When Do Financial Constraints Increase Preference for More Durable Goods: Making the Study of Utility's Third Derivative Great Again

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Abstract: Recent finding by Tully, Hershfield, and Meyvis (*JCR* 2015) that making people feel more financially constrained shifts their demand towards durable goods is consistent with some but not all standard microeconomic models of consumer behavior. This paper provides an easily verifiable sufficient condition for the underlying utility function to produce the observed preferences: marginal utility diminishing at an increasing rate. On the other hand, homothetic preferences commonly assumed by both analytical and empirical researchers cannot produce the observed findings.

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Introduction

In a series of consumer studies, Tully, Hershfield, and Meyvis (2015), hereafter "THM", have discovered that making people feel more financially constrained shifts their demand toward durable goods and away from perishable goods. THM's Study 6 provides a very clear example of their finding: subjects are instructed to imagine that they are walking around the city when it starts to rain, and they can either stop for a coffee in Starbucks or buy a poncho which is either described as "disposable" or "reusable" between subjects. Price is not an issue in the scenario because the coffee costs the same as the poncho in all conditions. In a control group, the subjects express approximately the same strength of relative (to coffee) preference for both types of poncho. However, asking the subjects to "keep in mind their financial constraints" before making the decision dramatically increases their relative preference for the reusable poncho while also decreasing their relative preference for the disposable poncho. This paper explores the implications of THM's finding for standard economic models of consumer preferences by characterizing the types of utility functions that do and do not give rise to the consumer preferences THM document.

One way to motivate this paper's research question is to ask whether the THM finding is consistent with standard microeconomics, or whether it should be considered a behavioral paradox. THM are silent regarding this question, and three fellow researchers I queried do not agree with each other: Avinash says "This result is obvious, poorer people should not waste their scarce money on coffee when they can get a durable poncho instead". Charles says "This result is surprising, richer people should just buy more of everything, they should not shift their demand towards one particular good", and his friend Paul agrees. Finally, Roy says "Coffee is a necessity, so making someone poorer shifts their demand to coffee, and the shift is faster with durable ponchos because they obviously represent a bigger chunk of the discretionary budget". I resolve this disagreement by showing that each of the above researchers can be correct under standard economic assumptions. Therefore, while the THM finding is consistent with some standard models, it is certainly not a generic property of standard preferences.

The main result of this paper is that the consumer preferences THM document are consistent with marginal utility diminishing at an increasing rate. Mathematically, if the consumer's utility u as a function of quantity consumed q is $u(q)$, then $u'''(q) \leq 0$ is a sufficient condition for tighter financial constraints to shift consumer demand towards durable goods and

away from perishable goods. The main result is robust to assumptions about complementarity or substitutability between the goods.

In addition to providing the above condition, I also show that the preferences THM document are not consistent with homothetic preferences such as the Cobb-Douglas utility function or the Constant Elasticity of Substitution (CES) function, and they are also not consistent with some popular non-homothetic preferences such as the Stone-Geary function. The results of this paper have clear and obvious implications for quantitative modeling of consumer behavior.

Model of the observed preferences

Let there be two goods, one called an experience and one called a material product.[1] Both goods cost the same per unit, and a consumer has a budget B – the total units of both goods he can afford. The product can be durable in that there is an expected number λ of future consumption opportunities during which a unit purchased today will still be available, with λ including any potential temporal discounting of future consumption. To determine his demand, the consumer selects the amount M^* of product and the amount E^* of the experience to purchase to maximize his utility $U(E,M;\lambda)$ such that the budget constraint $E+M \leq B$ holds:

$$\{M^*, E^*\} = \underset{M\geq 0, E\geq 0}{\arg\max}\ U(E,M;\lambda) \text{ subject to } M+E \leq B \tag{1}$$

In terms of the above notation, THM find that tightening the budget constraint B increases the difference between percentage demand M^*/B for a durable version (high λ) and the percentage demand M^*/B for a disposable version (low λ) of the material product. Considering a small change in durability so we can employ the tools of calculus, the finding is thus that the cross-partial of percentage demand for the material product in budget and durability is negative: $\frac{\partial^2}{\partial\lambda\,\partial B}\left(\frac{M^*}{B}\right) < 0$. The goal of this paper is to explore what does the sign of this cross-partial teach us about the form of $U(E,M;\lambda)$. The main result of this exploration is discussed next.

