since $N \cdot \alpha \cdot \beta \cdot p$ is the mean demand facing the central blood bank every day, the optimal inventory level at the central blood bank can supply about 1.67 days demands from all the hospitals.

The design aspect of a regional blood banking system shall now be discussed. Specifically, what is the optimal number of hospitals which vary in size in the system. It can be seen from Table 3 that the system cost per hospital day and the system cost per transfused unit are monotonic decreasing with the increase either in the number or the size of hospitals. Evidentally, the size is more influential to the system cost than the number. For instance, when the size is held at three, the cost per transfused unit reduces from 0.51 to 0.48 or 6% when the number of hospitals increased from two to six. But when the number of hospitals is held fixed, the cost reduces from 0.69 to 0.29 or 42% when the size increases from two to six. Figure 3 below summarizes the results concerning the sensitivity on system cost by varying (N, α, β).

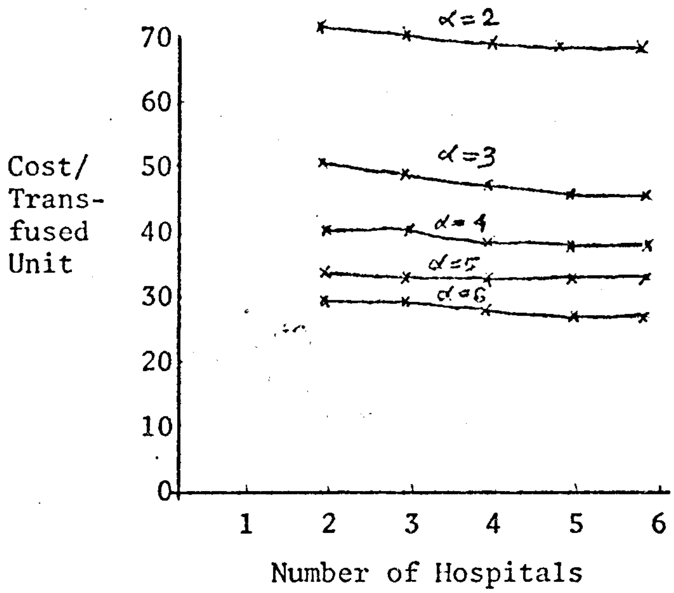

Fig. 3

V Summary:

A centralized blood banking system was studied in this chapter by a simulation approach. It is because the numerous possible combinations of all the input variables and decision alternatives, the exhaustive search for optimal policies, is cost prohibitive. We turn to the analytical approach interacted with our understanding of the system which was gained during the progress of the simulation to formulate a set of empirical equations. It is shown that this approach is fruitful. Two major measures of the system behavior, the number of outdated units and the number of shortage units are highly correlated between the simulation and the empirical equations.

A major result of the approach is that the optimal inventory levels at all locations for each set of input variables and decision alternatives can be easily found at a fraction of the simulation cost. Furthermore, several interpretations were derived on the system behavior from the empirical equations. It was found that neither outdate transhipment policy nor shortage transhipment policy have any apparent effect on the system outdate cost or the system shortage cost respectively. The supply policy that the amount received by the CBB is not unlimited but is a normal random variable with mean the amount ordered the standard deviation 20% of the mean also has no significant effect on the system cost either.

The results regarding the optimal inventory levels are two-fold. First, the optimal inventory levels at hospital blood banks depend only on the sizes of the hospitals regardless of the number of hospitals in the system. Second, the optimal inventory level at the central blood bank is fairly stable in the sense that it maintains a supply which is approximately 1.67 days demands from all the hospital blood banks. Neither the number nor the sizes of the hospitals' blood banks appear to have a significant effect on this number 1.67.

Finally, the system cost and the number and sizes of the hospital blood banks in the system was analyzed. It is found that size is a more influential factor than the number. This result should be helpful in the design of a regional blood banking system.

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