Main result: marginal utility diminishing at an increasing rate is a sufficient condition

Consider the bivariate utility function U as a function of each single argument. It is standard to

[1] Both the "experience" and the "product" are just generic goods in this paper, consumer framing of goods as either experiences or products is not modeled. However, I agree with THM's observation that material products tend to be more durable than experiences, so I label the good with variable durability a "material product".

assume that U is increasing in each argument with diminishing marginal returns. For tractability of the general case, I make an additional assumption that U is additively separable in its arguments: $U(E,M;\lambda)=(1+\lambda)u(M)+v(E)$. Note how durability ($\lambda$) enters the utility function as the expected number of additional consumption occasions – scaling the overall utility of M rather than adding to it. Given this assumption, the main result of this paper is:

Proposition 1: When $U(E,M;\lambda)=(1+\lambda)u(M)+v(E)$ with $u'>0, u''<0, v'>0, v''<0$, then $u'''\le 0$ and $v'''\le 0$ implies $\frac{\partial^2}{\partial\lambda\,\partial B}\left(\frac{M^*}{B}\right)<0$ as documented by THM.

Please see the Appendix for a detailed proof. The essence of the proof is a characterization of the demand M^* using first-order conditions, and expressing the cross-partial of this demand in terms of the two univariate utility functions that together determine U. An immediate corollary of Proposition 1 is that a large class of polynomial utility functions is consistent with the observed behavior:

Corollary 1: When $u(x)=x-cx^m$ and $v(y)=y-dy^n$ for some $c,d>0$ and x,y small enough for u and v to be increasing, then the consumer will exhibit the preferences found by THM when $\min(m,n)\ge 2$, that is when the polynomial term is at least quadratic.

The quadratic utility function with $m=n=2$ popularized by Dixit (1979) is especially tractable, and provides a nice closed-form example of the main result and its corollary:

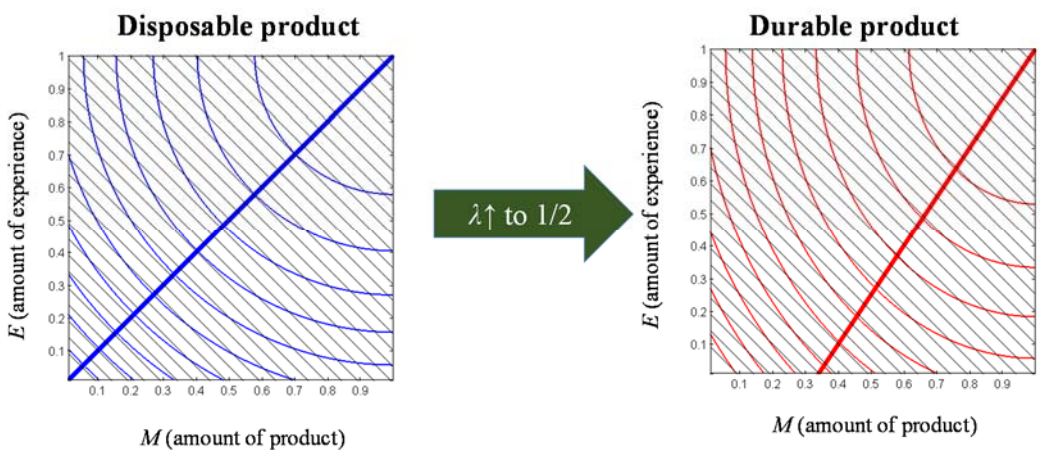
Quadratic example: Consider a quadratic utility $U(M,E)=a\left(M-\frac{M^2}{2}\right)+b\left(E-\frac{E^2}{2}\right)$, where $a,b>0$ are constants that weigh the relative importance of the two goods. Note that we are assuming away any substitutability or complementarity between M and E. The consumer solves

$$\max_{M,E}(1+\lambda)a\left(M-\frac{M^2}{2}\right)+b\left(E-\frac{E^2}{2}\right) \text{ subject to } M+E=B \tag{2}$$

The solution is: $M^*=\frac{bB+a(1+\lambda)-b}{a(1+\lambda)+b}$, so $\frac{\partial^2}{\partial\lambda\,\partial B}\left(\frac{M^*}{B}\right)=-\frac{2ab}{B^2\left[a(1+\lambda)+b\right]^2}<0$. To gain insight into quadratic preferences, consider the slope of relative demand in budget:

$\frac{\partial}{\partial B}\left(\frac{M^*}{B}\right)=\frac{b-a(1+\lambda)}{B^2\left[a(1+\lambda)+b\right]}<0 \Leftrightarrow \lambda>\frac{b}{a}-1$. In words: given sufficient durability, an increase in budget *de*creases the proportion of budget spent on the product. The right-hand panel of Figure 1 assumes that non-durable versions of the two goods are equally valued by the consumer, and shows what happens when budget increases and the product is durable: for small budgets, the consumer buys only the product, allocating 100 percent of the budget to it. As his budget increases, he adds some experience into the mix. In this sense, quadratic preferences capture the idea of perishable "experience" as a luxury, and the preferences capture the intuition the researcher named Avinash quoted in the Introduction. We now turn to preferences that do not satisfy the sufficient condition, and turn out to vindicate the second researcher.

Figure 1: Quadratic preferences with *a*=*b*=1



Note to Figure: The curves are indifference curves. The thin downward-sloping straight lines are budget constraints. The thick upward sloping line is the locus of solutions to equation 2.

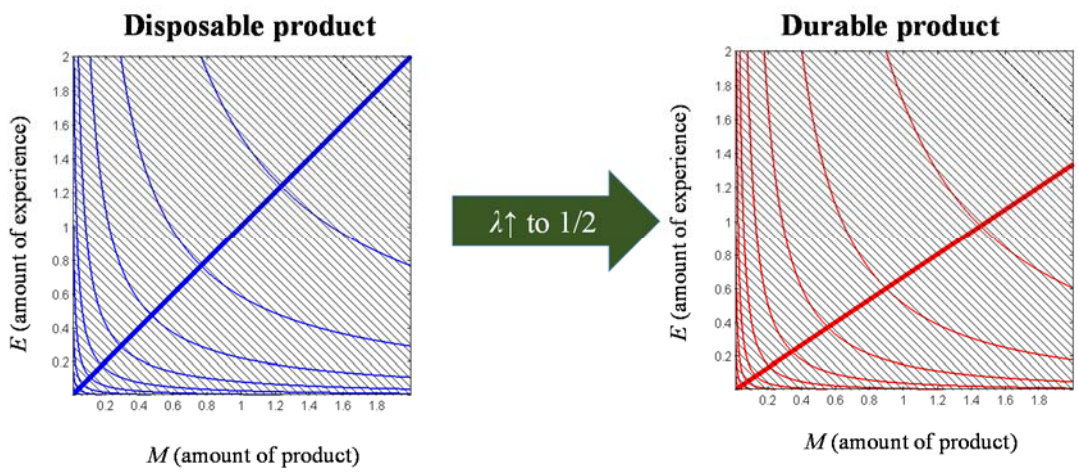
Cobb-Douglas example: Another textbook example of preferences is the Cobb-Douglas utility function $U(M,E)=\alpha\log(M)+\beta\log(E)$, where $\alpha>0$ and $\beta>0$ again represent weights of the individual goods within the total utility. The consumer solves:

$$\max_{M,E}(1+\lambda)\alpha\log(M)+\beta\log(E) \text{ subject to } M+E=B \tag{3}$$

It is well known that the solution to this problem is $M^*=\frac{\alpha(1+\lambda)}{\alpha(1+\lambda)+\beta}B$, so the consumer splits his budget according to the effective weight of each good in overall utility. Durability simply

increases the effective weight of the product. Clearly, these preferences support the intuition of Charles and Paul quoted in the introduction. Since the percentage demand does not depend on budget, it is immediate that $\frac{\partial^2}{\partial\lambda\,\partial B}\left(\frac{M^*}{B}\right)=\frac{\partial}{\partial B}\left(\frac{M^*}{B}\right)=0$. In fact, the type of preferences that imply $\frac{M^*}{B}$ does not depend on budget already have a name – they are called "homothetic". Formally, a utility function is homothetic when there exists a monotonic transformation of it (i.e. an alternative representation of the same underlining preferences) that is homogeneous of degree 1: $U(cM,cE)=cU(M,E)$. A well-known example of homothetic utility functions is the constant elasticity of substitution (CES) family. Graphically, homothetic preferences have indifference curves whose slopes are constant along rays beginning at the origin. Please see Figure 2 for an illustration.

Figure 2: Cobb-Douglas preferences are homothetic (example with *α*=*β*=0)



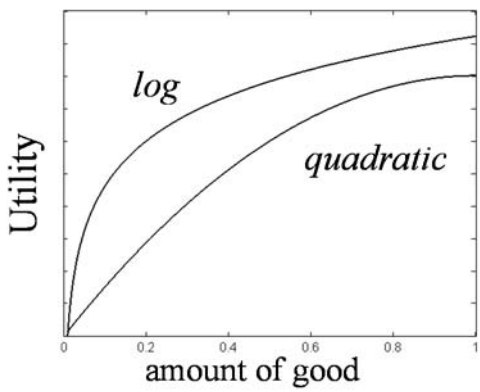
Note to Figure: see note to Figure 2, but replace equation 2 with equation 3.

One way to interpret the findings of THM is that demand for experiences and material products is not homothetic. Standard discrete-choice models, such as the logit, are homothetic. Therefore, THM effectively show that when personal wealth is changing within the data sample, empirical researchers should be using a non-homothetic demand system. Such models are rare in the literature, the seminal example is Allenby and Rossi (1991) extended in Allenby, Garratt and Rossi (2010). Moreover, the sensitivity of THM's cross-partial to the curvature of the utility function

suggests that functional forms of the non-homothetic models matter a lot for matching basic patterns of the data.

More relevant for Proposition 1 is that the Cobb-Douglas utility function does not satisfy the sufficient condition because its third derivative is $u'''(x) = 2x^{-3} > 0$. The log is thus an example of marginal utility diminishing at a decreasing rate. Figure 3 shows the difference between the log and the quadratic utility clearly: the log's curvature gradually relaxes, while the quadratic's increases. In the next and final example, I show that there are also *non*-homothetic preferences inconsistent with THM's finding.

Figure 3: What is the difference between the log and quadratic utilities?



Stone-Geary example: So far, we have seen two examples with the crucial cross-partial either negative or zero. One more example is needed to show that the cross-partial can also be positive, and so its sign is thus not apriori even weakly constrained by standard consumer theory. Consider the following generalization of the Cobb-Douglas preferences, due to Geary (1950) and used in Marketing by Iyengar et al (2011): $U(M,E) = \alpha \log(M-m) + \beta \log(E-e)$, where $m \geq 0$ and $e \geq 0$ represent minimum amounts of M and E that the consumer needs to purchase (Cobb-Douglas is the special case of $e=m=0$), with the utility only valid for $M>m$ and $E>e$. The solution to the consumer budget-allocation problem is: $M^* = m + \frac{\alpha(1+\lambda)}{\alpha(1+\lambda)+\beta}(B-e-m)$, and the key cross-partial is $\frac{\partial^2}{\partial\lambda\,\partial B}\left(\frac{M^*}{B}\right) = \frac{\alpha\beta(m+e)}{B^2\left[\alpha(1+\lambda)+\beta\right]^2} > 0$. Therefore, Stone-Geary preferences represent an entire family of preferences, for which the crucial cross-partial examined by THM is positive. As

in the Cobb-Douglas special case, the component utilities do not satisfy the sufficient condition of Proposition 1 because $u'''(x) = 2(M-m)^{-3} > 0$ and $v'''(x) = 2(E-e)^{-3} > 0$. When I set m=0<e, I obtain a model of a consumer whom the researcher Roy quated in the Introduction had in mind: increased durability makes this consumer spend a greater part of his *discretionary* budget (B-e) on ponchos while very financially constrained consumers (i.e., $B \approx e$) spend all their money on coffee. As a result, increasing the budget results in a greater difference between relative spending on durable product and relative spending on the disposable product.

Robustness check: What if the goods are substitutes or complements?

To allow for complementarity or substitution under the quadratic utility assumption, let a=b=0 for clarity, and include a cross-term:

$$U(M,E) = (1+\lambda)\left(M - \frac{M^2}{2}\right) + \left(E - \frac{E^2}{2}\right) + \frac{cEM}{2} \tag{4}$$

where c>0 captures complementarity, and c<0 captures substitution. The solution is: $M^* = \frac{B(1+c)+\lambda}{2(1+c)+\lambda}$, and so $\frac{\partial^2}{\partial\lambda\,\partial B}\left(\frac{M^*}{B}\right) = -\frac{2(c+1)}{B^2\left[2(c+1)+\lambda\right]^2}$ which is obviously negative as long as c>-1, i.e. as long the solution is valid. When c<-1, the substitution is so strong that the problem is not concave for small λ, and the consumer just spends all his budget on M. I conclude that quadratic utility is consistent with the THM finding even when the two goods are substitutes or complements.

To allow for complementarity or substitution under the Cobb-Douglas assumption while maintaining tractability, let the cross-term be log-transformed:

$$U(M,E) = (1+\lambda)\log(M) + \log(E) + c\log(EM) \tag{5}$$

The solution is: $M^* = \frac{B(1+c+\lambda)}{2(1+c)+\lambda}$, and so $\frac{\partial^2}{\partial\lambda\,\partial B}\left(\frac{M^*}{B}\right) = 0$ as before. I conclude that log utility is inconsistent with the THM finding even when the two goods are substitutes or complements. In summary, the acceleration or deceleration of diminishing returns seems to be the key determinant of whether the utility function implies the THM inequality even when the goods happen to be substitutes or complements.

Discussion

THM's finding that making people feel more financially constrained shifts their demand towards durable goods can be rationalized in a standard microeconomic model, but it is not a generic property of all standard preferences. Instead, the finding implies that consumer utility is non-homothetic (at least over the pairs of goods studied by THM), and must have a particular shape. This paper provides a simple sufficient condition on the utility function's shape for the resulting consumer preferences to exhibit the pattern found by THM: the third derivative of the function must be negative. In words, the pattern holds when the utility function has marginal utility diminishing at an increasing rate.

Many standard models of demand used both in analytical work (e.g. the Cobb-Douglas utility, the CES utility, and the Stone-Geary utility) and in empirical work (e.g. the multinomial logit) are inconsistent with the pattern discovered by THM. Future analytical and empirical modelers need to develop non-homothetic models similar to the quadratic utility, especially when attempting to model consumers with varying financial constraints.

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Appendix: Proof of Proposition 1

Since both utilities are increasing in quantity consumed, the budget constraint binds, and the consumer's problem is equivalent to $\max_M (1+\lambda)u(M)+v(B-M)$, which has the first-order condition

$$(1+\lambda)u'(M^*)=v'(B-M^*) \qquad \text{(FOC)}$$

It is easy to show that M^* increases in both λ (supermodularity) and B. Now consider the key cross-partial of interest:

$$\frac{\partial^2\left(\frac{M^*}{B}\right)}{\partial\lambda\,\partial B}=\left(\frac{1}{B}\right)\frac{\partial}{\partial\lambda}\left(\frac{\partial M^*}{\partial B}-\frac{M^*}{B}\right)=\left(\frac{1}{B}\right)\left(\frac{\partial^2 M^*}{\partial\lambda\,\partial B}-\frac{1}{B}\frac{\partial M^*}{\partial\lambda}\right)$$

Since M^* increases in λ, a sufficient condition for the cross-partial of interest to be negative is the cross-partial of raw demand (i.e. not as a percentage of budget) to be negative: $\frac{\partial^2 M^*}{\partial\lambda\,\partial B}<0$.

To derive the sign of the cross-partial of raw demand, let M^* be denoted by x for clarity and differentiate the FOC in both, starting with B.

$$B:(1+\lambda)u''(x)\frac{dx}{dB}=v''(B-x)\left(1-\frac{dx}{dB}\right)\Rightarrow\frac{dx}{dB}=\frac{v''(B-x)}{(1+\lambda)u''(x)+v''(B-x)}$$

$$\Rightarrow\frac{d^2x}{dBd\lambda}=\frac{-v'''(B-x)\frac{dx}{d\lambda}\left[(1+\lambda)u''(x)+v''(B-x)\right]}{\left[(1+\lambda)u''(x)+v''(B-x)\right]^2}\ldots$$

$$\ldots\frac{-\left[\delta u''(x)+(1+\lambda)u'''(x)\frac{dx}{d\lambda}-v'''(B-x)\frac{dx}{d\lambda}\right]v''(B-x)}{\left[(1+\delta\lambda)u''(x)+v''(B-x)\right]^2}$$

$$\propto -u''(x)v''(B-x)-(1+\lambda)\frac{dx}{d\lambda}\left[u''(x)v'''(B-x)+u'''(x)v''(B-x)\right]=$$

$$=\underbrace{-u''(x)v''(B-x)}_{<0}+\underbrace{\frac{(1+\lambda)u'(x)\left[u''(x)v'''(B-x)+u'''(x)v''(B-x)\right]}{(1+\lambda)u''(x)+v''(B-x)}}_{\text{denominator}<0}$$

where the last line substitutes for $\frac{dx}{d\lambda}$ based on the FOC. To guarantee that the entire expression is negative, it is sufficient that the numerator of the second part be positive. Since $u'(x)>0$, that numerator is positive whenever $\left[u''(x)v'''(B-x)+u'''(x)v''(B-x)\right]>0$. Since u and v are concave, this holds whenever $u'''\le 0$ and $v'''\le 0$. *QED Proposition* 1.